System Design for Ultra-Low-Power UWB-based Indoor Localization

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***Abstract*—In this paper, we propose a 3-tier ultra wide-band indoor localization system for autonomously powered sensor network applications. It consists of a large number of cost-effective tags, a number of cheap and low-power hubs and few synchronized base stations. Using the UWB characteristics and the hierarchical scheme, the localization system enables ultra-low-power, autonomous tags and precise positioning. We present how ambiguous solutions (coordinates and transmission time of an unknown node) can be eliminated with the help of the proper geometry of 4 reference nodes. A method for optimizing the number and placement of hubs is proposed. Different positioning algorithms are discussed and compared based on the position accuracy a.f.o range errors. We determine the optimal algorithm for different scenarios. Simulations are conducted to investigate the performance degradation due to timing errors.**

I. Introduction

Indoor localization has become an important field of research with its potential for applications both in private and industrial environments. Production, logistics, factory, security and safety applications are only a few examples. The major challenges for the indoor localization system are the cost, constrained energy and resolution. The device must be low cost to allow placement onto any objects of moderate value, such as shampoos and books. The amount of energy available for the device is quite limited, and could come from an attached battery cell. In many applications, replacing the batteries at regular instants however can be unacceptable or impractical. Autonomous, energy scavenging based devices will therefore enable many more applications. The high position accuracy (sub-meter) is regarded as a key point. So the system, as well as deployment strategies, must be developed according to these important requirements.

In recent years, the research on UWB communication reveals the possibility of low-power-consuming, low-cost systems [1] [2]. The IEEE 802.15 Task Group 4a has chosen impulse UWB radio (UWB-IR) as a good candidate for low-power radio in indoor applications like sensor networks. Many simple, low-power UWB-IR transmitter and receiver designs are proposed [3] [4] [5] [6]. Different methods can be applied to solve the localization problem. One possibility of position estimation involves two main steps: range measurement between the device and the reference nodes with known positions and positioning based on the measured ranges [7]. The time of arrival (TOA) estimation technique seems to be the most capable to exploit the high time resolution for the large bandwidth of the UWB signal. UWB offers a better ranging accuracy as well as superior capabilities in non-line-of-sight conditions compared to Wireless LAN [8]. Owning to its unique characteristics, UWB technology holds great promise to enable accurate indoor localization system.

Several low-power UWB-based indoor localization systems have been published [9] [10] [11]. The existing systems tried to prove the feasibility of UWB-based localization. Thus, they mainly focused on the device-level implementation and limited their performance evaluations to a small scale network (one unknown node, several synchronized reference nodes). The deployment of reference nodes is random. In this paper, instead of improving the physical-layer implementation, we concentrate on designing an efficient system scheme. The proposed 3-tier (tag-hub-base station) localization scheme enables the use of low-complexity, low-cost and autonomously powered UWB tags. This system can be extended to a large-scale network without increasing the number of synchronized base stations. We present how we eliminate the ambiguous solution (coordinates and transmission time of an unknown node) with the help of the proper geometry of 4 reference nodes. A method for optimizing the number of hubs and the hub placement is described. Different positioning algorithms are discussed and compared based on the position accuracy a.f.o range errors. The sensitivity of the overall performance to timing errors has been investigated.

The remainder of the paper is organized as follows. Section II gives a description how the localization system is conceived and the motivation. Section III highlights three positioning algorithms considered. Section IV describes the method for optimizing the number and deployment of hubs. Section V presents the simulation results. Section VI concludes the paper.

II. The conceived system and motivation

The overall system must be conceived such that there is as little power consumption as possible in the devices (tags) attached to the objects with unknown position. The reason is that we aim for tags that are autonomously powered, which scavenge their energy from the environment, store it on a capacitor and perform all the operations in burst mode as soon as enough energy is available. The choice of the tag is the first step in establishing a localization system. If a tag has very little energy available, then it preferably has no receiver that continuously scanning the ether. Among several candidates

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(TX+RX+μp, TX+RX, TX, RX), the UWB transmitter-only solution consumes the least power [4] [6]. In addition to the lower power consumption, the very little hardware complexity of an UWB transmitter offers the potential for a low-cost and highly integrated solution. Therefore, the tag is chosen as an UWB transmitter only.

