



Assignment 4.02: Projectiles - KEY

1. A cannon is placed on level ground. It is aimed 25 degrees above horizontal. The cannonball leaves the cannon with an initial speed of 300 m/s.

- (a) What is the horizontal component of the initial velocity (v_{ix})?

$$v_{ix} = v_i \cos(\theta) = 300 \text{ m/s} \cos(25^\circ) \approx 271.892 \text{ m/s}$$

- (b) What is the vertical component of the initial velocity (v_{iy})?

$$\text{Using up as Positive: } v_{iy} = v_i \sin(\theta) = 300 \text{ m/s} \sin(25^\circ) \approx 126.786 \text{ m/s}$$

- (c) What is the time it takes for the cannonball to reach its maximum height?

$$v_{fy} = v_{iy} + a_y t \longrightarrow t = \frac{v_{fy} - v_{iy}}{a_y} = \frac{0 \text{ m/s} - 126.786 \text{ m/s}}{-9.81 \text{ m/s}^2} \approx 12.924 \text{ s}$$

- (d) What is the maximum height of the cannonball?

$$d_y = v_{iy} t + \frac{1}{2} a_y t^2 = (126.786 \text{ m/s})(12.924 \text{ s}) + \frac{1}{2} (-9.81 \text{ m/s}^2)(12.924 \text{ s})^2 \approx 819.294 \text{ m}$$

- (e) What is the total time of flight for the cannonball?

$$t_{total} = 2 \times t = 2 \times 12.924 \text{ s} = 25.848 \text{ s}$$

- (f) How far from the cannon does the cannonball land?

$$d_x = v_{ix} t + \frac{1}{2} a_x t^2 = (271.892 \text{ m/s})(25.848 \text{ s}) \approx 7027.864 \text{ m}$$

2. A golfer hits a ball on a level golf-course at 35 m/s, 45° above horizontal.

- (a) What is the amount of time it takes the golf-ball to reach its maximum height (hint: find v_{ix} and v_{iy} first).

$$v_{ix} = v_i \cos(\theta) = 35 \text{ m/s} \cos(45^\circ) \approx 24.749 \text{ m/s}$$

$$v_{iy} = v_i \sin(\theta) = 35 \text{ m/s} \sin(45^\circ) \approx 24.749 \text{ m/s}$$

$$v_{fy} = v_{iy} + a_y t \longrightarrow t = \frac{v_{fy} - v_{iy}}{a_y} = \frac{0 \text{ m/s} - 24.749 \text{ m/s}}{-9.81 \text{ m/s}^2} \approx 2.523 \text{ s}$$

- (b) What is the total time the golf ball is in the air?

$$t_{total} = 2 \times t = 2 \times 2.523 \text{ s} = 5.046 \text{ s}$$

- (c) How far away does the golf ball land?

$$d_x = v_{ix} t + \frac{1}{2} a_x t^2 = (24.749 \text{ m/s})(5.046 \text{ s}) \approx 124.873 \text{ m}$$



3. Kay is attempting to kick a football through the field-goal posts. She kicks the ball at 18 m/s at a 35° angle to the ground. She is 20 meters from the goal-post.

- (a) What are the initial vertical and horizontal velocities of the football?

$$v_{ix} = v_i \cos(\theta) = 18\text{m/s} \cos(35^\circ) \approx 14.745 \text{ m/s}$$

$$v_{iy} = v_i \sin(\theta) = 18\text{m/s} \sin(35^\circ) \approx 10.324 \text{ m/s}$$

- (b) How long does it take the football to travel the distance to the goal post? (Hint does this depend on the vertical direction or the horizontal direction?)

$$d_x = v_{ix}t + \frac{1}{2}a_x t^2 \xrightarrow{0} t = \frac{d_x}{v_{ix}} = \frac{20\text{m}}{14.745\text{m/s}} \approx 1.356\text{s}$$

- (c) What is the height of the football when it passes the goal-post?

$$d_y = v_{iy}t + \frac{1}{2}a_y t^2 = (10.324\text{m/s})(1.356\text{s}) + \frac{1}{2}(-9.81\text{m/s}^2)(1.356\text{s})^2 \approx 4.980\text{m}$$

- (d) Assuming the football is kicked straight, does she score 3 points for her team?

Yes! (The field-goal crossbar in American Football is 10 feet, or a little over 3 meters high.)

4. Briana is hunting wild turkeys. She sees a turkey sitting on a branch at the top of a tree that is 35 meters away. She aims her bow at a 25° angle, and shoots the arrow with a speed of 65 m/s. The turkey is hit, and falls to the ground. Briana picks up the turkey and takes it home to save for thanksgiving dinner.

- (a) What are the initial vertical and horizontal velocities of the arrow? $v_{ix} = v_i \cos(\theta) =$

$$65\text{m/s} \cos(25^\circ) \approx 58.910 \text{ m/s}$$

$$v_{iy} = v_i \sin(\theta) = 65\text{m/s} \sin(25^\circ) \approx 27.470 \text{ m/s}$$

- (b) How long does it take for the arrow to hit the turkey?

$$d_x = v_{ix}t + \frac{1}{2}a_x t^2 \xrightarrow{0} t = \frac{d_x}{v_{ix}} = \frac{35\text{m}}{58.910\text{m/s}} \approx 0.594\text{s}$$

- (c) How high up was the turkey sitting?

$$d_y = v_{iy}t + \frac{1}{2}a_y t^2 = (27.470\text{m/s})(0.594\text{s}) + \frac{1}{2}(-9.81\text{m/s}^2)(0.594\text{s})^2 \approx 14.589\text{m}$$

- (d) How long does the turkey take to fall to the ground?

For the turkey, using up as positive:

$$d_y = v_{iy}t + \frac{1}{2}a_y t^2 \xrightarrow{0} t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times (-14.589\text{m})}{-9.81\text{m/s}^2}} \approx 1.725\text{s}$$