Collective Behaviors of Braitenberg Vehicles

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Abstract. Large collectives of artificial agents are quickly becoming a reality at the micro-scale for healthcare and biological research, and at the macro-scale for personal care, transportation, and environmental monitoring. However, the design space of reactive collectives and the resulting emergent behaviors are not well understood, especially with respect to different sensing models. Our work presents a well-defined model and simulation for study of such collectives, extending the Braitenberg Vehicle model to multi-agent systems with on-board stimulus. We define omnidirectional and directional sensing and stimulus models, and examine the impact of the modelling choices. We characterize the resulting behaviors with respect to spatial and kinetic energy metrics over the collective, and identify several behaviors that are robust to changes in the sensor model and other parameters. Finally, we provide a demonstration of how this approach can be used for control of a swarm using a single controllable agent and global mode switching.

1 Introduction

Valentino Braitenberg introduced his vehicles [2] as a thought experiment in *synthetic psychology*: a light-hearted demonstration of the human tendency to see order and intelligence in the collective behaviors of simple reactive agents. Sensors are "wired" directly to actuators, so the agents do not use on-board stateful representations or memory (Fig. 1(a)). What we term *intelligent behaviors*, such as aggregation, dispersal, and flocking, arise from structured interactions between agents and environmental stimuli. In this work, we introduce a simulated model of simple Braitenberg Vehicles (BVs) in a collective. Most demonstrations of BVs implement off-board stimuli sources; here, we extend the model to focus on the case where stimuli sources are on-board the agents, such that the environment of interest is the collective itself and its dynamics. We describe two stimuli models, analyze the emerging behaviors for robustness, and cast the behaviors in light of potential robotics applications.

From social organisms such as cellular slime mold, we know that collective intelligence is possible without highly complex nervous systems. At the micrometer-scale, researchers are developing engineered collectives for drug delivery and other applications. Robotics techniques at this scale are rapidly becoming a reality, with memristor-based circuits capable of sensing chemical and light signals [3] and logging and accessing physical memory of temporal events [20]. However, the agents are simply too small to use traditional onboard state-estimation and control algorithms, requiring careful design of primarily reactive strategies, uniform off-board controls for the entire collective [16], or a combination of the two

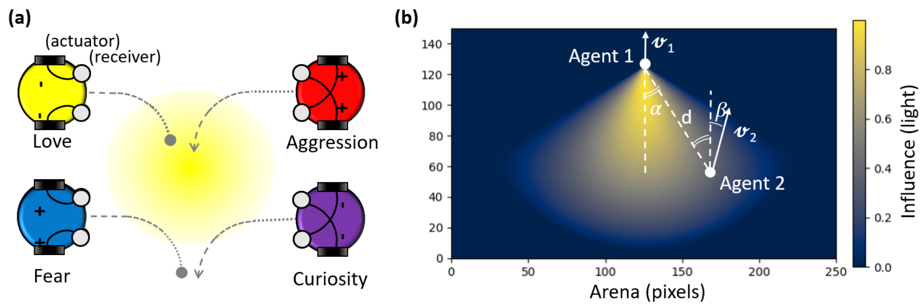

Fig. 1. (a) Overview of how the original BV react to environmental stimulus. (b) Overview of collective BVs used in this article.

approaches [7]. At the human scale, as robotic collectives begin assisting us with personal care, transportation, and environmental monitoring, the design of *reactive* collectives is appealing; the approach is more scalable than a centrally-controlled swarm, and could even help side-step some privacy and surveillance concerns. For example, a system that does not require cameras or global positioning will intrinsically provide a stronger guarantee of privacy, and can increase people's comfort around automated agents [6, 15]. The two-dimensional BV model provides a clear starting point for the principled design of reactive collectives from an embodied perspective, but to our knowledge, collectives of Braitenberg vehicles with onboard stimuli production have not been a first-class object of study in the robotics, control or autonomous systems literature.

