Scivault Physics

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Contents

Chapter 1

Introduction

1.1 Dimensional Analysis and SI units

1.1.1 SI Units

The **SI** system of units is the standard used by many scientists throughout the world. There are seven *fundamental* or *base* quantities from which all other measurements are derived. These quantities are listed below:

Table 1.1: **SI Units**

Quantity	**Unit**	**Unit Symbol**
time	second	s
length	meter	m
mass	kilogram	kg
electrical current	Ampere	A
temperature	Kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Of these quantities, mass, time and length are quite common. Thus, this system is sometimes called the MKS (meter, kilogram, second) system. In order to use any equations, all measurements must have correct units. For example, if a time is expressed in hours, it must first be converted into seconds before any calculations can be attempted.

1.1.2 Dimensional Analysis

Dimensional analysis is the process in which the units associated with quantities create *derived units*. For instance, when a distance is divided by a time, the units will be $\frac{m}{s}$ (read *meters per second*).

Dimensional analysis is an important part of solving physics problems. Often, correct dimensional analysis can help you determine if a problem has been solved correctly. One should not even attempt to calculate an answer to a problem until the correct units have been verified.

1.1.3 Unit Conversions

Often you will find that you need to convert a measurement from one unit to another. In order to do this, you must use a *conversion factor.* A conversion factor is a fraction that is based upon a statement of equality. For instance, since 60 seconds = 1 minute, a conversion factor will look like either $\frac{60s}{1min}$ or $\frac{1min}{60s}$. You should chose the version of the conversion factor that eliminates the units that you wish to convert.

Example 1.1.3

Problem: Convert 7.241 hours into seconds.

Solution: Begin by converting 7.241 hours into minutes. Since 60 minutes = 1 hour,

$$7.241\cancel{hr}\frac{60min}{1\cancel{hr}} = 434.46min$$

Knowing that 1 minute = 60 seconds, we can use a second conversion factor to obtain seconds:

$$434.46\cancel{min}\frac{60s}{1\cancel{min}} = \boxed{26067.6s}$$

This problem could be solved in one step if you know that 1 hour = 3600 seconds.

Example 1.1.3.2

Problem: A car travels with a speed of 20 m/s. What is this in miles per hour?

Solution: We begin by converting meters per second into meters per hour:

$$20\frac{m}{\cancel{s}} \cdot \frac{3600\cancel{s}}{1hr} = 72000\frac{m}{hr}$$

We also need to know that 1 mile = 1609.34 meters:

$$72000\frac{\cancel{m}}{hr} \cdot \frac{1mile}{1609.34\cancel{m}} \approx 44.739\frac{miles}{hr}$$

1.2 Vectors and Scalars

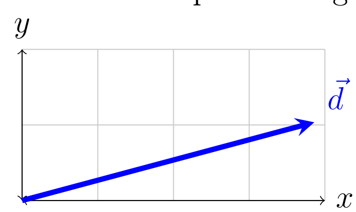
In the study of physics, there are two types of quantities that we will deal with on a regular basis: *scalars* and *vectors*.

A **scalar** is a quantity that you are already most likely very familiar with, as it is just a number; scalars have only a *magnitude* (a number that represents how big or strong it is), and can sometimes include units. Examples of scalars might be the number of people in a room, the mass of a car, or your age in years.

Vectors are different from scalars because in addition to a magnitude, they contain a direction as well. Examples of vectors might include 50 feet to the north, 5 m/s at a 33° angle, or 200 miles straight up. There are many ways of expressing vectors. Symbollically, they are often written with an arrow over them. For example, in the equation $\vec{F} = m\vec{a}$ both force and acceleration are vectors - meaning that both force and acceleration have a direction. Sometimes vectors will be expressed in **bold** typeface. Hence, the expression $\mathbf{F} = \mathrm{m}\ \mathbf{a}$ is equivalent to the expression shown above.

The direction for vectors in 1 dimension is easy - all you need is a positive or a negative. Usually, 1-dimensional motion takes place along the x-axis (left and right), but sometimes it will take place in the y- (forward and backward) and z- (up and down) dimensions. For the purposes of this book, positive is to the right and up unless otherwise stated. In two dimensions, a vector requires two pieces of information. One way of expressing a vector is in polar form. Polar form in two dimensions includes a magnitude of the vector and an angle, usually measured from the x-axis. There are several ways for writing this. 4 cm @ 15°, 4 cm $\angle 15°$ and 4 cm at 15° North of East all represent the following displacement vector:

Figure 1.1: A vector represented graphically.



The magnitude of the above vector is 4 cm. Mathematically, the magnitude of a vector can be written several ways, the most common being $|\vec{A}|$, though sometimes the magnitude of a vector can be written as the vector without the vector sign, as in A.

Unit vectors are vectors that have a length of one unit and are oriented along one axis. The unit vector for the x-direction is written as $\hat{\imath}$ (pronounced i-hat). $\hat{\imath}$ is a 1-unit long vector that is always parallel to the x-axis, and points in the direction of increasing x values. Likewise, the y-direction and z-direction unit vectors are written as $\hat{\jmath}$ and $\hat{k}$ respectively.

Because the surface of a paper is effectively 2-dimensional, it is very hard to draw lines that are oriented directly into or out of your paper. For this purpose, physicists have agreed to the following convention: vectors that point directly into your paper are notated by $\bigotimes$. Vectors that point directly out of your paper are shown by the symbol $\bigodot$.

Sometimes, vectors may be expressed in Cartesian coordinates. This vector could either be expressed as an ordered pair (or triple) with square brackets, such as [3, 4, 5] cm, or as a linear combination of the unit vectors shown above, such as $5\hat{\imath} + 12\hat{\jmath} + 3\hat{k}$ In each case, the distances in each direction are given by the numbers shown. The vector in figure 1.1 could be represented as: $\vec{d} \approx 3.86\hat{\imath} + 1.035\hat{\jmath}$.

When converting between polar and cartesian forms for two dimensional vectors, a little trigonometry shows:

$$x = r\cos(\theta) \tag{1.1}$$

$$y = r\sin(\theta) \tag{1.2}$$

$$r = \sqrt{x^2 + y^2} \tag{1.3}$$

$$\theta = \tan^{-1}(\frac{y}{x}) \tag{1.4}$$

In three dimensions, polar form comes in two types: **cylindrical** and **spherical**. In cylindrical coordinates, expressed as [r, θ, z], the above conversions are used, and the z-coordinate remains unchanged from Cartesian form. We will study spherical coordinates more in INSERT REFERNCE HERE!

1.3 Vector Mathematics

1.3.1 Vector Addition

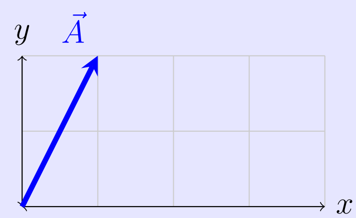
Graphical Addition of Vectors

When vectors are added graphically, they are added **head** to **tail**. This means that the arrowhead for a first vector becomes the origin for the second vector. The **resultant** vector is a straight line between the origin of the first vector and the head of the second.

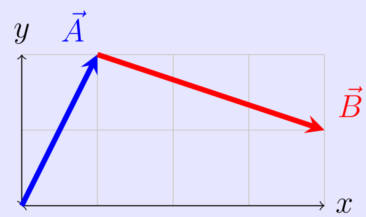
Example 1.3.1

Problem: If $\vec{A} = \hat{\imath} + 2\hat{\jmath}$ and $\vec{B} = 3\hat{\imath} - \hat{\jmath}$, find $\vec{C}$ graphically given $\vec{C} = \vec{A} + \vec{B}$

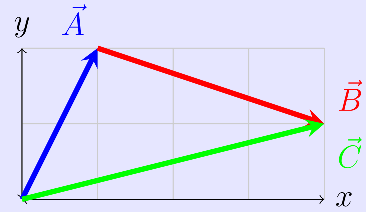
Solution: Begin by drawing $\vec{A}$:



Now use the head of $\vec{A}$ as the origin for $\vec{B}$:



The resulting vector is a straight line from the origin to the end of $\vec{B}$:



The resultant vector is given by $\boxed{\vec{C} = 4\hat{i} + \hat{j}}$

Mathematical Addition of Vectors

Mathematical addition of vectors in Cartesian form is quite easy - simply add corresponding x- and y- values together. For instance, in Example 1.3.1 we are given $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} - \hat{j}$. Adding these together would give $\vec{C} = (1+3)\hat{i} + (2-1)\hat{j} \longrightarrow \vec{C} = 4\hat{i} + \hat{j}$.

The easiest way to add vectors that are expressed in polar coordinates is to first convert them into cartesian coordinates using Equations (1.1) through (1.4) on page 4.

1.3.2 The Dot Product

In Cartesian Form

When multiplying vectors, it is sometimes necessary to obtain a scalar result. This is done through use of a dot product. A dot product is written as $\vec{A} \cdot \vec{B}$. This means to only multiply the components of the vectors that are in the same direction. In cartesian coordinates, this can be done by multiplying corresponding components, then adding the products.

Example 1.3.2

Problem: Given the vectors $\vec{Y} = 2\hat{i} + 3\hat{j}$ and $\vec{Z} = -4\hat{i} + 5\hat{j}$ find the dot product of vectors Y and Z.

Solution: Multiply coefficients from each vector, then add the products together:

$$2 \times (-4) + 3 \times 5 = -8 + 15 = \boxed{7}$$

In Polar Form

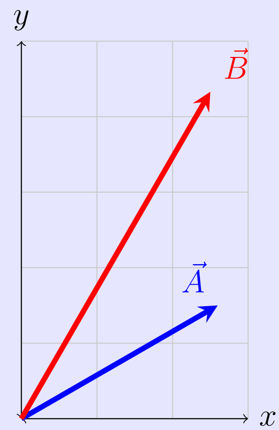
Often, vectors will be expressed in polar notation. If this is the case, the dot product can be found my multiplying the magnitude of the first vector times the magnitude of the second vector times the cosine of the angle between:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \tag{1.5}$$

Example 1.3.2.2

Problem: Given the vectors $\vec{D} = 3\angle 30°$ and $\vec{E} = 5\angle 60°$ find the dot product of vectors D and E.

Solution: We begin by graphing both vectors starting from a common origin:



Noticing that the angle between the two vectors is 30°, we can use equation 1.5 to calculate:

$$\vec{D} \cdot \vec{E} = |\vec{D}||\vec{E}| \cos\theta = (3)(5)\cos 30° \approx \boxed{12.990}$$

1.3.3 The Cross Product

Cross Products In Cartesian Form

Sometimes two vectors will be multiplied in such a way that they will result in another vector. This is called a **Cross Product**. A cross product is inherently a three-dimensional operation. A cross product can be found by calculating the determinant of a matrix:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.6}$$

The most common way to find the determinant of the above matrix is by using minors:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

This finding the determinate of each of the 2x2 matrices yields:

$$\vec{A} \times \vec{B} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

Example 1.3.3

Problem: Given the vectors $\vec{V} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{W} = -4\hat{\imath} + 5\hat{\jmath}$ find the cross product of vectors V and W.

Solution: Begin by creating a matrix, as shown in equation (1.6). Since both vectors lie in the X-Y plane, the Z-components for both vectors are zero:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix}$$

Now, use expansion by minors to create three two-by-two matrices:

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 5 & 0 \end{vmatrix} = \hat{\imath} \begin{vmatrix} 3 & 0 \\ 5 & 0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 3 \\ -4 & 5 \end{vmatrix}$$

Finding the determinate of each of the two-by-two matricies yields:

$$\vec{V} \times \vec{W} = \hat{\imath}(3 \cdot 0 - 0 \cdot 5) - \hat{\jmath}(2 \cdot 0 - 0 \cdot (-4)) + \hat{k}(2 \cdot 5 - 3 \cdot (-4))$$

Both the x-component and the y-components of this cross product are zero. Thus,

$$\boxed{\vec{V} \times \vec{W} = 22\hat{k}}$$

There are some important things you may wish to note.

1. The cross product is not commutative. That is, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$. In fact, the two cross products are in exactly opposite directions.

2. The cross product of two vectors is always a vector that is perpendicular to both of the original vectors.

Cross Products In Polar Form

When vectors are expressed in polar form, a cross product can often be found using two steps. To find the magnitude of the resulting vector, one would use the following equation:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta \tag{1.7}$$

In Equation 1.7, θ is the angle between the two vectors. To find the direction of the resulting vector, use the **First Right Hand Rule**. To use this rule, you point your index finger in the direction of the first vector to be multiplied. Your middle finger is then pointed in the direction of the second vector to be multiplied. Your thumb will point in the direction of the resultant vector, as shown in the image:

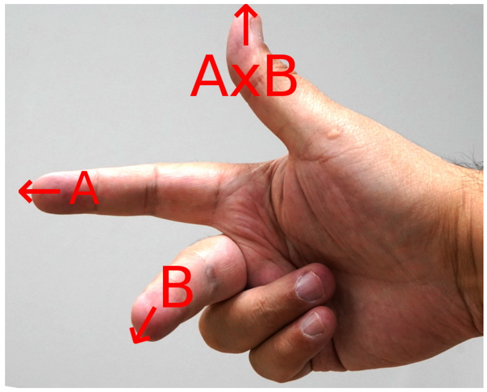


Figure 1.2: The First Right Hand Rule

The right had rule is easiest to use when the first two vectors are at 90° to each other. However, even if the first two vectors are not at right angles to each other, the resultant vector of a cross product will always be perpendicular to the plane of the first two vectors.

1.4 Exercises for Chapter 1

Section 1.1: Dimensional Analysis

1. What are the units for each of the following?

   (a) $\frac{5m}{2s}$

   (b) $\frac{10kg}{5s}$

   (c) $\frac{140kg \cdot 15m}{23s}$

   (d) $\frac{(\frac{15m}{15s})}{14s}$

Section 1.2: Vectors and Scalars

2. Given the vector $4\hat{\imath} - 3\hat{\jmath}$, represent the vector (a) graphically and (b) in polar form.

3. George walks 200 meters north, then walks 400 meters north. What is the vector from his starting position to his ending position in (a) cartesian form and (b) polar form (take north as 0 degrees).

