

Phase and Group velocity of EM waves

Aim: To understand the nature of EM waves travelling in a medium with the help of phase and group velocities.

Theory:

Any real signal consists of travelling waves of many diff. frequencies, which travel together as a group, at a speed that will always be less than or equal to speed of light in vacuum. To gain some insight into what may happen when real signal travels through a dispersive medium, we consider adding two waves of equal amplitude.

$f_1(z, t) = \cos(k_1 z - \omega_1 t)$ and $f_2(z, t) = \cos(k_2 z - \omega_2 t)$ are added we get

$$f_1(z, t) + f_2(z, t) = 2 \cos\left(\frac{\Delta k z - \Delta \omega t}{2}\right) \times \cos(\bar{k} z - \bar{\omega} t)$$

$$\bar{k} = \frac{k_1 + k_2}{2}, \quad \bar{\omega} = \frac{\omega_1 + \omega_2}{2}$$

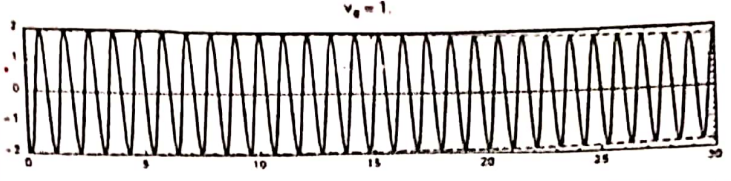
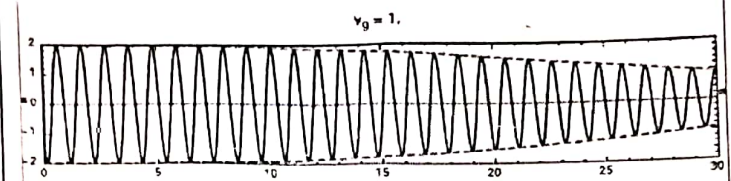
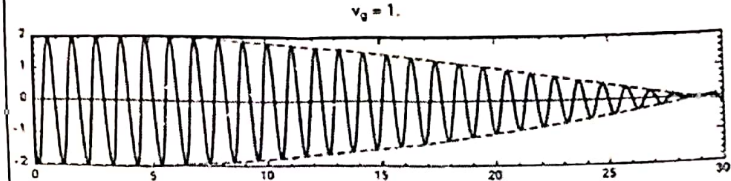
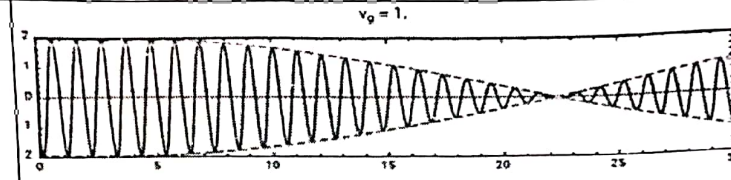
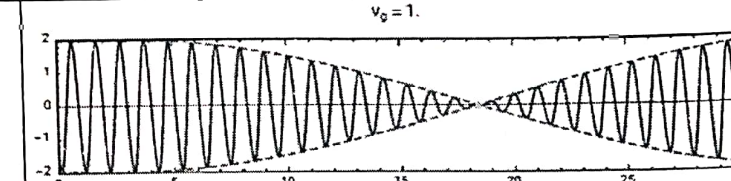
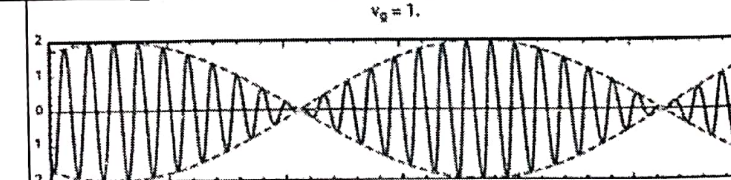
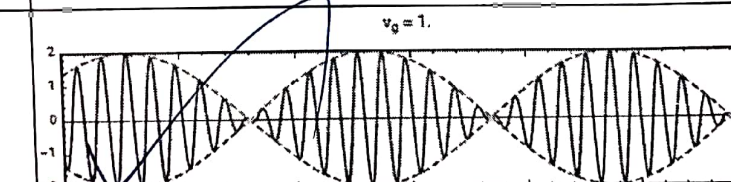
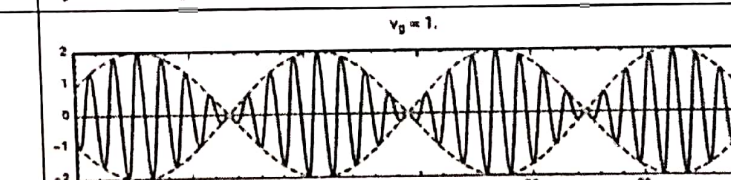
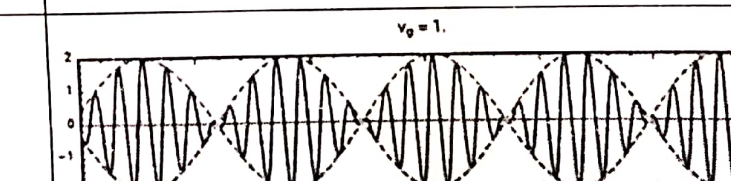
$$(\text{Note: } \underline{\Delta k} = \frac{k_1 + k_2}{2}, \quad \underline{\Delta \omega} = \frac{\omega_1 + \omega_2}{2})$$

