

Periodicities of binary Cepheids

Exploring a novel approach based on distance correlation

K. Barbey^{1*}, G. Viviani¹, R.I. Anderson¹

¹ Institute of Physics, EPFL, Chemin Peg 51b, CH-1290 Versoix

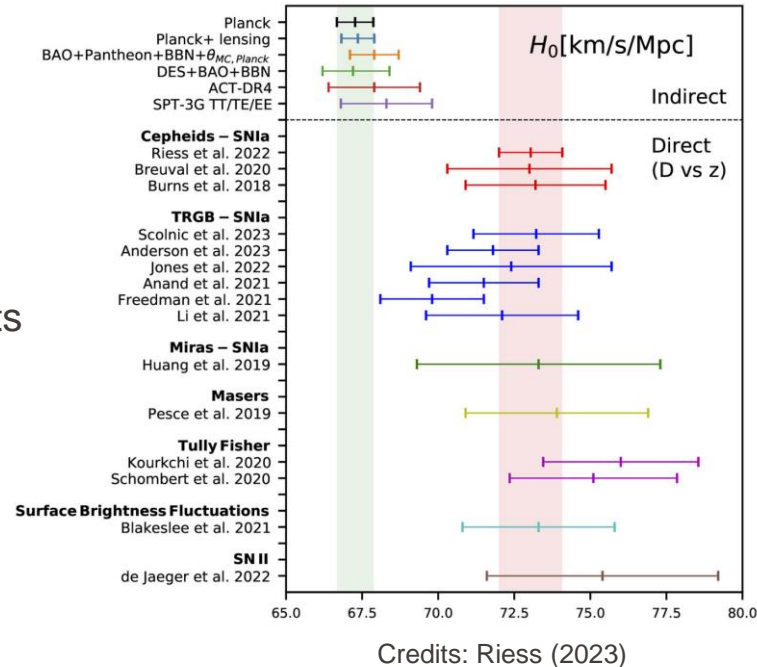
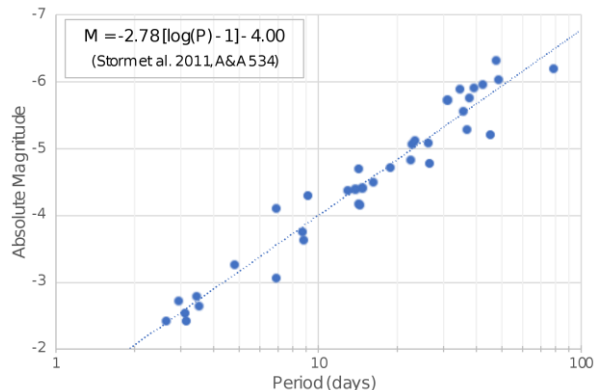
***** kent.barbey@epfl.ch

Photo Credits: GAIA Vari Citizen Science Project

Context I

Period, Luminosity and the Hubble Constant

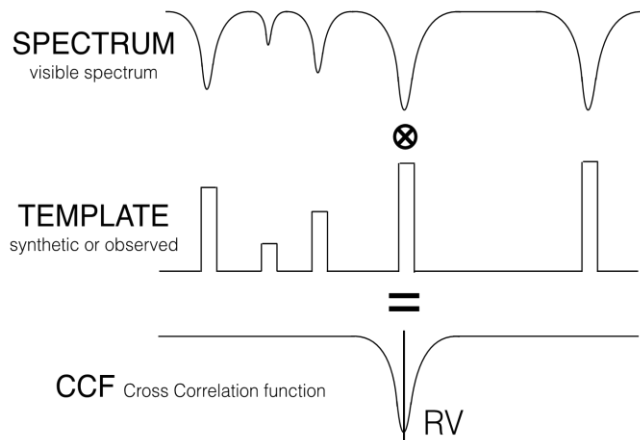
- H_0 : Local expansion rate of the universe
 - How do we compute it: early vs late universe
 - PL relation
- **Hubble tension**
 - Early universe: 67.4 ± 0.5 [km/s/Mpc]
 - Late universe: 73.04 ± 1.04 [km/s/Mpc]
 - Two possible causes: distance measurements or cosmological model



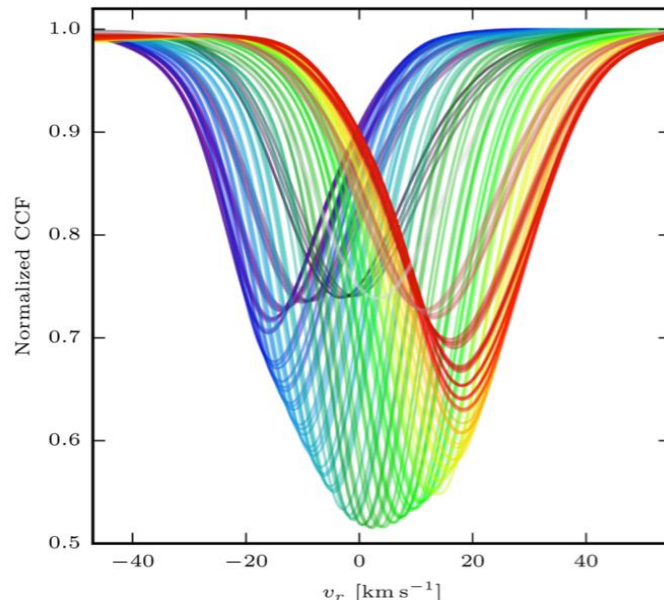
Context II

Radial velocities and periodicities

- **Radial velocity(RV):** velocity of a star to an observer projected on the line-of-sight.
 - Causes: orbital motion($\sim 10\text{-}20$ [km/s]) + pulsation of the star.
 - How do we find it: 1d-CCF for example.