Our contributions include a more formal definition of the BV model, an accessible Python implementation for simulating and characterizing the system, and an interpretation of the results and next steps toward application. We extend the BV model to include on-board stimuli and define two stimuli models, as would be found on multi-agent robotic systems: a symmetric *omni-directional* stimulus such as that enabled by radio frequency localization [11], and a *directional* stimulus modeled off a generic LED's spatial intensity. We also define two sensor models, an omnidirectional sensor model and a sensor model with an angular cutoff, such that agents cannot observe stimuli sources past a certain angle in their field of view. In this paper, we examine three combinations of these models: omnidirectional sensing and stimulus; directional sensing and omnidirectional stimulus; and directional sensing and directional stimulus. We characterize emergent behaviors with respect to metrics over the spatial distribution and kinetic energy of the collective. We characterize the behaviors of the resulting homogeneous collectives while varying three main design parameters:

- *Influence magnitude:* scale of the reaction control law that updates heading and velocity
- *Influence angle:* bounds on the relative angle between agents that allows for detection
- *Actuation noise:* We apply Gaussian noise to heading and velocity when each agent actuates, to simulate actuation noise or environment effects

1.1 Related Work

The premise that collectives of simple, reactive agents exhibit robust "behaviors" has been extensively validated in simulated and empirical work, from boid models of swarms [13] and particle-based optimization algorithms [5], to consensus in biological collectives [14]. However, the effort to *design* artificial multi-agent systems meeting certain standards of robustness and efficiency is still very much underway, with existing artificial systems falling far short of the performance seen in biological collectives [4]. Centralized methods are often more robust, especially in the face of robot-environment interactions, but decentralized methods are more scalable. In this work we focus on the BV model as it is a decentralized approach that unifies sensing and actuation, and serves as a good starting point for well-defined embodied design of reactive collectives.

The BV model has not often been a direct object of study by engineers, apart from use as a demonstration in education [9] and graphics [19]. However, due to the simplicity of the model, similar piecemeal insights can be found in the minimalist multi-robot systems literature. In [8] the authors perform a parameter sweep over the control space to find an optimal controller for aggregation, with a final strategy reminiscent of a discrete version of the BV *love* behavior. Researchers in control theory and applied mathematics have also shown formal stability proofs for emergent collective motion; for example, the Vicsek model of constant speed agents in the plane with a nearest-neighbor alignment feedback rule is a stable switched linear system [10, 18]; the Dubin's car model has been shown to reliably rendezvous given a simple reactive steering control law [21]. So while we have strong mathematical intuition that simple swarms can perform robust collective behaviors, there is a lack of understanding of the behavior space under different sensor models and sensor-actuator couplings - the BV model provides a starting point to unify these insights and advance the state-of-the-art in design of autonomous collectives.

Here, we present a thorough characterization of the BV model as a first step toward unification of largely qualitative work on BVs with other lines of work on formal system design using data structures in information space for single-robot planning and design problems [12], as well as work on co-design that aims to incorporate dynamic effects [22]. BVs have recently been used in developmental neurosimulation [1] and structured Bayesian techniques [17] as a useful testbed for embodied artificial intelligence techniques with a representation of sensors, actuators, and an artificial nervous "system," however these approaches generally lack a formal model or an accessible implementation. Our work fills this gap by implementing an open-source BV model with purely reactive behavior and several performance characterization metrics, allowing for comparison between approaches to reactive embodied collective design.

2 Model and Simulation

First, we detail the modelling choices made in the adaptation of the BV to our simulated implementation. We have emphasized directional sensing and stimulus production, motivated by practical constraints in robotic systems. Our interactive, Python-based implementation is available at github.com/CEI-lab/DARS2022-BVcollectives.

2.1 Agent Model

Each agent i has a position on the plane, x_i, and a linear velocity vector, v_i, describing the orientation of the agent and its speed. Agents have minimum and maximum speeds $[V_{min} = 0; V_{max}]$. Agents are considered to be points in the plane; we leave most considerations of morphology and collisions to future work, and assume that each agent is small compared to the workspace. Time progresses in discrete stages, where agents update their heading and speed at each time step as a memoryless, reactive function of their observations in the previous time step. We omit the time variable in our notation where operations occur within a single time step.

