Brno University of Technology
Faculty of Information Technology

VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ FAKULTA INFORMAČNÍCH TECHNOLOGIÍ

Modelling and Simulation

2022/2023

Project documentation

Ballistics in the military

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Brno, December 4, 2022

1. Introduction

This project required the study of external ballistics[1], the branch of ballistics that studies the movement of unguided objects.

This project deals with the implementation of a ballistic table[2], which contains a set of data about a specific bullet fired under certain conditions. To simulate the flight of a bullet, it was also necessary to study the factors and conditions that affect this movement.

Experiments were carried out, the purpose of which was to show how, using the data contained in the table, it is possible to calculate the flight of a bullet and the sight settings necessary for shooting at distances. It will find application in hunting, sports shooting, military and scientific purposes.

1.1 The authors

The authors of the project are Vladyslav Kovalets (xkoval21) and Evgeniya Taipova (xtaipo00) - 3rd year students of the Faculty of Information Technology of Brno University of Technology.

1.2 Validity of the simulation model

Our work is based on the physical formulas of external ballistics. The validity[3] of the simulation model[4] has been verified through several experiments and calculations using other ballistic tables for some of the bullets that are described in Chapter 5.

[1] https://en.wikipedia.org/wiki/External_ballistics

[2] https://en.wikipedia.org/wiki/Ballistic_table

[3] IMS: Peringer Petr a Hrubý Martin – Prezentace k předmětu IMS – str. 37, 2022

[4] IMS: Peringer Petr a Hrubý Martin – Prezentace k předmětu IMS – str. 7, 2022

2. Analysis of the topic and the methods and technologies used

2.1 Data necessary for the model

Muzzle velocity (m/s)

Muzzle velocity[5] is the speed of a projectile with respect to the muzzle at the moment it leaves the end of a gun's barrel. Firearm muzzle velocities range from approximately 120 m/s to 370 m/s in black powder muskets, to more than 1,200 m/s in modern rifles with high-velocity cartridges.

Bullet weight (g)

Bullet weight[6] is how heavy the projectile leaving your firearm is. The weight of the bullet is on average[7] from one gram to 50 grams.

Ballistic coefficient

The ballistic coefficient[8] (BC) of a body is a measure of its ability to overcome air resistance in flight. The values for BC can be as low as 0.12 and as high as 1.00 for commonly used bullets.

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[5] https://en.wikipedia.org/wiki/Muzzle_velocity

[6] https://www.wideners.com/blog/what-is-bullet-weight/

[7] https://www.luckygunner.com/lounge/choosing-right-bullet-weight/

[8] https://en.wikipedia.org/wiki/Ballistic_coefficient

Zero Range (m)

The range at which you wish the bullet to cross the line of sight. For example if you are shooting a rifle with a scope that is sighted in at 200 meters that means that if you want to shoot a target that is 200 meters away you should place the crosshair dead center on the target, because that is the range it is zeroed at.

Wind Speed (m/s)

Wind can have a significant effect on the trajectory of a bullet, particularly at long range, and this can affect the ability of the shooter to hit the intended aim point or target[9].

Wind Angle (deg)

Angle of the wind direction. A wind blowing downrange has an angle of zero, a wind blowing to the shooter's right has an angle of 90, a headwind, an angle of 180 and a wind blowing to the shooter's left has an angle of 270 degrees.

Temperature (°C)

The measured or estimated air temperature.

Altitude (m)

Used in estimating the atmospheric conditions. Increasing the temperature or altitude will increase the ballistic coefficient.

Pressure (mmhg)

Pressure is the force applied perpendicular to the surface of an object per unit area over which that force is distributed.

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[9] https://apps.dtic.mil/sti/pdfs/AD1063212.pdf

Scope Height (cm)

The distance in cm between your scope and your rifle measured from the center of the bore to the center of the reticle. This measurement is one of the variables used by most ballistic programs.[10]

Minute of Angle (MOA)

Essentially, MOA is used to help measure shooting in minutes since a bullet moves in an arc-shaped trajectory. The further a bullet has to travel (typically measured in yards), the greater effect gravity has as it decreases velocity. The farther away you are from a target, the lower your bullet might strike from where you intentionally aim. This is known as the bullet drop, which is measured in inches.[11]

Retardation coefficient rate

The Pejsa model[12] allows the slope factor to be tuned to account for subtle differences in the retardation rate of different bullet shapes and sizes. It ranges from 0.1 (flat-nose bullets) to 0.9 (very-low-drag bullets). If this slope or deceleration constant factor is unknown a default value of 0.5 is used. With the help of test firing measurements the slope constant for a particular bullet/rifle system/shooter combination can be determined.