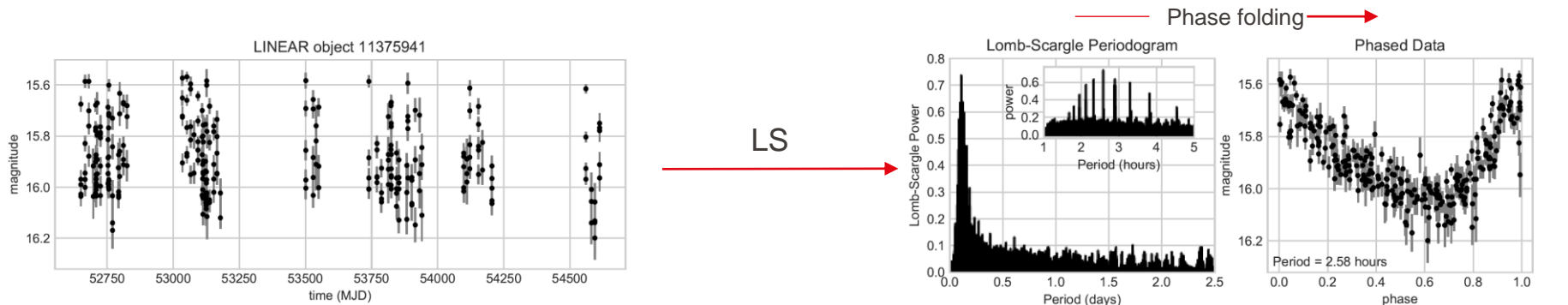
Credits: X. Dumusque



Credits: R.I. Anderson+2016

Finding periodicities with RVs.

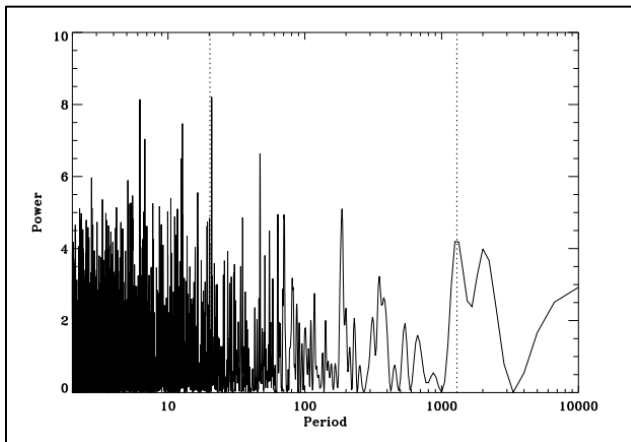
- **Phase-folding methods:** frequency that optimises a cost function
- **Least-squares methods:** frequency that maximises the likelihood of the fit.
- **Bayesian approaches.**
- **Fourier methods:** Schuster, *Lomb-Scargle*(LS) etc...



Motivation I

Why is the LS periodogram not always enough ?

- The case of Classical Cepheid variable stars.
 - Radial velocity: up to ~ 60 [km/s].
 - Often in binary systems.
 - Effects of pulsation: shift + shape variation of spectrum.
- What is the problem then ?
 - Time-varying shape modulations can
 - Induce errors in shift measurement
 - Mimic exoplanet reflex motion



Credits: Carolo et al. (2014)

Motivation II

Why this new approach and what does it do?

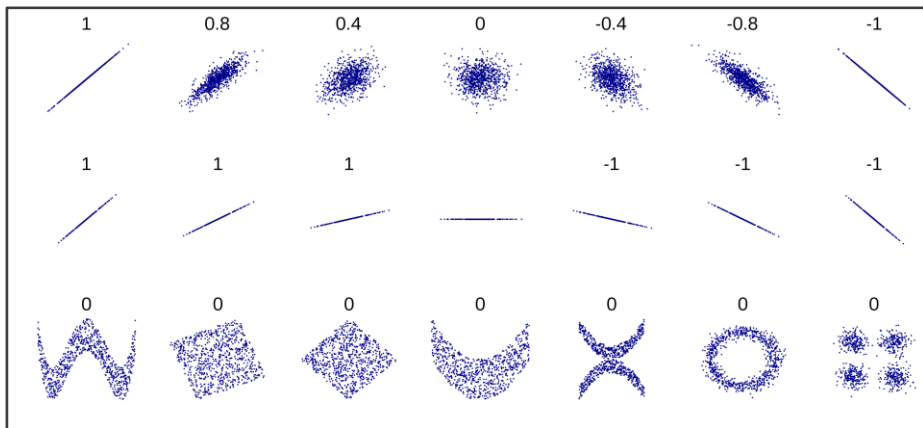
- Important to distinguish orbital from pulsational(stellar activity) motions.
 - Find companions to Cepheid binaries.
 - Uncover presence of non-radial pulsations.
 - Confirm older methods' results.
 - Find something else?
- Distance correlation-based periodograms:
 - No sinusoidal assumption
 - Shift: shape variations influence is controlled
 - Sensitive to line displacements.
 - Shape: shift variations influence is controlled
 - Sensitive to shape variations.

