Scivault Physics

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Version 2020213

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Chapter 1

Introduction

1.1 Dimensional Analysis and SI units

The **SI** system of units is the standard used by many scientists throughout the world. There are seven *fundamental* or *base* quantities from which all other measurements are derived. These quantities are listed below:

Table 1.1: **SI Units**

Quantity	**Unit**	**Unit Symbol**
time	second	s
length	meter	m
mass	kilogram	kg
electrical current	Ampere	A
temperature	Kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Of these quantities, mass, time and length are quite common. Thus, this system is sometimes called the MKS (meter, kilogram, second) system. In order to use any equations, all measurements must have correct units. For example, if a time is expressed in hours, it must first be converted into seconds before any calculations can be attempted.

Dimensional analysis is the process in which the units associated with quantities create *derived units*. For instance, when a distance is divided by a time, the units will be $\frac{m}{s}$ (read *meters per second*).

Dimensional analysis is an important part of solving physics problems. Often, correct dimensional analysis can help you determine if a problem has been solved correctly. One should not even attempt to calculate an answer to a problem until the correct units have been verified.

1.2 Vectors and Scalars

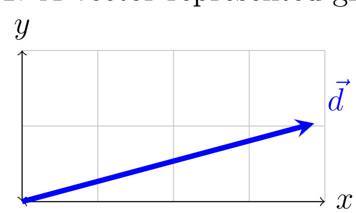
In the study of physics, there are two types of quantities that we will deal with on a regular basis: *scalars* and *vectors*.

A **scalar** is a quantity that you are already most likely very familiar with, as it is just a number; scalars have only a *magnitude* (a number that represents how big or strong it is), and can sometimes include units. Examples of scalars might be the number of people in a room, the mass of a car, or your age in years.

Vectors are different from scalars because in addition to a magnitude, they contain a direction as well. Examples of vectors might include 50 feet to the north, 5 m/s at a 33° angle, or 200 miles straight up. There are many ways of expressing vectors. Symbollically, they are often written with an arrow over them. For example, in the equation $\vec{F} = m\vec{a}$ both force and acceleration are vectors - meaning that both force and acceleration have a direction. Sometimes vectors will be expressed in **bold** typeface. Hence, the expression $\mathbf{F} = \mathrm{m}\ \mathbf{a}$ is equivalent to the expression shown above.

The direction for vectors in 1 dimension is easy - all you need is a positive or a negative. Usually, 1-dimensional motion takes place along the x-axis (left and right), but sometimes it will take place in the y- (forward and backward) and z- (up and down) dimensions. For the purposes of this book, positive is to the right and up unless otherwise stated. In two dimensions, a vector requires two pieces of information. One way of expressing a vector is in polar form. Polar form in two dimensions includes a magnitude of the vector and an angle, usually measured from the x-axis. There are several ways for writing this. 4 cm @ 15°, 4 cm ∠15° and 4 cm at 15° North of East all represent the following displacement vector:

Figure 1.1: A vector represented graphically.



The magnitude of the above vector is 4 cm. Mathematically, the magnitude of a vector can be written several ways, the most common being $|\vec{A}|$, though sometimes the magnitude of a vector can be written as the vector without the vector sign, as in A.

Unit vectors are vectors that have a length of one unit and are oriented along one axis. The unit vector for the x-direction is written as $\hat{\imath}$ (pronounced i-hat). $\hat{\imath}$ is a 1-unit long vector that is always parallel to the x-axis, and points in the direction of increasing x values. Likewise, the y-direction and z-direction unit vectors are written as $\hat{\jmath}$ and $\hat{k}$ respectively.

Because the surface of a paper is effectively 2-dimensional, it is very hard to draw lines that are oriented directly into or out of your paper. For this purpose, physicists have agreed to the following convention: vectors that point directly into your paper are notated by $\bigotimes$. Vectors that point directly out of your paper are shown by the symbol $\bigodot$.

Sometimes, vectors may be expressed in Cartesian coordinates. This vector could either be expressed as an ordered pair (or triple) with square brackets, such as [3, 4, 5] cm, or as a linear combination of the unit vectors shown above, such as $5\hat{i} + 12\hat{j} + 3\hat{k}$ In each case, the distances in each direction are given by the numbers shown. The vector in figure 1.1 could be represented as: $\vec{d} \approx 3.86\hat{i} + 1.035\hat{j}$.