Section 1.3: Vector Mathematics

4. Addition - cartesian

5. Addition - polar

6. Dot Product - Cartesian

7. Dot Product - polar

8. Cross Product - cartesian

9. Cross Product - polar

Chapter 2

Kinematics in One Dimension

2.1 Distance and Displacement

You are probably already familiar with the concept of **distance** - you might get in your car and drive a total of 1.2 miles to school, turning right after 0.45 miles, according to your car's odometer. Distance is a scalar that tells you how far something traveled. The symbol d usually represents distance.

While you may have traveled a total distance of 1.2 miles from your school, you are significantly less than 1.2 miles away from home; in fact, you are approximately 0.874 miles from home, following a direct path directly from your home to the school, at an angle of 59°(not worrying that this path might take you through someone's back yard or kitchen).

Displacement is a vector that tell you how far something is from the origin, and is independent of the path taken to get there. The displacement vector is commonly symbolized by $\vec{r}$ though sometimes it may be written as $\vec{d}$ or $\vec{x}$.

2.2 Average and Instantaneous Speed and Velocity

Speed is a scalar value that represents the change in distance per change in time of an object. Speed is usually represented with the symbol v, without the vector sign. You are probably already familiar with this quantity, since the speedometer on your family car measures speed. For the purposes of physics, speed has little value because it is a scalar that tells us nothing of direction. Much more useful is the concept called velocity. Velocity and speed are related much like distance and displacement.

Velocity is the change in displacement of an object per unit time, and as such is a vector. Positive velocities indicate that the object is moving forward, relative to the axis in question, and negative velocities generally mean that the object is moving backward, relative the the axis. The average velocity of an object is given by:

$$\overrightarrow{v_{avg}} = \frac{\Delta \vec{r}}{\Delta t} \tag{2.1}$$

Average velocity is useful if an object's velocity is not changing. However, many times it is more useful to talk about instantaneous velocity. Instantaneous velocity tells us how fast an object is moving at a given instant in time. In order to calculate instantaneous velocity, we must allow our time interval in the above formula to become infinitesimally small. In this case, a little calculus proves:

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{2.2}$$

Calculation of average velocity is rather straightforward, assuming you know both distance traveled and the time it took. If an object is not speeding up or slowing down during a specific time interval, the instantaneous velocity at any time during this interval is equal to the average velocity. If the object does speed up or slow down during the time interval in question, the average velocity and the instantaneous velocity at a certain time during the interval are not necessarily the same.

Example 2.2.1

Problem: You ride your bicycle in a straight line for a distance of 73 meters in 12.5 second. What is your average speed?

Solution: Begin by drawing a diagram:

$$v_{avg} = \frac{d}{t} = \frac{73m\ \hat{i}}{12.5s} = 5.84\frac{m}{s}\ \hat{i}$$

Example 2.2.2

Problem: A bicyclist rides his bike to the east. His position (in meters) is given by the following expression:

$$\vec{r} = (0.5t^2 + 4t)\hat{i}$$

a. What is his average velocity from t = 0 to t = 5 seconds?

b. What is his instantaneous velocity at t=3 seconds?

Solution:

a. The total displacement (in meters) after five seconds is given by:

$$\hat{r} = (0.5 \times (5s)^2 + 4 \times 5s)\ m\ \hat{i} = 32.5\ m\ \ \hat{i}$$

Thus, the average velocity is -

$$\overrightarrow{v_{avg}} = \frac{\vec{d}}{t} = \frac{32.5m\ \hat{i}}{5s} = \boxed{6.5\frac{m}{s}\ \hat{i}}$$

b. The instantaneous velocity of an object is found using a derivative with respect to time. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(0.5t^2 + 4t)\hat{i} = (t+4)\hat{i}$$

Evaluating this at t=3s yields:

$$\vec{v} = (3+4)\hat{i} = 7\frac{m}{s}\hat{i}$$

2.3 Relative Motion at Constant Velocity

2.4 Acceleration

2.4.1 Average Acceleration

Velocity is not always constant. For instance, when you are driving through a city, there are times when you might be going 30 mph, and there are times when you might be stopped at a streetlight. City driving requires you to speed up at some times, and slow down at other times âĂŞ your velocity changes as a function of time. Change in velocity per change in time is called acceleration. Keep in mind, both speeding up and slowing down are forms of acceleration. In the case that an object is traveling with a positive velocity, slowing down causes negative acceleration (or deceleration). Like velocity, acceleration comes in two basic types âĂŞ average and instantaneous. To find average velocity, we calculate the change in velocity per change in time:

$$\overrightarrow{a_{avg}} = \frac{\Delta\vec{v}}{\Delta t} \tag{2.3}$$

Keep in mind that this can be expressed as:

$$\overrightarrow{a_{avg}} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{\Delta t} \tag{2.4}$$

where v_f and v_i are final velocity and initial velocity, respectively. Sometimes final velocity is expressed as v and initial velocity is symbolized as v_0.

2.4.2 Instantaneous Acceleration

Instantaneous velocity is found by letting the time interval in question become infinitesimally small. A little calculus proves that:

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{2.5}$$

Combining this with equation 2.2, we find:

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} \tag{2.6}$$

Example 2.4

Problem: A car is traveling 20 m/s in the positive x direction. The driver sees a red light, and applies the brakes, causing the vehicle to come to a stop in 4 seconds. What is the average acceleration caused by the brakes?

Solution: Using the definition of Average Acceleration from Equation 2.4, we find:

$$\overrightarrow{a_{avg}} = \frac{\overrightarrow{v_f} - \overrightarrow{v_i}}{\Delta t} = \frac{0\frac{m}{s}\hat{i} - 20\frac{m}{s}\hat{i}}{4s} = -5\frac{m}{s^2}\hat{i}$$

2.5 The Kinematic Equations

Using the definitions above, and a little calculus (or a lot of algebra) we can prove the following four equations:

The Kinematic Equations

$$\vec{d} = \frac{\overrightarrow{v_f} + \overrightarrow{v_i}}{2}t \tag{2.7}$$

$$\vec{d} = \overrightarrow{v_i}t + \frac{1}{2}\vec{a}t^2 \tag{2.9}$$

$$\overrightarrow{v_f} = \overrightarrow{v_i} + \vec{a}t \tag{2.8}$$

$$\overrightarrow{v_i}^2 = \overrightarrow{v_i}^2 + 2\vec{a}\vec{d} \tag{2.10}$$

These equations enable you to solve the vast majority of kinematic problems. Keep in mind that these equations should only be applied in one direction at at time âĂŞ meaning that 2-dimensional and 3-dimensional problems will require you to split all the quantities into component vectors before you can solve these equations in each direction separately.

2.6 Vertical Motion and Gravity

One of the most common types of acceleration we experience every day is the acceleration due to gravity. Acceleration due to gravity is given a special symbol: g. On Earth, $g \approx 9.81\frac{m}{s^2}$. Keep in mind that this value is only useful for calculations involving gravity that take place on the Earth's surface. All planets, stars, and celestial bodies (in fact, all objects with any mass) have their own gravitational acceleration. Acceleration due to gravity on the moon, for instance, is $1.62\frac{m}{s^2}$.

Objects undergo free fall when they are allowed to continue to accelerate due to gravity until they impact something that breaks their fall (often this is the ground). In order to make the calculations easier, we often ignore air resistance.

Because gravity at the Earth's surface is downward, sign conventions become a little more important than previously. If upward is positive, g will have a negative sign attached to it to indicate the direction of acceleration.

Example 2.6

Problem: You are standing on top of a building. You drop a rock from the top of the building, and let it free fall until it hits the ground, 3.2 seconds later.

a. What is the height of the building?

b. An identical building is build on the moon. How long does it take the rock to fall in this case?

Solution:

a. To solve for distance, we apply equation 1.5.3 in the y-direction, and substitute ay = g and vi = 0 m/s,:

b. Because we know that vi = 0 m/s, and ay = gm = 1.62 m/s2, we can let the final term drop out of equation 1.5.3 to yield:

Chapter 3

Graphing Motion

3.1 Position vs Time Graphs

3.1.1 Constant Position

3.1.2 Constant Velocity

3.1.3 Constant Acceleration

3.2 Velocity vs Time Graphs

3.2.1 Constant Position

3.2.2 Constant Velocity

3.2.3 Constant Acceleration

3.3 Acceleration vs Time Graphs

3.3.1 Constant Position

3.3.2 Constant Velocity

3.3.3 Constant Acceleration

Chapter 4

Kinematics in Two Dimensions

4.1 Horizontal Launch Projectiles

4.2 Projectiles Launched at an Arbitray Angle

Chapter 5

Newtons Laws

5.1 Newton's First Law

> *Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.*
>
> *Every body continues in its state of being at rest or moving uniformly in a direction, except insofar as it is compelled to change its state by means of an imparted force.*
>
> – Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

You may have heard Sir Isaac Newton's first law of physics stated in different ways than the above. Often in grade school, students are taught a phrase beginning with "objects in motion...". Sometimes this law is called the "Law of Inertia". This is a very basic understanding of the complexity of this law. In fact, all non-accelerating systems are governed by this law. As long as the vector sum of the forces upon an object is zero, the object will continue in a state of uniform motion (remaining at rest is a type of uniform motion) until something causes the equilibrium of the system to be lost. Likewise, if an object is known to have an acceleration of zero, we can state that the vector sum of the forces is equal to zero. We can use this law to characterize non-accelerating systems:

5.2 Newton's Second Law

Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

The change in motion is proportional to the amount of force of motion imparted, and according to the straight line made by the force impressed.

-Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

5.3 Newton's Third Law

Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi.

For an action there is always an equal and opposite reaction: or the two bodies on each other are always equal and in opposite directions.

-Newton, Isaac. *Philosophiae Naturalis Principia Mathematica.* tr. J. Williamson

5.4 Applications of Newton's Laws

5.4.1 Friction

5.4.2 Elevators

5.4.3 Pulleys

Chapter 6

Work and Energy

6.1 Work

$$W = \vec{F} \cdot \vec{d} \tag{6.1}$$

6.2 Energy

6.2.1 Kinetic Energy

$$K = \frac{1}{2}mv^2 \tag{6.2}$$

6.2.2 Potential energy

Gravitational Potential Energy

$$U_g = mhg \tag{6.3}$$

Elastic Potential Energy

$$\overrightarrow{F_s} = -k\vec{x} \tag{6.4}$$

$$U_s = \frac{1}{2}kx^2 \tag{6.5}$$

While all real springs convert mechanical energy into heat when stretching or compressing, the amount of energy lost as heat (a process called hysteresis) is usually negligible. However, some stretchable objects, such as rubber bands, lose significant amounts of energy as heat. Thus, equations 6.4 and 6.5 are not applicable to these objects.

6.3 The Work-Energy Theorem

The *Work-Energy Theorem* states that doing work on an object causes that object's energy to change by the same amount as the work done. This means that an object has 8 Joules of energy, and 2 Joules of work is done on the object, the object will have 10 Joules of work at the end of the process. While this is often associated with a change in kinetic energy, the energy change associated with work can also be associated with gravitational potential energy, thermal energy, or any other form of energy.

6.4 The Law of Conservation of Energy

States that Energy cannot be created or destroyed (this isn't entirely true. This law will be tweaked in chapter INSERT REFERENCE HERE.

6.5 Springs

$$T_p = 2\pi\sqrt{\frac{m}{k}} \tag{6.6}$$

6.6 Pendulums

A pendulum is any weight on the end of an arm that is free to swing back and forth. You may have seen pendulums in old-fashioned clocks, and the swings at a park also act like a pendulum. When a pendulum swings at a small angle ($\theta \lessapprox 5\,\text{deg}$), the period of a pendulum is given by:

$$T_p = 2\pi\sqrt{\frac{l}{g}} \tag{6.7}$$

Notice that the period of a pendulum does not depend on the mass of the bob. The only two variables that affect its period (assuming a small angle) are the length of the arm and gravity.

Chapter 7

Impulse and Momentum

7.1 Momentum

Linear momentum is defined by the following equation:

$$\vec{p} \equiv m\vec{v} \tag{7.1}$$

where $\vec{p}$ is momentum, m is mass, and $\vec{v}$ is velocity. Determining an object's momentum can be extremely useful in solving problems involving collisions or explosions.

Example 7.1.1

Problem: A 1400 kg car is traveling at 12 m/s. What is its momentum?

Solution: Begin by drawing a diagram and identifying variables:

$$\vec{p} = m\vec{v} = 1400kg \times 12\frac{m}{s} = 16800\frac{kg \cdot m}{s}$$

You may notice that the units for momentum are $\frac{kg \cdot m}{s}$. This is read as "kilogram meters per second," and there is no special name for this unit.

7.2 Impulse

When a force is applied to an object for a certain amount of time, an *impulse* is delivered to that object. Impulse is defined by the following equation:

$$\vec{J} \equiv \vec{F}t \tag{7.2}$$

where $\vec{J}$ is impulse, $\vec{F}$ is force, and t is time. The units for impulse are $\mathrm{N} \cdot \mathrm{s}$, which are equivalent to $\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}}$.

7.3 The Impulse-Momentum Theorem

Just as work causes an object's energy to change, Impulse causes an object's momentum to change:

$$\vec{J} = \Delta\vec{p} \tag{7.3}$$

7.4 The Law of Conservation of Momentum

The law of momentum states that momentum cannot be created or destroyed. Thus, whatever momentum as system has in its initial state, plus any changes in momentum from outside the system (accounted for as impulse) must be the momentum it has in its final state.

