Slide 1

MINIMAX SEARCH

CSE 511A: Introduction to Artificial Intelligence

1

Slide 2

MINIMAX GAME TREE

MAX

MIN

MAX

MIN

2

Slide 3

MINIMAX GAME TREE

MIN

MAX

terminal nodes

utility   -1   0   1

3

Slide 4
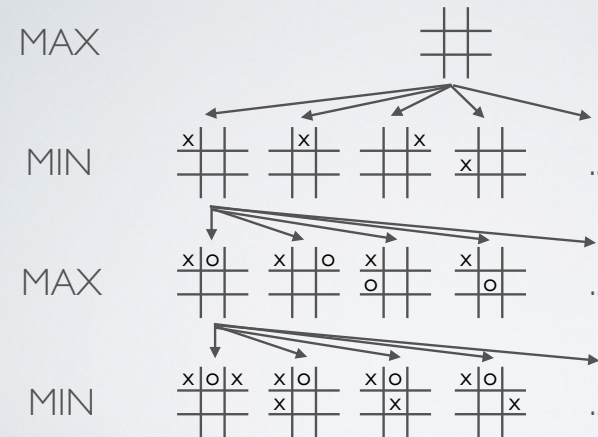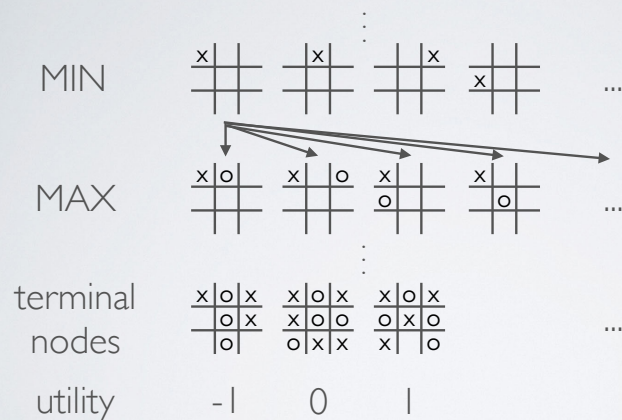
MINIMAX SEARCH

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the agent is MAX: return max-value(state)
    if the agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

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Slide 5

MAX

MIN

MAX

MIN

```
def max-value(state):
  initialize v = -∞
  for each successor of state:
    v = max(v, min-value(successor))
  return v
```

```
def min-value(state):
  initialize v = +∞
  for each successor of state:
    v = min(v, max-value(successor))
  return v
```

Leaf values: 3 4 1 2 7 8 5 6

5

Slide 6
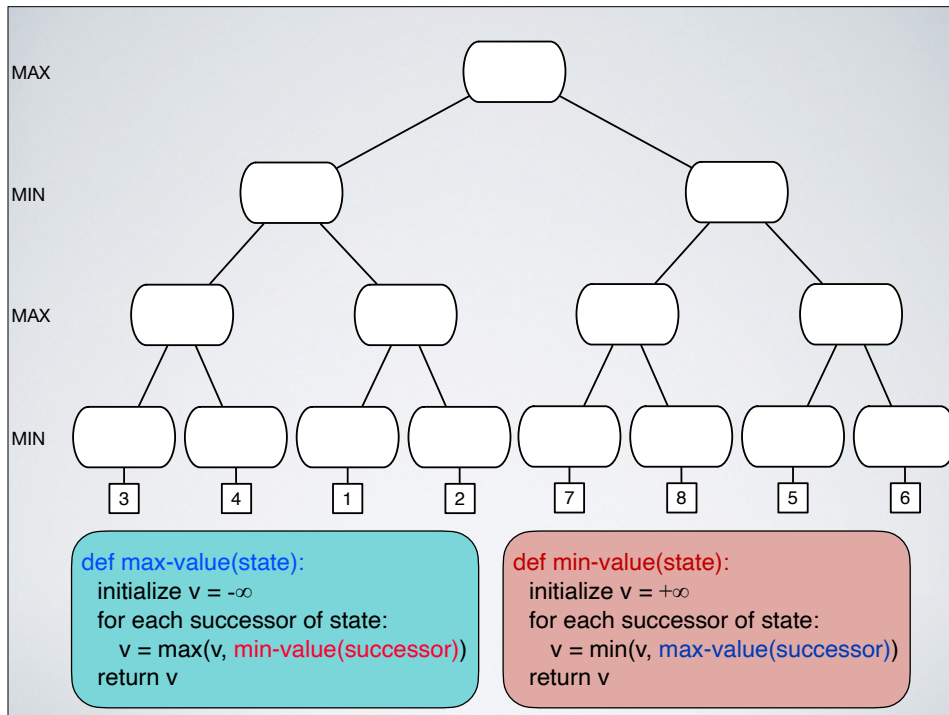
MINIMAX SEARCH

	Minimax
Correct the solution it finds is optimal	Yes
Complete it terminates	Yes
Space Complexity max nodes in memory	$O(bm)$
Time Complexity max nodes generated	$O(b^m)$

branching factor b
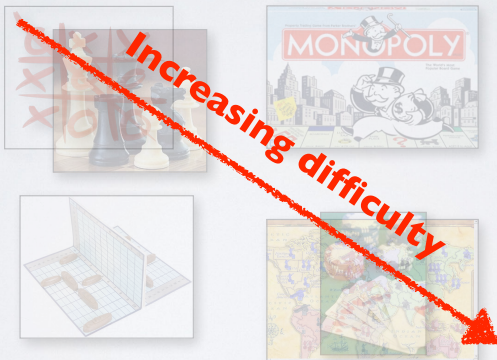depth of the goal d
depth of tree m