When converting between polar and cartesian forms for two dimensional vectors, a little trigonometry shows:

$$x = r\cos(\theta) \tag{1.1}$$

$$y = r\sin(\theta) \tag{1.2}$$

$$r = \sqrt{x^2 + y^2} \tag{1.3}$$

$$\theta = \tan^{-1}(\frac{y}{x}) \tag{1.4}$$

In three dimensions, polar form comes in two types: **cylindrical** and **spherical**. In cylindrical coordinates, expressed as [r, θ, z], the above conversions are used, and the z-coordinate remains unchanged from Cartesian form. We will study spherical coordinates more in INSERT REFERNCE HERE!

1.3 Vector Mathematics

1.3.1 Vector Addition

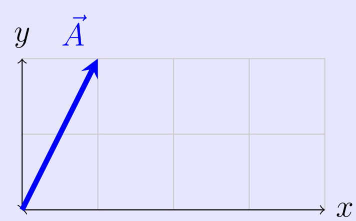
Graphical Addition of Vectors

When vectors are added graphically, they are added **head** to **tail**. This means that the arrowhead for a first vector becomes the origin for the second vector. The **resultant** vector is a straight line between the origin of the first vector and the head of the second.

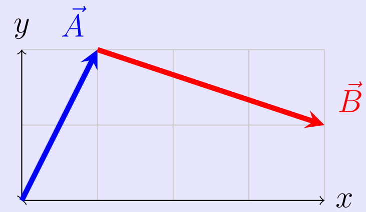
Example 1.3.1

Problem: If $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} - \hat{j}$, find $\vec{C}$ graphically given $\vec{C} = \vec{A} + \vec{B}$

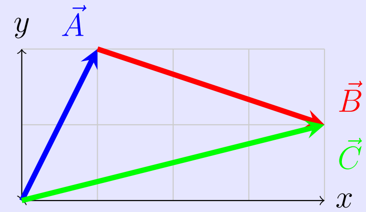
Solution: Begin by drawing $\vec{A}$:



Now use the head of $\vec{A}$ as the origin for $\vec{B}$:



The resulting vector is a straight line from the origin to the end of $\vec{B}$:



The resultant vector is given by $\boxed{\vec{C} = 4\hat{i} + \hat{j}}$

Mathematical Addition of Vectors

Mathematical addition of vectors in Cartesian form is quite easy - simply add corresponding x- and y- values together. For instance, in Example 1.3.1 we are given $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} - \hat{j}$. Adding these together would give $\vec{C} = (1+3)\hat{i} + (2-1)\hat{j} \longrightarrow \vec{C} = 4\hat{i} + \hat{j}$.

The easiest way to add vectors that are expressed in polar coordinates is to first convert them into cartesian coordinates using Equations (1.1) through (1.4) on page 3.

1.3.2 The Dot Product

In Cartesian Form

When multiplying vectors, it is sometimes necessary to obtain a scalar result. This is done through use of a dot product. A dot product is written as $\vec{A} \cdot \vec{B}$. This means to only multiply the components of the vectors that are in the same direction. In cartesian coordinates, this can be done by multiplying corresponding components, then adding the products.

Example 1.3.2

Problem: Given the vectors $\vec{Y} = 2\hat{i} + 3\hat{j}$ and $\vec{Z} = -4\hat{i} + 5\hat{j}$ find the dot product of vectors Y and Z.

Solution: Multiply coefficients from each vector, then add the products together:

$$2 \times (-4) + 3 \times 5 = -8 + 15 = \boxed{7}$$

In Polar Form

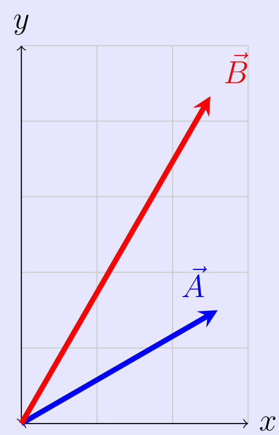
Often, vectors will be expressed in polar notation. If this is the case, the dot product can be found my multiplying the magnitude of the first vector times the magnitude of the second vector times the cosine of the angle between:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \tag{1.5}$$

Example 1.3.2.2

Problem: Given the vectors $\vec{D} = 3\angle 30°$ and $\vec{E} = 5\angle 60°$ find the dot product of vectors D and E.