Chapter 8

Circular Motion and Orbits

8.1 Centripetal Forces and Accelerations

8.1.1 Centripetal Force

We have already learned that an object in motion will continue to move in a straight line, assuming no forces are acting on the object. In order for an object to move along a circular path, there must therefore be a force acting on the object to keep it from moving in a straight line. If you whirl a mass on a string around in a circle, tension in the string keeps the mass from continuing to move in a straight line. As the Moon orbits the Earth, the gravitational attraction between the Moon and the Earth keeps the Moon in its orbit around the Earth. Any force that keeps an object moving along a circular path is called a **Centripetal Force** (Centripetal literally translates from Latin as "center seeking"). Any centripetal force can be described as:

$$F_c = \frac{mv^2}{r} \tag{8.1}$$

The direction of a centripetal force is always toward the center of the circle.

8.1.2 Centripetal Acceleration

When an object moves in a circle, even if its speed remains constant, its velocity is constantly changing due to its constant change in direction of motion. Thus the object must be constantly accelerating in a direction toward the center of the circle. Using Equation 8.1 and Newton's Second Law, it is possible to prove that centripetal acceleration is given by:

$$a_c = \frac{v^2}{r} \tag{8.2}$$

Just as centripetal force is always directed toward the center of the circle, centripetal acceleration is also always directed toward the center of the circle.

Example 8.1.2

Problem: A children's toy consists of a 0.5kg ball attached to the end of a light 0.3-meter-long rope. A child grabs the toy from the end of the rope and swings the ball around in a circle above his head with a tangential speed of 2 m/s. What is the tension in the rope?

Solution: In order to keep moving in a circle, tension in the rope must act as the centripetal force. Therefore, the tension in the rope is given by:

$$F_T = F_c = \frac{mv^2}{r} = \frac{0.5kg \cdot (2m/s)^2}{0.3m} \approx \boxed{6.667N}$$

Example 8.1.2.2

Problem: A toy car travels around a loop of diameter 0.3 meters. What is the minimum speed the car needs to travel in order to make it around the loop?

Solution: When the car is traveling around the loop, it is most likely to fall off at the top. If gravity is stronger than the needed centripetal force, the car falls. If gravity is equal to, or even less than the required centripetal force, the car stays on the track. Thus:

$$F_g \leq F_c$$

Substituting equations shows:

$$\cancel{m}g \leq \frac{\cancel{m}v^2}{r}$$

Solving for v yields:

$$\sqrt{gr} \leq v$$

Note that the diameter is given in the problem, but the formula requires the radius. Thus, substituting numbers gives:

$$\sqrt{(9.81\frac{m}{s^2})(0.15m)} \leq v$$

Thus:

$$\boxed{1.213\frac{m}{s} \lessapprox v}$$

8.2 Kepler's Laws of Planetary Motion

8.2.1 Kepler's First Law

Kepler's first law states that the orbit of a planet is an ellipse with the Sun at one of the two foci:

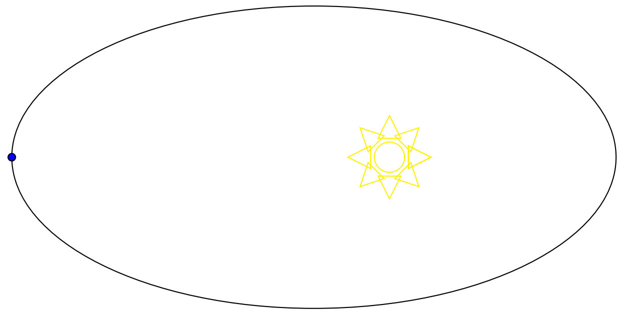


Figure 8.1: Planets orbit the sun along elliptical path. This diagram exaggerates the eccentricity of the orbital path to show the placement of the sun.

One should note that most planet's orbital path is much closer to circular than the diagram above. For instance, here is earth's orbit compared to a perfect circle:

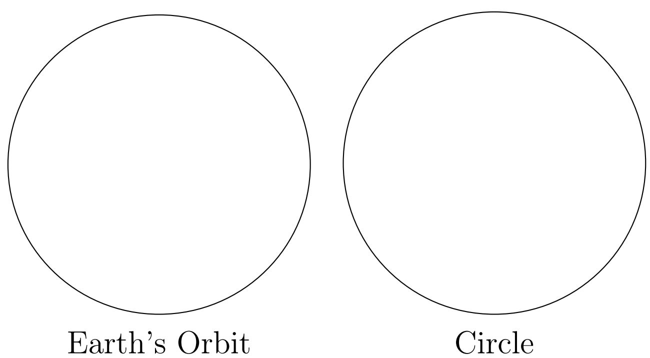


Figure 8.2: Earth's orbit shape compared to a circle.

The Eccentricity of an ellipse can be found using:

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}} \tag{8.3}$$

where a is the semimajor axis and b is the semiminor axis, as shown in the diagram:

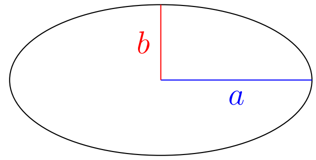


Figure 8.3: The semimajor axis (a) and semiminor axis (b) of an ellipse.

8.2.2 Kepler's Second Law

A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

This can be expressed as:

$$\frac{A_1}{t_1} = \frac{A_2}{t_2} \tag{8.4}$$

It can be shown that this expression is equivalent to:

$$r_1 v_1 = r_2 v_2 \tag{8.5}$$

Where r_1 and r_2 are the average radii for each segment of arc, and v_1 and v_2 are the average velocities, respectively.

8.2.3 Kepler's Third Law

The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit. Thus, for any two planets orbiting the sun,

$$\frac{{T_1}^2}{{R_1}^3} = \frac{{T_2}^2}{{R_2}^3} \tag{8.6}$$

8.3 Newton's Law of Universal Gravitation

Newton's Law of Universal Gravitation is a way of calculating the gravitational force between any two objects with mass. It is given by:

Newton's Law of Universal Gravtation

$$F_g = \frac{G m_1 m_2}{r^2} \tag{8.7}$$

where G is the Universal Gravitational Constant:

$$G = 6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$$

The masses of the two objects are m_1 and m_2, and r is the distance between these objects.

Sometimes, you will see equation 8.7 written with a negative sign in order to make it consistent with some of the laws of electrostatics, studied in chapter 13. However, you will likely end up assigning a sign to the force, according to the coordinate system you have defined for the problem.

8.4 Orbital Motion

Whenever a object orbits another that has a much larger mass, if the orbit can be approximated by a circle, the gravitational attraction between the two bodies acts as a centripetal force. Thus, a fundamental realization of orbital mechanics is:

Chapter 9

Rotational Mechanics

9.1 Angular Velocity and Acceleration

An object that is spinning can be described using angular velocity and angular acceleration. Angular velocity is a way of expressing how much an object rotates in a given time. It could be measured in Rotations per Minute (rpms), Degrees per hour, or any other measurement of an angle divided by any measurement of time. However, it is advantageous to use Radians per Second.

Just like velocity measures how fast an object is moving in a line, angular velocity measures how fast an object is rotating. Average ngular velocity is given by the following equation:

$$\vec{\omega}_{avg} = \frac{\Delta\vec{\theta}}{\Delta t} \tag{9.1}$$

and instantaneous angular velocity is given by:

$$\vec{\omega} = \frac{d\vec{\theta}}{dt} \tag{9.2}$$

9.2 Angular Kinematics

9.3 Moment of Inertia

9.4 Torque

9.5 Angular Kinetic Energy

9.6 Angular Momentum

9.6.1 The Definition of Angular Momentum

Angular momentum can be calculated using the formula:

$$\vec{L} = I\vec{\omega} \tag{9.3}$$

where $\vec{L}$ is angular momentum, I is the object's moment of Inertia and $\vec{\omega}$ is the object's angular velocity. The SI units for angular momentum are $\frac{kgm^2}{s}$.

Example 9.6.1

Problem: A bicycle wheel has a mass of 0.3 kg, and can be thought of as a thin ring with a radius of 0.33m. When the wheel is turning at a rate of 2 rotations per second, what its its angular momentum?

Solution: Begin by converting the angular velocity ω to appropriate units:

$$\vec{\omega} = 2\frac{rotations}{s} = 4\pi\frac{rad}{s}$$

Then calculate the moment of inertia. Using the formula for a thin ring:

$$I = mr^2 = 0.3kg(0.33m)^2 \approx 0.033kgm^2$$

Finally, use equation 9.3 to find the angular momentum.

$$\vec{L} = I\vec{\omega} = 0.011kgm^2 \cdot 4\pi\frac{rad}{s} \approx \boxed{0.411\frac{kgm^2}{s}}$$

9.6.2 Conservation of Angular Momentum

Just like linear momentum[1], angular momentum is a quantity that is conserved. Thus, whatever angular momentum a closed system has in its initial state will be equal to the angular momentum the system has in its final state.

The classic example of the Law of Conservation of Momentum is an ice skater who enters a spin. By changing the positioning of his or her arms and legs, an ice skater can change their moment of inertia. When they bring their arms and legs closer to their axis of rotation, their moment of inertia decreases. Since angular momentum is conserved, their angular velocity must increase as their moment of inertia decreases, and thus the ice skater is able to spin very fast.

[1] see momentum in section 9.6

Chapter 10

Waves

10.1 Fundamentals of Waves

You have probably heard of waves in the context of the ocean, a lake, or other bodies of liquid. Waves are also found in earthquakes, sound, light, and even at the stadium when people do "the wave." A *wave* is a distortion that transfers energy from one place to another without the permanent transfer of mass.

The material that a wave travels through is called a *medium*. For instance, the medium for an ocean wave is water, while the medium for light could be air, glass, water, or even empty space (no medium).

10.1.1 Types of Waves

Waves can be categorized into several basic types:

- *Transverse Waves* - are waves that are displaced perpendicular to the direction of travel. For instance, ocean waves are a type of transverse wave because their displacement is vertical, though they travel horizontally. Normally, a transverse wave is drawn similar to the figure below:

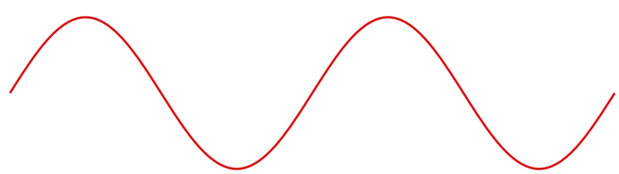


Figure 10.1: A simple transverse wave

- *Longitudinal Waves* - are waves that are displaced in the same direction as the direction of travel. You can think of this as a compression, or shock wave that travels through a medium. The places where a material is closer together than normal are called compressions, while places that are spaced farther apart are called rarefractions, as seen in figure 10.2

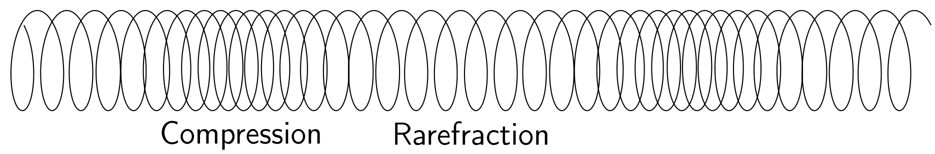


Figure 10.2: A simple longitudinal wave

- *Electromagnetic Waves* - are waves that are made up of oscillating electric and magnetic fields. These are usually modeled as transverse waves, but they are just representations of the strength of the electric and magnetic fields.

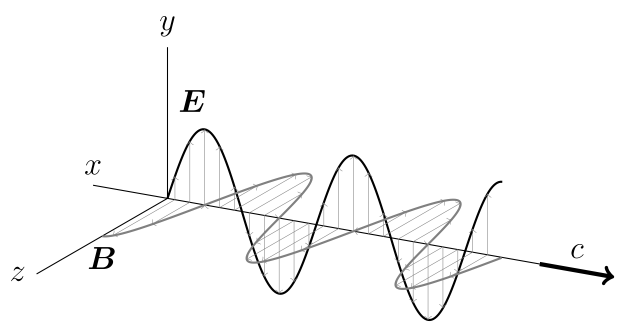


Figure 10.3: An electromagnetic wave

Electromagnetic waves are what make up the electromagnetic spectrum. Radio waves, microwaves, infrared, visible light, ultraviolet, X-rays and γ-rays are all electromagnetic waves. They are categorized into different types based on their frequency.

There are other types of waves, such as matter waves and gravitational waves that are beyond the scope of this text.

10.1.2 Basic Wave Characteristics and Vocabulary

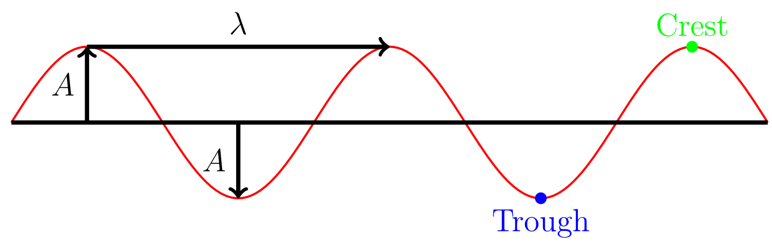


Figure 10.4: The measurements of a wave

Extrema

Crests are each of the highest points of a wave, and **troughs** (rhymes with coughs) are the lowest points of a wave, as seen in the diagram 10.4. The word **extrema** refers to all crests and troughs, as they are the most extreme points of the wave.

Amplitude

Amplitude measures how large or how strong a wave is. It is measured from the center of a wave to one of the extrema - either upward to a crest, or downward to a trough. For a physical, transverse wave, amplitude can be measured in meters. The variable for amplitude is A. For sound, we experience the amplitude of the wave as *volume*, and for light, we experience it as *brightness*.

Wavelength

The **wavelength** of a wave is measured from any point to an identical point on the wave, along the axis of propagation. Wavelength is measured in meters, and the symbol for wavelength is λ (lambda). The easiest way to measure wavelength is from crest to crest or from trough to trough.

Period

The **period** of a wave measures how long it takes a wave to repeat itself. It is measured in seconds, and uses the variable T.

