(a)

We find the following relation between D ,$(x-\xi)$ and θ:

$cos\theta = \frac{D}{(D^2+(x-\xi)^2)^{1/2}}$ $sin\theta = \frac{(x-\xi)}{(D^2+(x-\xi)^2)^{1/2}}$

The directional vector: $\hat{r} = \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix}$

$\Delta F = \begin{bmatrix} M\Delta g_z \\ M\Delta g_x \end{bmatrix} = \frac{GM\Delta m}{r^2}\hat{r} = \frac{GM\Delta m}{D^2+(x-\xi)^2} \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} = \begin{bmatrix} \frac{GM\Delta mD}{(D^2+(x-\xi)^2)^{3/2}} \\ \frac{GM\Delta m(x-\xi)}{(D^2+(x-\xi)^2)^{3/2}} \end{bmatrix}$

so $\Delta g_z = \frac{G\Delta mD}{(D^2+(x-\xi)^2)^{3/2}}$

(b)

Because the anomalies are at 0.1 meter intervals from 0 to 10, we consider the k anomaly:

$\xi = 0.1(k-1)$

$g(x, \Delta m_k) = \frac{GD\Delta m_k}{[D^2+(x-0.1(k-1))^2]^{3/2}}$

so the total contribution of all anomalies can be calculated as

$g(x, m) = \sum_{k=1}^{101} \frac{GD\Delta m_k}{[D^2+(x-0.1(k-1))^2]^{3/2}}$ where $m = [\Delta m_1 \Delta m_2 ... \Delta m_{101}]'$

(c)

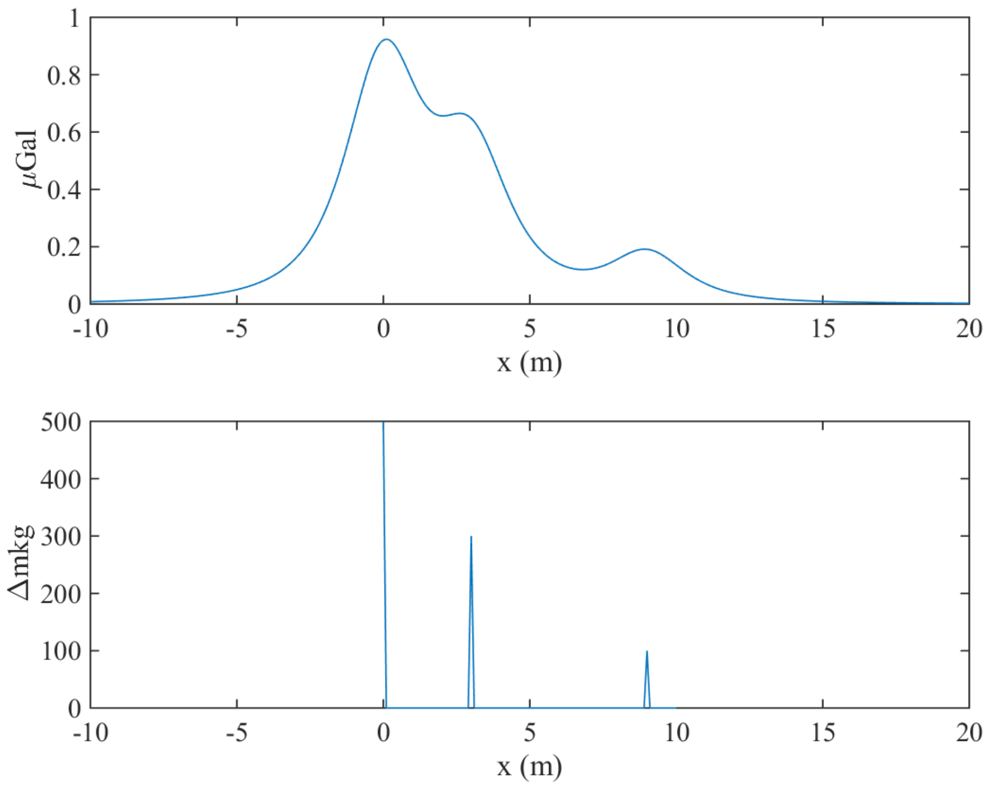
Linear. Because the model predication is a linear function of m:

