Math Formulation

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1 State Space

We define the state vector for a multi-agent system with n robots. We can define a substate $\hat{x}_n(t)$ that stores the substate for each individual robot. The system state will be the collection of these substates.

$$x_n(t) = \begin{bmatrix} x_n(t) & y_n(t) & \theta_n(t) & b_n(t) & (V_C)_n(t) & (V_p)_n(t) \end{bmatrix} \tag{1}$$

This substate of robot n has these six real number values: $x_n(t)$ for the x position, $y_n(t)$ for the y position, $\theta_n(t)$ for the heading, $b_n(t)$ for the button sensor state (0 or 1), $(V_C)_n(t)$ for the capacitor voltage, and $(V_p)_n(t)$ for the photoresistor voltage. These are continuous substates. So we form the system state as the collection of n substates.

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{2}$$

This state has a size of $x(t) \in \mathbf{X}, |\mathbf{X}| = \mathbb{R}^{6n}$ for the $6n$ real numbers represented by our state.

2 Action Space

The action space is the movements for all the robots. We have no system dynamics for our simple map and only need to define the control function $u(t)$. We also include a Gaussian noise term $w(t)$ associated with the process noise. This is drawn from a Gaussian with covariance Q. $w(t) \sim \mathcal{N}(0, Q)$

$$x(t+1) = x(t) + Bu(t) + w(t) \tag{3}$$

3 Output (Observation) Space

We define the output space based on the UWB communication data and the BLE data transmitted by robots. We have measurement noise $v(t) \sim \mathcal{N}(0, R)$

$$z(t) = Cx(t) + v(t) \tag{4}$$

The C matrix will map the state to a measured state based on the BLE and UWB communications. We represent these as separate operators $\tilde{U}$ and $\tilde{B}$.

$$C = \tilde{U}\tilde{B} \tag{5}$$

4 State Estimation (Kalman Filter)

The Kalman filter is estimating the localization of an individual robot. Each robot will have their own Kalman filter as described here (indicated by subscript n). This is represented as the position and heading values in the state. We first have the state/covariance prediction step:

$$\hat{x}_{n,t+1|t} = \hat{x}_{n,t|t} + Bu_n(t) \tag{6}$$
$$P_{n,t+1|t} = P_{n,t|t} + Q_n \tag{7}$$

then the update step:

$$\hat{x}_{n,t|t} = \hat{x}_{n,t|t-1} + K_n(t)[z_n(t) - C\hat{x}_{n,t|t-1}] \tag{8}$$

with Kalman gain

$$K_n(t) = P_{n,t|t-1}C_n^T(C_nP(n,t|t-1)C_n^T + R_n)^{-1} \tag{9}$$

and covariance update

$$P_{n,t|t} = (I - K_n(t)C_n)P_{n,t|t-1} \tag{10}$$

5 UWB Localization

An ultra-wideband (UWB) localization system uses a combination of two components, anchors and tags, to localize robots. Each robot is fitted with a tag that emits UWB signals that are received by multiple anchors that are placed in the operational space of the robots. The time of flight of the signals between the tags and anchors can be used to precisely localize the robots. Since UWB uses ToF flight calculations for localization, it is resistant to localization confusion based on reflections from obstacles and walls in the environment. [1].

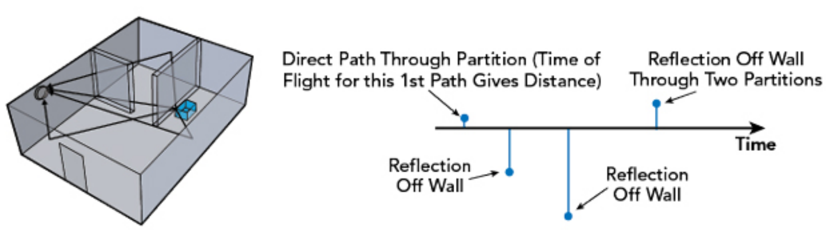

Figure 1: UWB resistance to multipath [1]

Utilizing multiple anchors, the absolute position can be determined using these ToF measurements; this requires two-way communication. A time difference of arrival (TDoA) method with multiple anchors can instead applied so only one-way communication (from tag to anchors) is necessary. A UWB blink uses around 50 micro-watts of power. Cutting down on power requirements for the tags which is advantageous given the power constraints of our robots.

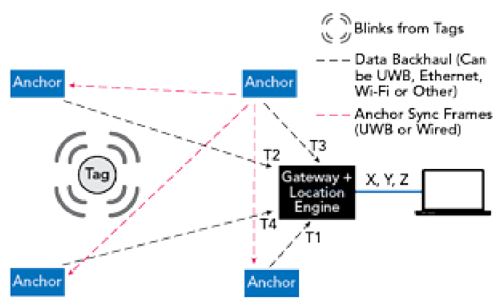

Figure 2: Time Difference of Arrival (TDoA) UWB Localization [1]

With the TDoA measurements and three or more anchors, a multilateration algorithm [3] can determine the robot position with on the scale of ± 10 cm [2].

6 Bluetooth Low Energy Communication

The Bluetooth Low Energy (BLE) communication is only used for the hub-robot intercommunication. The robots send their measured quantities (sensor value, capacitor voltage, photoresistor voltage) to the hub and the hub transmits back movement commands based on the estimated state of the robot. BLE communication is used for this communication due to its low power consumption for the robots, on the order of 5 micro-watts. This allows us to effectively communicate with our robots without significant power draw on the capacitors of the robots.

References

[1] Mickael Viot, Alexis Bizalion and Jervais Seegars. "Exploring Ultra-Wideband Technology for Micro-Location-Based Services." Microwave Journal, Microwave Journal, 15 June 2021, www.microwavejournal.com/articles/36143-exploring-ultra-wideband-technology-for-micro-location-based-services.

[2] Qorvo, www.qorvo.com/products/p/QM33120W#documents. Accessed 19 Nov. 2024.

[3] “Multilateration.” Multilateration - an Overview — ScienceDirect Topics, www.sciencedirect.com/topics/engineering/multilateration. Accessed 19 Nov. 2024.