Frequency

The **frequency** of a wave measures how many times a wave repeats itself in one second. The symbol for frequency is f and the units for frequency are $\frac{cycles}{second}$. We give this unit the name *hertz*, abbreviated Hz. For sound, we experience frequency as *pitch*, and for light we experience frequency as *color*.

The frequency of a wave and the period of a wave are inverses of each other. Thus:

$$f = \frac{1}{T} \tag{10.1}$$

Example 10.1.1

Problem: A wave has a frequency of 102.1 MHz. What is the period of the wave?

Solution:
We know that $f = 102.1\,\text{MHz}$ Converting into scientific notation gives:

$$f = 102.1\,\text{MHz} = 102.1 \times 10^6\,\text{Hz} = \boxed{1.021 \times 10^8\,\text{Hz}}$$

Begin by using equation 10.1:

$$f = \frac{1}{T}$$

Solving for T yields:

$$T = \frac{1}{f}$$

Then, substitute numbers:

$$T = \frac{1}{1.021 \times 10^8\,\text{Hz}} \approx \boxed{9.794 \times 10^{-9}\,\text{s}}$$

10.1.3 Velocity of a Wave

We already know from equation 2.1 that the average velocity of any object is given by $\overrightarrow{v_{avg}} = \frac{\vec{d}}{\Delta t}$. In the case of a wave, the time it takes the wave to repeat is the period, T, and distance the wave must move in order to repeat is one wavelength, λ. Thus, $\overrightarrow{v_{avg}} = \frac{\vec{d}}{\Delta t} = \frac{\vec{\lambda}}{T}$. Using equation 10.1, it can be proven that:

$$v = f\lambda \tag{10.2}$$

Example 10.1.2

Problem: An ocean wave has a period of 15 seconds, and is traveling at a speed of 2 meters per second. How far is it from one crest of the wave to another?

Solution: The question asks for the distance from one crest to another, which is the wavelength. Given values:

$$T = 15\,\mathrm{s}$$
$$v = 2\,\mathrm{m/s}$$

First, we use equation 10.1 to find the frequency:

$$f = \frac{1}{T} = \frac{1}{15\,\mathrm{s}} \approx 0.067\,\mathrm{s}$$

We now use equation 10.2:

$$v = f\lambda$$

Solving for wavelength gives:

$$\frac{v}{f} = \lambda$$

Substituting gives:

$$\lambda = \frac{v}{f} \approx \frac{2\,\mathrm{m/s}}{0.067\,\mathrm{s}} \approx \boxed{30\,\mathrm{m}}$$

10.2 The Doppler Effect

We have all experienced a car driving past us while it is honking its horn. As the car drives past, there is a significant change in the pitch of the horn. In fact, when either the source of a wave or an observer of a wave is moving, it causes the observer's perception of the frequency of that wave to change. This is called the **Doppler Effect**.

When the source of a wave moves *toward* an observer, its frequency is shifted higher.

Figure 10.5: Observers hear a higher pitch

When the source of a wave moves *away* from an observer, its frequency is shifted lower.

Figure 10.6: Observers hear a lower pitch

Likewise, when an observer moves *toward* the source of a sound, its frequency is shifted higher.

Figure 10.7: Observer hears a higher pitch

When an observer moves *away* from a source of sound, its frequency is shifted lower.

Figure 10.8: Observer hears a lower pitch

The equation for the Doppler effect is given by equation 10.3:

$$f_{observed} = f_{source}\left(\frac{v_{wave} \pm v_{observer}}{v_{wave} \pm v_{source}}\right) \tag{10.3}$$

In this equation, do not consider the $\pm$ sign to represent both operations. Instead, you must chose which operation to use based on the given situation. In the numerator of the equation it is exactly what is expected: add for a higher frequency and subtract for a lower frequency. In the denominator, it is backward - add for lower frequency, and subtract for higher frequency.

In the air, the speed of sound depends on air pressure, temperature, and even humidity. The standard value for speed of sound in air is **343 m/s**, though in reality this number can change quite significantly depending on atmospheric conditions.

Example 10.2.1

Problem: A car's horn has a pitch of 550 Hz. It is driving at 15 m/s toward a stationary observer.

a. What is the frequency that the observer hears at the car approaches.

b. The car then passes the observer. What is the frequency that the observer hears as the car travels away from him?

Solution: A car's horn produces sound, therefore the velocity of the wave is the speed of sound.

Part a: Given values:

$$f_{source} = 550\,\mathrm{Hz}$$
$$v = 343\,\mathrm{m/s}$$
$$v_{source} = 15\,\mathrm{m/s}$$
$$v_{observer} = 0\,\mathrm{m/s}$$

We can use equation 10.3 to find the frequency the observer hears:

$$f_{observed} = f_{source}(\frac{v_{wave} \pm v_{observer}}{v_{wave} \pm v_{source}})$$

In this case, the observer is not moving, so it does not matter whether we chose a plus or minus. The source is moving toward the observer, causing the observer to hear a higher pitch. Therefore, since the source is in the denominator, we chose a minus. Substituting numbers and chosing correct signs gives:

$$f_{observed} = 550\,\mathrm{Hz}(\frac{343\,\mathrm{m/s} + 0\,\mathrm{m/s}}{343\,\mathrm{m/s} - 15\,\mathrm{m/s}})$$

Evaluating this expression gives:

$$\boxed{f_{observed} \approx 575.152\,\mathrm{Hz}}$$

Part b: After the car has passed the observer, it is now traveling away from the observer. Thus, the only change that needs to be made is that v_{source} should now have a plus sign:

$$f_{observed} = 550\,\mathrm{Hz}(\frac{343\,\mathrm{m/s} + 0\,\mathrm{m/s}}{343\,\mathrm{m/s} + 15\,\mathrm{m/s}})$$

Evaluating this expression yields:

$$\boxed{f_{observed} \approx 526.955\,\mathrm{Hz}}$$

10.3 The Principle of Superposition and Interference

10.3.1 The Principle of Superposition

The **Principle of Superposition** is the idea that waves can overlap. Consider, for instance, a swimming pool where a light breeze creates ripples in the water as shown below:

Figure 10.9: Ripples on a swimming pool

You could also imagine in the same swimming pool on a calm day, a person could jump into the water creating waves that look similar to the ones below:

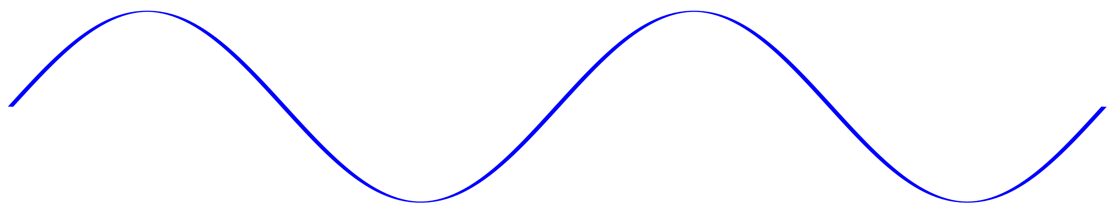


Figure 10.10: Large waves in a swimming pool

Thus, we can use the principle of superposition to predict what the wave will look like should a person jump into the swimming pool on a day when there are ripples in the water. By combining the two types of waves, we would see something like the figure below:

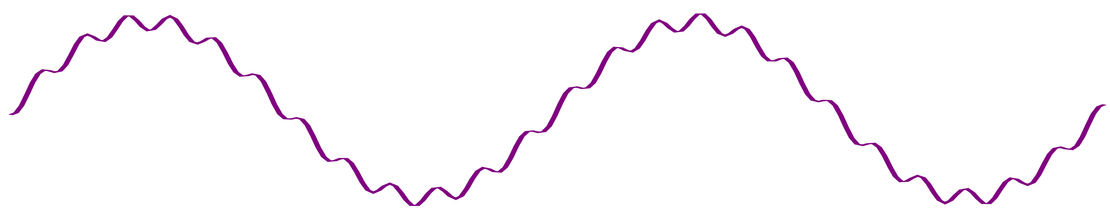


Figure 10.11: Ripples and waves combine

10.3.2 Interference

Constructive Interference

Sometimes, waves of approximately the same amplitude may overlap, causing the amplitude of the resulting wave to become larger. This is called **Constructive Interference**:

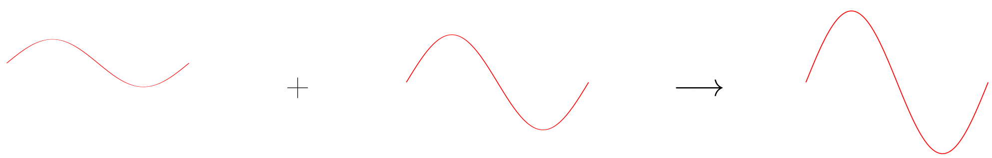


Figure 10.12: Constructive interference

In order for constructive interference to happen, the two waves must be *in phase* - that is, the displacement of the waves must be in the same direction.

When two waves that are continuous overlap, they will interfere constructively if they align perfectly:

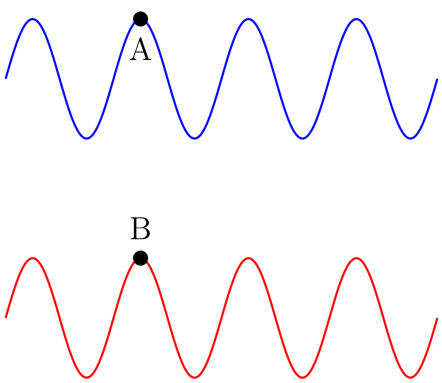


Figure 10.13: Two waves aligned perfectly

Likewise, constructive interference will occur if the wave is shifted by a distance of one wavelength.

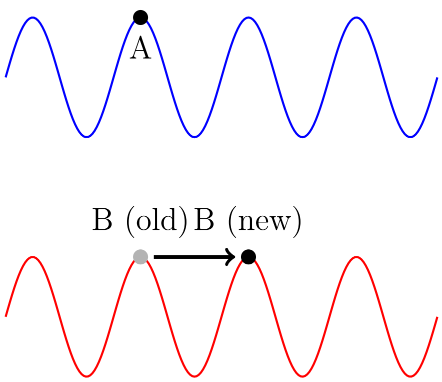


Figure 10.14: The red wave has been shifted one wavelength to the right.

The wave could be shifted in a similar manner by two, three, or any integer number of wavelengths right or left, and the two waves will still cause constructive interference. Thus, the amount of shift between the waves, $\Delta\ell$, is given by:

$$\Delta\ell = m\lambda \tag{10.4}$$

where $\Delta\ell$ is how far the waves are shifted, and m is any integer.

Destructive Interference

At other times, waves may overlap while they are **out of phase** - that is, their displacement is in opposite directions, causing the wave to become smaller. This is called **destructive interference**.

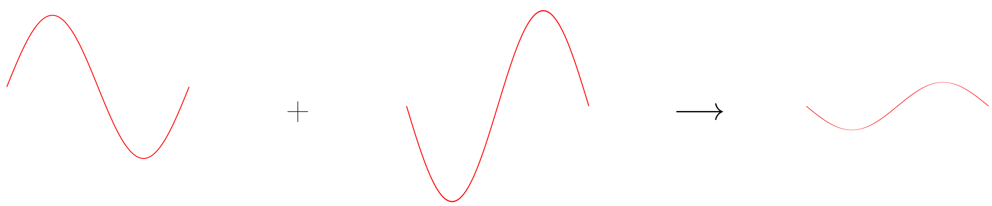


Figure 10.15: Destructive Interference

If two waves are displaced by the same amount in opposite directions, they can even cancel out completely. This would be **perfectly destructive interference**:

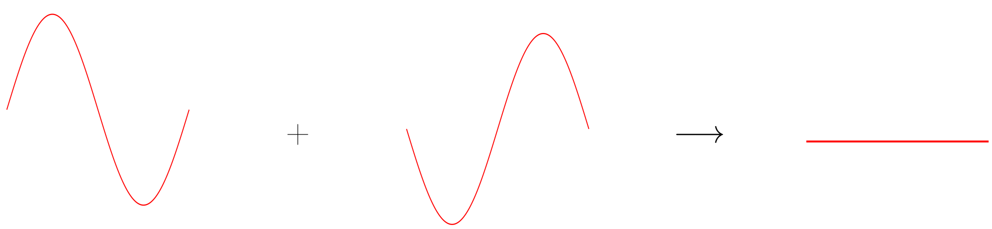


Figure 10.16: Perfectly Destructive Interference

In order for destructive interference to happen, the two waves must be perfectly *out of phase* - that is, the displacement of the waves must be in opposite direction, meaning the waves are already shifted by a half wavelength:

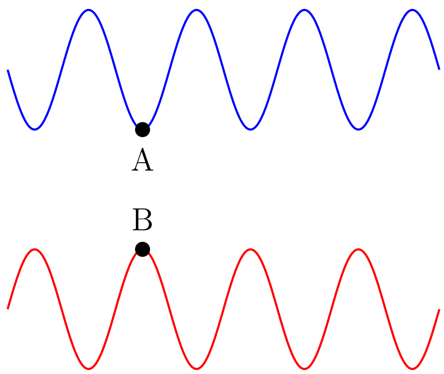


Figure 10.17: Two waves aligned perfectly

Likewise, destructive interference will occur if the wave is shifted by a distance of one wavelength (after already being off by half a wavelength).

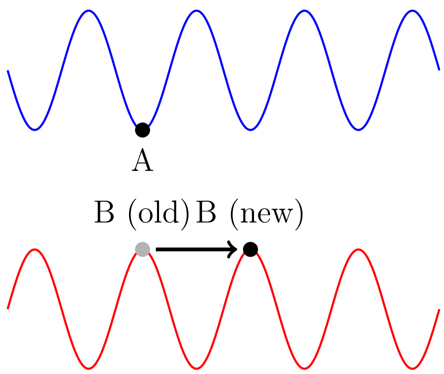


Figure 10.18: The red wave has been shifted one wavelength to the right.