Solution: We begin by graphing both vectors starting from a common origin:



Noticing that the angle between the two vectors is 30°, we can use equation 1.5 to calculate:

$$\vec{D} \cdot \vec{E} = |\vec{D}||\vec{E}| \cos\theta = (3)(5)\cos 30° \approx \boxed{12.990}$$

1.3.3 The Cross Product

Cross Products In Cartesian Form

Sometimes two vectors will be multiplied in such a way that they will result in another vector. This is called a **Cross Product**. A cross product is inherently a three-dimensional operation. A cross product can be found by calculating the determinant of a matrix:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.6}$$

The most common way to find the determinant of the above matrix is by using minors:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

This finding the determinate of each of the 2x2 matrices yields:

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

Example 1.3.3

Problem: Given the vectors $\vec{V} = 2\hat{i} + 3\hat{j}$ and $\vec{W} = -4\hat{i} + 5\hat{j}$ find the cross product of vectors V and W.

Solution: Begin by creating a matrix, as shown in equation (1.6). Since both vectors lie in the X-Y plane, the Z-components for both vectors are zero:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix}$$

Now, use expansion by minors to create three two-by-two matrices:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix} = \hat{i} \begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 3 \\ -4 & 5 \end{vmatrix}$$

Finding the determinate of each of the two-by-two matricies yields:

$$\vec{V} \times \vec{W} = \hat{i}(3 \cdot 0 - 0 \cdot 5) - \hat{j}(2 \cdot 0 - 0 \cdot (-4)) + \hat{k}(2 \cdot 5 - 3 \cdot (-4))$$

Both the x-component and the y-components of this cross product are zero. Thus,

$$\boxed{\vec{V} \times \vec{W} = 22\hat{k}}$$

There are some important things you may wish to note.

1. The cross product is not commutative. That is, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$. In fact, the two cross products are in exactly opposite directions.

2. The cross product of two vectors is always a vector that is perpendicular to both of the original vectors.

Cross Products In Polar Form

When vectors are expressed in polar form, a cross product can often be found using two steps. To find the magnitude of the resulting vector, one would use the following equation:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta \tag{1.7}$$

In Equation 1.7, θ is the angle between the two vectors. To find the direction of the resulting vector, use the **First Right Hand Rule**. To use this rule, you point your index finger in the direction of the first vector to be multiplied. Your middle finger is then pointed in the direction of the second vector to be multiplied. Your thumb will point in the direction of the resultant vector, as shown in the image:

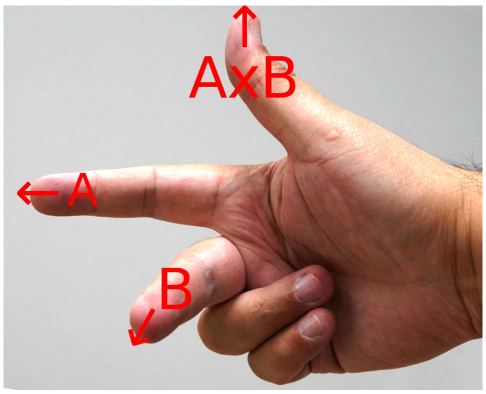


Figure 1.2: The First Right Hand Rule

The right had rule is easiest to use when the first two vectors are at 90° to each other. However, even if the first two vectors are not at right angles to each other, the resultant vector of a cross product will always be perpendicular to the plane of the first two vectors.

1.4 Exercises for Chapter 1

Section 1.1: Dimensional Analysis

1. What are the units for each of the following?

   (a) $\frac{5m}{2s}$

   (b) $\frac{10kg}{5s}$

   (c) $\frac{140kg \cdot 15m}{23s}$

   (d) $\frac{(\frac{15m}{15s})}{14s}$

Section 1.2: Vectors and Scalars

2. Given the vector $4\hat{\imath} - 3\hat{\jmath}$, represent the vector (a) graphically and (b) in polar form.

3. George walks 200 meters north, then walks 400 meters north. What is the vector from his starting position to his ending position in (a) cartesian form and (b) polar form (take north as 0 degrees).

Section 1.3: Vector Mathematics

4. Addition - cartesian

5. Addition - polar

6. Dot Product - Cartesian

7. Dot Product - polar

8. Cross Product - cartesian

9. Cross Product - polar

Chapter 2

Kinematics in One Dimension

2.1 Distance and Displacement

You are probably already familiar with the concept of **distance** - you might get in your car and drive a total of 1.2 miles to school, turning right after 0.45 miles, according to your car's odometer. Distance is a scalar that tells you how far something traveled. The symbol d usually represents distance.

While you may have traveled a total distance of 1.2 miles from your school, you are significantly less than 1.2 miles away from home; in fact, you are approximately 0.874 miles from home, following a direct path directly from your home to the school, at an angle of 59°(not worrying that this path might take you through someone's back yard or kitchen).