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[10] https://www.accurateshooter.com/optics/determining-scope-height-above-bore/

[11] https://www.pelican.com/us/en/discover/pelican-flyer/post/what-does-moa-mean-/

[12] https://en.wikipedia.org/wiki/External_ballistics#Pejsa_model

2.2 Data from ballistic table

Data that can be obtained thanks to this ballistic table.

Range (m)

Distance from the shooter along the line of sight.

Velocity (m/s)

The speed of the bullet in meters per second at a certain distance.

Energy (J)

Kinetic energy of the bullet. The kinetic energy is a measure of the maximum amount of work (force time distance) an object can do.

Drop (cm)

The distance above or below the line of sight. Listed in centimeters. Drop is positive above the line of sight and negative below the line of sight.

Path (cm)

Path shows where the bullet should strike at the various ranges.

Elevation (MOA)

The vertical angle the barrel makes with the line of sight.

Windage (MOA)

The distance to the right or left of the line of sight. Windage is positive to the shooters right and negative to the shooters left.

Time (sec)

The time of flight of the bullet in seconds. [13]

2.3 Mathematical calculations

The formulas[14] of Arthur J. Pejsa[15] were used for calculations.

Formulas are used with units of the American number system. For convenience, the input and output data in our calculator are converted to the metric system.

Speed of the bullet

$$V = v0 \cdot \left(1 - 3 \cdot n \cdot \frac{R}{Rc}\right)^{\frac{1}{n}}$$

V - speed [fp/s]

v0 - muzzle speed [fp/s]

n - retardation coefficient rate

R - distance at the moment [yards]

Rc - adjusted retardation coefficient

Energy of the bullet

$$E = \frac{mv^2}{450380}$$

E - energy [ft-lb]

m - bullet mass [gr]

v - velocity of a bullet [fp/s]

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[13] http://www.jbmballistics.com/ballistics/calculators/help/traj/traj_exp.shtml

[14] https://www.mathscinotes.com/2015/05/pejsa-bullet-height-versus-distance-formula-for-a-zeroed-rifle/ and https://www.mathscinotes.com/page/39/

[15] https://www.startribune.com/obituaries/detail/14011087/

Path of the bullet

$$H = -(D + S) + \frac{(Dz + S) \cdot R}{Z}$$

H - path [in]

Dz- drop at zero range [in]

D - drop at this distance [in]

S- scope height [in]

R- distance at the moment [yards]

Z- at zero range [yards]

Drop of the bullet

$$Fm = \left(\frac{\frac{G}{v0}}{\frac{1}{1+R}} - \left(0.75 + 0.00006 \cdot R\right) \cdot n \cdot R \right)^2$$

G - constant with value 41.68

v0 - muzzle speed [fp/s]

R - distance at the moment [yards]

n - retardation coefficient rate

3. Conception

In our model, we rely on the sources that were mentioned in Chapter 2.

The simulation[16] takes into account the parameters that the user enters. The program calculates the data needed for the table using formulas. And then with the help of them the user receives graphics.

4. Simulating model architecture

The simulation model is implemented using the C programming language using the *pbPlots* library for plotting. The program is characterized by the parametrization of data in the model, which can be changed in order to conduct experiments and obtain the desired answers to the questions posed at the beginning of the work.

The launch of the program was tested on the server Eva of the Faculty of Information Technology at VUT in Brno.

4.1 Use of the simulation model

Launch preparation:

```
$ unzip 09_xkoval21_xtaipo00.zip
$ cd  09_xkoval21_xtaipo00
$ make
```

To run the program with default values, use this command

```
$ make run
```

[16] IMS: Peringer Petr a Hrubý Martin – Prezentace k předmětu IMS – str. 33, 2022

Or you can set the parameters that suit you using the options and specify stdout for the table:

$ out/ballistic_table -b 7.62x51mm -m 860 -w 9.7 -c 0.397 -r 330 -s 0 -i 50 -z 100 -n 10 -d 180 -t 15 -a 170 -e 750 -g 2 -o 1 -l 0.5 -v 450 > out/ballistic_table.txt