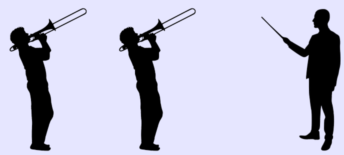
Since destructive interference occurs whenever the waves are shifted by half-integer multiples of the wavelength (ie, 0.5, 1.5, 2.5, etc), the amount of shift between the waves, $\Delta\ell$, is given by:

$$\boxed{\Delta\ell = (m + \frac{1}{2})\lambda \qquad (10.5)}$$

where $\Delta\ell$ is how far the waves are shifted, and m is any integer.

Example 10.3.1

Problem: Two trombone players stand in a single file line, with their conductor directly in front of them, as shown in the diagram:



Both trombones play a low-$B\flat$ ($f = 116.54\,\text{Hz}$). What is the smallest, non-zero distance that the players should stand apart in order for the conductor to hear the loudest possible sound? (Assume both players are perfectly in tune and the waves they create are perfectly in phase.)

Solution: We already know:

$$f = 116.54\,\text{Hz}$$
$$v = 343\,\text{m/s}$$

Thus, we can use equation 10.2 to find the wavelength:

$$v = f\lambda \longrightarrow \lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{116.54\,\text{Hz}} \approx 2.943\,\text{m}$$

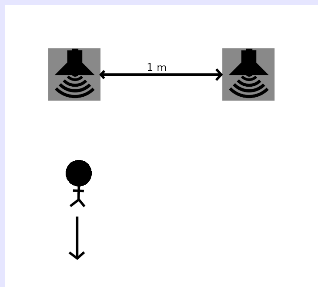
Since we are asked for the loudest sound, we know that this is constructive interference. We know that if $m = 0$, the distance between the two players will be zero. Thus, the smallest non-zero distance is given when $m = 1$. Using equation 10.4, we find:

$$\Delta\ell = m\lambda = (1)(2.943\,\text{m}) = 2.943\,\text{m}$$

Example 10.3.2

Problem: Two speakers are aligned along an east-west line, and are placed 1 meter apart. A single frequency is played on both speakers. A person starts in front of the left-most speaker, and begins to walk to the south. When the person has walked 2 meters to the south, she hears the sound get significantly quieter. As she continues to walk, he hears the sound get louder again. What is the frequency of the sound?

Solution: Begin by drawing a diagram of the situation:



We begin by calculating the difference in distances between the speakers. The distance to the west speaker is 2 meters. The distance to the right speaker can be found using the Pythagorean Theorem:

$$c = \sqrt{a^2 + b^2} = \sqrt{(2\,\mathrm{m})^2 + (1\,\mathrm{m})^2} = \sqrt{5\,\mathrm{m}^2} \approx 2.236\,\mathrm{m}$$

Therefore, $\Delta\ell$, the distance between the two paths the sound is traveling is given by:

$$\Delta\ell = 2.236\,\mathrm{m} - 2\,\mathrm{m} \approx 0.236\,\mathrm{m}$$

Since this problem states that sound gets quieter, we know that this is destructive interference, and we can use equation 10.5:

$$\Delta\ell = (m + \frac{1}{2})\lambda$$

We know that the smallest value m can be is 0, so our equation reduces to:

$$\Delta\ell = (\cancelto{0}{m} + \frac{1}{2})\lambda \longrightarrow \Delta\ell = \frac{\lambda}{2}$$

Solving for λ gives:

$$\lambda = 2\Delta\ell = (2)(0.236\,\mathrm{m}) = 0.472\,\mathrm{m}$$

Knowing that the speed of sound is 343 m/s, we can use equation 10.2 to determine the frequency:

$$v = f\lambda \longrightarrow f = \frac{v}{\lambda} = \frac{343\,\mathrm{m/s}}{0.472\,\mathrm{m}} = \boxed{726.540\,\mathrm{Hz}}$$

10.4 Resonance

Chapter 11

Optics

Optics is the study of light and how that light interacts with matter. There are two general fields of Optics: *Geometric Optics* and *Physical Optics*. Geometric optics studies how rays of light travel, while physical optics studies how multiple rays of light interact as waves.

An important constant in the field of optics is the speed of light. The speed of light in empty space is exactly:

$$\boxed{c = 299\,792\,458\,\mathrm{m/s}}$$

The symbol used is c because the speed of light is constant in all frames of reference. This fact will be discussed in more detail in (Insert REFERENCE HERE.) You will often see this number rounded to 2.998×10^8 m/s or even 3.0×10^8m/s.

11.1 Geometric Optics

While light in empty space always travels at the same speed, c, light can be slowed down when it travels through a medium. This leads to some phenomena that we may encounter in everyday life.

11.1.1 Refraction

Refraction is a phenomenon associated with how light changes direction as it moves from one medium to another, due to the change in the speed that light travels at in each of the media. This is often demonstrated by looking at a pencil in a glass of water, or a fish in a pond.

The Index of Refraction

The index of refraction is the ratio of the speed of light in a vacuum to the speed of light in a material. It can be calculated as follows:

$$n \equiv \frac{c}{v_m} \tag{11.1}$$

where n is the index of refraction, c is the speed of light in empty space, and v_m is the speed of light in the material.

Example 11.1.1

Problem: Light travels at a speed of 2.254×10^8 m/s in water. What is the index of refraction of water?

Solution: Using the definition of index of refraction, we find:

$$n \equiv \frac{c}{v_m} = \frac{2.997 \times 10^8\,\mathrm{m/s}}{2.254 \times 10^8\,\mathrm{m/s}} = 1.330 \tag{11.2}$$

You may notice that the index of refraction is unitless, since all units cancel in the calculation. A list of indices of refraction can be found in Appendix B.3 on page 76.

Snell's Law

Snell's Law, named for the Dutch physicist Willebrord Snell, explains that light always takes the path of least time between two points. When light travels in a single medium of constant optical density, it travels in a straight line. However, when light changes medium, it will change direction.

There are several components and measurements that should be included on a diagram in this situation.

- The *interface* is the boundary where the two materials meet.
- The *normal* is an imaginary line perpendicular to the surface.
- The *incident ray* is the ray of light that is traveling toward the interface.
- The *refracted ray* is the ray of light traveling away from the interface.
- The *incident angle*, θ_i is the angle between the incident ray and the normal.
- The *refracted angle* θ_r is the angle between the refracted ray and the normal.

A diagram that shows a ray of light traveling from air into water might look like this:

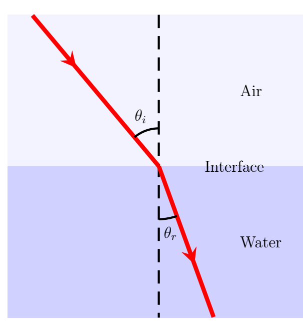


Figure 11.1: A diagram of light traveling from air into water.

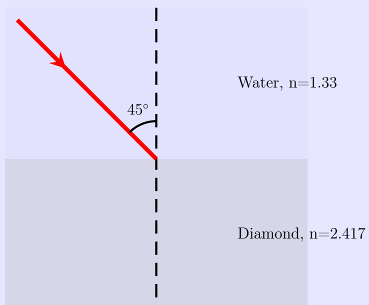
In a diagram such as above, it can be seen that the path of least time is given by the following mathematical representation of Snell's Law:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r) \tag{11.3}$$

Example 11.1.2

Problem: Light travels from water (n=1.33) into diamond (n=2.417). If the angle of incidence is 45°, what is the refracted angle?

Solution: Begin by drawing a (partial) diagram:



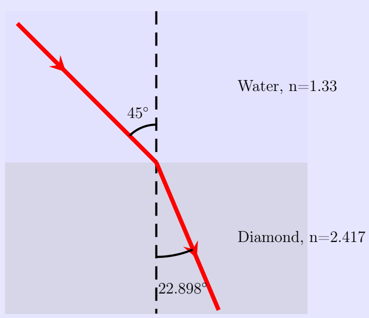
Snell's law states:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Solving this for θ_r yields:

$$\theta_r = \sin^{-1}\left(\frac{n_i \sin(\theta_i)}{n_r}\right) = \sin^{-1}\left(\frac{1.33 \sin(45°)}{2.417}\right) \approx \boxed{22.898°} \tag{11.4}$$

The completed diagram would look like this:



Total Internal Reflection In the specific case that light is traveling from a material with a higher index of refraction into a material with a lower index of refraction, the refracted angle will be larger than the incident angle. In this case, it is possible that the refracted angle could refract at exactly 90°. The incident angle that causes a ray to be refracted at 90° is called the *critical angle*. If the angle of the incident ray exceeds the critical angle, the ray does not refract out of the material. Instead, it reflects back into the material.

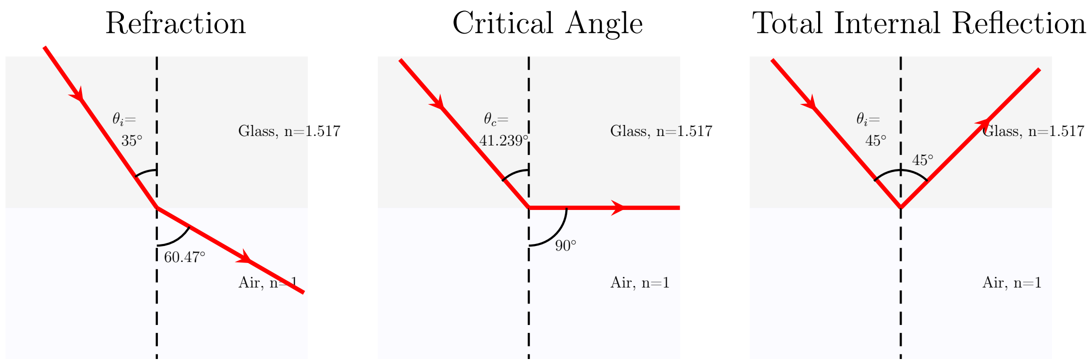


Figure 11.2: Total internal reflection occurs when the critical angle is exceeded.

Example 11.1.3

Problem: Light travels from glass (n=1.517) into water (n=1.33). Calculate the critical angle, and draw a diagram of the situation.

Solution: We know that the refracted ray must travel at a 90° angle to the normal, so $\theta_r = 90°$.

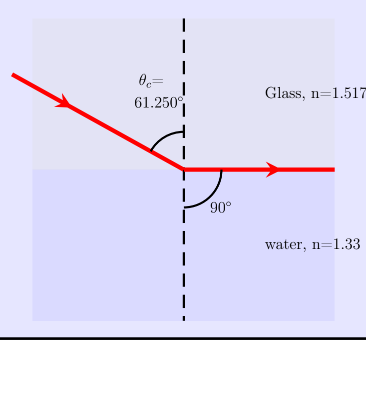
Snell's Law states:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

Solving for θ_i gives:

$$\theta_i = \sin^{-1}(\frac{n_r \sin(\theta_r)}{n_i}) = \sin^{-1}(\frac{1.33\sin(90)}{1.517}) \approx \boxed{61.250°} \tag{11.5}$$

The corresponding diagram should look like the following:



Lenses

There are two basic types of lenses that are discussed in this text:

- *Convex* lenses, sometimes called *converging* lenses are larger in the center than at the edges. These lenses are often used as magnifying glasses.

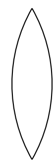


Figure 11.3: A Simple diagram representation of a convex lens

- *Concave* lenses, sometimes called *diverging* lenses are larger at the edges than the center. They are found in lenses for near-sightedness.

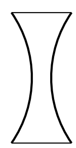


Figure 11.4: A Simple diagram representation of a concave lens

Convex Lenses When parallel rays of light strike a convex lens, the light can be focused into a very small area. The point at which the light is focused is called the *focal point*

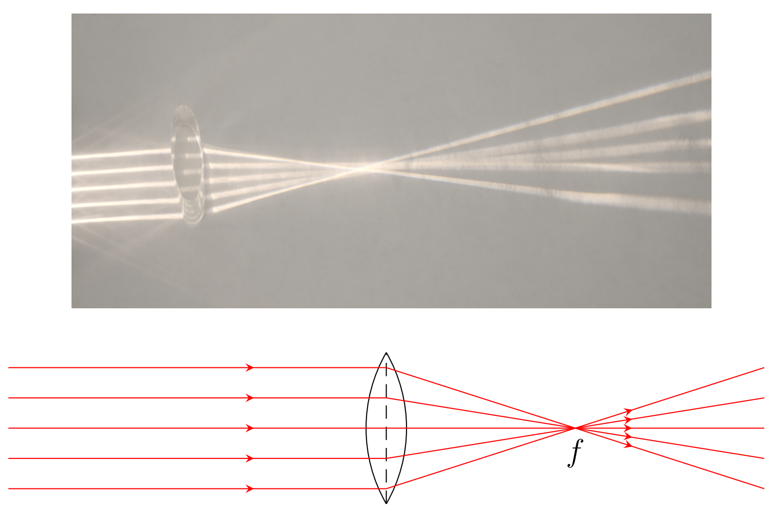


Figure 11.5: Light focused by a convex lens. The focal point is labeled f.

The distance from the center of the lens to the focal point is called the *Focal Length.* Because light call pass through the lens either way, lenses have two focal points, both equidistant from the center of the lens.

Image Formation with Convex Lenses When a lens interacts with an object, an image is normally formed. There are two often-used methods for determining where images form: *Ray Tracing* and use of *the lens equation.*

The method of ray tracing is performed in the following example:

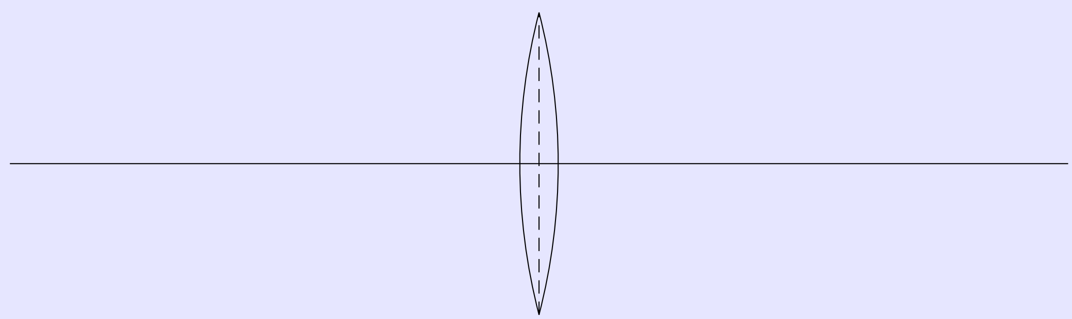
Example 11.1.

