CMPT 440 – Spring 2019: Quantum Finite Automata

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Theoretical Background

Quantum finite automata, or QFAs, were introduced by Moore and Crutchfield and Kondacs and Watrous. A quantum automaton Q consists of: a hilbert space H, an initial state vector, a subspace and an operator that projects into it, an input alphabet, and finally a unitary transition matrix for each symbol in the alphabet. Quantum finite automata are able to do many of the things that regular finite automata can do, but tend to be more space-efficient in their calculations. This finite automata implements quantum computing, the combination of quantum mechanics and computer science. By using quantum mechanics, a larger set of problems can be solved, for example, factoring. Just as finite automata have one and two way automata, so does the quantum version. The main advantage of quantum finite automata is that they can recognize all regular languages, but represent them in quite a few less states than typically seen in finite automata. That being said, quantum finite automata are more intimidating to work with and require a good grasp on the subject of quantum computing.

An Example

One type of QFA is known as the one-way quantum finite automata. This type has a tape head that moves one cell only to the right at each evolution and represents a theoretical model for a quantum computer. This can be defined as a 6-tuple $M = (Q, \sum, \delta,$q0, $Qacc,$Qrej). As an example, we can use a one letter alphabet where $\sum = \{\ a\}$. The state space is $Q = \{q0, q1, qacc, qrej\}$ with δ being defined as:

$$\delta(q0, a, q0) = 1/2,$$
$$\delta(q0, a, q1) = 1/2,$$
$$\delta(q0, a, qacc) = 0,$$
$$\delta(q0, a, qrej) = 1/2.$$

At the end of this one-way QFA we find both the probability of accepting to be $1/4$ and the probability of rejecting to be $1/2 + 1/4 = 3/4$.

References

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