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7

Velocity in Mechanisms

(Relative Velocity Method)

Features

7.1. Introduction

We have discussed, in the previous chapter, the instantaneous centre method for finding the velocity of various points in the mechanisms. In this chapter, we shall discuss the relative velocity method for determining the velocity of different points in the mechanism. The study of velocity analysis is very important for determining the acceleration of points in the mechanisms which is discussed in the next chapter.

7.2. Relative Velocity of Two Bodies Moving in Straight Lines

Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 7.1 (*a*) and 7.2 (*a*) respectively.

Consider two bodies *A* and *B* moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig. 7.1 (*a*). The relative velocity of *A* with respect to *B*,

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B} \qquad \ldots(i)$$

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From Fig. 7.1 (*b*), the relative velocity of *A* with respect to *B* (*i.e.* v_{AB}) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

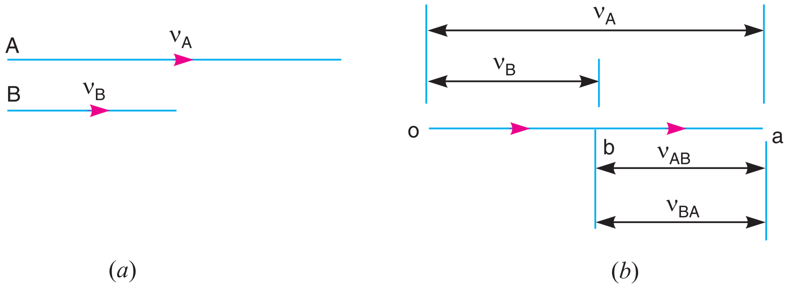


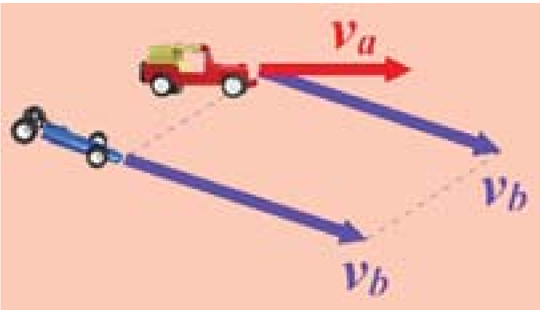
Fig. 7.1. Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of *B* with respect to *A*,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$ ...(*ii*)

or $\overline{ab} = \overline{ob} - \overline{oa}$

Now consider the body *B* moving in an inclined direction as shown in Fig. 7.2 (*a*). The relative velocity of *A* with respect to *B* may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point *o* and draw vector *oa* to represent v_A in magnitude and direction to some suitable scale. Similarly, draw vector *ob* to represent v_B in magnitude and direction to the same scale. Then vector *ba* represents the relative velocity of *A* with respect to *B* as shown in Fig. 7.2 (*b*). In the similar way as discussed above, the relative velocity of *A* with respect to *B*,



v_{AB} = Vector difference of v_A and $v_B = \overline{v_A} - \overline{v_B}$

or $\overline{ba} = \overline{oa} - \overline{ob}$

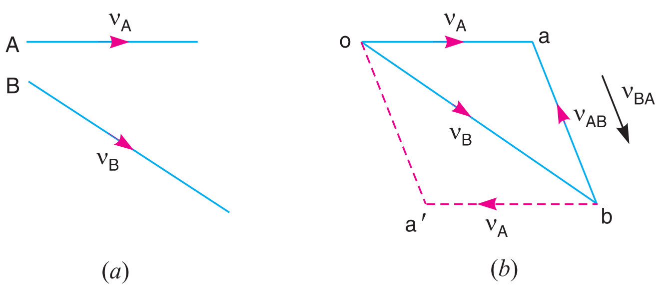


Fig. 7.2. Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of *B* with respect to *A*,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

or $\overline{ab} = \overline{ob} - \overline{oa}$

From above, we conclude that the relative velocity of point A with respect to B (v_{AB}) and the relative velocity of point B with respect A (v_{BA}) are equal in magnitude but opposite in direction, *i.e.*

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Note: It may be noted that to find v_{AB}, start from point b towards a and for v_{BA}, start from point a towards b.

7.3. Motion of a Link

Consider two points A and B on a rigid link AB, as shown in Fig. 7.3 (*a*). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.

Hence ***velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.***

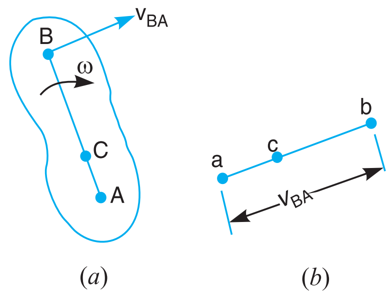


Fig. 7.3. Motion of a Link.

The relative velocity of B with respect to A (*i.e.* v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig. 7.3 (*b*).

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \qquad \text{...}(i)$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad \text{...}(iii)$$

Thus, we see from equation (*iii*), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

Note: The relative velocity of A with respect to B is represented by ba, although A may be a fixed point. The motion between A and B is only relative. Moreover, it is immaterial whether the link moves about A in a clockwise direction or about B in a clockwise direction.

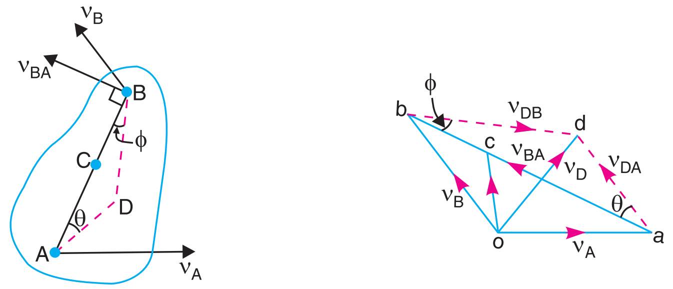
7.4. Velocity of a Point on a Link by Relative Velocity Method

The relative velocity method is based upon the relative velocity of the various points of the link as discussed in Art. 7.3.

Consider two points A and B on a link as shown in Fig. 7.4 (*a*). Let the absolute velocity of the point A *i.e.* v_A is known in magnitude and direction and the absolute velocity of the point B *i.e.* v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig. 7.4 (*b*). The velocity diagram is drawn as follows :

1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to v_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB of Fig. 7.4 (*a*). This line will represent the velocity of B with respect to A, *i.e.* v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b.

5. Measure *ob*, which gives the required velocity of point *B* (v_B), to the scale.



(*a*) Motion of points on a link. (*b*) Velocity diagram.

Fig. 7.4

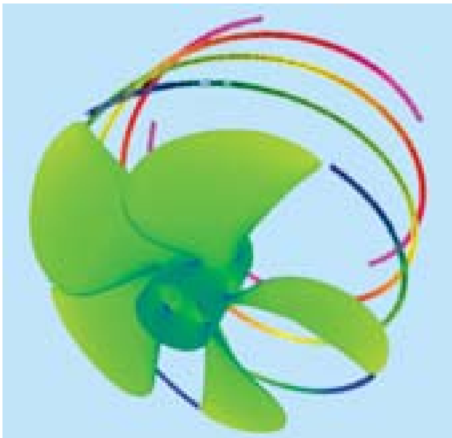
Notes : 1. The vector *ab* which represents the velocity of *B* with respect to *A* (v_{BA}) is known as velocity of image of the link *AB*.

2. The absolute velocity of any point *C* on *AB* may be determined by dividing vector *ab* at *c* in the same ratio as *C* divides *AB* in Fig. 7.4 (*a*).

In other words

$$\frac{ac}{ab} = \frac{AC}{AB}$$

Join *oc*. The *vector *oc* represents the absolute velocity of point *C* (v_C) and the vector *ac* represents the velocity of *C* with respect to *A* *i.e.* v_{CA}.



3. The absolute velocity of any other point *D* outside *AB*, as shown in Fig. 7.4 (*a*), may also be obtained by completing the velocity triangle *abd* and similar to triangle *ABD*, as shown in Fig. 7.4 (*b*).

4. The angular velocity of the link *AB* may be found by dividing the relative velocity of *B* with respect to *A* (*i.e.* v_{BA}) to the length of the link *AB*. Mathematically, angular velocity of the link *AB*,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

7.5. Velocities in Slider Crank Mechanism

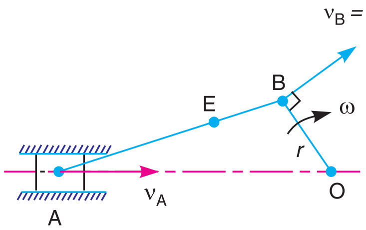
In the previous article, we have discused the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.

A slider crank mechanism is shown in Fig. 7.5 (*a*). The slider *A* is attached to the connecting rod *AB*. Let the radius of crank *OB* be *r* and let it rotates in a clockwise direction, about the point *O* with uniform angular velocity ω rad/s. Therefore, the velocity of *B* *i.e.* v_B is known in magnitude and direction. The slider reciprocates along the line of stroke *AO*.

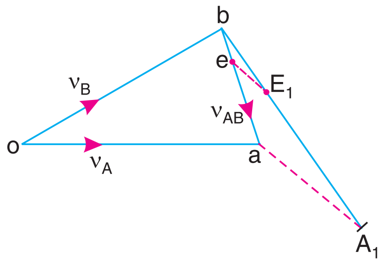
The velocity of the slider *A* (*i.e.* v_A) may be determined by relative velocity method as discussed below :

1. From any point *o*, draw vector *ob* parallel to the direction of v_B (or perpendicular to *OB*) such that $ob = v_B = \omega.r$, to some suitable scale, as shown in Fig. 7.5 (*b*).

* The absolute velocities of the points are measured from the pole (*i.e.* fixed points) of the velocity diagram.



(*a*) Slider crank mechanism.



(*b*) Velocity diagram.

Fig. 7.5

2. Since *AB* is a rigid link, therefore the velocity of *A* relative to *B* is perpendicular to *AB*. Now draw vector *ba* perpendicular to *AB* to represent the velocity of *A* with respect to *B* *i.e.* v_{AB}.

3. From point *o*, draw vector *oa* parallel to the path of motion of the slider *A* (which is along *AO* only). The vectors *ba* and *oa* intersect at *a*. Now *oa* represents the velocity of the slider *A* *i.e.* v_A, to the scale.

The angular velocity of the connecting rod *AB* (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

The direction of vector *ab* (or *ba*) determines the sense of ω_{AB} which shows that it is anticlockwise.

Note : The absolute velocity of any other point *E* on the connecting rod *AB* may also be found out by dividing vector *ba* such that $be/ba = BE/BA$. This is done by drawing any line bA_1 equal in length of *BA*. Mark $bE_1 = BE$. Join aA_1. From E_1 draw a line E_1e parallel to aA_1. The vector *oe* now represents the velocity of *E* and vector *ae* represents the velocity of *E* with respect to *A*.

7.6. Rubbing Velocity at a Pin Joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig. 7.6.

Let ω_1 = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to *O*.

ω_2 = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and

r = Radius of the pin.

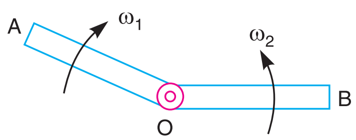


Fig. 7.6. Links connected by pin joints.