In this case, *ballistic_table.txt* from *out* founder looks like this:

```
7.62x51mm
INPUT DATA:
860      Muzzle speed (m/s)
9.70     Bullet wt (g)
0.397    Ballistic coefficient
330      Special range (m)
0        Start range (m)
50       Increment (m)
100      at Zero range (m)
4        Wind speed (m/s)
90.0     Wind dir. (deg)
15.0     Temp. (Celcius)
170      Altitude (m)
750      Pressure (mmhg)
2.00     Scope Ht. (cm)
1.00     in./MOA @ 100 yds
0.50     Retard. Coeff. rate
450      Break Velocity (m/s)

Calculated data & constants:
0.406    Adjusted BC
3487     Retardation coeff.
3563     Adj. Retard. Coeff.
1000     Standard pressure
246      Mayewski constant
59       Temp. (Faht.)
2.78     Drop at Zero

Range[m]       Speed[m/s]     Energy[J]      Drop[cm]       Path[cm]       Elevn[MOA]     Windage[MOA]   Time[sec]
330            619            1856           90.21          -62.30         6.80           3.00           0.45
0              860            3588           0.00           -2.00                                        0.00
50             821            3268           1.71           0.82           -0.59          0.39           0.06
100            783            2971           7.06           -0.00          0.00           0.81           0.12
150            745            2694           16.42          -4.83          1.16           1.24           0.19
200            709            2438           30.19          -14.07         2.53           1.70           0.26
250            673            2200           48.85          -28.20         4.06           2.18           0.33
300            639            1980           72.92          -47.74         5.73           2.68           0.40
350            605            1776           103.00         -73.29         7.54           3.22           0.49
400            572            1589           139.78         -105.54        9.50           3.78           0.57
450            541            1417           184.05         -145.28        11.62          4.38           0.66
500            510            1260           236.72         -193.41        13.93          5.01           0.76
550            480            1116           298.82         -250.99        16.43          5.68           0.86
600            450            984            371.58         -319.22        19.15          6.39           0.96
650            422            865            456.42         -399.52        22.13          7.15           1.08
```

Graphs are saved regardless of the output method. Graphs can be found in the *out* folder:

graph_range_speed.png
graph_range_energy.png
graph_range_drop.png
graph_range_path.png
graph_range_elevn.png
graph_range_windage.png
graph_range_time.png

5. The essence of simulation experiments and their course

The purpose of the experiments was to understand how the output changes when changing various bullet parameters and external factors.

5.1 Experiment 1

The purpose of the first experiment was to understand how temperature affects the flight of a bullet.[17]

To do this, the temperature of +40 degrees Celsius and -40 was selected, the remaining conditions are the same (for example, the weight of the bullet, speed).

Range[m]	Speed[m/s]	Energy[J]	Drop[cm]	Path[cm]	Elevn[MOA]	Windage[MOA]	Time[sec]
0	914	2507	0.00	-4.50			0.00
50	884	2346	1.50	-0.68	0.47	0.19	0.06
100	855	2193	6.14	-0.00	0.00	0.38	0.11
150	826	2048	14.13	-2.67	0.61	0.58	0.17
200	798	1911	25.71	-8.94	1.53	0.79	0.23
250	770	1780	41.15	-19.06	2.61	1.01	0.30
300	743	1656	60.72	-33.31	3.81	1.23	0.36
350	716	1539	84.74	-52.01	5.09	1.46	0.43
400	690	1428	113.54	-75.49	6.47	1.70	0.50
450	664	1323	147.50	-104.13	7.93	1.95	0.58
500	639	1224	187.02	-138.34	9.49	2.21	0.65
550	614	1131	232.57	-178.56	11.13	2.48	0.73
600	590	1043	284.62	-225.30	12.87	2.76	0.82
650	566	960	343.75	-279.11	14.72	3.05	0.90
700	542	882	410.55	-340.60	16.68	3.36	0.99
750	519	810	485.71	-410.44	18.76	3.67	1.09
800	497	741	569.98	-489.39	20.97	4.01	1.19
850	475	678	664.21	-578.29	23.33	4.35	1.29
900	454	618	769.32	-678.09	25.83	4.72	1.40
950	433	562	886.38	-789.83	28.51	5.10	1.51
1000	413	511	1016.58	-914.71	31.36	5.50	1.63
1050	393	463	1161.24	-1054.06	34.42	5.92	1.75

