Scivault Physics

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Contents

Chapter 1

Introduction

1.1 Dimensional Analysis and SI units

1.1.1 SI Units

The SI System of Units is the standard used by many scientists throughout the world. There are seven *fundamental* or *base* quantities from which all other measurements are derived. These quantities are listed below:

Table 1.1: **SI Units**

Quantity	**Unit**	**Unit Symbol**
time	second	s
length	meter	m
mass	kilogram	kg
electrical current	Ampere	A
temperature	Kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Of these quantities, mass, time and length are quite common. Thus, this system is sometimes called the MKS (meter, kilogram, second) system. In order to use any equations, all measurements must have correct units. For example, if a time is expressed in hours, it must first be converted into seconds before any calculations can be attempted.

1.1.2 Dimensional Analysis

Dimensional Analysis is the process in which the units associated with quantities create *derived units*. For instance, when a distance is divided by a time, the units will be $\frac{m}{s}$ (read *meters per second*).

Dimensional analysis is an important part of solving physics problems. Often, correct dimensional analysis can help you determine if a problem has been solved correctly. One should not even attempt to calculate an answer to a problem until the correct units have been verified.

1.1.3 Unit Conversions

Often you will find that you need to convert a measurement from one unit to another. In order to do this, you must use a conversion factor. A conversion factor is a fraction that is based upon a statement of equality. For instance, since 60 seconds = 1 minute, a conversion factor will look like either $\frac{60s}{1min}$ or $\frac{1min}{60s}$. You should chose the version of the conversion factor that eliminates the units that you wish to convert.

Example 1.1.3

Problem: Convert 7.241 hours into seconds.

Solution: Begin by converting 7.241 hours into minutes. Since 60 minutes = 1 hour,

$$7.241\cancel{hr}\frac{60min}{1\cancel{hr}} = 434.46min$$

Knowing that 1 minute = 60 seconds, we can use a second conversion factor to obtain seconds:

$$434.46\cancel{min}\frac{60s}{1\cancel{min}} = \boxed{26067.6s}$$

This problem could be solved in one step if you know that 1 hour = 3600 seconds.

Example 1.1.3.2

Problem: A car travels with a speed of 20 m/s. What is this in miles per hour?

Solution: We begin by converting meters per second into meters per hour:

$$20\frac{m}{\cancel{s}} \cdot \frac{3600\cancel{s}}{1hr} = 72000\frac{m}{hr}$$

We also need to know that 1 mile = 1609.34 meters:

$$72000\frac{\cancel{m}}{hr} \cdot \frac{1mile}{1609.34\cancel{m}} \approx 44.739\frac{miles}{hr}$$

1.2 Vectors and Scalars

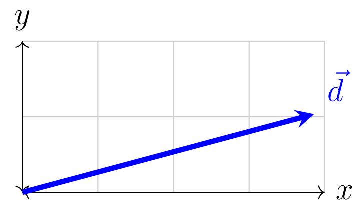
In the study of physics, there are two types of quantities that we will deal with on a regular basis: *scalars* and *vectors*.

A scalar is a quantity that you are already most likely very familiar with, as it is just a number; scalars have only a magnitude (a number that represents how big or strong it is), and can sometimes include units. Examples of scalars might be the number of people in a room, the mass of a car, or your age in years.

Vectors are different from scalars because in addition to a magnitude, they contain a direction as well. Examples of vectors might include 50 feet to the north, 5 m/s at a 33° angle, or 200 miles straight up. There are many ways of expressing vectors. Symbollically, they are often written with an arrow over them. For example, in the equation $\vec{F} = m\vec{a}$ both force and acceleration are vectors - meaning that both force and acceleration have a direction. Sometimes vectors will be expressed in **bold** typeface. Hence, the expression $\mathbf{F} = \mathrm{m}\,\mathbf{a}$ is equivalent to the expression shown above.

The direction for vectors in 1 dimension is easy - all you need is a positive or a negative. Usually, 1-dimensional motion takes place along the x-axis (left and right), but sometimes it will take place in the y- (forward and backward) and z- (up and down) dimensions. For the purposes of this book, positive is to the right and up unless otherwise stated. In two dimensions, a vector requires two pieces of information. One way of expressing a vector is in polar form. Polar form in two dimensions includes a magnitude of the vector and an angle, usually measured from the x-axis. There are several ways for writing this. 4 cm @ 15°, 4 cm $\angle 15°$ and 4 cm at 15° North of East all represent the following displacement vector:

Figure 1.1: A vector represented graphically.



The magnitude of the above vector is 4 cm. Mathematically, the magnitude of a vector can be written several ways, the most common being $|\vec{A}|$, though sometimes the magnitude of a vector can be written as the vector without the vector sign, as in A.

1.2.1 Unit Vectors

Unit vectors are vectors that have a length of one unit and are oriented along one axis. The unit vector for the x-direction is written as $\hat{\imath}$ (pronounced i-hat). $\hat{\imath}$ is a 1-unit long vector that is always parallel to the x-axis, and points in the direction of increasing x values. Likewise, the y-direction and z-direction unit vectors are written as $\hat{\jmath}$ and $\hat{k}$ respectively.

Because the surface of a paper is effectively 2-dimensional, it is very hard to draw lines that are oriented directly into or out of your paper. For this purpose, physicists have agreed to the following convention: vectors that point directly into your paper are notated by $\bigotimes$. Vectors that point directly out of your paper are shown by the symbol $\odot$.

Sometimes, vectors may be expressed in Cartesian coordinates. This vector could either be expressed as an ordered pair (or triple) with square brackets, such as [3, 4, 5] cm, or as a linear combination of the unit vectors shown above, such as $5\hat{\imath} + 12\hat{\jmath} + 3\hat{k}$ In each case, the distances in each direction are given by the numbers shown. The vector in figure 1.1 could be represented as: $\vec{d} \approx 3.86\hat{\imath} + 1.035\hat{\jmath}$.

When converting between polar and cartesian forms for two dimensional vectors, a little trigonometry shows:

$$x = r\cos(\theta) \quad (1.1)$$

$$r = \sqrt{x^2 + y^2} \quad (1.3)$$

$$y = r\sin(\theta) \quad (1.2)$$

$$\theta = \tan^{-1}(\frac{y}{x}) \quad (1.4)$$

In three dimensions, polar form comes in two types: **cylindrical** and **spherical**. In cylindrical coordinates, expressed as [r, θ, z], the above conversions are used, and the z-coordinate remains unchanged from Cartesian form. In spherical coordinates, the vector [r, θ, ϕ] includes a radial distance, an azimuthal angle and a polar angle.

1.3 Vector Mathematics

1.3.1 Vector Addition

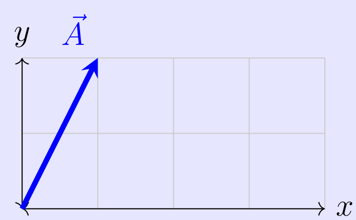
Graphical Addition of Vectors

When vectors are added graphically, they are added **head** to **tail**. This means that the arrowhead for a first vector becomes the origin for the second vector. The resultant vector is a straight line between the origin of the first vector and the head of the second.

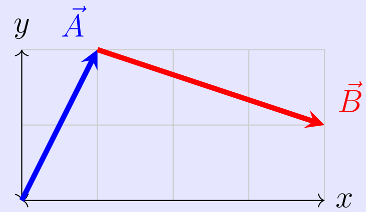
Example 1.3.1

Problem: If $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} - \hat{j}$, find $\vec{C}$ graphically given $\vec{C} = \vec{A} + \vec{B}$

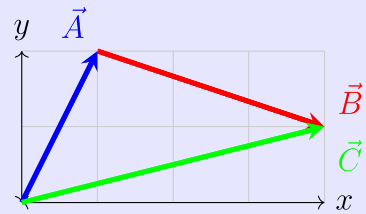
Solution: Begin by drawing $\vec{A}$:



Now use the head of $\vec{A}$ as the origin for $\vec{B}$:



The resulting vector is a straight line from the origin to the end of $\vec{B}$:



The resultant vector is given by $\boxed{\vec{C} = 4\hat{\imath} + \hat{\jmath}}$

Mathematical Addition of Vectors

Mathematical addition of vectors in Cartesian form is quite easy - simply add corresponding x- and y- values together. For instance, in Example 1.3.1 we are given $\vec{A} = \hat{\imath} + 2\hat{\jmath}$ and $\vec{B} = 3\hat{\imath} - \hat{\jmath}$. Adding these together would give $\vec{C} = (1+3)\hat{\imath} + (2-1)\hat{\jmath} \longrightarrow \vec{C} = 4\hat{\imath} + \hat{\jmath}$.

The easiest way to add vectors that are expressed in polar coordinates is to first convert them into cartesian coordinates using Equations (1.1) through (1.4) on page 4.

1.3.2 The Dot Product

In Cartesian Form

When multiplying vectors, it is sometimes necessary to obtain a scalar result. This is done through use of a dot product. A dot product is written as $\vec{A} \cdot \vec{B}$. This means to only multiply the components of the vectors that are in the same direction. In cartesian coordinates, this can be done by multiplying corresponding components, then adding the products.

Example 1.3.2

Problem: Given the vectors $\vec{Y} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{Z} = -4\hat{\imath} + 5\hat{\jmath}$ find the dot product of vectors Y and Z.

Solution: Multiply coefficients from each vector, then add the products together:

$$2 \times (-4) + 3 \times 5 = -8 + 15 = \boxed{7}$$

In Polar Form

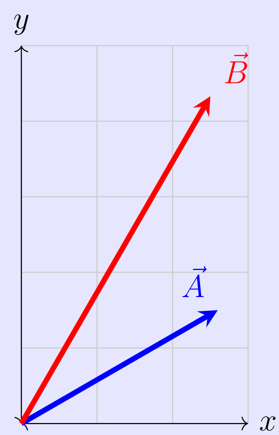
Often, vectors will be expressed in polar notation. If this is the case, the dot product can be found my multiplying the magnitude of the first vector times the magnitude of the second vector times the cosine of the angle between:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \tag{1.5}$$

Example 1.3.2.2

Problem: Given the vectors $\vec{D} = 3\angle 30°$ and $\vec{E} = 5\angle 60°$ find the dot product of vectors D and E.

Solution: We begin by graphing both vectors starting from a common origin:



Noticing that the angle between the two vectors is 30°, we can use equation 1.5 to calculate:

$$\vec{D} \cdot \vec{E} = |\vec{D}||\vec{E}| \cos\theta = (3)(5)\cos 30° \approx \boxed{12.990}$$

1.3.3 The Cross Product

Cross Products In Cartesian Form

Sometimes two vectors will be multiplied in such a way that they will result in another vector. This is called a cross product. A cross product is inherently a three-dimensional operation. A cross product can be found by calculating the determinant of a matrix:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.6}$$

The most common way to find the determinant of the above matrix is by using minors:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

This finding the determinate of each of the 2x2 matrices yields:

$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x) + \hat{k}(A_x B_y - A_y B_x)$$

Example 1.3.3

Problem: Given the vectors $\vec{V} = 2\hat{i} + 3\hat{j}$ and $\vec{W} = -4\hat{i} + 5\hat{j}$ find the cross product of vectors V and W.

Solution: Begin by creating a matrix, as shown in equation (1.6). Since both vectors lie in the X-Y plane, the Z-components for both vectors are zero:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix}$$

Now, use expansion by minors to create three two-by-two matrices:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix} = \hat{i} \begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 3 \\ -4 & 5 \end{vmatrix}$$

Finding the determinate of each of the two-by-two matricies yields:

$$\vec{V} \times \vec{W} = \hat{i}(3 \cdot 0 - 0 \cdot 5) - \hat{j}(2 \cdot 0 - 0 \cdot (-4)) + \hat{k}(2 \cdot 5 - 3 \cdot (-4))$$

Both the x-component and the y-components of this cross product are zero. Thus,

$$\boxed{\vec{V} \times \vec{W} = 22\hat{k}}$$

There are some important things you may wish to note.

1. The cross product is not commutative. That is, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$. In fact, the two cross products are in exactly opposite directions.

2. The cross product of two vectors is always a vector that is perpendicular to both of the original vectors.

Cross Products In Polar Form

When vectors are expressed in polar form, a cross product can often be found using two steps. To find the magnitude of the resulting vector, one would use the following equation:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta \tag{1.7}$$

In Equation 1.7, θ is the angle between the two vectors. To find the direction of the resulting vector, use the first right hand rule. To use this rule, you point your index finger in the direction of the first vector to be multiplied. Your middle finger is then pointed in the direction of the second vector to be multiplied. Your thumb will point in the direction of the resultant vector, as shown in the image:

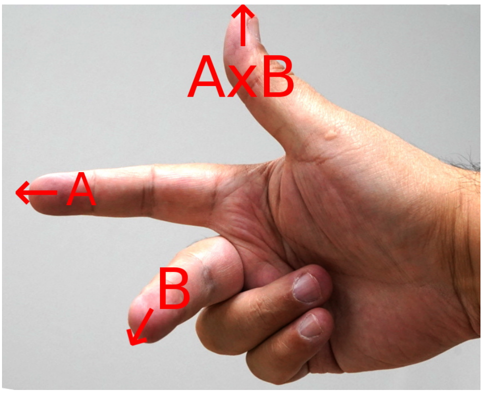


Figure 1.2: The First Right Hand Rule

The right had rule is easiest to use when the first two vectors are at 90° to each other. However, even if the first two vectors are not at right angles to each other, the resultant vector of a cross product will always be perpendicular to the plane of the first two vectors.

Chapter 2

Kinematics in One Dimension

2.1 Distance and Displacement

You are probably already familiar with the concept of distance - you might get in your car and drive a total of 1.2 miles to school, turning right after 0.45 miles, according to your car's odometer. Distance is a scalar that tells you how far something traveled. The symbol d usually represents distance.

While you may have traveled a total distance of 1.2 miles from your school, you are significantly less than 1.2 miles away from home; in fact, you are approximately 0.874 miles from home, following a direct path directly from your home to the school, at an angle of 59°(not worrying that this path might take you through someone's back yard or kitchen).

Displacement is a vector that tell you how far something is from the origin, and is independent of the path taken to get there. The displacement vector is commonly symbolized by $\vec{d}$ though sometimes it may be written as $\vec{r}$ or $(\vec{x} - \vec{x_0})$.

2.2 Average and Instantaneous Speed and Velocity

Speed is a scalar value that represents the change in distance per change in time of an object. Speed is usually represented with the symbol v, without the vector sign. You are probably already familiar with this quantity, since the speedometer on your family car measures speed. For the purposes of physics, speed has little value because it is a scalar that tells us nothing of direction. Much more useful is the concept called velocity. Velocity and speed are related much like distance and displacement.

Velocity is the change in displacement of an object per unit time, and as such is a vector. Positive velocities indicate that the object is moving forward, relative to the axis in question, and negative velocities generally mean that the object is moving backward, relative the the axis. The average velocity of an object is given by:

$$\overrightarrow{v_{avg}} = \frac{\Delta \vec{d}}{\Delta t} \tag{2.1}$$

Average velocity is useful if an object's velocity is not changing. However, many times it is more useful to talk about instantaneous velocity. Instantaneous velocity tells us how fast an object is moving at a given instant in time. In order to calculate instantaneous velocity, we must allow our time interval in the above formula to become infinitesimally small. In this case, a little calculus proves:

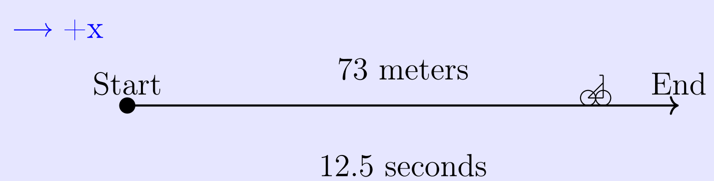
$$\vec{v} = \frac{d\vec{r}}{dt} \tag{2.2}$$

Calculation of average velocity is rather straightforward, assuming you know both distance traveled and the time it took. If an object is not speeding up or slowing down during a specific time interval, the instantaneous velocity at any time during this interval is equal to the average velocity. If the object does speed up or slow down during the time interval in question, the average velocity and the instantaneous velocity at a certain time during the interval are not necessarily the same.

Example 2.2.1

Problem: You ride your bicycle in a straight line for a distance of 73 meters in 12.5 second. What is your average speed?

Solution: Begin by drawing a diagram:



Since we have both distance and time, average speed can be easily calculated:

$$v_{avg} = \frac{d}{t} = \frac{73m\ \hat{i}}{12.5s} = 5.84\frac{m}{s}\ \hat{i}$$

Example 2.2.2

Problem: A bicyclist rides his bike to the east. His position (in meters) is given by the following expression:

$$\vec{r}(t) = (0.5t^2 + 4t)\hat{i}$$

a. What is his average velocity from t = 0 to t = 5 seconds?

b. What is his instantaneous velocity at t=3 seconds?

Solution:

a. The total displacement (in meters) after five seconds is given by:

$$\vec{d} = \vec{r}(5\,\mathrm{s}) - \vec{r}(0\,\mathrm{s}) = (0.5 \times (5s)^2 + 4 \times 5s)\ m\ \hat{i} - 0 = 32.5\ m\ \ \hat{i}$$

Thus, the average velocity is -

$$\overrightarrow{v_{avg}} = \frac{\vec{d}}{t} = \frac{32.5m\ \hat{i}}{5s} = \boxed{6.5\frac{m}{s}\ \hat{i}}$$

b. The instantaneous velocity of an object is found using a derivative with respect to time. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(0.5t^2 + 4t)\hat{i} = (t+4)\hat{i}$$

Evaluating this at t=3s yields:

$$\vec{v} = (3+4)\hat{i} = 7\frac{m}{s}\hat{i}$$

2.3 Relative Motion at Constant Velocity

2.4 Acceleration

2.4.1 Average Acceleration

Velocity is not always constant. For instance, when you are driving through a city, there are times when you might be going 30 mph, and there are times when you might be stopped at a streetlight. City driving requires you to speed up at some times, and slow down at other times – your velocity changes as a function of time. Change in velocity per change in time is called acceleration. Keep in mind, both speeding up and slowing down are forms of acceleration. In the case that an object is traveling with a positive velocity, slowing down causes negative acceleration (or deceleration). Like velocity, acceleration comes in two basic types – average and instantaneous. To find average velocity, we calculate the change in velocity per change in time:

$$\overrightarrow{a_{avg}} = \frac{\Delta\vec{v}}{\Delta t} \tag{2.3}$$

Keep in mind that this can be expressed as:

$$\overrightarrow{a_{avg}} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{\Delta t} \tag{2.4}$$

where v_f and v_i are final velocity and initial velocity, respectively. Sometimes final velocity is expressed as v and initial velocity is symbolized as v_0.

2.4.2 Instantaneous Acceleration

Instantaneous velocity is found by letting the time interval in question become infinitesimally small. A little calculus proves that:

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{2.5}$$

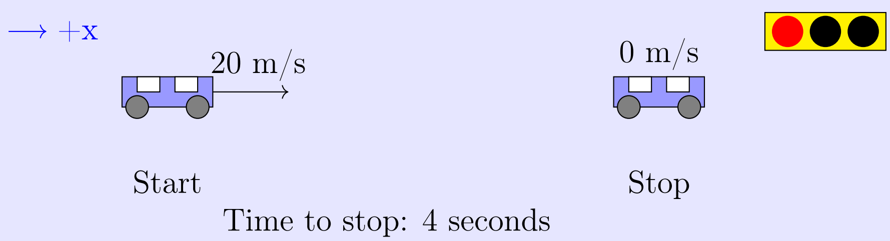
Combining this with equation (2.2), we find:

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} \tag{2.6}$$

Example 2.4

Problem: A car is traveling 20 m/s in the positive x direction. The driver sees a red light, and applies the brakes, causing the vehicle to come to a stop in 4 seconds. What is the average acceleration caused by the brakes?

Solution: First, draw the diagram:



Using the definition of Average Acceleration from Equation 2.4, we find:

$$\overrightarrow{a_{avg}} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{\Delta t} = \frac{0\frac{m}{s}\hat{i} - 20\frac{m}{s}\hat{i}}{4s} = -5\frac{m}{s^2}\hat{i}$$

2.5 The Kinematic Equations

2.5.1 The Kinematic Variables

There are five variables that are often used to solve problems involving constant acceleration. These variables are listed below:

Table 2.1: **The Kinematic Variable**

Quantity	**Variable**	**Units**
Displacement	$\vec{d}$ or $\vec{x} - \vec{x_0}$	m
Initial Velocity	$\vec{v_i}$ or $\vec{v_0}$	m/s
Final Velocity	$\vec{v_f}$ or $\vec{v}$	m/s
Acceleration	$\vec{a}$	m/s^2
time	t	s

2.5.2 The Kinematic Equations

Using the definitions above, and a little calculus (or a lot of algebra) we can prove the following four equations:

The Kinematic Equations

$$\vec{x} - \vec{x_0} = \frac{\vec{v} + \vec{v_0}}{2} t \tag{2.7}$$

$$\vec{x} - \vec{x_0} = \vec{v_0} t + \frac{1}{2} \vec{a} t^2 \tag{2.9}$$

$$\vec{v} = \vec{v_0} + \vec{a} t \tag{2.8}$$

$$\vec{v}^2 = \vec{v_0}^2 + 2\vec{a}(\vec{x} - \vec{x_0}) \tag{2.10}$$

These equations enable you to solve the vast majority of kinematic problems. Keep in mind that these equations should only be applied in one direction at at time – meaning that 2-dimensional and 3-dimensional problems will require you to split all the quantities into component vectors before you can solve these equations in each direction separately. It should also be noted that these equations assume constant acceleration. If the acceleration of the object is changing, the equations are not valid.

2.6 Vertical Motion and Gravity

One of the most common types of acceleration we experience every day is the acceleration due to gravity. Acceleration due to gravity is given a special symbol: g. On Earth, $g \approx 9.81\,\mathrm{m/s^2}$. Keep in mind that this value is only useful for calculations involving gravity that take place on the Earth's surface. All planets, stars, and celestial bodies (in fact, all objects with any mass) have their own gravitational acceleration. Acceleration due to gravity on the moon, for instance, is $1.62\,\mathrm{m/s^2}$.

Objects undergo free fall when the only force acting on the object is gravity. In order to make the calculations easier, we often ignore air resistance.

Because gravity at the Earth's surface is downward, sign conventions become a little more important than previously. If upward is positive, g will have a negative sign attached to it to indicate the direction of acceleration.

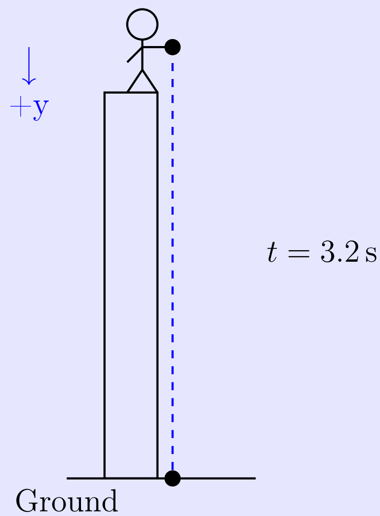
Example 2.6

Problem: You are standing on top of a building. You drop a rock from the top of the building, and let it free fall until it hits the ground, 3.2 seconds later.

a. What is the height of the building?

b. An identical building is build on the moon ($g_{moon} = 1.62\,\mathrm{m/s^2}$). How long does it take the rock to fall in this case?

Solution: First, draw the diagram:



Note that we have chosen downward to be positive. This means that gravity, displacement, and final velocity will all be positive. Next, create a table of kinematic variables:

$y - y_0 =$	
$v_{0y} =$	$0\,\mathrm{m/s^2}$
$v_y =$	
$a_y =$	$9.81\,\mathrm{m/s^2}$
$t =$	$3.2\,\mathrm{s}$

a. To solve for distance, we apply equation 1.5.3 in the y-direction:

$$y - y_0 = \cancelto{0}{v_{0y}t} + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.81\,\mathrm{m/s^2})(3.2\,\mathrm{s})^2 \approx 50.227\,\mathrm{m}$$

b. On the moon, we have new kinematic variables:

$y - y_0 =$	$50.227\,\mathrm{m}$
$v_{0y} =$	$0\,\mathrm{m/s^2}$
$v_y =$	
$a_y =$	$1.62\,\mathrm{m/s^2}$
$t =$	

We can use equation 1.5.3 again, this time solving for time:

$$y - y_0 = \cancelto{0}{v_{0y}t} + \frac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(50.227\,\mathrm{m})}{1.62\,\mathrm{m/s^2}}} \approx 7.875\,\mathrm{s}$$

Chapter 3

Graphing Motion

3.1 Introduction

Creating graphs of an object's motion is a useful tool for several reasons. Graphs do all the of the following:

- Graphs allow us to visualize relationships within collected data.
- Graphs allow for detailed analysis, allowing us to find regression lines, slope, and area.
- Graphs allow for easy and intuitive extrapolation - that is, prediction of future behavior.
- Graphs help us to understand complex forms of motion.
- Graphs help us to compare different types of motion.

In general, there are three types of graphs that physicists use routinely when studying motion of objects: *Position vs Time graphs, Velocity vs Time graphs,* and *Acceleration vs Time* graphs. Using each of these types of graph, we will analyze the characteristics of different types of motion.

It may be helpful to review some math. The formula for a line, in slope-intercept format is given by:

$$y = mx + b \tag{3.1}$$

where m is the slope of the line and b is the y-intercept.

To determine the slope of a line, use the slope formula:

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \tag{3.2}$$

3.2 Position vs Time Graphs

There are some important things to know about position vs time graphs:

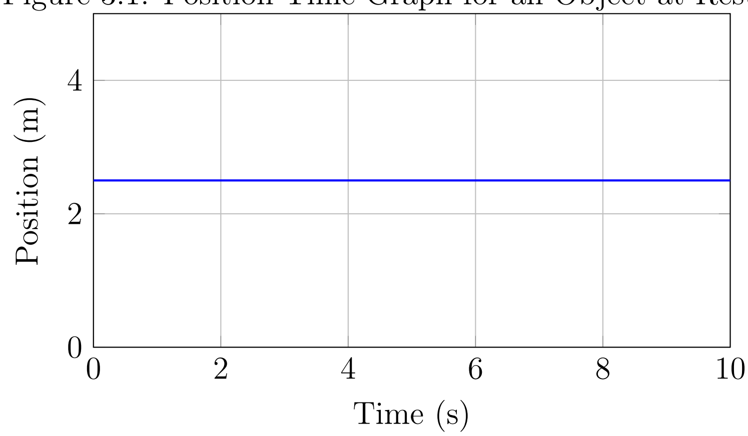
- The slope of a position vs time graph is equal to the object's velocity.
- Curves on a position-time graph indicate acceleration.

With these ideas in mind, let us examine several common types of motion.