As a result, each tag i must have a direct range estimate to at least three reference nodes (2D) or four reference nodes (3D). For each reference node j, this range estimate d_{ij} can be used to relate the unknown coordinates (x_i,y_i,z_i) of the tag to the known coordinates (x_j,y_j,z_j) of the reference nodes using the standard distance formula:

$$d_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \tag{1}$$

The coordinates of the tag can then be derived by solving (1).

Among the most common range measurement techniques (time of arrival (TOA), received signal strength intensity (RSSI), angle of arrival (AOA)), the TOA estimation technique seems to be the most capable to exploit the high time resolution given by the large bandwidth of the UWB signal. It estimates the delay of the signal at the receiver site due to the propagation. And this delay defines the distance between the transmitter and the receiver. Many detection techniques have been proposed to obtain fast synchronization, which is very important for the delay-based ranging. Non-coherent receivers such as energy detection have been proposed for low-complexity and low-cost UWB TOA estimation solutions [12]. In addition, the precise detection of the direct line of sight (LOS) path in multi-paths environment is a key point for a more accurate ranging.

For cost reasons, an accurate time reference should be avoided in the tags. As a result, the tag transmission time is unknown. The unknown transmission time turns (1) into:

$$C^2(t_i - t_j)^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \tag{2}$$

where C is the speed of light, t_i is the tag transmission time. Adding t_i as unknown requires an extra receiving node to get a unique solution (x_i,y_i,z_i,t_i).

This 2-tier localization scheme requires many synchronized reference nodes (base stations) covering the whole monitored area to communicate with tags which only have a limited radio range. It is not trivial and too expensive to have a localization system like that. Therefore, a secondary set of nodes (hubs) are introduced to interface between the tags and a limited set of base stations. The system (see Fig. 1) consists of a large number of autonomously powered tags, a number of hubs (battery-powered) and few synchronized base stations (mains-powered). Tags transmit their signal (ID) at a random time (i.e. whenever there is enough energy available for the transmission). Hubs within the tags' radio range receive and forward the tag signals with the added hub information to base stations directly or in a multi-hop fashion. Techniques are used to minimize possible collisions of transmission. All information is then collected by a central controller, which solves the tag positions. 2-step calculations are conducted to localize an object attached to a tag. First, the hubs' position and transmission time t_{tx}, which refer to the absolute time reference are solved. Secondly, the tag position and its transmission time are derived.

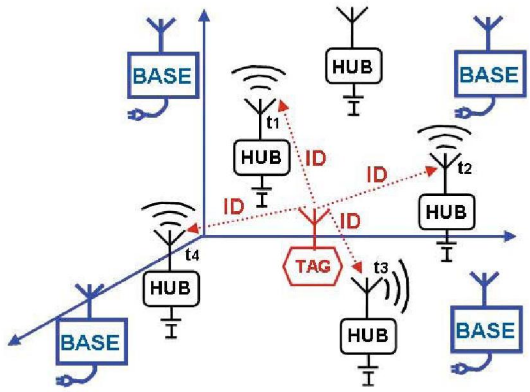

Fig. 1. A 3-tier wireless localization system with autonomously powered tags transmitting to hubs which transmits to base stations

Clock synchronization between hubs is not required. By sending the recorded hub transmission time (t'_{tx}) and receiving time (t'_{rx}), which may refer to different time references for different hubs, the clock difference can be calibrated:

$$t_{rx} = t_{tx} - t'_{tx} + t'_{rx} \tag{3}$$

where t_{tx} is the calculated hub transmission time via (2). By calculating that the arrival time of the tag signal at different hubs are calibrated to the same time reference. It works well if the hub's clock is stable during the period of receiving and transmitting the signal. Since this period is quite short, the drift of the hubs' clock will not be an issue in practice.

The advantages of this 3-tier approach are that: UWB tags can be made very simple and cheap; there is no need for a large number of synchronized expensive base stations and we use extra, cheaper hubs instead, thus this can be more practical due to cost constraint; the network can be extended easily to a large scale; the tags can be ultra low power, autonomous.

III. Positioning Algorithm

Position estimation involves solving (2) for (x_i, y_i, z_i, t_i).

