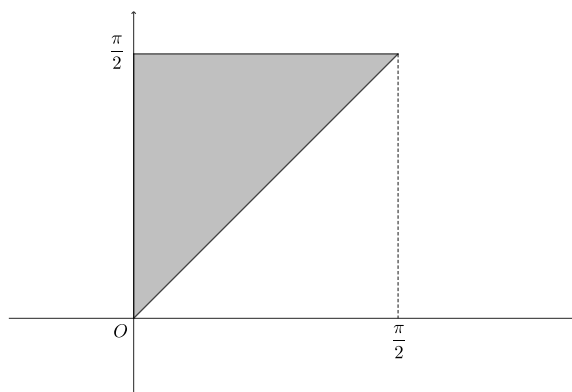


$$(1) \iint_D \sin(x+y) \, dx dy, \quad D = \left\{ (x, y) \mid 0 \leq y \leq \frac{\pi}{2}, 0 \leq x \leq y \right\}$$

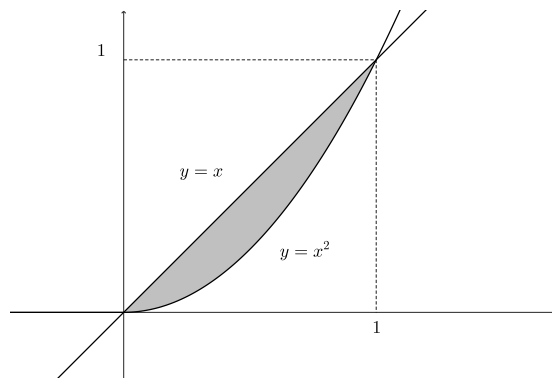
$$\begin{aligned} \iint_D \sin(x+y) \, dx dy &= \int_0^{\frac{\pi}{2}} \left(\int_0^y \sin(x+y) \, dx \right) dy = \int_0^{\frac{\pi}{2}} \left[-\cos(x+y) \right]_{x=0}^{x=y} dy \\ &= \int_0^{\frac{\pi}{2}} (-\cos(2y) + \cos y) dy = 1 \end{aligned}$$



$$(2) \iint_D x \, dx dy, \quad D = \{ (x, y) \mid y \leq x \leq \sqrt{y} \}$$

D は横線集合として $D = \{ (x, y) \mid 0 \leq x \leq 1, x^2 \leq y \leq x \}$ と書ける.

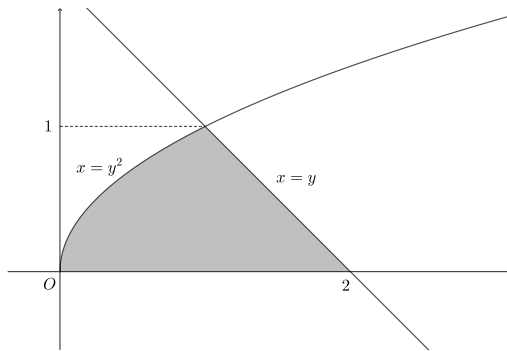
$$\iint_D x \, dx dy = \int_0^1 \left(\int_{x^2}^x x \, dy \right) dx = \int_0^1 [xy]_{y=x^2}^{y=x} dx = \int_0^1 (x^2 - x^3) dx = \frac{1}{12}$$



$$(3) \iint_D xy \, dxdy, \quad D = \{ (x, y) \mid x + y \leq 2, y^2 \leq x, 0 \leq y \}$$

D は横線集合として $D = \{ (x, y) \mid 0 \leq y \leq 1, y^2 \leq x \leq 2 - y \}$ と書ける.

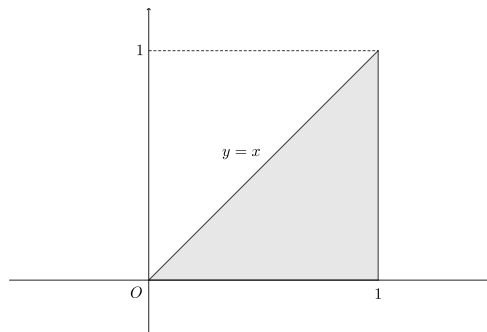
$$\begin{aligned} \iint_D xy \, dxdy &= \int_0^1 \left(\int_{y^2}^{2-y} xy \, dx \right) dy = \int_0^1 \left[\frac{x^2 y}{2} \right]_{x=y^2}^{x=2-y} dy \\ &= \frac{1}{2} \int_0^1 (-y^5 + y^3 - 4y^2 + 4y) dy = \frac{3}{8} \end{aligned}$$



$$(4) \iint_D \sin(\pi x^2) \, dxdy, \quad D = \{ (x, y) \mid y \leq x \leq 1, 0 \leq y \leq 1 \}$$

D は縦線集合として $D = \{ (x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x \}$ と書ける.

$$\begin{aligned} \iint_D \sin(\pi x^2) \, dxdy &= \int_0^1 \left(\int_0^x \sin(\pi x^2) dy \right) dx = \int_0^1 \sin(\pi x^2) \left[y \right]_{y=0}^{y=x} dx \\ &= \int_0^1 x \sin(\pi x^2) dx = \frac{1}{\pi} \end{aligned}$$