According to the definition,

Rubbing velocity at the pin joint *O*

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

Note : When the pin connects one sliding member and the other turning member, the angular velocity of the sliding member is zero. In such cases,

Rubbing velocity at the pin joint = $\omega.r$

where ω = Angular velocity of the turning member, and

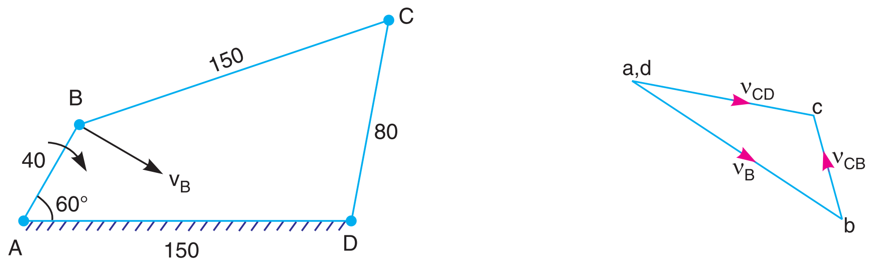
r = Radius of the pin.

Example 7.1. *In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.*

Solution. Given : N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\,\pi \times 120/60 = 12.568$ rad/s

Since the length of crank AB = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



(*a*) Space diagram (All dimensions in mm). (*b*) Velocity diagram.

Fig. 7.7

First of all, draw the space diagram to some suitable scale, as shown in Fig. 7.7 (*a*). Now the velocity diagram, as shown in Fig. 7.7 (*b*), is drawn as discussed below :

1. Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (*i.e.* v_{BA} or v_B) such that

$$\text{vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$

2. Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (*i.e.* v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (*i.e.* v_{CD} or v_C). The vectors bc and dc intersect at c.

By measurement, we find that

$$v_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

We know that CD = 80 mm = 0.08 m

∴ Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D) \quad \textbf{Ans.}$$

Example 7.2. *The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine :* **1.** *velocity of piston,* **2.** *angular velocity of connecting rod,* **3.** *velocity of point E on the connecting rod 1.5 m from the gudgeon pin,* **4.** *velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively,* **5.** *position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.*

Solution. Given : $N_{BO} = 180$ r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s

Since the crank length $OB = 0.5$ m, therefore linear velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

. . . (Perpendicular to BO)

1. *Velocity of piston*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.8 (*a*). Now the velocity diagram, as shown in Fig. 7.8 (*b*), is drawn as discussed below :

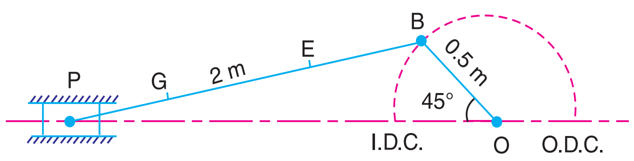
1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or velocity of *B* such that

$$\text{vector } ob = v_{BO} = v_B = 9.426 \text{ m/s}$$

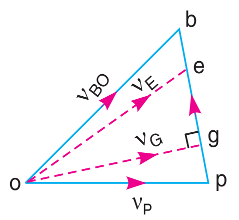
2. From point *b*, draw vector *bp* perpendicular to *BP* to represent velocity of *P* with respect to *B* (*i.e.* v_{PB}) and from point *o*, draw vector *op* parallel to *PO* to represent velocity of *P* with respect to *O* (*i.e.* v_{PO} or simply v_P). The vectors *bp* and *op* intersect at point *p*.

By measurement, we find that velocity of piston *P*,

$$v_P = \text{vector } op = 8.15 \text{ m/s} \quad \textbf{Ans.}$$



(*a*) Space diagram.



(*b*) Velocity diagram.

Fig. 7.8

2. *Angular velocity of connecting rod*

From the velocity diagram, we find that the velocity of *P* with respect to *B*,

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

Since the length of connecting rod *PB* is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

3. *Velocity of point E on the connecting rod*

The velocity of point *E* on the connecting rod 1.5 m from the gudgeon pin (*i.e.* $PE = 1.5$ m) is determined by dividing the vector *bp* at *e* in the same ratio as *E* divides *PB* in Fig. 7.8 (*a*). This is done in the similar way as discussed in Art 7.6. Join *oe*. The vector *oe* represents the velocity of *E*. By measurement, we find that velocity of point *E*,

$$v_E = \text{vector } oe = 8.5 \text{ m/s} \quad \textbf{Ans.}$$

Note : The point *e* on the vector *bp* may also be obtained as follows :

$$\frac{BE}{BP} = \frac{be}{bp} \quad \text{or} \quad be = \frac{BE \times bp}{BP}$$

4. *Velocity of rubbing*

We know that diameter of crank-shaft pin at *O*,

$$d_O = 50 \text{ mm} = 0.05 \text{ m}$$

Diameter of crank-pin at B,

$$d_B = 60 \text{ mm} = 0.06 \text{ m}$$

and diameter of cross-head pin,

$$d_C = 30 \text{ mm} = 0.03 \text{ m}$$

We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_O}{2} \times \omega_{BO} = \frac{0.05}{2} \times 18.85 = 0.47 \text{ m/s} \quad \textbf{Ans.}$$

Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = \frac{0.06}{2}(18.85 + 3.4) = 0.6675 \text{ m/s} \quad \textbf{Ans.}$$

...($\because$ ω_{BO} is clockwise and ω_{PB} is anticlockwise.)

and velocity of rubbing at the pin of cross-head

$$= \frac{d_C}{2} \times \omega_{PB} = \frac{0.03}{2} \times 3.4 = 0.051 \text{ m/s} \quad \textbf{Ans.}$$

...($\because$ At the cross-head, the slider does not rotate and only the connecting rod has angular motion.)

5. *Position and linear velocity of point G on the connecting rod which has the least velocity relative to crank-shaft*

The position of point G on the connecting rod which has the least velocity relative to crank-shaft is determined by drawing perpendicular from o to vector bp. Since the length of og will be the least, therefore the point g represents the required position of G on the connecting rod.

By measurement, we find that

$$\text{vector } bg = 5 \text{ m/s}$$

The position of point G on the connecting rod is obtained as follows:

$$\frac{bg}{bp} = \frac{BG}{BP} \quad \text{or} \quad BG = \frac{bg}{bp} \times BP = \frac{5}{6.8} \times 2 = 1.47 \text{ m} \quad \textbf{Ans.}$$

By measurement, we find that the linear velocity of point G,

$$v_G = \text{vector } og = 8 \text{ m/s} \quad \textbf{Ans.}$$

Example 7.3. *In Fig. 7.9, the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.*

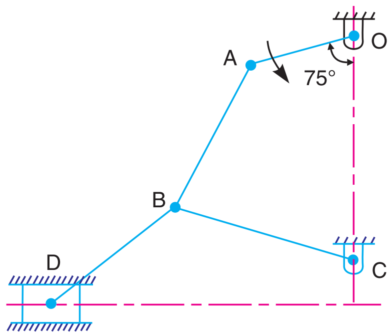


Fig. 7.9

Solution. Given: $N_{AO} = 600$ r.p.m. or

$$\omega_{AO} = 2\pi \times 600/60 = 62.84 \text{ rad/s}$$

Since $OA = 28$ mm $= 0.028$ m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

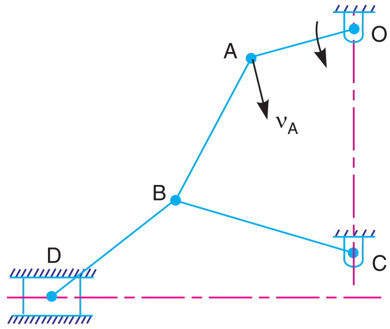
. . . (Perpendicular to OA)

Linear velocity of the slider D

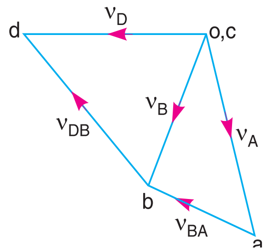
First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.10 (a). Now the velocity diagram, as shown in Fig. 7.10 (b), is drawn as discussed below :

1. Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$



(*a*) Space diagram.



(*b*) Velocity diagram.

Fig. 7.10

2. From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect A (*i.e.* v_{BA}) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (*i.e.* v_{BC} or v_B). The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (*i.e.* v_{DB}) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (*i.e.* v_D). The vectors bd and od intersect at d.

By measurement, we find that velocity of the slider D,

$$v_D = \text{vector } od = 1.6 \text{ m/s} \quad \textbf{Ans.}$$

Angular velocity of the link BD

By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link BD = 46 mm = 0.046 m, therefore angular velocity of the link BD,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B) \quad \textbf{Ans.}$$

Example 7.4. *The mechanism, as shown in Fig. 7.11, has the dimensions of various links as follows :*

AB = DE = 150 mm ; BC = CD = 450 mm ; EF = 375 mm.

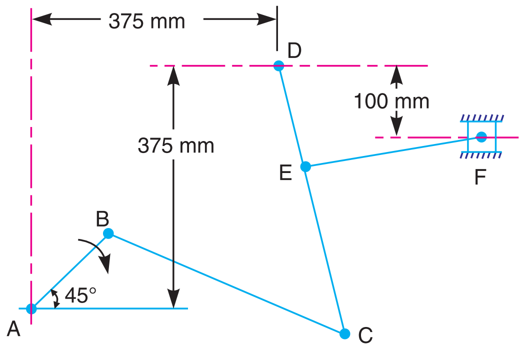


Fig. 7.11

The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC.

The block F moves in the horizontal guides, being driven by the link EF. Determine: **1.** *velocity of the block F,* **2.** *angular velocity of DC, and* **3.** *rubbing speed at the pin C which is 50 mm in diameter.*

Solution. Given : $N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 4\pi$ rad/s

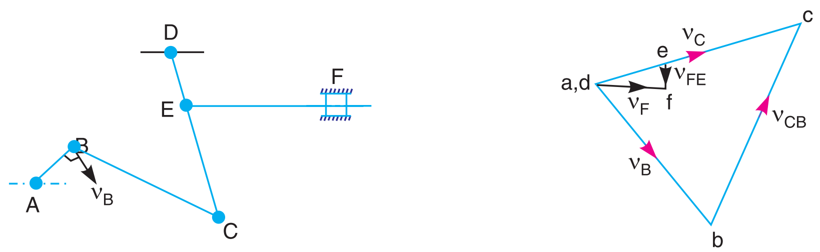
Since the crank length $AB = 150$ mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885 \text{ m/s}$$

... (Perpendicular to AB)

1. *Velocity of the block F*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.12 (*a*). Now the velocity diagram, as shown in Fig. 7.12 (*b*), is drawn as discussed below:



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.12

1. Since the points A and D are fixed, therefore these points are marked as one point* as shown in Fig. 7.12 (*b*). Now from point *a*, draw vector *ab* perpendicular to AB, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

$$\text{vector } ab = v_{BA} = v_B = 1.885 \text{ m/s}$$

2. The point C moves relative to B and D, therefore draw vector *bc* perpendicular to BC to represent the velocity of C with respect to B (*i.e.* v_{CB}), and from point *d*, draw vector *dc* perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (*i.e.* v_{CD} or v_C). The vectors *bc* and *dc* intersect at *c*.

3. Since the point E lies on DC, therefore divide vector *dc* in *e* in the same ratio as E divides CD in Fig. 7.12 (*a*). In other words

$$ce/cd = CE/CD$$

The point *e* on *dc* may be marked in the same manner as discussed in Example 7.2.

