

7.5 練習問題 解答

(1)

$$\begin{aligned}\iint_{[-1,1] \times [1,2]} (x^3 - y^2) \, dx dy &= \int_{-1}^1 \left(\int_1^2 (x^3 - y^2) \, dy \right) dx = \int_{-1}^1 \left[x^3 y - \frac{y^3}{3} \right]_{y=1}^{y=2} dx \\ &= \int_{-1}^1 \left(x^3 - \frac{7}{3} \right) dx = -\frac{14}{3}\end{aligned}$$

(2)

$$\begin{aligned}\iint_{[0,1] \times [-1,0]} e^{2x-3y} \, dx dy &= \int_0^1 \left(\int_{-1}^0 e^{2x-3y} \, dy \right) dx = \int_0^1 \left[-\frac{e^{2x-3y}}{3} \right]_{y=-1}^{y=0} dx \\ &= \frac{1}{3} \int_{-1}^0 (e^{2x+3} - e^{2x}) \, dx = \frac{1 - e^2 - e^3 + e^5}{6}\end{aligned}$$

(3)

$$\begin{aligned}\iint_{[0,2] \times [0,1]} x e^{xy} \, dx dy &= \int_0^2 \left(\int_0^1 x e^{xy} \, dy \right) dx = \int_0^2 \left[e^{xy} \right]_{y=0}^{y=1} dx \\ &= \int_0^2 (e^x - 1) \, dx = e^2 - 3\end{aligned}$$

(4)

$$\begin{aligned}\iint_{[0,\pi/2] \times [1,e]} (\cos x) \log y \, dx dy &= \int_1^e \left(\int_0^{\pi/2} (\cos x) \log y \, dx \right) dy \\ &= \int_1^e \left((\log y) \int_0^{\pi/2} \cos x \, dx \right) dy \\ &= \left(\int_0^{\pi/2} \cos x \, dx \right) \left(\int_1^e \log y \, dy \right) = 1 \cdot 1 = 1\end{aligned}$$