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = 2$cm. Use ray-tracing to determine the position, size, and orientation of the image.

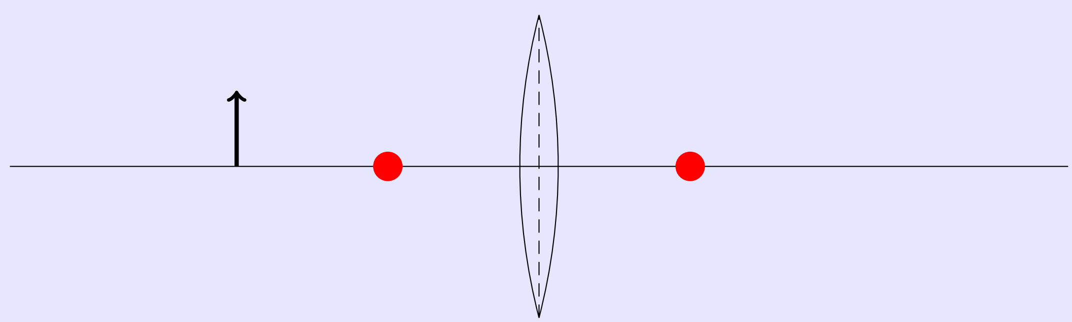
Solution:

1. Begin by drawing a horizontal line across your paper. This is your optical axis:

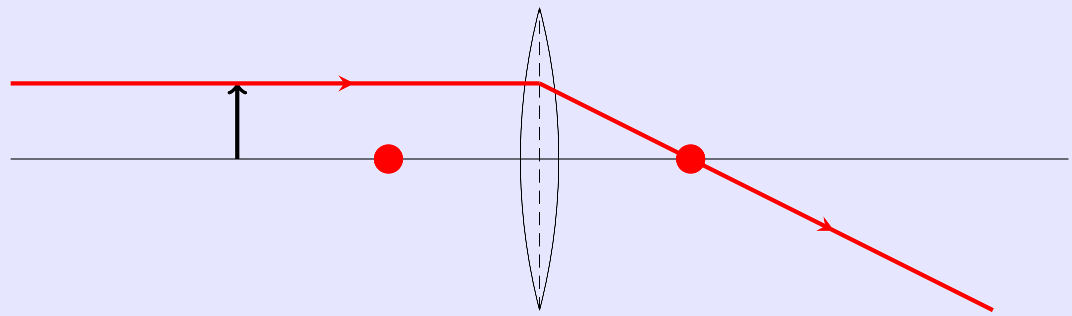
2. Draw a lens on the optical axis. Use a dotted line to represent the center of the lens.



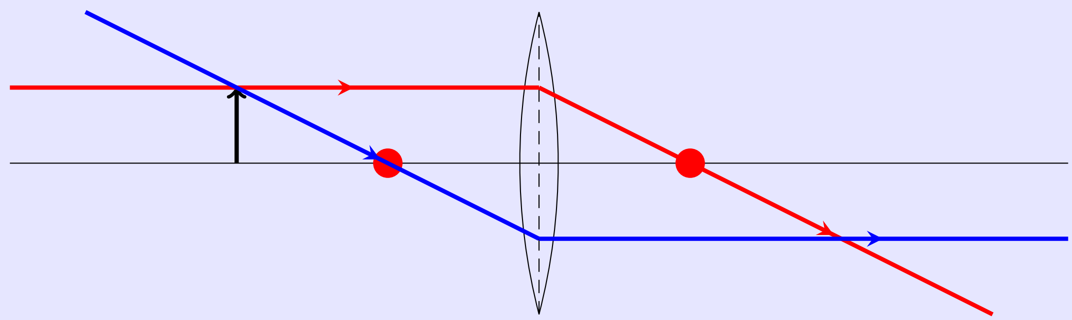
3. Measure and label focal points from the center of the lens, along the optical axis, and draw the object as an arrow with its base on the optical axis.



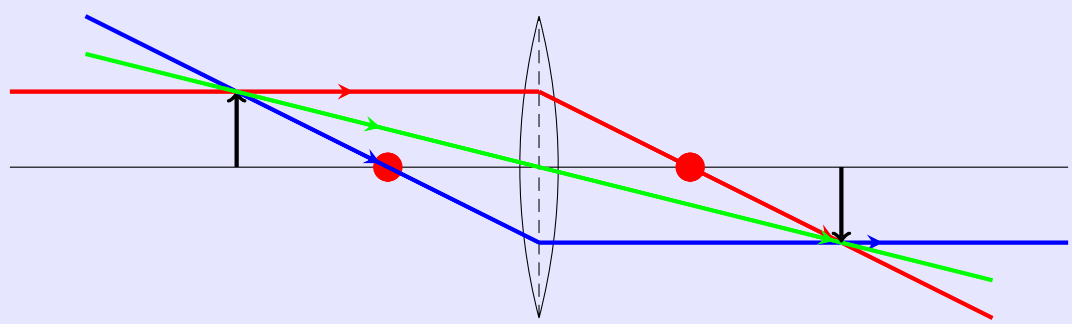
4. The first ray of light will begin on the left of the diagram, graze the top of the object, and continue to the center of the lens. It will then be directed down, through the far focal point.



5. The second ray of light will graze the top of the object, pass through the near focal point, continuing until it hits the center of the lens. There, the ray will be directed parallel to the optical axis.



6. The final ray of light will graze the top of the object, directed toward the intersection of center of the lens and the optical axis. This ray will not change direction. The intersection of the three rays is where the top of the image will form.



The diagram can then be measured. In this case, the image is 4 cm to the right of the lens, inverted, and the same size as the object.

The *lens equation* can also be used to determine the distance to the image. The lens equation is:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{11.6}$$

where f is the focal length of the lens, o is the object distance from the lens, and i is the image distance from the lens.

The magnification of an image can be determined by the following equation:

$$m = -\frac{i}{o} = \frac{h_i}{h_o} \tag{11.7}$$

where h_i is the image height, and h_o is the object height.

Example 11.1.5

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = 2\text{cm}$. Use the lens equation and the magnification formula to determine the position, size, and orientation of the image.

Solution: In this case, we are given $h_o = 1\text{cm}$, $o = 4\text{cm}$ and $f = 2\text{cm}$. First, the lens equation is solved for i:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$i = \frac{1}{\frac{1}{f} - \frac{1}{o}} = \frac{1}{\frac{1}{2\text{cm}} - \frac{1}{4\text{cm}}} = \boxed{4\text{cm}}$$

Calculating the magnification gives:

$$m = -\frac{i}{o} = -\frac{4\text{cm}}{4\text{cm}} = \boxed{-1}$$

The negative magnification means that the image is inverted. You may also notice that magnification is a unitless quantity. Finally, calculating the height of the image shows that the image is the same size (thought inverted).

$$m = \frac{h_i}{h_o} \longrightarrow h_i = mh_o = (-1)(1\text{cm}) = \boxed{-1\text{cm}}$$

Concave Lenses Instead of being focused to a point, parallel rays of light that are incident upon a concave lens diverge, as shown below:

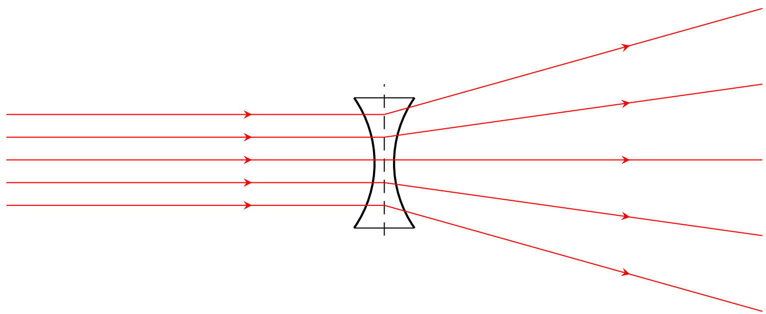


Figure 11.6: A concave lens causes light rays to diverge.

Though the rays do not converge on the right side of the lens in figure 11.6, the rays can be extended backward to show an apparent focal point on the wrong side of the lens:

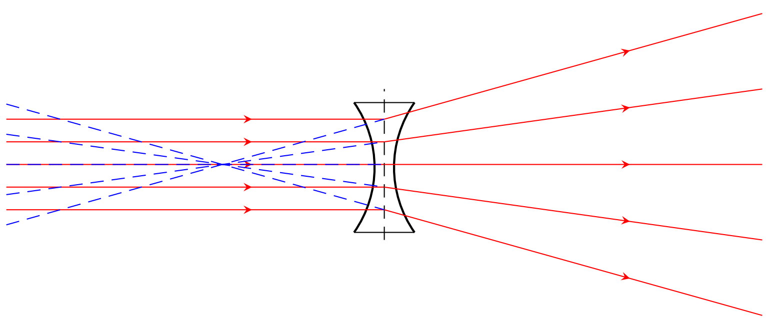


Figure 11.7: A concave lens appears to have a focal point on the other side of the lens.

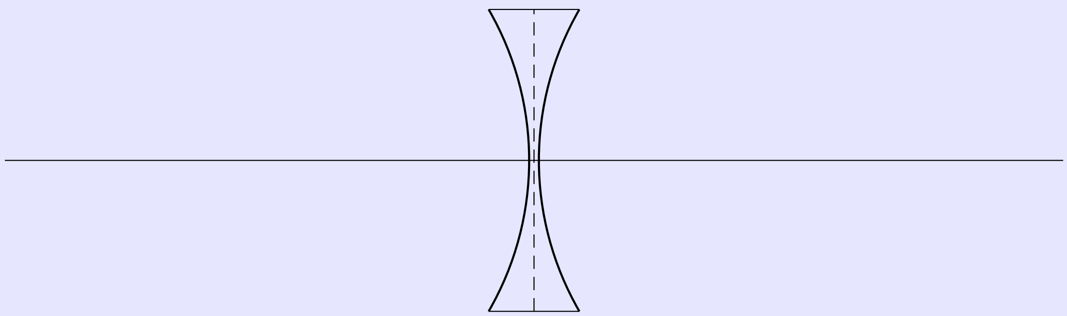
Because concave lenses appear to have a focal point on the wrong side of the lens, their focal length is negative.

Example 11.1.6

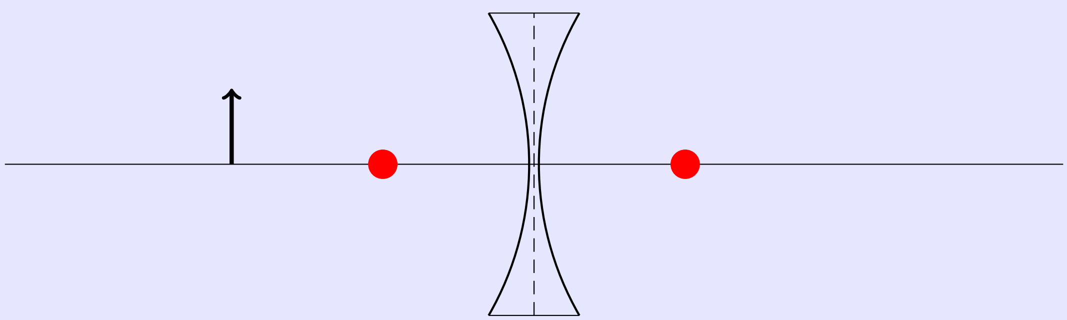
Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = -2$cm. Use ray-tracing to determine the position, size, and orientation of the image.

Solution:

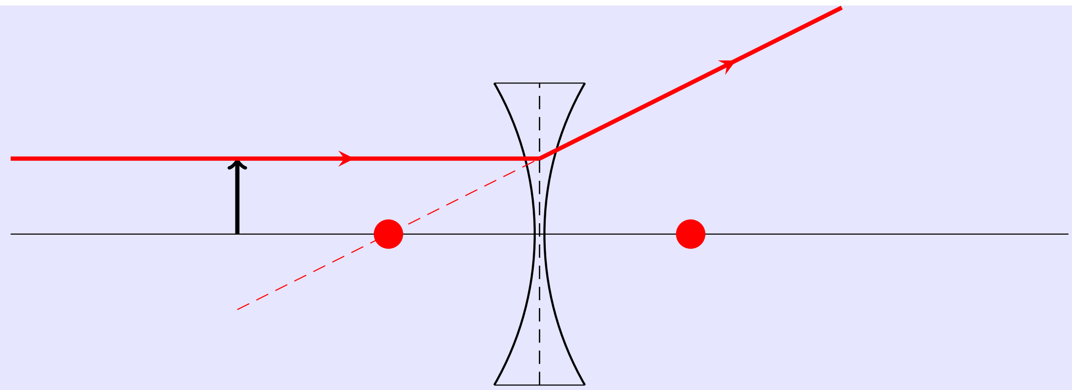
1. Begin by drawing the optical axis and lens. Note that since the focal length is negative, this is a concave lens.