Inspired by Braitenberg's two forward-facing sensors, we define a two-part "windshield" sensor model. Each agent can "see" other agents within a circular sector, centered at x_i and symmetric across v_i, with a maximum sensing radius of d_{max}. Each agent then has two high level sensors, each of which reports the *total* intensity of stimuli observed in that half of the field of view. We let $y_{i,r}$ and $y_{i,l}$ denote the right-hand and left-hand sensor readings for agent i. In the omnidirectional sensing case, the field of view is the entire left and right hemispheres of the agents. In the directional sensing case, the field of view is limited by the angle constraint θ_{max} such that if the angle between the heading of the robot v_i and the source of stimulus is greater than θ_{max}, the stimulus is not visible.

We implement two different stimuli models, an *omnidirectional* source and a *directed* source. The omnidirectional source has no angular dependence, and the stimulus intensity is defined as

$$I_o(d) = I_{max}\left(1 - (\frac{d}{d_{max}})^2\right), \tag{1}$$

where I_{max} is the maximum intensity stimulus as would be observed if agents were immediately adjacent, and d_{max} can be thought of as the distance cutoff beyond which the stimulus is undetectable. The directional source is modelled off a generic LED intensity profile, and uses a similar scaling law in the angular direction, such that

$$I_d(d,\theta) = I_{max}\left(1 - (\frac{d}{d_{max}})^2\right)\left(1 - (\frac{\theta}{\theta_{max}})^2\right). \tag{2}$$

Finally, we define how the maximal visibility angle θ_{max} is implemented in the case with directional stimulus and directional sensing. As in Figure 1, without loss of generality, take two agents with headings v_1 and v_2, and let agent 2 be observing agent 1. Let α be the angular coordinate of Agent 2 within Agent 1's reference frame, where directed stimulus intensity at Agent 2 is maximized when $\alpha = 0$. Let β be the internal angle between the two headings, such that the agents are aligned when $\beta = 0$. Agent 1 is *visible* to Agent 2 if the angular constraint $\alpha + \beta \leq \theta_{max}$ is satisfied, creating a linear dependence of the sensor on the angle-to-stimuli. For example, if Agent 2 is at the edge of the stimulus field ($\alpha = \theta_{max}$), it can only observe Agent 1 if the agents are perfectly aligned, and the observed intensity from Agent 1 is given by Eq. 2. If Agent 2 is in the center of the stimulus field ($\alpha = 0$), it can observe Agent 1 as long as θ_1 is within the range $\theta_2 \pm \theta_{max}$. This implementation simulates a scenario where as the stimulus signal becomes weaker, more exact alignment is required for successful sensing (a common limitation of photodiodes and radio frequency receivers). Of course these variables could be separated in future work to study the effects of sensing and actuating ranges separately, but here it is an efficient means to study a range of directional system designs using a single variable. See Fig. 1(B) for an illustration of the directional stimulus profile for $\theta_{max} = \pi/3$.

Next, we define reactive, proportional control laws according to Braitenberg's schema, where we vary *kinesis* (k) to move faster or slower in response to stimuli, and vary *taxis* (t) to turn toward or away from stimuli. For agent i, we control the speed v_i and the orientation θ_i according to

$$\dot{v_i} = k(y_{i,l}, y_{i,r})$$
$$\dot{\theta_i} = t(y_{i,l}, y_{i,r})$$

where $y_{i,l}$ and $y_{i,r}$ are the left and right sensor response magnitudes, respectively. For readability, we define $\ell_i = y_{i,l} + y_{i,r}$ for the total stimulus observed by an agent, and $\delta_i = y_{i,l} - y_{i,r}$ for the difference between the two sensing regimes, with a positive δ_i indicating more stimulus on the left hand side of the agent than the right.

The basic reactive "behaviors" that we consider here are love, fear, aggression, and curiosity, and each correspond to different simple control law implementations for k and t. For example, in the love behavior, agents turn toward stimuli and slow down as intensity increases, leading to reliable aggregations. The discretized control laws for love are then

$$v_i(t+1) = v_i(t) - \gamma\ell_i(t) \quad (3)$$
$$\theta_i(t+1) = \theta_i(t) + \gamma\delta_i(t) \quad (4)$$

where γ is a scaling factor and design parameter we call the *influence*, governing how strongly the agents actuate in each time step. In our discrete model and simulation, it combines the design parameters of gain in the control law and the discretization of time.