A. Direct calculation

The unknown position and transmission time are directly calculated by solving (2), where $j = 1, 2, 3, 4$ [10]. Basically by subtracting the first equation from the other three, three linear equations can be derived:

$$\begin{aligned} &C^2(t_1 - t_j)(t_1 + t_j - 2t) = (x_1 - x_j) \\ &(x_1 + x_j - 2x) + (y_1 - y_j)(y_1 + y_j - 2y) + \\ &(z_1 - z_j)(z_1 + z_j - 2z) \qquad (j = 2, 3, 4) \end{aligned} \tag{4}$$

It is possible to express three unknowns in terms of the fourth. Then, substitution in one of the original quadratic equations will produce a quadratic equation in one variable. As is well-known, solving this gives two sets of solutions. [10] proposed to get rid of one set of solutions (x, y, z, t) with no physical

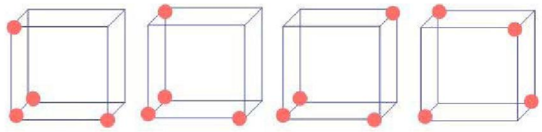

Fig. 2. 4 reference points are in any 4 (not in a plane) of the 8 vertices of a cube

meaning or beyond the monitored area. But based on our simulations, the chance of two possible physical solutions is quite high. By adding one extra reference point, a unique solution can be derived but at a higher system cost.

Fortunately, the problem can still be solved using four reference points with the help of proper geometry. We consider a cubic area ($5 \times 5 \times 5\ m^3$) as an example. Simulations are conducted with different placements of four hubs: randomly located in the cube, on the surface of the cube, in the vertices of the cube. The whole cube is scanned with 10 cm resolution. The percentages of having two meaningful solutions are 26.02%, 14.56%, 0 for the aforementioned three placements. The results show that the ambiguity can be eliminated by removing the solution with no physical meaning or beyond the cube area, when the four reference points are located in any four (not in a plane) of the eight vertices of a cube (see Fig. 2). Therefore, the cube can be regarded as a basic cell. The whole monitored area can then be subdivided into many of these basic cells (cubes). The body diagonal of the cube, i.e. the longest distance a tag signal may travel in the cube, is then limited by its radio range.

B. Linear least-squares

We assume there are n+1 reference nodes. In (2), where $j = 1, ..., n+1$, by subtracting the first equation from the rest, a linear system of n equations is obtained. This can be written as:

$$AX = b \tag{5}$$

where

$$A = 2\begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 & -C^2(t_1 - t_2) \\ \vdots & \vdots & \vdots & \vdots \\ x_1 - x_n & y_1 - y_n & z_1 - z_n & -C^2(t_1 - t_n) \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

$$b = \begin{bmatrix} -C^2(t_1^2 - t_2^2) + x_1^2 - x_2^2 + y_1^2 - y_2^2 + z_1^2 - z_2^2 \\ \vdots \\ -C^2(t_1^2 - t_n^2) + x_1^2 - x_n^2 + y_1^2 - y_n^2 + z_1^2 - z_n^2 \end{bmatrix}$$

This method can only be used if the number of reference nodes is larger than four. The X can be determined as:

$$X = (A^T A)^{-1} A^T b \tag{6}$$

The singular value decomposition (SVD) of A is:

$$A = U \Sigma V^T \tag{7}$$

The SVD may be used to find a minimum norm solution to a (possibly) rank-deficient linear least squares problem. The effective rank, k, of A can be determined as the number of singular values which exceed a suitable threshold. The solution is given by:

$$X = V \Sigma^{-1} U^T b \tag{8}$$

C. Nonlinear optimization

The position estimation can be achieved by solving a non-linear optimization problem. The objective function is defined as:

$$f(x,y,z,t) = min \sum_{j=1}^{n}(((x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2)^{\frac{1}{2}} - C(t_j - t))^2 \tag{9}$$

where (x, y, z) are coordinates to be estimated, t is the unknown transmission time, (x_j,y_j,z_j) are the coordinates of the reference nodes, t_j are the known receiving time of the jth reference node. Clearly, the objective function is the summation of the squared range errors. The purpose is to find a solution (x, y, z, t) that minimizes this objective function. Many algorithms are proposed to solve this problem. We use Nelder-Mead simplex search method of [13]. This is a direct search method that does not use numerical or analytic gradients. If n is the length of x, a simplex in n-dimensional space is characterized by the n+1 distinct vectors that are its vertices. In two dimensional space, a simplex is a triangle; in three dimensional space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance. Matlab provides a function based on this algorithm. We modified this function such that the lower and upper bound of the variables can be set which results in a faster convergence and more accurate position estimation. The initial estimation values of the position coordinates are chosen to be the mean position of the reference nodes. The initial estimated transmission time is chosen to be some time earlier than the earliest receiving time [10]. This method requires at least five reference nodes. Its iterative nature requires large computational resources.

