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1 Reduce

Consider the reduce function:

```
template < typename T, typename op >
T reduce (T* array , size_t n) {
	T result = array [0];
	for (i0nt i=1; i<n; ++i)
		result = op (result , array [i]);
	return result ;
}
```

Do not be scared by the syntax! In C++, templates allow you to replace types and values in a piece of code by a type or a value known at compilation time. This is similar to generics in Java. So if you dene T as int and op as sum, it boils down to computing the sum of the array. You could use
op as max and compute the maximum value of the array.

1.1 int, sum

Consider rst the int, sum case which computes the sum of an array of integers.
We have already shown that there is no parallelism in the parallel task graph extracted from the algorithm
as it is written.

Question: Assuming you have P processors, rewrite the code to introduce one local variable per processor to store partial computation so as to achieve more parallelism. What is the width, critical path and work ?

Ans:

Q.1)

1.1 Program

```
template <typename T, typename op>
    T reduce (T* array , size_t n)
        T result = array [0]       # Task 1
        for (int i = 1 ; i<n ; ++i)
            result = op (result, array [i] ); #Task 2
        return result                      # Task 3
```

Let say N = 4 and array = [1, 2, 3, 4]

① result = 1 (reduce_pre)

↓ int i = 1

② result = op(result, array[1])
= op(1, 2) # since op is sum
= 3
[reduce_result_array[1]]

↓ result

③ return result
return 3 (reduce_post)

④ result = 3 (reduce_pre)

↓ i = 2

⑤ result = op(result, array[2])
= op(3, 3)
= 6
(reduce_result, array[2])

↓ result

⑥ return result
return 6 (reduce_post)

⑦ result = 6
(reduce_pre)

↓ i = 3

⑧ result = op(result, array[3])
= op(6, 4)
= 10
(reduce_result_array[3])

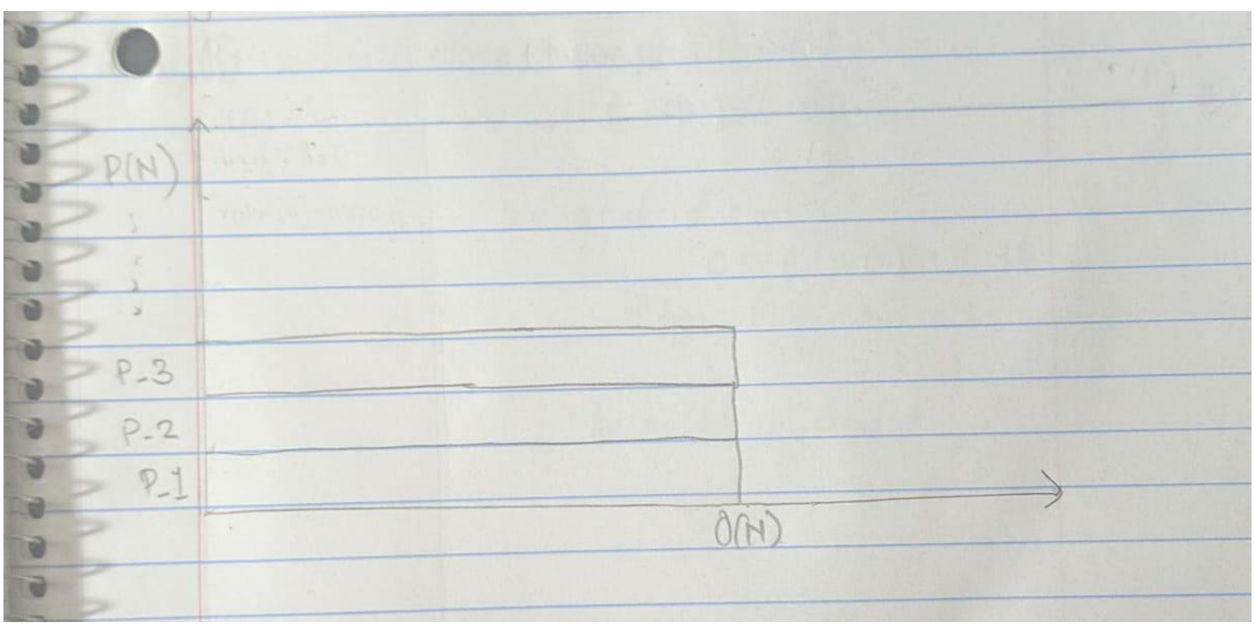
↓ result

⑨ return result
return 10 (reduce_post)

Since there is no parallelism, we will be changing the code as below:

```
Steps   template <typename T, typename op>
          T reduce (T* array, size_t n) {
①            T value [P]        # Processor array to stored value
          for (int i=0; P<P; ++i) {
②            value [i] = array[0]   # Storing inital value of array[0]
                                         # in result
             for (int j=1; j<N j++j) {
③              value [i] = op (value [i], array [j]); # Storing the value
                                   by suming value[i] and
                                   array[j] in value[i]
             }
          }
④          return value        # returning value
        }
P(N)
```

Question: What does a schedule look like on P processors?
Ans:



What is the width, critical path and work ?
Ans: Width should be theta(P),Critical Path theta(N) and Work O(P*N)

1.2 Variants

(Note that by correct we mean that the parallel version would produce the exact same result as the sequential
version in all cases.)

