Q1. [10 pts] First-Order Logic

Using *only* the following predicates:

- IsMan(x), which returns true iff x is a man.
- IsBarber(x), which returns true iff x is a barber. (Note that x doesn't have to be a man.)
- Shaves(x, y), which returns true iff x shaves y. (Note that neither x nor y have to be a man or a barber.)

Translate the following sentences into first-order logic:

(a) [2 pts] All men are barbers.

$$\forall x \ \text{IsMan}(x) \Rightarrow \text{IsBarber}(x)$$

(b) [2 pts] All barbers shave themselves.

$$\forall x \ \text{IsBarber}(x) \Rightarrow \text{Shaves}(x, x)$$

(c) [2 pts] All men need to be shaved by a barber (any barber).

$$\forall x \ \exists y \ \text{IsBarber}(y) \wedge (\text{IsMan}(x) \Rightarrow \text{Shaves}(y, x))$$

(d) [2 pts] There is a barber who shaves all men.

$$\exists y \ \forall x \ \text{IsBarber}(y) \wedge (\text{IsMan}(x) \Rightarrow \text{Shaves}(y, x))$$

(e) [2 pts] There is a barber who shaves all men who do not shave themselves.

$$\exists y \ \forall x \ \text{IsBarber}(y) \wedge ((\text{IsMan}(x) \wedge \neg(\text{Shaves}(x, x)) \Rightarrow \text{Shaves}(y, x))$$