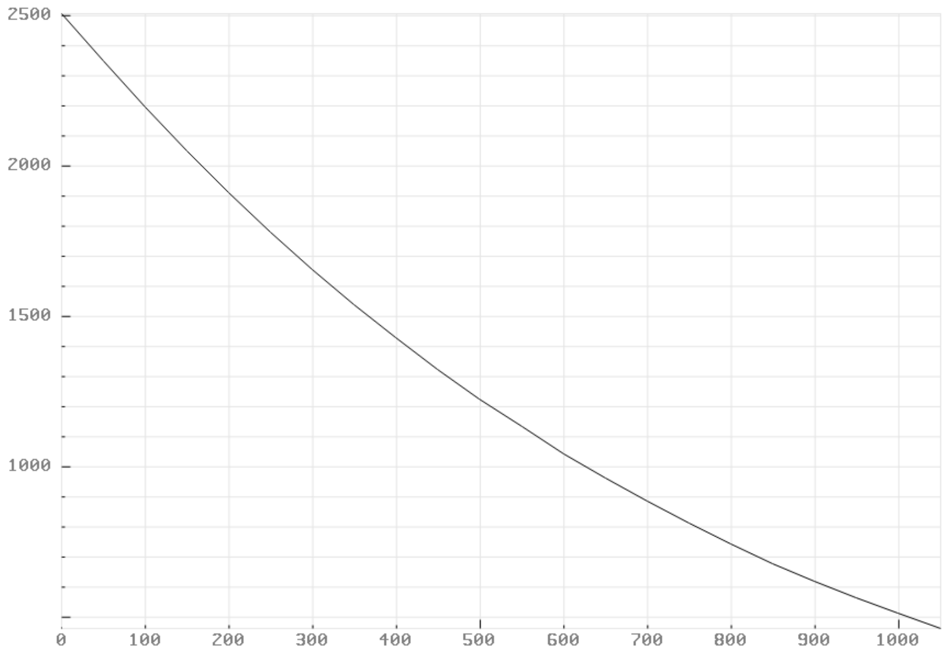
Output of the program for the first experiment:
ballistic table for temperature +40 degrees

[17] https://www.ronspomeroutdoors.com/blog/air-temperature-changes-bullet-trajectory

Range[m]	Speed[m/s]	Energy[J]	Drop[cm]	Path[cm]	Elevn[MOA]	Windage[MOA]	Time[sec]
0	914	2507	0.00	-4.50			0.00
50	874	2293	1.51	-0.64	0.44	0.25	0.06
100	835	2093	6.23	-0.00	0.00	0.52	0.11
150	797	1907	14.47	-2.87	0.66	0.80	0.18
200	760	1733	26.57	-9.61	1.65	1.09	0.24
250	724	1571	42.92	-20.59	2.82	1.39	0.31
300	688	1422	63.96	-36.25	4.14	1.71	0.38
350	654	1283	90.17	-57.10	5.59	2.05	0.45
400	620	1154	122.12	-83.69	7.17	2.41	0.53
450	587	1035	160.45	-116.65	8.89	2.78	0.61
500	556	926	205.89	-156.73	10.75	3.18	0.70
550	525	826	259.28	-204.75	12.76	3.60	0.79
600	495	734	321.57	-261.67	14.95	4.04	0.89
650	465	650	393.89	-328.62	17.33	4.52	1.00
700	437	573	477.51	-406.88	19.93	5.02	1.11
750	410	504	573.95	-497.94	22.76	5.55	1.23
800	383	441	684.95	-603.58	25.87	6.13	1.35