4. From point *e*, draw vector *ef* perpendicular to EF to represent the velocity of F with respect to E (*i.e.* v_{FE}) and from point *d* draw vector *df* parallel to the path of motion of F, which is horizontal, to represent the velocity of F *i.e.* v_F. The vectors *ef* and *df* intersect at *f*.

By measurement, we find that velocity of the block F,

$$v_F = \text{vector } df = 0.7 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocity of DC*

By measurement from velocity diagram, we find that velocity of C with respect to D,

$$v_{CD} = \text{vector } dc = 2.25 \text{ m/s}$$

* When the fixed elements of the mechanism appear at more than one place, then all these points lie at one place in the velocity diagram.

Since the length of link DC = 450 mm = 0.45 m, therefore angular velocity of DC,

$$\omega_{DC} = \frac{v_{CD}}{DC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \qquad \text{. . . (Anticlockwise about } D\text{)}$$

3. *Rubbing speed at the pin C*

We know that diameter of pin at C,

$$d_C = 50 \text{ mm} = 0.05 \text{ m} \quad \text{or} \quad \text{Radius}, r_C = 0.025 \text{ m}$$

From velocity diagram, we find that velocity of C with respect to B,

$$v_{CB} = \text{vector } bc = 2.25 \text{ m/s} \qquad \text{. . . (By measurement)}$$

$$\text{Length } BC = 450 \text{ mm} = 0.45 \text{ m}$$

∴ Angular velocity of BC,

$$\omega_{CB} = \frac{v_{CB}}{BC} = \frac{2.25}{0.45} = 5 \text{ rad/s} \qquad \text{... (Anticlockwise about } B\text{)}$$

We know that rubbing speed at the pin C

$$= (\omega_{CB} - \omega_{CD})\, r_C = (5 - 5)\, 0.025 = 0 \textbf{ Ans.}$$

Example 7.5. *In a mechanism shown in Fig. 7.13, the crank OA is 100 mm long and rotates clockwise about O at 120 r.p.m. The connceting rod AB is 400 mm long.*

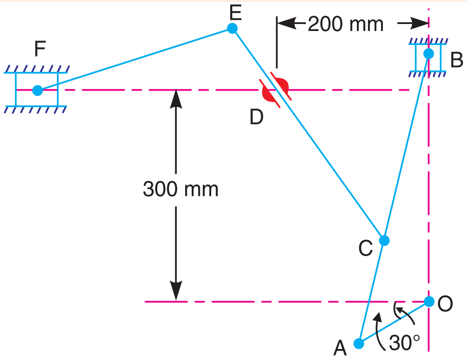


Fig. 7.13.

At a point C on AB, 150 mm from A, the rod CE 350 mm long is attached. This rod CE slides in a slot in a trunnion at D. The end E is connected by a link EF, 300 mm long to the horizontally moving slider F.

For the mechanism in the position shown, find ***1.*** *velocity of F,* ***2.*** *velocity of sliding of CE in the trunnion, and* ***3.*** *angular velocity of CE.*

Solution. Given : v_{AO} = 120 r.p.m. or $\omega_{AO} = 2\,\pi \times 120/60 = 4\,\pi$ rad/s

Since the length of crank OA = 100 mm = 0.1 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 4\,\pi \times 0.1 = 1.26 \text{ m/s}$$

. . . (Perpendicular to AO)

1. *Velocity of F*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.14 (*a*). Now the velocity diagram, as shown in Fig. 7.14 (*b*), is drawn as discussed below :

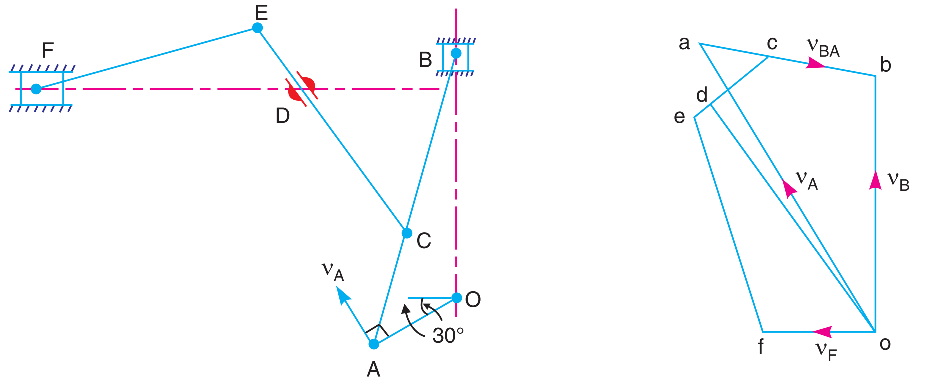
An aircraft uses many mechanisms in engine, power transmission and steering.

Note : This picture is given as additional information and is not a direct example of the current chapter.

1. Draw vector *oa* perpendicular to *AO*, to some suitable scale, to represent the velocity of *A* with respect to *O* or simply velocity of *A* (*i.e.* v_{AO} or v_A), such that

$$\text{vector } oa = v_{AO} = v_A = 1.26 \text{ m/s}$$

2. From point *a*, draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* *i.e.* v_{BA}, and from point *o* draw vector *ob* parallel to the motion of *B* (which moves along *BO* only) to represent the velocity of *B* *i.e.* v_B. The vectors *ab* and *ob* intersect at *b*.



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.14

3. Since the point *C* lies on *AB*, therefore divide vector *ab* at *c* in the same ratio as *C* divides *AB* in the space diagram. In other words,

$$ac/ab = AC/AB$$

4. From point *c*, draw vector *cd* perpendicular to *CD* to represent the velocity of *D* with respect to *C* *i.e.* v_{DC}, and from point *o* draw vector *od* parallel to the motion of *CD*, which moves along *CD* only, to represent the velocity of *D*, *i.e.* v_D.

5. Since the point *E* lies on *CD* produced, therefore divide vector *cd* at *e* in the same ratio as *E* divides *CD* in the space diagram. In other words,

$$cd/ce = CD/CE$$

6. From point *e*, draw vector *ef* perpendicular to *EF* to represent the velocity of *F* with respect to *E* *i.e.* v_{FE}, and from point *o* draw vector *of* parallel to the motion of *F*, which is along *FD* to represent the velocity of *F* *i.e.* v_F.

By measurement, we find that velocity of *F*,

$$v_F = \text{vector } of = 0.53 \text{ m/s} \quad \textbf{Ans.}$$

2. *Velocity of sliding of CE in the trunnion*

Since velocity of sliding of *CE* in the trunnion is the velocity of *D*, therefore velocity of sliding of *CE* in the trunnion

$$= \text{vector } od = 1.08 \text{ m/s} \quad \textbf{Ans.}$$

3. *Angular velocity of CE*

By measurement, we find that linear velocity of *C* with respect to *E*,

$$v_{CE} = \text{vector } ec = 0.44 \text{ m/s}$$

Since the length *CE* = 350 mm = 0.35 m, therefore angular velocity of *CE*,

$$\omega_{CE} = \frac{v_{CE}}{CE} = \frac{0.44}{0.35} = 1.26 \text{ rad/s (Clockwise about } E) \quad \textbf{Ans.}$$

Example 7.6. *In a mechanism as shown in Fig. 7.15, the various dimensions are : OC = 125 mm ; CP = 500 mm ; PA = 125 mm ; AQ = 250 mm and QE = 125 mm.*

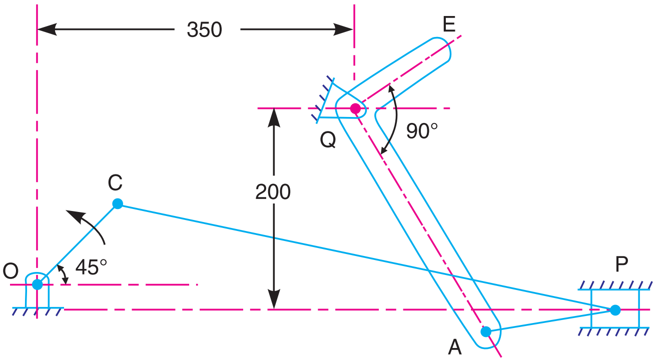


Fig. 7.15. All dimensions in mm.

The slider P translates along an axis which is 25 mm vertically below point O. The crank OC rotates uniformly at 120 r.p.m. in the anti-clockwise direction. The bell crank lever AQE rocks about fixed centre Q.

Draw the velocity diagram and calculate the absolute velocity of point E of the lever.

Solution. Given : N_{CO} = 120 r.p.m. or $\omega_{CO} = 2\pi \times 120/60 = 12.57$ rad/s ; OC = 125 mm = 0.125 m

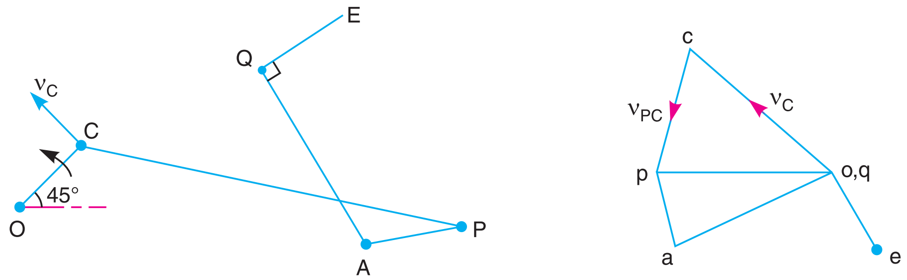
We know that linear velocity of C with respect to O or velocity of C, (because O is as fixed point)

$$v_{CO} = v_C = \omega_{CO} \times OC = 12.57 \times 0.125 = 1.57 \text{ m/s}$$

First of all, draw the space diagram, as shown in Fig. 7.16 (*a*), to some suitable scale. Now the velocity diagram, as shown in Fig. 7.16 (*b*) is drawn as discussed below :

1. Since the points O and Q are fixed, therefore these points are taken as one point in the velocity diagram. From point o, draw vector oc perpendicular to OC, to some suitable scale, to represent the velocity of C with respect to O or velocity of C, such that

$$\text{vector } oc = v_{CO} = v_C = 1.57 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.16

2. From point c, draw vector cp perpendicular to CP to represent the velocity of P with respect to C (*i.e.* v_{PC}) and from point o, draw vector op parallel to the path of motion of slider P (which is horizontal) to represent the velocity of P (*i.e.* v_P). The vectors cp and op intersect at p.

3. From point p, draw vector pa perpendicular to PA to represent the velocity of A with respect to P (*i.e.* v_{AP}) and from point q, draw vector qa perpendicular to QA to represent the velocity of A (*i.e.* v_A). The vectors pa and qa intersect at a.