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S.No.	Δu	ΔK	wave pattern of resultant waves	V_g
1	0.02	0.02		
2	0.04	0.04		
3	0.06	0.06		
4	0.08	0.08		
5	0.1	0.1		
6	0.2	0.2		
7	0.3	0.3		
8	0.4	0.4		
9	0.5	0.5		

no.	$\Delta\omega$	Δk	Wave pattern of the resultant waves	V_g
1	0.02	0.02		1m/s
2	0.04	0.04		1m/s
3	0.06	0.06		1m/s
4	0.08	0.08		1m/s
5	0.1	0.1		1m/s
6	0.2	0.2		1m/s
7	0.3	0.3		1m/s
8	0.4	0.4		1m/s
9	0.5	0.5		1m/s

The result is fast oscillating waves that travels with phase velocity $V_p = \omega/k$ and the amplitude of this wave is being modulated in space and time by $2 \cos(\Delta k z - \Delta \omega t)$. This modulated wave ~~exists~~ travels at the group velocity is given by $V_g = \frac{\Delta \omega}{\Delta k}$.

Inference:-

- ① Are the wave patterns for various values of $\Delta \omega$ and Δk same? if not, why?
- ② Comment on the phase velocity (V_p) of the waves for increased values of $\Delta \omega$ and Δk .
- ③ When did we see V_p and V_g being same.
- ④ Draw a typical dispersion relation ~~curve~~ $(\omega(k))$ curve.

?

Answers -

- ① No, the wave patterns of various values of $\Delta \omega$ and Δk are different because the modulation of the wave propagation through space and time and this is written as $2 \cos\left(\frac{\Delta k z}{2} - \frac{\Delta \omega t}{2}\right)$
- ② There will be no change of phase velocity (V_p) for various increased values of $\Delta \omega$ and Δk are related, so, phase velocity for the waves for increased values of $\Delta \omega$ and Δk is same.
- ③ In non-dispersive medium, we see V_p and V_g being the same or equal because of which two monochromatic waves are propagating in ~~not~~ vacuum, they have the same velocity c , then speed of light in vacuum and the super posed waves phase velocity is equal to group velocity.
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- ω
- $V_g = V_s$
- 45°
- Non-dispersive medium
- ω
- Case (i)
- $V_g > V_p$
- ω
- Case (ii)
- $V_g < V_p$
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