Likewise, we define fear as the emergent behavior when the agents turn away from stimuli, and slow down in response to stimuli; agents "fear" the stimuli and wish to avoid it. Aggression emerges when agents turn toward stimuli and speed up ("attacking"), and curiosity emerges when agents turn away from stimuli and speed up (on their way to find the next stimulus signal). The control laws are summarized in Table 1.

We implement two independent sources of noise, on the actuation direction and speed. During each timestep, the agent updates its nominal heading and speed according to the rules in Table 1. Then an additive Gaussian noise term is applied both to the nominal speed and nominal heading. The default noise has a mean of zero for both heading and speed, and a standard deviation of $\sigma_\theta = \frac{\pi}{72} = 5°$ for the heading and $\sigma_v = \frac{v_{max}}{72}$ for speed. In later sections, we increase these standard deviations to study system response to actuation noise.

2.2 World

We define a two-dimensional 500x500 unit environment. We give the agents a maximum velocity of 4 units per second. To study collective behavior without

Response to Increasing Stimulus	Negative Kinesis Slow Down	Positive Kinesis Speed Up
Positive Taxis Turn Toward	**Love** $v_i(t+1) = v_i(t) - \gamma\ell_i(t)$ $\theta_i(t+1) = v_i(t) + \gamma\delta_i(t)$	**Aggression** $v_i(t+1) = v_i(t) + \gamma\ell_i(t)$ $\theta_i(t+1) = v_i(t) + \gamma\delta_i(t)$
Negative Taxis Turn Away	**Fear** $v_i(t+1) = v_i(t) - \gamma\ell_i(t)$ $\theta_i(t+1) = v_i(t) - \gamma\delta_i(t)$	**Curiosity** $v_i(t+1) = v_i(t) + \gamma\ell_i(t)$ $\theta_i(t+1) = v_i(t) - \gamma\delta_i(t)$

Table 1. The reaction rules for each of the four regimes of interest.

the influence of boundary conditions, we identify the edges $[0, y] \sim [500, y]$ and $[x, 0] \sim [x, 500]$, creating an impossible torus environment. Agents are able to sense across these identified edges. We include the possibility for obstacles to be present, with a default behavior of an elastic collision when an agent crosses an obstacle boundary. However, the computation of occlusions and shading are beyond the scope of this work, so agents are able to detect each other through obstacles if the angle and distance constraints are met.

2.3 Metrics

We define two metrics for evaluating the emergent behavior in terms of spatial- and velocity distribution.
The *distance* metric is defined as the population average of the average distance from one agent to all other agents, or

$$d_{pop} = \frac{1}{N}\sum_{i=0}^{N}\frac{1}{N}\sum_{j=0}^{N} d(a_i, a_j) \tag{5}$$

where $d(a_i, a_j)$ is the minimum Euclidean distance along the surface of the torus world between agents i and j. For context, if 20 agents are uniformly distributed on a 500x500 unit environment, we would expect this average distance to be ~250 units.
We also compute the population average for a *kinetic energy* metric, such that

$$KE_{pop} = \frac{1}{N}\sum_{i=0}^{N} v_i^2 \tag{6}$$

normalizing the mass of each agent to 1.

3 Emergent Behaviors

For the models with directional sensing, we characterize each of the four emergent behaviors, with respect to the distance and energy metrics, by performing parameter sweeps across the influence angle θ_{max} and the influence magnitude γ. For the influence angle, we vary $\theta_{max} \in [\pi/8, \pi/2]$, and we vary the influence scale in $[0.2, 1]$. We found that a nonlinear influence scale more clearly shows trends, so we plot data for $\gamma = [1/x \text{ for } x \text{ in } [1., 1.5, \ldots, 5.5]]$. For each pair of angle/influence variables, we simulated 20 randomly initialized collectives of 20 agents, collecting the metrics at each timestep after a convergence period $T_{conv} = 490$ for a duration of $T_{record} = 10$ timesteps, for a total simulation runtime of 500 timesteps. We then average each metric over time during the converged behavior, and report the value on the heat maps in Figures 2, 3, 4, and 5. We also show a representative time-lapse of agent positions for the two directional models over the last 100 frames of the simulation. Finally, we show the system response to noise for all three models, the two directional models and the model with omnidirectional sensing and stimuli. For the noise analysis for each mode, we found the mean and standard deviation over 20 iterations with $T_{conv} = 490$ and $T_{record} = 10$, the same as the heat maps. Because the omnidirectional model is overpowered compared to the directional models, if we use the same d_{max} for all three models, the omnidirectional model agents tend to simply spin in place in all modes. To more clearly show the different responses to noise, we set $d_{max} = 100$ for omnidirectional model ($d_{max} = 200$ for the directed models always). We leave further investigation of the effect of sensing radius to future work.