} Two periodograms for the price of one

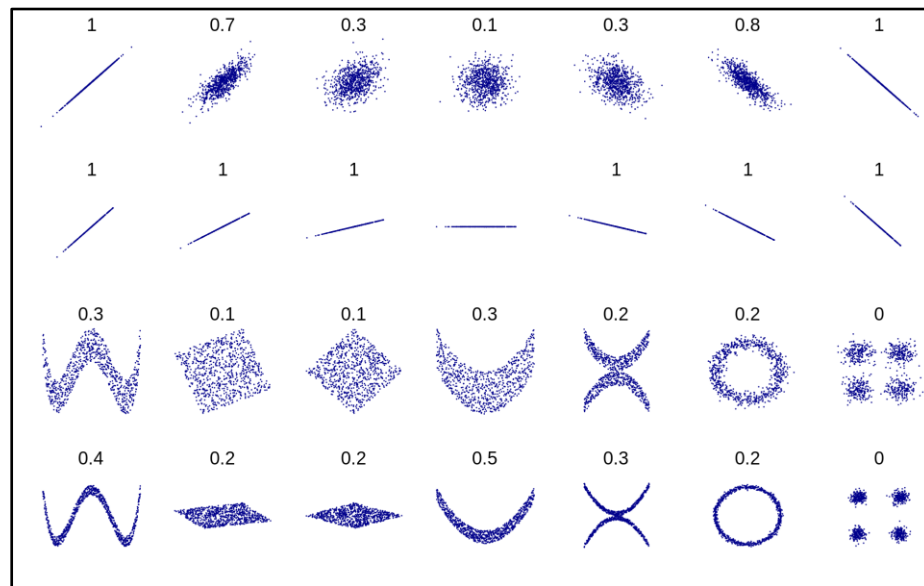
Mathematical framework I

Semi-partial distance correlation: A didactic example

- **Pearson's correlation:** measure of linear statistical relationship between two random variables → linear dependence
- **Distance correlation (Székely, 2005):** measure of both linear and non linear relationships between two random variables.
→ General dependence



Correlation coefficient:
 $Cor(X, Y)$



Distance correlation coefficient:
 $dCor(X, Y)$

Mathematical framework II

Semi-partial distance correlation: A didactic example

Experiment:

- **GOAL** : Assess the relation between color(x) and effective temperature(y) for stars.
- Take n stars and measurements of color, T_{eff} and line-of-sight dust column density(z).

$$\mathbf{x} = \begin{pmatrix} x_1 - \bar{x} \\ \vdots \\ x_n - \bar{x} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{pmatrix}, \quad \mathbf{z} = \begin{pmatrix} z_1 - \bar{z} \\ \vdots \\ z_n - \bar{z} \end{pmatrix}$$

- Naive idea: compute $Cor(x, y) = \frac{1}{n} \frac{\langle x, y \rangle}{\sigma_x \sigma_y}$. Problem: \mathbf{x} and \mathbf{y} may be correlated with \mathbf{z} !
→ \mathbf{z} : nuisance parameter.
- Solution: Partial correlation: $pCor(x, y) = \frac{1}{n} \frac{\langle e_x, e_y \rangle}{\sigma_{e_x} \sigma_{e_y}}$ or semi-partial: $\frac{1}{n} \frac{\langle e_x, y \rangle}{\sigma_{e_x} \sigma_y}$
- From correlation to distance correlation: $\mathbf{x}, \mathbf{y}, \mathbf{z} \xrightarrow{\text{red}} \bar{\bar{x}}, \bar{\bar{y}}, \bar{\bar{z}}$ (distance matrices)

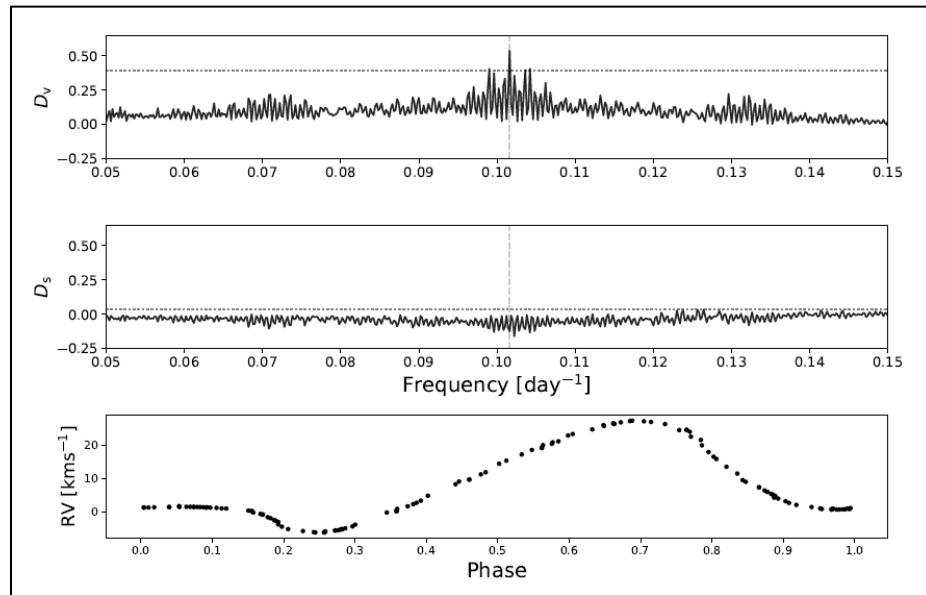
For us: $\bar{\bar{\phi}}$: phase, $\bar{\bar{v}}$: RV, $\bar{\bar{s}}$: shape(spectra)

