Math Formulation

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1 State Space

We define the state vector for a multi-agent system with n robots. We can define a substate $\hat{x}_n(t)$ that stores the substate for each individual robot. The system state will be the collection of these substates.

$$x_n(t) = \begin{bmatrix} x_n(t) & y_n(t) & \theta_n(t) & b_n(t) & (V_C)_n(t) & (V_p)_n(t) \end{bmatrix} \tag{1}$$

This substate of robot n has these six real number values: $x_n(t)$ for the x position, $y_n(t)$ for the y position, $\theta_n(t)$ for the heading, $b_n(t)$ for the button sensor state (0 or 1), $(V_C)_n(t)$ for the capacitor voltage, and $(V_p)_n(t)$ for the photoresistor voltage. These are continuous substates. So we form the system state as the collection of n substates.

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{2}$$

This state has a size of $x(t) \in \mathbf{X}, |\mathbf{X}| = \mathbb{R}^{6n}$ for the $6n$ real numbers represented by our state.

2 Action Space

The action space is the movements for all the robots. We have no system dynamics for our simple map and only need to define the control function $u(t)$. We also include a Gaussian noise term $w(t)$ associated with the process noise. This is drawn from a Gaussian with covariance Q. $w(t) \sim \mathcal{N}(0, Q)$

$$x(t+1) = x(t) + Bu(t) + w(t) \tag{3}$$

3 Output (Observation) Space

We define the output space based on the UWB communication data and the BLE data transmitted by robots. We have measurement noise $v(t) \sim \mathcal{N}(0, R)$

$$z(t) = Cx(t) + v(t) \tag{4}$$

The C matrix will map the state to a measured state based on the BLE and UWB communications. We represent these as separate operators $\tilde{U}$ and $\tilde{B}$.

$$C = \tilde{U}\tilde{B} \tag{5}$$

4 State Estimation (Kalman Filter)

The Kalman filter is estimating the localization of an individual robot. Each robot will have their own Kalman filter as described here (indicated by subscript n). This is represented as the position and heading values in the state. We first have the state/covariance prediction step:

$$\hat{x}_{n,t+1|t} = \hat{x}_{n,t|t} + Bu_n(t) \tag{6}$$

$$P_{n,t+1|t} = P_{n,t|t} + Q_n \tag{7}$$

then the update step:

$$\hat{x}_{n,t|t} = \hat{x}_{n,t|t-1} + K_n(t)[z_n(t) - C\hat{x}_{n,t|t-1}] \tag{8}$$

with Kalman gain

$$K_n(t) = P_{n,t|t-1}C_n^T(C_nP(n,t|t-1)C_n^T + R_n)^{-1} \tag{9}$$

and covariance update

$$P_{n,t|t} = (I - K_n(t)C_n)P_{n,t|t-1} \tag{10}$$