4. Now draw vector qe perpendicular to vector qa in such a way that

$$QE/QA = qe/qa$$

By measurement, we find that the velocity of point E,

$$v_E = \text{vector } oe = 0.7 \text{ m/s} \quad \textbf{Ans.}$$

Example 7.7. *A quick return mechanism of the crank and slotted lever type shaping machine is shown in Fig. 7.17.*

The dimensions of the various links are as follows :

$$O_1O_2 = 800 \text{ mm} \; ; \; O_1B = 300 \text{ mm} \; ;$$

$$O_2D = 1300 \text{ mm} \; ; \; DR = 400 \text{ mm}.$$

*The crank O_1B makes an angle of 45° with the vertical and rotates at 40 r.p.m. in the counter clockwise direction. Find : **1.** velocity of the ram R, or the velocity of the cutting tool, and **2.** angular velocity of link O_2D.*

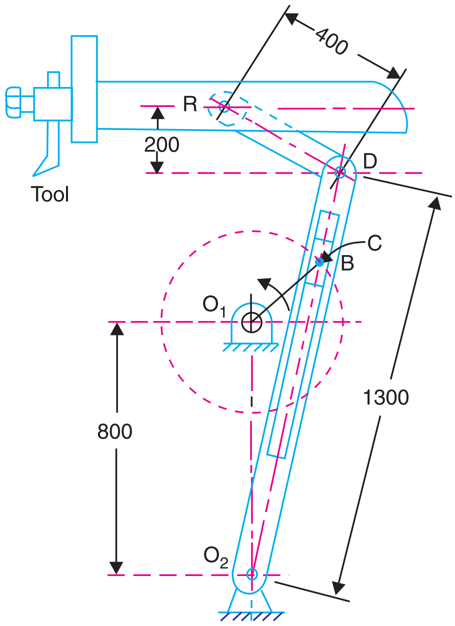


Fig. 7.17. All dimensions in mm.

Solution. Given: $N_{BO1} = 40$ r.p.m. or $\omega_{BO1} = 2\,\pi \times 40/60 = 4.2$ rad/s

Since the length of crank $O_1B = 300$ mm $= 0.3$m, therefore velocity of B with respect to O_1 or simply velocity of B (because O_1 is a fixed point),

$$v_{BO1} = v_B = \omega_{BO1} \times O_1B = 4.2 \times 0.3 = 1.26 \text{ m/s} \qquad \text{. . . (Perpendicular to } O_1B)$$

1. Velocity of the ram R

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.18 (*a*). Now the velocity diagram, as shown in Fig. 7.18 (*b*), is drawn as discussed below :

1. Since O_1 and O_2 are fixed points, therefore these points are marked as one point in the velocity diagram. Draw vector o_1b perpendicular to O_1B, to some suitable scale, to represent the velocity of B with respect to O_1 or simply velocity of B, such that

$$\text{vector } o_1b = v_{BO1} = v_B = 1.26 \text{ m/s}$$

2. From point o_2, draw vector o_2c perpendicular to O_2C to represent the velocity of the coincident point C with respect to O_2 or simply velocity of C (*i.e.* v_{CO2} or v_C), and from point b, draw vector bc parallel to the path of motion of the sliding block (which is along the link O_2D) to represent the velocity of C with respect to B (*i.e.* v_{CB}). The vectors o_2c and bc intersect at c.

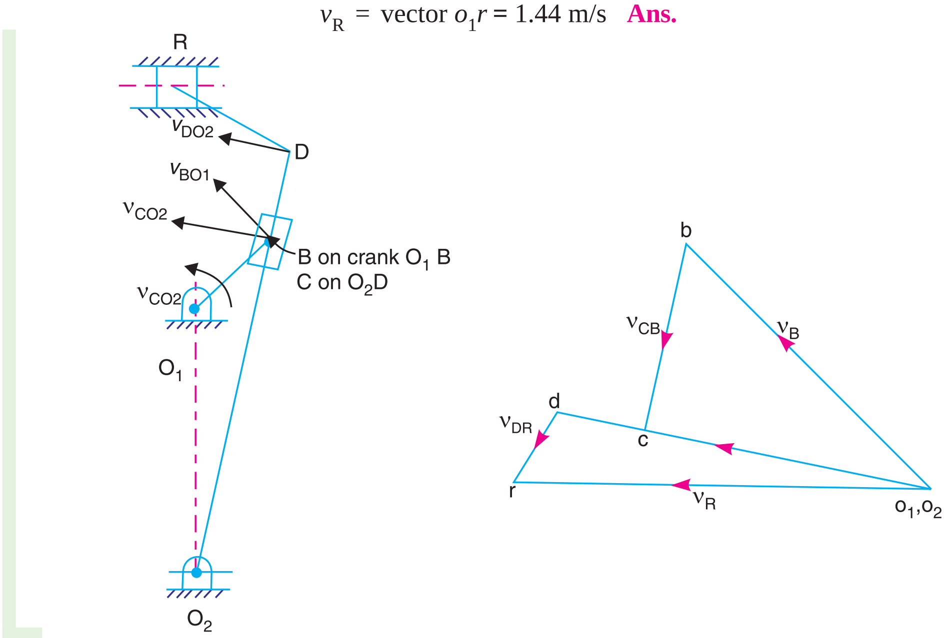
3. Since the point D lies on O_2C produced, therefore divide the vector o_2c at d in the same ratio as D divides O_2C in the space diagram. In other words,

$$cd / o_2d = CD/O_2D$$

4. Now from point d, draw vector dr perpendicular to DR to represent the velocity of R with respect to D (*i.e.* v_{RD}), and from point o_1 draw vector o_1r parallel to the path of motion of R (which is horizontal) to represent the velocity of R (*i.e.* v_R). The vectors dr and o_1r intersect at r.

By measurement, we find that velocity of the ram R,

$$v_R = \text{vector } o_1r = 1.44 \text{ m/s} \quad \textbf{Ans.}$$



(*a*) Space diagram (*b*) Velocity diagram.

Fig. 7.18

2. *Angular velocity of link O_2D*

By measurement from velocity diagram, we find that velocity of D with respect to O_2 or velocity of D,

$$v_{DO2} = v_D = \text{vector } o_2d = 1.32 \text{ m/s}$$

We know that length of link O_2D = 1300 mm = 1.3 m. Therefore angular velocity of the link O_2D,

$$\omega_{DO2} = \frac{v_{DO2}}{O_2D} = \frac{1.32}{1.3} = 1.015 \text{ rad/s (Anticlockwise about } O_2) \quad \textbf{Ans.}$$

The above picture shows prototype of an industrial steam engine. Before to the invention of electricity, steam engines used to provide the power needed to turn wheels in the factories.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 7.8. *In the mechanism, as shown in Fig. 7.19, the crank O_1A rotates at a speed of 60 r.p.m. in a clockwise direction imparting vertical reciprocating motion to the rack R, by means of toothed quadrant Q. O_1 and O_2 are fixed centres and the slotted bar BC and quadrant Q are rocking on O_2.*

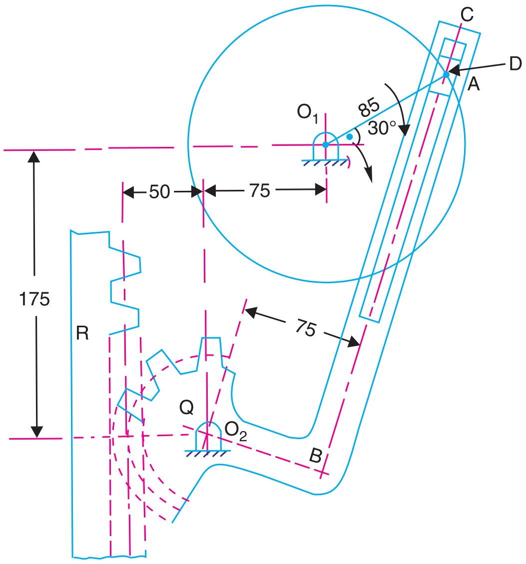


Fig. 7.19. All dimensions are in mm.

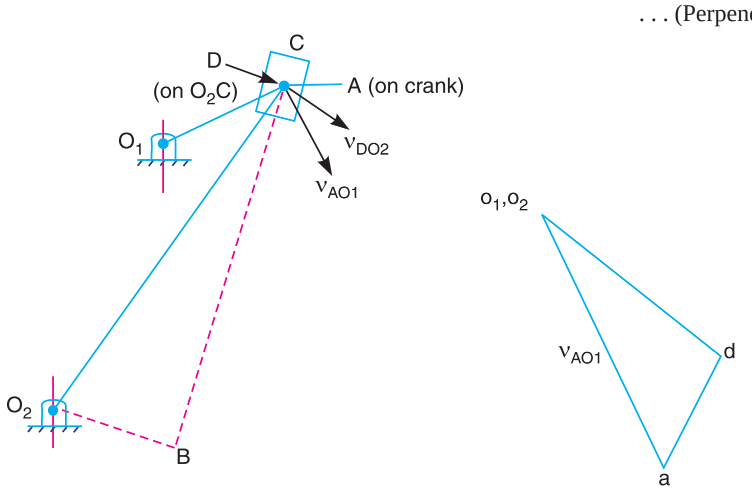
*Determine : **1.** the linear speed of the rack when the crank makes an angle of 30° to the horizontal, **2.** the ratio of the times of lowering and raising the rack, and **3.** the length of the stroke of the rack.*

Solution. Given : N_{AO1} = 60 r.p.m. or $\omega_{AO1} = 2\,\pi \times 60/60 = 6.28$ rad/s

Since crank length $O_1A = 85$ mm, therefore velocity of A with respect to O_1 or velocity of A, (because O_1 is a fixed point),

$$v_{AO1} = v_A = \omega_{AO1} \times O_1A = 6.28 \times 85 = 534 \text{ mm/s}$$

. . . (Perpendicular to O_1A)



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.20

1. *Linear speed of the rack*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.20 (*a*). Now the velocity diagram, as shown in Fig. 7.20 (*b*), is drawn as discussed below :

1. Since O_1 and O_2 are fixed points, therefore they are marked as one point in the velocity diagram. From point o_1, draw vector o_1a perpendicular to O_1A, to some suitable scale, to represent the velocity of A with respect to O_1 or simply velocity of A, such that

$$\text{vector } o_1a = v_{AO1} = v_A = 534 \text{ mm/s}$$

2. From point a, draw vector ad parallel to the path of motion of D (which is along the slot in the link BC) to represent the velocity D with respect to A (*i.e.* v_{DA}), and from point o_2 draw vector o_2d perpendicular to the line joining the points O_2 and D (because O_2 and D lie on the same link) to represent the velocity of D (*i.e.* v_{DO_2} or v_D). The vectors ad and o_2d intersect at d.

Note : The point A represents the point on the crank as well as on the sliding block whereas the point D represents the coincident point on the lever O_2C.

By measurement, we find that

$$v_{DO2} = v_D = \text{vector } o_2d = 410 \text{ mm/s, and } O_2D = 264 \text{ mm}$$

We know that angular velocity of the quadrant Q,

$$\omega_Q = \frac{v_{DO2}}{O_2D} = \frac{410}{264} = 1.55 \text{ rad/s} \quad (\text{Clockwise about } O_2)$$

Radius of the quadrant Q,

$$r_Q = 50 \text{ mm}$$

Since the rack and the quadrant have a rolling contact, therefore the linear velocity at the points of contact will be same as that of quadrant.