6

Slide 7

MINIMAX SEARCH

deterministic   stochastic

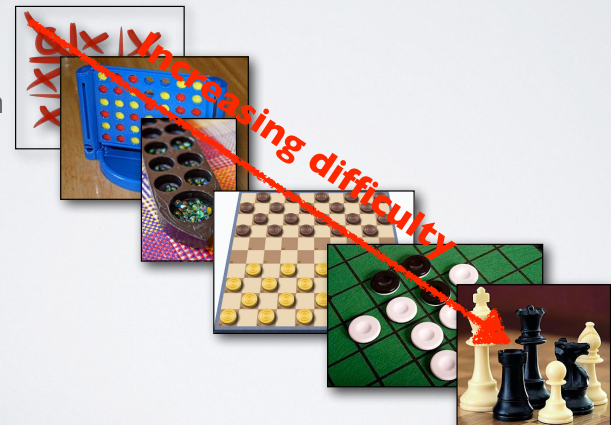perfect information

imperfect information

Increasing difficulty

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Slide 8

MINIMAX SEARCH

deterministic

perfect information

Increasing difficulty

8

Slide 9

MINIMAX SEARCH

deterministic

perfect information

Increasing difficulty

Tic-tac-toe: $\sim 10^5$
Connect Four: $\sim 10^{21}$
Congkak-6: $\sim 10^{33}$

Draughts: $\sim 10^{54}$
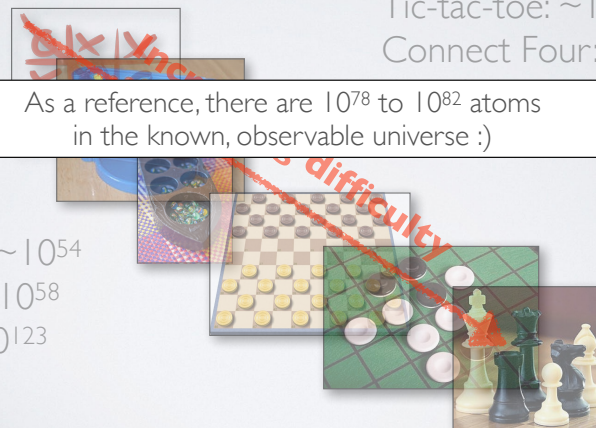Othello: $\sim 10^{58}$
Chess: $\sim 10^{123}$

Slide 10

MINIMAX SEARCH

deterministic

perfect information

Tic-tac-toe: $\sim 10^5$
Connect Four: $\sim 10^{21}$

As a reference, there are 10^{78} to 10^{82} atoms
in the known, observable universe :)

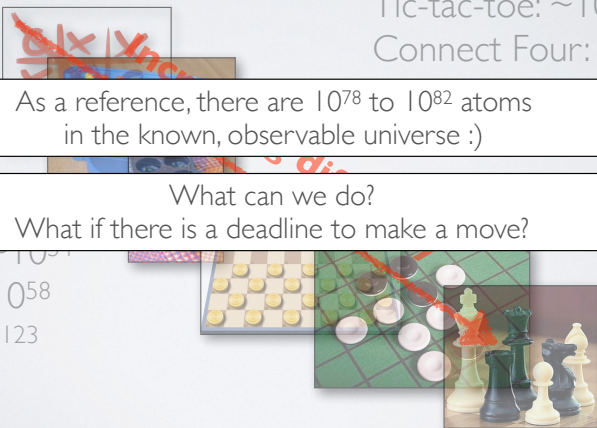Draughts: $\sim 10^{54}$
Othello: $\sim 10^{58}$
Chess: $\sim 10^{123}$

Slide 11

MINIMAX SEARCH

deterministic

perfect information

Tic-tac-toe: $\sim 10^5$
Connect Four: $\sim 10^{21}$

As a reference, there are 10^{78} to 10^{82} atoms
in the known, observable universe :)