3.1 Love

Loving agents turn toward each other and slow down. In the robotics space, this behavior is useful if we wish to aggregate agents in a low-energy state, perhaps to await a new task or to exchange information between each other with a static network.

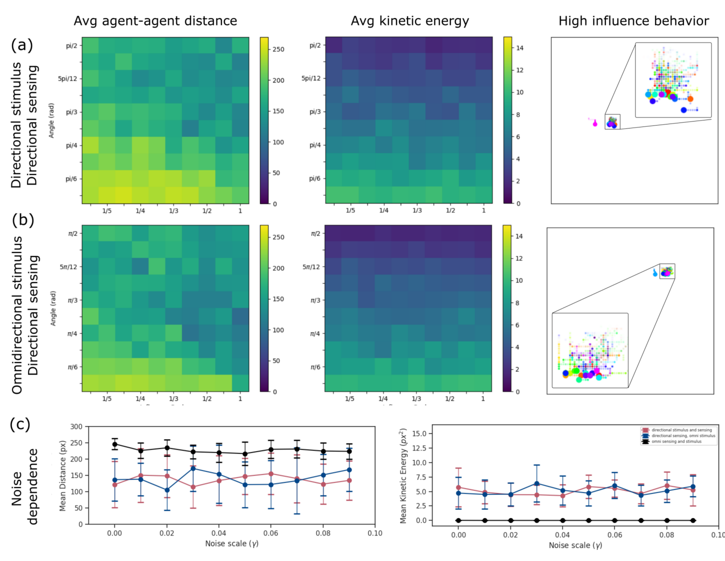

Fig. 2. Love after convergence. (a)-(b) Behaviors in the design space for the two directional models. (c) The response of each sensor-stimuli model of average agent-agent distance and kinetic energy in response to increasing noise.

As seen in the inter-agent distance plots in Fig. 2(a) and (b), clusters generally become more tightly packed when angle and influence increase, as expected, however the visibility angle appears to have a stronger effect. If the field of view is too narrow, the agents may never reach an aggregated state, as seen by the nonzero kinetic energy at lower angles.

3.2 Aggression

Aggressive agents turn toward stimuli and speed up. This behavior is useful for clustering and aggregation in spaces where agents wish to keep a high momentum for quick dispersion after aggregation, or when agents are desired to aggregate very quickly. We've found that this strategy leads to robust clustering of agents: as the agents reach full speed near an existing cluster, they overshoot and immediately turn back toward the cluster. These clusters appear to be dynamically stable, as seen in the time-lapse plot in Fig. 3 in the regions with low agent-agent distance but high kinetic energy. The angle and influence parameters appear equally important in this case, indicating that a large influence parameter can overcome some of the limitation of directed stimulus. In the directed light or sensing cases, if the influence is high enough and noise is very low, the system falls into an aligning state where the agents follow each other in a line around the environment. This scenario is useful if you have one or a few controllable agents and wish to have the rest of the agents follow in a given direction. Interestingly, as seen in Fig. 3(c), the agents with omnidirectional sensing and stimulus are the worst at forming tight aggregations. Since all their observations are binned into the left and right hemispheres, they are less sensitive to the locations of other agents and may turn past nearby agents, or reach an equilibrium state where they see the same amount of light in all directions while all moving, essentially chasing each other around the torus.