∴ Linear speed of the rack,

$$v_R = w_Q.r_Q = 1.55 \times 50 = 77.5 \text{ mm/s} \quad \textbf{Ans.}$$

2. *Ratio of the times of lowering and raising the rack*

The two extreme positions of the rack (or AB) are when the tangent to the circle with centre O_1 is also a tangent to the circle with centre O_2, as shown in Fig. 7.21. The rack will be raising when the crank moves from A_1 to A_2 through an angle α and it will be lowering when the crank moves from A_2 to A_1 through an angle β. Since the times of lowering and raising the rack is directly proportional to their respective angles, therefore

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\beta}{\alpha} = \frac{240°}{120°} = 2 \quad \textbf{Ans.}$$

. . . (By measurement)

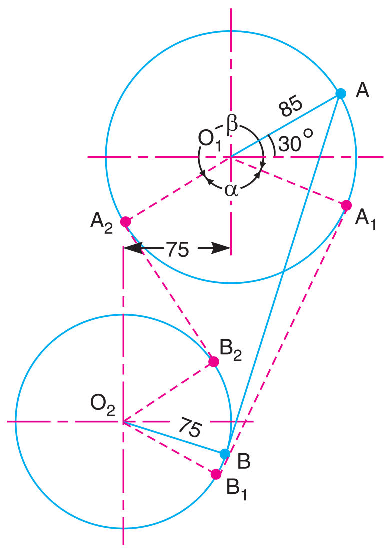


Fig. 7.21. All dimensions in mm.

3. *Length of stroke of the rack*

By measurement, we find that angle $B_1O_2B_2 = 60° = 60 \times \pi / 180 = 1.047$ rad

We know that length of stroke of the rack

= Radius of the quadrant × Angular rotation of the quadrant in radians

$= r_Q \times \angle B_1O_2B_2$ in radians $= 50 \times 1.047 = 52.35$ mm **Ans.**

Example 7.9. *Fig. 7.22 shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows :*

OQ = 100 mm ; OP = 200 mm, RQ = 150 mm and RS = 500 mm.

The crank OP makes an angle of 60° with the vertical. Determine the velocity of the slider S (cutting tool) when the crank rotates at 120 r.p.m. clockwise.

Find also the angular velocity of the link RS and the velocity of the sliding block T on the slotted lever QT.

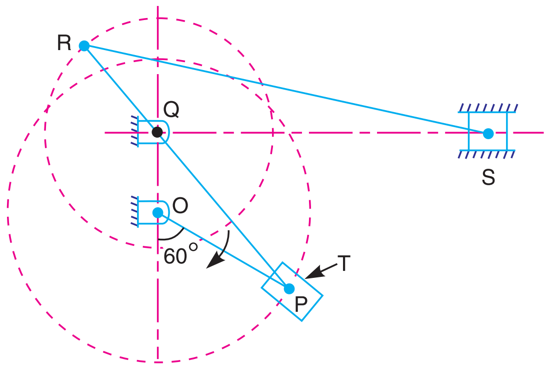


Fig. 7.22

Solution. Given : N_{PO} = 120 r.p.m. or $\omega_{PO} = 2\,\pi \times 120/60 = 12.57$ rad/s

Since the crank OP = 200 mm = 0.2 m, therefore velocity of P with respect to O or velocity of P (because O is a fixed point),

$$v_{PO} = v_P = \omega_{PO} \times OP = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

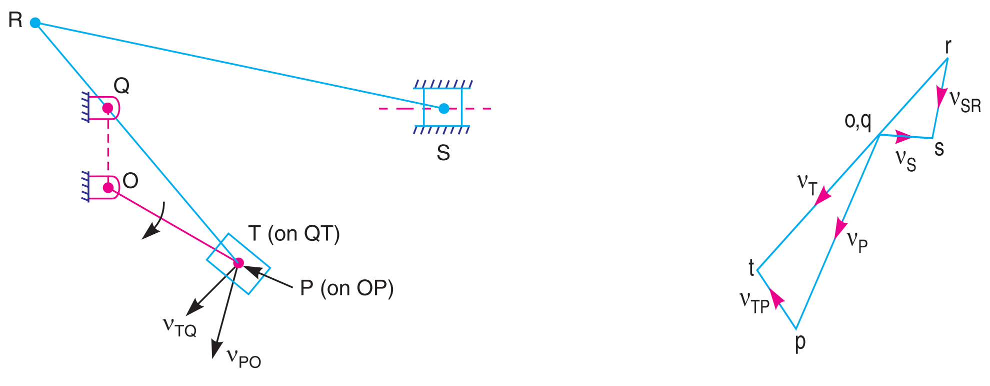
. . . (Perpendicular to PO)

Velocity of slider S (cutting tool)

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.23 (*a*). Now the velocity diagram, as shown in Fig. 7.23 (*b*) is drawn as discussed below :

1. Since O and Q are fixed points, therefore they are taken as one point in the velocity diagram. From point o, draw vector op perpendicular to OP, to some suitable scale, to represent the velocity of P with respect to O or simply velocity of P, such that

$$\text{vector } op = v_{PO} = v_P = 2.514 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.23

2. From point q, draw vector qt perpendicular to QT to represent the velocity of T with respect to Q or simply velocity of T (*i.e.* v_{TQ} or v_T) and from point p draw vector pt parallel to the path of motion of T (which is parallel to TQ) to represent the velocity of T with respect to P (*i.e.* v_{TP}). The vectors qt and pt intersect at t.

Note : The point T is a coincident point with P on the link QT.

3. Since the point R lies on the link TQ produced, therefore divide the vector tq at r in the same ratio as R divides TQ, in the space diagram. In other words,

$$qr/qt = QR/QT$$

The vector qr represents the velocity of R with respect to Q or velocity of R (*i.e.* v_{RQ} or v_R).

4. From point r, draw vector rs perpendicular to RS to represent the velocity of S with respect to R and from point o draw vector or parallel to the path of motion of S (which is parallel to QS) to represent the velocity of S (*i.e* v_S). The vectors rs and os intersect at s.

By measurement, we find that velocity of the slider S (cutting tool),

$$v_S = \text{vector } os = 0.8 \text{ m/s} \quad \textbf{Ans.}$$

Angular velocity of link RS

From the velocity diagram, we find that the linear velocity of the link RS,

$$v_{SR} = \text{vector } rs = 0.96 \text{ m/s}$$

Since the length of link RS = 500 mm = 0.5 m, therefore angular velocity of link RS,

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 0.92 \text{ rad/s (Clockwise about } R) \quad \textbf{Ans.}$$

Velocity of the sliding block T on the slotted lever QT

Since the block T moves on the slotted lever with respect to P, therefore velocity of the sliding block T on the slotted lever QT,

$$v_{TP} = \text{vector } pt = 0.85 \text{ m/s} \quad \textbf{Ans.} \qquad \text{. . . (By measurement)}$$

7.7. Forces Acting in a Mechanism

Consider a mechanism of a four bar chain, as shown in Fig. 7.24. Let force F_A newton is acting at the joint A in the direction of the velocity of A (v_A m/s) which is perpendicular to the link DA. Suppose a force F_B newton is transmitted to the joint B in the direction of the velocity of B (*i.e.* v_B m/s) which is perpendicular to the link CB. If we neglect the effect of friction and the change of kinetic energy of the link (*i.e.*), assuming the efficiency of transmission as 100%), then by the principle of conservation of energy,

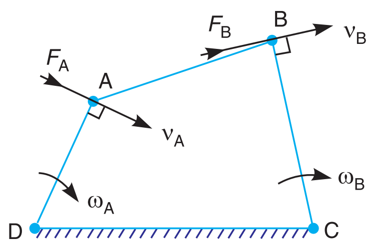


Fig. 7.24. Four bar mechanism.

Input work per unit time

= Output work per unit time

∴ Work supplied to the joint A

= Work transmitted by the joint B

or
$$F_A.v_A = F_B.v_B \quad \text{or} \quad F_B = \frac{F_A.v_A}{v_B} \qquad \text{. . . }(i)$$

If we consider the effect of friction and assuming the efficiency of transmission as η, then

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{F_B.v_B}{F_A.v_A} \quad \text{or} \quad F_B = \frac{\eta.F_A.v_A}{v_B} \qquad \text{. . . }(ii)$$

Notes : 1. If the turning couples due to the forces F_A and F_B about D and C are denoted by T_A (known as driving torque) and T_B (known as resisting torque) respectively, then the equations (*i*) and (*ii*) may be written as

$$T_A.\omega_A = T_B.\omega_B, \text{ and } \eta = \frac{T_B.\omega_B}{T_A.\omega_A} \qquad \text{. . . }(iii)$$

where ω_A and ω_B are the angular velocities of the links DA and CB respectively.

2. If the forces F_A and F_B do not act in the direction of the velocities of the points A and B respectively, then the component of the force in the direction of the velocity should be used in the above equations.

7.8. Mechanical Advantage

It is defined as the ratio of the load to the effort. In a four bar mechanism, as shown in Fig. 7.24, the link DA is called the driving link and the link CB as the driven link. The force F_A acting at A is the effort and the force F_B at B will be the load or the resistance to overcome. We know from the principle of conservation of energy, neglecting effect of friction,

$$F_A \times v_A = F_B \times v_B \quad \text{or} \quad \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

$\therefore$ Ideal mechanical advantage,

$$\text{M.A.}_{(ideal)} = \frac{F_B}{F_A} = \frac{v_A}{v_B}$$

If we consider the effect of friction, less resistance will be overcome with the given effort. Therefore the actual mechanical advantage will be less.

Let η = Efficiency of the mechanism.

$\therefore$ Actual mechanical advantage,

$$\text{M.A.}_{(actual)} = \eta \times \frac{F_B}{F_A} = \eta \times \frac{v_A}{v_B}$$

Note : The mechanical advantage may also be defined as the ratio of output torque to the input torque.

Let T_A = Driving torque,

T_B = Resisting torque,

ω_A and ω_B = Angular velocity of the driving and driven links respectively.

$\therefore$ Ideal mechanical advantage,

$$\text{M.A.}_{(ideal)} = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B} \qquad \text{. . . (Neglecting effect of friction)}$$

and actual mechanical advantage,

$$\text{M.A.}_{(actual)} = \eta \times \frac{T_B}{T_A} = \eta \times \frac{\omega_A}{\omega_B} \qquad \text{. . . (Considering the effect of friction)}$$

Example 7.10. *A four bar mechanism has the following dimensions :*

DA = 300 mm ; CB = AB = 360 mm ; DC = 600 mm. The link DC is fixed and the angle ADC is 60°. The driving link DA rotates uniformly at a speed of 100 r.p.m. clockwise and the constant driving torque has the magnitude of 50 N-m. Determine the velocity of the point B and angular velocity of the driven link CB. Also find the actual mechanical advantage and the resisting torque if the efficiency of the mechanism is 70 per cent.

Solution. Given : N_{AD} = 100 r.p.m. or $\omega_{AD} = 2\,\pi \times 100/60 = 10.47$ rad/s ; T_A = 50 N-m

Since the length of driving link, DA = 300 mm = 0.3 m, therefore velocity of A with respect to D or velocity of A (because D is a fixed point),

$$v_{AD} = v_A = \omega_{AD} \times DA = 10.47 \times 0.3 = 3.14 \text{ m/s}$$

. . . (Perpendicular to DA)

Velocity of point B

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.25 (a). Now the velocity diagram, as shown in Fig. 7.25 (b), is drawn as discussed below :

1. Since the link DC is fixed, therefore points d and c are taken as one point in the velocity diagram. Draw vector da perpendicular to DA, to some suitable scale, to represent the velocity of A with respect to D or simply velocity of A (*i.e.* v_{AD} or v_A) such that

$$\text{vector } da = v_{AD} = v_A = 3.14 \text{ m/s}$$

2. Now from point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A (*i.e.* v_{BA}), and from point c draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (*i.e.* v_{BC} or v_B). The vectors ab and cb intersect at b.