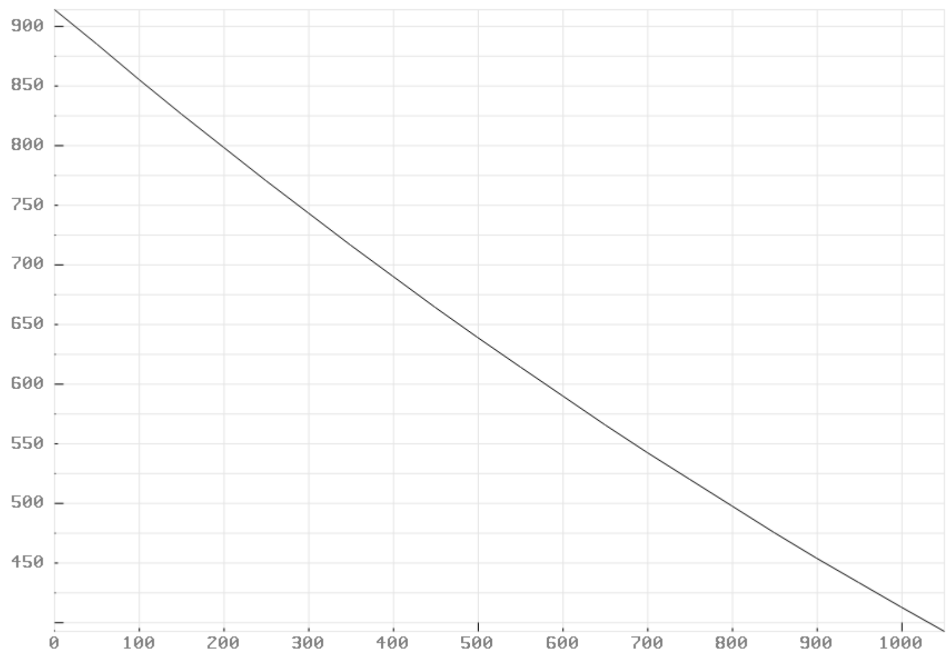
Output of the program for the first experiment:
ballistic table for temperature -40 degrees

Graph showing the dependence of energy [J] (vertical axis) on distance [m] (horizontal axis), at a temperature of +40 degrees



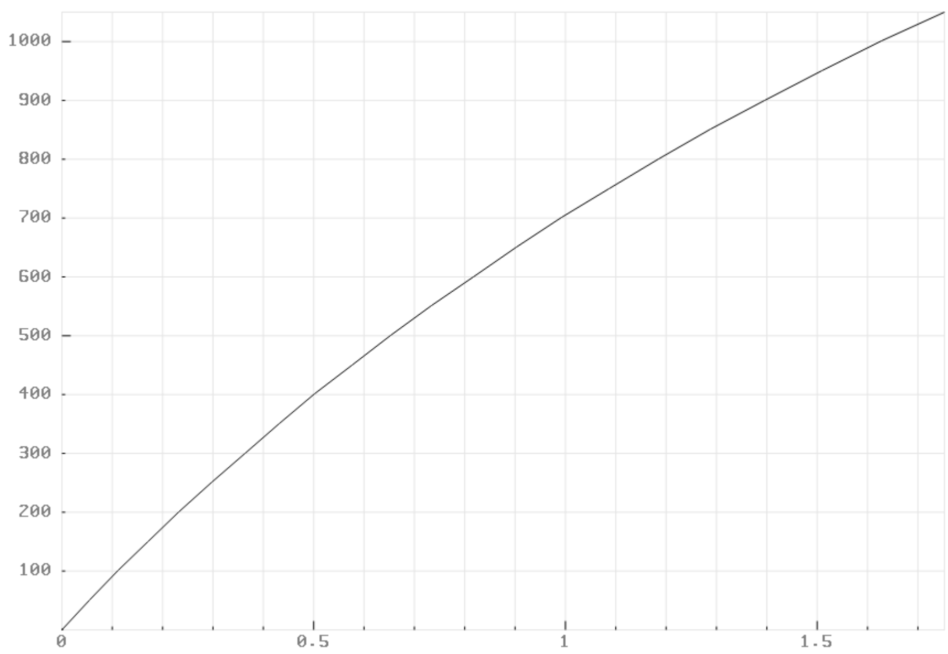
Output of the program for the first experiment:
graph_range_energy.png

Graph of speed [m/s] (vertical axis) versus distance [m] (horizontal axis) at a temperature of +40 degrees



Output of the program for the first experiment:
graph_range_speed.png

Graph showing how long in seconds (horizontal axis) it takes to reach a certain distance in meters (vertical axis) at a temperature of +40 degrees



Output of the program for the first experiment:
graph_range_time.png

Thus, having studied the table that came out, it was found that at a colder temperature, the bullet loses energy faster and requires a little longer to reach a certain distance, and the bullet speed also drops faster. For example, at a distance of 700 meters, the speed at a warmer temperature will be 542 m/s and energy of 882 Joules, and with a cold 437 m/s and 573 Joules.

They, in turn, affect Elevation and Windage. At a distance of 400 meters, elevation and windage of a bullet at a temperature of +40 are 6.47 MOA and 1.7 MOA, and at temperatures - 40 they are 7.17 MOA and 2.41 MOA. Thanks to this, we can conclude that at a colder temperature, the probability of hitting a target decreases, and the probability of a miss increases.

