Length is the foundation of modern statistics.

Until recently, computers were small and slow. Computational shortcomings hindered our ability to model random phenomena in the real world, even when statistical theory was sufficiently developed. Now, with advanced computers, we can finally realize the full power of statistical modeling.

The theory underlying statistics is simple. In fact, you're intuitively familiar with it. The most important idea in statistics is the idea of *measure*. A *measure* is an abstract notion of length. *Measures* have three basic properties that hold the statistical world together.

To make these properties concrete, let's consider measuring a piece of string with a ruler. Immediately, we know one thing for sure: the length of the string is not less than zero. If this seems obvious—good. It doesn't make sense to say something has negative length. Mathematicians call this the *non-negativity* property of measure. Now imagine cutting the string with a pair of scissors, and measuring the length of either resulting piece. What can we say about the length of either piece? Intuitively, we know each length is less than the length of the original string. That's *monotonicity*. And lastly, imagine you joined two separate pieces of string at one end. What can we say about the length of the joined string? We know its length should be the sum of the two smaller lengths. That's *countable subadditivity*. To summarize, measures have three basic properties derived from the notion of length: (1) *non-negativity*, (2) *monotonicity*, and (3) *countable subadditivity*.

Probabilities behave in the same way. Consider a deck of playing cards. What is the probability of pulling the jack of diamonds? Similar to the length of a string, this probability is never less than zero—*non-negativity*. We also know that the probability of pulling the jack of diamonds is less than the probability of pulling a jack of any suit—*monotonicity*. And lastly, we know that the probability of pulling a jack, any jack, is the sum of the probabilities of pulling each suited jack—*countable subadditivity*. Additionally, probabilities have a special property that distinguishes them from other measures. The probability (length) of the whole sample space (string) is 1.

Statistical questions such as: how does the brain compute decisions? should you go for it on 4th and 5? what drugs can cure Alzheimer's? are the subject of countless time, money, and energy. Yet they can be boiled down to what everyone intuitively knows about lengths and strings.