3.2.1 Constant Position

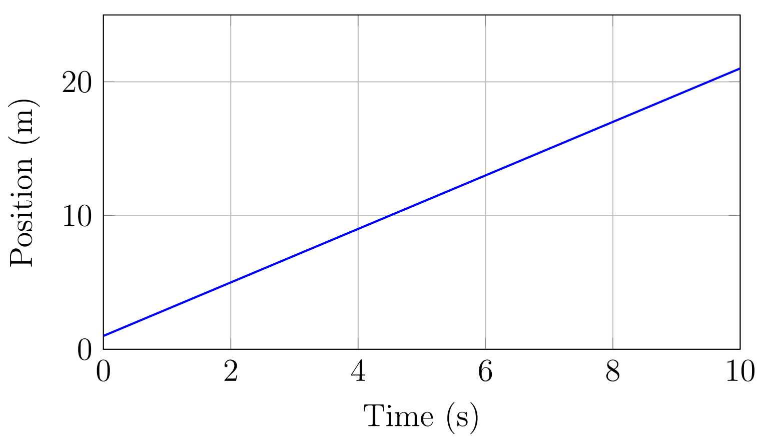
An object with a constant position will be at rest; it is not moving. Thus, a position-time graph for a non-moving object might look something like this:

Figure 3.1: Position-Time Graph for an Object at Rest

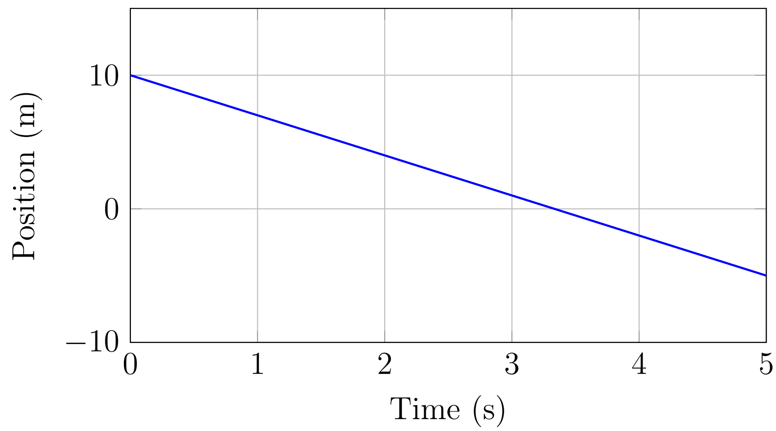


3.2.2 Constant Velocity

Position-Time Graph for an Object Moving at Constant Speed

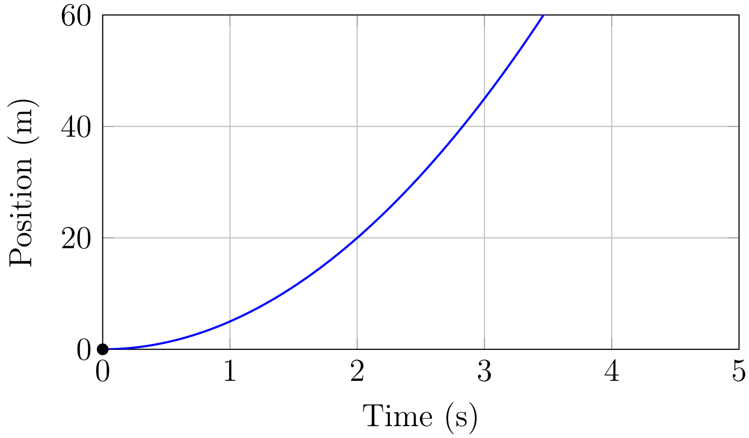


Position-Time Graph for an Object Moving with Constant Negative Velocity



3.2.3 Constant Acceleration

Position-Time Graph for an Object with Constant Acceleration



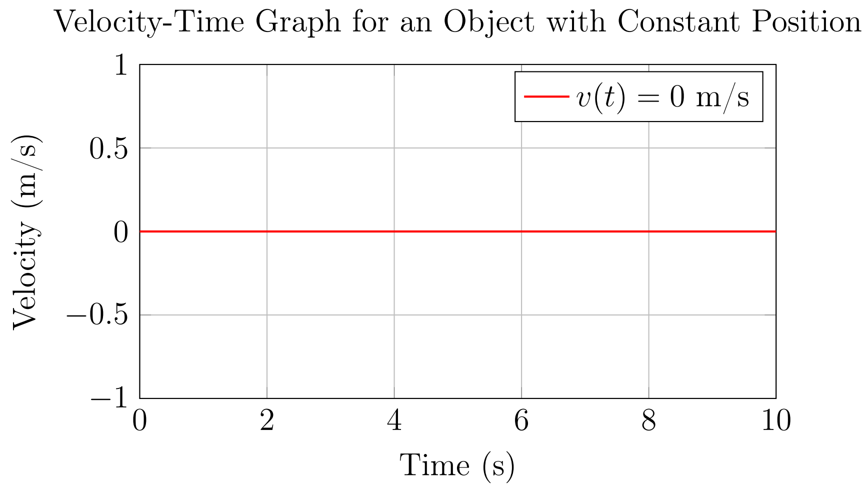
3.3 Velocity vs Time Graphs

Velocity vs Time graphs have different properties than position vs time graphs. Important characteristics of this type of graph include:

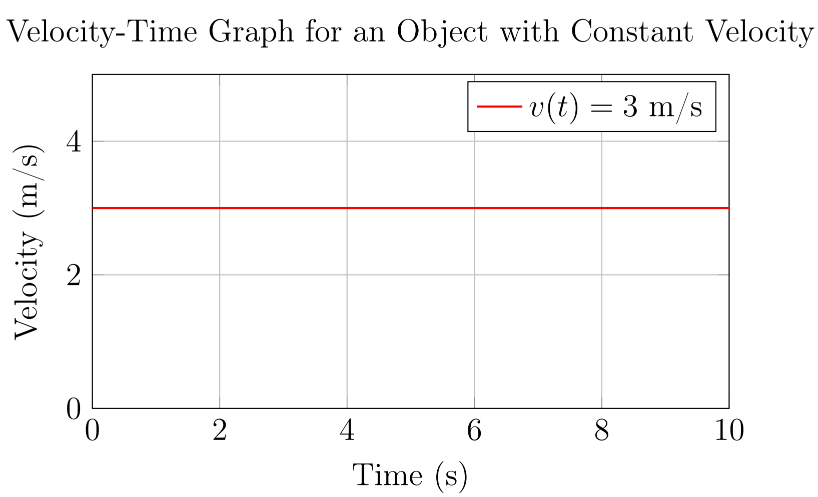
- The slope of a velocity vs time graph is the acceleration of the object.

- The distance traveled by an object is given by the area under the line or curve.

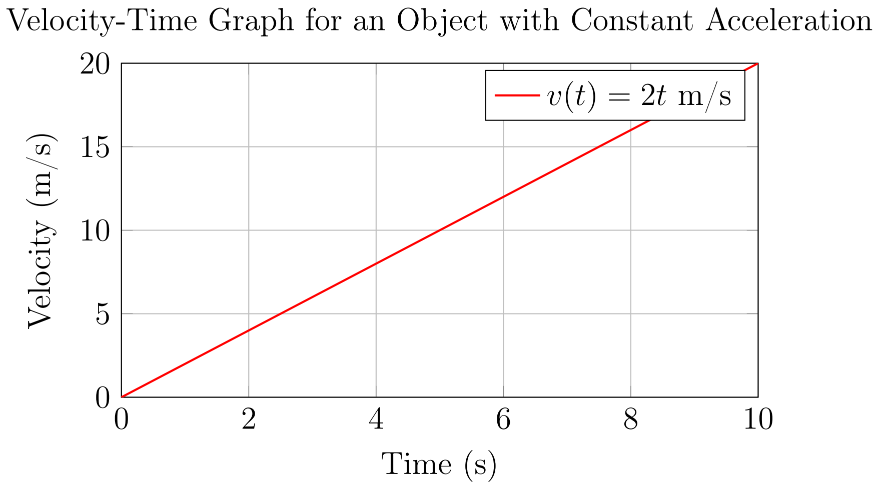
3.3.1 Constant Position



3.3.2 Constant Velocity



3.3.3 Constant Acceleration

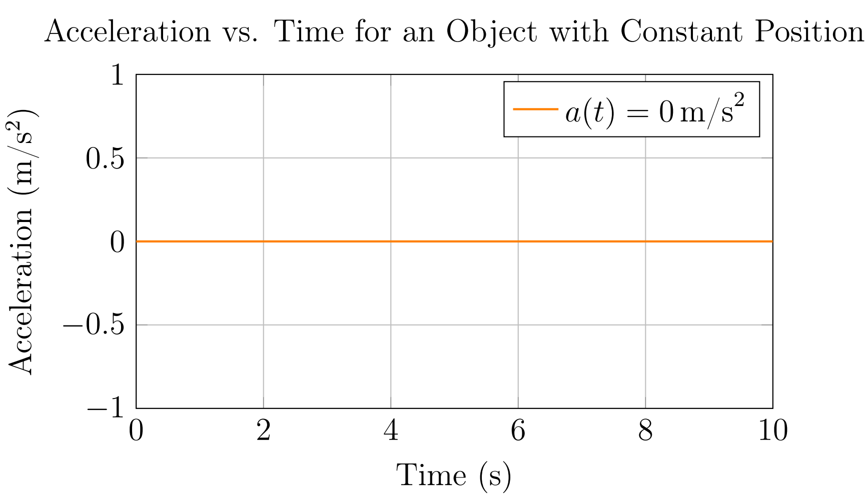


3.4 Acceleration vs Time Graphs

For acceleration vs time graphs,

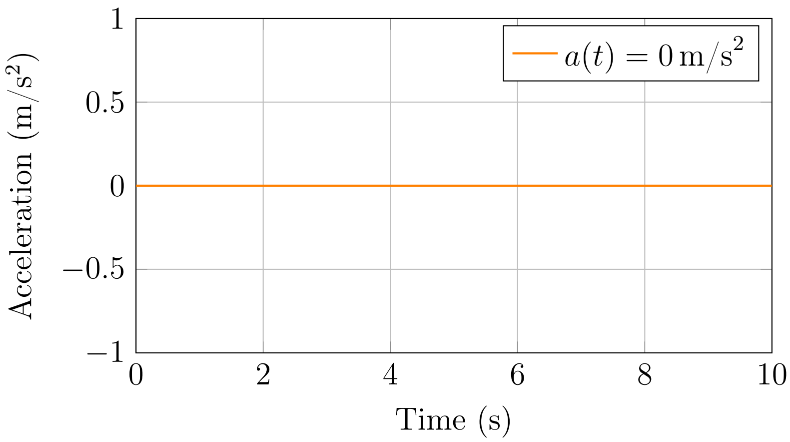
- The area under the line is equal to the change in velocity.
- The slope of the line is called jerk, but isn't often used.

3.4.1 Constant Position



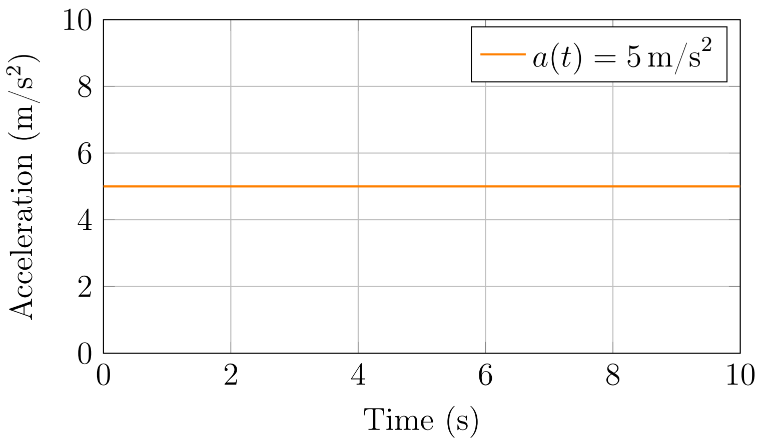
3.4.2 Constant Velocity

Acceleration vs. Time for an Object with Constant Velocity



3.4.3 Constant Acceleration

Acceleration vs. Time for an Object with Constant Acceleration



Chapter 4

Kinematics in Two Dimensions

4.1 Introduction

When an object is moving in two (or three) dimensions at once - like when it moves both horizontally and vertically at the same time - the motion of the object in each dimension is completely independent. This means that each dimension will have its own set of kinematic variables. The only kinematic variable that can be used in all directions is time, since time is a scalar and does not have a direction. Thus, instead of only five kinematic variables, problems will have sets of five variables in each direction, with time counting for every dimension.

Table 4.1: **Kinematic Variables in Multiple Dimensions**

Quantity	**Variable**	**Quantity**	**Variable**
Horizontal Displacement	$\vec{d_x}$ or $\vec{x} - \vec{x_0}$	Vertical Displacement	$\vec{d_y}$ or $\vec{y} - \vec{y_0}$
Horizontal Initial Velocity	$\vec{v_{ix}}$ or $\vec{v_{0x}}$	Vertical Initial Velocity	$\vec{v_{iy}}$ or $\vec{v_{0y}}$
Horizontal Final Velocity	$\vec{v_{fx}}$ or $\vec{v_x}$	Vertical Final Velocity	$\vec{v_{fy}}$ or $\vec{v_y}$
Horizontal Acceleration	$\vec{a_x}$	Vertical Acceleration	$\vec{a_y}$
Time		t	

4.2 Projectiles

A **projectile** is any object that meets the following critera:

- The object is in *free-fall.* That is, Gravity is the only force that acts on the object (all other forces are negligable).
- The object is moving in two dimensions at the same time. Most often, describe it as moving both horizontally and vertically at the same time.

4.2.1 Horizontally Launched Projectiles

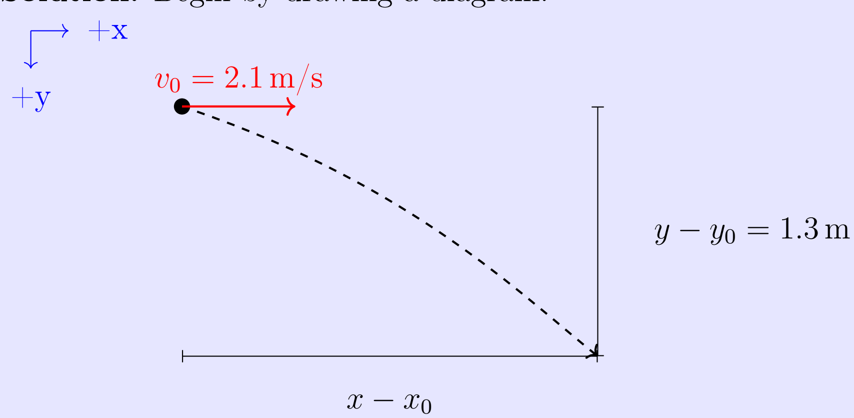
Often, projectiles will be launched horizontally, such as when a ball rolls off a table, or when an archer shoots a perfectly level arrow. In this case, the math is somewhat easier to deal with. The initial velocity stated in the problem will be entirely horizontal, and the initial vertical velocity will be zero.

Example 4.2.1

Problem: A ball is rolling 2.1 m/s when it rolls off the edge of a 1.3 meter high table.

1. How long is the ball in the air?
2. How far, horizontally, does the ball land from the edge of the table?
3. What is the magnitude of the final velocity of the ball?
4. What is the angle of impact?

Solution: Begin by drawing a diagram:



You may notice from the coordinate system that the downward direction has been chosen as +y. This will help us to avoid needing negatives in the problem.

Next, we create a table with each of the kinematic variables for each dimension:

Horizontal		Vertical	
$\vec{x} - \vec{x_0} =$		$\vec{y} - \vec{y_0} =$	$1.3\,\mathrm{m}$
$\vec{v_{0x}} =$	$2.1\,\mathrm{m/s}$	$\vec{v_{0y}} =$	$0\,\mathrm{m/s}$
$v_x =$		$v_y =$	
$a_x =$	$0\,\mathrm{m/s^2}$	$a_y =$	$9.81\,\mathrm{m/s^2}$
$t =$			

1. We see that the vertical direction has three variables, and can be used to calculate the time the ball is in the air, using equation eq. (2.9), applied in the vertical direction:

$$y - y_0 = \cancelto{0}{v_{0y}t} + \frac{1}{2}a_y t^2$$

   Solving for t yields:

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(1.3\,\mathrm{m})}{9.81\,\mathrm{m/s^2}}} \approx 0.515\,\mathrm{s}$$

2. To find the distance the ball has traveled horizontally, we can use the same kinematic equation as the previous step, only this time, applied in the horizontal direction:

$$x - x_0 = v_{0x}t + \cancelto{0}{\frac{1}{2}a_x t^2}$$

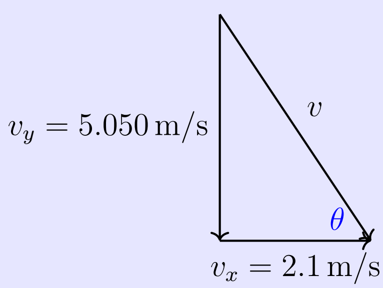
$$x - x_0 = (2.1\,\mathrm{m/s})(0.515\,\mathrm{s}) \approx 1.081\,\mathrm{m}$$

3. Finding the final velocity of the ball requires finding both the x- and y- components of the final velocity:

$$v_x = v_{0x} + \cancelto{0}{a_x t} = 2.1\,\mathrm{m/s}$$

$$v_y = \cancelto{0}{v_{0y}} + a_y t = 9.81\,\mathrm{m/s^2} 0.515\,\mathrm{s} \approx 5.050\,\mathrm{m/s}$$

   We can use these components of the final velocity to determine v according to the following diagram:



   Using the pythagorean theorum, we find:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.1\,\mathrm{m/s})^2 + (5.050\,\mathrm{m/s})^2} \approx 5.470\,\mathrm{m/s}$$

4. The angle of impact is marked as θ in the diagram for part 3. We can find the angle of impact by using trigonometry. Using tangent yields:

$$\tan\theta = \frac{opp}{adj}$$

$$\theta = \tan^{-1}(\frac{opp}{adj}) = \tan^{-1}(\frac{v_y}{v_x}) = \tan^{-1}(\frac{5.050\,\mathrm{m/s}}{2.1\,\mathrm{m/s}}) \approx 67.420°$$

4.2.2 Projectiles Launched at an Arbitrary Angle

When projectiles are launched at an angle, the initial horizontal and vertical components of the initial velocity can be determined using trigonometry.

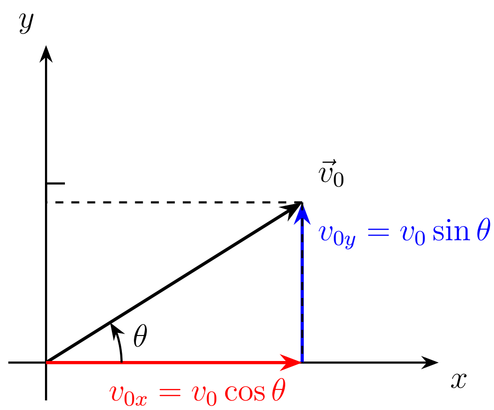


Figure 4.1: The horizontal And vertical components of initial velocity.

In general, when the angle of elevation from horizontal is provided, as shown in Figure 4.1, the following equations allow you to deterine the initial and horizontal components of the initial velocity:

$$\overrightarrow{v_{ox}} = \overrightarrow{v_o}\cos\theta \tag{4.1}$$

$$\overrightarrow{v_{oy}} = \overrightarrow{v_o}\sin\theta \tag{4.2}$$

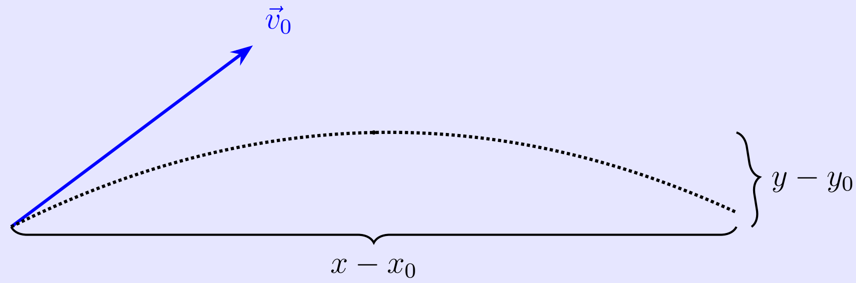
You can then solve the problem using normal kinematic equations.

Example 4.2.1

Problem: A cannon launches a cannonball into the air with an initial speed of 125 m/s at an angle of 32 degrees above horizontal.

1. How long does the cannonball take to get to the top of its path?
2. How high does the cannonball go?
3. How far does the cannonball land from its launchpoint?

Solution: Begin by drawing a diagram:



1. To find the time it takes to get to the top, we will us the rising portion of the motion only. We can find the initial x- and y- component velocities by using trigonometry:

$$\overrightarrow{v_{ox}} = \overrightarrow{v_o} \cos\theta = (125\,\mathrm{m/s})\cos(32\,\mathrm{deg}) \approx 106.006\,\mathrm{m/s}$$

$$\overrightarrow{v_{oy}} = \overrightarrow{v_o} \sin\theta = (125\,\mathrm{m/s})\sin(32\,\mathrm{deg}) \approx 66.240\,\mathrm{m/s}$$

We know that at the top, the projectile's vertical velocity will be zero, so we can write the following variables:

Horizontal		**Vertical**	
$\vec{x} - \vec{x_0} =$		$\vec{y} - \vec{y_0} =$	
$\vec{v_{0x}} =$	$106.006\,\mathrm{m/s}$	$\vec{v_{0y}} =$	$66.240\,\mathrm{m/s}$
$v_x =$		$v_y =$	$0\,\mathrm{m/s}$
$a_x =$	$0\,\mathrm{m/s^2}$	$a_y =$	$-9.81\,\mathrm{m/s^2}$
$t =$			

We can now calculate the time to the top by using equation 2.8 in the y-direction:

$$\overrightarrow{v_y} = \overrightarrow{v_{0y}} + \overrightarrow{a_y}t \longrightarrow t = \frac{\overrightarrow{v_y} - \overrightarrow{v_{oy}}}{\overrightarrow{a_y}} = \frac{0\,\mathrm{m/s} - 66.240\,\mathrm{m/s}}{-9.81\,\mathrm{m/s^2}} \approx 6.752\,\mathrm{s}$$

2. We can now calculate the maximum height, $\overrightarrow{y} - \overrightarrow{y_0}$ by using any of equations 2.7, 2.9, or 2.10 in the y-direction. While all of these equations will yield the same result, in order to minimize rounding error, the author chooses equation 2.10 in this situation.

$$\overrightarrow{v_y}^2 = \overrightarrow{v_{0y}}^2 + 2\overrightarrow{a_y}(\overrightarrow{y} - \overrightarrow{y_0}) \longrightarrow \overrightarrow{y} - \overrightarrow{y_0} = \frac{\overrightarrow{v_y}^2 - \overrightarrow{v_{0y}}^2}{2\overrightarrow{a_y}}$$

$$\overrightarrow{y} - \overrightarrow{y_0} = \frac{(0\,\mathrm{m/s})^2 - (66.240\,\mathrm{m/s})^2}{2(-9.81\,\mathrm{m/s^2})} \approx 223.635\,\mathrm{m}$$

3. It should be noted that the time calculated in part (1) of this problem is the time it takes for the ball to reach its highest point - which is when the ball has only traveled half its horizontal distance. Due to the symmetry of the situation, we can double the time to find the total time of flight. This can be used to calculate the distance the ball travels in the x- direction.

$$t_{total} = 2t_{rising} = 2 \times 6.752\,\mathrm{s} \approx 13.505\,\mathrm{s} \tag{4.3}$$

Using equation 2.9 in the x-direction gives:

$$\overrightarrow{x} - \overrightarrow{x_0} = \overrightarrow{v_0}t + \cancelto{0}{\frac{1}{2}at^2} = \overrightarrow{v_0}t = (106.006\,\mathrm{m/s})(13.505\,\mathrm{s}) \approx 1431.565\,\mathrm{m} \tag{4.4}$$

Chapter 5

Forces and Newtons Laws

5.1 Forces

Force is a vector quantity that measures how hard a push or a pull on an object is. Though all forces we encounter in everyday life can be explained in terms of the four fundamental forces, these forces manifest themselves in different ways. In fact, with the exception of gravity, almost all forces humans deal with are electromagnetic.

5.2 Free Body Diagrams

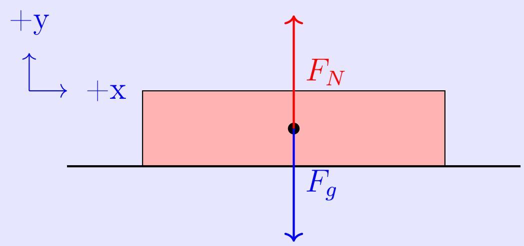
A free body diagram is a diagram that includes all external forces acting on an object or system. In general, free body diagrams should follow the following rules:

- Draw a coordinate system, which shows what direction is +x and +y.
- Objects should be represented by a dot, a box, or a sketch of the object. Systems can be represented by multiple dots, boxes, or sketches.
- Unless otherwise stated, forces act on the center of mass.
- Forces are represented as arrows. Arrows should start at the center of mass (or other point they act on) and point away from the object.
- The length of force arrows should be proportional to the strength of the force.
- All forces should be labeled.
- No arrows other than forces should appear on the diagram. (Don't include accleration, displacements, velocities, etc.)

Example 5.2.1

Problem: A book is at rest on top of a table. Draw the free-body diagram of the book on the table.

Solution: The diagram should look as follows:

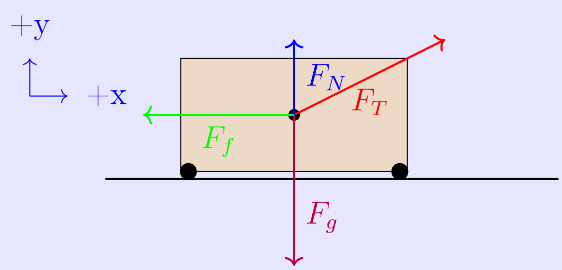


In this diagram, the normal force and the gravitational force are balanced. Thus, the object remains at rest, according to Newton's 1st Law.

Example 5.2.2

Problem: As a person walks through an airport, she pulls a suitcase at a constant speed using a strap at a 25 degree angle above horizontal. Friction is significant. Draw a free body diagram of the situation.

Solution: The diagram should look as follows:



In this diagram, the sum of the forces that act in the x-direction (friction, and the x-component of tension) must equal zero to ensure the suitcase maintains a constant horizontal speed. In addition, the forces that act in the y-direction (Gravity, Normal Force, and the Y-component of tension) must sum to equal zero as well so that the suitcase does not accelerate vertically.

5.3 Newton's First Law

> *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*
>
> *Every body continues in its state of being at rest or moving uniformly in a direction, except insofar as it is compelled to change its state by means of an imparted force.*
>
> – Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

You may have heard Sir Isaac Newton's first law of physics stated in different ways than the above. Often in grade school, students are taught a phrase beginning with "objects in motion...". Sometimes this law is called the "Law of Inertia". This is a very basic understanding of the complexity of this law. In fact, all non-accelerating systems are governed by this law. As long as the vector sum of the forces upon an object is zero, the object will continue in a state of uniform motion (remaining at rest is a type of uniform motion) until something causes the equilibrium of the system to be lost. Likewise, if an object is known to have an acceleration of zero, we can state that the vector sum of the forces is equal to zero. We can use this law to characterize non-accelerating systems:

$$\Sigma\vec{F} = 0 \tag{5.1}$$

5.4 Newton's Second Law

Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

The change in motion is proportional to the amount of force of motion imparted, and according to the straight line made by the force impressed.

-Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

Newton's Second Law describes objects and systems that have a constant acceleration. The vector sum of the forces on an object is equal to the object's mass times its acceleration:

$$\Sigma\vec{F} = m\vec{a} \tag{5.2}$$

It should be noted that forces are vectors. Thus, when forces are combined, their directions should be taken into account. The direction of the net force is always in the same direction as the acceleration of the object.

It should also be noted that if all the forces on an object cancel, the acceleration of the object will be zero - which means that the first law is just a special case of the second law when $\Sigma F = 0N$.

Often, if the forces on an object of known mass can be determined, the acceleration of the object can also be determined. The acceleration found using Newton's second law can then be used in kinematic equations to determine other important quantities.

5.5 Newton's Third Law

Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

For an action there is always an equal and opposite reaction: or the two bodies on each other are always equal and in opposite directions.

-Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

5.6 Applications of Newton's Laws

5.6.1 Friction

Friction occurs whenever two surfaces are in contact, and it is a force that has two effects.

- Friction always opposes the motion of an object, or even the tendency to move.
- Friction dissipates energy into heat.

There are two types of friction that one should know about: **static friction** and **kinetic friction**. Static friction is present when two surfaces are in contact, but not sliding. Kinetic friction is occurs when two surfaces are in contact and sliding past each other. One should note that when an object is rolling, there is static friction between the outer edge of the wheel and the surface it is rolling on, whereas static or kinetic friction may be present where the wheel is connected to the axle, depending on the type of joint that is used.

The **coefficient of static friction** and the **coefficient of kinetic friction**, symbolized by μ_s and μ_k respectively, measure how hard it is to slide two surfaces past each other. A surface that is very slippery will have low coefficients of friction, whereas surfaces that grip each other well will very high coefficients of friction. A perfectly frictionless surface would have a coefficient of friction of 0. Some examples of coefficients of friction can be found in insert reference here. One should also note that the coefficients of friction are unitless.

To calculate the force of static friction, the following formula is used:

$$|F_f| \leq \mu_s |F_N| \tag{5.3}$$

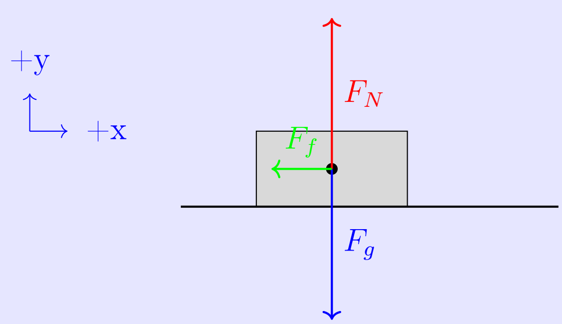
The force of kinetic friction can be calculated by using this formula:

$$|F_f| = \mu_k |F_N| \tag{5.4}$$

Example 5.6.1

Problem: In the game of curling, a stone of mass 19.1 kg is pushed across the ice with an initial speed of 4 m/s. The coefficient of kinetic friction is 0.045. How far does the rock slide until it comes to a stop?

Solution: First, draw a free body diagram:



Since the mass of the rock is known, we can find the force of gravity:

$$\vec{F}_g = mg = (19.1\,\mathrm{kg})(-9.81\,\mathrm{m/s^2}) \approx -187.371\,\mathrm{N}$$

Since the rock is not accelerating in the y-direction, the normal force must be equal in magnitude to gravity, and in the opposite direction.

$$\vec{F}_N \approx 187.371\,\mathrm{N}$$

The normal force is now known, so we can find the force of friction on the rock:

$$|F_f| = \mu_k |F_N| = (0.045)(187.371\,\mathrm{N}) \approx 8.432\,\mathrm{N}$$

Since the force of friction is to the left, we assign a negative value to it:

$$F_f \approx -8.432\,\mathrm{N}$$

Newton's 2nd Law allows us to find the acceleration of the rock, since the only unbalanced force is friction.