By measurement, we find that velocity of point B,

$$v_B = v_{BC} = \text{vector } cb = 2.25 \text{ m/s} \quad \textbf{Ans.}$$

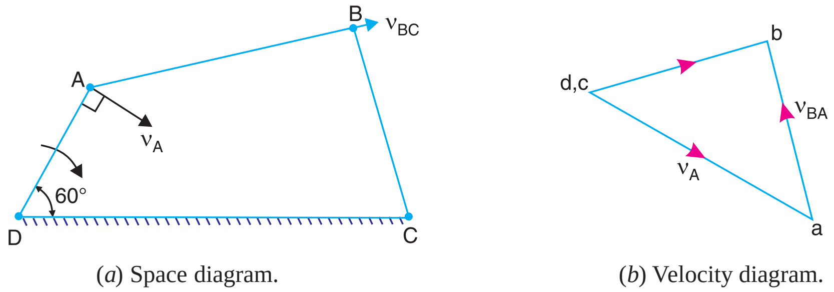


Fig. 7.25

Angular velocity of the driven link CB

Since CB = 360 mm = 0.36 m, therefore angular velocity of the driven link CB,

$$\omega_{BC} = \frac{v_{BC}}{BC} = \frac{2.25}{0.36} = 6.25 \text{ rad/s (Clockwise about } C) \quad \textbf{Ans.}$$

Actual mechanical advantage

We know that the efficiency of the mechanism,

$$\eta = 70\% = 0.7 \qquad \ldots \text{(Given)}$$

$\therefore$ Actual mechanical advantage,

$$\text{M.A.}_{(actual)} = \eta \times \frac{\omega_A}{\omega_B} = 0.7 \times \frac{10.47}{6.25} = 1.17 \quad \textbf{Ans.}$$

$$\ldots (\because \omega_A = \omega_{AD}; \text{ and } \omega_B = \omega_{BC})$$

Resisting torque

Let T_B = Resisting torque.

We know that efficiency of the mechanism (η),

$$0.7 = \frac{T_B \cdot \omega_B}{T_A \cdot \omega_A} = \frac{T_B \times 6.25}{50 \times 10.47} = 0.012\, T_B$$

$$\therefore \quad T_B = 58.3 \text{ N–m} \quad \textbf{Ans.}$$

Example 7.11. *The dimensions of the various links of a pneumatic riveter, as shown in Fig. 7.26, are as follows :*

OA = 175 mm ; AB = 180 mm ; AD = 500 mm ; and BC = 325 mm.

Find the velocity ratio between C and ram D when OB is vertical. What will be the efficiency of the machine if a load of 2.5 kN on the piston C causes a thrust of 4 kN at the ram D ?

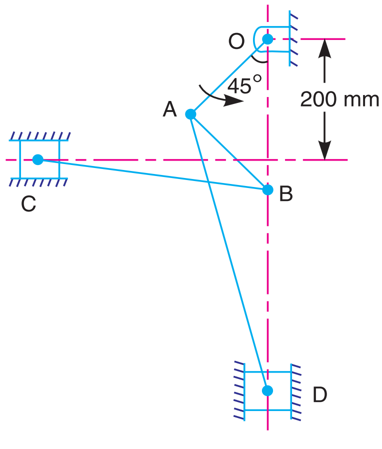


Fig. 7.26

Solution. Given : W_C = 2.5 kN = 2500 N ; W_D = 4 kN = 4000 N

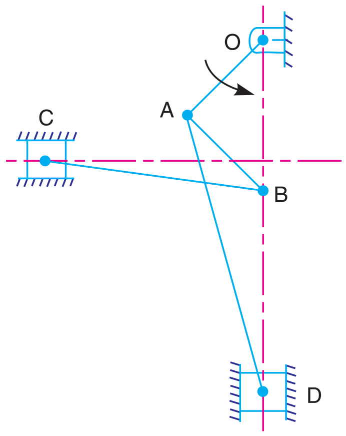
Let N = Speed of crank OA.

∴ Angular velocity of crank OA,

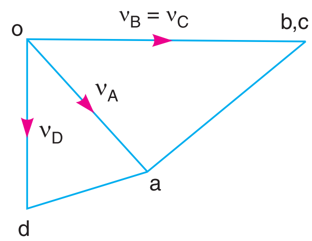
$$\omega_{AO} = 2\,\pi\,N/60 \text{ rad/s}$$

Since the length of crank OA = 175 mm = 0.175 m, therefore velocity of A with respect to O (or velocity of A) (because O is a fixed point),

$$v_{AO} = v_A = \frac{2\,\pi\,N}{60} \times 0.175 = 0.0183\ N \text{ m/s} \qquad \text{. . . (Perpendicular to } OA)$$



(*a*) Space diagram.



(*b*) Velocity diagram.

Fig. 7.27

Velocity ratio between C and the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.27 (*a*), Now the velocity diagram, as shown in Fig. 7.27 (*b*), is drawn as discussed below :

1. Draw vector *oa* perpendicular to OA to represent the velocity of A (*i.e.* v_A) such that

vector $oa = v_A$ = 0.0183 N m/s

Since the speed of crank (N) is not given, therefore let we take vector *oa* = 20 mm.

2. From point *a*, draw a vector *ab* perpendicular to AB to represent the velocity of B with respect to A (*i.e.* v_{BA}), and from point *o* draw vector *ob* perpendicular to *OB* to represent the velocity of B with respect to A or simply velocity of B (*i.e.* v_{BO} or v_B). The vectors *ab* and *ob* intersect at *b*.

3. Now from point *b*, draw vector *bc* perpendicular to *BC* to represent the velocity of C with respect to B (*i.e.* v_{CB}) and from point *o* draw vector *oc* parallel to the path of motion of C to represent the velocity of C (*i.e.* v_C). The vectors *bc* and *oc* intersect at *c*. We see from Fig. 7.27 (*b*) that the points b and c coincide. Therefore velocity of B with respect to C is zero and velocity of B is equal to velocity of C, *i.e.*

$$v_{BC} = 0 \qquad \ldots (\because \ b \text{ and } c \text{ coincide})$$

and $$v_B = v_C \qquad \ldots (\because \text{ vector } ob = \text{vector } oc)$$

4. From point a, draw vector ad perpendicular to AD to represent velocity of D with respect to A *i.e.* v_{DA}, and from point o draw vector ob parallel to the path of motion of D to represent the velocity of D *i.e.* v_D. The vectors ad and od intersect at d.

By measurement from velocity diagram, we find that velocity of C,

$$v_C = \text{vector } oc = 35 \text{ mm}$$

and $$\text{velocity of } D, v_D = \text{vector } od = 21 \text{ mm}$$

∴ Velocity ratio between C and the ram D

$$= v_C / v_D = 35/21 = 1.66 \quad \textbf{Ans.}$$

Efficiency of the machine

Let η = Efficiency of the machine,

We know that work done on the piston C or input,

$$= W_C \times v_C = 2500\, v_C$$

and work done by the ram D or output,

$$= W_D \times v_D = 4000\, v_D$$

$$\therefore \quad \eta = \frac{\text{Output}}{\text{Input}} = \frac{4000\, v_D}{2500\, v_C} = \frac{4000}{2500} \times \frac{1}{1.66} \qquad \ldots \left(\because \frac{v_C}{v_D} = 1.66 \right)$$

$$= 0.96 \text{ or } 96\% \quad \textbf{Ans.}$$

Example 7.12. *In the toggle mechanism, as shown in Fig. 7.28, the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed of 180 r.p.m.*

The dimensions of various links are as follows :

OA = 180 mm ; CB = 240 mm ; AB = 360 mm ; and BD = 540 mm.

For the given configuration, find : ***1.*** *Velocity of slider D,* ***2.*** *Angular velocity of links AB, CB and BD;* ***3.*** *Velocities of rubbing on the pins of diameter 30 mm at A and D, and* ***4.*** *Torque applied to the crank OA, for a force of 2 kN at D.*

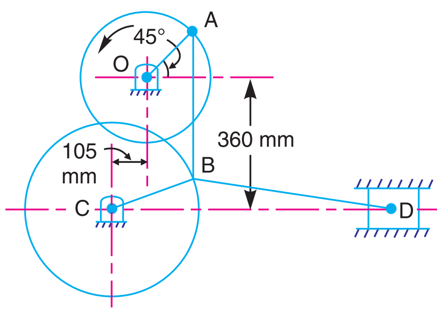


Fig. 7.28

Solution. Given : $N_{AO} = 180$ r.p.m. or $\omega_{AO} = 2\pi \times 180/60 = 18.85$ rad/s

Since the crank length $OA = 180$ mm $= 0.18$ m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

$$v_{AO} = v_A = \omega_{AO} \times OA = 18.85 \times 0.18 = 3.4 \text{ m/s}$$

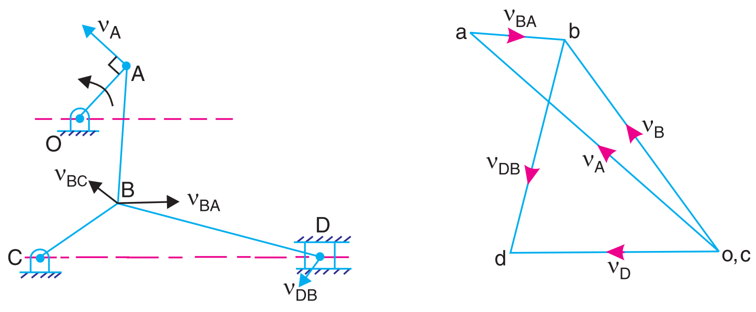
$$\ldots (\text{Perpendicular to } OA)$$

1. *Velocity of slider D*

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.29 (a). Now the velocity diagram, as shown in Fig. 7.29 (b), is drawn as discussed below :

1. Draw vector *oa* perpendicular to *OA*, to some suitable scale, to represent the velocity of *A* with respect to *O* or velocity of *A* (*i.e.* v_{AO} or v_A,) such that

$$\text{vector } oa = v_{AO} = v_A = 3.4 \text{ m/s}$$



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.29

2. Since point *B* moves with respect to *A* and also with respect to *C*, therefore draw vector *ab* perpendicular to *AB* to represent the velocity of *B* with respect to *A* *i.e.* v_{BA}, and draw vector *cb* perpendicular to *CB* to represent the velocity of *B* with respect to *C*, *i.e.* v_{BC}. The vectors *ab* and *cb* intersect at *b*.

3. From point *b*, draw vector *bd* perpendicular to *BD* to represent the velocity of *D* with respect to *B* *i.e.* v_{DB}, and from point *c* draw vector *cd* parallel to the path of motion of the slider *D* (which is along *CD*) to represent the velocity of *D*, *i.e.* v_D. The vectors *bd* and *cd* intersect at *d*.

By measurement, we find that velocity of the slider *D*,

$$v_D = \text{vector } cd = 2.05 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocities of links AB, CB and BD*

By measurement from velocity diagram, we find that

Velocity of *B* with respect to *A*,

$$v_{BA} = \text{vector } ab = 0.9 \text{ m/s}$$

Velocity of *B* with respect to *C*,

$$v_{BC} = v_B = \text{vector } cb = 2.8 \text{ m/s}$$

and velocity of *D* with respect to *B*,

$$v_{DB} = \text{vector } bd = 2.4 \text{ m/s}$$

We know that *AB* = 360 mm = 0.36 m ; *CB* = 240 mm = 0.24 m and *BD* = 540 mm = 0.54 m.