Question: Would that parallel version be correct for int, max? Why?
Ans: The parallel version will be correct for int,max. For each value in the value array which is int here, the program will run P times and replaces the max value each time. The max value will be decided between value[i] and array[j]. For each iteration once the max value is derived , it stores into the value array by returning value. Since each process is independent from each other there will be no dependencies. Hence the parallel version will be correct for int,max.

Question: Would that parallel version be correct for string, concat? Why?
Ans: The parallel version will be correct for string,concat. For each value in the value array which is string here, the program will run P times and replaces the concatenated value each time. The concatenated value will be obtained by joining the value at value[i] position and array[j] position. For each iteration once the concatenated value is derived , it stores that value in the value array. Since each process is independent from each other there will be no dependencies. Hence the parallel version will be correct for string,concat.

Question: Would that parallel version be correct for float, sum? Why?
Ans:The parallel version will be correct for float,sum. For each value in the value array which is float here, the program will run P times and replaces the summation value each time. The summation value will be obtained by summing the value at value[i] position and array[j] position. For each iteration once the summation value is derived , it stores that value in the value array. Since each process is independent from each other there will be no dependencies. Hence the parallel version will be correct for float,sum

Question: Would that parallel version be correct for float, max? Why?
Ans:The parallel version will be correct for float,max. For each value in the value array which is float here, the program will run P times and replaces the max value each time. The max value will be obtained by considering maximum value at value[i[position and array[j] position. For each iteration once the max value is derived , it stores that value in the value array. Since each process is independent from each other there will be no dependencies. Hence the parallel version will be correct for float,max.

2 Prex sum

```
void prefixsum (int * arr , int n, int * pr) {
        pr [0] = 0;
        for (int i=0; i<n; ++i)
                pr[i+1] = pr[i] + arr [i];
}
```

We have seen that this writing of prex sum generates a parallel task graph that is one long chain.

Question: Rewrite this algorithm to make it parallel on P processors. (Hint: What goes wrong if you were blindly parallelising the code by cutting the work in say 3 chunks? You may have to add some work without changing the complexity in Big-Oh notation. A single pass on the array is not enough.)

Ans:

```
void prefixSum(int *arr, int n, int *pr){
        int lastNum;
        for(int height=0; height<=log(n-1); height++){
                for(int i=0; i<=n-1; i = 2^(d+1)){
                        arr[i+2^(d+1)-1] = arr[i+2^(d)-1] + arr[i+2^(d+1)-1];
                }
        }
        lastNum = arr[n-1];
        arr[n-1] = 0;
        for(int height=log(n-1); height>=0; height--){
                for(int i=0; i<=n-1; i=2^(d+1)){
                        int temp = arr[i+2^(d)-1];
                        arr[i+2^(d)-1] = arr[i+2^(d+1)-1];
                        arr[i+2^(d+1)-1] = temp + arr[i+2^(d+1)-1];
                }
        }
        for(int j=1;j<n;j++){
                pr[i] = arr[i];
        }
        pr[n-1] = lastNum;
}
```

Question: What is the work, width, and critical path of the algorithm you created?

Ans:

Work is **O(n)**.

Width is **n/2**. As in first and for loop we add left element value in right element which is like a binary tree. And in last for loop we copy each element from arr and store it in pr so there it can be (n-1).

Critical Path is **O(log n).**

3 Merge Sort

There is limited parallelism in merge sort because the merge operation is naturally sequential.
Question: Rewrite merge to make it suitable to execute with P processors. (Hint: it is one of these cases where you may have to increase complexity slightly.)
Ans:

```
Function mergeSort(Array, Length)
        If Length == 1 return Array
        //Parallel Sections
        //Start
        leftArray = mergeSort(0, Length/2) //Run on processor 0
        rightArray = mergeSort(Length/2 + 1, Length) // Run on processor 1
        //End
        Return merge(leftArray, rightArray)

Function merge(leftArray, rightArray)
        B = new Array
        While leftArray =! Empty AND rightArray != Empty
                If leftArray[0] < rightArray[0]
                        B.insrtAtLast(leftArray[0])
                        Remove leftArray[0]
                Else
                        B.insrtAtLast(rightArray[0])
                        Remove rightArray[0]
        While leftArray != Empty
                B.insrtAtLast(leftArray[eachElement])
        While rightArray != Empty
                B.insrtAtLast(rightArray[eachElement])
        Return B
```

Question: What are the work and critical path of the merge algorithm you created?
Ans:
Work is O(n) and critical path would be O(2*log n).

Question: If you used that parallel merge in merge sort, what would the work and critical path of merge sort become?
Ans:
Work will be the O(n). Critical path will be O(log n).