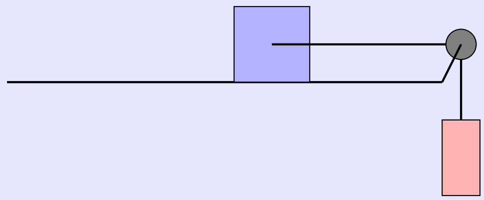
$$\Sigma\vec{F} = m\vec{a} \longrightarrow \vec{a} = \frac{\Sigma\vec{F}}{m} = \frac{\vec{F}_f}{m} = \frac{-8.432\,\mathrm{N}}{19.1\,\mathrm{kg}} \approx -0.442\,\mathrm{m/s^2}$$

Using kinematic equations allows us to find the distance:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x}-\vec{x}_0) \longrightarrow (\vec{x}-\vec{x}_0) = \frac{\vec{v}^2 - \vec{v}_0^2}{2\vec{a}} = \frac{(0\,\mathrm{m/s})^2 - (4\,\mathrm{m/s})^2}{2(-0.442\,\mathrm{m/s^2})} \approx 18.120\,\mathrm{m}$$

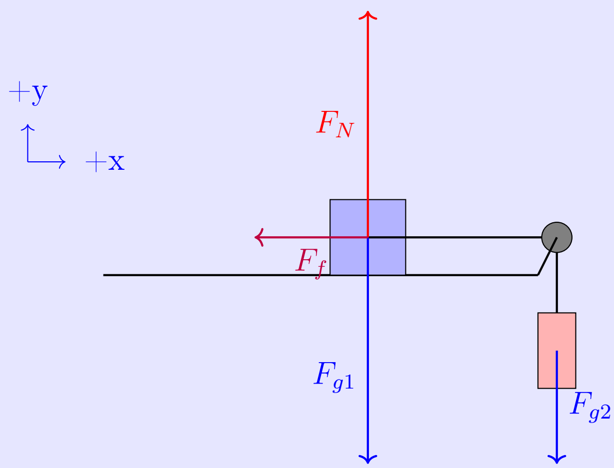
Example 5.6.2

Problem: A 2-kg mass is placed on top of a table. It is connected to a string that runs through a pulley at the edge of the table, which subsequently is connected to a mass that hangs off the table, as shown in the diagram.



A student finds that the largest mass that can be held by the string without the mass on the table moving is 0.423 kg. What is the coefficient of static friction between the table and the 2 kg mass?

Solution: First, draw a free body diagram:



One should note that tension in the string is not shown on the free body diagram, because tension in the string is a force that is internal to the system.

We can find the force of gravity on the 2 kg mass:

$$\vec{F}_{g1} = m_1 g = (2\,\mathrm{kg})(-9.81\,\mathrm{m/s^2}) \approx -19.62\,\mathrm{N}$$

The 2 kg mass does not accelerate vertically, so we know that the normal force must be equal to the force of gravity, F_{g1}:

$$\vec{F}_N \approx 19.62\,\mathrm{N}$$

We can also find the force of gravity on the hanging mass.

$$\vec{F}_{g2} = m_2 g = (0.423\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) \approx 4.150\,\mathrm{N}$$

You can think of the x-axis of the coordinate system as wrapped around the pulley in the same way the string goes around it, so this force is in the +x direction. This means that static friction is the same magnitude, but in the -x direction.

$$\vec{F}_f \approx -4.150\,\mathrm{N}$$

Now, we can use equation 5.3 to calculate the coefficient of friction.

$$|F_f| \leq \mu_s |F_N|$$

Because this is the greatest amount of mass that can be attached without causing the 2 kg mass to slide, the force of friction must be at its greatest amount, so:

$$|F_f| = \mu_s |F_N| \longrightarrow \mu_s = \frac{|F_f|}{|F_N|} = \frac{|-4.150\,\mathrm{N}|}{|19.62\,\mathrm{N}|} \approx 0.212$$

5.6.2 Inclined Planes

The normal force that a surface exerts on an object is always perpendicular to the surface. Thus, when an object is placed on an inclined plane, the normal force exerted on the object will not be vertical. Similarly, any frictional forces will be parallel to the inclined plane, and the motion of the object will be along the plane as well. Thus, it is easier to deal with inclined planes by creating a coordinate system that is rotated to align with the inclined plane, as seen in the figure below:

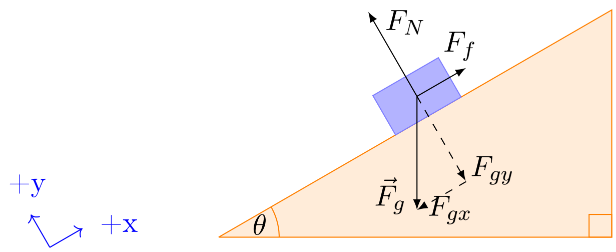
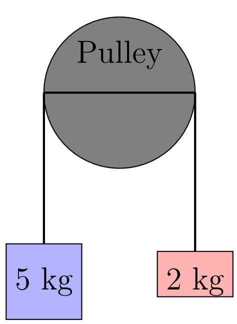


Figure 5.1: An inclined plane

In this case, we have chosen to make the positive x direction up the ramp, and the positive y direction upward and to the left, perpendicular to the ramp. This means that the normal force is in the postive y direction, and the frictional force is in the positive x direction. The only force that does not align to the coordinate system is gravity. However, it is easy to decompose gravity into two forces, one in the x-direction and one in the y-direction, as shown by the dashed arrows in the diagram. A little geometry proves that $F_{gx} = F_g \sin(\theta)$ and $F_{gx} = F_g \cos(\theta)$. We can then assign positives and negatives based on the coordinate system we have chosen.

5.6.3 Elevators

5.6.4 Pulleys and Atwood Machines



Chapter 6

Work and Energy

6.1 Energy

Energy is the ability of an object or system to create a change. In the SI system, the unit for energy is $\frac{kgm^2}{s^2}$, which is given the name joules (J).

Though energy can be found in many forms, there are three basic categories of energy: mechanical energy, chemical energy, and Electromagnetic Energy (Light). This chapter focuses on the forms of mechanical energy , which is the sum of kinetic energy and potential energy.

6.1.1 Kinetic Energy

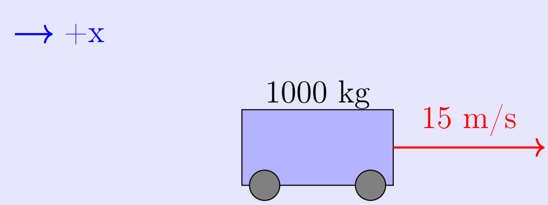
Kinetic energy is energy of motion. Any object that is moving has kinetic energy. Kinetic energy cannot be negative. The formula for kinetic energy is:

$$K = \frac{1}{2}mv^2 \tag{6.1}$$

Example 6.1.1

Problem: A 1000 kg car is traveling at 15 m/s. What is its kinetic energy?

Solution: Begin by drawing a diagram:



Since we know both the mass and the speed of the object, we can use equation (6.1) to solve the problem:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1000\,\text{kg})(15\,\text{m/s})^2 = 225\,000\,\text{J}$$

6.1.2 Potential energy

Potential energy is energy that is stored. Though there are many forms of potential energy, we will concentrate on two forms of stored energy in this section: gravitational potential energy and elastic potential energy. In this text, potential energy is symbolized by the variable U, though it is quite common to find potential energy symbolized by the letters PE as well.

A common misconception is that kinetic energy and potential energy cannot exist at the same time. This is not the case, and object often have both kinetic energy and potential energy simultaneously.

Gravitational Potential Energy

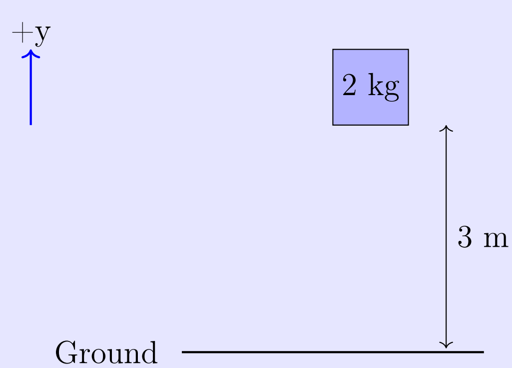
Gravitational potential energy is energy that is stored due to the interaction of an object with gravitational field. When a relatively small object is in a uniform gravitational field (such as near the surface of a planet), the gravitational potential energy is given by:

$$U_g = mgh \tag{6.2}$$

Example 6.1.1

Problem: A 2 kg block is held 3 meters above the ground. What is the gravitational potential energy of the block?

Solution: Begin by drawing a diagram:



We can now calculate potential energy using equation (6.2):

$$U_g = mgh = (2\,\text{kg})(9.81\,\frac{\text{m}}{\text{s}^2})(3\,\text{m}) = 58.86\,\text{J}$$

Elastic Potential Energy

Springs and other stretchable materials that return to their original shape when not subjected to external forces store elastic potential energy. For an ideal spring, the force the spring exerts is proportional to the distance it has been stretched. This is called Hooke's law:

$$\overrightarrow{F_s} = -k\vec{x} \tag{6.3}$$

where k is the spring constant, and x is the distance the spring has been stretched or compressed from its equilibrium position.

Some calculus can show that the energy stored in a spring that obeys Hooke's Law is:

$$U_s = \frac{1}{2}kx^2 \tag{6.4}$$

While all real springs convert mechanical energy into heat when stretching or compressing, the amount of energy lost as heat (a process called hysteresis) is usually negligible. However, some stretchable objects, such as rubber bands, lose significant amounts of energy as heat. Thus, equations 6.3 and 6.4 are not applicable to these objects.

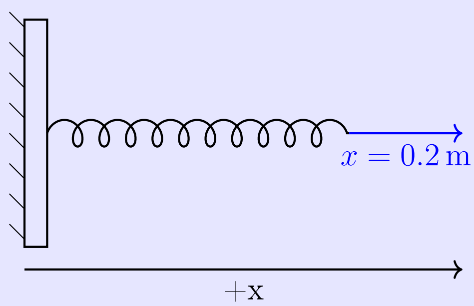
Example 6.1.1

Problem: Spring is streched a distance of 0.2 m using a force of 20 N.

(a) What is the spring constant of the spring?

(b) What is the elastic portential energy stored in the spring?

Solution: Begin by drawing a diagram:



(a) To find the spring constant, we can use Hooke's Law, Equation 6.3:

$$\overrightarrow{F_s} = -k\vec{x} \longrightarrow k = \frac{\overrightarrow{F_s}}{\vec{x}} = \frac{20\,\mathrm{N}}{0.2\,\mathrm{m}} = 100\,\mathrm{N/m}$$

(b) Now that the spring constant is known, we can use equation 6.4 to find the elastic potential energy.

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}(100\,\mathrm{N/m})(0.2\,\mathrm{m})^2 = 2\,\mathrm{J}$$

6.2 Work

Work is the amount of energy that is transferred into or out of an object due to the application of a force that results in a displacement. The formula for work is:

$$W = \vec{F} \cdot \vec{d} \tag{6.5}$$

Note that both force and displacement are vectors, and work is the dot product of the two vectors. Thus, referencing equation 1.5

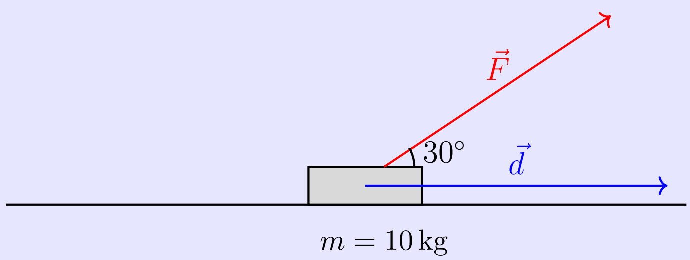
$$W = \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|cos(\theta) \tag{6.6}$$

It should also be noted that for work to be done, there must be a non-zero displacement. No work is done on an object that does not move.

Example 6.2.1

Problem: A person pulls a 10-kg sled along a flat surface by applying a force of 50 N at an angle of 30° above the horizontal. The sled moves a distance of 5 m. Assume there is no friction. How much work is done on the sled by the person?

Solution: Begin by drawing a diagram:



The work done on the sled can be found by applying equation 6.6.

$$W = \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|cos(\theta) = (50\,\mathrm{N})(5\,\mathrm{m})cos(30°) \approx 216.506\,\mathrm{J}$$

6.3 The Work-Energy Theorem

The *Work-Energy Theorem* states that doing work on an object causes that object's energy to change by the same amount as the work done. This means that an object has 8 Joules of energy, and 2 Joules of work is done on the object, the object will have 10 Joules of work at the end of the process. While this is often associated with a change in kinetic energy, the energy change associated with work can also be associated with gravitational potential energy, thermal energy, or any other form of energy.

$$W = \Delta E \tag{6.7}$$

6.4 Power

Power is defined as how quickly work is done. Power is given by:

$$P = \frac{W}{t} \tag{6.8}$$

Since the work-energy theorem states that work is a change in energy, power is also used to describe how fast energy is flowing into or out of a system.

$$P = \frac{\Delta E}{t} \tag{6.9}$$

6.5 The Law of Conservation of Energy

The Law of Conservation of Energy states that energy cannot be created or destroyed (this isn't entirely true. This law will be tweaked in chapter 19.

6.6 Springs

We have already seen that springs follow Hooke's Law, given in equation 6.3. They also store elastic potential energy according to equation 6.4. When a spring is set into oscillatory motion, the period of oscillation is given by the following formula.

$$T_p = 2\pi\sqrt{\frac{m}{k}} \tag{6.10}$$

Example 6.6.1

Problem: A 0.25 kg mass is attached to a spring and set into oscillatory motion. The period of the spring's motion is 0.45 seconds. What is the spring constant of the spring?

Solution: We can solve equation 6.10 for k to answer this question.

$$T_p = 2\pi\sqrt{\frac{m}{k}} \longrightarrow k = \frac{4\pi^2 m}{{T_p}^2} = \frac{4\pi^2(0.25\,\mathrm{kg})}{(0.45\,\mathrm{s})^2} \approx 48.739\,\mathrm{N/m}$$

6.7 Pendulums

A pendulum is any weight on the end of an arm that is free to swing back and forth. You may have seen pendulums in old-fashioned clocks, and the swings at a park also act like a pendulum. When a pendulum swings at a small angle ($\theta \lessapprox 5\,\mathrm{deg}$), the period of a pendulum is given by:

$$T_p = 2\pi\sqrt{\frac{l}{g}} \tag{6.11}$$

Notice that the period of a pendulum does not depend on the mass of the bob. The only two variables that affect its period (assuming a small angle) are the length of the arm and gravity.

Chapter 7

Impulse and Momentum

7.1 Momentum

Linear momentum is defined by the following equation:

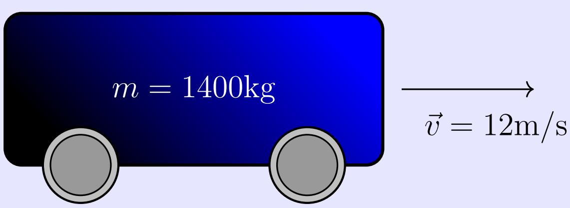
$$\vec{p} \equiv m\vec{v} \tag{7.1}$$

where $\vec{p}$ is momentum, m is mass, and $\vec{v}$ is velocity. Determining an object's momentum can be extremely useful in solving problems involving collisions or explosions.

Example 7.1.1

Problem: A 1400 kg bus is traveling at 12 m/s. What is its momentum?

Solution: Begin by drawing a diagram and identifying variables:



Using equation 7.1, we see:

$$\vec{p} = m\vec{v} = 1400kg \times 12\frac{m}{s} = \boxed{16800\frac{kg \cdot m}{s}}$$

You may notice that the units for momentum are $\frac{kg \cdot m}{s}$. This is read as "kilogram meters per second," and there is no special name for this unit.

7.2 Impulse

When a force is applied to an object for a certain amount of time, an *impulse* is delivered to that object. Impulse is defined by the following equation:

$$\vec{J} \equiv \vec{F}t \tag{7.2}$$

where $\vec{J}$ is impulse, $\vec{F}$ is force, and t is time. The units for impulse are $\mathrm{N} \cdot \mathrm{s}$, which are equivalent to $\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$.

7.3 The Impulse-Momentum Theorem

Just as work causes an object's energy to change, impulse causes an object's momentum to change:

$$\vec{J} = \Delta\vec{p} \tag{7.3}$$

7.4 The Law of Conservation of Momentum

The Law of Conservation of Momentum states that momentum cannot be created or destroyed. Thus, the total momentum a system has in its initial state, plus any changes in momentum from outside the system (accounted for as impulse) must be equal to the total momentum the system has in its final state.

$$\vec{p_i} + \vec{J} = \vec{p_f} \tag{7.4}$$

Example 7.4.1

Problem: A 1000 kg car is traveling to the right at 12 m/s. It collides head-on with an 800 kg car traveling to the left at 8 m/s. The two cars get stuck together. What is the velocity of the wreckage immediately after the collision?

Solution: Begin by drawing a diagram and identifying variables:

$$\vec{p} = m\vec{v} = 1400kg \times 12\frac{m}{s} = 16800\frac{kg \cdot m}{s}$$

Chapter 8

Circular Motion and Orbits

8.1 Centripetal Forces and Accelerations

8.1.1 Centripetal Force

We have already learned that an object in motion will continue to move in a straight line, assuming no forces are acting on the object. In order for an object to move along a circular path, there must therefore be a force acting on the object to keep it from moving in a straight line. If you whirl a mass on a string around in a circle, tension in the string keeps the mass from continuing to move in a straight line. As the Moon orbits the Earth, the gravitational attraction between the Moon and the Earth keeps the Moon in its orbit around the Earth. Any force that keeps an object moving along a circular path is called a **Centripetal Force** (Centripetal literally translates from Latin as "center seeking"). Any centripetal force can be described as:

$$F_c = \frac{mv^2}{r} \tag{8.1}$$

The direction of a centripetal force is always toward the center of the circle.

8.1.2 Centripetal Acceleration

When an object moves in a circle, even if its speed remains constant, its velocity is constantly changing due to its constant change in direction of motion. Thus the object must be constantly accelerating in a direction toward the center of the circle. Using Equation 8.1 and Newton's Second Law, it is possible to prove that centripetal acceleration is given by:

$$a_c = \frac{v^2}{r} \tag{8.2}$$

Just as centripetal force is always directed toward the center of the circle, centripetal acceleration is also always directed toward the center of the circle.

Example 8.1.2

Problem: A children's toy consists of a 0.5kg ball attached to the end of a light 0.3-meter-long rope. A child grabs the toy from the end of the rope and swings the ball around in a circle above his head with a tangential speed of 2 m/s. What is the tension in the rope?

Solution: In order to keep moving in a circle, tension in the rope must act as the centripetal force. Therefore, the tension in the rope is given by:

$$F_T = F_c = \frac{mv^2}{r} = \frac{0.5kg \cdot (2m/s)^2}{0.3m} \approx \boxed{6.667N}$$

Example 8.1.2.2

Problem: A toy car travels around a loop of diameter 0.3 meters. What is the minimum speed the car needs to travel in order to make it around the loop?

Solution: When the car is traveling around the loop, it is most likely to fall off at the top. If gravity is stronger than the needed centripetal force, the car falls. If gravity is equal to, or even less than the required centripetal force, the car stays on the track. Thus:

$$F_g \leq F_c$$

Substituting equations shows:

$$\cancel{m}g \leq \frac{\cancel{m}v^2}{r}$$

Solving for v yields:

$$\sqrt{gr} \leq v$$

Note that the diameter is given in the problem, but the formula requires the radius. Thus, substituting numbers gives:

$$\sqrt{(9.81\frac{m}{s^2})(0.15m)} \leq v$$

Thus:

$$\boxed{1.213\frac{m}{s} \lessapprox v}$$

8.2 Kepler's Laws of Planetary Motion

8.2.1 Kepler's First Law

Kepler's first law states that the orbit of a planet is an ellipse with the Sun at one of the two foci:

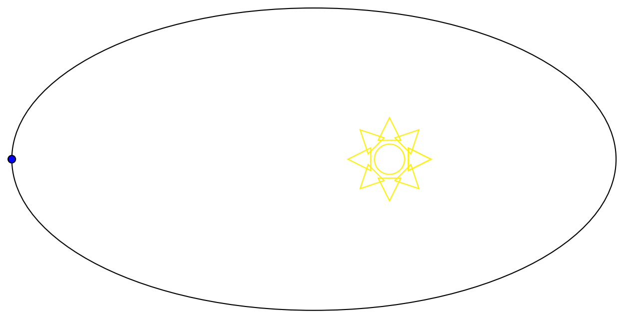


Figure 8.1: Planets orbit the sun along elliptical path. This diagram exaggerates the eccentricity of the orbital path to show the placement of the sun.

One should note that most planet's orbital path is much closer to circular than the diagram above. For instance, here is earth's orbit compared to a perfect circle:

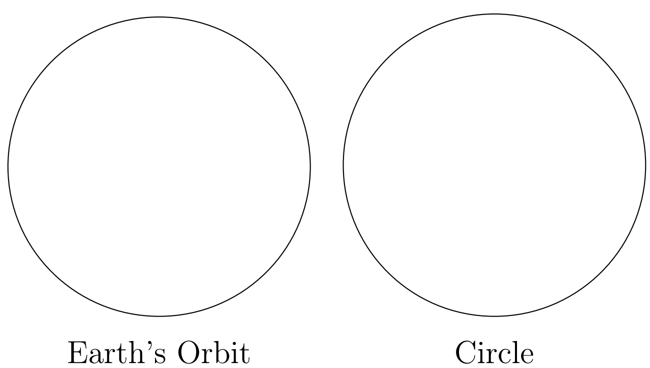


Figure 8.2: Earth's orbit shape compared to a circle.

The Eccentricity of an ellipse can be found using:

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}} \tag{8.3}$$

where a is the semimajor axis and b is the semiminor axis, as shown in the diagram:

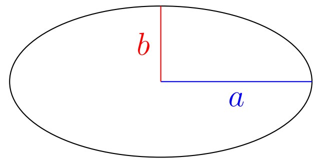


Figure 8.3: The semimajor axis (a) and semiminor axis (b) of an ellipse.

8.2.2 Kepler's Second Law

A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

This can be expressed as:

$$\frac{A_1}{t_1} = \frac{A_2}{t_2} \tag{8.4}$$

It can be shown that this expression is equivalent to:

$$r_1 v_1 = r_2 v_2 \tag{8.5}$$

Where r_1 and r_2 are the average radii for each segment of arc, and v_1 and v_2 are the average velocities, respectively.

8.2.3 Kepler's Third Law

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit. Thus, for any two planets orbiting the sun,

$$\frac{{T_1}^2}{{R_1}^3} = \frac{{T_2}^2}{{R_2}^3} \tag{8.6}$$

8.3 Newton's Law of Universal Gravitation

Newton's Law of Universal Gravitation is a way of calculating the gravitational force between any two objects with mass. It is given by:

Newton's Law of Universal Gravtation

$$F_g = \frac{G m_1 m_2}{r^2} \tag{8.7}$$

where G is the Universal Gravitational Constant:

$$G = 6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$$

The masses of the two objects are m_1 and m_2, and r is the distance between these objects.

Sometimes, you will see equation 8.7 written with a negative sign in order to make it consistent with some of the laws of electrostatics, studied in chapter 15. However, you will likely end up assigning a sign to the force, according to the coordinate system you have defined for the problem.

8.4 Orbital Motion

Whenever a object orbits another that has a much larger mass, if the orbit can be approximated by a circle, the gravitational attraction between the two bodies acts as a centripetal force. Thus, a fundamental realization of orbital mechanics is:

Chapter 9

Rotational Mechanics

9.1 Angular Velocity and Acceleration

An object that is spinning can be described using angular velocity and angular acceleration. Angular velocity is a way of expressing how much an object rotates in a given time. It could be measured in Rotations per Minute (rpms), Degrees per hour, or any other measurement of an angle divided by any measurement of time. However, it is advantageous to use Radians per Second.

Just like velocity measures how fast an object is moving in a line, angular velocity measures how fast an object is rotating. Average angular velocity is given by the following equation:

$$\vec{\omega}_{avg} = \frac{\Delta\vec{\theta}}{\Delta t} \tag{9.1}$$

and instantaneous angular velocity is given by:

$$\vec{\omega} = \frac{d\vec{\theta}}{dt} \tag{9.2}$$

9.2 Angular Kinematics

Using the definitions of angular velocity and angular acceleration and a little calculus (or a lot of algebra) we can prove the following four equations:

The Angular Kinematic Equations

$$\vec{\theta} = \frac{\overrightarrow{\omega_f} + \overrightarrow{\omega_i}}{2} t \tag{9.3}$$

$$\vec{\theta} = \overrightarrow{\omega_i} t + \frac{1}{2} \vec{\alpha} t^2 \tag{9.5}$$

$$\overrightarrow{\omega_f} = \overrightarrow{\omega_i} + \vec{\alpha} t \tag{9.4}$$

$$\overrightarrow{\omega_f}^2 = \overrightarrow{\omega_i}^2 + 2\vec{\alpha}\vec{\theta} \tag{9.6}$$

You may notice that the form of the angular kinematic equations is equivalent to the form of the original, linear kinematic equations introduced in section 2.5.2. This means that all the intuition and skills developed previously are still valid.

9.3 Moment of Inertia

The **Moment of Inertia** (sometimes called **rotational inertia**), symbolized I, of an object can be thought of as the rotational equivalent of mass. An object of lesser mass will accelerate more than an object of greater mass when the same amount of force is applied. Likewise, an object with a smaller moment of inertia will tend to have a greater angular acceleration than an object with a greater moment of inertia when the same forces are applied to the objects.

The moment of inertia of an object depends both on the mass of the object and the way in which the mass is distributed. Some common moments of inertia can be found in the attached table.

Solid Sphere	$I = \frac{2}{5}mr^2$	Hollow Sphere	$I = \frac{2}{3}mr^2$

9.4 Torque

Torque is the rotational equivalent to force. That is, it is a force that is applied to an object somewhere other than the center of mass, causing the object to rotate.

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{9.7}$$

You may note that the two vectors are multiplied as a cross product. The magnitude of the torque is given by:

$$|\vec{\tau}| = r \cdot F \cdot sin(\theta) \tag{9.8}$$

and the direction is given by the 1st Right Hand Rule (see section 1.3.3).

Example 9.4.1

Problem: A force of 10N acts tangentially to the right at the top of a wheel for radius r=0.1 m. What is the torque exerted on the wheel and in what direction?

Solution: Begin by drawing the diagram:

We can use Equation 9.8 to determine the torque:

$$|\vec{\tau}| = r \cdot F \cdot sin(\theta) = 0.1\text{m} \cdot 10\text{N} \cdot \sin(90°) = \boxed{1\text{m} \times \text{N}}$$

The direction of the torque is given by the 1st right hand rule. Your index finger points along the direction of $\vec{r}$. Your middle finger bends 90 degrees and aligns with $\vec{F}$. Your thumb shows the resulting direction of the torque: **into the page.**

9.4.1 Newton's Laws in a Rotational Setting

Newton's First Law for Rotation

Objects in uniform rotational motion will remain in uniform rotational motion and objects at rotational rest will remain at rotational rest until acted upon by an external, unbalanced torque.

Newton's Second Law for Rotation

$$\vec{\tau} = I \cdot \vec{\alpha} \tag{9.9}$$

Newton's Third Law for Rotation

For every torque, there is an equal opposite torque.

9.5 Angular Kinetic Energy

Angular Kinetic Energy, sometimes called rotational kinetic energy, is energy of rotational motion. An object that is only rotating - like a fan - has angular kinetic energy only. An object that is rolling - like the wheel of a car - has both translational kinetic energy (found using $k = \frac{1}{2}mv^2$) and angular kinetic energy, found using:

$$K = \frac{1}{2}I\omega^2 \tag{9.10}$$

9.6 Angular Momentum

9.6.1 The Definition of Angular Momentum

Angular momentum can be calculated using the formula:

$$\vec{L} = I\vec{\omega} \tag{9.11}$$

where $\vec{L}$ is angular momentum, I is the object's moment of Inertia and $\vec{\omega}$ is the object's angular velocity. The SI units for angular momentum are $\frac{kgm^2}{s}$.

Example 9.6.1

Problem: A bicycle wheel has a mass of 0.3 kg, and can be thought of as a thin ring with a radius of 0.33m. When the wheel is turning at a rate of 2 rotations per second, what its its angular momentum?

Solution: Begin by converting the angular velocity ω to appropriate units:

$$\vec{\omega} = 2\frac{rotations}{s} = 4\pi\frac{rad}{s}$$

Then calculate the moment of inertia. Using the formula for a thin ring:

$$I = mr^2 = 0.3kg(0.33m)^2 \approx 0.033kgm^2$$

Finally, use equation 9.11 to find the angular momentum.

$$\vec{L} = I\vec{\omega} = 0.011kgm^2 \cdot 4\pi\frac{rad}{s} \approx \boxed{0.411\frac{kgm^2}{s}}$$

9.6.2 Conservation of Angular Momentum

Just like linear momentum[1], angular momentum is a quantity that is conserved. Thus, whatever angular momentum a closed system has in its initial state will be equal to the angular momentum the system has in its final state.