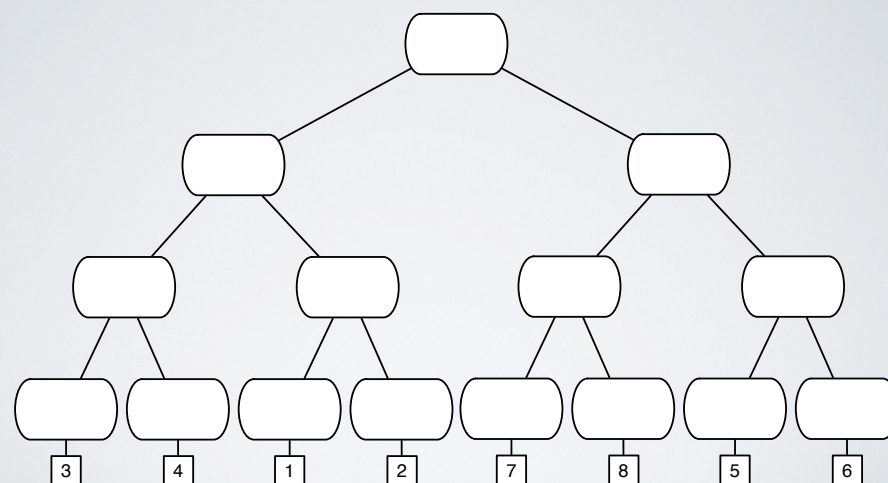What can we do?
What if there is a deadline to make a move?

Draughts: $\sim 10^{54}$
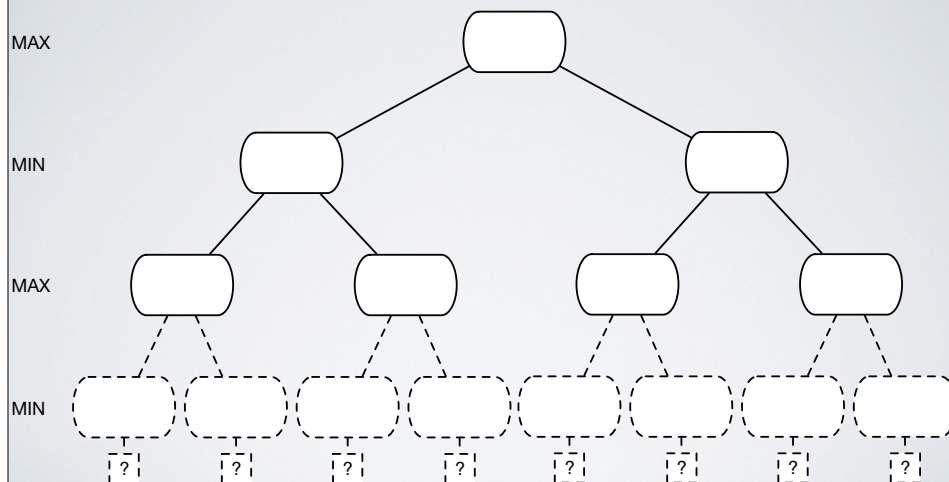Othello: $\sim 10^{58}$
Chess: $\sim 10^{123}$

Slide 12

MINIMAX SEARCH

MAX

MIN

MAX

MIN

3 4 1 2 7 8 5 6

MINIMAX SEARCH

MAX

MIN

MAX

MIN

? ? ? ? ? ? ? ?

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76 CS188 Pacman

SCORE: 0

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76 CS188 Pacman
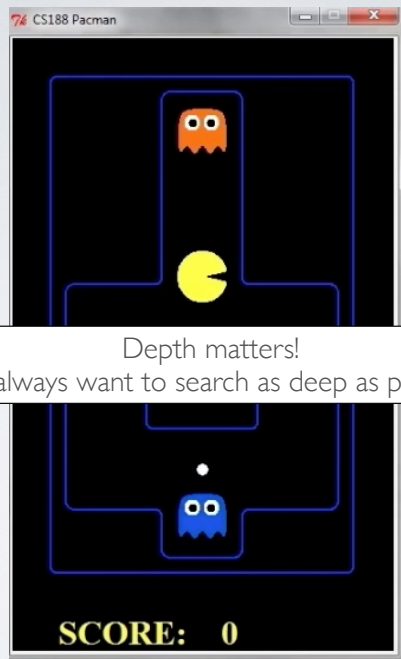
Depth matters!
You always want to search as deep as possible
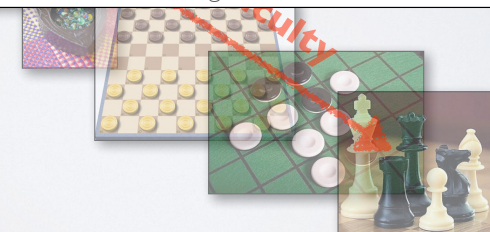
SCORE: 0

MINIMAX SEARCH

deterministic

Tic-tac-toe: $\sim 10^5$
Connect Four: $\sim 10^{21}$

perfect

in

What can we do?
What if there is a deadline to make a move?
Use heuristics! Estimates on how "good" or "bad" the state is

Draughts: $\sim 10^{54}$
Othello: $\sim 10^{58}$
Chess: $\sim 10^{123}$