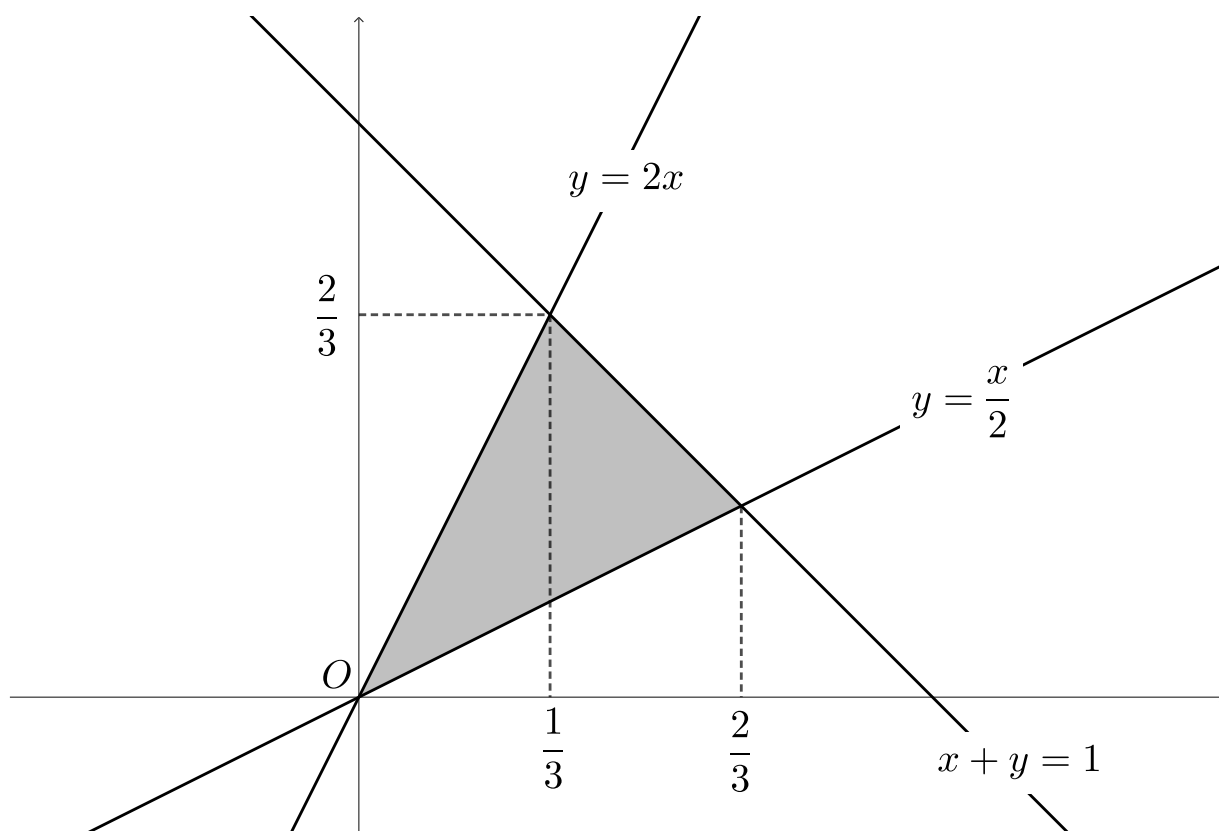
$$(5) \iint_D y \, dx dy, \quad D = \left\{ (x, y) \mid \frac{y}{2} \leq x \leq 2y, x + y \leq 1 \right\}$$

D は縦線集合として

$$D = \left\{ (x, y) \mid 0 \leq x \leq \frac{2}{3}, \frac{x}{2} \leq y \leq \varphi(x) \right\}, \quad \varphi(x) = \begin{cases} 2x & (0 \leq x \leq \frac{1}{3}) \\ 1-x & (\frac{1}{3} \leq x \leq \frac{2}{3}) \end{cases}$$

と書ける.

$$\begin{aligned} \iint_D y \, dx dy &= \int_0^{\frac{2}{3}} \left(\int_{\frac{x}{2}}^{\varphi(x)} y \, dy \right) dx = \int_0^{\frac{1}{3}} \left(\int_{\frac{x}{2}}^{2x} y \, dy \right) dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(\int_{\frac{x}{2}}^{1-x} y \, dy \right) dx \\ &= \int_0^{\frac{1}{3}} \left[\frac{y^2}{2} \right]_{y=\frac{x}{2}}^{y=2x} dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left[\frac{y^2}{2} \right]_{y=\frac{x}{2}}^{y=1-x} dx \\ &= \int_0^{\frac{1}{3}} \frac{15}{8} x^2 \, dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(\frac{3}{8} x^2 - x + \frac{1}{2} \right) dx = \frac{5}{216} + \frac{7}{216} = \frac{1}{18} \end{aligned}$$



8.3 練習問題解答

(6) $\iint_D x \, dx dy, \quad D = \{ (x, y) \mid -2 \leq x \leq 1, x^2 + 4x + 1 \leq y \leq -x^2 + 2x + 1 \}$

D は縦線集合として $D = \{ (x, y) \mid -1 \leq x \leq 0, x^2 + 4x + 1 \leq y \leq -x^2 + 2x + 1 \}$ と書ける.

$$\begin{aligned} \iint_D x \, dx dy &= \int_{-1}^0 \left(\int_{x^2+4x+1}^{-x^2+2x+1} x \, dy \right) dx = \int_{-1}^0 [xy]_{y=x^2+4x+1}^{-x^2+2x+1} dx \\ &= \int_{-1}^0 (-2x^3 - 2x^2) dx = -\frac{1}{6} \end{aligned}$$