The classic example of the Law of Conservation of Momentum is an ice skater who enters a spin. By changing the positioning of his or her arms and legs, an ice skater can change their moment of inertia. When they bring their arms and legs closer to their axis of rotation, their moment of inertia decreases. Since angular momentum is conserved, their angular velocity must increase as their moment of inertia decreases, and thus the ice skater is able to spin very fast.

[1] see momentum in section 9.6

Chapter 10

Waves

10.1 Fundamentals of Waves

You have probably heard of waves in the context of the ocean, a lake, or other bodies of liquid. Waves are also found in earthquakes, sound, light, and even at the stadium when people do "the wave." A wave is a distortion that transfers energy from one place to another without the permanent transfer of mass.

The material that a wave travels through is called a medium. For instance, the medium for an ocean wave is water, while the medium for light could be air, glass, water, or even empty space (no medium).

10.1.1 Types of Waves

Waves can be categorized into several basic types:

- Transverse Wave - are waves that are displaced perpendicular to the direction of travel. For instance, ocean waves are a type of transverse wave because their displacement is vertical, though they travel horizontally. Normally, a transverse wave is drawn similar to the figure below:

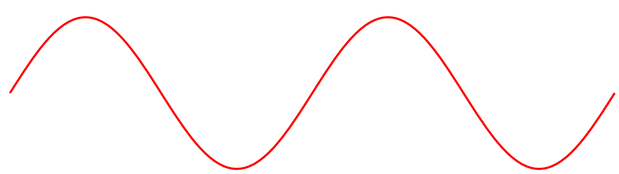


Figure 10.1: A simple transverse wave

- Longitudinal Wave - are waves that are displaced in the same direction as the direction of travel. You can think of this as a compression, or shock wave that travels through a medium. The places where a material is closer together than normal are called compressions, while places that are spaced farther apart are called rarefractions, as seen in figure 10.2

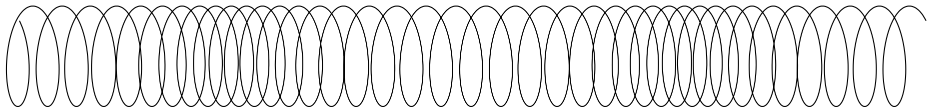


Figure 10.2: A simple longitudinal wave

- Electromagnetic Wave - are waves that are made up of oscillating electric and magnetic fields. These are usually modeled as transverse waves, but they are just representations of the strength of the electric and magnetic fields.

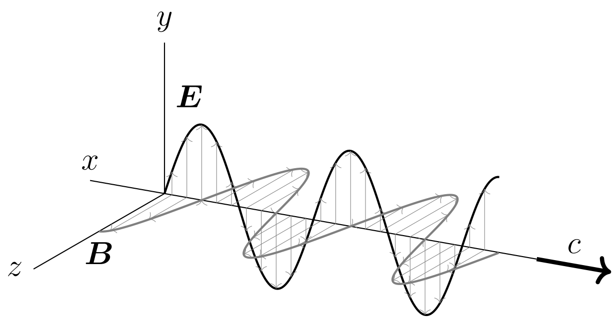


Figure 10.3: An electromagnetic wave

Electromagnetic waves are what make up the electromagnetic spectrum. Radio waves, microwaves, infrared, visible light, ultraviolet, X-rays and γ-rays are all electromagnetic waves. They are categorized into different types based on their frequency.

There are other types of waves, such as matter waves and gravitational waves that are beyond the scope of this text.

10.1.2 Basic Wave Characteristics and Vocabulary

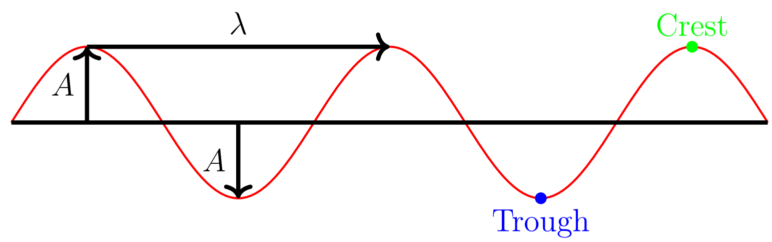


Figure 10.4: The measurements of a wave

Extrema

Crests are each of the highest points of a wave, and **troughs** (rhymes with coughs) are the lowest points of a wave, as seen in the diagram 10.4. The word **extrema** refers to all crests and troughs, as they are the most extreme points of the wave.

Amplitude

Amplitude measures how large or how strong a wave is. It is measured from the center of a wave to one of the extrema - either upward to a crest, or downward to a trough. For a physical, transverse wave, amplitude can be measured in meters. The variable for amplitude is A. For sound, we experience the amplitude of the wave as *volume*, and for light, we experience it as *brightness*.

Wavelength

The **wavelength** of a wave is measured from any point to an identical point on the wave, along the axis of propagation. Wavelength is measured in meters, and the symbol for wavelength is λ (lambda). The easiest way to measure wavelength is from crest to crest or from trough to trough.

Period

The **period** of a wave measures how long it takes a wave to repeat itself. It is measured in seconds, and uses the variable T.

Frequency

The **frequency** of a wave measures how many times a wave repeats itself in one second. The symbol for frequency is f and the units for frequency are $\frac{cycles}{second}$. We give this unit the name *hertz*, abbreviated Hz. For sound, we experience frequency as *pitch*, and for light we experience frequency as *color*.

The frequency of a wave and the period of a wave are inverses of each other. Thus:

$$f = \frac{1}{T} \tag{10.1}$$

Example 10.1.1

Problem: A wave has a frequency of 102.1 MHz. What is the period of the wave?

Solution:
We know that $f = 102.1\,\mathrm{MHz}$ Converting into scientific notation gives:

$$f = 102.1\,\mathrm{MHz} = 102.1 \times 10^6\,\mathrm{Hz} = \boxed{1.021 \times 10^8\,\mathrm{Hz}}$$

Begin by using equation 10.1:

$$f = \frac{1}{T}$$

Solving for T yields:

$$T = \frac{1}{f}$$

Then, substitute numbers:

$$T = \frac{1}{1.021 \times 10^8\,\mathrm{Hz}} \approx \boxed{9.794 \times 10^{-9}\,\mathrm{s}}$$

10.1.3 Velocity of a Wave

We already know from equation 2.1 that the average velocity of any object is given by $\overrightarrow{v_{avg}} = \frac{\vec{d}}{\Delta t}$. In the case of a wave, the time it takes the wave to repeat is the period, T, and distance the wave must move in order to repeat is one wavelength, λ. Thus, $\overrightarrow{v_{avg}} = \frac{\vec{d}}{\Delta t} = \frac{\vec{\lambda}}{T}$. Using equation 10.1, it can be proven that:

$$v = f\lambda \tag{10.2}$$

Example 10.1.2

Problem: An ocean wave has a period of 15 seconds, and is traveling at a speed of 2 meters per second. How far is it from one crest of the wave to another?

Solution: The question asks for the distance from one crest to another, which is the wavelength. Given values:

$$T = 15\,\mathrm{s}$$
$$v = 2\,\mathrm{m/s}$$

First, we use equation 10.1 to find the frequency:

$$f = \frac{1}{T} = \frac{1}{15\,\mathrm{s}} \approx 0.067\,\mathrm{s}$$

We now use equation 10.2:

$$v = f\lambda$$

Solving for wavelength gives:

$$\frac{v}{f} = \lambda$$

Substituting gives:

$$\lambda = \frac{v}{f} \approx \frac{2\,\mathrm{m/s}}{0.067\,\mathrm{s}} \approx \boxed{30\,\mathrm{m}}$$

10.2 The Doppler Effect

We have all experienced a car driving past us while it is honking its horn. As the car drives past, there is a significant change in the pitch of the horn. In fact, when either the source of a wave or an observer of a wave is moving, it causes the observer's perception of the frequency of that wave to change. This is called the **Doppler Effect**.

When the source of a wave moves *toward* an observer, its frequency is shifted higher.

Figure 10.5: Observers hear a higher pitch

When the source of a wave moves *away* from an observer, its frequency is shifted lower.

Figure 10.6: Observers hear a lower pitch

Likewise, when an observer moves *toward* the source of a sound, its frequency is shifted higher.

Figure 10.7: Observer hears a higher pitch

When an observer moves *away* from a source of sound, its frequency is shifted lower.

Figure 10.8: Observer hears a lower pitch

The equation for the Doppler effect is given by equation 10.3:

$$f_{observed} = f_{source}\left(\frac{v_{wave} \pm v_{observer}}{v_{wave} \pm v_{source}}\right) \tag{10.3}$$

In this equation, do not consider the $\pm$ sign to represent both operations. Instead, you must chose which operation to use based on the given situation. In the numerator of the equation it is exactly what is expected: add for a higher frequency and subtract for a lower frequency. In the denominator, it is backward - add for lower frequency, and subtract for higher frequency.

In the air, the speed of sound depends on air pressure, temperature, and even humidity. The standard value for speed of sound in air is **343 m/s**, though in reality this number can change quite significantly depending on atmospheric conditions.

Example 10.2.1

Problem: A car's horn has a pitch of 550 Hz. It is driving at 15 m/s toward a stationary observer.

a. What is the frequency that the observer hears at the car approaches.

b. The car then passes the observer. What is the frequency that the observer hears as the car travels away from him?

Solution: A car's horn produces sound, therefore the velocity of the wave is the speed of sound.

Part a: Given values:

$$f_{source} = 550\,\text{Hz}$$
$$v = 343\,\text{m/s}$$
$$v_{source} = 15\,\text{m/s}$$
$$v_{observer} = 0\,\text{m/s}$$

We can use equation 10.3 to find the frequency the observer hears:

$$f_{observed} = f_{source}\left(\frac{v_{wave} \pm v_{observer}}{v_{wave} \pm v_{source}}\right)$$

In this case, the observer is not moving, so it does not matter whether we chose a plus or minus. The source is moving toward the observer, causing the observer to hear a higher pitch. Therefore, since the source is in the denominator, we chose a minus. Substituting numbers and chosing correct signs gives:

$$f_{observed} = 550\,\text{Hz}\left(\frac{343\,\text{m/s} + 0\,\text{m/s}}{343\,\text{m/s} - 15\,\text{m/s}}\right)$$

Evaluating this expression gives:

$$\boxed{f_{observed} \approx 575.152\,\text{Hz}}$$

Part b: After the car has passed the observer, it is now traveling away from the observer. Thus, the only change that needs to be made is that v_{source} should now have a plus sign:

$$f_{observed} = 550\,\text{Hz}\left(\frac{343\,\text{m/s} + 0\,\text{m/s}}{343\,\text{m/s} + 15\,\text{m/s}}\right)$$

Evaluating this expression yields:

$$\boxed{f_{observed} \approx 526.955\,\text{Hz}}$$

10.3 The Principle of Superposition and Interference

10.3.1 The Principle of Superposition

The **Principle of Superposition** is the idea that waves can overlap. Consider, for instance, a swimming pool where a light breeze creates ripples in the water as shown below:

Figure 10.9: Ripples on a swimming pool

You could also imagine in the same swimming pool on a calm day, a person could jump into the water creating waves that look similar to the ones below:

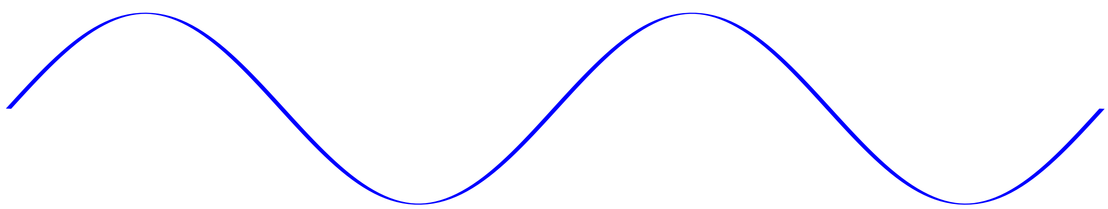


Figure 10.10: Large waves in a swimming pool

Thus, we can use the principle of superposition to predict what the wave will look like should a person jump into the swimming pool on a day when there are ripples in the water. By combining the two types of waves, we would see something like the figure below:

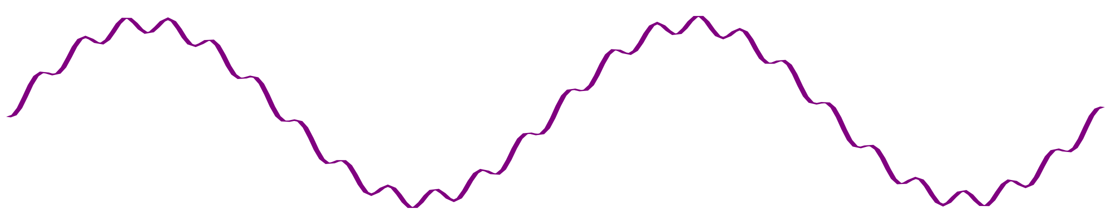


Figure 10.11: Ripples and waves combine

10.3.2 Interference

Constructive Interference

Sometimes, waves of approximately the same amplitude may overlap, causing the amplitude of the resulting wave to become larger. This is called **Constructive Interference**:

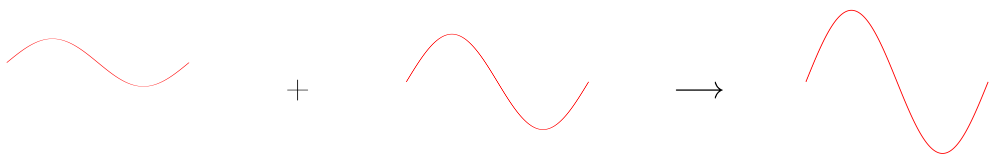


Figure 10.12: Constructive interference

In order for constructive interference to happen, the two waves must be *in phase* - that is, the displacement of the waves must be in the same direction.

When two waves that are continuous overlap, they will interfere constructively if they align perfectly:

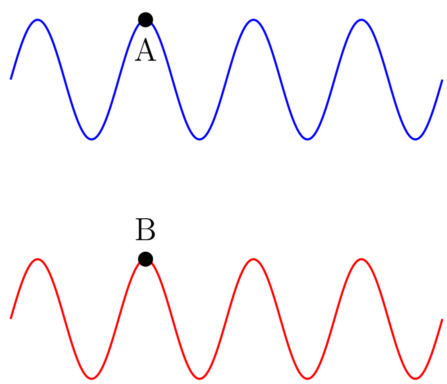


Figure 10.13: Two waves aligned perfectly

Likewise, constructive interference will occur if the wave is shifted by a distance of one wavelength.

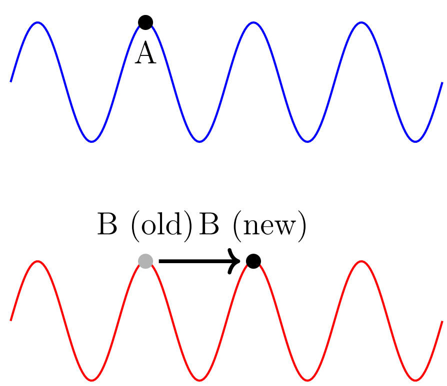


Figure 10.14: The red wave has been shifted one wavelength to the right.

The wave could be shifted in a similar manner by two, three, or any integer number of wavelengths right or left, and the two waves will still cause constructive interference. Thus, the amount of shift between the waves, $\Delta\ell$, is given by:

$$\Delta\ell = m\lambda \tag{10.4}$$

where $\Delta\ell$ is how far the waves are shifted, and m is any integer.

Destructive Interference

At other times, waves may overlap while they are **out of phase** - that is, their displacement is in opposite directions, causing the wave to become smaller. This is called **destructive interference**.

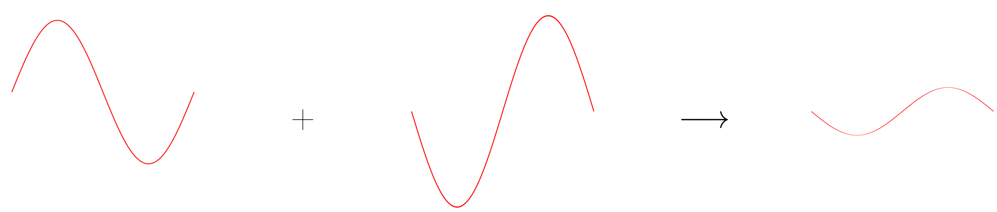


Figure 10.15: Destructive Interference

If two waves are displaced by the same amount in opposite directions, they can even cancel out completely. This would be **perfectly destructive interference**:

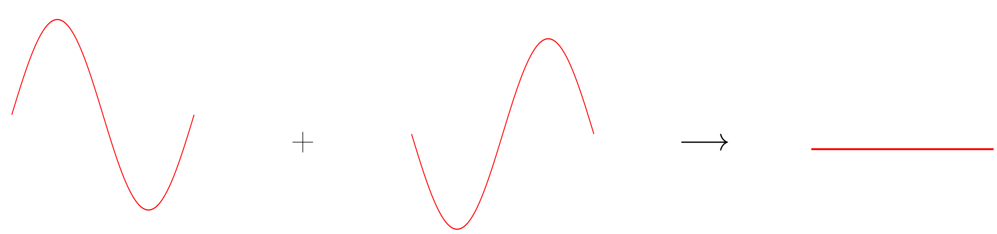


Figure 10.16: Perfectly Destructive Interference

In order for destructive interference to happen, the two waves must be perfectly *out of phase* - that is, the displacement of the waves must be in opposite direction, meaning the waves are already shifted by a half wavelength:

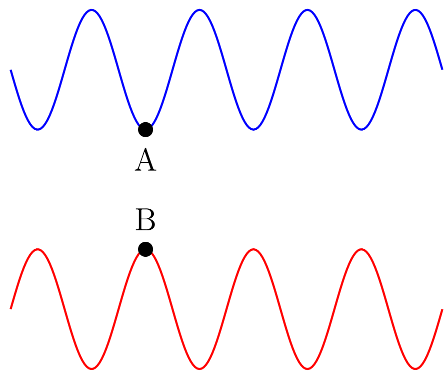


Figure 10.17: Two waves aligned perfectly

Likewise, destructive interference will occur if the wave is shifted by a distance of one wavelength (after already being off by half a wavelength).

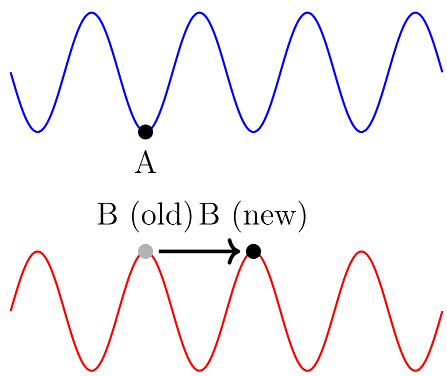


Figure 10.18: The red wave has been shifted one wavelength to the right.

Since destructive interference occurs whenever the waves are shifted by half-integer multiples of the wavelength (ie, 0.5, 1.5, 2.5, etc), the amount of shift between the waves, $\Delta\ell$, is given by:

$$\Delta\ell = (m + \frac{1}{2})\lambda \tag{10.5}$$

where $\Delta\ell$ is how far the waves are shifted, and m is any integer.

Example 10.3.1

Problem: Two trombone players stand in a single file line, with their conductor directly in front of them, as shown in the diagram:

Both trombones play a low-$B\flat$ ($f = 116.54\,\mathrm{Hz}$). What is the smallest, non-zero distance that the players should stand apart in order for the conductor to hear the loudest possible sound? (Assume both players are perfectly in tune and the waves they create are perfectly in phase.)

Solution: We already know:

$$f = 116.54\,\mathrm{Hz}$$
$$v = 343\,\mathrm{m/s}$$

Thus, we can use equation 10.2 to find the wavelength:

$$v = f\lambda \longrightarrow \lambda = \frac{v}{f} = \frac{343\,\mathrm{m/s}}{116.54\,\mathrm{Hz}} \approx 2.943\,\mathrm{m}$$

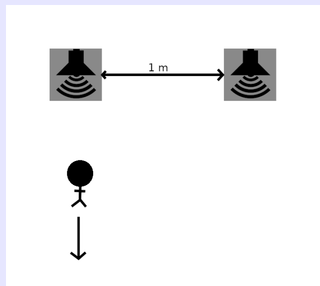
Since we are asked for the loudest sound, we know that this is constructive interference. We know that if $m = 0$, the distance between the two players will be zero. Thus, the smallest non-zero distance is given when $m = 1$. Using equation 10.4, we find:

$$\Delta\ell = m\lambda = (1)(2.943\,\mathrm{m}) = 2.943\,\mathrm{m}$$

Example 10.3.2

Problem: Two speakers are aligned along an east-west line, and are placed 1 meter apart. A single frequency is played on both speakers. A person starts in front of the left-most speaker, and begins to walk to the south. When the person has walked 2 meters to the south, she hears the sound get significantly quieter. As she continues to walk, he hears the sound get louder again. What is the frequency of the sound?

Solution: Begin by drawing a diagram of the situation:



We begin by calculating the difference in distances between the speakers. The distance to the west speaker is 2 meters. The distance to the right speaker can be found using the Pythagorean Theorem:

$$c = \sqrt{a^2 + b^2} = \sqrt{(2\,\mathrm{m})^2 + (1\,\mathrm{m})^2} = \sqrt{5\,\mathrm{m}^2} \approx 2.236\,\mathrm{m}$$

Therefore, $\Delta\ell$, the distance between the two paths the sound is traveling is given by:

$$\Delta\ell = 2.236\,\mathrm{m} - 2\,\mathrm{m} \approx 0.236\,\mathrm{m}$$

Since this problem states that sound gets quieter, we know that this is destructive interference, and we can use equation 10.5:

$$\Delta\ell = (m + \frac{1}{2})\lambda$$

We know that the smallest value m can be is 0, so our equation reduces to:

$$\Delta\ell = (\cancelto{0}{m} + \frac{1}{2})\lambda \longrightarrow \Delta\ell = \frac{\lambda}{2}$$

Solving for λ gives:

$$\lambda = 2\Delta\ell = (2)(0.236\,\mathrm{m}) = 0.472\,\mathrm{m}$$

Knowing that the speed of sound is 343 m/s, we can use equation 10.2 to determine the frequency:

$$v = f\lambda \longrightarrow f = \frac{v}{\lambda} = \frac{343\,\mathrm{m/s}}{0.472\,\mathrm{m}} = \boxed{726.540\,\mathrm{Hz}}$$

10.4 Resonance

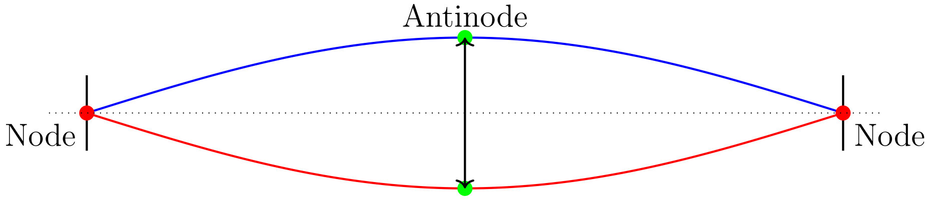
Resonance is a process in which waves reinforce each other through constructive interference, resulting in a wave with much greater amplitude than the original wave. Musical instruments, lasers, microwave ovens, and radio antennas are a few examples of devices that make use of resonance.

10.4.1 Resonance of a String that is Fixed on Both Ends

Figure 10.19: Stringed instruments like the Ukelele make use of resonance.

Stringed instruments make music by resonating at specific frequencies. A table of musical notes and their frequencies can be found in table B.2 on page 137.

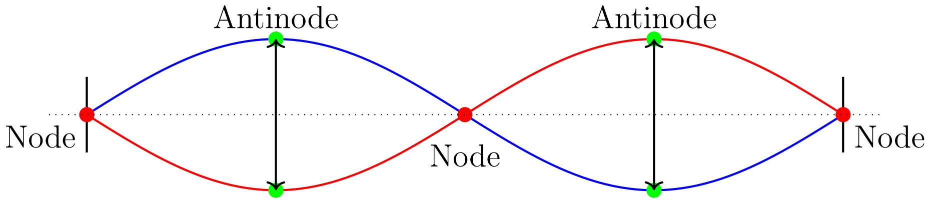
The strings on string instruments, such as the Ukelele in figure 10.19, are typically fixed at both ends. One side is held in place by the bridge, while the other side is held in place either at the neck of the instrument, or by a finger placed somewhere along the string. When energy is introduced into the string (typically by plucking or the use of a bow), the string typically creates a standing wave that looks like this:



This mode of vibration has only one antinode. Because the string is held in place at the ends, the ends are nodes. It is called the Fundamental mode, the first harmonic, or the n=1 mode. In this mode, the length of the string, L must be equal to a half wavelength. Therefore,

$$L = \frac{\lambda}{2} \tag{10.6}$$

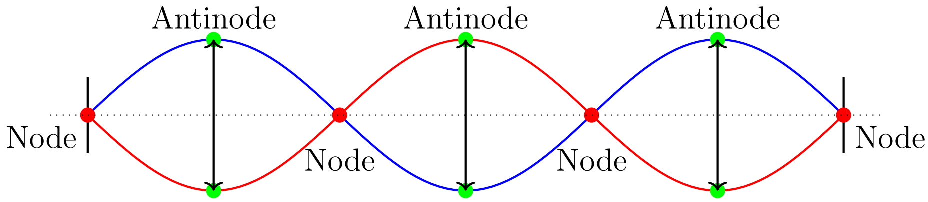
It is possible to make this wave resonate differently. Many string players know how to create "harmonics" by lightly placing their finger on the string in specific places. A string vibrating in the second harmonic, or the n=2 mode would look like this:



In this case, we can see that the length of the string must be a full wavelength now:

$$L = \lambda \tag{10.7}$$

Making the string vibrate in the third harmonic, or the n=3 mode, would look like this:



Here we can see that there is 3/2 of a wave present. Therefore,

$$L = \frac{3\lambda}{2} \tag{10.8}$$

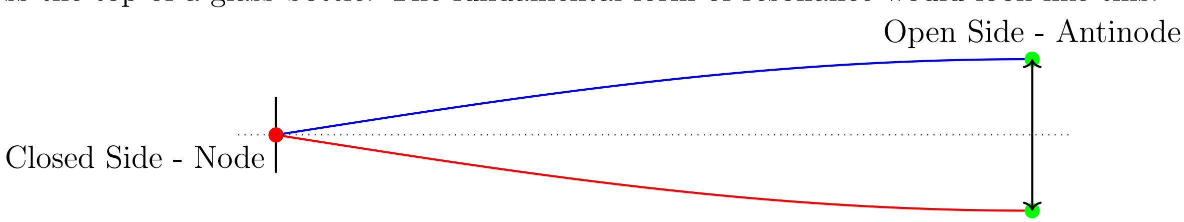
From the previous examples, we can generalize the relationship between string length and wavelength. For strings fixed on both ends,

$$L = \frac{n\lambda}{2} \tag{10.9}$$

Where n = any positive integer (1,2,3,...)

10.4.2 Resonance of a Tube that is Open on One End and Closed on the Other

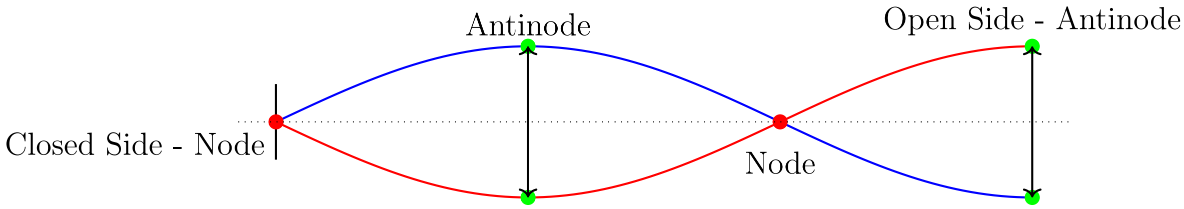
Resonance can also be achieved from tubes that are closed or fixed on one end and open on the other. Examples of this type of resonance would include flutes, pan-flutes, and blowing across the top of a glass bottle. The fundamental form of resonance would look like this:



From this picture, it can be seen that there is 1/4 wavelength present. So,

$$L = \frac{\lambda}{4} \tag{10.10}$$

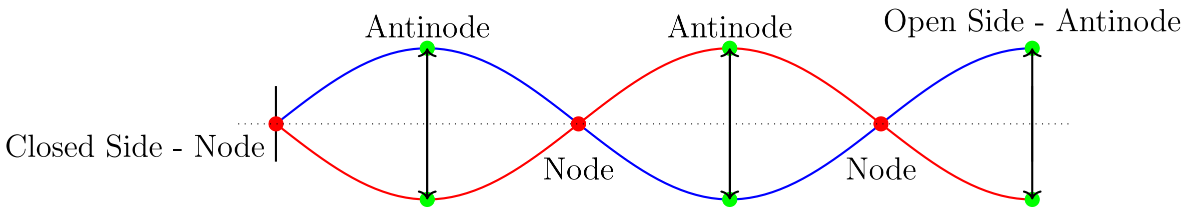
The next mode of resonance would look like this:



We can see that in this mode of resonance, $\frac{3}{4}$ of a wavelength is visible. So,

$$L = \frac{3\lambda}{4} \tag{10.11}$$

The next form of resonance would look like this:



Which results in $\frac{5}{4}$ of the wave.

$$L = \frac{5\lambda}{4} \tag{10.12}$$

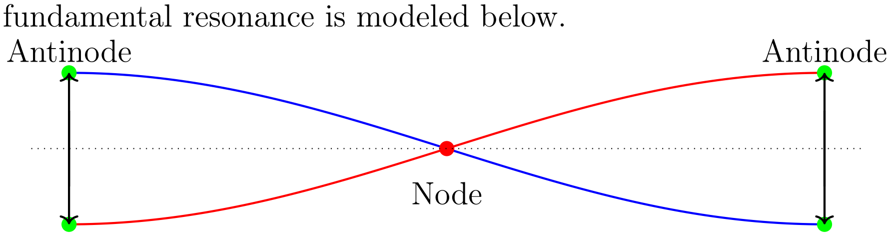
Thus, from these three expressions, we can generalized the following equation for any more of resonance for a tube open on one end and closed on the other.

$$L = \frac{n\lambda}{4} \tag{10.13}$$

Where n is odd (1,3,5,...)