These three methods will be compared based on the position accuracy a.f.o TOA errors in Section V.

IV. The optimal number and placement of hubs

The number of hubs should be as small as possible for reasons of cost. At least four hubs (in four vertices of a cube) are required to solve the position of a tag inside the cube as discussed in Section III. The total monitored area is subdivided into several cubes. Each cube must have at least four hubs in its vertices to make sure that tags anywhere in the room can be seen by at least four hubs. For a given room size the required number of hubs in the monitored area has a minimal number for a certain radio range of the tag. Four times the number of cubes is much more than the minimal. We implemented

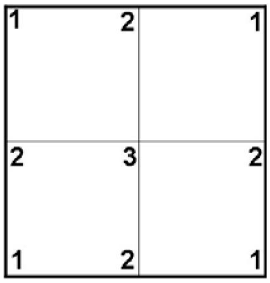

Fig. 3. Weight of points in 2D example

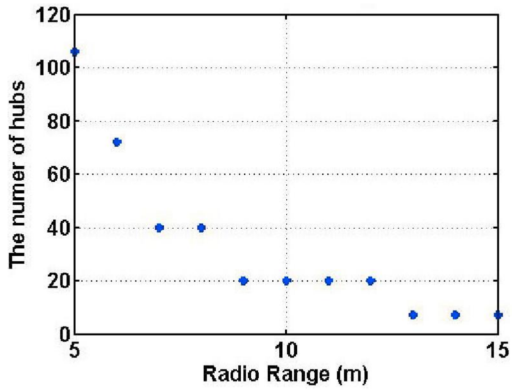

Fig. 4. Minimum required number of hubs versus tag radio range for a $15 \times 15 \times 10\ m^3$ room

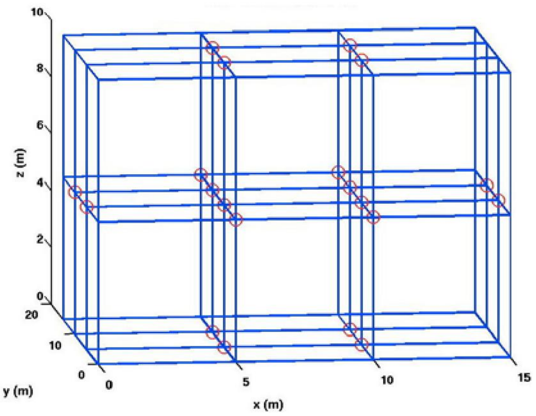

Fig. 5. Minimum number of hubs in a $15 \times 15 \times 10\ m^3$ room

a method to calculate the minimum number of hubs. The purpose is to let the hubs be shared by as many cubes as possible. As illustrated in 2D in Fig. 3, there are three types of points. First we assign weights to the points. Points shared by four squares have weight 3, points shared by two squares have weight 2, and points only in one square have weight 1. The second step is to allocate hubs to all the squares, making sure that there are at least three hubs in each square. We start from the first square, locate the point with the higher weight first. In case of the same weight, we locate the point farther away from the origin first. By doing so for all the squares, the hubs are shared maximally. As a result, we get the minimal number of hubs.

The minimum number of hubs is a function of the room volume, the tag radio range and the geometry of the room. Fig. 4 shows the relationship between the minimum number of hubs and the tag radio range in a $15 \times 15 \times 10\ m^3$ room. Indeed the larger the tag radio range, the less hubs are needed. However, according to the Friis transmission equation, the distance that a tag signal can travel is limited by the power of the tag signal, the hub sensitivity and the antenna gain. In addition, the transmitted UWB pulses from the tag must meet the FCC Spectrum Mask. The low-date-rate system can operate over significantly longer ranges indoors, as compared with a system designed for high-data-rate communications, that is because it can utilize significantly higher peak powers than those allowable for high-data-rate system. Around the target radio range value of 10 m, a reasonable number of hubs in our example (20) is required. For a radio range of 15 m, this drops to 8.