$$\boldsymbol{d_i} = \boldsymbol{G_{i,:}m}$$

where

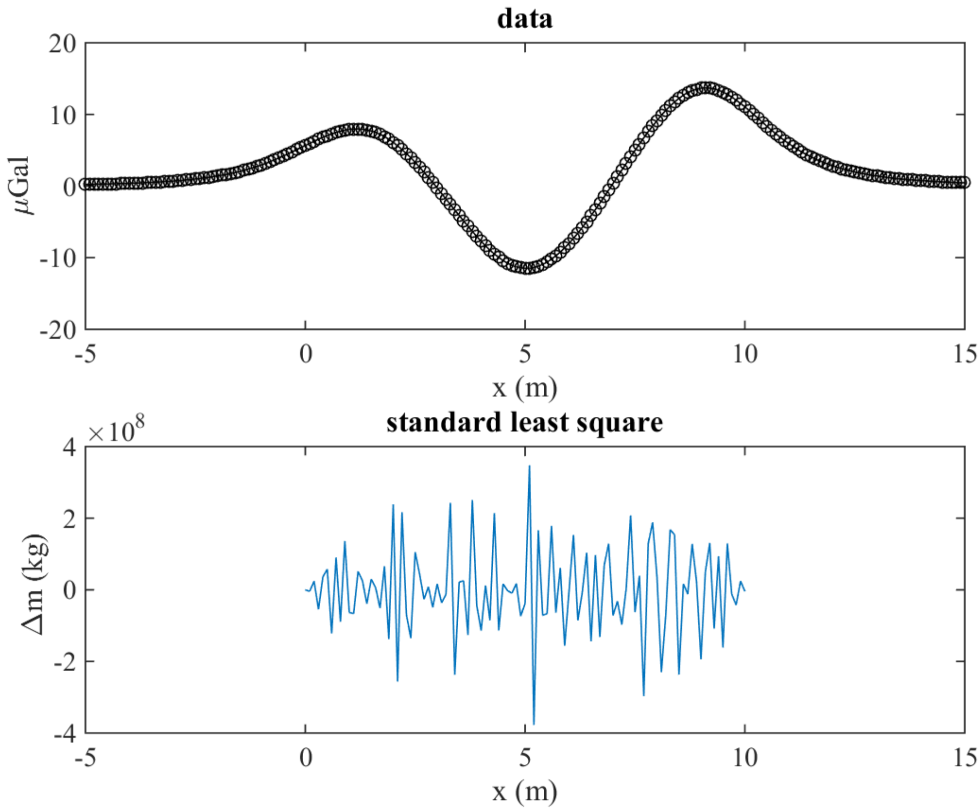
$$\boldsymbol{G_{ij}} = \frac{GD}{[D^2+(x_i-0.1\cdot(j-1))^2]^{3/2}}$$

(d)

