5.4 練習問題 解答

1. $y = \sin x,\ y = \cos x,\ x = \frac{\pi}{4},\ x = \frac{5\pi}{4}$ で囲まれる部分の面積を求めればよい.

$$\int_{\frac{pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx = \Big[-\cos x - \sin x \Big]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} = 2\sqrt{2}$$

2. (1)

$$\begin{aligned}
\int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt &= \int_0^{\pi} \sqrt{(1-\cos t)^2 + (-\sin t)^2}\ dt \\
&= \int_0^{\pi} \sqrt{2 - 2\cos t}\ dt = \int_0^{\pi} \sqrt{4\sin^2 \frac{t}{2}}\ dt \\
&= 2\int_0^{\pi} \sin\frac{t}{2}\ dt = -4\left[\cos\frac{t}{2}\right]_0^{\pi} = 4
\end{aligned}$$

(2)

$$\begin{aligned}
\int_0^{\log 2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx &= \int_0^{\log 2} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2}\ dx \\
&= \int_0^{\log 2} \sqrt{\left(\frac{e^x + e^{-x}}{2}\right)^2}\ dx = \int_0^{\log 2} \frac{e^x + e^{-x}}{2}\ dx \\
&= \left[\frac{e^x - e^{-x}}{2}\right]_0^{\log 2} = \frac{2 - \frac{1}{2}}{2} - \frac{0-0}{2} = \frac{3}{4}
\end{aligned}$$

双曲線関数 $\sinh, \cosh$ の性質を使うなら

$$\begin{aligned}
\int_0^{\log 2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx &= \int_0^{\log 2} \sqrt{1 + \sinh^2 x}\ dx = \int_0^{\log 2} \sqrt{\cosh^2 x}\ dx \\
&= \int_0^{\log 2} \cosh x\ dx = [\sinh x]_0^{\log 2} = \sinh(\log 2) = \frac{3}{4}
\end{aligned}$$