“Oganesson versus Uranium Hydrogen-like Ions from the Viewpoint of Old Quantum Mechanics”

arXiv:2509.06249v2[quant-ph] 11 Sept 2025

https://arxiv.org/pdf/2509.06249

(Last modified/executed on September 12, 2025; 14:30 PM Arizona time.)

In[22]:=
```
SetDirectory[NotebookDirectory[]];
```

In[23]:=
```
(*---Constants and Setup---*)
(*Pretty print subscript-style variables for display only*)
MakeBoxes[nt, StandardForm] := SubscriptBox["n", "t"]
MakeBoxes[nr, StandardForm] := SubscriptBox["n", "r"]

(*Physical constants*)
Z = 122; (*Atomic number of a hypotetical element Unbibium, Ubb*)
α = 1 / 137.036; (*Fine structure constant*)

(*Quantum numbers*)
ntVal = 1;
nrVal = 1;

(*---Define symbolic formulas as functions---*)

(*Eq.(3.19)*)
ωFunc[nt_] := Sqrt[nt^2 - Z^2 α^2] / nt

(*Eq.(3.20)*)
ϵFunc[nt_, nr_] :=
 Sqrt[nr] Sqrt[(nr + 2 Sqrt[nt^2 - Z^2 α^2])] / (nr + Sqrt[nt^2 - Z^2 α^2])

(*Eq.(3.21)*)
aFunc[nt_, nr_] :=
 ((nr + Sqrt[nt^2 - Z^2 α^2]) Sqrt[Z^2 α^2 + (nr + Sqrt[nt^2 - Z^2 α^2])^2]) / Z

(*---Compute numeric values---*)

ωVal = ωFunc[ntVal] // N
ϵVal = ϵFunc[ntVal, nrVal] // N
aVal = aFunc[ntVal, nrVal] // N
{rMin = aVal * (1 - ϵVal) // N, rMax = aVal * (1 + ϵVal) // N}
DeltaTheta = 2 * Pi * ((1 / ωVal) - 1) // N
PeriodTheta = 2 * Pi * (1 / ωVal) // N

(*---Define relativistic radial orbit r(t) from Eq.(3.9)---*)
r[t_] := (aVal (1 - ϵVal^2)) / (1 + ϵVal Cos[ωVal t])

PolarPlot[r[t], {t, 0, PeriodTheta}, PlotLabel → "Relativistic 'Elliptical' Orbit",
 AxesLabel → {"x", "y"}, PlotRange → All, PlotStyle → Thick]
PolarPlot[r[t], {t, 0, 1.45 * PeriodTheta},
 PlotLabel → "Relativistic 'Elliptical' Orbit",
 AxesLabel → {"x", "y"}, PlotRange → All, PlotStyle → Thick]
PolarPlot[r[t], {t, 0, 1.75 * PeriodTheta},
 PlotLabel → "Relativistic 'Elliptical' Orbit",
 AxesLabel → {"x", "y"}, PlotRange → All, PlotStyle → Thick]
PolarPlot[r[t], {t, 0, 2 * PeriodTheta},
 PlotLabel → "Relativistic 'Elliptical' Orbit",
 AxesLabel → {"x", "y"}, PlotRange → All, PlotStyle → Thick]
```

