Before we get into the proof, we need to remember two things.
Remember: a measure preserving transformation is some function $T : X \to X$ on a set X such that $\mu(x) = \mu(T^{-1}x)$ for $x \in X$.
Also remember: an element $x \in B$ is said to be *B-recurrent* if there exists some $k \geq 1$ such that $T^k x \in B$.
Claim. For probability space (X, σ, μ), if $B \in \sigma$ and $\mu(B) > 0$, then almost every $x \in B$ is B-recurrent, i.e. returns to B at least once.

Proof. We will prove the claim via contradiction. Let F be the set of all elements in B that are not B-current. We want to show that $\mu(F) = 0$. We have

$$F = \{x \in B : T^k x \notin B \text{ for any k} \geq 1\}.$$

So $F \cap T^{-k}x = \emptyset$ for any $k \geq 1$. By construction, you can't reach a point in F from another point in F via transformations. It follows that $T^{-l}F \cap T^{-m}F = \emptyset$ for $l, m \geq 1$ and $l \neq m$. Concretely, the preimages of F under at least one transformation T don't share any elements (if the preimages did share elements, it would mean that you could get from F back into F in $|m - l|$ transformations, but we've constructed F precisely so you can't return to it). Thus, F, $T^{-1}F$, $T^{-2}F$, ... are pairwise disjoint. Since T is measure preserving, we have $\mu(F) = \mu(T^{-k})$ for any $k \geq 1$. Now we assume that $\mu(F) > 0$ to get our desired contradiction. If $\mu(F) > 0$, then

$$\mu(\bigcup_{k=1}^{\infty} T^{-k}F) \leq \sum_{k=1}^{\infty} \mu(T^{-k}F) = \infty$$

by countable subadditivity. Since we're working on a probability space, we know

$$\bigcup_{k=1}^{\infty} T^{-k}F \subseteq X,$$

and by taking measures we get the contradiction

$$1 = \mu(X) \geq \mu(\bigcup_{k=1}^{\infty} T^{-k}F) = \infty.$$

By contradiction, it must be that $\mu(F) = 0$. Almost every element of B is B-recurrent.

□