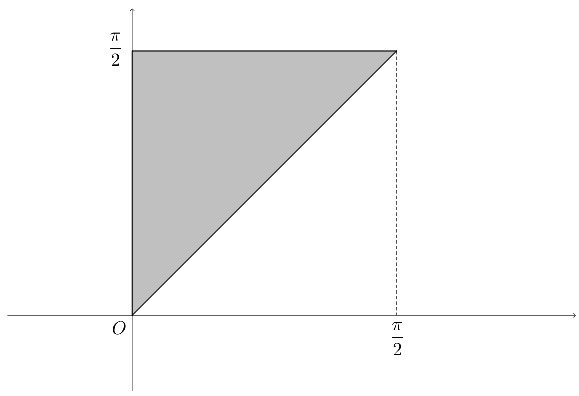
(1) $\displaystyle\iint_D \sin(x+y)\ dxdy, \quad D = \left\{\ (x,y)\ \middle|\ 0 \leqq y \leqq \frac{\pi}{2},\ 0 \leqq x \leqq y\ \right\}$

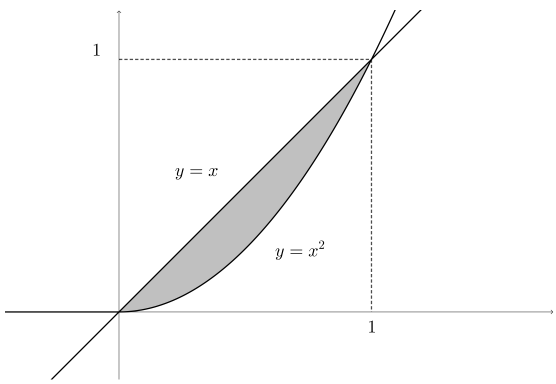
$$\iint_D \sin(x+y)\ dxdy = \int_0^{\frac{\pi}{2}} \left(\int_0^y \sin(x+y)\ dx\right) dy = \int_0^{\frac{\pi}{2}} \Big[-\cos(x+y)\Big]_{x=0}^{x=y} dy$$
$$= \int_0^{\frac{\pi}{2}} (-\cos(2y) + \cos y)\, dy = 1$$



(2) $\displaystyle\iint_D x\ dxdy, \quad D = \{\ (x,y) \mid y \leqq x \leqq \sqrt{y}\ \}$

D は横線集合として $D = \left\{\ (x,y)\ \middle|\ 0 \leqq x \leqq 1,\ x^2 \leqq y \leqq x\ \right\}$ と書ける.

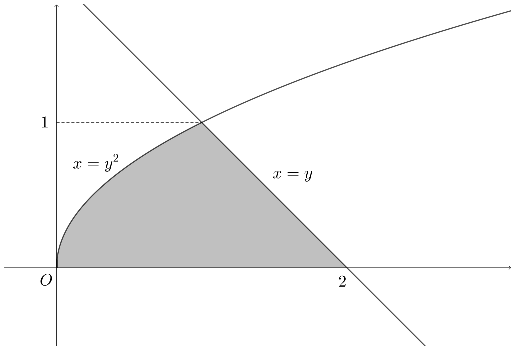
$$\iint_D x\ dxdy = \int_0^1 \left(\int_{x^2}^x x\ dy\right) dx = \int_0^1 \Big[xy\Big]_{y=x^2}^{y=x}\ dx = \int_0^1 \left(x^2 - x^3\right) dx = \frac{1}{12}$$



(3) $\displaystyle\iint_D xy\ dxdy, \quad D = \left\{\ (x,y)\ \middle|\ x+y \leqq 2,\ y^2 \leqq x,\ 0 \leqq y\ \right\}$

D は横線集合として $D = \left\{\ (x,y)\ \middle|\ 0 \leqq y \leqq 1,\ y^2 \leqq x \leqq 2-y\ \right\}$ と書ける．

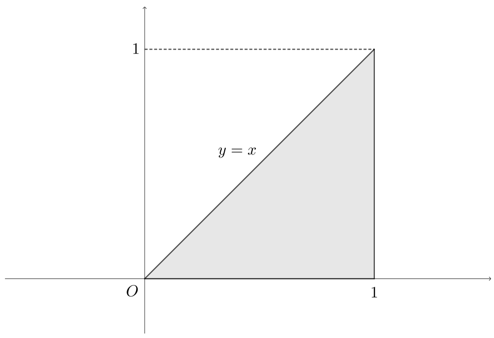
$$\begin{aligned}\iint_D xy\ dxdy \int_0^1 \left(\int_{y^2}^{2-y} xy\ dx\right) dy &= \int_0^1 \left[\frac{x^2y}{2}\right]_{x=y^2}^{x=2-y}\ dy \\ &= \frac{1}{2}\int_0^1 \left(-y^5+y^3-4y^2+4y\right) dy = \frac{3}{8}\end{aligned}$$



(4) $\displaystyle\iint_D \sin\left(\pi x^2\right)\ dxdy, \quad D = \{\ (x,y) \mid y \leqq x \leqq 1,\ 0 \leqq y \leqq 1\ \}$

D は縦線集合として $D = \{\ (x,y) \mid 0 \leqq x \leqq 1,\ 0 \leqq y \leqq x\ \}$ と書ける．

$$\begin{aligned}\iint_D \sin\left(\pi x^2\right)\ dxdy &= \int_0^1 \left(\int_0^x \sin\left(\pi x^2\right) dy\right) dx = \int_0^1 \sin\left(\pi x^2\right)\Big[y\Big]_{y=0}^{y=x}\ dx \\ &= \int_0^1 x\sin\left(\pi x^2\right) dx = \frac{1}{\pi}\end{aligned}$$