∴ Angular velocity of the link *AB*,

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{0.9}{0.36} = 2.5 \text{ rad/s} \quad (\text{Anticlockwise about } A) \quad \textbf{Ans.}$$

Similarly angular velocity of the link *CB*,

$$\omega_{CB} = \frac{v_{BC}}{CB} = \frac{2.8}{0.24} = 11.67 \text{ rad/s} \quad (\text{Anticlockwise about } C) \quad \textbf{Ans.}$$

and angular velocity of the link *BD*,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{2.4}{0.54} = 4.44 \text{ rad/s} \quad (\text{Clockwise about } B) \quad \textbf{Ans.}$$

3. *Velocities of rubbing on the pins A and D*

Given : Diameter of pins at A and D,

$$D_A = D_D = 30 \text{ mm} = 0.03 \text{ m}$$

$\therefore$ Radius, $r_A = r_D = 0.015$ m

We know that relative angular velocity at A

$$= \omega_{BC} - \omega_{BA} + \omega_{DB} = 11.67 - 2.5 + 4.44 = 13.61 \text{ rad/s}$$

and relative angular velocity at D

$$= \omega_{DB} = 4.44 \text{ rad/s}$$

$\therefore$ Velocity of rubbing on the pin A

$$= 13.61 \times 0.015 = 0.204 \text{ m/s} = 204 \text{ mm/s}$$ **Ans.**

and velocity of rubbing on the pin D

$$= 4.44 \times 0.015 = 0.067 \text{ m/s} = 67 \text{ mm/s}$$ **Ans.**

4. *Torque applied to the crank OA*

Let T_A = Torque applied to the crank OA, in N-m

$\therefore$ Power input or work supplied at A

$$= T_A \times \omega_{AO} = T_A \times 18.85 = 18.85\, T_A \text{ N-m}$$

We know that force at D,

$$F_D = 2 \text{ kN} = 2000 \text{ N} \qquad \text{. . . (Given)}$$

$\therefore$ Power output or work done by D,

$$= F_D \times v_D = 2000 \times 2.05 = 4100 \text{ N-m}$$

Assuming 100 per cent efficiency, power input is equal to power output.

$\therefore$ $18.85\, T_A = 4100$ or $T_A = 217.5$ N-m **Ans.**

Example 7.13. *The dimensions of the mechanism, as shown in Fig. 7.30, are as follows : AB = 0.45 m; BD = 1.5 m : BC = CE = 0.9 m.*

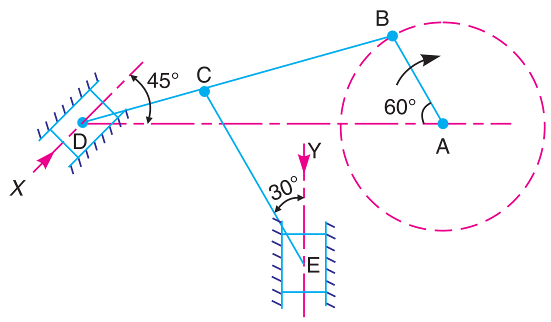


Fig. 7.30

The crank AB turns uniformly at 180 r.p.m. in the clockwise direction and the blocks at D and E are working in frictionless guides.

Draw the velocity diagram for the mechanism and find the velocities of the sliders D and E in their guides. Also determine the turning moment at A if a force of 500 N acts on D in the direction of arrow X and a force of 750 N acts on E in the direction of arrow Y.

Solution. Given : $N_{BA} = 180$ r.p.m. or $\omega_{BA} = 2\pi \times 180/60 = 18.85$ rad/s

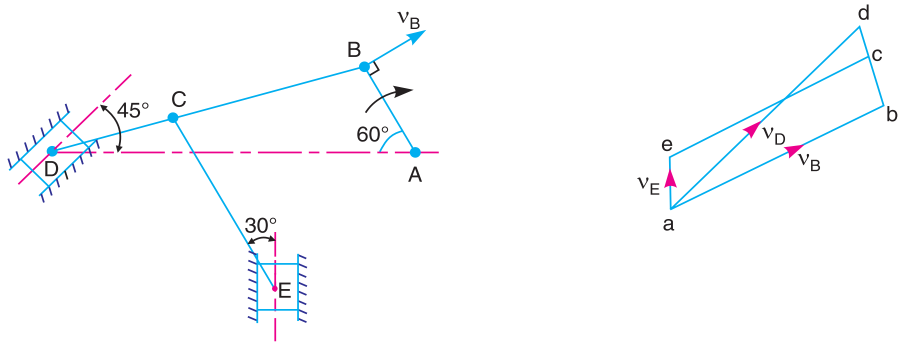
Since $AB = 0.45$ m, therefore velocity of B with respect to A or velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 18.85 \times 0.45 = 8.5 \text{ m/s}$$

. . . (Perpendicular to AB)

Velocities of the sliders D and E

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.31 (*a*). Now the velocity diagram, as shown in Fig. 7.31 (*b*), is drawn as discussed below :



(*a*) Space diagram. (*b*) Velocity diagram.

Fig. 7.31

1. Draw vector *ab* perpendicular to AB, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (*i.e.* v_{BA} or v_B), such that

$$\text{vector } ab = v_{BA} = v_B = 8.5 \text{ m/s}$$

2. From point *b*, draw vector *bd* perpendicular to *BD* to represent the velocity of D with respect to B (*i.e.* v_{DB}) and from point *a* draw vector *ad* parallel to the motion of D to represent the velocity of D (v_D). The vectors *bd* and *ad* intersect at *d*.

3. Since the point C lies on *BD*, therefore divide vector *bd* at *c* in the same ratio as C divides *BD* in the space diagram. In other words,

$$bc/bd = BC/BD$$

4. Now from point *c*, draw vector *ce* perpendicular to *CE* to represent the velocity of E with respect to C (*i.e.* v_{EC}) and from point *a* draw vector *ae* parallel to the path of E to represent the velocity of E (*i.e.* v_E). The vectors *ce* and *ae* intersect at *e*.

By measurement, we find that

Velocity of slider D, v_D = vector $ad = 9.5$ m/s **Ans.**

Velocity of slider E, v_E = vector $ae = 1.7$ m/s **Ans.**

Turning moment at A

Let T_A = Turning moment at A (or at the crank-shaft).

We know that force at D, $F_D = 500$ N . . . (Given)

and Force at E, $F_E = 750$ N . . . (Given)

∴ Power input $= F_D \times v_D - F_E \times v_E$

. . . (– ve sign indicates that F_E opposes the motion)

$$= 500 \times 9.5 - 750 \times 1.7 = 3475 \text{ N-m/s}$$

Power output $= T_A.\omega_{BA} = T_A \times 18.85\, T_A$ N-m/s

Neglecting losses, power input is equal to power output.

∴ $3475 = 18.85\, T_A$ or $T_A = 184.3$ N-m **Ans.**

EXERCISES

1. In a slider crank mechanism, the length of crank *OB* and connecting rod *AB* are 125 mm and 500 mm respectively. The centre of gravity *G* of the connecting rod is 275 mm from the slider *A*. The crank speed is 600 r.p.m. clockwise. When the crank has turned 45° from the inner dead centre position, determine: 1. velocity of the slider *A*, 2. velocity of the point *G*, and 3. angular velocity of the connecting rod *AB*. **[Ans. 6.45 m/s ; 6.75 m/s ; 10.8 rad/s]**

2. In the mechanism, as shown in Fig. 7.32, *OA* and *OB* are two equal cranks at right angles rotating about *O* at a speed of 40 r.p.m. anticlockwise. The dimensions of the various links are as follows :

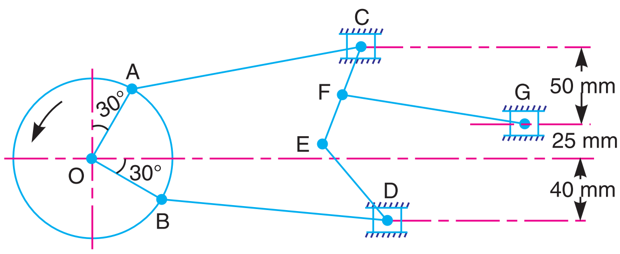


Fig. 7.32

OA = *OB* = 50 mm ; *AC* = *BD* = 175 mm ; *DE* = *CE* = 75 mm ; *FG* = 115 mm and *EF* = *FC*.

Draw velocity diagram for the given configuration of the mechanism and find velocity of the slider *G*. **[Ans. 68 mm/s]**

3. The dimensions of various links in a mechanism, as shown in Fig. 7.33, are as follows :

AB = 60 mm ; *BC* = 400 mm ; *CD* = 150 mm ; *DE* = 115 mm ; and *EF* = 225 mm.

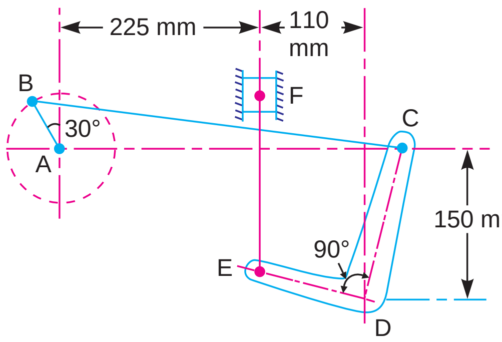


Fig. 7.33

Find the velocity of the slider *F* when the crank *AB* rotates uniformly in clockwise direction at a speed of 60 r.p.m. **[Ans. 250 mm/s]**

4. In a link work, as shown in Fig. 7.34, the crank *AB* rotates about *A* at a uniform speed of 150 r.p.m. The lever *DC* oscillates about the fixed point *D*, being connected to *AB* by the connecting link *BC*. The block *F* moves, in horizontal guides being driven by the link *EF*, when the crank *AB* is at 30°. The dimensions of the various links are :

AB = 150 mm ; *BC* = 450 mm ; *CE* = 300 mm ; *DE* = 150 mm ; and *EF* = 350 mm.

Find, for the given configuration, 1. velocity of slider *F*, 2. angular velocity of *DC*, and 3. rubbing speed at pin *C* which is 50 mm in diameter. **[Ans. 500 mm/s ; 3.5 rad/s ; 2.4 m/s]**

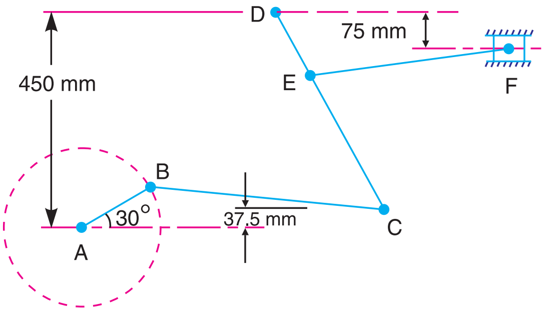


Fig. 7.34

5. The oscillating link *OAB* of a mechanism, as shown in Fig. 7.35, is pivoted at *O* and is moving at 90 r.p.m. anticlockwise. If *OA* = 150 mm ; *AB* = 75 mm, and *AC* = 250 mm, calculate

1. the velocity of the block *C*;
2. the angular velocity of the link *AC*; and
3. the rubbing velocities of the pins at *O*, *A* and *C*, assuming that these pins are of equal diameters of 20 mm.