10.4.3 Resonance of a Tube that is Open on Both Ends

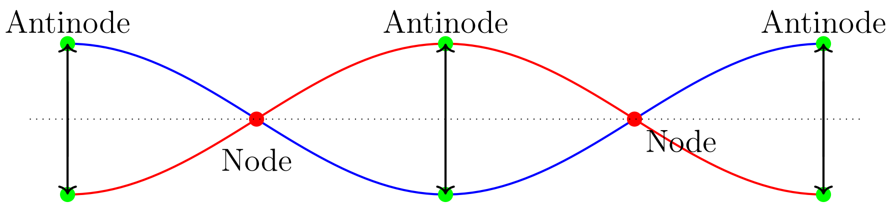
Objects can also resonate when they are open on both ends. For instance, many brass and woodwind instruments have openings on both sides. For a tube that is open on both ends, the fundamental resonance is modeled below.



We can see that one-half wavelength is present in this mode of resonance. This means:

$$L = \frac{\lambda}{2} \tag{10.14}$$

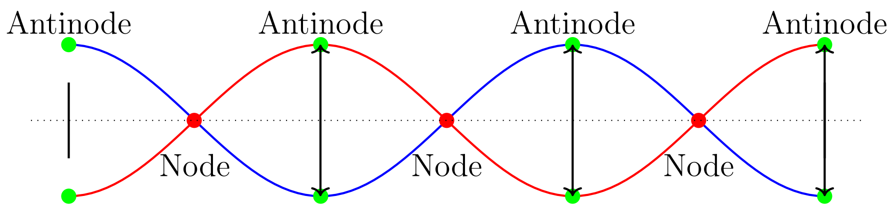
The second mode would look like this:



In this case, we can see that the length of the string must be a full wavelength now:

$$L = \lambda \tag{10.15}$$

Making the string vibrate in the third harmonic, or the n=3 mode, would look like this:



Here we can see that there is 3/2 of a wave present. Therefore,

$$L = \frac{3\lambda}{2} \tag{10.16}$$

Generalizing these allows us to create the following formula:

$$L = \frac{n\lambda}{2} \tag{10.17}$$

Where n = any positive integer (1,2,3,...)

You may notice that is result is the same as for a string that is fixed on both ends.

Chapter 11

Optics

Optics is the study of light and how that light interacts with matter. There are two general fields of Optics: *Geometric Optics* and *Physical Optics*. Geometric optics studies how rays of light travel, while physical optics studies how multiple rays of light interact as waves.

An important constant in the field of optics is the speed of light. The speed of light in empty space is exactly:

$$c = 299\,792\,458\,\text{m/s}$$

The symbol used is c because the speed of light is constant in all frames of reference. This fact will be discussed in more detail in chapter 21 You will often see this number rounded to 2.998×10^8 m/s or even 3.0×10^8m/s.

11.1 Geometric Optics

While light in empty space always travels at the same speed, c, light can be slowed down when it travels through a medium. This leads to some phenomena that we may encounter in everyday life.

11.1.1 Refraction

Refraction is a phenomenon associated with how light changes direction as it moves from one medium to another, due to the change in the speed that light travels at in each of the media. This is often demonstrated by looking at a pencil in a glass of water, or a fish in a pond.

The Index of Refraction

The index of refraction is the ratio of the speed of light in a vacuum to the speed of light in a material. It can be calculated as follows:

$$n \equiv \frac{c}{v_m} \tag{11.1}$$

where n is the index of refraction, c is the speed of light in empty space, and v_m is the speed of light in the material.

Example 11.1.1

Problem: Light travels at a speed of 2.254×10^8 m/s in water. What is the index of refraction of water?

Solution: Using the definition of index of refraction, we find:

$$n \equiv \frac{c}{v_m} = \frac{2.997 \times 10^8\,\text{m/s}}{2.254 \times 10^8\,\text{m/s}} = 1.330 \tag{11.2}$$

You may notice that the index of refraction is unitless, since all units cancel in the calculation. A list of indices of refraction can be found in Appendix B.5 on page 138.

Snell's Law

Snell's Law, named for the Dutch physicist Willebrord Snell, explains that light always takes the path of least time between two points. When light travels in a single medium of constant optical density, it travels in a straight line. However, when light changes medium, it will change direction.

There are several components and measurements that should be included on a diagram in this situation.

- The *interface* is the boundary where the two materials meet.
- The *normal* is an imaginary line perpendicular to the surface.
- The *incident ray* is the ray of light that is traveling toward the interface.
- The *refracted ray* is the ray of light traveling away from the interface.
- The *incident angle*, θ_i is the angle between the incident ray and the normal.
- The *refracted angle* θ_r is the angle between the refracted ray and the normal.

A diagram that shows a ray of light traveling from air into water might look like this:

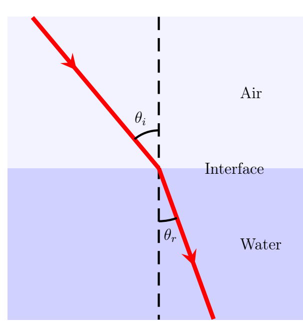


Figure 11.1: A diagram of light traveling from air into water.

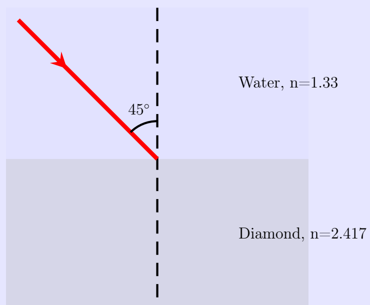
In a diagram such as above, it can be seen that the path of least time is given by the following mathematical representation of Snell's Law:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r) \tag{11.3}$$

Example 11.1.2

Problem: Light travels from water (n=1.33) into diamond (n=2.417). If the angle of incidence is 45°, what is the refracted angle?

Solution: Begin by drawing a (partial) diagram:



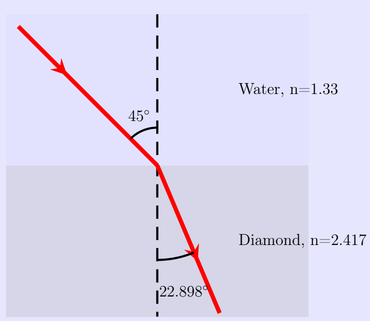
Snell's law states:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Solving this for θ_r yields:

$$\theta_r = \sin^{-1}\left(\frac{n_i \sin(\theta_i)}{n_r}\right) = \sin^{-1}\left(\frac{1.33 \sin(45°)}{2.417}\right) \approx \boxed{22.898°} \tag{11.4}$$

The completed diagram would look like this:



Total Internal Reflection In the specific case that light is traveling from a material with a higher index of refraction into a material with a lower index of refraction, the refracted angle will be larger than the incident angle. In this case, it is possible that the refracted angle could refract at exactly 90°. The incident angle that causes a ray to be refracted at 90° is called the *critical angle*. If the angle of the incident ray exceeds the critical angle, the ray does not refract out of the material. Instead, it reflects back into the material.

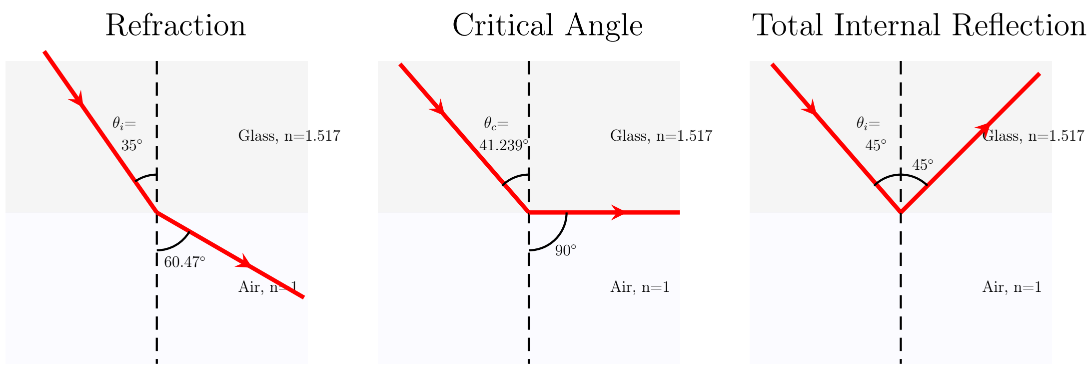


Figure 11.2: Total internal reflection occurs when the critical angle is exceeded.

Example 11.1.3

Problem: Light travels from glass (n=1.517) into water (n=1.33). Calculate the critical angle, and draw a diagram of the situation.

Solution: We know that the refracted ray must travel at a 90° angle to the normal, so $\theta_r = 90°$.

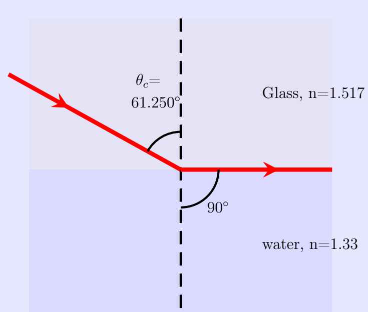
Snell's Law states:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Solving for θ_i gives:

$$\theta_i = \sin^{-1}(\frac{n_r \sin(\theta_r)}{n_i}) = \sin^{-1}(\frac{1.33 \sin(90)}{1.517}) \approx \boxed{61.250°} \tag{11.5}$$

The corresponding diagram should look like the following:



Lenses

There are two basic types of lenses that are discussed in this text:

- *Convex* lenses, sometimes called *converging* lenses are larger in the center than at the edges. These lenses are often used as magnifying glasses.

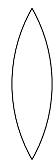


Figure 11.3: A Simple diagram representation of a convex lens

- *Concave* lenses, sometimes called *diverging* lenses are larger at the edges than the center. They are found in lenses for near-sightedness.

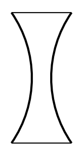


Figure 11.4: A Simple diagram representation of a concave lens

Convex Lenses When parallel rays of light strike a convex lens, the light can be focused into a very small area. The point at which the light is focused is called the *focal point*

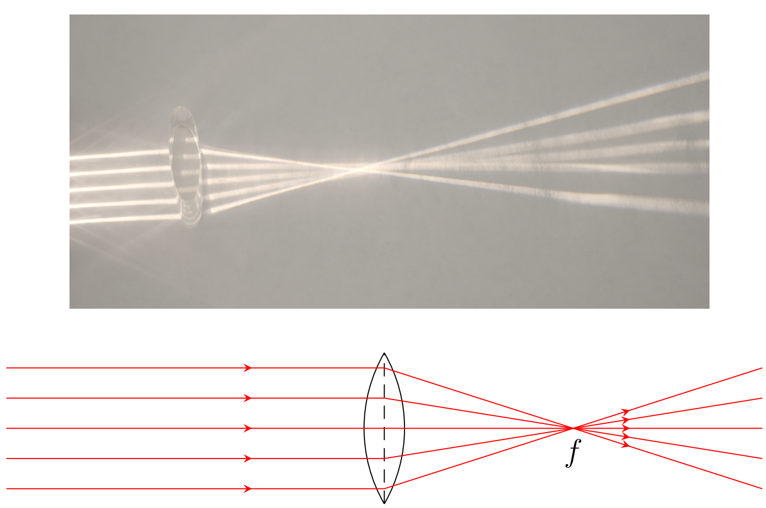


Figure 11.5: Light focused by a convex lens. The focal point is labeled f.

The distance from the center of the lens to the focal point is called the *Focal Length.* Because light call pass through the lens either way, lenses have two focal points, both equidistant from the center of the lens.

Image Formation with Convex Lenses When a lens interacts with an object, an image is normally formed. There are two often-used methods for determining where images form: *Ray Tracing* and use of *the lens equation.*

The method of ray tracing is performed in the following example:

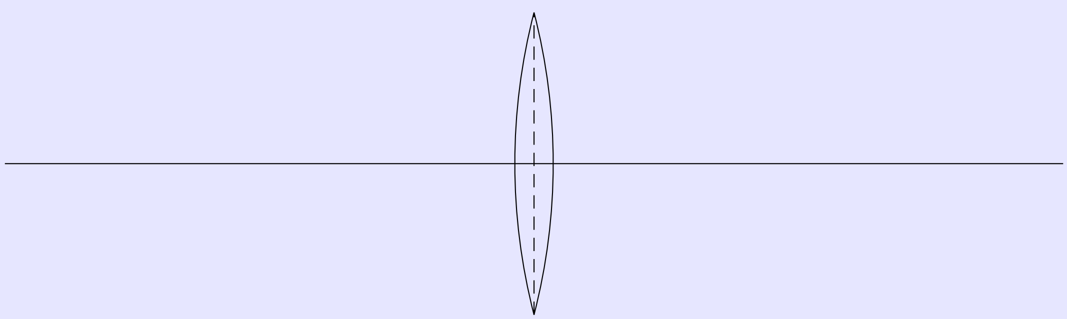
Example 11.1.

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = 2$cm. Use ray-tracing to determine the position, size, and orientation of the image.

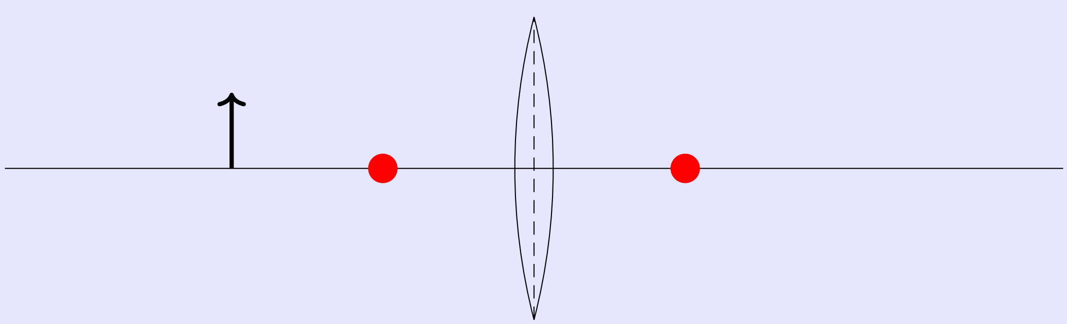
Solution:

1. Begin by drawing a horizontal line across your paper. This is your optical axis:

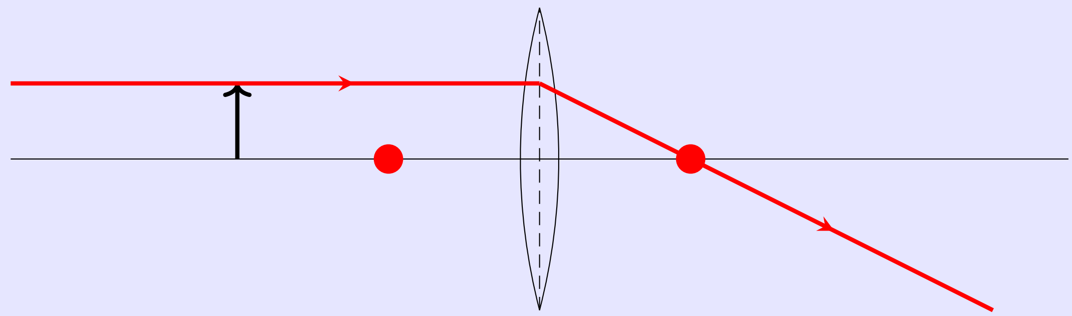
2. Draw a lens on the optical axis. Use a dotted line to represent the center of the lens.



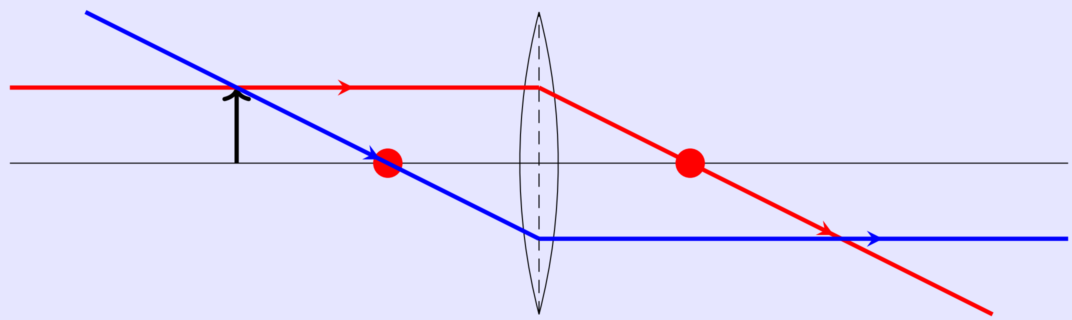
3. Measure and label focal points from the center of the lens, along the optical axis, and draw the object as an arrow with its base on the optical axis.



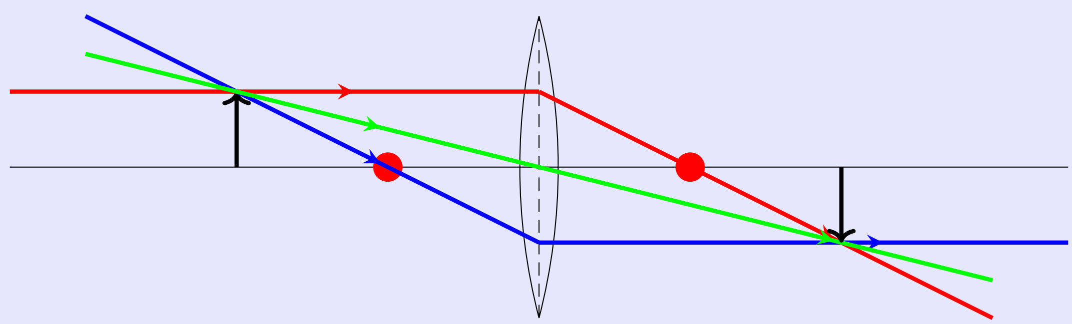
4. The first ray of light will begin on the left of the diagram, graze the top of the object, and continue to the center of the lens. It will then be directed down, through the far focal point.



5. The second ray of light will graze the top of the object, pass through the near focal point, continuing until it hits the center of the lens. There, the ray will be directed parallel to the optical axis.



6. The final ray of light will graze the top of the object, directed toward the intersection of center of the lens and the optical axis. This ray will not change direction. The intersection of the three rays is where the top of the image will form.



The diagram can then be measured. In this case, the image is 4 cm to the right of the lens, inverted, and the same size as the object.

The *lens equation* can also be used to determine the distance to the image. The lens equation is:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{11.6}$$

where f is the focal length of the lens, o is the object distance from the lens, and i is the image distance from the lens.

The magnification of an image can be determined by the following equation:

$$m = -\frac{i}{o} = \frac{h_i}{h_o} \tag{11.7}$$

where h_i is the image height, and h_o is the object height.

Example 11.1.5

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = 2\text{cm}$. Use the lens equation and the magnification formula to determine the position, size, and orientation of the image.

Solution: In this case, we are given $h_o = 1\text{cm}$, $o = 4\text{cm}$ and $f = 2\text{cm}$. First, the lens equation is solved for i:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$i = \frac{1}{\frac{1}{f} - \frac{1}{o}} = \frac{1}{\frac{1}{2\text{cm}} - \frac{1}{4\text{cm}}} = \boxed{4\text{cm}}$$

Calculating the magnification gives:

$$m = -\frac{i}{o} = -\frac{4\text{cm}}{4\text{cm}} = \boxed{-1}$$

The negative magnification means that the image is inverted. You may also notice that magnification is a unitless quantity. Finally, calculating the height of the image shows that the image is the same size (thought inverted).

$$m = \frac{h_i}{h_o} \longrightarrow h_i = mh_o = (-1)(1\text{cm}) = \boxed{-1\text{cm}}$$

Concave Lenses Instead of being focused to a point, parallel rays of light that are incident upon a concave lens diverge, as shown below:

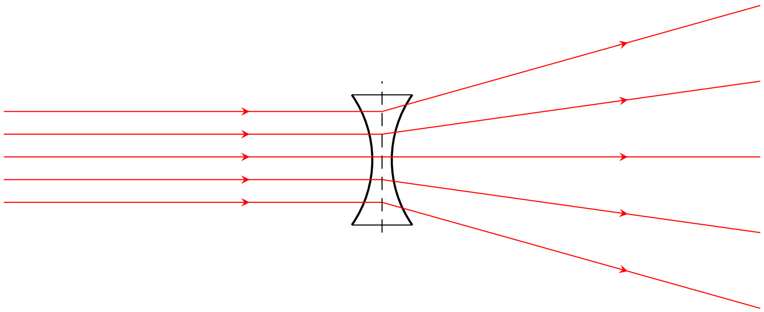


Figure 11.6: A concave lens causes light rays to diverge.

Though the rays do not converge on the right side of the lens in figure 11.6, the rays can be extended backward to show an apparent focal point on the wrong side of the lens:

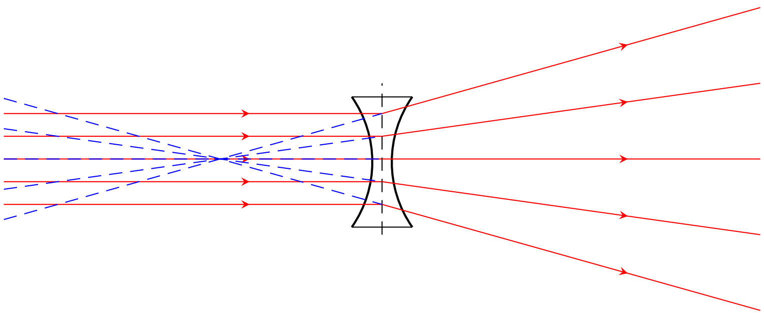


Figure 11.7: A concave lens appears to have a focal point on the other side of the lens.

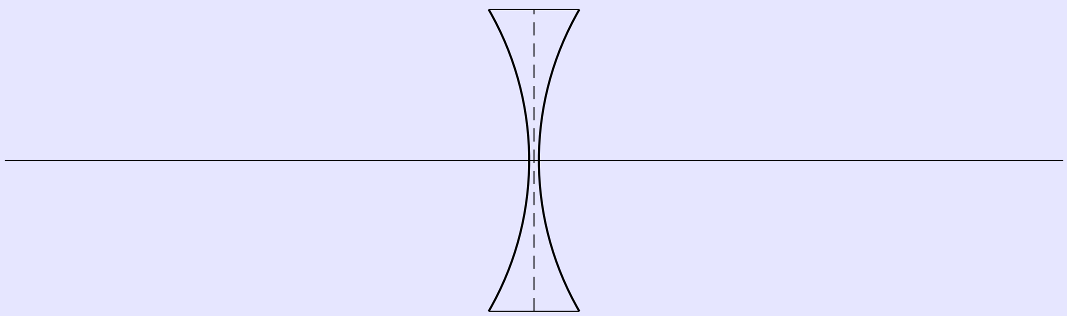
Because concave lenses appear to have a focal point on the wrong side of the lens, their focal length is negative.

Example 11.1.6

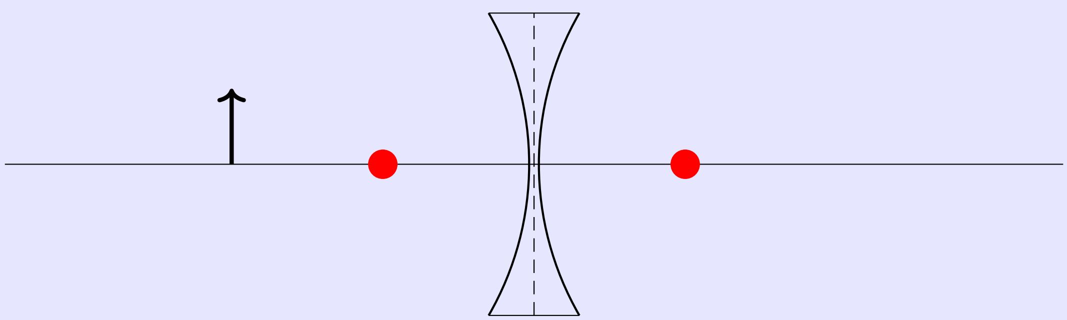
Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = -2$cm. Use ray-tracing to determine the position, size, and orientation of the image.

Solution:

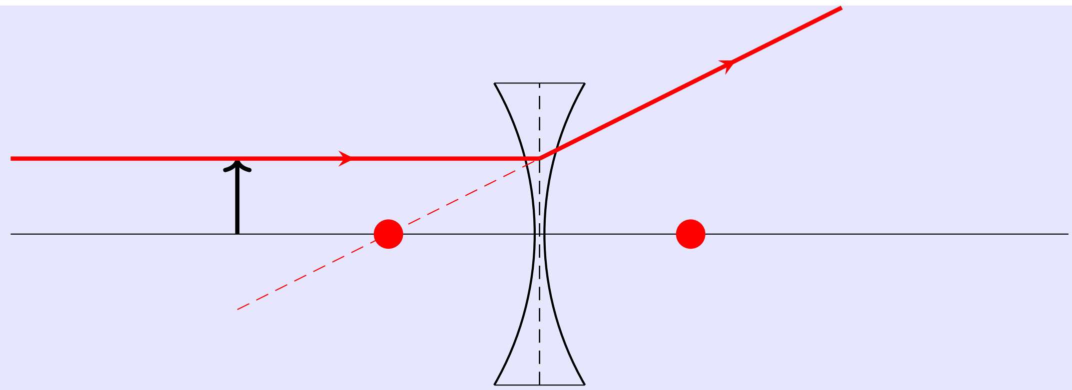
1. Begin by drawing the optical axis and lens. Note that since the focal length is negative, this is a concave lens.



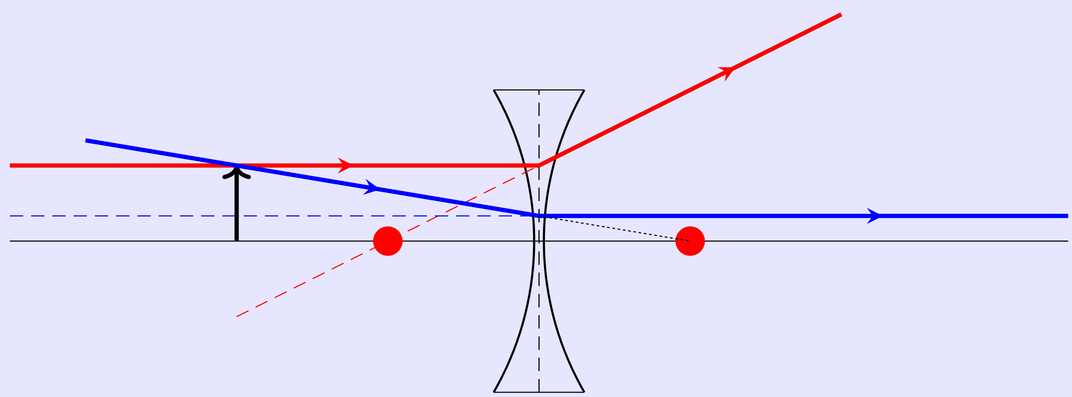
2. Measure and label focal points from the center of the lens, along the optical axis, and draw the object as an arrow with its base on the optical axis. Remember, that because the focal length is negative, the focal point for the right side is on the left, and the focal point for the left side is on the right.



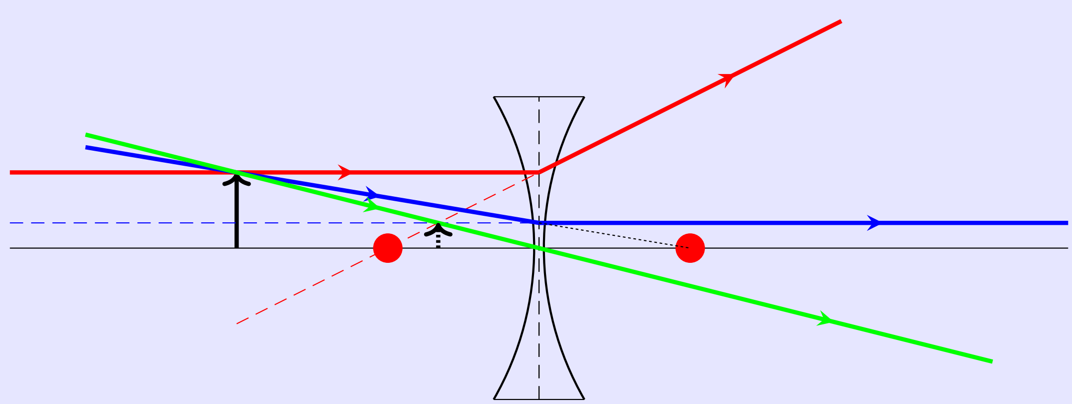
3. The first ray of light will begin on the left of the diagram, graze the top of the object, and continue to the center of the lens. It will then be directed upward, as if it came from the right focal point (which is actually on the left).