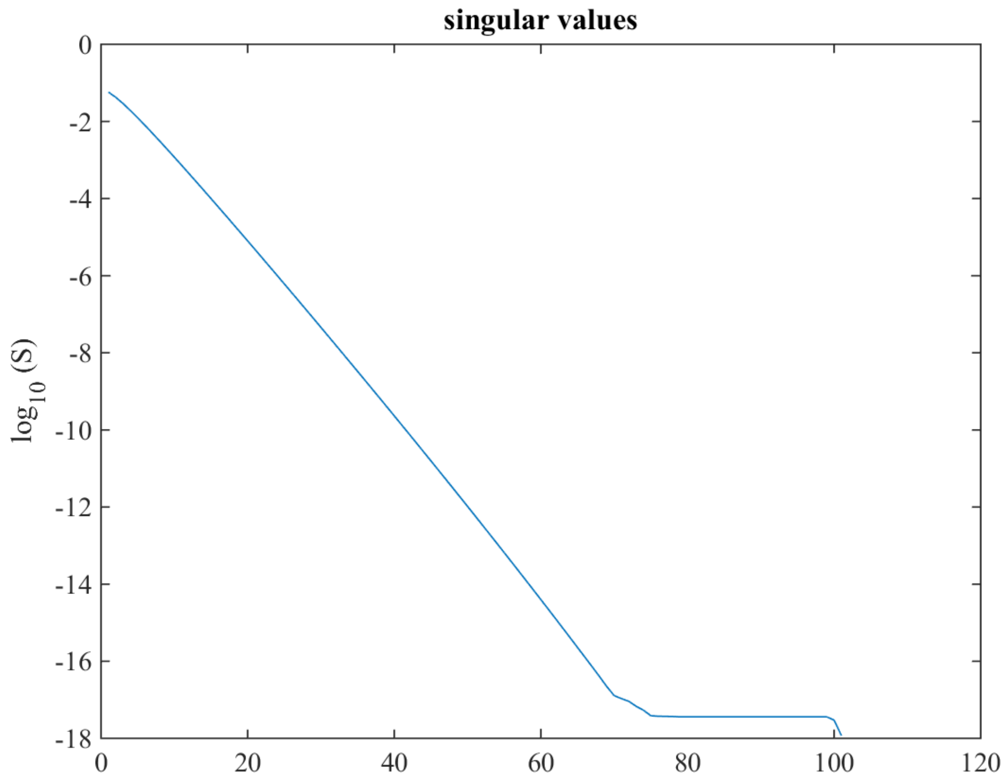


1
0.8
0.6
0.4
0.2
0
μGal
-10
-5
0
5
10
15
20
x (m)
500
400
300
200
100
0
Δmkg
-10
-5
0
5
10
15
20
x (m)

(e)

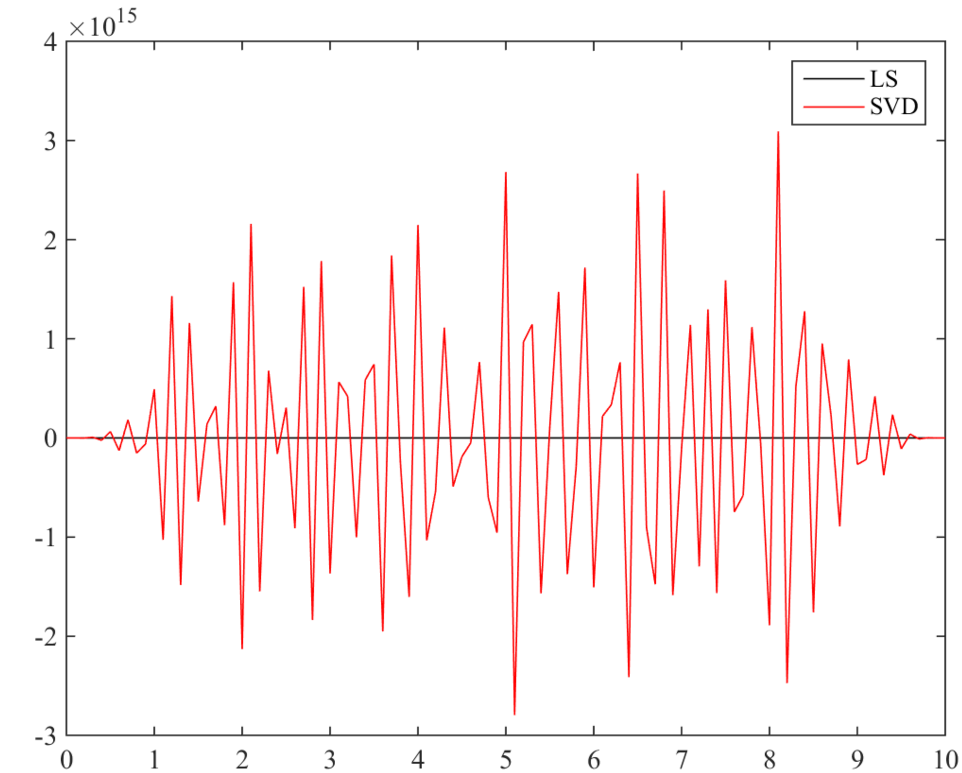


We see that the inversion result is messy. To understand it, we calculate the singular values, and plot them in logarithm.