- One distance metric per matrix: $d_\phi = \phi_{ij}(P - \phi_{ij})$, $d_v = |v_i - v_j|$, $d_s = (1 - C_{ij})^{\frac{1}{2}}$
- Constructing the periodograms: $D_{\phi v}, D_{\phi s}$

Some examples:

β Doradus

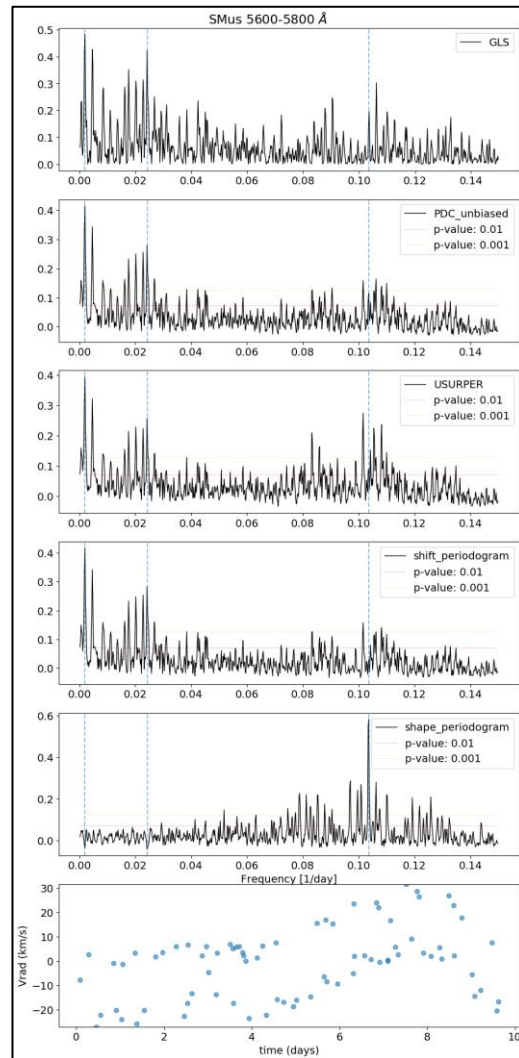
- Classical Cepheid (no known companion)
- 135 CORALIE observations
- Range tested: 4900-5150 Å
- Periods recovered:
 - Radial pulsation periodicity: ~ 9.84 days



Credits: Binnendijk et al. (2021)
(badly labeled)

Some examples: S Muscae

- Binary classical Cepheid
- 60 CORALIE observations: 01.2012 → 05.2018
- Range tested: 5600-5800 Å
- Periods recovered:
 - Radial pulsation periodicity: ~9.66 days
 - Orbital periodicity: ~505 days
 - Window function: 40 days



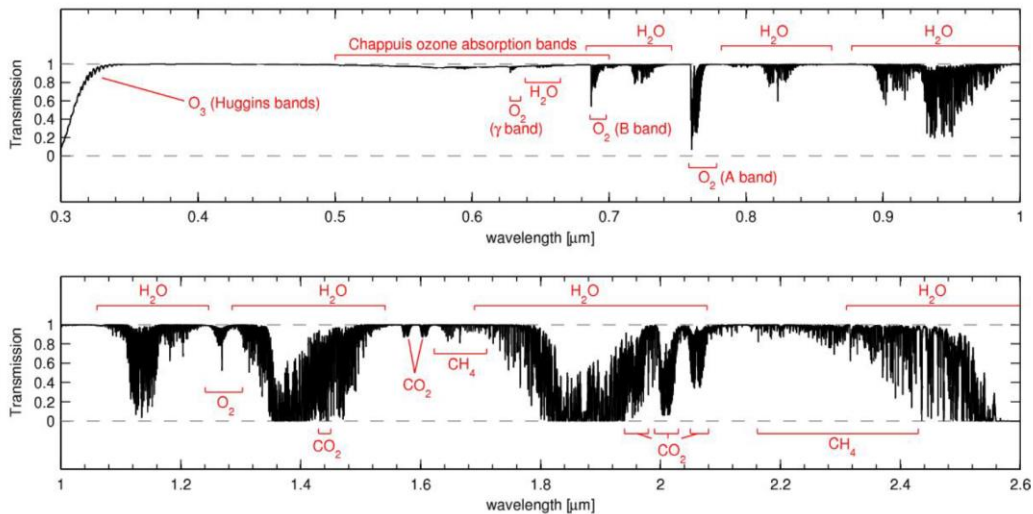
What have I been doing apart from calceito and cycling?

- Finding the right peak
 - Harmonics, aliases, window function.
- What wavelength ranges should be avoided?
 - Impact of tellurics (« high-frequencies » of Earth's atmosphere)
- What range size should be used?
 - Low frequency variation of earth's atmosphere.
 - Preprocessing
- What range gives the best results?
- Asserting the statistical significance of a peak.
 - Can a FAP be computed without implementing bootstrap?