Out[32]=

0.455419

Out[33]=

0.949782

Out[34]=

0.0203534

Out[35]=

{0.00102211, 0.0396847}

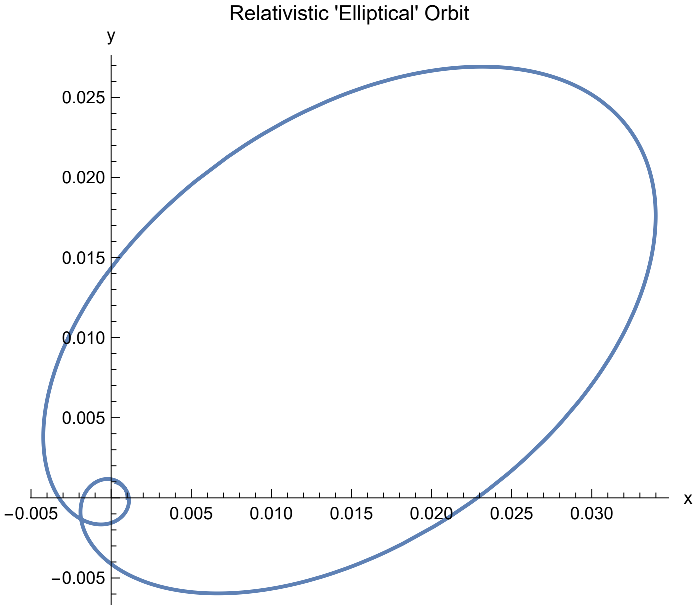
Out[36]=

7.5133

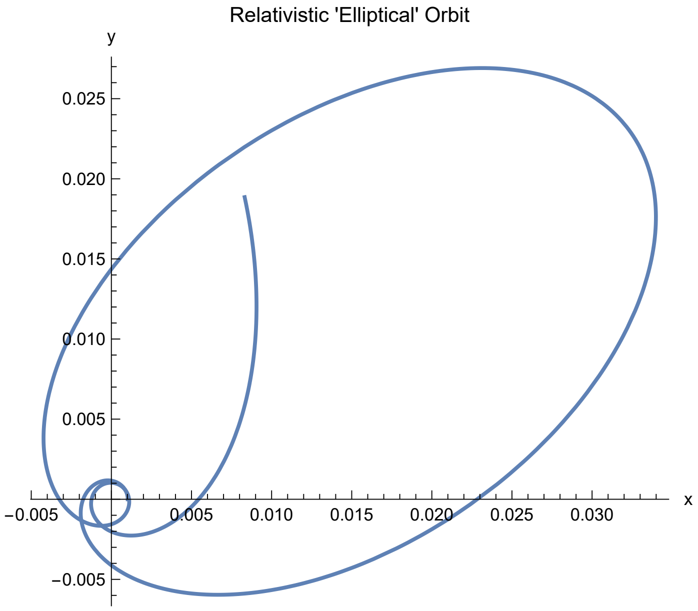
Out[37]=

13.7965

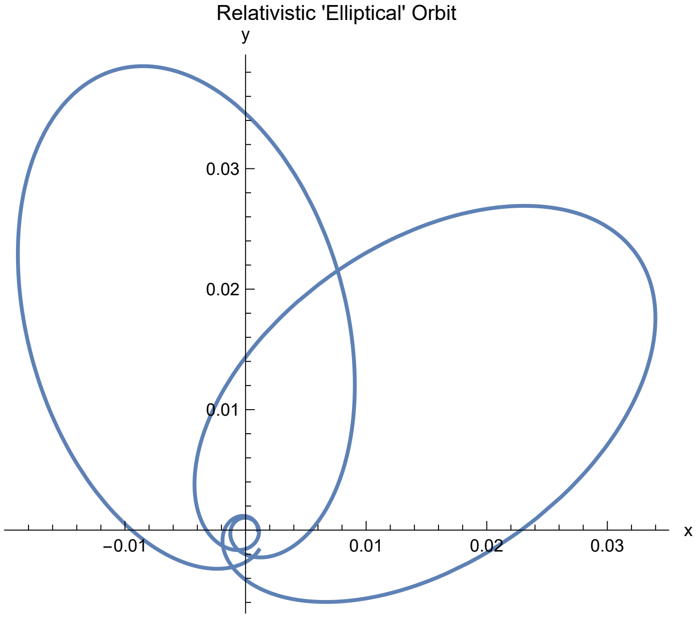
Out[39]=



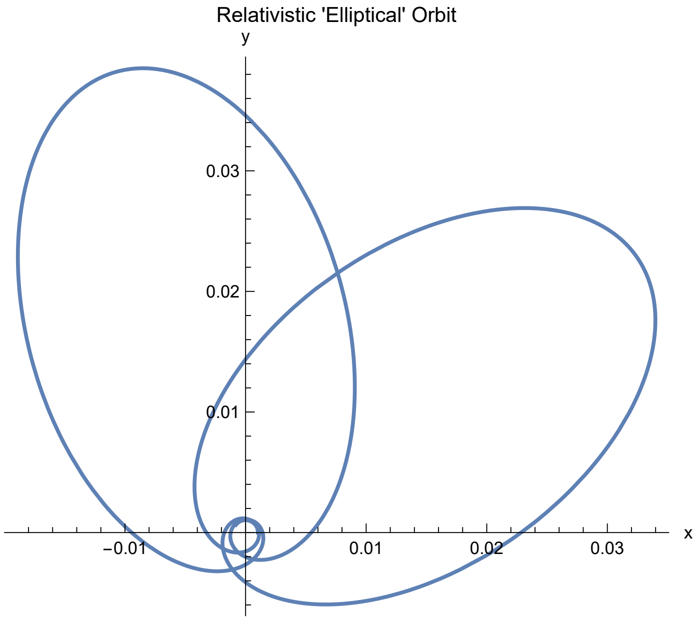
Out[40]=



Out[41]=



Out[42]=



In[43]:=

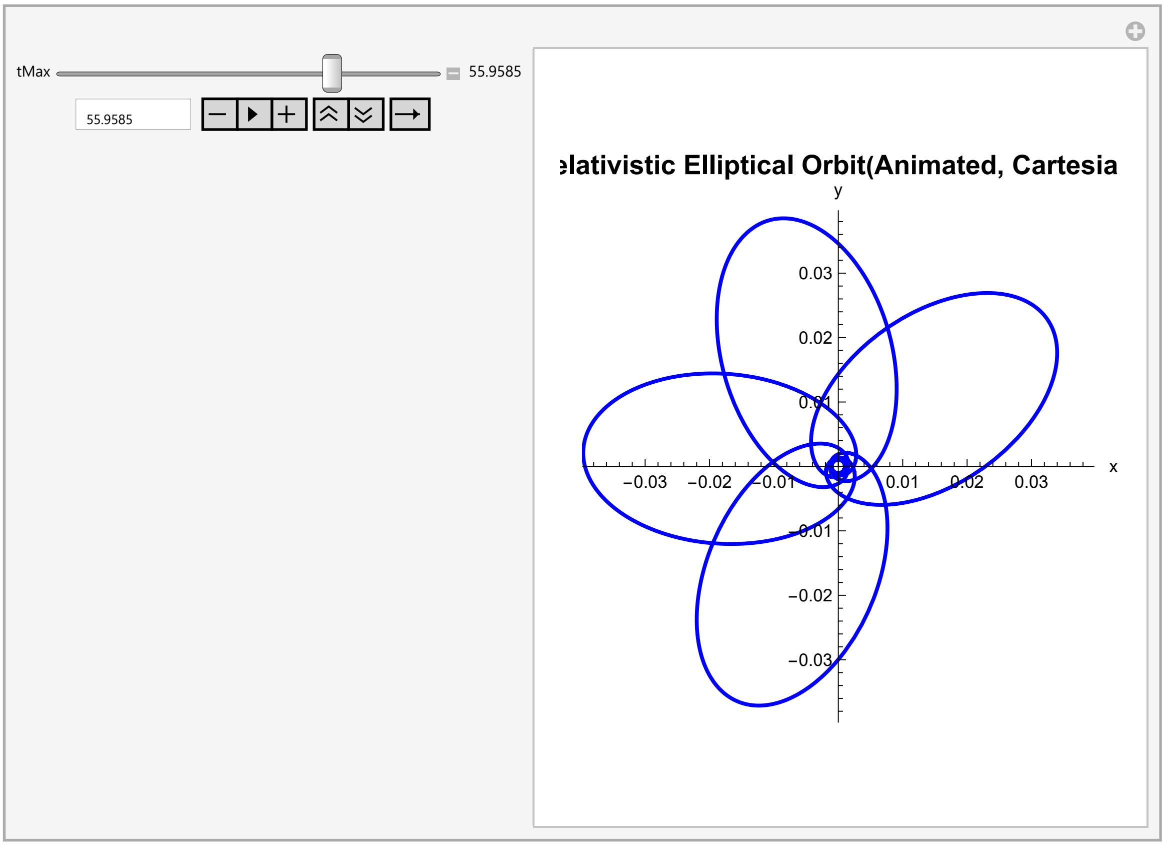
```
Export["UnunenniumIon.pdf", 40]
```

Out[43]=

```
UnunenniumIon.pdf
```

In[44]:=
```
(*Animation in Cartesian Coordinates*)
Manipulate[ParametricPlot[{r[t] Cos[t], r[t] Sin[t]},
  {t, 0, tMax}, PlotStyle → {Blue, Thick}, AxesLabel → {"x", "y"},
  PlotLabel → Style["Relativistic Elliptical Orbit(Animated, Cartesian)", 14, Bold],
  PlotRange → {{-rMax, rMax}, {-rMax, rMax}},
  (*Fixed range ensures full visibility from r_min=0.00257988 to r_max=0.0581747*)
  AspectRatio → 1, PlotPoints → 1000, PerformanceGoal → "Quality"],
 {tMax, 0.01, 24 Pi, Appearance → "Labeled"}]
```

Out[44]=



Summary:

In the above animation of the relativistic Kepler motion of an electron in hydrogen-like Unbibium, Ubb, when Z=122, the nucleus is situated in the fixed focus at the origin. For this quantum state, we choose n_r =nr=1 and n_θ=nt=1 in the fine structure formula (3.18). By (3.19)-(3.21), one gets: ω= 0.455419, ϵ=0.949782, and a=0.0203534, in Bohr's atomic units. The 'winding' number is 3!
The perihelion and aphelion move along two concentric circles around the nucleus with radii: $r_{min} = a\ (1 - \epsilon)$ =0.00102211 and $r_{max} = a\ (1 + \epsilon) = 0.0396847$, respectively. In the animation, the geometrical loci of the successive perihelia and aphelia, the outer and inner envelopes of the orbit, see Figure 3, are not shown for simplicity.
Rotation of the ellipse over one period is given by $\Delta\theta$=2π(1/ω-1)=7.5133.