4. The second ray of light will graze the top of the object and be directed toward the near focal point (which is on the far side of the lens). Upon encountering the optical axis, the ray will be directed parallel to the optical axis.



5. The final ray of light will graze the top of the object, directed toward the intersection of center of the lens and the optical axis. This ray will not change direction. The intersection of the three rays is where the top of the image will form.



In this case, the image is behind the lens, meaning it is virtual. It is upright, and smaller than the original. Measuring the image distance yields $i = -1.333\text{cm}$ (that is, the image is 1.333 cm to the left of the lens).

The same problem can be solved using the lens equation as well:

Example 11.1.5

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = -2\text{cm}$. Use the lens equation and the magnification formula to determine the position, size, and orientation of the image.

Solution: In this case, we are given $h_o = 1\text{cm}$, $o = 4\text{cm}$ and $f = -2\text{cm}$. First, the lens equation is solved for i:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$i = \frac{1}{\frac{1}{f} - \frac{1}{o}} = \frac{1}{\frac{1}{-2\text{cm}} - \frac{1}{4\text{cm}}} = \boxed{-\frac{4}{3}\text{cm} \approx -1.333\text{cm}}$$

Calculating the magnification gives:

$$m = -\frac{i}{o} = -\frac{-\frac{4}{3}\text{cm}}{4\text{cm}} = \boxed{\frac{1}{3}}$$

The positive magnification means that the image is inverted. Since the magnification of this image is less than one, the image will appear smaller. Finally, we calculate image height:

$$m = \frac{h_i}{h_o} \longrightarrow h_i = mh_o = (\frac{1}{3})(1\text{cm}) = \boxed{\frac{1}{3}\text{cm}}$$

11.1.2 Reflection

The Law of Reflection

Compared to refraction, the law of reflection is quite simple: the angle of incidence is equal to the angle of reflection.

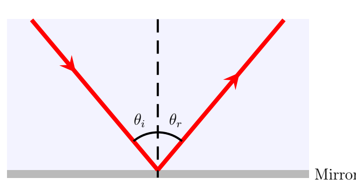


Figure 11.8: The angle of incidence is equal to the angle of reflection

This law of reflection can be expressed as the following equation.

$$\theta_i = \theta_r \tag{11.8}$$

Mirrors

Flat Mirrors Flat mirrors are quite easy to work with. The images produced by a flat mirror are always virtual, always the same size as the original object, and always upright. For this reason, flat mirrors are often used in bathrooms and fitting rooms to give an accurate assessment of one's appearance.

Concave Mirrors Like convex lenses, Concave mirrors are converging optical devices - that is, they focus parallel rays of light to a point. Unlike lenses, mirrors only have one focal point, since light can only interact with the mirror from one direction.

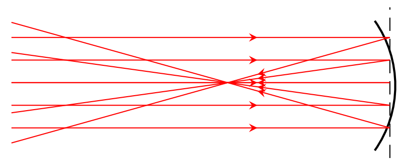


Figure 11.9: A concave mirror focuses light to a point.

Convex Mirrors

11.2 Physical Optics

11.2.1 Young's Double Slit Experiment

11.2.2 Thin Film Interference

Thin film interference is a phenomenon that takes place when a layer of material causes the reflections of light to interfere with each other. This usually only happens when the thickness of the material is on the same order as the wavelength of light. Common examples of thin film interference include it irridescent colors that are seen reflecting from soap bubbles and oil slicks. In addition, anti-reflective coatings on eyeglasses and other optical devices often make use of thin film interference.

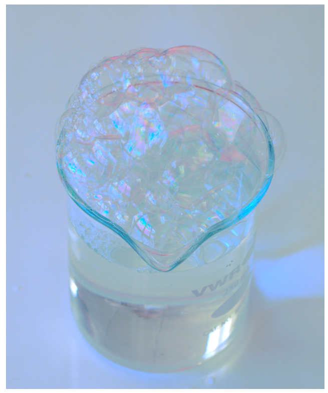


Figure 11.10: Iridescent colors are visible on soap bubbles due to thin film interference.

Chapter 12

Fluids

12.1 Density

The density of an object measures mass per unit volume. It is calculated using the following formula:

$$\rho = \frac{m}{V} \tag{12.1}$$

Where ρ is density in $\frac{kg}{m^3}$, m is mass in kg, and V is volume in m^3. The density of an object depends on the material from which it is made. The density of pure water is $1000\frac{kg}{m^3}$. The densities for various other materials can be found in Appendix B.3 on page 137.

12.2 Buoyant Force

When an object is placed in a fluid, it displaces some of the fluid - that is, it pushes the fluid out of the way. *Archimedes Principle* states that the weight of the fluid displaced is equal to the buoyant force that acts on an object.

$$F_B = \rho V g \tag{12.2}$$

In this equation, ρ is the density of the **fluid**, V is the volume displaced by the object, and g is the acceleration due to gravity. One should note that buoyant force is present whenever an object interacts with a fluid, even when the object is completely immersed or sinks.

Example 12.2.1

Problem: A ball has a radius of 0.1 meters, and is held in place completely submerged in a (freshwater) lake. What is the buoyant force that acts on the ball?

Solution: First, calculate the volume of the ball. The volume of a sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.1\,\mathrm{m})^3 \approx 4.189 \times 10^{-3}\mathrm{m}^3$$

The buoyant force on the ball is given by Equation 12.2:

$$F_B = \rho V g = 1000\,\frac{\text{kg}}{\text{m}^3} \times 4.189 \times 10^{-3}\text{m}^3 \times 9.81\,\text{m/s}^2 \approx 41.092\,\text{N}$$

12.2.1 Percent Submerged

If the density of a fluid and the density of a solid object are known, one can easily determine if an object will float in the fluid by comparing the densities. If the density of the fluid is greater than the density of the object, the object will float, However, if the density of the object is greater than the density of the fluid, the object will sink. If the density of the object is the same as the density of the fluid, the object will display *neutral buoyancy* - in which the object will neither sink nor float. Instead it will "hang" in the fluid until its equilibrium is disturbed.

When an object floats in a fluid, the percentage of the volume of the object that is submerged can be determined using the following formula:

$$\% = \frac{\rho_{solid}}{\rho_{liquid}} \times 100 \tag{12.3}$$

Example 12.2.2

Problem: You may have heard that "90% of an iceberg is below the water. Determine the percentage of an iceberg that is below the waterline. The density of ice is $917\,\frac{\text{kg}}{\text{m}^3}$ and the density of ocean water is $1025\,\frac{\text{kg}}{\text{m}^3}$.

Solution: Use equation 12.3 to determine the percent submerged:

$$\% = \frac{\rho_{ice}}{\rho_{water}} = \frac{917\,\frac{\text{kg}}{\text{m}^3}}{1025\,\frac{\text{kg}}{\text{m}^3}} \times 100 = 89.463\%$$

12.3 Pressure

Pressure is defined as force per unit area:

$$P = \frac{F}{A} \tag{12.4}$$

The units for pressure are *Pascals* (abbreviated Pa) which are equivalent to $\frac{\text{N}}{\text{m}^2}$.

Example 12.3.1

Problem: A box has a mass of 2.5 kg. Its has an area of $0.2\,\mathrm{m}^2$ at its base. What is the pressure the box puts on the table?

Solution: The force due to gravity is:

$$F_g = mg = 2.5\,\mathrm{kg}9.81\,\mathrm{m/s}^2 = 24.525\,\mathrm{N}$$

Using equation 12.4, we determine:

$$P = \frac{F}{A} = \frac{24.525\,\mathrm{N}}{0.2\,\mathrm{m}^2} = 122.625\,\mathrm{Pa}$$

12.3.1 Hydrostatic Pressure

Hydrostatic pressure is the pressure exerted on an object due to a non-moving fluid. The hydrostatic pressure on a submerged object can be calculated using the following formula. This is often called **gauge pressure**.

$$P = \rho g h \tag{12.5}$$

There are some important things you should note about this equation. First, the h represents depth under the surface, not height. Therefore, h should be measured from the surface of the fluid downward. Secondly, this formula assumes that the fluid is incompressible, and it will not work for compressible fluids like air, because the density of the fluid will change as it compresses. Finally, the pressure calculated by this formula is the gauge pressure. That is, it is only the pressure due to the fluid, and does not take any additional atmospheric pressure into account. To find absolute pressure, one must add the external atmospheric pressure to the formula:

$$P_{absolute} = \rho g h + P_{atm} \tag{12.6}$$

Example 12.3.2

Problem: A scuba diver is in the ocean, where the density of the water is $1025\,\mathrm{kg/m}^3$ when he finds his pressure gauge reads $2.56 \times 10^5\mathrm{Pa}$. What is the depth of the diver?

Solution: Normally, a pressure gauge will read 0 Pa at the surface of the water, so we usre the is Using equation 12.5, we can solve for h.

$$P = \rho g h \longrightarrow h = \frac{P}{\rho g} = \frac{2.56 \times 10^5\mathrm{Pa}}{(1025\,\mathrm{kg/m}^3)(9.81\,\mathrm{m/s}^2)} \approx 25.459\,\mathrm{m}$$

12.4 The Fluid Continuity Equation

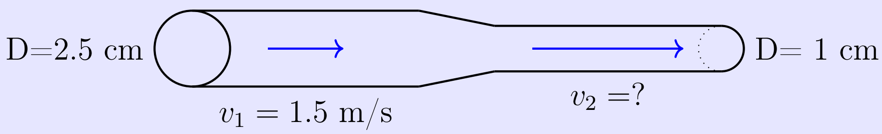
When a an incompressible fluid (ie. a liquid) is flowing through a pipe, and the pipe is completely full, the mass that enters one side of the pipe must be equal to the mass that exits the other side of the pipe. From the law of conservation of mass, it can be shown that the relationship between the cross-sectional areas and the speed of the fluid are given by the fluid *continuity equation*:

$$A_1 v_1 = A_2 v_2 \tag{12.7}$$

Example 12.4.1

Problem: Water flows in a pipe with a 2.5 cm diameter at a speed of 1.5 m/s. It enters a smaller area of pipe, where the diameter is 1 cm in diameter. What is the speed of the pipe in the smaller section?

Solution: First, draw a diagram:



In order to apply the continuity equation, we need to know the radius of each side. Since we are given the diameter, $r_1 = \frac{2.5\,\text{cm}}{2} = 1.25\,\text{cm}$ and $r_2 = \frac{1\,\text{cm}}{2} = 0.5\,\text{cm}$. We can now apply the continuity equation, seen above as equation 12.7. Solving for v_2 yields:

$$A_1 v_1 = A_2 v_2 \longrightarrow v_2 = \frac{A_1 v_1}{A_2}$$

Since both sides have a circular cross section, we can substitude $A = \pi r^2$.

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\cancel{\pi} {r_1}^2 v_1}{\cancel{\pi} {r_2}^2} = \frac{(1.25\,\text{cm})^2 (1.5\,\text{m/s})}{(0.5\,\text{cm})^2} \approx 9.375\,\text{m/s}$$

12.5 Bernouli's Equation

Just as the continuity equation is derived from the law of conservation of mass, one can derive another equation from the law of conservation of energy. This equation is named for Daniel Bernouli, who was the first to derive and publish it.

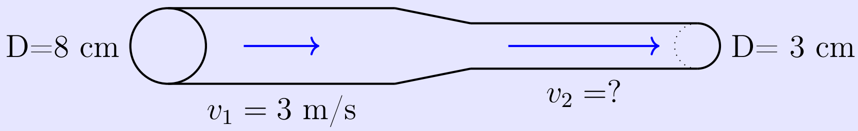
$$\rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2 \tag{12.8}$$

While Bernouli's equation may look intimidating, one should always start by determining if there are equivilant terms on each side of the equation. Equivalent terms on both side of the equation can be subtracted from both sides, making the equation significantly easier to solve.

Example 12.5.2

Problem: Water flows through a horizontal pipe with a diameter of 8 cm at a speed of 3 m/s. The pipe then narrows to a 3 cm diameter section. Assuming incompressible, non-viscous flow, what is the pressure difference between the wider and narrower sections of the pipe?

Solution: First, draw a diagram:



Step 1: Solve for v_2 using the continuity equation Since the flow is incompressible, we use the continuity equation:

$$A_1 v_1 = A_2 v_2$$

where the cross-sectional area of a pipe is given by:

$$A = \pi \left(\frac{d}{2}\right)^2.$$

Substituting for both pipe sections:

$$\pi \left(\frac{d_1}{2}\right)^2 v_1 = \pi \left(\frac{d_2}{2}\right)^2 v_2.$$

Canceling π and solving for v_2:

$$v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2$$

Substituting $d_1 = 8\,\text{cm}$ and $d_2 = 3\,\text{cm}$:

$$v_2 = (3\,\text{m/s}) \left(\frac{8\,\text{cm}}{3\,\text{cm}}\right)^2 \approx 21.333 \text{ m/s}$$

Step 2: Solve for $P_1 - P_2$ using Bernoulli's equation
We apply **Bernoulli's equation** between the two sections of the pipe:

$$\rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2$$

Since the pipe is horizontal, there is no change in the height of the pipe. Therefore, the *rhogh* can be subtracted from each side.

$$\cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 + P_1 = \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2 + P_2$$

Rearranging Bernoulli's equation:

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right).$$

Substituting known values:

$$P_1 - P_2 = \frac{1}{2}(1000)\left(21.33^2 - 3^2\right) \approx 2.23 \times 10^5 \text{ Pa}.$$

Thus, the pressure **decreases by $\mathbf{2.23 \times 10^5}$ Pa** in the narrow section of the pipe.

Chapter 13

Heat and Thermodynamics

Heat is thermal energy that is transferred from one object to another. Most often, heat energy is symbolized by the variable Q.

13.1 Modes of Heat Transfer

There are three modes in which heat is transferred from one object to another.

- Conduction occurs when objects are physically touching each other. Metals tend to be good conductors of heat.
- Convection occurs when a moving fluid causes heat to be transferred. Convection currents in fluids can occur naturally, or they can be forced.
- Thermal radiation (not to be confused with Ionizing Radiation, which will be studied in chapter 20) occurs when heat is transferred via electromagnetic waves. While infrared light is most often associated with heat transfer, all electromagnetic rays transfer energy - and that energy often ends up as thermal energy. Additionally, thermal radiation is the only mode of heat transfer that does not require a medium - it can happen through empty space.

13.2 Laws of Thermodynamics

There are a total of four laws of thermodynamics. Each law of thermodynamics focuses on a particular aspect of how heat can be transferred.

13.2.1 0th Law

The 0th law of thermodynamics deals with the idea of thermal equilibrium. Let us suppose there are three objects: A, B, and C, and each object has a temperature, T_A, T_B, and T_C, respectively. If $T_A = T_B$ and $T_B = T_C$, then $T_A = T_C$.

While this may seem to go without saying, it is important to realize the implications of this law. First, when objects are in thermal equilibrium (at the same temperature), no heat will flow. Second, heat energy is fungible, meaning that all heat energy is the same and will have the same effect on the temperature of an object, despite the origin or process that delivers that thermal energy to an object.

13.2.2 1st Law

The First Law of Thermodynamics can be thought of as a restatement of the Law of Conservation of Energy. The change in internal thermal energy of a system (ΔU) will be equal to the amount of energy that entered or left the system. Energy can enter or leave a system as either work (W) or heat (Q). Thus,

$$\Delta U = Q + W \tag{13.1}$$

This equation is often printed with various sign conventions that can sometimes make it harder to understand. Rather than memorizing sign conventions, the author of this book suggest developing an understanding that energy entering the system should increase the internal energy and energy exiting the system should decrease the internal energy. Positives and negatives can then be chosen based on the situation.

13.2.3 2nd Law

The Second Law of Thermodynamics states that systems tend to become more chaotic over time. That is, the *entropy* of a closed system will always increase.

This concept is fairly easy to understand. The probability of an organized stack of books falling down into a disorganized pile is much greater than the probability of a disorganized pile of books spontaneously arranging themselves into an organized stack. Thus, the Second Law of Thermodynamics merely states that events that are more probable also tend to be more disorganized.

There are two important implications of the Second Law of Thermodynamics:

1. Heat always flows from objects of higher temperature to objects of lower temperature.

2. The direction of time can be determined by the increase in entropy of closed systems.

13.2.4 3rd Law

The Third Law of Thermodynamics states that the entropy of a system approaches a constant value as the temperature of a system approaches absolute zero.

When combined with the 2nd law of thermodynamics, which states that the entropy of a closed system always increases, this means that the temperature of absolute zero can never be reached.

13.3 Specific Heat Capacity

Have you ever gone to the beach and found that the sand is extremely hot, while the water is quite cold? This is an example of a difference in specific heat capacity. Different materials respond to heat by warming at different rates; it takes more energy to warm 1 kg of water by 1°C than it takes to warm 1 kg of sand by 1°C. Thus, the sand tends to warm up faster than the water.

The amount of heat, Q, needed to cause a mass of m to change its temperature by ΔT is given by the following formula:

$$Q = mc\Delta T \tag{13.2}$$

The specific heat capacity of the material is c. A list of specific heat capacities for common materials can be found in Appendix B.7 on page 139.

Example 13.3.1

Problem: How much energy is required to heat 2.5 kg of water from 20°C to 60°C?

Solution: The specific heat of water is $4180\frac{\mathrm{J}}{\mathrm{kg}^\circ\mathrm{C}}$. Applying equation 13.2 yields:

$$Q = mc\Delta T = (2.5\,\mathrm{kg})(4180\frac{\mathrm{J}}{\mathrm{kg}^\circ\mathrm{C}})(60^\circ\mathrm{C} - 20^\circ\mathrm{C}) = 418\,000\,\mathrm{J}$$

13.4 Phase Changes and Latent Heat

13.4.1 The States of Matter

There are 4 common states of matter that exist abundantly in the universe:

1. **Solid** - in which matter has both a definite shape and a definite volume. Subatomically, atoms are packed together tightly, causing them to be stuck in place. While they can vibrate, they are unable to move significantly.

2. **Liquid** - in which matter has a definite volume, but not a definite shape. Atoms are still packed together relatively tightly, but are free to move around.

3. **Gas** - in which matter has no definite shape or size; gasses will expand to fill their container. Individual atoms are far apart and rarely interact with each other.

4. **Plasma** - in which atoms become ionized. That is, electrons and nuclei are completely separated from each other. Like gasses, plasmas have no definite shape or volume.

It is important to note that there are other states of matter, such as Bose-Einstein condensates, that are beyond the scope of this text.

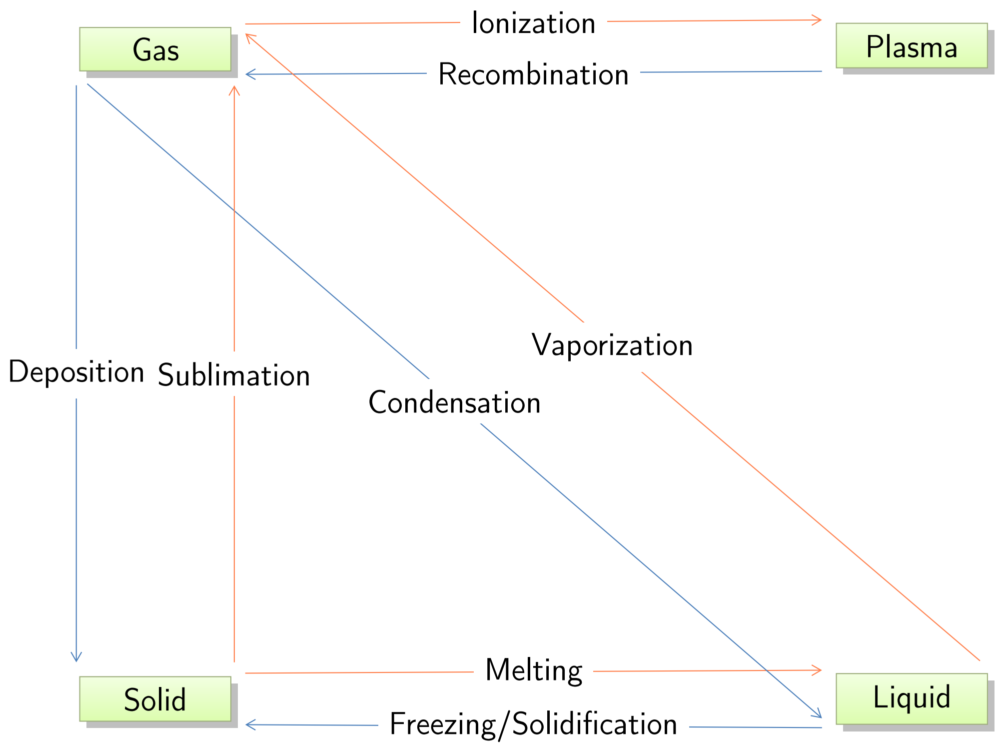


Figure 13.1: The Phase Changes of Matter

13.4.2 Latent Heat of Fusion

In order change from a solid to a liquid, an object must absorb energy. This energy does not contribute to a change in temperature. Instead, it is used to break intermolecular bonds in the solid. This energy is called the latent heat of fusion. One can calculate the energy required to melt an object by using the following equation:

$$Q = mL \tag{13.3}$$

Example 13.4.1

Problem: How much energy does it take to turn 0.25 kg of ice at -20°C into water at 10°C? Use the following numbers in your calculation:

$c_{ice} = 2090\frac{\mathrm{J}}{\mathrm{kg°C}}$
$c_{water} = 4090\frac{\mathrm{J}}{\mathrm{kg°C}}$
$L_{f_{water}} = 3.34 \times 10^5 \frac{\mathrm{J}}{\mathrm{kg}}$

Solution: There are three parts to this transition. First, the ice heats up from -10°C to 0°C, absorbing energy (Q_{ice}). Then the ice melts, absorbing the latent heat of fusion (Q_{melt}). Finally the water warms up from 0°C to 10°C, absorbing energy (Q_{water}).

So the total amount of energy is:

$$Q_{total} = Q_{ice} + Q_{melt} + Q_{water}$$

Using equations 13.2 and 13.3, we can make the following substitutions:

$$Q_{total} = mc_{ice}\Delta T_{ice} + mL_{f_{water}} + mc_{water}\Delta T_{water}$$

$$Q_{total} = (0.25\,\mathrm{kg})(2090\frac{\mathrm{J}}{\mathrm{kg^\circ C}})(20^\circ\mathrm{C}) + (0.25\,\mathrm{kg})(3.34\times10^5\frac{\mathrm{J}}{\mathrm{kg}}) + (0.25\,\mathrm{kg})(4180\frac{\mathrm{J}}{\mathrm{kg^\circ C}})(10^\circ\mathrm{C})$$

Evaluating this expression yields:

$$Q_{total} = 104\,400\,\mathrm{J}$$

It should be noted that this process is reversible. Removing mL_f from a liquid that is at its freezing point will cause the liquid to freeze (solidify).

13.4.3 Latent Heat of Vaporization

The energy required to convert a liquid into a gas is called the latent heat of vaporization, and is symbolized as L_v. Like the latent heat of fusion, adding the latent heat of vaporization to a liquid that is at its boiling point does not cause its temperature to change. Instead, this energy breaks intermolecular bonds, causing the liquid to become a gas at the same temperature.

Chapter 14

Gas Laws

Unlike liquids, gasses are easily compressible. Thus, the compressibility of the gasses must be taken into account when discussing how the gasses behave in relation to temperature and pressure. There are four terms that must be defined with relation to gasses:

- An adiabatic process process is one in which no heat flows. Adiabatic processes take place in either a perfectly insulated environment or happen quickly enough that the amount of heat that flows into or out of the surrounding environment is negligible.
- An isothermal process process is one in which the temperature remains constant throughout the process. This is done by adding or removing heat from the system.
- In an isobaric process process, the pressure of the sample remains constant.
- Finally, in an isochoric process process, the volume of the gas does not change. This is sometimes called isovolumetric.

Before we begin our study of gas laws, it is important to note that all temperatures must be on an absolute scale. This text will use the Kelvin scale. To convert from celsius to kelvin, simply add 273.15:

$$T_K = T_C + 273.15\text{K} \tag{14.1}$$

14.1 Boyle's, Charles's, Gay-Lussac's, and The Combined Gas Law

You may remember from chemistry that the temperature (T), pressure (P), and volume (V) of a gas are all related. In fact, an isothermal process will be governed by *Boyle's Law*:

$$P_1V_1 = P_2V_2 \tag{14.2}$$

Similarly, an isobaric process is governed by *Charles's Law*:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \tag{14.3}$$

And the *Gay-Lussac Law* describes isochoric processes:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \tag{14.4}$$

After the discovery of these three laws, scientists used these to create the ***Combined Gas Law***:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \tag{14.5}$$

Rather than trying to remember each relationship, it is often easier to always apply the combined gas law to a situation. Whatever variable is held constant will appear on both sides of the equation, and thus be eliminated.

Example 14.1.1

Problem: A balloon has a volume of 0.1m^3 at room temperature (20°C). What would its volume be if placed into a blast chiller at −40°C?

Solution: Begin by converting all temperatures to kelvin. By Equation 14.1,

$$T_1 = T_{C1} + 273.15\text{K} = 20°\text{C} + 273.15\text{K} = 293.15\text{K}$$

$$T_2 = T_{C2} + 273.15\text{K} = -40°\text{C} + 273.15\text{K} = 233.15\text{K}$$

Since there is no mention of changing pressure, we can assume that both states occur at atmospheric pressure, and thus the process is isobaric. Using the combined gas law:

$$\frac{\cancel{P_1} V_1}{T_1} = \frac{\cancel{P_2} V_2}{T_2}$$

Solving for V_2 yields:

$$V_2 = \frac{V_1 T_2}{T_1} = \frac{0.1\text{m}^3 \times 233.15\text{K}}{293.15\text{K}} \approx \boxed{7.953 \times 10^{-2}\text{m}^3}$$

14.2 The Ideal Gas Law

The ideal gas law states:

$$PV = nRT \tag{14.6}$$

where P is pressure, V is volume, T is temperature, n is the number of moles of gas that make up the sample and R is the universal gas constant:

$$R = 8.314\,462\,618\,153\,24\,\frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$$

Sometimes, it is useful to know the exact number of molecules in a sample without having to convert from moles. The following formula allows one to determine N, the number of molecules in a sample.

$$PV = Nk_BT \tag{14.7}$$

In this equation, k_B is known as **Boltzmann's Constant** , which is the Universal Gas Constant divided by Avagadro's Number:

$$k_B = 1.380649 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}$$

Example 14.2.1

Problem: A sample of gas occupies $0.5\mathrm{m}^3$ at a pressure of 101325Pa and a temperature of 300K. Find

(a) the number of moles the sample contains.

(b) the number of molecules the sample contains.

Solution:

(a) begin by using Equation 14.6:

$$PV = nRT$$

Solving for n and substituting gives:

$$n = \frac{PV}{RT} = \frac{(101325\mathrm{Pa})(0.5\mathrm{m}^3)}{(8.314\,\frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}})(300\mathrm{K})} \approx \boxed{20.312\,\mathrm{mol}}$$

(b) to solve for the number of molecules, use Equation 14.7:

$$PV = Nk_BT$$

Solving for N and substituting gives:

$$n = \frac{PV}{k_BT} = \frac{(101325\mathrm{Pa})(0.5\mathrm{m}^3)}{(k_B = 1.381 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}})(300\mathrm{K})} \approx \boxed{1.228 \times 10^{25}\mathrm{molecules}}$$

14.3 Internal Energy of a Gas

14.3.1 Absolute Internal Energy

The internal energy of an ideal gas can be thought of as the total of the kinetic energies of all molecules in the sample. The internal energy of the gas is given by:

$$U = \frac{3}{2}nRT = \frac{3}{2}Nk_BT \tag{14.8}$$

14.3.2 Change in Internal Energy

Removing all the internal energy from a sample of gas would require the sample to reach a temperature of absolute zero, which is prohibited by the 3rd law of thermodynamics. Therefore, it is often more important to think about the change in internal energy of the sample. The internal energy of a gas will change when either heat (Q) or work (W) flow into or out of the system:

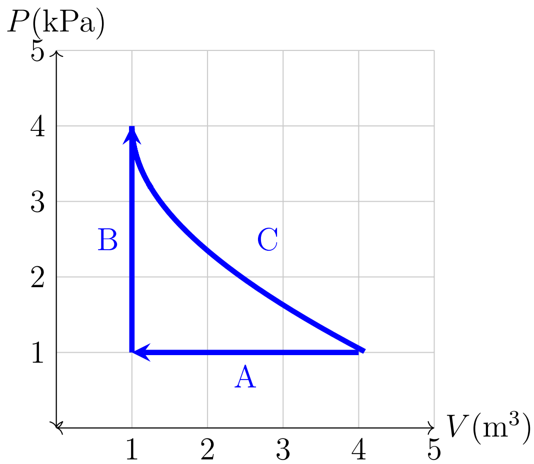
$$\Delta U = Q + W \tag{14.9}$$

There are several sign conventions associated with Equation 14.9 that are sometimes confusing to students. The easiest way to remember is that energy that flows *into* a system causes its internal energy to *increase*, and energy that flows *out* of a system causes its energy to decrease. Positive and negative signs should be assigned to work and heat that cause this to be true, despite any sign conventions of an equation. Remember to *understand* the process that is occurring, and do not just blindly follow an equation.