Fig. 5 shows the placement of hubs in 3D. The size of the basic cell is $5 \times 5 \times 5\ m^3$ (i.e. the tag radio range is $5\sqrt{3}$ m). Although we get the optimal number of hubs, the position of the hubs is not optimal: hubs are more concentrated in the center. That may cause more conflicts of signals in the central area, which will result in a longer detection time of the tags and even a non-working system when collisions happen all the time. Secondly, that may cause unbalanced position accuracy.

To improve the hub location without adding more hubs, we have implemented an iterative method to maximize the spread of the hubs across the room as to reduce the number of collisions. The variance of the number of hubs in a cube is a measure of how spreaded the hubs are:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2 = \overline{x^2} - (\overline{x})^2 \qquad (10)$$

where N is the number of cubes in a room, x_i is the number of hubs in the cube i, $\overline{x}$ is the mean of x_i. The goal is to minimize σ^2. We find the cube with the maximum number of hubs, then try to move a hub around in the cube if possible to reduce σ^2. We do this iteratively until no points can be moved or σ^2 is 0. The scattered hubs after applying this method to the example of Fig. 5 are shown in Fig. 6.

The consequence of having the minimum number of hubs is the necessity of locating the hubs in some specific positions for avoiding ambiguity in the solutions. In a realistic environment it is often difficult to have complete freedom in the determination of the hubs locations. The sensitivity of the overall performance to nonideal hub placement has been investigated in [14].

V. Simulation Results

The positioning algorithms described in Section III use the arrival time of UWB pulses to measure distance. Therefore, the TOA measurement accuracy will influence the position

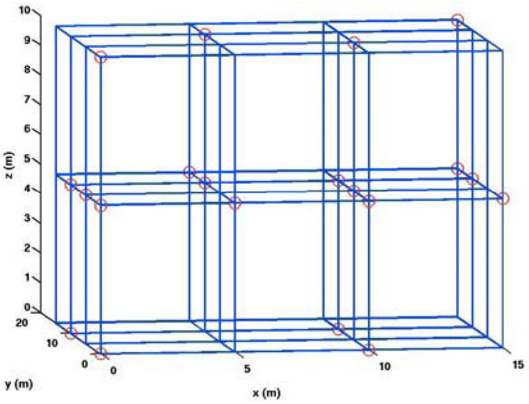

Fig. 6. Scattered hubs in a $15 \times 15 \times 10\ m^3$ room

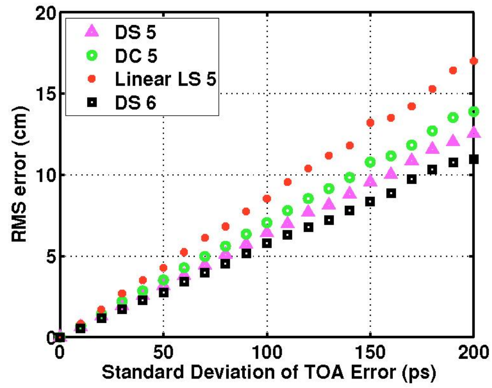

Fig. 7. Root mean squared error of a tag position estimation in a $10 \times 10 \times 5\ m^3$ room. DS 5 and DS 6 denote using the simplex direct search method with 5 and 6 receiving hubs respectively. DC 5 and Linear ls 5 are the direct calculation and the linear least-squares method with 5 receiving hubs.

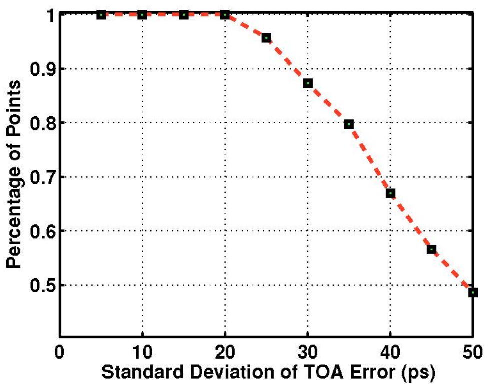

Fig. 8. Percentage of points that satisfy 10 cm position accuracy in a $5 \times 5 \times 5\ m^3$ cube