3.3 Fear

Fearful agents turn away from stimuli and slow down, and the resulting emergent behavior is a space-filling formation. This behavior lends itself well to applications that need automatic organization of a collective into a mostly static network of agents monitoring the environment or passing messages.

In the torus environment, the agents all eventually align and the entire collective slowly drifts in the same direction around the environment. Essentially, they are all running away from each other at a uniform speed, unable to speed up or turn without increasing their locally apparent stimulus. The same behavior may be engineered to create flows in environments where the topology allows, or avoided by careful environment design or other measures to stabilize the collective if so desired.

We see from Figures 4(a) and (c) that this behavior is more robust to changes in the angular and influence variables. When comparing directed and omnidirectional stimulus models, the omnidirectional stimulus increases the distancing from other agents, as we would expect as perceived stimulus from neighbors is stronger. The only way to decrease performance is to have a prohibitively narrow field of view. Interestingly, the influence variable has less impact than the angle. From the control laws, we see that when the agents slow down upon encountering more stimulus the system stabilizes to zero or a constant uniform velocity at a rate depending on the influence, but shrinking influence cannot destabilize the system.

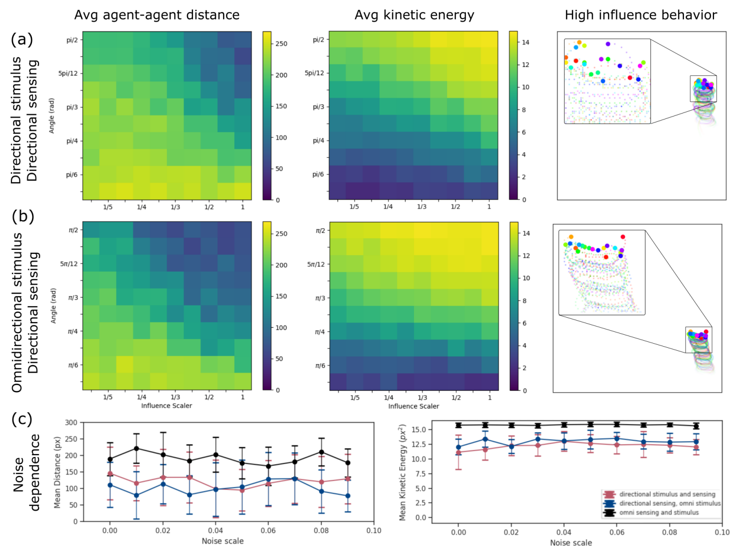

Fig. 3. Aggression after convergence. (a)-(b) Behaviors in the design space for the two directional models. (c) The response of each sensor-stimuli model of average agent-agent distance and kinetic energy in response to increasing noise.

3.4 Curiosity

Finally, in the curiosity mode, agents turn away from stimulus and speed up. The agents disperse, similarly to fear, but maintain high kinetic energy. This would be applicable to an engineered collective that needed a high mixing rate between agents, or where slowing down destabilizes the agents. In Fig. 5, we see extremely robust space-filling behavior, even with the most restrictive, physically realistic directed sensing and stimuli. There is a different mode that emerges for low influence and angle, where agents do not perceive enough stimuli from other agents and eventually slow down and stop. However, even in this case, given uniformly distributed initial configurations, they maintain a nearly uniform distribution in the environment. The effect of initial conditions, and conditions necessary for successful switching between modes, is an exciting future avenue of inquiry.

4 Conclusion and Outlook

We have presented a novel model of Braitenberg Vehicles with onboard stimulus production, and characterized the behavior of the system with respect to collective spatial distribution and kinetic energy, under three sensing and stimulus models inspired by robotics applications. We identified several unexpected modes, such as a low kinetic energy mode for curiosity. We also analyzed for response to noise, and identified interesting modes in the low-noise regime, such as the follow-the-leader behavior in aggression. It is also interesting to note that noise aids convergence for certain behaviors, such as aggression, where if agents are too far from a