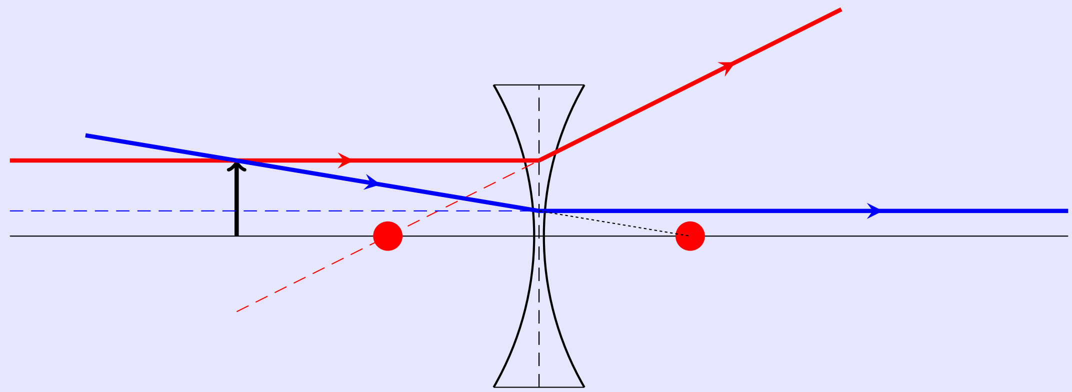
2. Measure and label focal points from the center of the lens, along the optical axis, and draw the object as an arrow with its base on the optical axis. Remember, that because the focal length is negative, the focal point for the right side is on the left, and the focal point for the left side is on the right.



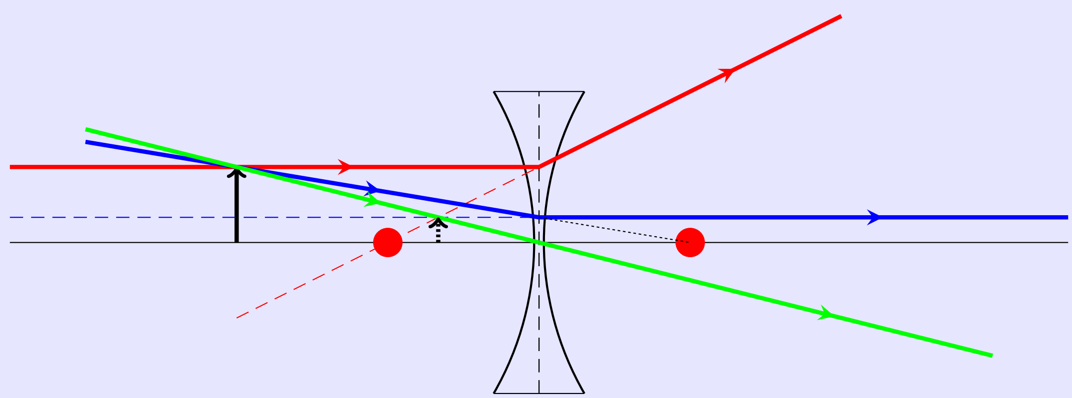
3. The first ray of light will begin on the left of the diagram, graze the top of the object, and continue to the center of the lens. It will then be directed upward, as if it came from the right focal point (which is actually on the left).



4. The second ray of light will graze the top of the object and be directed toward the near focal point (which is on the far side of the lens). Upon encountering the optical axis, the ray will be directed parallel to the optical axis.



5. The final ray of light will graze the top of the object, directed toward the intersection of center of the lens and the optical axis. This ray will not change direction. The intersection of the three rays is where the top of the image will form.



In this case, the image is behind the lens, meaning it is virtual. It is upright, and smaller than the original. Measuring the image distance yields $i = -1.333$cm (that is, the image is 1.333 cm to the left of the lens).

The same problem can be solved using the lens equation as well:

Example 11.1.5

Problem: A 1-cm tall object is placed 4 cm to the left of a lens of focal length $f = -2$cm. Use the lens equation and the magnification formula to determine the position, size, and orientation of the image.

Solution: In this case, we are given $h_o = 1$cm, $o = 4$cm and $f = -2$cm. First, the lens equation is solved for i:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

$$i = \frac{1}{\frac{1}{f} - \frac{1}{o}} = \frac{1}{\frac{1}{-2\text{cm}} - \frac{1}{4\text{cm}}} = \boxed{-\frac{4}{3}\text{cm} \approx -1.333\text{cm}}$$

Calculating the magnification gives:

$$m = -\frac{i}{o} = -\frac{-\frac{4}{3}\text{cm}}{4\text{cm}} = \boxed{\frac{1}{3}}$$

The positive magnification means that the image is inverted. Since the magnification of this image is less than one, the image will appear smaller. Finally, we calculate image height:

$$m = \frac{h_i}{h_o} \longrightarrow h_i = mh_o = (\frac{1}{3})(1\text{cm}) = \boxed{\frac{1}{3}\text{cm}}$$

11.1.2 Reflection

The Law of Reflection

Compared to refraction, the law of reflection is quite simple: the angle of incidence is equal to the angle of reflection.

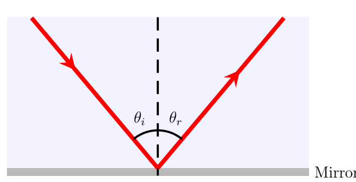


Figure 11.8: The angle of incidence is equal to the angle of reflection

This law of reflection can be expressed as the following equation.

$$\theta_i = \theta_r \tag{11.8}$$

Mirrors

Flat Mirrors Flat mirrors are quite easy to work with. The images produced by a flat mirror are always virtual, always the same size as the original object, and always upright. For this reason, flat mirrors are often used in bathrooms and fitting rooms to give an accurate assessment of one's appearance.

Concave Mirrors Like convex lenses, Concave mirrors are converging optical devices - that is, they focus parallel rays of light to a point. Unlike lenses, mirrors only have one focal point, since light can only interact with the mirror from one direction.

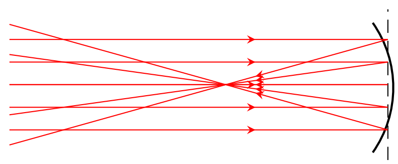


Figure 11.9: A concave mirror focuses light to a point.

Convex Mirrors

11.2 Physical Optics

11.2.1 Young's Double Slit Experiment

11.2.2 Thin Film Interference

Thin film interference is a phenomenon that takes place when a layer of material causes the reflections of light to interfere with each other. This usually only happens when the thickness of the material is on the same order as the wavelength of light. Common examples of thin film interference include it irridescent colors that are seen reflecting from soap bubbles and oil slicks. In addition, anti-reflective coatings on eyeglasses and other optical devices often make use of thin film interference.

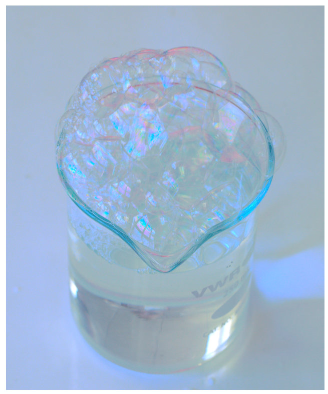


Figure 11.10: Iridescent colors are visible on soap bubbles due to thin film interference.

Chapter 12

Heat and Thermodynamics

12.1 Specific Heat

Chapter 13

Electrostatics

13.1 Electrostatic Charge

You've probably experienced electrostatic charges - sometimes we just call it static - in everyday life. Dragging your shoes on carpet, rubbing balloon on your hair, and even just putting on a piece of clothing can cause electrostatic charges to build up, causing articles to cling together, your hair to get frizzy, and can even cause small sparks.

Most electrical charges we encounter in everyday life are due to imbalances of protons and electrons. Electrons are capable of moving from atom to atom under the right conditions. However, protons are stuck in the nucleus of an atom and do not move unless nuclear reactions are taking place. Thus, most electrical charges we encounter in actual life are due to electrons being transferred from one object to another.

The units for charge are Coulombs (C), and it is represented by the variable q. You may remember from chemistry that protons have small positive charges and electrons have small negative charges. Table 13.1 shows the charge of each of the elementary particles that make up normal matter:

Table 13.1: Elementary Charges

Particle	**Charge**	**Mass**
Proton	1.602×10^{-19} C	1.673×10^{-27} kg
Electron	-1.602×10^{-19} C	9.109×10^{-31} kg
Neutron	0 C	1.675×10^{-31} kg

You may notice that the charge of an electron and a proton are exactly the same with the only difference being a negative sign for the electron. Thus a Hydrogen atom, made of 1 proton and 1 electron will have a total charge of zero coulombs. In fact, any combination of the same number of protons and electrons will have no charge.

You should also note that 1 coulomb of electrostatic charge is a large amount. Most charges we encounter in everyday life might be 1 millionth or even 1 billionth of a coulomb.

13.2 Coulomb's Law and Electrostatic Force

When two charged objects interact, it generates a force. This force, often called electrostatic force, is repulsive for charges with the same sign, whereas it is attractive for two charges with opposite signs. So two positive charges will repel, as will two negative charges, but a positive charge will be attracted to a negative charge.

To determine the about of force that will be exerted one charged particle due to another charged particle, we use **Coulomb's Law**:

$$F = \frac{kq_1q_2}{r^2} \tag{13.1}$$

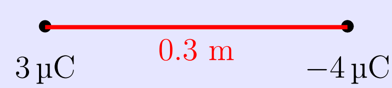
In this equation, q_1 and q_2 are the charges of the particles, r is the distance between the charges, and k is **Coulomb's Constant**. Coulomb's Constant is a universal constant, meaning its value does not change. The value of coulomb's constant is $k \approx 9 \times 10^9\,\mathrm{Nm^2/C^2}$.

It is also important to note that this equation has a non-intuitive sign convention. Repulsive forces will be postive, and attractive forces will be negative. If forces calculated using Coulomb's Law need to be used in further calculations, be sure assign a positive or negative to the calculated force according to the direction the force is actually in.

Example 13.2.1

Problem: A positive charge of $3\,\mu\mathrm{C}$ and a negative charge of $-4\,\mu\mathrm{C}$ are separated by a distance of 0.3 meters. What is the electrostatic force felt by the positive charge due to the negative charge?

Solution: Begin by drawing a diagram of the situation:



We also need to convert the charges into scientific notation:

$$q_1 = 3\,\mu\mathrm{C} = 3 \times 10^{-6}\,\mathrm{C}$$
$$q_2 = -4\,\mu\mathrm{C} = -4 \times 10^{-6}\,\mathrm{C}$$

Note: It does not matter which charge is q_1 and which is q_2. Either way will yield the same answer.

Now, we can use Coulomb's Law, equation 13.1 to calculate the force:

$$F = \frac{kq_1q_2}{r^2} = \frac{(9 \times 10^9\,\mathrm{Nm^2/C^2})(3 \times 10^{-6}\,\mathrm{C})(-4 \times 10^{-6}\,\mathrm{C})}{(0.3\,\mathrm{m})^2} = \boxed{-1.2\,\mathrm{N}}$$

The negative on the answer does not indicate that the force it to the left. Rather, it indicates that this is an attractive force. Thus, the $3\,\mu\mathrm{C}$ charge feels a 1.2 N force directed to the *right*, whereas the $-4\,\mu\mathrm{C}$ charge feels a 1.2 N force directed to the *left*.

13.3 Electrostatic Potential Energy

When two electrostatically charged objects are brought near each other, Coulomb's Law states that they will put a force on each other. If the charged particles are then released, they will accelerate. This means that two charged particles that are close enough to affect each other must have potential energy. The formula to calculate electrostatic potential energy is:

$$U_E = \frac{kq_1q_2}{r} \tag{13.2}$$

13.4 Electric Field

$$E = \frac{kq}{r^2} \tag{13.3}$$

13.5 Electric Potential and Voltage

$$V = \frac{kq}{r} \tag{13.4}$$

13.6 Capacitors

13.6.1 Construction of Capacitors

13.6.2 Capacitors in Circuits

Chapter 14

Circuits

14.1 Circuits Symbols

14.2 Resistors

Chapter 15

Magnetic Forces and Fields

15.1 Types of Magnetism

15.1.1 Permanent Magnetism

Paramagnetism

Ferromagnetism

Dimagnetism

15.1.2 Electromagnetism

15.2 Magnetic Force on a Charged Particle

15.3 Magnetic Force on a Current-Carrying Wire

15.4 Magnetic Field Produced by a Current-Carrying Wire

Chapter 16

Magnetic Induction

16.1 Lenz's Law

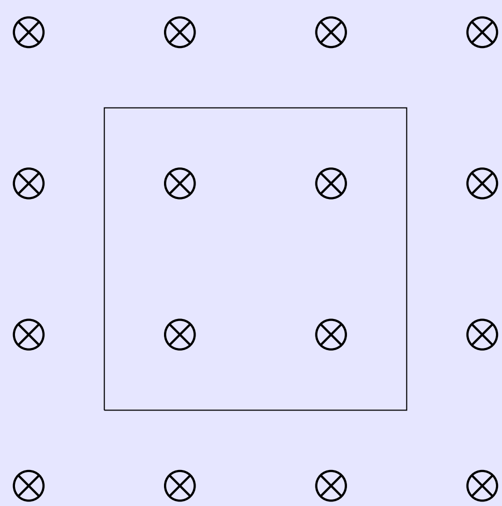
Lenz's law states that in the presence of a changing magnetic field, a current will flow in a loop that generates a magnetic field opposite to the direction of the change.

Several examples are shown below:

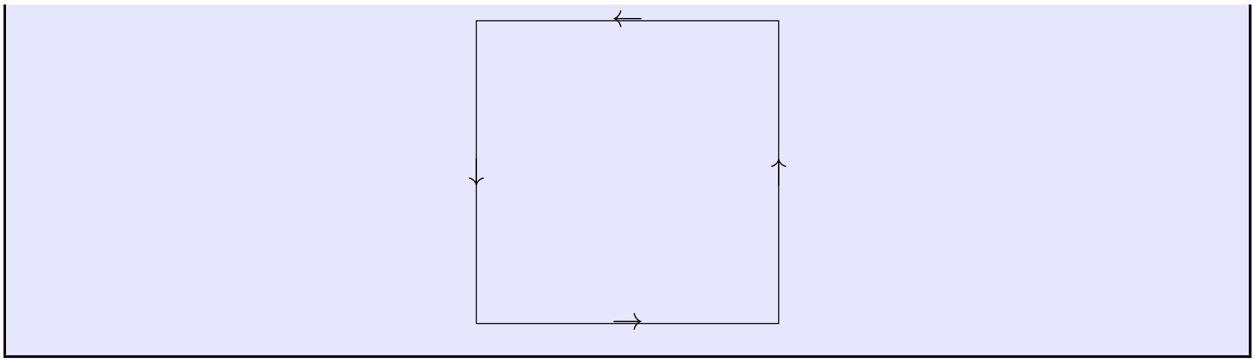
Example 16.1.1

Problem: A square of metal is located along the same plane as the page. A magnetic field is directed into the page, and is getting stronger. In what direction will current flow?