[Ans. 1.2 m/s; 1.6 rad/s^2 clockwise; 21 200 mm/s, 782 mm/s, 160 mm/s]

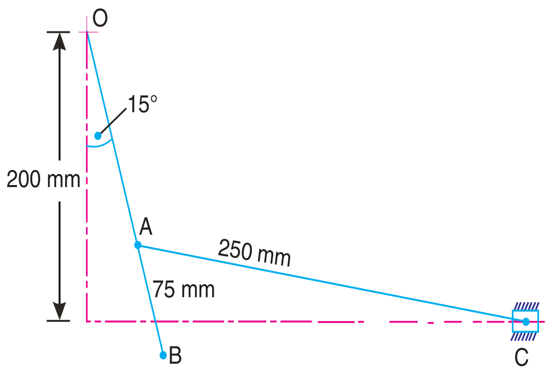


Fig. 7.35

6. The dimensions of the various links of a mechanism, as shown in Fig. 7.36, are as follows :

AB = 30 mm ; *BC* = 80 mm ; *CD* = 45 mm ; and *CE* = 120 mm.

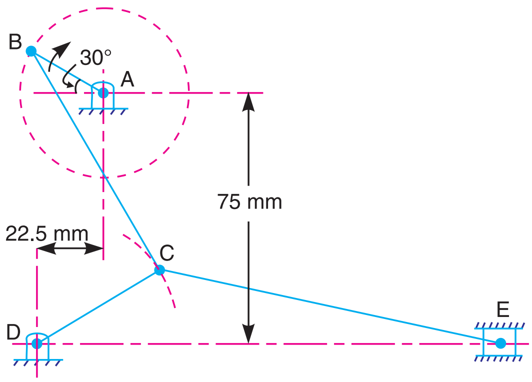


Fig. 7.36

The crank *AB* rotates uniformly in the clockwise direction at 120 r.p.m. Draw the velocity diagram for the given configuration of the mechanism and determine the velocity of the slider *E* and angular velocities of the links *BC*, *CD* and *CE*.

Also draw a diagram showing the extreme top and bottom positions of the crank *DC* and the corresponding configurations of the mechanism.

Find the length of each of the strokes.

[Ans. 120 mm/s ; 2.8 rad/s ; 5.8 rad/s ; 2 rad/s ; 10 mm ; 23 mm]

7. Fig. 7.37 shows a mechanism in which the crank OA, 100 mm long rotates clockwise about O at 130 r.p.m. The connecting rod AB is 400 mm long. The rod CE, 350 mm long, is attached to AB at C, 150 mm from A. This rod slides in a slot in a trunnion at D. The end E is connected by a link EF, 300 mm long, to the horizontally moving slider F.

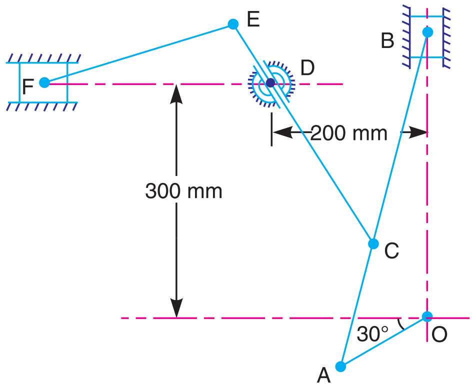


Fig. 7.37

Determine, for the given configuration : 1. velocity of F, 2. velocity of sliding of CE in the trunnion, and 3. angular velocity of CE. **[Ans. 0.54 m/s ; 1.2 m/s ; 1.4 rad/s]**

8. Fig. 7.38 shows the mechanism of a quick return motion of the crank and slotted lever type shaping machine. The dimensions of the various links are as follows :

OA = 200 mm ; AB = 100 mm ; OC = 400 mm ; and CR = 150 mm.

The driving crank AB makes 120° with the vertical and rotates at 60 r.p.m. in the clockwise direction. Find : 1. velocity of ram R, and 2. angular velocity of the slotted link OC.

[Ans. 0.8 m/s ; 1.83 rad/s]

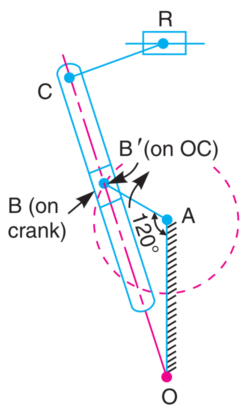


Fig. 7.38

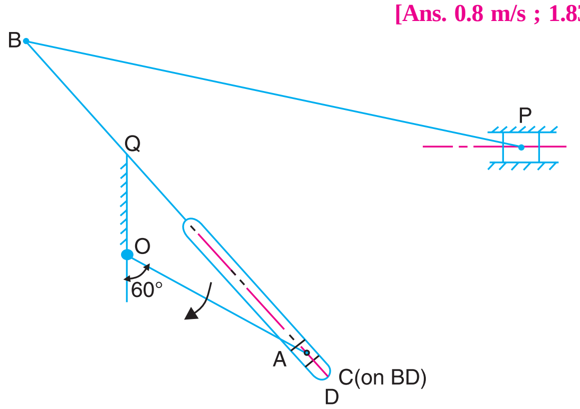


Fig. 7.39

9. In a Whitworth quick return motion mechanism, as shown in Fig. 7.39, the dimensions of various links are as follows :

OQ = 100 mm ; OA = 200 mm ; BQ = 150 mm and BP = 500 mm.

If the crank OA turns at 120 r.p.m. in clockwise direction and makes an angle of 120° with OQ, Find : 1. velocity of the block P, and 2. angular velocity of the slotted link BQ.

[Ans. 0.63 m/s ; 6.3 rad/s]

10. A toggle press mechanism, as shown in Fig. 7.40, has the dimensions of various links as follows : OP = 50 mm ; RQ = RS = 200 mm ; PR = 300 mm.

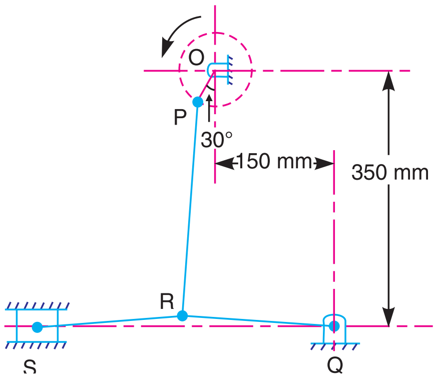


Fig. 7.40

Find the velocity of S when the crank OP rotates at 60 r.p.m. in the anticlockwise direction. If the torque on P is 115 N-m, what pressure will be exerted at S when the overall efficiency is 60 per-cent.

[Ans. 400 m/s ; 3.9 kN]

11. Fig. 7.41 shows a toggle mechanism in which link D is constained to move in horizontal direction. For the given configuration, find out : 1. velocities of points band D; and 2. angular velocities of links AB, BC, and BD.

The rank OA rotates at 60 r.p.m. in anticlockwise direction.

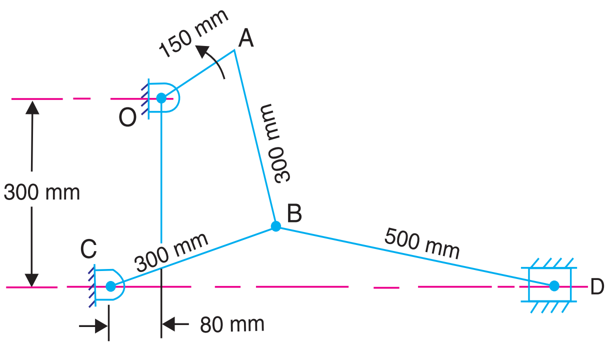


Fig. 7.41

[Ans. 0.9 m/s; 0.5 m/s; 0.0016 rad/s (anticlockwise) 0.0075 rad/s (anti-clockwise),0.0044 rad/s (anti-clockwise)]

12. A riveter, as shown in Fig. 7.42, is operated by a piston F acting through the links EB, AB and BC. The ram D carries the tool. The piston moves in a line perpendicular to the line of motion of D. The length of link BC is twice the length of link AB. In the position shown, AB makes an angle of 12° with AC and BE is at right angle to AC. Find the velocity ratio of E to D. If, in the same position, the total load on the piston is 2.2 kN, find the thrust exerted by D when the efficiency of the mechanism is 72 per cent,

Ans. [3.2 ; 5 kN]

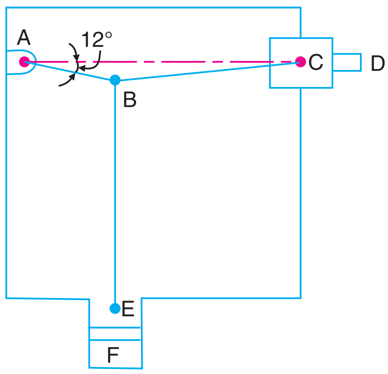


Fig. 7.42

DO YOU KNOW ?

1. Describe the method to find the velocity of a point on a link whose direction (or path) is known and the velocity of some other point on the same link in magnitude and direction is given.
2. Explain how the velocities of a slider and the connecting rod are obtained in a slider crank mechanism.

3. Define rubbing velocity at a pin joint. What will be the rubbing velocity at pin joint when the two links move in the same and opposite directions ?

4. What is the difference between ideal mechanical advantage and actual mechanical advantage ?

OBJECTIVE TYPE QUESTIONS

1. The direction of linear velocity of any point on a link with respect to another point on the same link is

(*a*) parallel to the link joining the points (*b*) perpendicular to the link joining the points
(*c*) at 45° to the link joining the points (*d*) none of these

2. The magnitude of linear velocity of a point *B* on a link *AB* relative to point *A* is

(*a*) ωAB (*b*) $\omega (AB)^2$
(*c*) $\omega^2 . AB$ (*d*) $(\omega . AB)^2$

where ω = Angular velocity of the link *AB*.

3. The two links *OA* and *OB* are connected by a pin joint at *O*. If the link *OA* turns with angular velocity ω_1 rad/s in the clockwise direction and the link *OB* turns with angular velocity ω_2 rad/s in the anti-clockwise direction, then the rubbing velocity at the pin joint *O* is

(*a*) $\omega_1 . \omega_2 . r$ (*b*) $(\omega_1 - \omega_2)\, r$
(*c*) $(\omega_1 + \omega_2)\, r$ (*d*) $(\omega_1 - \omega_2)\, 2\, r$

where *r* = Radius of the pin at *O*.

4. In the above question, if both the links *OA* and *OB* turn in clockwise direction, then the rubbing velocity at the pin joint *O* is

(*a*) $\omega_1 . \omega_2 . r$ (*b*) $(\omega_1 - \omega_2)\, r$
(*c*) $(\omega_1 + \omega_2)\, r$ (*d*) $(\omega_1 - \omega_2)\, 2\, r$

5. In a four bar mechanism, as shown in Fig. 7.43, if a force F_A is acting at point *A* in the direction of its velocity v_A and a force F_B is transmitted to the joint *B* in the direction of its velocity v_B, then the ideal mechanical advantage is equal to

(*a*) $F_B . v_A$ (*b*) $F_A . v_B$

(*c*) $\dfrac{F_B}{v_B}$ (*d*) $\dfrac{F_B}{F_A}$

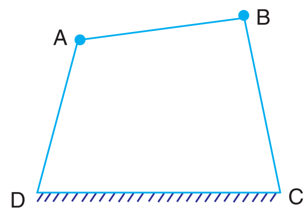


Fig. 7.43

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)

GO To FIRST