14.4 PV-Diagrams

A **PV-Diagram** will show the relationship of pressure and volume for a sample of gas, with pressure on the y-axis and volume on the x-axis.

Figure 14.1: A PV Diagram for a Sample of Gas

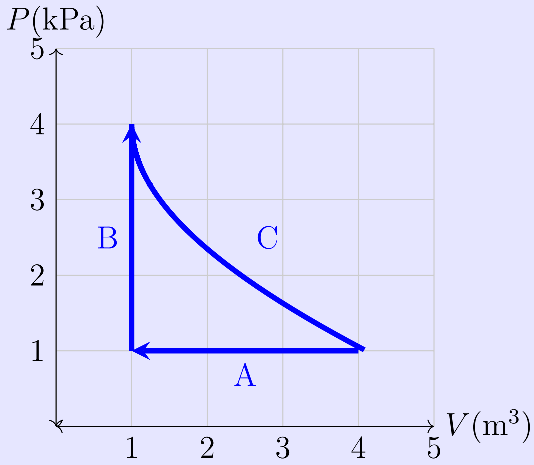


Different processes will have different characteristic shapes on the PV Diagram. In figure 14.1, the process labeled A is isobaric, because the pressure does not change. Process B is Isochoric, since the volume is constant. Process C would likely be either adiabatic or isothermal, since pressure and volume follow a hyperbolic curve.

The area under any process on a PV diagram is equal to the amount of work that flows either into or out of the gas. In general, in processes pointing toward the left, work is done *on* the gas (and thus the internal energy increases), whereas in processes pointing toward the right, work is done *by* the gas (causing the internal energy to decrease). One should note that there is no area under a vertical line. Thus, in isochoric processes - such as process B in figure 14.1, in which the volume does not change, there is no work done.

Example 14.4.1

Problem: A sample of Gas is taken through the process ABC, as shown in the PV diagram below:



The temperature of the gas at the starting point is 300K. Calculate:

(a) The temperature of the gas at the end of Process A.

(b) The number of molecules in the sample.

(c) The amount of work done in process A.

(d) the amount of work done in process B.

Solution:

(a) Use the combined gas law, Equation 14.5 to find the temperature at the end of process A:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Since the pressure does not change, this equation reduces to Charles's Law:

$$\frac{\cancel{P_1}V_1}{T_1} = \frac{\cancel{P_2}V_2}{T_2} \rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Solving for T_2 gives:

$$T_2 = \frac{V_2T_1}{V_1} = \frac{(1\text{m}^3)(300\text{K})}{4\text{m}^3} = \boxed{75\text{K}}$$

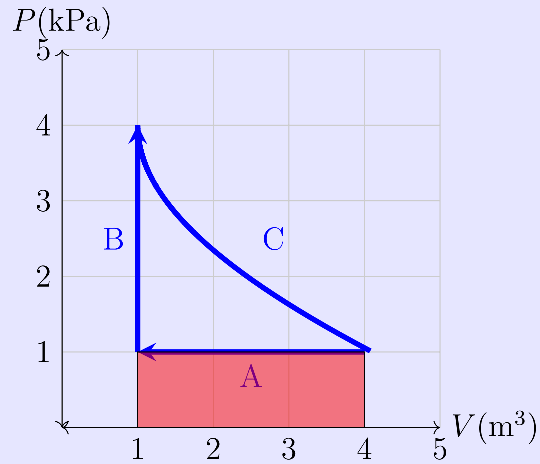
(b) The Ideal Gas Law, Equation 14.7, can be applied to the gas at any point on the diagram. Here, it is applied in the initial state:

$$PV = Nk_BT$$

Solving for N gives:

$$N = \frac{PV}{k_B T} = \frac{(1 \times 10^3 \text{Pa})(4\text{m}^3)}{(1.381 \times 10^{-23}\text{J/K})(300\text{K})} \approx \boxed{9.655 \times 10^{23} \text{molecules}}$$

(c) The work done by process A is equal to the area under the line for process A:



Since this shape is a rectangle, we merely multiply base times height:

$$W = A = BH = (3\text{m}^3)(1 \times 10^3 \text{Pa}) = \boxed{3 \times 10^3 \text{J}}$$

(d) Since process B is a vertical segment on the PV-Diagram, there is no area underneath this segment. Therefore, the are and the work done are both 0 Joules.

Chapter 15

Electrostatics

15.1 Electrostatic Charge

You've probably experienced electrostatic charges - sometimes we just call it static - in everyday life. Dragging your shoes on carpet, rubbing balloon on your hair, and even just putting on a piece of clothing can cause electrostatic charges to build up, causing articles to cling together, your hair to get frizzy, and can even cause small sparks.

Most electrical charges we encounter in everyday life are due to imbalances of protons and electrons. Electrons are capable of moving from atom to atom under the right conditions. However, protons are stuck in the nucleus of an atom and do not move unless nuclear reactions are taking place. Thus, most electrical charges we encounter in actual life are due to electrons being transferred from one object to another.

The units for charge are Coulombs (C), and it is represented by the variable q. You may remember from chemistry that protons have small positive charges and electrons have small negative charges. Table 15.1 shows the charge of each of the elementary particles that make up normal matter:

Table 15.1: Elementary Charges

Particle	**Charge**	**Mass**
Proton	1.602×10^{-19} C	1.673×10^{-27} kg
Electron	-1.602×10^{-19} C	9.109×10^{-31} kg
Neutron	0 C	1.675×10^{-31} kg

You may notice that the charge of an electron and a proton are exactly the same with the only difference being a negative sign for the electron. Thus a Hydrogen atom, made of 1 proton and 1 electron will have a total charge of zero coulombs. In fact, any combination of the same number of protons and electrons will have no charge.

You should also note that 1 coulomb of electrostatic charge is a large amount. Most charges we encounter in everyday life might be 1 millionth or even 1 billionth of a coulomb.

15.2 Coulomb's Law and Electrostatic Force

When two charged objects interact, it generates a force. This force, often called electrostatic force, is repulsive for charges with the same sign, whereas it is attractive for two charges with opposite signs. So two positive charges will repel, as will two negative charges, but a positive charge will be attracted to a negative charge.

To determine the about of force that will be exerted one charged particle due to another charged particle, we use **Coulomb's Law**:

$$F = \frac{kq_1q_2}{r^2} \tag{15.1}$$

In this equation, q_1 and q_2 are the charges of the particles, r is the distance between the charges, and k is **Coulomb's Constant**. Coulomb's Constant is a universal constant, meaning its value does not change. The value of coulomb's constant is:

$$k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9\,\mathrm{Nm^2/C^2}$$

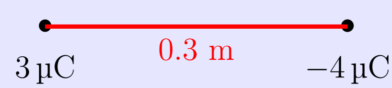
where ϵ_0 is the permitivity of free space.

It is also important to note that this equation has a non-intuitive sign convention. Repulsive forces will be postive, and attractive forces will be negative. If forces calculated using Coulomb's Law need to be used in further calculations, be sure assign a positive or negative to the calculated force according to the direction the force is actually in.

Example 15.2.1

Problem: A positive charge of $3\,\mu\mathrm{C}$ and a negative charge of $-4\,\mu\mathrm{C}$ are separated by a distance of 0.3 meters. What is the electrostatic force felt by the positive charge due to the negative charge?

Solution: Begin by drawing a diagram of the situation:



We also need to convert the charges into scientific notation:

$$q_1 = 3\,\mu\mathrm{C} = 3 \times 10^{-6}\,\mathrm{C}$$
$$q_2 = -4\,\mu\mathrm{C} = -4 \times 10^{-6}\,\mathrm{C}$$

Note: It does not matter which charge is q_1 and which is q_2. Either way will yield the same answer.

Now, we can use Coulomb's Law, equation 15.1 to calculate the force:

$$F = \frac{kq_1q_2}{r^2} = \frac{(9 \times 10^9\,\mathrm{Nm^2/C^2})(3 \times 10^{-6}\,\mathrm{C})(-4 \times 10^{-6}\,\mathrm{C})}{(0.3\,\mathrm{m})^2} = \boxed{-1.2\,\mathrm{N}}$$

The negative on the answer does not indicate that the force it to the left. Rather, it indicates that this is an attractive force. Thus, the $3\,\mu\mathrm{C}$ charge feels a 1.2 N force directed to the *right*, whereas the $-4\,\mu\mathrm{C}$ charge feels a 1.2 N force directed to the *left*.

15.3 Electrostatic Potential Energy

When two electrostatically charged objects are brought near each other, Coulomb's Law states that they will put a force on each other. If the charged particles are then released, they will accelerate. This means that two charged particles that are close enough to affect each other must have potential energy. The formula to calculate electrostatic potential energy is:

$$U_E = \frac{kq_1q_2}{r} \tag{15.2}$$

15.4 Electric Field

$$E = \frac{kq}{r^2} \tag{15.3}$$

15.5 Electric Potential and Voltage

$$V = \frac{kq}{r} \tag{15.4}$$

15.6 Capacitors

15.6.1 Construction of Capacitors

15.6.2 Capacitors in Circuits

Chapter 16

Circuits

16.1 ohm's Law

16.2 Kirchoff's Laws

16.3 Circuits Symbols

16.4 Series Circuits

16.5 Parallel Circuits

16.6 Complex Circuits

Chapter 17

Magnetic Forces and Fields

17.1 Introduction to Magnetism

All magnetic fields are created by moving electrically charged particles. Since all atoms contain protons that can rotate or oscillate, as well as electrons that can move around outside the nucleus, all atoms have magnetic fields. Most of the time, the effects of many atoms cancel out due to their random orientations. However, sometimes atoms in areas can align, creating permanent magnetic properties in an object.

17.2 Types of Magnetism

17.2.1 Permanent Magnetism

Permanent magnetism occurs when an object exhibits magnetic properties without any external voltages or currents being applied. There are three basic types of permanent magnetism:

Ferromagnetism

Ferromagnetism is the type of magnetism most of us are familiar with. It occurs in iron, nickle, cobalt, and some types of rare-earth metals, such as neodymium. Ferromagnetic materials are strongly attracted to external magnetic fields. In addition, ferromagnetic materials become magnetized themselves when exposed to an external magnetic field. That magnetization can remain even when the external magnetic field is removed.

Paramagnetism

Paramagnetic materials are weakly attracted to magnets, but do not become magnetized themselves. Examples of paramagnetic materials include aluminum, tungsten, and platinum.

Dimagnetism

Diamagnetic materials have a weak and opposite response to magnetic fields, meaning they are repelled by a magnet. Examples of diamagnetic materials include copper, silver, and gold.

17.2.2 Electromagnetism

17.3 Magnetic Force on a Charged Particle

17.4 Magnetic Force on a Current-Carrying Wire

17.5 Magnetic Field Produced by a Current-Carrying Wire

Chapter 18

Magnetic Induction

18.1 Lenz's Law

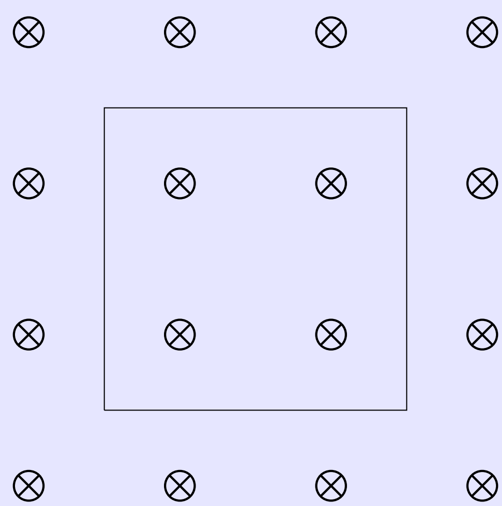
Lenz's law states that in the presence of a changing magnetic field, a current will flow in a loop that generates a magnetic field opposite to the direction of the change.

Several examples are shown below:

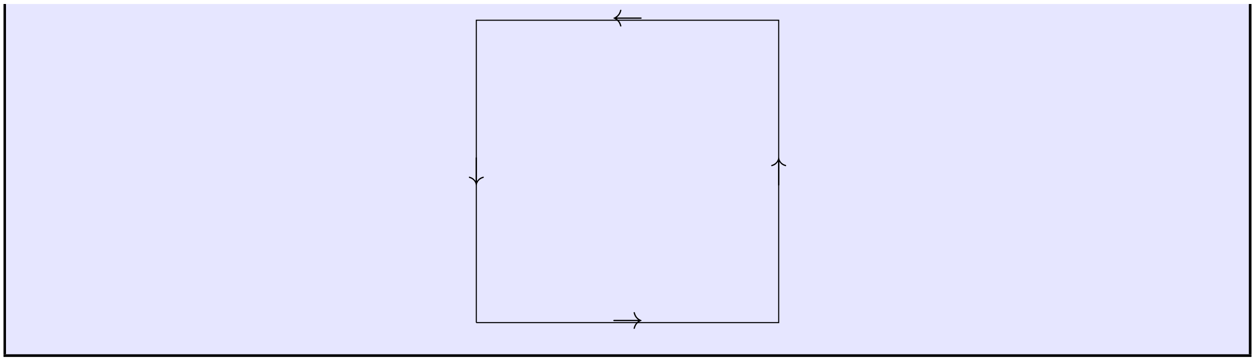
Example 18.1.1

Problem: A square of metal is located along the same plane as the page. A magnetic field is directed into the page, and is getting stronger. In what direction will current flow?

Solution: We begin by drawing a diagram:



Since the magnetic field is getting stronger, the current induced will oppose this change by creating a magnetic field directed out of the page (thereby cancelling some of the growing strength). Using the third right hand rule, current will flow counterclockwise, as shown below:



18.2 Faraday's Law

$$\varepsilon = -\frac{\partial \Phi}{\partial t} \tag{18.1}$$

Chapter 19

Modern and Atomic Physics

19.1 The Dual Nature of Light

19.1.1 Young's Double Slit Experiment

Young's Double Slit Experiment proved once and for all that light has a wave nature.

19.1.2 The Photoelectric Effect

When metals are exposed to light (in a vacuum), electrons are ejected from the metal. While experimenting with the photoelectric effect, the following observations were made:

- Increasing the brightness of the light (the amplitude of the electromagnetic wave) causes more electrons to be ejected, but their maximum speed remains the same.
- Increasing the frequency of the light (changing the color toward the violet end of the spectrum) while keeping the intensity the same causes faster electrons to be emitted.

These observations are inconsistent with the wave model of light. Albert Einstein proposed understanding the photoelectric effect as a collision between a particle of light, called a *photon*, and an electron. The energy of a photon is given by:

$$E = hf \tag{19.1}$$

In this equation f is the frequency of the light, and h is Planck's Constant. Planck's constant is:

$$h = 6.626 \times 10^{34}\mathrm{J} \cdot \mathrm{s} = 4.136 \times 10^{-15}\mathrm{eV} \cdot \mathrm{s}$$

$$E = hf - \phi \tag{19.2}$$

Thus, Einstein's explanation of the photoelectric effect proved once and for all that light has a particle nature. When combined with the results from Young's Double Slit Experiment, we find that light can act as either a stream of particles or waves, depending on the experiment and the observations that we make.

19.2 The Dual Nature of Matter

19.2.1 Mass-Energy Equivalence

The masses of the elementary particles are given in the table below:

Table 19.1: Mass of Elementary Particles

Particle	**Mass** (u)
Proton	1.00727647
Neutron	1.008665
Electron	0.00055

When the mass of elementary particles had been determined, scientists noticed that the masses of various elements did not equal what would be predicted by adding up constituent particles. For example, a helium atom is made of two protons, two neutrons, and two electrons. The combined mass of all these particles is 4.03298294 u, but the mass of a hydrogen atom is 4.002603 u - a difference of a bit more than 0.03 u. This “missing" mass is called a *mass defect*.

Albert Einstein proposed that the missing mass was completely destroyed, and released in the form of energy. In what is often recognized as his most famous equation, Albert Einstein derived the following expression:

$$E = mc^2 \tag{19.3}$$

Simultaneously, the Law of Conservation of Mass and the Law of Conservation of Energy are now combined into a single, unified Law of Conservation of Mass-Energy. Simply put, mass-energy equivalence states that mass and energy are the same thing.

19.2.2 deBroigle Wavelength

$$\lambda = \frac{h}{p} \tag{19.4}$$

19.3 Atomic Physics

For hydrogen, these energy levels are given by the following formula:

$$E_n = \frac{-13.6\text{eV}}{n^2} \tag{19.5}$$

This energy can be thought of as the electrostatic potential energy of the electron-proton configuration in the atom. In order to return the electron to zero potential energy (an infinite distance away from the proton), one would need to somehow provide 13.6 eV of energy to the electron.

19.3.1 Energy Level Diagrams and Atomic Spectra

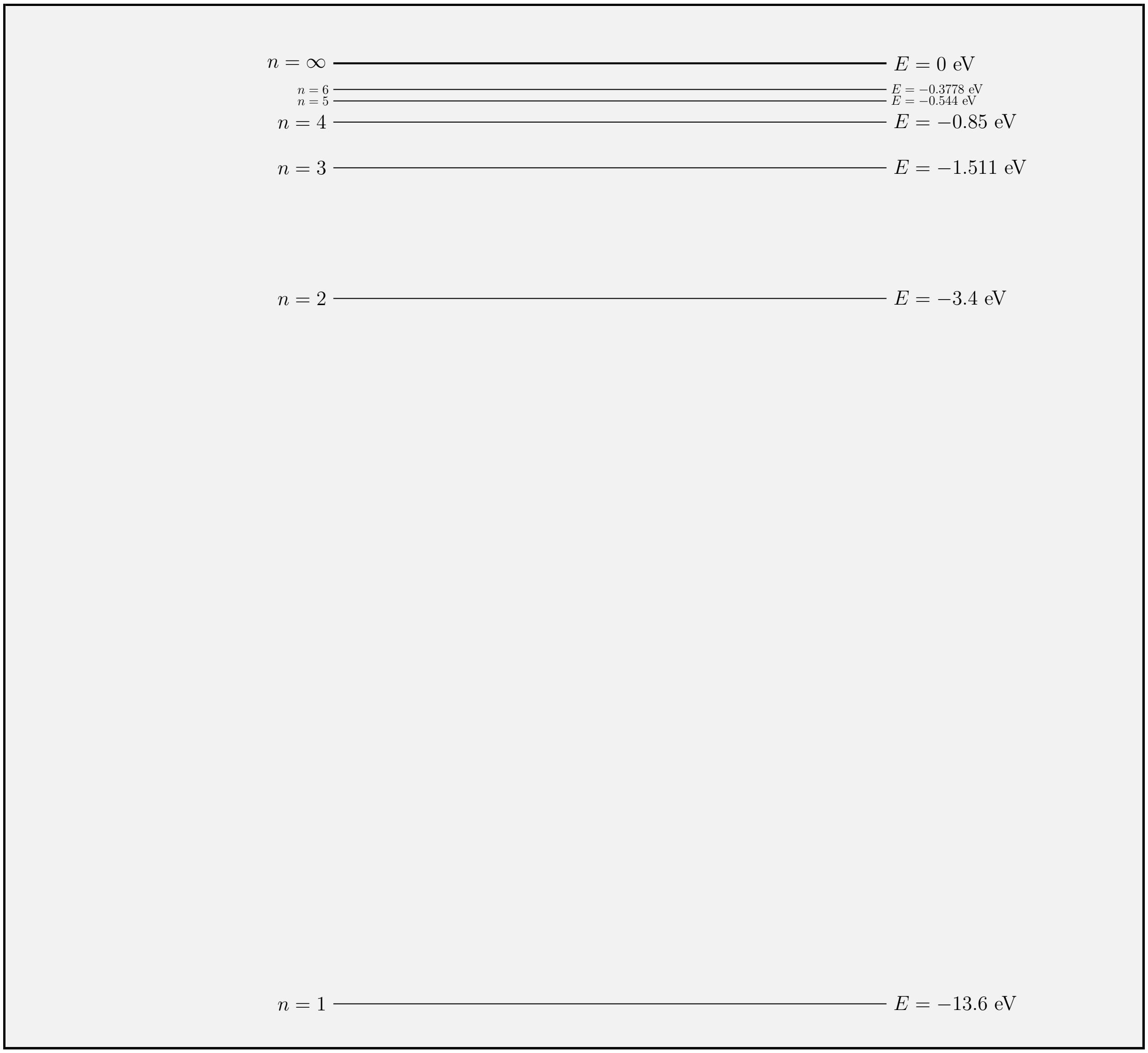


Figure 19.1: A simple energy level diagram for Hydrogen

Chapter 20

Nuclear Physics

20.1 Elements, Isotopes, and Ions

You may remember some of the information contained on the periodic table of elements from chemistry. A sample box from the periodic table is shown below:

11
Na
Sodium
22.989769

Figure 20.1: An example of a box from the Periodic Table of Elements.

The number of protons in an atom determines what element it is. In the example above, the atomic number of sodium is 11. This means that any atom that has 11 protons is a sodium atom. The element number is written to the bottom-left of the elements symbol when writing nuclear equations: ${}_{11}\text{Na}$, though this is redundant, since the symbol tells you what element it is.

An atom's isotope is determined by the number of protons and neutrons in that atom. Collectively, protons and neutrons are called *nucleons*. Thus, increasing the number of neutrons present in an atom will change its isotope without changing its element. While Sodium only has one stable isotope with 23 nucleons, other elements sometimes have multiple isotopes that are stable. Thus, when referring to a specific isotopes, we often state the element's name and isotope number, such as Sodium-29 or Carbon-12. The isotope is often written to the upper-left of the isotope's symbol: ${}^{23}\text{Na}$.

A neutral atom has the same number of protons and electrons, but not all atoms are electrically neutral. An atom that has lost or gained one or more electrons is called an ion. Ions are indicated by placing a charge on the upper-right of the element symbol: Na^{2+}.

In nuclear reactions, it is often useful to see element, isotope, and ion information at the same time. The symbol ${}^{59}_{27}\text{Co}^{-}$ would refer to a cobalt-59 atom that has one extra electron.

20.2 Radioactive Decay

20.2.1 Types of Radioactive Decay

Alpha Decay

Alpha decay is when an atom emits an α particle (alpha particle). *α-particles* are really just a helium nucleus. That is, in alpha decay, a group of two protons and two neutrons spontaneously breaks off of the nucleus.

Example 20.2.1

Problem: A Uranium-238 atom undergoes alpha decay.

1. Write a formula for this reaction.
2. How much energy is released in this reaction?

Solution: We know that the Uranium-238 atom will break up into an alpha particle and another unknown atom, so we begin by writing the equation with a placeholder element:

$$^{238}_{92}U \longrightarrow ^{4}_{2}\alpha^{2+} + X$$

Applying conservation laws allows us to determine the unknown element's isotope and atomic number. This element must have a mass-number of 234, as well as 90 protons. Likewise, since the initial atom was electrically neutral, the final product must have a charge of -2 in order to cancel the positive charge of the alpha particle. Since Element 90 is Thorium, we can write:

$$\boxed{^{238}_{92}U \longrightarrow ^{4}_{2}\alpha^{2+} + ^{234}_{90}Th^{2-}}$$

In order to determine the amount of energy released in this process, we must find the difference in mass between the two sides of the equation.

$$m_{U238} = 238.05078826\text{u}$$

$$m_{He4} = 4.002602\text{u}$$

$$m_{Th234} = 234.0436\text{u}$$

Subtracting the two sides shows:

$$\Delta m = m_{U238} - (m_{He4} + m_{Th234}) = 238.05078826\text{u} - (4.002602\text{u} + 234.0436\text{u}) = 0.00458626\text{u}$$

We can convert atomic mass units to kilograms since we know $1\text{u} = 1.6605402 \times 10^{-27}\text{kg}$:

$$\Delta m = 0.00458626\text{u} \times \frac{1.6605402 \times 10^{-27}\text{kg}}{1\text{u}} = 7.616 \times 10^{-30}\text{kg}$$

Applying $E = mc^2$ yields:

$$E = mc^2 = 7.616 \times 10^{-30}\text{kg} \cdot (3 \times 10^8\text{m/s})^2 = \boxed{6.854 \times 10^{-13}\text{J}} \tag{20.1}$$

Beta Decay

Gamma Decay

Gamma decay is a type of radioactive decay that occurs when an atomic nucleus emits a gamma ray - that is, a high-energy photon. Gamma decay can occur on its own or as a result of other types of radioactive decay, such as alpha or beta decay.

During gamma decay, the atomic nucleus undergoes a transition from a higher energy state to a lower energy state, releasing energy in the form of a gamma ray. Gamma rays are extremely energetic and can penetrate through many materials, including human tissue, making them potentially harmful to living organisms and hard to shield against. However, they are also useful in many areas, such as medical imaging, radiation therapy, and nuclear physics.

Gamma decay does not change the atomic mass or atomic number of the nucleus, because gamma rays have no mass or charge. However, it can lead to the emission of other particles, such as electrons, as a result of the interaction between the gamma ray and the surrounding matter. Gamma decay is often accompanied by other types of radioactive decay, such as alpha or beta decay, as the unstable nucleus decays toward a more stable configuration.

20.2.2 Half-Life

Half-life represents the time it takes for the number of radioactive atoms in a sample to decrease by half, due to the decay of those atoms, and it is usually denoted with the symbol $t_{1/2}$. Each isotope has its own specific half-life, some of which can be found in (Insert reference here). These half-lifes can range from fractions of a second to many billions of years. Some isotopes with short half-lives are used in medical imaging, while others with long half-lives are used in geology and astrophysics.

The half-life of a radioactive isotope can be used to determine the age of a rock or other geological material by measuring the ratio of the parent isotope to its decay products, since the isotope will decay according to the following formula:

$$A = A_0 e^{\frac{-\ln(2)\cdot t}{t_{1/2}}} \tag{20.2}$$

20.3 Fission and Fusion

20.3.1 Fission

20.3.2 Fusion

Chapter 21

Relativity

21.1 Special Relativity

21.1.1 Length Contraction

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}} \tag{21.1}$$

21.1.2 Time Dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{21.2}$$

21.1.3 Mass Increase

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{21.3}$$

Appendices

Appendix A

Math Skills

A.1 Scientific Notation

- Scientific Notation always has three parts: the *coefficient*, the *base*, and the *exponent*:

$$\text{Coefficient} \rightarrow 6.022 \times \underset{\underset{\text{Base}}{\uparrow}}{10}^{23} \leftarrow \text{Exponent}$$

- In scientific notation the base is always 10.
- A negative in front of the coefficient means the whole number is negative.
- A negative exponent means the number is very small (close to zero).
- The exponent counts how many places the decimal moved, NOT the number of zeroes.
- When comparing numbers in scientific notation, look at (in order):
  1. Negatives in front of the coefficient.
  2. Exponents
  3. Coefficients
- To multiply, multiply coeffients, then ADD exponents.
- To divide, divide coefficients, then SUBTRACT exponents.
- To raise to a power, raise the coefficient to the power, then MULTIPLY exponents.
- To enter scientific notation on most calculators use the "EE" key. 6.022×10^{23} is entered as 6.022E23. Calculator notation should <u>**never**</u> be handwritten.
- Metric Prefixes are really just scientific notation:

Prefix	Letter	Power of 10
nano	n	$\times 10^{-9}$
micro	μ	$\times 10^{-6}$
milli	m	$\times 10^{-3}$
centi	c	$\times 10^{-2}$
deci	d	$\times 10^{-1}$
Deka	D	$\times 10^{1}$
Hecto	H	$\times 10^{2}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$

A.2 Algebra

To study physics, you should have some knowledge of algebra. Though this appendix is too small to include all algebraic techniques, there are some things that are repeated often in physics. These commonly recurring algebraic themes are highlighted here.

A.2.1 Solve for a Variable

Solving for a variable is one of the most fundamental skills in algebra and physics. Whether you're isolating a variable to find a force, velocity, or time, this process involves rearranging equations using inverse operations.

Example A.2.1

Problem: Solve the Equation $2x + 3 = 12x + 1$ for x.

Solution: To solve this equation, we begin by combining like terms. Terms involving x should be moved to one side, while terms involving only numbers are moved to the other.

$$2x + 3 = 12x + 1$$
$$-10x + 3 = 1$$
$$-10x = -2$$

Now, dividing by -10 gives:

$$x = 5$$

Solve for a Variable in the Denominator of a Fraction

Sometimes, the variable you're solving for appears in the denominator. This can often be confusing at first, but the trick is to eliminate the fraction by multiplying both sides by the denominator.

