“Old Quantum Mechanics by Bohr and Sommerfeld from a Modern Perspective”

arXiv:2506.00408v1 [quant-ph] 31 May 2025

(Last modified on June 2, 2025; 8:30 PM Arizona time.)

In[1]:=
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(* Here, from (3.17)-(3.18),
we derive the expansion (3.19) for the relativistic Schroedinger equation (3.14). *)
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In[2]:= EnergyRelSchr[μ_] :=
$$\frac{1}{\sqrt{1+\frac{\mu^2}{\left(n+\frac{1}{2}+\sqrt{\left(\frac{1}{2}+l\right)^2-\mu^2}\right)^2}}}$$

In[3]:= `Simplify[PowerExpand[Series[EnergyRelSchr[μ], {μ, 0, 5}, Assumptions → 2 * l + 1 > 0]]]`

Out[3]=
$$1-\frac{\mu^2}{2\,(1+l+n)^2}-\frac{(5+2\,l+8\,n)\,\mu^4}{8\,\left((1+2\,l)\,(1+l+n)^4\right)}+O[\mu]^6$$

In[4]:= `FullSimplify[% /. n → n - l - 1]`

Out[4]=
$$1-\frac{\mu^2}{2\,n^2}+\frac{\left(3-\frac{8\,n}{1+2\,l}\right)\mu^4}{8\,n^4}+O[\mu]^6$$

In[5]:= `(* This is the nonrelativistic expansion (3.19). *)`

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(* Here, from (3.22),
we derive the expansion (3.24) for the Dirac equation (3.20). *)
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In[7]:=
$$\frac{1}{\mathrm{Sqrt}\left[1+\mu^2/(n+\nu)^2\right]}$$

Out[7]=
$$\frac{1}{\sqrt{1+\frac{\mu^2}{(n+\nu)^2}}}$$

In[8]:= `EnergyDirac[μ_] = % /. ν → Sqrt[(j + 1/2)^2 - μ^2]`

Out[8]=
$$\frac{1}{\sqrt{1+\frac{\mu^2}{\left(n+\sqrt{\left(\frac{1}{2}+j\right)^2-\mu^2}\right)^2}}}$$

In[9]:= `Simplify[PowerExpand[Series[EnergyDirac[μ], {μ, 0, 5}, Assumptions → 2 * j + 1 > 0]]]`

Out[9]=
$$1-\frac{2\,\mu^2}{(1+2\,j+2\,n)^2}-\frac{2\,(1+2\,j+8\,n)\,\mu^4}{(1+2\,j)\,(1+2\,j+2\,n)^4}+O[\mu]^6$$

In[10]:= `FullSimplify[% /. n → n - j - 1/2]`

Out[10]=
$$1-\frac{\mu^2}{2\,n^2}+\frac{\left(3-\frac{8\,n}{1+2\,j}\right)\mu^4}{8\,n^4}+O[\mu]^6$$

In[11]:= `(* This is the expansion (3.24). *)`