5.2 Experiment 2

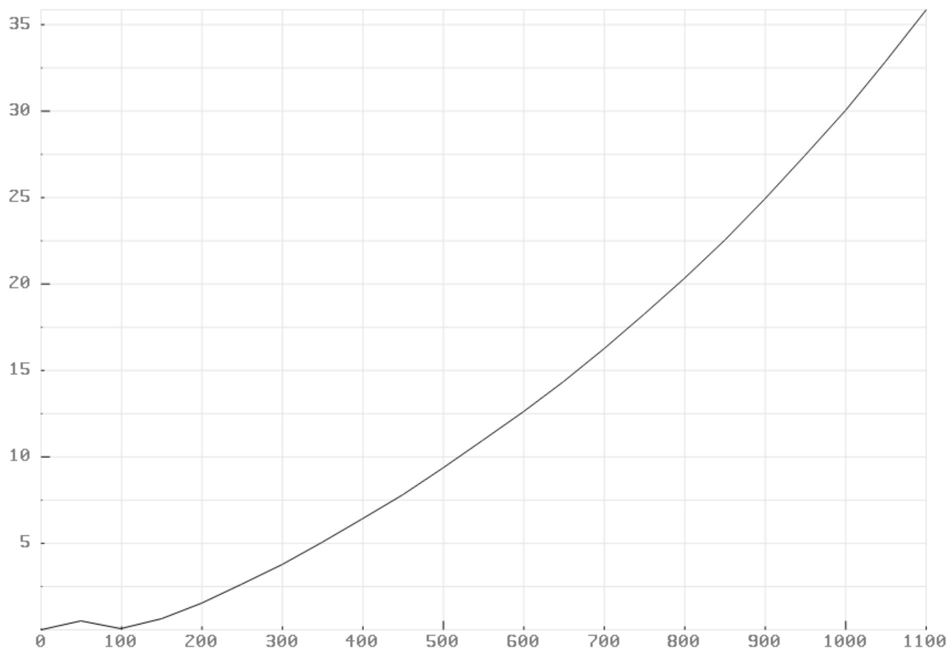
The second experiment is based on the change again of an external factor - atmospheric pressure. After conducting an experiment, you can understand that it affects all output data. So, the lower the atmospheric pressure, the slower the speed drops, the energy therefore the bullet reaches a certain target faster and can fly farther. Also, windage and elevation change more slowly.

For the experiment, we use standard parameters and change only the atmospheric pressure.

Range[m]	Speed[m/s]	Energy[J]	Drop[cm]	Path[cm]	Elevn[MOA]	Windage[MOA]	Time[sec]
0	914	2507	0.00	-4.50			0.00
50	886	2355	1.50	-0.69	0.47	0.18	0.06
100	858	2211	6.12	-0.00	0.00	0.36	0.11
150	831	2073	14.07	-2.64	0.60	0.55	0.17
200	804	1942	25.57	-8.83	1.51	0.74	0.23
250	778	1817	40.87	-18.81	2.58	0.94	0.30
300	752	1698	60.21	-32.85	3.75	1.15	0.36
350	727	1585	83.89	-51.21	5.02	1.37	0.43
400	702	1478	112.21	-74.23	6.36	1.59	0.50
450	677	1376	145.51	-102.22	7.79	1.82	0.57
500	653	1280	184.17	-135.56	9.30	2.06	0.65
550	629	1189	228.57	-174.66	10.89	2.31	0.73
600	606	1103	279.18	-219.95	12.57	2.56	0.81
650	583	1021	336.47	-271.93	14.34	2.83	0.89
700	561	945	400.97	-331.13	16.22	3.11	0.98
750	539	872	473.29	-398.13	18.20	3.40	1.07
800	518	804	554.06	-473.59	20.30	3.70	1.16
850	497	740	644.01	-558.24	22.52	4.01	1.26
900	476	680	743.95	-652.86	24.87	4.34	1.36
950	456	623	854.75	-758.36	27.37	4.68	1.47
1000	436	571	977.42	-875.71	30.02	5.04	1.58
1050	417	521	1113.05	-1006.04	32.85	5.41	1.70
1100	398	475	1262.89	-1150.56	35.86	5.80	1.82

