CSE 412A
Spring 2022

Introduction to Artificial Intelligence

Exercise 7

- You have approximately as many minutes as there are points.
- Mark your answers ON THE EXERCISE ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.
- For True/False questions, please *circle* your answer.

First name	
Last name	
WUSTL ID	

For staff use only:

Q1.	Bayesian Networks, HMMs, and Particle Filtering	/20
	Total	/20

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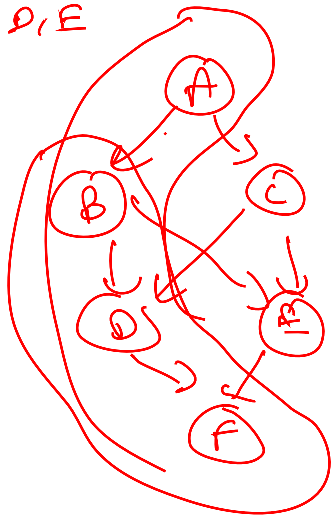
F

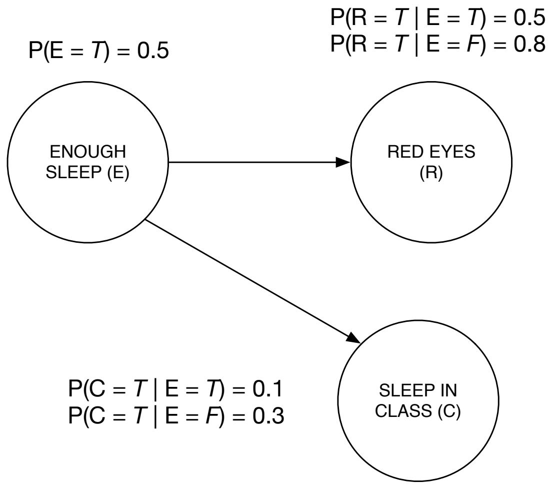
Each node is conditionally independent of all its ancestors
A, B, C

given its parents.
D, E



$$p(F \mid D, E, C, B, A) = p(F \mid D, E)$$

Q1. [20 pts] Bayesian Networks, HMMs, and Particle Filtering



Consider the Bayesian network above, where E, R, and C are random variables indicating if a student has enough sleep, has red eyes, and sleeps in class, respectively.

(a) [2 pts] Calculate the probability that a student has enough sleep, has red eyes, and sleeps in class. Write down the equations before replacing them with numbers.

$$P(E,R,C) = P(C|E,R)\,P(E,R)$$
$$= P(C|E)\,P(R|E)\,P(E)$$
$$= 0.1 \times 0.5 \times 0.5$$
$$= 0.025$$

(b) [2 pts] Calculate the probability that a student has red eyes and does not sleep in class. Write down the equations before replacing them with numbers.

$$P(R,\neg C) = P(R,\neg C,E) + P(R,\neg C,\neg E) = 0.505$$
$$P(R,\neg C,E) = P(\neg C|E)\,P(R|E)\,P(E) = 0.9 \times 0.5 \times 0.5 = 0.225$$
$$P(R,\neg C,\neg E) = P(\neg C|\neg E)\,P(R|\neg E)\,P(\neg E) = 0.7 \times 0.8 \times 0.5 = 0.28$$

(c) [2 pts] Calculate the probability that a student has enough sleep given that ~~he~~ they sleeps in class and does not have red eyes.

$$P(E|C,\neg R) = P(E,C,\neg R) / P(C,\neg R) = 0.025/0.055 = 0.455$$
$$\Rightarrow P(E,C,\neg R) = P(C|E)\,P(\neg R|E)\,P(E) = 0.1 \times 0.5 \times 0.5 = 0.025$$
$$P(C,\neg R) = P(C,\neg R,E) + P(C,\neg R,\neg E) = 0.025 + 0.03 = 0.055$$
$$P(C,\neg R,\neg E) = P(C|\neg E)\,P(\neg R|\neg E)\,P(\neg E) = 0.3 \times 0.2 \times 0.5 = 0.03$$

Imagine that 95% of the time Red Eye Disease (RED) causes red eyes in those with the disease. At any point in time 5% of the population have red eyes and at any point in time 1% of the population has RED. You have red eyes.

(d) [3 pts] Represent all the facts above using the probability notations.

RED : has RED disease

red : has red eyes

$P(\text{red} \mid \text{RED}) = 0.95$

$P(\text{red}) = 0.05$

$P(\text{RED}) = 0.01$

$P(\text{red}=? \mid \text{RED}=?) = 0.95$

$P(X = \text{false}) \equiv P(\neg X)$

(e) [3 pts] What is the probability that you have RED? Justify your answer mathematically. Write down the equations before replacing them with numbers.

$$P(\text{RED} \mid \text{red}) = \frac{P(\text{RED}, \text{red})}{P(\text{red})} = \frac{P(\text{red} \mid \text{RED})\, P(\text{RED})}{P(\text{red})}$$

$$= \frac{0.95 \times 0.01}{0.05}$$

$$= 0.19$$

(f) Each question is worth 1 point. Leaving a question blank is worth 0 points. **Answering a question incorrectly is worth −1 point.** This gives you an expected value of 0 for random guessing.

(i) [1 pt] [*true* or *false*] A fair coin is flipped three times. Assume that it must land on either "heads" or "tails". The probability of seeing three "tails" is less than 0.2.

(ii) [1 pt] [*true* or *false*] If X, Y, and Z are binary random variables (i.e., their possible values are *true* and *false* only), then the conditional probability table for $\Pr(X \mid Y, Z)$ can be fully specified by $3 = 2^2 - 1$ values.

(iii) [1 pt] [*true* or *false*] A joint probability distribution can always be factored into a product of conditional distributions.

(iv) [1 pt] [*true* or *false*] An edge in a Bayesian network indicates a causal relationship between two random variables.

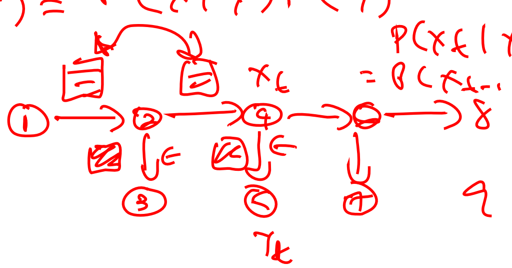
(v) [1 pt] [*true* or *false*] A Bayesian network is allowed to have a directed cycle.

(vi) [1 pt] [*true* or *false*] A hidden Markov model can be interpreted as a Bayesian network.

(vii) [1 pt] [*true* or *false*] Particle filtering is based on the concept of representing a probability distribution with samples from the distribution.

(viii) [1 pt] [*true* or *false*] Every run of particle filtering will produce exactly the same output.

$P(X, Y) = P(X \mid Y)\, P(Y)$



$P(x_t \mid x_{t-1}) = P(x_{t-1} \mid x_{t-2})$

YZ	$\Pr(X \mid Y, Z)$
TT	0.3
TF	0.4
FT	0.5
FF	0.6