Solution: We begin by drawing a diagram:



Since the magnetic field is getting stronger, the current induced will oppose this change by creating a magnetic field directed out of the page (thereby cancelling some of the growing strength). Using the third right hand rule, current will flow counterclockwise, as shown below:



16.2 Faraday's Law

$$\varepsilon = -\frac{\partial \Phi}{\partial t} \tag{16.1}$$

Chapter 17

Nuclear Physics

17.1 Elements, Isotopes, and Ions

You may remember some of the information contained on the periodic table of elements from chemistry. A sample box from the periodic table is shown below:

11 Na Sodium 22.989769

Figure 17.1: An example of a box from the Periodic Table of Elements.

The number of protons in an atom determines what element it is. In the example above, the atomic number of sodium is 11. This means that any atom that has 11 protons is a sodium atom. The element number is written to the bottom-left of the elements symbol when writing nuclear equations: ${}_{11}\mathrm{Na}$, though this is redundant, since the symbol tells you what element it is.

An atom's isotope is determined by the number of protons and neutrons in that atom. Collectively, protons and neutrons are called *nucleons*. Thus, increasing the number of neutrons present in an atom will change its isotope without changing its element. While Sodium only has one stable isotope with 23 nucleons, other elements sometimes have multiple isotopes that are stable. Thus, when referring to a specific isotopes, we often state the element's name and isotope number, such as Sodium-29 or Carbon-12. The isotope is often written to the upper-left of the isotope's symbol: ${}^{23}\mathrm{Na}$.

A neutral atom has the same number of protons and electrons, but not all atoms are electrically neutral. An atom that has lost or gained one or more electrons is called an ion. Ions are indicated by placing a charge on the upper-right of the element symbol: Na^{2+}.

In nuclear reactions, it is often useful to see element, isotope, and ion information at the same time. The symbol ${}^{59}_{27}\mathrm{Co}^{-}$ would refer to a cobalt-59 atom that has one extra electron.

17.2 Radioactive Decay

17.2.1 Types of Radioactive Decay

Alpha Decay

Alpha decay is when an atom emits an α (alpha) particle. *α-particles* are really just a helium nucleus. That is, in alpha decay, a group of two protons and two neutrons spontaneously breaks off of the nucleus.

Example 17.2.1

Problem: A Uranium-238 atom undergoes alpha decay.

1. Write a formula for this reaction.
2. How much energy is released in this reaction?

Solution: We know that the Uranium-238 atom will break up into an alpha particle and another unknown atom, so we begin by writing the equation with a placeholder element:

$$^{238}_{92}U \longrightarrow ^{4}_{2}\alpha^{2+} + X$$

Applying conservation laws allows us to determine the unknown element's isotope and atomic number. This element must have a mass-number of 234, as well as 90 protons. Likewise, since the initial atom was electrically neutral, the final product must have a charge of -2 in order to cancel the positive charge of the alpha particle. Since Element 90 is Thorium, we can write:

$$\boxed{^{238}_{92}U \longrightarrow ^{4}_{2}\alpha^{2+} + ^{234}_{90}Th^{2-}}$$

In order to determine the amount of energy released in this process, we must find the difference in mass between the two sides of the equation.

$$m_{U238} = 238.05078826\text{u}$$

$$m_{He4} = 4.002602\text{u}$$

$$m_{Th234} = 234.0436\text{u}$$

Subtracting the two sides shows:

$$\Delta m = m_{U238} - (m_{He4} + m_{Th234}) = 238.05078826\text{u} - (4.002602\text{u} + 234.0436\text{u}) = 0.00458626\text{u}$$

We can convert atomic mass units to kilograms since we know $1\text{u} = 1.6605402 \times 10^{-27}\text{kg}$:

$$\Delta m = 0.00458626\text{u} \times \frac{1.6605402 \times 10^{-27}\text{kg}}{1\text{u}} = 7.616 \times 10^{-30}\text{kg}$$

Applying $E = mc^2$ yields:

$$E = mc^2 = 7.616 \times 10^{-30}\text{kg} \cdot (3 \times 10^8 \text{m/s})^2 = \boxed{6.854 \times 10^{-13}\text{J}} \tag{17.1}$$

Beta Decay

Gamma Decay

17.2.2 Half-Life

17.3 Fission and Fusion

17.3.1 Fission

17.3.2 Fusion

Appendices

Appendix A

Math Skills

A.1 Scientific Notation

- Scientific Notation always has three parts: the *coefficient*, the *base*, and the *exponent*:

$$\text{Coefficient} \rightarrow 6.022 \times \underset{\substack{\uparrow \\ \text{Base}}}{10}^{23} \leftarrow \text{Exponent}$$

- In scientific notation the base is always 10.
- A negative in front of the coefficient means the whole number is negative.
- A negative exponent means the number is very small (close to zero).
- The exponent counts how many places the decimal moved, NOT the number of zeroes.
- When comparing numbers in scientific notation, look at (in order):
  1. Negatives in front of the coefficient.
  2. Exponents
  3. Coefficients
- To multiply, multiply coeffients, then ADD exponents.
- To divide, divide coefficients, then SUBTRACT exponents.
- To raise to a power, raise the coefficient to the power, then MULTIPLY exponents.
- To enter scientific notation on most calculators use the "EE" key. 6.022×10^{23} is entered as 6.022E23. Calculator notation should **<u>never</u>** be handwritten.
- Metric Prefixes are really just scientific notation:

Prefix	Letter	Power of 10
nano	n	$\times 10^{-9}$
micro	μ	$\times 10^{-6}$
milli	m	$\times 10^{-3}$
centi	c	$\times 10^{-2}$
deci	d	$\times 10^{-1}$
Deka	D	$\times 10^{1}$
Hecto	H	$\times 10^{2}$
Kilo	k	$\times 10^{3}$
Mega	M	$\times 10^{6}$
Giga	G	$\times 10^{9}$

A.2 Algebra

A.3 Trigonometry

A.4 Radians and Arc Length

A.4.1 Radians

Just like there are many different units that measure distance (meters, feet, inches, miles, furlongs, etc.), there are also different ways of measuring angles. You are probably already familiar with degrees. A right angle is 90°, and a full circle is 360°. When calculating arc length or using the angular kinematic equations, the standard units for angles are *radians*[1] There are 2π radians in a complete circle, so,

$$360^\circ = 2\pi\,\text{radians}$$

This equation can be used to convert an angle from radians to degrees or vice-versa.

Example A.4.1

Problem: An angle measures 34°. What is this angle in radians?

Solution: Begin by creating a conversion factor. In this case, since we have degrees and want radians, we create a fraction with 2π radians on top of the fraction, and 360° on the bottom. Multiplying by this fraction gives:

$$34^\circ \times \frac{2\pi\,\text{rad}}{360^\circ} = \frac{17\pi}{90}\text{rad} \approx 0.593\,\text{rad}$$

It should be noted that the fraction $\frac{2\pi\,\text{rad}}{360^\circ}$ can easily be reduced to $\frac{\pi\,\text{rad}}{180^\circ}$. Using this as your conversion factor will yield the same results. It is also often easier and more accurate to leave measurements involving radians in terms of π.

Example A.4.2

Problem: An angle measures $\frac{\pi}{2}$rad. What is this angle in degrees?

Solution: Again, we begin by creating a conversion factor. Because we have a measurement in radians and are asked to find degrees, we create a fraction with 360° on top of the fraction, and 2π radians on the bottom:

$$\frac{\pi}{2} \times \frac{360^\circ}{2\pi\,\text{rad}} = 90^\circ$$

[1]It should be noted that strictly speaking, radians are not a unit; since the definition of a radian has to do with a ratio, all numbers with radians as the unit are actually unitless.

A.4.2 Arc Length

The distance along the circumference of a circle that corresponds to an internal angle of the circle is called *Arc Length*. Though arc-length is a measurement of length, the symbol used for Arc Length is s.

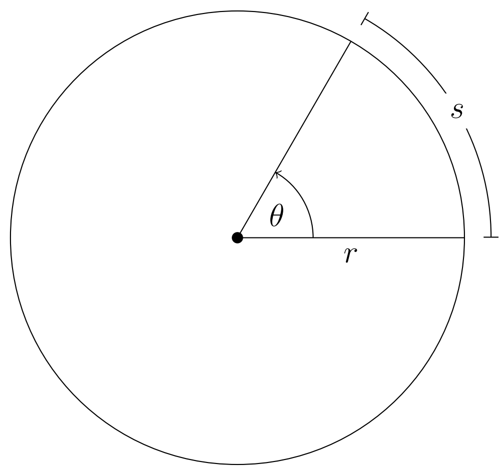


Figure A.1: The relationship between arc-length, radius, and angle

Arc Length can be found using the following equation:

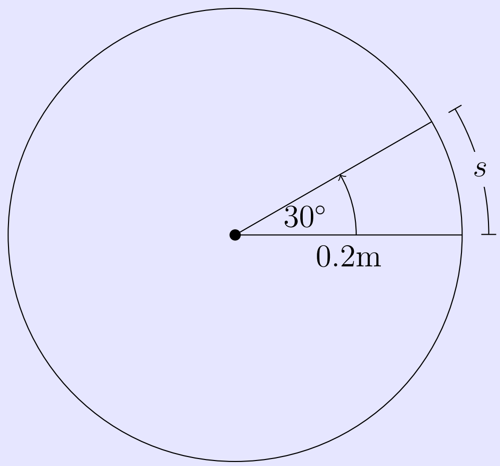
$$\boxed{s = r\theta} \tag{A.1}$$

where r is the radius of the circle and θ is the internal angle, measured in radians. Additionally, while meters are the proper SI units for these measurements, this formula will work with any units of length as long as both arc-length, s, and radius, r are measured using the same units.

Example A.4.2

Problem: Find the arc length of 30° of a circle with a radius of 0.2 meters.

Solution: Begin by drawing a diagram:



In order to find arc length, we must first convert the angle from degrees to radians:

$$\theta = 30^\circ \times \frac{2\pi\text{rad}}{360^\circ} = \frac{\pi}{6}\text{rad}$$

Now, arc length can be found using equation A.1.

$$s = r\theta = (0.2\text{m})(\frac{\pi}{6}\text{rad}) \approx 0.105\text{m}$$

Appendix B

Reference Tables

B.1 Greek Letters

Name	**Captial**	**Lower Case**	**Alternate versions**
alpha	A	α	
beta	B	β	
gamma	Γ	γ	
delta	Δ	δ	
epsilon	E	ε	ϵ
zeta	Z	ζ	
eta	H	η	
theta	Θ	θ	$\mathit{\Theta}$, ϑ
iota	I	ι	
kappa	K	κ	
lambda	Λ	λ	$\mathit{\Lambda}$
mu	M	μ	
nu	N	ν	
xi	Ξ	ξ	$\mathit{\Xi}$
omicron	O	o	
pi	Π	π	$\mathit{\Pi}\varpi$
rho	P	ρ	ϱ
sigma	Σ	σ, ς	
tau	T	τ	
upsilon	Υ	υ	$\mathit{\Upsilon}$
phi	Φ	ϕ	$\mathit{\Phi}$, φ
chi	X	χ	
psi	Ψ	ψ	$\mathit{\Psi}$
omega	Ω	ω	$\mathit{\Omega}$

B.2 Musical Notes and Frequencies

Table B.2: Frequencies of Musical Notes, in Hz

Octave:	0	1	2	3	4	5	6	7	8
C	16.35	32.70	65.41	130.81	261.63	523.25	1046.50	2093.00	4186.01
C♯/D♭	17.32	34.65	69.30	138.59	277.18	554.37	1108.733	2217.46	4434.92
D	18.35	36.71	73.42	146.83	293.66	587.33	1174.66	2349.32	4698.64
D♯/E♭	19.45	38.89	77.78	155.56	311.13	622.25	1244.51	2489.02	4978.03
E	20.60	41.20	82.41	164.81	329.63	659.26	1318.51	2637.02	5274.04
F	21.83	43.65	87.31	174.61	349.23	698.46	1396.91	2793.83	5587.65
F♯/G♭	23.12	46.25	92.50	185.00	369.99	739.99	1479.98	2959.96	5919.91
G	24.50	49.00	98.00	196.00	392.00	783.99	1567.98	3135.96	6271.93
G♯/A♭	25.96	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44	6644.88
A	27.50	55.00	110.00	220.00	440.00	880.00	1760.00	3520.00	7040.00
A♯/B♭	29.14	58.27	116.54	233.08	466.16	932.33	1864.66	3729.31	7458.62
B	30.87	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07	7902.13

B.3 Common Indices of Refraction

Table B.3: Table of Common Indices of Refraction

Material	**Index of Refraction**
Air	1.000273
Acrylic	1.495
Cubic Zirconium	2.15-2.18
Diamond	2.417
Crown Glass	1.485-1.755
Flint Glass	1.60-1.62
Vacuum (Empty Space)	1
Vegetable Oil	1.47
Water	1.333

Source: Wikipedia contributors. (2021, January 8). List of refractive indices. In Wikipedia, The Free Encyclopedia

B.4 Physical Constants

Table B.5: Table of Common Physical Constants

Quantity	Symbol	Value
Charge of an electron	e^-	$1.6 \times 10^{-19}\mathrm{C}$
Speed of light	c	$2.99792458 \times 10^8\mathrm{m/s}$
Universal Gravitational Constant	G	$6.67 \times 10^{-11}\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}$

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