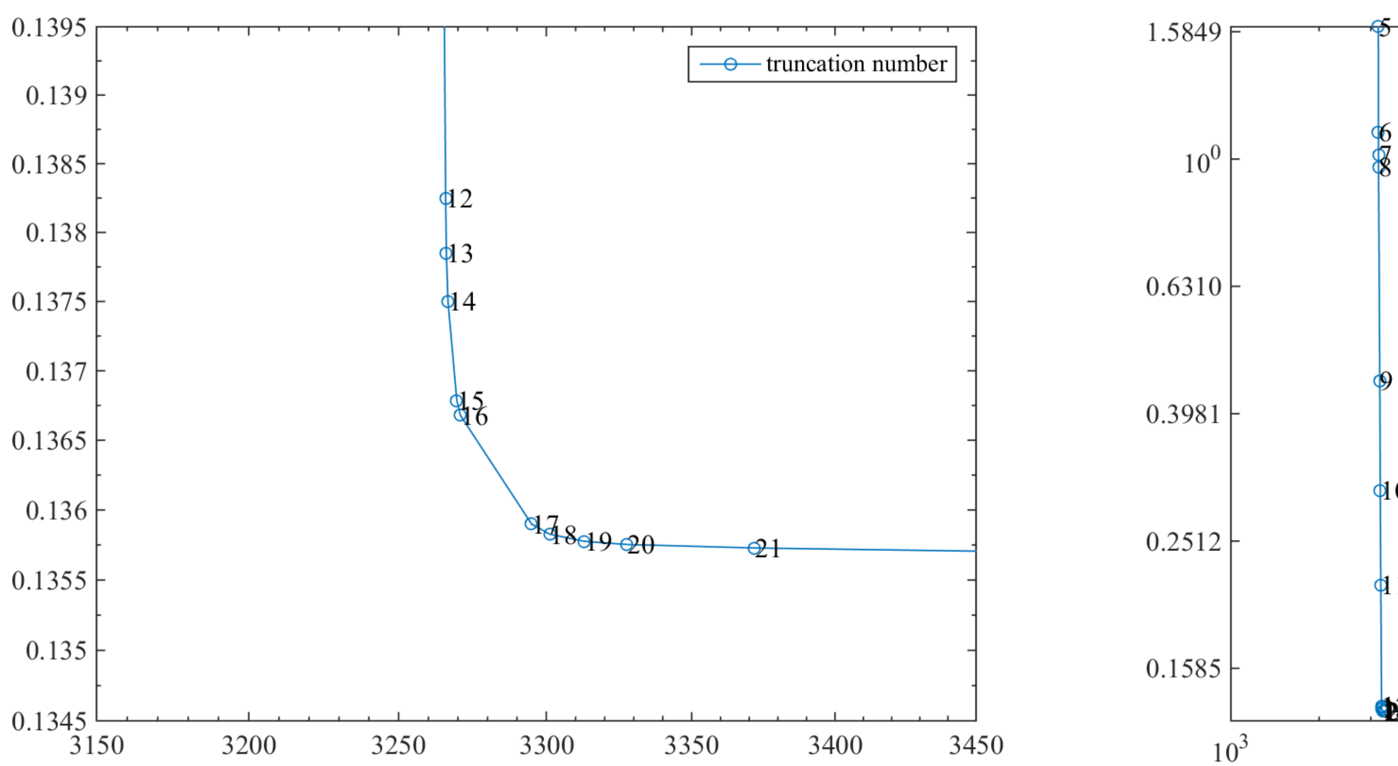

singular values
log10 (S)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
0
20
40
60
80
100
120

We see that, ..

(f)


×10^15
LS
SVD
4
3
2
1
0
-1
-2
-3
0 1 2 3 4 5 6 7 8 9 10

(g)


truncation number
0.1395
0.139
0.1385
0.138
0.1375
0.137
0.1365
0.136
0.1355
0.135
0.1345
3150 3200 3250 3300 3350 3400 3450
12
13
14
15
16
17
18
19
20
21
1.5849
10^0
0.6310
0.3981
0.2512
0.1585
10^3