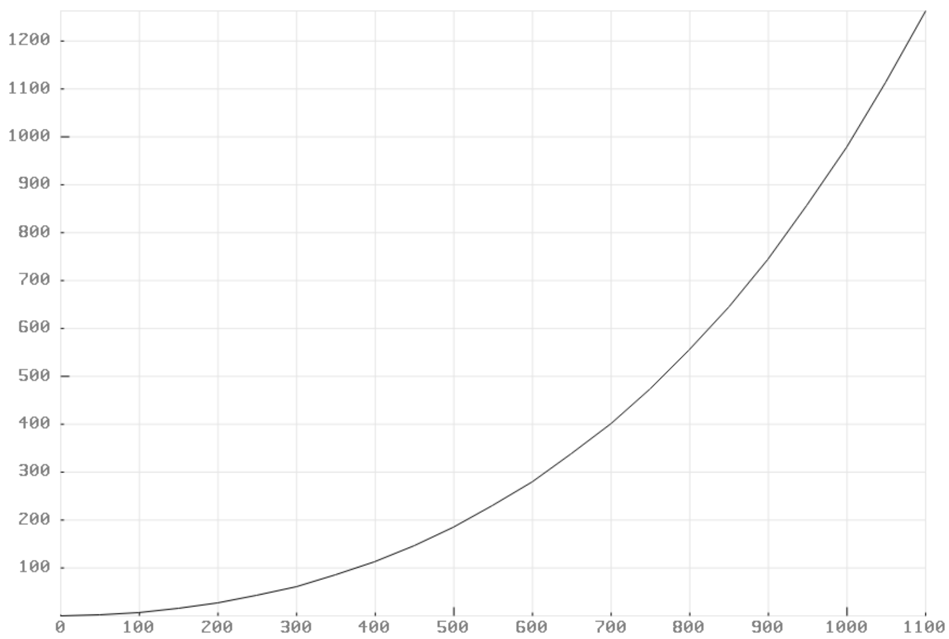
Output of the program for the second experiment:
ballistic table for atmospheric pressure 650 mmhg

Graph of elevation [cm] (vertical axis) versus range (horizontal axis) with atmospheric pressure 650 mmhg.



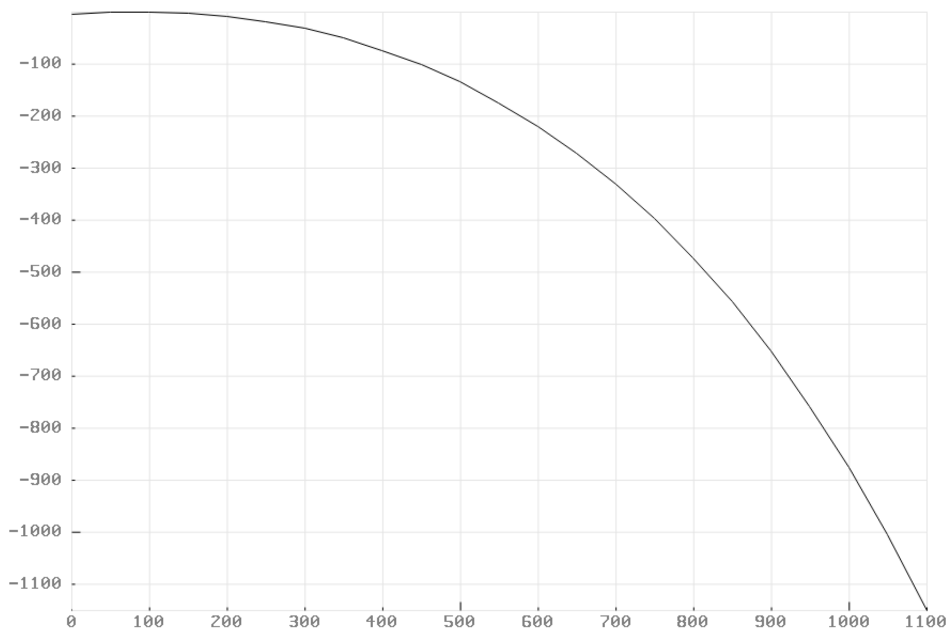
Output of the program for the second experiment:
graph_range_elevn.png

The graph shows how the drop [cm] of a bullet (vertical axis) changes at a certain distance [m] (horizontal axis) with atmospheric pressure 650 mmhg.