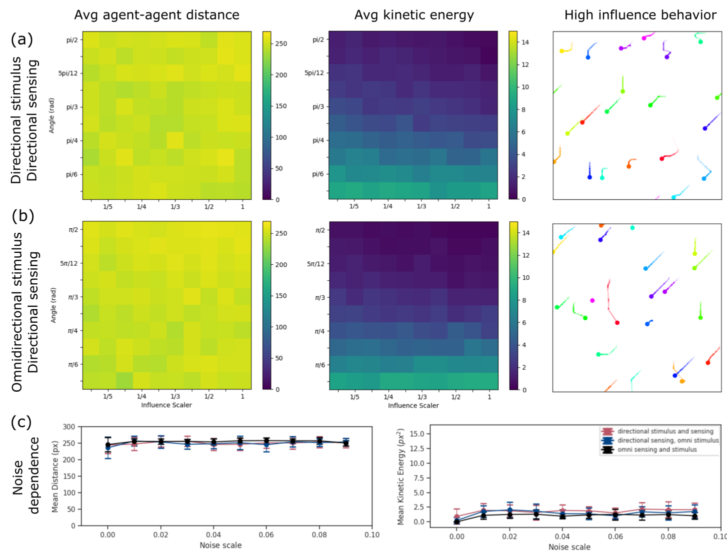

Fig. 4. Fear after convergence. (a)-(b) Behaviors in the design space for the two directional models. (c) The response of each sensor-stimuli model of average agent-agent distance and kinetic energy in response to increasing noise.

cluster they may become stationary and "lost." We identified that when the agents can only observe stimulus originating within a limited angle range, the emergent behavior is different from the omnidirectional sensing case. However, adding an LED-inspired directional stimulus model does not appear to change behavior qualitatively, and only slightly shrinks the region of angle and influence that leads to the expected behavior in that mode if it has an effect, indicating that with respect to sensor and stimulus models, mutual visibility is the main factor driving the type of emergent behavior. Other interesting design parameters that we have not considered here include agent density in the space, environment geometries and strategies for agent-environment interactions, heterogenous collectives, and the influence of controllable or unreliable agents. In the supplemental video, available at `https://youtu.be/17GNopGu4EE`, we show a demonstration of how a single controllable agent can be used along with global mode-switching to guide a swarm through narrow passageways and explore a multi-room environment.

Many of these behaviors that we have shown quantitatively in our simulation or qualitatively through observation are likely amenable to dynamical system or control proofs. We also aim to use this platform for comparison of more stateful on-board computation to the baseline reactive behaviors, as well as extend the model to include a fully embodied representation of the agents that allow for agent-agent collisions and more physical agent-environment interactions. The advantage of this approach is allowing for scalable and minimal methods for collective control, enabling the collective to switch between several useful modes using only a few global parameters.

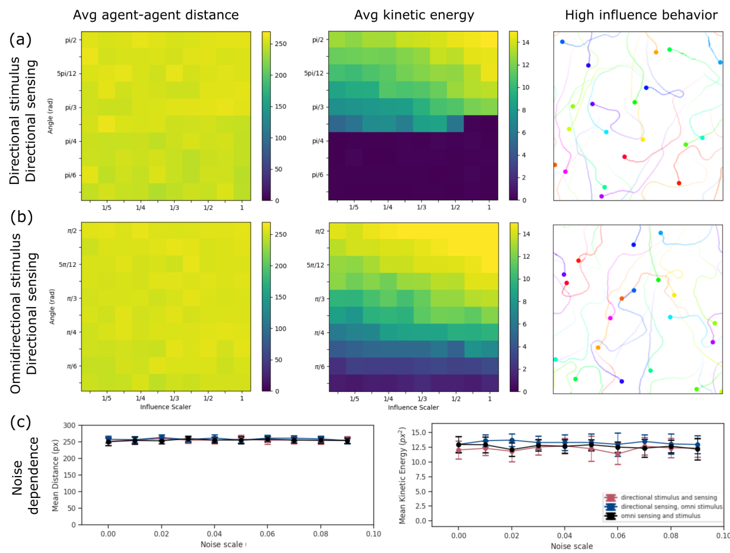

Fig. 5. Curiosity after convergence. (a)-(b) Behaviors in the design space for the two directional models. (c) The response of each sensor-stimuli model of average agent-agent distance and kinetic energy in response to increasing noise.

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