Tellurics

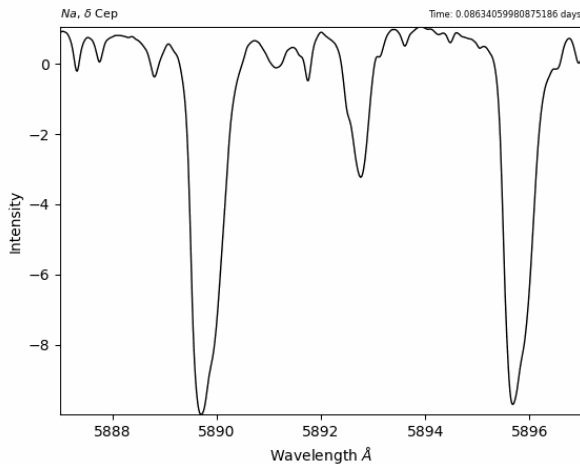
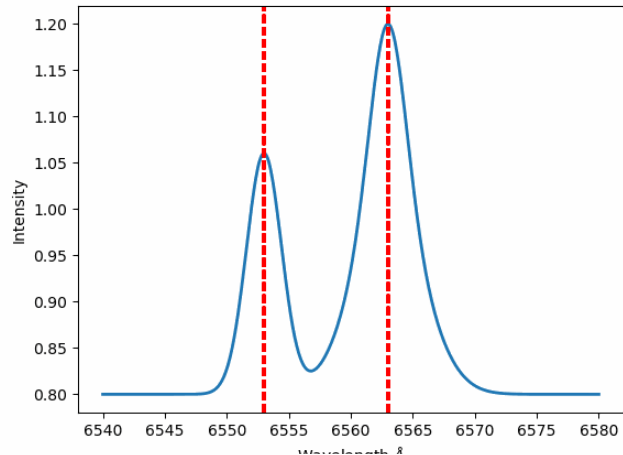
The sky is a limit

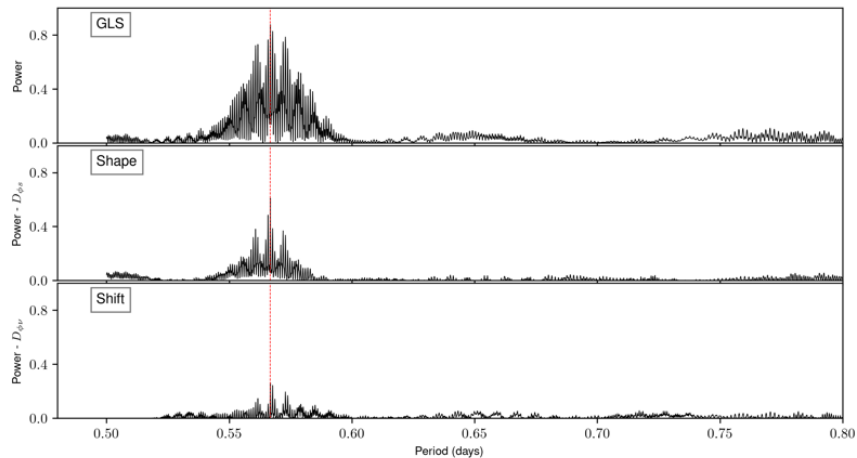
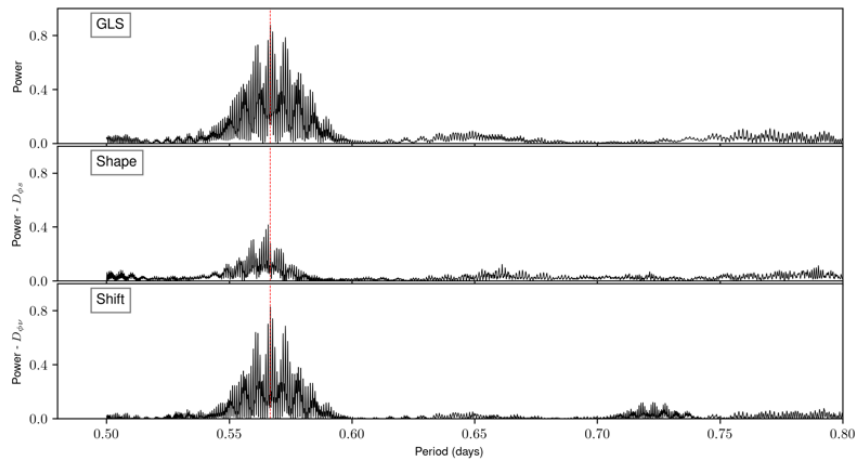
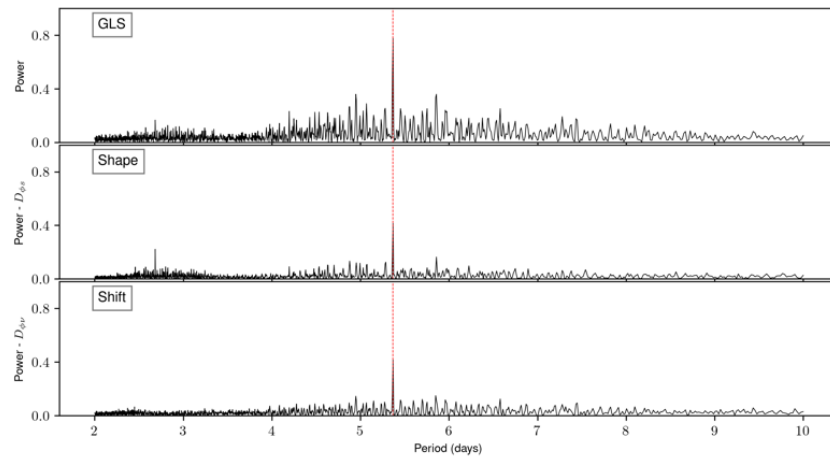
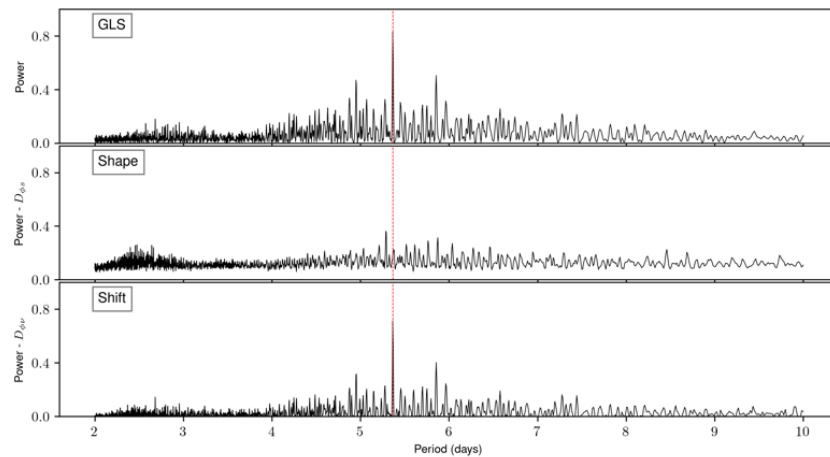
- Question
- Goal: : finding the right wavelength ranges



