Assignment 7.03 - Question 5 - Solution

The Uranium-235 and Uranium-238 on earth are assumed to have been formed in a supernova long before the earth was formed. Currently, only 0.72% of the uranium in earth is ^{235}U while the remaining uranium is ^{238}U. If we assume that the two isotopes were created in equal proportions, how long ago did this supernova take place?

We know the following information from the question and the table of half-lives:

$t_{1/2-235} = 703.8$ Million Years$= 7.038 \times 10^8$ Years
$t_{1/2-238} = 4.468$ Billion Years$= 4.468 \times 10^9$ Years
$A_{235} = 0.72\% = 0.0072$
$A_{238} = 99.28\% = 0.9928$

The question states that we should assume that the two isotopes were created in equal proportions. Thus:

$$A_{0-235} = A_{0-238} \tag{1}$$

The radioactive decay equation states:

$$A = A_0 e^{\frac{-\ln(2)\cdot t}{t_{1/2}}} \tag{2}$$

Solving this equation for A_0 yields:

$$A_0 = \frac{A}{e^{\frac{-\ln(2)\cdot t}{t_{1/2}}}} \tag{3}$$

Plugging this into both sides of equation 1 gives:

$$\frac{A_{235}}{e^{\frac{-\ln(2)\cdot t}{t_{1/2-235}}}} = \frac{A_{238}}{e^{\frac{-\ln(2)\cdot t}{t_{1/2-238}}}} \tag{4}$$

Next, we take the natural log of both sides:

$$\ln\left(\frac{A_{235}}{e^{\frac{-\ln(2)\cdot t}{t_{1/2-235}}}}\right) = \ln\left(\frac{A_{238}}{e^{\frac{-\ln(2)\cdot t}{t_{1/2-238}}}}\right) \tag{5}$$

Remembering that:

$$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B) \tag{6}$$

Equation 5 can be written as:

$$\ln(A_{235}) - \ln\left(e^{\frac{-\ln(2)\cdot t}{t_{1/2-235}}}\right) = \ln(A_{238}) - \ln\left(e^{\frac{-\ln(2)\cdot t}{t_{1/2-238}}}\right) \tag{7}$$

Since taking the natural log of a number and raising e to a power are inverse operations, this can be simplified to:

$$\ln(A_{235}) - \frac{-\ln(2)\cdot t}{t_{1/2-235}} = \ln(A_{238}) - \frac{-\ln(2)\cdot t}{t_{1/2-238}} \tag{8}$$

Combining negatives gives:

$$\ln(A_{235}) + \frac{\ln(2)\cdot t}{t_{1/2-235}} = \ln(A_{238}) + \frac{\ln(2)\cdot t}{t_{1/2-238}} \tag{9}$$

We then move terms with a t to the left side, and other terms to the right side:

$$\frac{\ln(2)\cdot t}{t_{1/2-235}} - \frac{\ln(2)\cdot t}{t_{1/2-238}} = \ln(A_{238}) - \ln(A_{235}) \tag{10}$$

Factoring t from the left side of the equation gives:

$$\left(\frac{\ln(2)}{t_{1/2-235}} - \frac{\ln(2)}{t_{1/2-238}}\right) t = \ln(A_{238}) - \ln(A_{235}) \tag{11}$$

Finally, we divide by the expression in parenthesis:

$$t = \frac{\ln(A_{238}) - \ln(A_{235})}{\left(\frac{\ln(2)}{t_{1/2-235}} - \frac{\ln(2)}{t_{1/2-238}}\right)} \tag{12}$$

While this expression is a bit messy and could be made a little nicer, our calculator should be able to handle it. So, substituting numbers gives:

$$t = \frac{\ln(99.28\%) - \ln(0.72\%)}{\left(\frac{\ln(2)}{7.038\times10^{8}\text{yrs}} - \frac{\ln(2)}{4.468\times10^{9}\text{yrs}}\right)} \tag{13}$$

Evaluating this gives the final answer of:

$$\boxed{t = 5.937\times10^{9}\text{yrs}} \tag{14}$$

Since the solar system is estimated to be 4.6×10^{9} years old, this is a reasonable estimate. While this problem assumes that the two isotopes were created in equal amounts, this may not be a reasonable assumption. A better understanding of the probability of the creation of each isotope is necessary in order to refine this estimate.