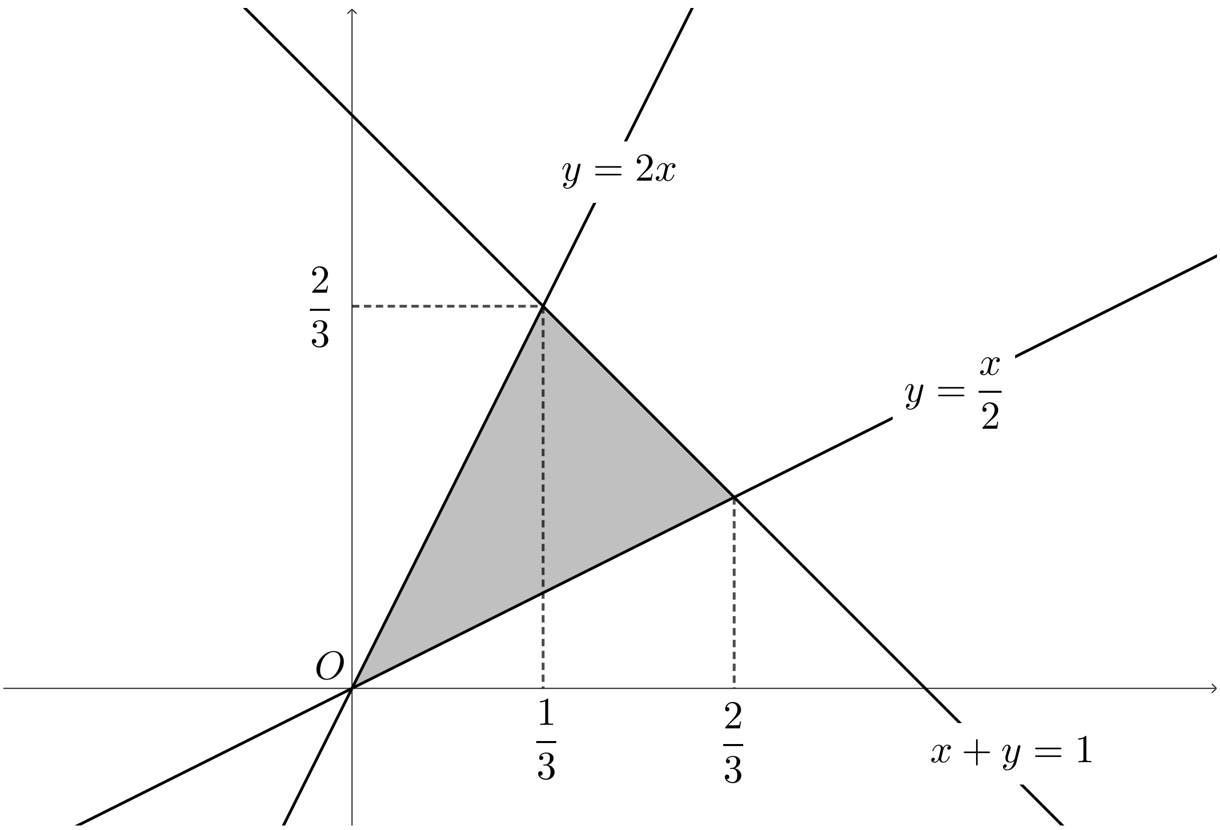
(5) $\displaystyle\iint_D y\ dxdy, \quad D = \left\{ (x,y) \;\middle|\; \frac{y}{2} \leqq x \leqq 2y,\ x+y \leqq 1 \right\}$

D は縦線集合として

$$D = \left\{ (x,y) \;\middle|\; 0 \leqq x \leqq \frac{2}{3},\ \frac{x}{2} \leqq y \leqq \varphi(x) \right\}, \quad \varphi(x) = \begin{cases} 2x & \left(0 \leqq x \leqq \frac{1}{3}\right) \\ 1-x & \left(\frac{1}{3} \leqq x \leqq \frac{2}{3}\right) \end{cases}$$

と書ける．

$$\begin{aligned}
\iint_D y\ dxdy &= \int_0^{\frac{2}{3}} \left(\int_{\frac{x}{2}}^{\varphi(x)} y\ dy \right) dx = \int_0^{\frac{1}{3}} \left(\int_{\frac{x}{2}}^{2x} y\ dy \right) dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(\int_{\frac{x}{2}}^{1-x} y\ dy \right) dx \\
&= \int_0^{\frac{1}{3}} \left[\frac{y^2}{2} \right]_{y=\frac{x}{2}}^{y=2x} dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left[\frac{y^2}{2} \right]_{y=\frac{x}{2}}^{y=1-x} dx \\
&= \int_0^{\frac{1}{3}} \frac{15}{8} x^2\ dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \left(\frac{3}{8}x^2 - x + \frac{1}{2} \right) dx = \frac{5}{216} + \frac{7}{216} = \frac{1}{18}
\end{aligned}$$



(6) $\displaystyle\iint_D x\ dxdy, \quad D = \{\,(x,y)\mid -2 \leqq x \leqq 1,\ x^2+4x+1 \leqq y \leqq -x^2+2x+1\,\}$

D は縦線集合として $D = \{\,(x,y)\mid -1 \leqq x \leqq 0,\ x^2+4x+1 \leqq y \leqq -x^2+2x+1\,\}$ と書ける．

$$\iint_D x\ dxdy = \int_{-1}^{0}\left(\int_{x^2+4x+1}^{-x^2+2x+1} x\ dy\right)dx = \int_{-1}^{0}\Big[xy\Big]_{y=x^2+4x+1}^{-x^2+2x+1}dx$$
$$= \int_{-1}^{0}\left(-2x^3-2x^2\right)dx = -\frac{1}{6}$$