Credits: Przybilla

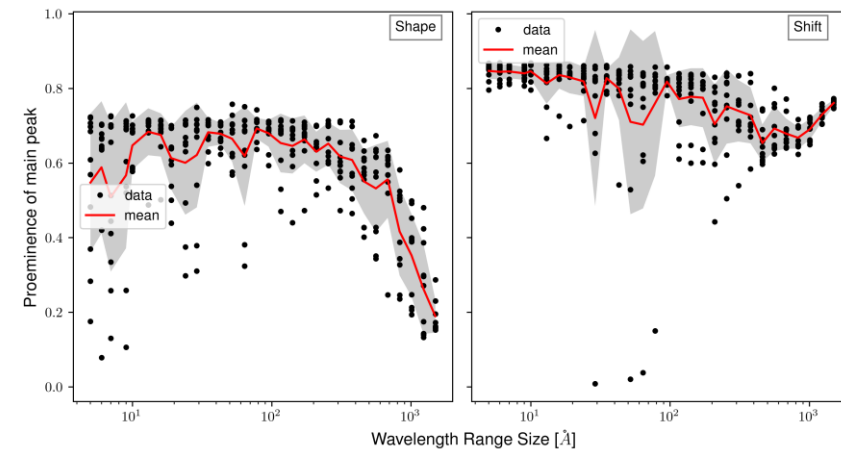
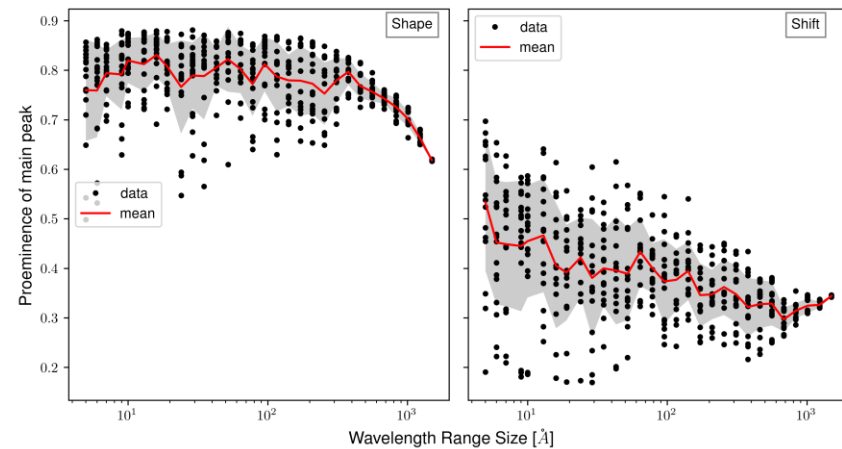
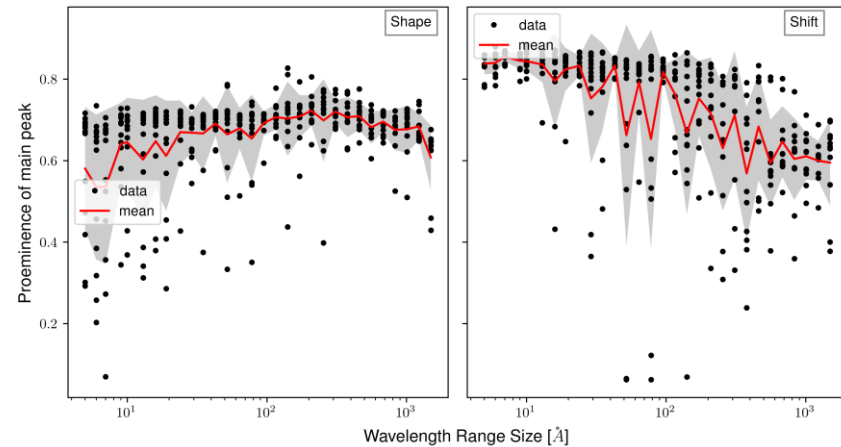
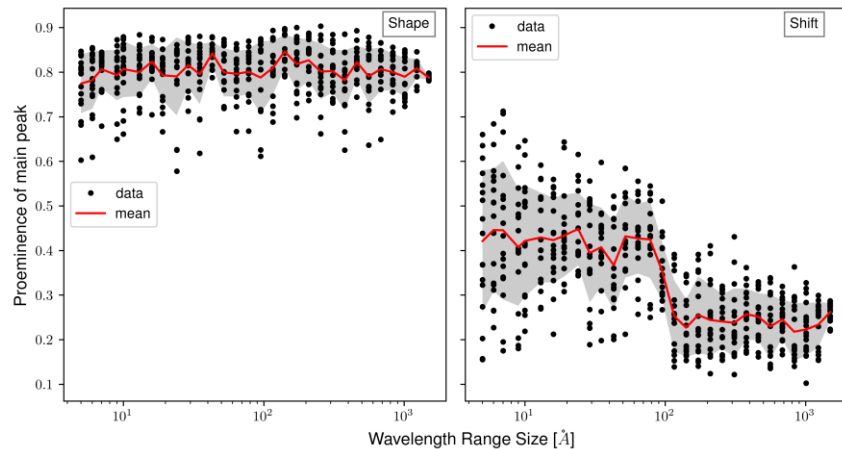
Blending of Telluric Line and Pulsating Star Absorption Peak