Example A.2.1.2

Problem: Solve for R in the equation:

$$V = \frac{I}{R}$$

Solution: Multiply both sides by R to eliminate the denominator:

$$V \cdot R = I$$

Then divide both sides by V to isolate R:

$$R = \frac{I}{V}$$

Solve for a Variable using the Quadratic Formula

In some physics problems, especially those involving kinematics or energy, you might encounter equations that take the form of a quadratic:

$$ax^2 + bx + c = 0$$

The solution is given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example A.2.1.3

Problem: Solve for t:

$$0 = 5t^2 - 20t + 15$$

Solution: Identify $a = 5$, $b = -20$, and $c = 15$. Plug into the quadratic formula:

$$t = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(5)(15)}}{2(5)} = \frac{20 \pm \sqrt{400 - 300}}{10} = \frac{20 \pm \sqrt{100}}{10}$$

$$t = \frac{20 \pm 10}{10} = 3 \text{ or } 1$$

It should be noted that negative answers can arise in physics, and negatives often indicate direction. However, context tells you which solutions are meaningful. While a negative velocity may just mean an object is traveling opposite the expected direction, a negative time is likely meaningless.

A.2.2 Solve a System of Equations

Many physics problems involve more than one equation and variable. These systems can be solved using several algebraic techniques.

Solve a System of Equations by Combination

Also called the addition or elimination method, this approach involves adding or subtracting equations to eliminate one variable. This is particularly useful when coefficients are opposites or can easily be made opposites.

Example A.2.1.4

Problem: Solve the system:

$$2x + 3y = 12$$
$$4x - 3y = 6$$

Solution: Add the equations to eliminate y:

$$\begin{array}{rl} & 2x + 3y = 12 \\ + & 4x - 3y = 6 \\ \hline & 6x \qquad = 18 \\ & \qquad x = 3 \end{array}$$

Substitute $x = 3$ into the first equation:

$$2(3) + 3y = 12 \Rightarrow 6 + 3y = 12 \Rightarrow y = 2$$

Solve a System of Equations by Substitution

This method is ideal when one of the equations is already solved for one variable, or can easily be rearranged to do so.

Example A.2.1.5

Problem: Solve the system:

$$\begin{aligned} y &= 2x + 1 \\ 3x + y &= 16 \end{aligned}$$

Solution: Substitute the expression for y into the second equation:

$$3x + (2x + 1) = 16$$

$$5x + 1 = 16$$

$$5x = 15$$

$$x = 3$$

Now substitute back to find y:

$$y = 2(3) + 1 = 7$$

Solve a System of Equations using Matrices

Matrices are useful for solving larger systems of equations, especially in physics problems involving circuits or forces in multiple directions. If you have three or more variables an equal number of equations, using a matrix to solve the problem keeps your work organized as well. You may use any of the following operations:

- Multiply or divide any row by a constant.
- Add two rows together, and replace either row with the result.
- Swap the order of rows.

Example A.2.1.5

Problem:
Solve the system using matrices:

$$\begin{aligned} 2x + 3y + z &= 8 \\ x + y - z &= 1 \\ 3x + 2y + z &= 15 \end{aligned}$$

Solution: Begin by writing the equations as an augmented matrix:

$$\begin{bmatrix} 2 & 3 & 1 & 8 \\ 1 & 1 & -1 & 1 \\ 3 & 2 & 1 & 15 \end{bmatrix}$$

First, we can multiply the 2nd row by -2.

$$R_2 = -2R_2 \longrightarrow \begin{bmatrix} 2 & 3 & 1 & 8 \\ -2 & -2 & 2 & -2 \\ 3 & 2 & 1 & 15 \end{bmatrix}$$

Now we add Row 1 to Row 2:

$$R_2 = R_1 + R_2 \longrightarrow \begin{bmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 6 \\ 3 & 2 & 1 & 15 \end{bmatrix}$$

Next, we multiply Row 1 by 3 and row 3 by -2:

$$\begin{matrix} R_1 = 3R_1 \\ R_3 = -2R_3 \end{matrix} \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & 1 & 3 & 6 \\ -6 & -4 & -2 & -30 \end{bmatrix}$$

Now we can add Row 1 to Row 3:

$$R_3 = R_1 + R_3 \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & 1 & 3 & 6 \\ 0 & 5 & 1 & -6 \end{bmatrix}$$

Multiplying row 2 by -5 gives:

$$R_2 = -5R_2 \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & -5 & -15 & -30 \\ 0 & 5 & 1 & -6 \end{bmatrix}$$

Adding Row 2 and row 3 yields:

$$R_3 = R_2 + R_3 \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & -5 & -15 & -30 \\ 0 & 0 & -14 & -36 \end{bmatrix}$$

Now we can divide Row 3 by -14:

$$R_3 = \frac{R_3}{-14} \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & -5 & -15 & -30 \\ 0 & 0 & 1 & \frac{18}{7} \end{bmatrix}$$

This means that $z = \frac{18}{7}$. Continuing to solve the problem, we can multiply row 3 by 15 and add it to row 2 to to get:

$$R_2 = R_2 + 15R_3 \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & -5 & 0 & \frac{60}{7} \\ 0 & 0 & 1 & \frac{18}{7} \end{bmatrix}$$

Now, divide row 2 by -5:

$$R_2 = \frac{R_2}{5} \longrightarrow \begin{bmatrix} 6 & 9 & 3 & 24 \\ 0 & 1 & 0 & \frac{-12}{7} \\ 0 & 0 & 1 & \frac{18}{7} \end{bmatrix}$$

Which means we now know $y = \frac{-12}{7}$. We can now use row 2 and row 3 to eliminate variables in row 1:

$$R_1 = R1 + (-9R_2) + (-3R_3) \longrightarrow \begin{bmatrix} 6 & 0 & 0 & \frac{222}{7} \\ 0 & 1 & 0 & \frac{-12}{7} \\ 0 & 0 & 1 & \frac{18}{7} \end{bmatrix}$$

Finally, dividing Row 1 by 6 gives:

$$R_1 = \frac{R_1}{6} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{37}{7} \\ 0 & 1 & 0 & \frac{-12}{7} \\ 0 & 0 & 1 & \frac{18}{7} \end{bmatrix}$$

A.3 Proportional Reasoning

A.3.1 Proportional Reasoning

Proportional reasoning is a powerful tool in physics. Often, you don't need to plug in numbers to figure out how a change in one variable affects another. You only need to understand how variables are related in a formula.

The idea: If a formula contains multiple variables, and you know how one variable changes, you can predict how the output will change—assuming all other variables stay constant or have known changes.

The force of gravity between two masses is given by:

$$F = G\frac{m_1 m_2}{r^2}$$

A planet has twice Earth's mass and three times Earth's radius. If the gravitational force on a mass on Earth is F_E, what is the gravitational force on the same mass on the new planet, in terms of F_E?

Solution: Let Earth have mass M, and radius R, so:

$$F_E = G\frac{mM}{R^2}$$

On the new planet:

$$F' = G\frac{m(2M)}{(3R)^2} = G\frac{2mM}{9R^2} = \frac{2}{9}F_E$$

Answer: $\frac{2}{9}F_E$

Comment: The trick is to substitute the new values into the formula and simplify. Don't calculate anything you don't need.

The period T of a pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

If the length L of a pendulum is quadrupled, by what factor does the period T change?

Solution:

$$T' = 2\pi\sqrt{\frac{4L}{g}} = 2\pi\cdot 2\sqrt{\frac{L}{g}} = 2T$$

Answer: The period doubles.

Comment: When a variable is inside a square root, its effect is weaker. Quadrupling L only doubles T.

The kinetic energy of an object is:

$$K = \frac{1}{2}mv^2$$

If the velocity of the object triples, how does its kinetic energy change?

Solution:

$$K' = \frac{1}{2}m(3v)^2 = \frac{1}{2}m \cdot 9v^2 = 9K$$

Answer: The kinetic energy increases by a factor of 9.
Comment: Pay attention to exponents—tripling v squares the effect in v^2.

The electric force between two charges is given by:

$$F = k\frac{q_1 q_2}{r^2}$$

If both charges are doubled and the distance is halved, what happens to the electric force?

Solution:

$$F' = k\frac{(2q_1)(2q_2)}{(r/2)^2} = k\frac{4q_1 q_2}{r^2/4} = k\frac{4q_1 q_2 \cdot 4}{r^2} = 16F$$

Answer: The force increases by a factor of 16.
Comment: Changing more than one variable? Just multiply each effect together.

The pressure at the bottom of a fluid is given by:

$$P = \rho g h$$

If the height of the fluid is doubled and the fluid is twice as dense, what happens to the pressure?

Solution:

$$P' = (2\rho)g(2h) = 4\rho g h = 4P$$

Answer: The pressure increases by a factor of 4.
Comment: Linear relationships like this are easy to reason through—just multiply the scaling factors.

A.4 Trigonometry

A.5 Radians and Arc Length

A.5.1 Radians

Just like there are many different units that measure distance (meters, feet, inches, miles, furlongs, etc.), there are also different ways of measuring angles. You are probably already familiar with degrees. A right angle is 90°, and a full circle is 360°. When calculating arc length or using the angular kinematic equations, the standard units for angles are *radians*[1] There are 2π radians in a complete circle, so,

$$360^\circ = 2\pi\,\text{radians}$$

This equation can be used to convert an angle from radians to degrees or vice-versa.

Example A.5.1

Problem: An angle measures 34°. What is this angle in radians?

Solution: Begin by creating a conversion factor. In this case, since we have degrees and want radians, we create a fraction with 2π radians on top of the fraction, and 360° on the bottom. Multiplying by this fraction gives:

$$34^\circ \times \frac{2\pi\,\text{rad}}{360^\circ} = \frac{17\pi}{90}\text{rad} \approx 0.593\,\text{rad}$$

It should be noted that the fraction $\frac{2\pi\,\text{rad}}{360^\circ}$ can easily be reduced to $\frac{\pi\,\text{rad}}{180^\circ}$. Using this as your conversion factor will yield the same results. It is also often easier and more accurate to leave measurements involving radians in terms of π.

Example A.5.2

Problem: An angle measures $\frac{\pi}{2}$rad. What is this angle in degrees?

Solution: Again, we begin by creating a conversion factor. Because we have a measurement in radians and are asked to find degrees, we create a fraction with 360° on top of the fraction, and 2π radians on the bottom:

$$\frac{\pi}{2} \times \frac{360^\circ}{2\pi\,\text{rad}} = 90^\circ$$

[1]It should be noted that strictly speaking, radians are not a unit; since the definition of a radian has to do with a ratio, all numbers with radians as the unit are actually unitless.

A.5.2 Arc Length

The distance along the circumference of a circle that corresponds to an internal angle of the circle is called *Arc Length*. Though arc-length is a measurement of length, the symbol used for Arc Length is s.

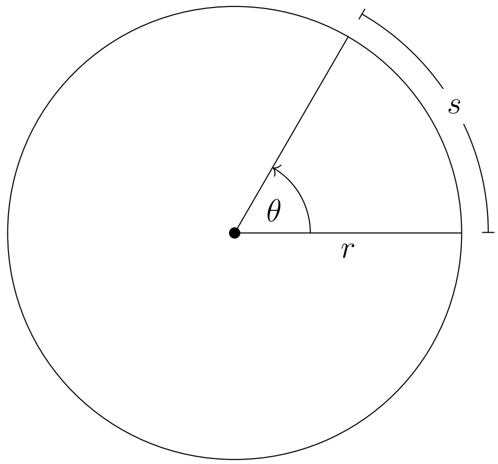


Figure A.1: The relationship between arc-length, radius, and angle

Arc Length can be found using the following equation:

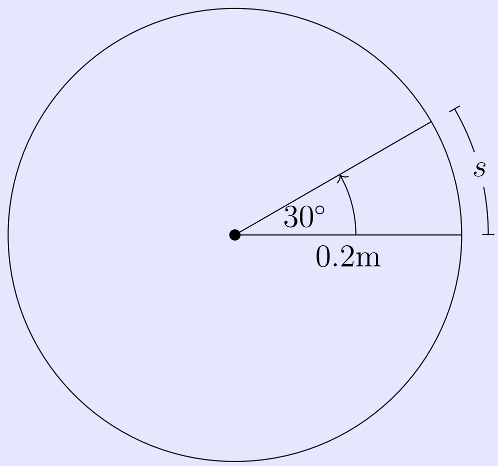
$$s = r\theta \tag{A.1}$$

where r is the radius of the circle and θ is the internal angle, measured in radians. Additionally, while meters are the proper SI units for these measurements, this formula will work with any units of length as long as both arc-length, s, and radius, r are measured using the same units.

Example A.5.2

Problem: Find the arc length of 30° of a circle with a radius of 0.2 meters.

Solution: Begin by drawing a diagram:



In order to find arc length, we must first convert the angle from degrees to radians:

$$\theta = 30^\circ \times \frac{2\pi\text{rad}}{360^\circ} = \frac{\pi}{6}\text{rad}$$

Now, arc length can be found using equation A.1.

$$s = r\theta = (0.2\text{m})(\frac{\pi}{6}\text{rad}) \approx 0.105\text{m}$$

Appendix B

Reference Tables

B.1 Greek Letters

Name	**Captial**	**Lower Case**	**Alternate versions**
alpha	A	α	
beta	B	β	
gamma	Γ	γ	
delta	Δ	δ	
epsilon	E	ε	ϵ
zeta	Z	ζ	
eta	H	η	
theta	Θ	θ	$\mathit{\Theta}$, ϑ
iota	I	ι	
kappa	K	κ	
lambda	Λ	λ	$\mathit{\Lambda}$
mu	M	μ	
nu	N	ν	
xi	Ξ	ξ	$\mathit{\Xi}$
omicron	O	o	
pi	Π	π	$\mathit{\Pi}\varpi$
rho	P	ρ	ϱ
sigma	Σ	σ, ς	
tau	T	τ	
upsilon	Υ	υ	$\mathit{\Upsilon}$
phi	Φ	ϕ	$\mathit{\Phi}$, φ
chi	X	χ	

psi	Ψ	ψ	$\mathit{\Psi}$
omega	Ω	ω	$\mathit{\Omega}$

B.2 Musical Notes and Frequencies

Table B.2: Frequencies of Musical Notes, in Hz

Octave:	0	1	2	3	4	5	6	7	8
C	16.35	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00	4186.01
C♯/D♭	17.32	34.65	69.30	138.59	277.18	554.37	1108.733	2217.46	4434.92
D	18.35	36.71	73.42	146.83	293.66	587.33	1174.66	2349.32	4698.64
D♯/E♭	19.45	38.89	77.78	155.56	311.13	622.25	1244.51	2489.02	4978.03
E	20.60	41.20	82.41	164.81	329.63	659.26	1318.51	2637.02	5274.04
F	21.83	43.65	87.31	174.61	349.23	698.46	1396.91	2793.83	5587.65
F♯/G♭	23.12	46.25	92.50	185.00	369.99	739.99	1479.98	2959.96	5919.91
G	24.50	49.00	98.00	196.00	392.00	783.99	1567.98	3135.96	6271.93
G♯/A♭	25.96	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44	6644.88
A	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00
A♯/B♭	29.14	58.27	116.54	233.08	466.16	932.33	1864.66	3729.31	7458.62
B	30.87	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07	7902.13

B.3 Densities of Common materials

The table below shows the density of some common materials:

Table B.3: Table of Densities of Common Materials

Material	**Density** $(\frac{kg}{m^3})$
Ethanol	789
Ice	917
Olive Oil	929
Water (pure)	1000
Water (ocean)	1025

B.4 Common Indices of Refraction

Table B.5: Table of Common Indices of Refraction

Material	**Index of Refraction**
Air	1.000273
Acrylic	1.495
Cubic Zirconium	2.15-2.18
Diamond	2.417
Crown Glass	1.485-1.755
Flint Glass	1.60-1.62
Vacuum (Empty Space)	1
Vegetable Oil	1.47
Water	1.333

Source: Wikipedia contributors. (2021, January 8). List of refractive indices. In Wikipedia, The Free Encyclopedia

Table B.7: Table of Specific Heat Capacities of Common Materials

Material	Specific Heat Capacity
ice	df
water	34
Water Vapor	43

B.5 Physical Constants

Table B.9: Table of Common Physical Constants

Quantity	Symbol	Value
Charge of an electron	e^-	$1.6 \times 10^{-19}\mathrm{C}$
Speed of light	c	$2.99792458 \times 10^{8}\mathrm{m/s}$
Universal Gravitational Constant	G	$6.67 \times 10^{-11}\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$

Glossary

Acceleration The rate of change of velocity with respect to time. 11

Acceleration due to Gravity The acceleration of an object caused by a planet's gravity. Earth's gravity is typically measured as $9.81\,\mathrm{m/s^2}$. 13

Adiabatic Process A thermodynamic process in which no heat is exchanged with the surroundings ($Q = 0$). Changes in internal energy result only from work done on or by the system. Adiabatic processes are common in rapid expansions or compressions, such as in gas dynamics and atmospheric science. 93

Alpha Decay A type of radioactive decay in which an unstable nucleus emits an alpha particle (${}^4_2\mathrm{He}$), reducing its atomic number by 2 and mass number by 4. Alpha decay occurs in heavy elements such as uranium and radium. 112

Alpha Particle A positively charged particle consisting of two protons and two neutrons, identical to a helium-4 nucleus (${}^4_2\mathrm{He}$). Alpha particles are emitted in alpha decay and have low penetration power, being stopped by paper or human skin. 112

Amplitude The maximum displacement of a wave from its equilibrium position. It represents the wave's energy and is measured in units appropriate to the wave type (e.g., meters for mechanical waves, volts per meter for electromagnetic waves). 53

Antinode A point in a standing wave where the displacement is maximum due to constructive interference. Antinodes alternate with nodes along the medium and represent points of highest oscillation in systems such as vibrating strings and sound waves. 64

Conduction The transfer of heat through a material without the bulk movement of matter. It occurs due to collisions between particles in a solid or fluid and is governed by Fourier's law, where the heat transfer rate is proportional to the temperature gradient. 88

Convection The transfer of heat through the bulk movement of a fluid (liquid or gas). Convection can be natural (due to density differences) or forced (due to external forces such as fans or pumps). It plays a key role in atmospheric circulation, ocean currents, and heat dissipation in engineering systems. 88

Conversion Factor A ratio used to convert a quantity expressed in one unit to another unit. 2

Cross Product A vector product of two vectors, resulting in a vector that is perpendicular to both of the original vectors. 6

Density A measure of mass per unit volume of a substance. It is mathematically expressed as $\rho = \frac{m}{V}$, where ρ (rho) is the density, m is the mass, and V is the volume. The SI unit of density is kilograms per cubic meter (kg/m^3), though grams per cubic centimeter (g/cm^3) is also commonly used. 82

Determinant A scalar value that can be computed from the elements of a square matrix and encodes certain properties of the matrix. 6

Dimensional Analysis A mathematical technique used to check the consistency of equations and to derive relationships between physical quantities by analyzing their dimensions. 1

Displacement A vector quantity representing the change in position of an object, with both magnitude and direction. 9

Distance A scalar quantity that represents the total path length traveled by an object, regardless of direction. 9

Dot Product A scalar product of two vectors, equal to the product of their magnitudes and the cosine of the angle between them. 5

Elastic Potential Energy The energy stored in an elastic material, such as a spring, when it is stretched or compressed. Elastic potential energy is calculated as $PE = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement from the equilibrium position. 37

Element A pure substance consisting of only one type of atom, characterized by a unique number of protons in its nucleus (atomic number, Z). Elements are the fundamental building blocks of matter and are organized in the periodic table. 111

Energy The capacity of a system to perform work or produce change, typically measured in joules (J). Energy exists in various forms, including kinetic, potential, thermal, and chemical energy, and can be transferred or transformed but not created or destroyed (according to the law of conservation of energy). 35

Force A vector quantity that represents a push or pull on an object, causing it to accelerate, decelerate, or maintain its motion. It is typically measured in newtons (N) and described by both its magnitude and direction. 26

Free Body Diagram A graphical representation used to visualize the forces acting on an object, where the object is typically represented as a point or a simple shape, and the forces are represented as arrows showing both magnitude and direction. 26

Free Fall The motion of an object under the influence of gravitational force only. 13

Frequency The number of complete wave cycles that pass a given point per unit time. It is denoted by f and is measured in hertz (Hz), where 1 Hz = 1/s. Frequency and period are related by $f = \frac{1}{T}$. 53

Friction A resistive force that occurs when two surfaces move or attempt to move across each other. It opposes motion and acts parallel to the surfaces in contact, with two main types: static friction (preventing motion) and kinetic friction (opposing ongoing motion). 30

Fundamental Forces The four basic forces that govern interactions in the universe: gravitational force, electromagnetic force, strong nuclear force, and weak nuclear force. These forces are responsible for the behavior of objects at both macroscopic and subatomic levels. 26

Gravitational Potential Energy The energy an object possesses due to its position in a gravitational field, commonly calculated as $PE = mgh$, where m is the object's mass, g is the acceleration due to gravity, and h is the height above a reference point. 36

Half-Life The time required for half of a radioactive substance to decay. It is a characteristic property of a radioactive isotope and is denoted by $t_{1/2}$. The remaining quantity of a radioactive substance after n half-lives is given by $N = N_0 \left(\frac{1}{2}\right)^n$. 113

Harmonic A frequency component of a wave that is an integer multiple of the fundamental frequency. Harmonics are observed in vibrating systems such as strings, air columns, and electrical circuits. The first harmonic (fundamental frequency) corresponds to the lowest frequency standing wave, while higher harmonics (overtones) correspond to additional modes of vibration. 65

Heat A form of energy transfer due to a temperature difference between two objects or systems. Heat flows spontaneously from higher temperature to lower temperature regions and is measured in joules (J) in the SI system. 88

Heat, Latent Heat of Fusion The amount of heat required to change a substance from a solid to a liquid at constant temperature and pressure. It is denoted by L_f and measured in joules per kilogram (J/kg). The latent heat of fusion represents the energy needed to overcome intermolecular forces during melting without changing the temperature. 91

Hooke's Law A principle describing the behavior of springs and other elastic materials, stating that the force exerted by a spring is proportional to its displacement from the equilibrium position. Mathematically, $F = -kx$, where F is the restoring force, k is the spring constant, and x is the displacement. 37

Hydrostatic Pressure The pressure exerted by a fluid at rest due to the force of gravity. It is given by the equation $P = \rho gh$, where P is the hydrostatic pressure, ρ is the fluid density, g is the acceleration due to gravity, and h is the depth of the fluid. Hydrostatic pressure increases with depth and acts equally in all directions at a given point in the fluid. 84

Ion An atom or molecule that has gained or lost one or more electrons, resulting in a net electric charge. A positively charged ion is called a cation, while a negatively charged ion is called an anion. Ions play essential roles in chemical reactions, conductivity, and biological processes. 111

Isobaric Process A thermodynamic process in which the pressure remains constant ($\Delta P = 0$). Any heat added to the system results in a change in both internal energy and volume. Isobaric processes are common in atmospheric and engineering applications, such as heating gases at constant pressure. 93

Isochoric Process A thermodynamic process in which the volume remains constant ($\Delta V = 0$), meaning no work is done by the system. Any heat transfer results in a change in internal energy. Isochoric processes are common in rigid, sealed containers where expansion is restricted. 93

Isothermal Process A thermodynamic process in which the temperature of the system remains constant ($\Delta T = 0$). Any heat added to the system is fully converted into work, maintaining a constant internal energy. Isothermal processes often occur in systems that exchange heat with a large thermal reservoir. 93

Isotope A variant of an element that has the same number of protons but a different number of neutrons. Isotopes of an element share the same chemical properties but may have different atomic masses and nuclear stability. Some isotopes are stable, while others are radioactive. 111

Jerk The rate of change of acceleration with respect to time, often referred to as the third derivative of position with respect to time. 20

Joule The SI unit of energy, represented by the symbol J. One joule is defined as the amount of work done when a force of one newton displaces an object by one meter in the direction of the force, equivalent to $1\,\mathrm{J} = 1\,\mathrm{N} \cdot \mathrm{m}$. 35

Kinetic Energy The energy an object possesses due to its motion, calculated as $KE = \frac{1}{2}mv^2$, where m is the object's mass and v is its velocity. Kinetic energy depends on both mass and speed, and it increases with the square of the speed. 35

Magnitude The size or quantity of a vector, representing its length regardless of direction. 2

Mechanical Energy The sum of kinetic and potential energy in a system, representing the energy associated with the motion and position of an object. Mechanical energy is conserved in an isolated system with no non-conservative forces, such as friction. 35

Medium The substance or material through which a mechanical wave travels. Examples include air for sound waves, water for ocean waves, and solid materials for seismic waves. 51

Node A point in a standing wave where there is no displacement due to destructive interference. Nodes remain stationary and occur at fixed positions along the medium in systems with standing waves, such as vibrating strings and air columns. 64

Period The time required for one complete cycle of a wave to pass a given point. It is denoted by T and is measured in seconds (s). The period is related to frequency by the equation $T = \frac{1}{f}$. 53

Potential Energy The stored energy in a system due to the position, arrangement, or state of its components. Potential energy can take various forms, including gravitational potential energy (due to height in a gravitational field) and elastic potential energy (due to stretching or compressing an elastic material). 36

Pressure, Absolute The total pressure measured relative to a perfect vacuum (zero pressure). It is given by $P_{\text{absolute}} = P_{\text{gauge}} + P_{\text{atm}}$, where P_{gauge} is the gauge pressure and P_{atm} is the atmospheric pressure. Absolute pressure is always positive and is used in many scientific and engineering calculations. 84

Pressure, Gauge The pressure measured relative to atmospheric pressure. It is defined as $P_{\text{gauge}} = P_{\text{absolute}} - P_{\text{atm}}$, where P_{absolute} is the absolute pressure and P_{atm} is the atmospheric pressure. Gauge pressure is positive when it is above atmospheric pressure and negative when it is below (such as in a vacuum). 84

Resonance A phenomenon that occurs when an external force matches the natural frequency of a system, causing it to oscillate with maximum amplitude. Resonance is observed in mechanical systems, electrical circuits, and acoustic waves, and is responsible for phenomena such as the amplification of sound in musical instruments. 64

Resultant A vector that represents the sum of two or more vectors. 4

Right Hand Rule, First A rule used to determine the direction of the cross product of two vectors. Point the index fingers of the right hand in the direction of the first vector and the middle in the direction of the second; the thumb points in the direction of the resultant vector. 8

Scalar A physical quantity that has only magnitude and no direction, such as temperature or mass. 2

SI System of Units The International System of Units, a standardized system of measurement based on seven base units including the meter, kilogram, second, and ampere. 1

Slope A measure of the steepness or incline of a line, calculated as the ratio of the vertical change to the horizontal change between two points on the line. 16

Slope-Intercept Format A way of expressing the equation of a straight line in the form $y = mx + b$, where m is the slope and b is the y-intercept. 16

Speed A scalar quantity representing the distance traveled per unit of time. 9

Spring Constant A parameter that quantifies the stiffness of a spring or elastic material. It is denoted by k and has units of force per unit length (e.g., N/m). The spring constant determines the proportionality between the restoring force exerted by the spring and its displacement from the equilibrium position, as described by Hooke's Law. 37

Thermal Radiation The transfer of heat in the form of electromagnetic waves, primarily infrared radiation. Unlike conduction and convection, thermal radiation does not require a medium for transfer. It follows Stefan-Boltzmann's law, which states that the power radiated by an object is proportional to the fourth power of its absolute temperature. 88

Unit Vector A vector with a magnitude of one, used to indicate direction without scale. 3

Vector A physical quantity that has both magnitude and direction, such as velocity or force. 3

Velocity The rate of change of displacement with respect to time, a vector quantity. 9

Wave A disturbance that transfers energy through space or a medium without a net movement of matter. Waves can be classified into mechanical and electromagnetic waves, depending on whether they require a medium for propagation. 51

Wave, Electromagnetic A wave that consists of oscillating electric and magnetic fields and does not require a medium to propagate. Electromagnetic waves travel at the speed of light in a vacuum and include radio waves, microwaves, infrared, visible light, ultraviolet, X-rays, and gamma rays. 52

Wave, Longitudinal A type of wave in which the disturbance is parallel to the direction of wave propagation. Examples include sound waves and pressure waves in fluids. Longitudinal waves consist of compressions and rarefactions. 51

Wave, Transverse A type of wave in which the disturbance is perpendicular to the direction of wave propagation. Examples include water waves and electromagnetic waves. In a transverse wave, crests and troughs represent areas of maximum displacement. 51

Wavelength The distance between two consecutive points in phase on a wave, such as crest to crest or trough to trough in a transverse wave. It is denoted by λ (lambda) and is measured in meters (m). The wavelength is related to wave speed and frequency by $v = f\lambda$. 53

Work A measure of energy transfer that occurs when an object is moved by a force. Calculated as $W = Fd\cos\theta$, where W is the work done, F is the applied force, d is the displacement, and θ is the angle between the force and displacement directions. 38

y-Intercept The point where a line crosses the y-axis of a graph, representing the value of y when x is zero. 16

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