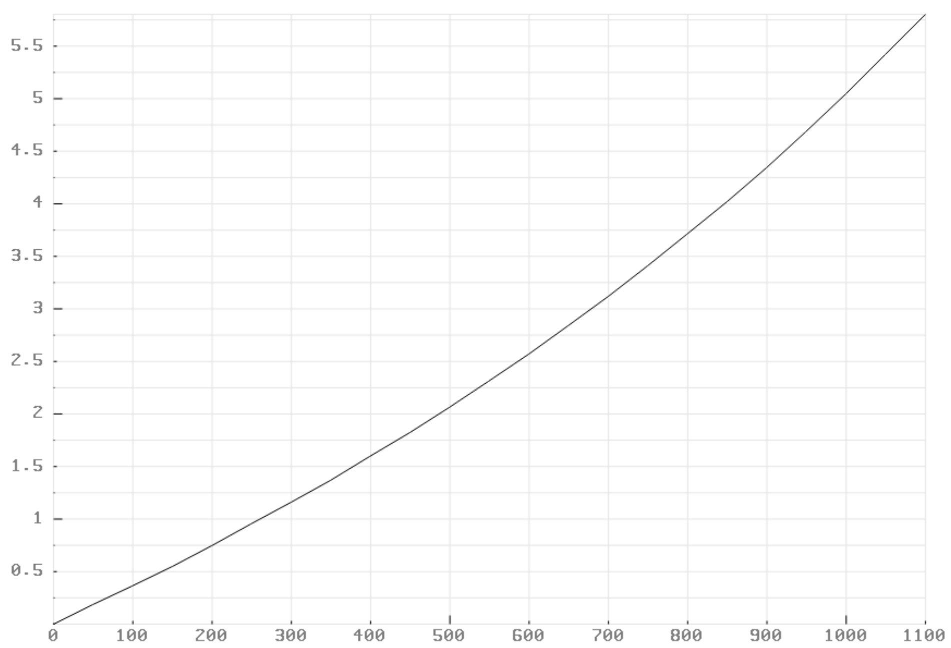
Output of the program for the second experiment:
graph_range_drop.png

The graph shows how the path [cm] of a bullet (vertical axis) changes at a certain distance [m] (horizontal axis) with atmospheric pressure 650 mmhg.



Output of the program for the second experiment:
graph_range_path.png

A graph that shows windage [cm] (vertical axis) at a certain range [m] (horizontal axis) with atmospheric pressure 650 mmhg.



Output of the program for the second experiment:
graph_range_windage.png

Range[m]	Speed[m/s]	Energy[J]	Drop[cm]	Path[cm]	Elevn[MOA]	Windage[MOA]	Time[sec]
0	914	2507	0.00	-4.50			0.00
50	879	2320	1.51	-0.66	0.46	0.22	0.06
100	845	2145	6.18	-0.00	0.00	0.45	0.11
150	812	1979	14.29	-2.77	0.63	0.68	0.17
200	780	1824	26.12	-9.25	1.59	0.93	0.24
250	748	1677	41.98	-19.78	2.71	1.19	0.30
300	716	1540	62.23	-34.69	3.96	1.46	0.37
350	686	1411	87.27	-54.38	5.33	1.74	0.44
400	656	1291	117.51	-79.28	6.80	2.03	0.52
450	627	1178	153.46	-109.89	8.37	2.34	0.59
500	598	1073	195.66	-146.75	10.06	2.66	0.68
550	570	976	244.72	-190.47	11.87	2.99	0.76
600	543	885	301.33	-241.73	13.81	3.35	0.85
650	516	800	366.26	-301.33	15.89	3.72	0.95
700	490	722	440.41	-370.13	18.13	4.11	1.05
750	465	649	524.75	-449.13	20.53	4.52	1.15
800	441	583	620.44	-539.47	23.12	4.95	1.26
850	417	521	728.76	-642.46	25.91	5.41	1.38
900	393	465	851.21	-759.57	28.94	5.90	1.50

Output of the program for the second experiment:
ballistic table for atmospheric pressure 812 mmhg

From this experiment, it follows that a bullet that slows down more due to high atmospheric pressure will take longer to reach the target, and it will be more displaced from its original path by gravity and wind.

6. Summary of simulation experiments and conclusion

The experiment shows that in order to correctly calculate the trajectory of the flight of a bullet, many factors must be taken into account and that one factor can completely influence other data.

Therefore, within the framework of the project, a ballistic calculator was created, which is based on physical formulas for calculation and implemented using a program written in the C programming language.

7. References

https://www.fit.vutbr.cz/study/courses/IMS/public/prednasky/IMS.pdf

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