H_{β} : 4861 Å $O_2 \gamma$ -band: 6260-6300 Å H_{β} : 4861 Å $O_2 \gamma$ -band: 6260-6300 Å

R Crucis - Wavelength Range Size Analysis: 4000-5600 Å - Data points: 505

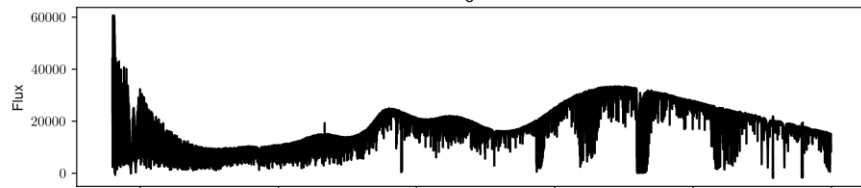
RR Lyrae - Wavelength Range Size Analysis: 4000-5600 Å - Data points: 383



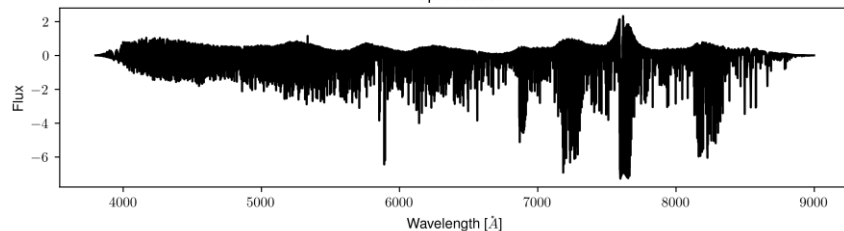
Finding the right wavelength range: The effect of preprocessing

