Assignment 0.05 - Dot and Cross Products - KEY

1. Use the given vectors to calculate each of the following products:

$$\vec{A} = 2\hat{i} + 6\hat{j}$$
$$\vec{B} = 3\hat{i} + 4\hat{j}$$

Find:

(a) $\vec{A} \cdot \vec{B} = 2 \cdot 3 + 6 \cdot 4 = \boxed{30}$

(b) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 0 \\ 3 & 4 & 0 \end{vmatrix} = \hat{i} \cdot \begin{vmatrix} 6 & 0 \\ 4 & 0 \end{vmatrix} - \hat{j} \cdot \begin{vmatrix} 2 & 0 \\ 3 & 0 \end{vmatrix} + \hat{k} \cdot \begin{vmatrix} 2 & 6 \\ 3 & 4 \end{vmatrix}$

$= \hat{i} \cdot (6 \cdot 0 - 0 \cdot 4) - \hat{j} \cdot (2 \cdot 0 - 0 \cdot 3) + \hat{k} \cdot (2 \cdot 4 - 6 \cdot 3) = \boxed{0\hat{i} + 0\hat{j} - 10\hat{k}}$

(c) $\vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 2 & 6 & 0 \end{vmatrix} = \hat{i} \cdot \begin{vmatrix} 4 & 0 \\ 6 & 0 \end{vmatrix} - \hat{j} \cdot \begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} + \hat{k} \cdot \begin{vmatrix} 3 & 4 \\ 2 & 6 \end{vmatrix}$

$= \hat{i} \cdot (4 \cdot 0 - 0 \cdot 6) - \hat{j} \cdot (3 \cdot 0 - 0 \cdot 2) + \hat{k} \cdot (3 \cdot 6 - 4 \cdot 2) = \boxed{0\hat{i} + 0\hat{j} + 10\hat{k}}$

2. Use the given vectors to calculate each of the following products:

$$\vec{F} = 1\hat{i} + 2\hat{j} - 4\hat{k}$$
$$\vec{G} = -5\hat{i} - 3\hat{j} + 2\hat{k}$$

Find:

(a) $\vec{F} \cdot \vec{G} = 1 \cdot (-5) + 2 \cdot (-3) + (-4) \cdot 2 = \boxed{-19}$

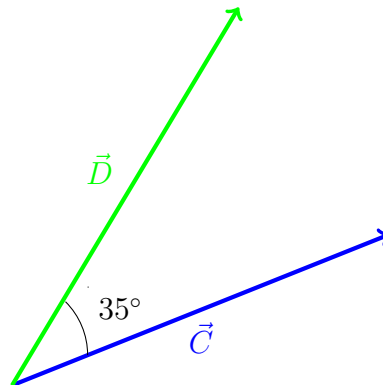(b) $\vec{G} \cdot \vec{F} = (-5) \cdot 1 + (-3) \cdot 2 + 2 \cdot (-4) = \boxed{-19}$

(c) $\vec{F} \times \vec{G} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -4 \\ -5 & -3 & 2 \end{vmatrix} = \hat{i} \cdot \begin{vmatrix} 2 & -4 \\ -3 & 2 \end{vmatrix} - \hat{j} \cdot \begin{vmatrix} 1 & -4 \\ -5 & 2 \end{vmatrix} + \hat{k} \cdot \begin{vmatrix} 1 & 2 \\ -5 & -3 \end{vmatrix}$

$= \hat{i} \cdot (2 \cdot 2 - (-3) \cdot (-4)) - \hat{j} \cdot (3 \cdot 0 - 0 \cdot 2) + \hat{k} \cdot (3 \cdot 6 - 4 \cdot 2) = \boxed{0\hat{i} + 0\hat{j} + 10\hat{k}}$

(d) $(\vec{F} \times \vec{G}) \cdot F$

3. In the following diagram, $|\vec{C}| = 5m$ and $|\vec{D}| = 5.8m$

Find:

(a) $\vec{C} \cdot \vec{D}$

(b) $|\vec{C} \times \vec{D}|$

4. Use the Right Hand Rule to determine the direction of the cross-product $\vec{L} \times \vec{M}$ in each of the following situations. *(Note: All answers will be "Toward the left of the page," "Toward the right of the page," "Toward the top of the page," "Toward the bottom of the page," "Into the page," or "out of the page.")*

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