accuracy. In our 2-step calculations, the hub locations are static and determined once for any given configuration. Averaging over time can be used to improve the accuracy of the hub location. In addition, the base stations are equipped with very accurate synchronized clocks. As a starting point, we assume the first-step (hub to base station) TOA measurements are accurate. We can solve accurately the hub transmission time and position. The performance of our localization system is firstly evaluated only considering that the second-step TOA measurements (tag to hub) can be erroneous due to timing errors. Whereas another error due to the TOA estimator itself has been extensively addressed in the literature. The monitored area has a dimension of $10 \times 10 \times 5\ m^3$. Simulations are conducted to compare the different algorithms discussed in Section III. Assume the erroneous TOA at the jth hub is $t_j + \Delta t_j$, where the Δt_j is a random variable of the normal distribution ($N(0, \sigma^2)$) with zero mean and variance σ^2 (e.g. jitter). In Monte-Carlo simulations, 1000 random samples of $N(0, \sigma^2)$ are generated for a tag position. At each test σ, the root mean square error (RMS) of the coordinates estimation has been calculated for the different algorithms. The results are depicted in Fig. 7. We conclude that if the tag can be seen by 5 hubs, the simplex direct search method provides a higher position accuracy, and the position accuracy can be improved by even more receiving hubs.

Consider now a tag somewhere in the room (Fig. 6) transmitting a signal. The number of hubs receiving this tag signal can be 4,5,6... depending on the position of the tag. The chance of having redundant information (more than 4 receiving hubs) to solve a tag position is quite high, which is good for position accuracy. According to the simulation results depicted in Fig. 7, the positioning algorithms applied to different scenarios in our localization system are summarized:

- the tag is seen by 4 hubs: direct calculation of 4 quadratic equations is the best method
- the tag is seen by >4 hubs: a simplex direct search is the best method.

The geometrical constellation of hubs and tags influences the accuracy of the position estimation as well. Although [15] shows that the tetrahedron constellation (four vertices of a cube) is a good choice for installing hubs for accurate, three-dimensional localization, range measurement errors are still not equally translated to position errors at all locations of the area. At each test σ, the position accuracy for 216 tags uniformly located in the $5 \times 5 \times 5\ m^3$ cubic area has been evaluated. 1000 random samples of $N(0, \sigma^2)$ are generated for each tag position. Fig. 8 depicts the percentage of points that satisfy ($\geq$99.7%) 10 cm position accuracy versus the standard deviation of TOA error. It clearly shows that the standard deviation of TOA error ($\leq$20 ps) is a guarantee that the whole cubic area satisfies 10 cm position accuracy. This value will in turn determine the design specifications, i.e. the hub's clock jitter.

To derive the overall performance, we assess RMS errors of 100 tags in the monitored $10 \times 10 \times 5\ m^3$ area (shown in Fig. 9). At each test σ, 1000 runs are conducted with new random positions of 100 tags. The RMS errors are averaged to give an average performance. Sub-meter accuracy (e.g. 10cm) can be achieved if the standard deviation of TOA error is less than 170 ps. This value is more relaxed due to the redundant

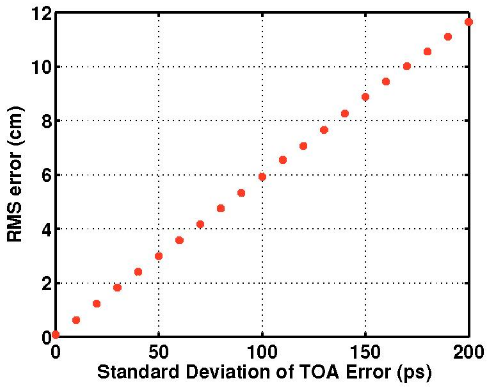

Fig. 9. Root mean squared error of 100 tags' position estimation in a $10 \times 10 \times 5\ m^3$ room

information and RMS averaging nature.

VI. Conclusion

In this paper we presented a 3-tier ultra wide-band indoor localization system, which consists of tags, hubs and base stations. This scheme enables the use of low-cost, small-size, ultra-low-power tags, which are attached to the objects with unknown position. In addition, we don't need a large number of synchronized expensive base stations but extra cheaper hubs instead, which makes the system more practical due to cost. We have explained that at least four reference nodes are required to solve the coordinates of an unknown point and its transmission time. A method to determine the optimal number and deployment of hubs has been proposed. Different positioning algorithms have been discussed and compared. We propose using different algorithms for different scenarios to give the overall best position accuracy. Simulations are conducted to investigate the performance degradation due to timing errors. The results from our system design will guide the hardware implementation of our UWB localization system.

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