SPARTA preprocessing

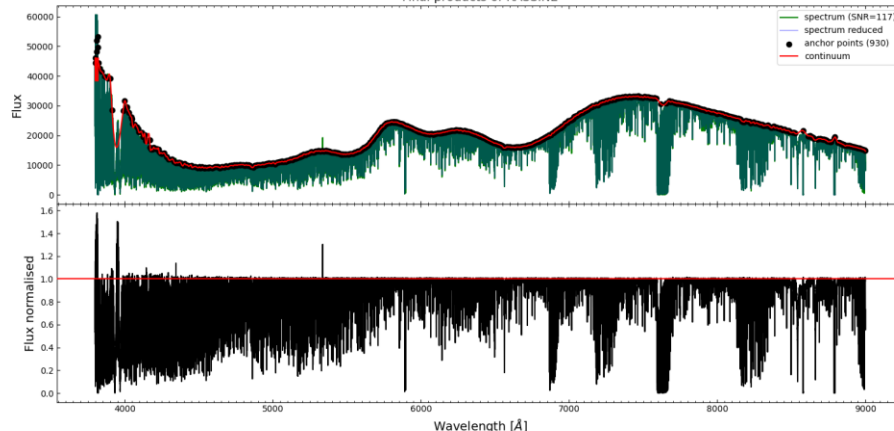
Original



Preprocessed

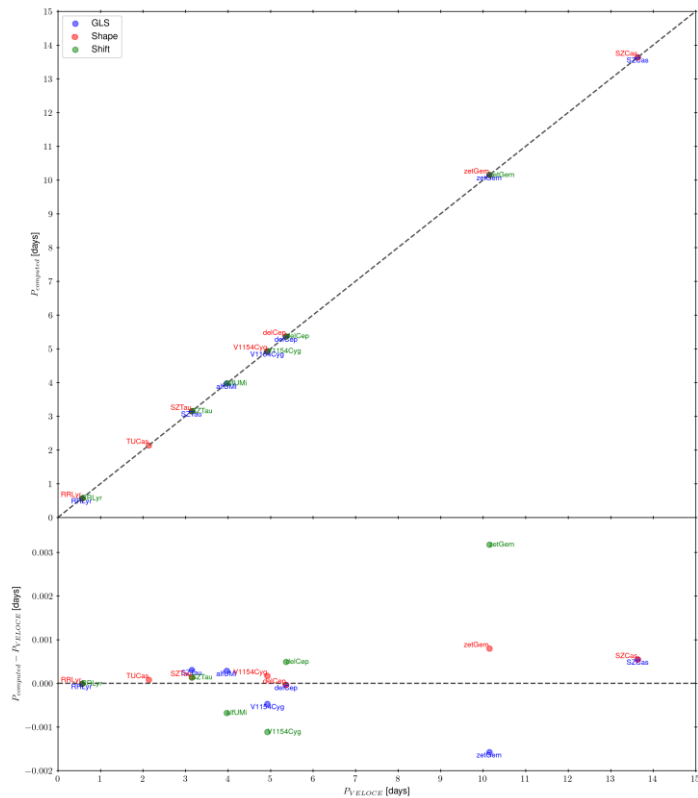


Final products of RASSINE

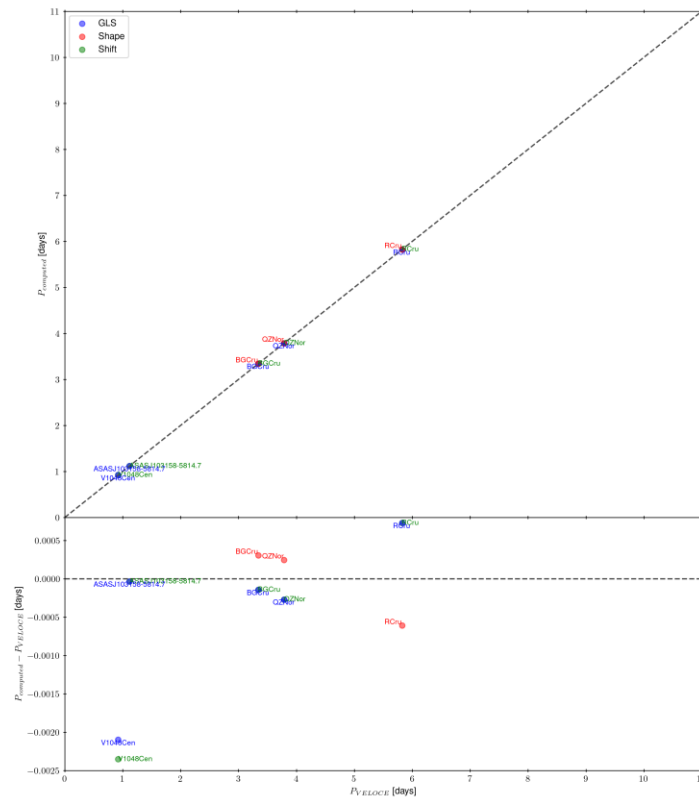


- Hopes
 - Get cleaner periodograms (avoiding Earth's atmosphere low-frequencies)
 - Enable bigger wavelength ranges

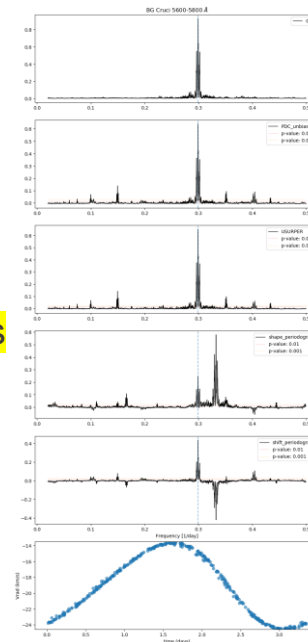
Hermes vs VELOCE/AAVSO - minimised residuals



Coralie14 vs VELOCE/AAVSO - minimised residuals



- Complete Polaris data set.
- RASSINE time series preprocessing.
- Error analysis on periodicities found
- Detecting new periodicities: from the variable star and from companions
 - BG Cru: stability of the 3.00 days additional periodicity
 - R Cru: orbital periodicity
 - ASAS: orbital periodicity
- Shift periodogram showing a peak at the pulsational frequency.
 - RV signal not properly removed
 - Estimating the accuracy of the RV?
- Asserting the statistical significance of a peak
 - Spurious peaks: harmonics and aliases
 - Implement bootstrap? Is there another way?
 - Baluev, Davies, Süveges not applicable(or so I think)
- Which range should be used?
- Optimisation the computation of multiple periodograms at the same time(see with Giordano and Mauricio for multiprocessing).
- $O(n \log(m))$



Conclusion

- **Goal of the algorithm:**
Separately identify Doppler and spectral-shape modulations.
- **Results:**
 - Avoid tellurics
 - Use wavelength ranges of ~100-200 angströms with original preprocessing.
 - All Hermes except Polaris stars have their main period found with the shape periodogram at max power
 - For Coralie14, only R Cru, BG Cru and QZ Nor have their main period found with the shape periodogram at max power.
 - Shift periodogram shouldn't show a peak at the shape periodogram's stellar pulsation peaks -> could be a way to know if the RVs are accurately measured.
- **Questions on my mind:**
 - Does this shift problem pose a problem to finding orbital companions?
 - Does it make the shape periodogram less efficient?
 - Why does BG Cru's second periodicity show a hole in the shift periodogram and its main periodicity don't even though it has a higher power?
 - Why is ASAS' shape periodogram flat?
 - Polaris/BG Cru/zetGem: upward, downward trend.

- A. Binnenfeld, S. Shahaf, R. I. Anderson and S. Zucker.
New Periodograms Separating Orbital Radial Velocities and Spectral
Shape Variations.
A&A 659 A189 (2022)
- J. T. VanderPlas.
Understanding the Lomb-Scargle Periodogram.
ApJS 236 16 (2018)
- R. I. Anderson, G. Viviani et al.
VELOCE I. High-Precision Radial Velocities of Cepheids.
[Manuscript submitted for publication], IPHYS EPFL Switzerland.
- M. Cretignier, J. Francfort, X. Dumusque, R. Allart and F. Pepe.
RASSINE: Interactive Tool for Normalising Stellar Spectra.
A&A 640 A42 (2020)

RR Lyrae - Wavelength Range Size Analysis: 4802-4880 Å

