CSE 412A Spring 2022 Introduction to Artificial Intelligence

Exercise 9

- You have approximately as many minutes as there are points.
- Mark your answers ON THE EXERCISE ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation. All short answer sections can be successfully answered in a few sentences AT MOST.
- For True/False questions, please *circle* your answer.

First name	
Last name	
WUSTL ID	

For staff use only:

Q1.	Propositional Logic	/20
	Total	/20

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Q1. [20 pts] Propositional Logic

1 Either taxes are increased or if expenditures rise then the debt ceiling is raised. 2 If taxes are increased, then the cost of collecting taxes rises. 3 If a rise in expenditures implies that the government borrows more money, then if the debt ceiling is raised, then interest rates increase. 4 If taxes are not increased and the cost of collecting taxes does not increase, then if the debt ceiling is raised, then the government borrows more money. 5 The cost of collecting taxes does not increase. 6 Either interest rates do not increase or the government does not borrow more money.

(a) [8 pts] Translate the statements above into propositional logic, using *only* the propositional symbols **T** (for increased taxes), **E** (for increased expenditures), **D** (for increased debt ceiling), **C** (for increased cost of collecting taxes), **G** (for increased government borrowing), and **I** (for increased interest rates).

Note: *The "either or" statements above are inclusive or statements.*

1. $T \vee (E \rightarrow D)$
2. $T \rightarrow C$
3. $(E \rightarrow G) \rightarrow (D \rightarrow I)$
4. $(\neg T \wedge \neg C) \rightarrow (D \rightarrow G)$
5. $\neg C$
6. $\neg I \vee \neg G$

(b) [8 pts] Translate the statements into conjunctive normal form (CNF). $= (x_1 \vee x_2) \wedge (x_3 \vee x_4) \wedge \dots$

$x_1 \rightarrow x_2 \equiv \underbrace{\neg x_1 \vee x_2}_{\text{CNF}}$

1. $T \vee (\neg E \vee D)$
2. $\neg T \vee C$
3. $(\neg E \vee G) \rightarrow (\neg D \vee I) \equiv \neg(\neg E \vee G) \vee (\neg D \vee I) =$
   $= (E \wedge \neg G) \vee (\neg D \vee I) =$
   $= (\neg D \vee I \vee E) \wedge (\neg D \vee I \vee \neg G)$
4. $\neg(\neg T \wedge \neg C) \vee (\neg D \vee G) = T \vee C \vee \neg D \vee G$
5. $\neg C$
6. $\neg I \vee \neg G$

(c) Each question is worth 1 point. Leaving a question blank is worth 0 points. **Answering a question incorrectly is worth −1 point.** This gives you an expected value of 0 for random guessing.

(i) [1 pt] [*true* or *false*] $(A \to B) \vee (A \vee B)$ is a valid sentence.

(ii) [1 pt] [*true* or *false*] $(A \to B) \vee (A \vee B)$ is a satisfiable sentence.

(iii) [1 pt] [*true* or *false*] A sentence cannot be both valid and unsatisfiable.

(iv) [1 pt] [*true* or *false*] All satisfiable sentences are valid sentences.

$\neg A \vee B \vee A = \neg A \vee A \vee B$

tautology