Displacement is a vector that tell you how far something is from the origin, and is independent of the path taken to get there. The displacement vector is commonly symbolized by $\vec{r}$ though sometimes it may be written as $\vec{d}$.

2.2 Average and Instantaneous Speed and Velocity

Speed is a scalar value that represents the change in distance per change in time of an object. Speed is usually represented with the symbol v, without the vector sign. You are probably already familiar with this quantity, since the speedometer on your family car measures speed. For the purposes of physics, speed has little value because it is a scalar that tells us nothing of direction. Much more useful is the concept called velocity. Velocity and speed are related much like distance and displacement.

Velocity is the change in displacement of an object per unit time, and as such is a vector. Positive velocities indicate that the object is moving forward, relative to the axis in question, and negative velocities generally mean that the object is moving backward, relative the the axis. The average velocity of an object is given by:

$$\overrightarrow{v_{avg}} = \frac{\Delta \vec{r}}{\Delta t} \tag{2.1}$$

Average velocity is useful if an object's velocity is not changing. However, many times it is more useful to talk about instantaneous velocity. Instantaneous velocity tells us how fast an object is moving at a given instant in time. In order to calculate instantaneous velocity, we must allow our time interval in the above formula to become infinitesimally small. In this case, a little calculus proves:

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{2.2}$$

Calculation of average velocity is rather straightforward, assuming you know both distance traveled and the time it took. If an object is not speeding up or slowing down during a specific time interval, the instantaneous velocity at any time during this interval is equal to the average velocity. If the object does speed up or slow down during the time interval in question, the average velocity and the instantaneous velocity at a certain time during the interval are not necessarily the same.

Example 2.2.1

Problem: You ride your bicycle in a straight line for a distance of 73 meters in 12.5 second. What is your average speed?

Solution: Begin by drawing a diagram:

$$v_{avg} = \frac{d}{t} = \frac{73m\ \hat{i}}{12.5s} = 5.84\frac{m}{s}\ \hat{i}$$

Example 2.2.2

Problem: A bicyclist rides his bike to the east. His position (in meters) is given by the following expression:

$$\vec{r} = (0.5t^2 + 4t)\hat{i}$$

1. What is his average velocity from t = 0 to t = 5 seconds?
2. What is his instantaneous velocity at t=3 seconds?

Solution:

1. The total displacement (in meters) after five seconds is given by:

$$\hat{r} = (0.5 \times (5s)^2 + 4 \times 5s)\ m\ \hat{i} = 32.5\ m\ \ \hat{i}$$

Thus, the average velocity is -

$$\overrightarrow{v_{avg}} = \frac{\vec{d}}{t} = \frac{32.5m\ \hat{i}}{5s} = \boxed{6.5\frac{m}{s}\ \hat{i}}$$

2. The instantaneous velocity of an object is found using a derivative with respect to time. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(0.5t^2 + 4t)\hat{i} = (t+4)\hat{i}$$

Evaluating this at t=3s yields:

$$\vec{v} = (3+4)\hat{i} = 7\frac{m}{s}\hat{i}$$

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Circular Motion and Orbits

8.1 Centripetal Forces and Accelerations

8.1.1 Centripetal Force

We have already learned that an object in motion will continue to move in a straight line, assuming no forces are acting on the object. In order for an object to move along a circular path, there must therefore be a force acting on the object to keep it from moving in a straight line. If you whirl a mass on a string around in a circle, tension in the string keeps the mass from continuing to move in a straight line. As the Moon orbits the Earth, the gravitational attraction between the Moon and the Earth keeps the Moon in its orbit around the Earth. Any force that keeps an object moving along a circular path is called a **Centripetal Force** (Centripetal literally translates from Latin as "center seeking"). Any centripetal force can be described as:

$$F_c = \frac{mv^2}{r} \tag{8.1}$$

The direction of a centripetal force is always toward the center of the circle.

8.1.2 Centripetal Acceleration

When an object moves in a circle, even if its speed remains constant, its velocity is constantly changing due to its constant change in direction of motion. Thus the object must be constantly accelerating in a direction toward the center of the circle. Using Equation 8.1 and Newton's Second Law, it is possible to prove that centripetal acceleration is given by:

$$a_c = \frac{v^2}{r} \tag{8.2}$$

Just as Centripetal force is always directed toward the center of the circle, centripetal acceleration is also always directed toward the center of the circle.

Example 8.1.2

Problem: A children's toy consists of a 0.5kg ball attached to the end of a light 0.3 meter rope. A child grabs the toy from the end of the rope and swings the ball around in a circle above his head. What is the tension in the rope?

Solution:

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