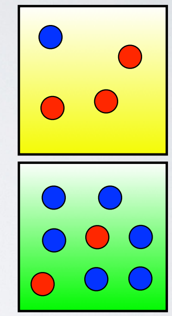
PROBABILITIES

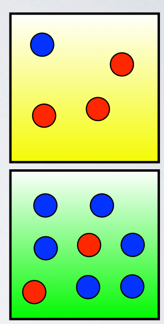
CSE 511A: Introduction to Artificial Intelligence

MARBLES AND BOXES



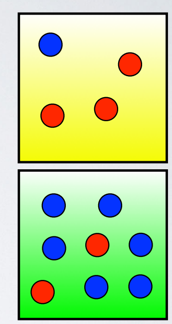
- Boxes: Yellow (y) and Green (g)
- Marbles: Red (r) and Blue (b)
- If you randomly select a box and randomly pick a marble from that box, then
  - the identity of the box is a *random variable* B
  - the identity of the marble is a *random variable* M
- B can take 2 possible values: y and g
- M can take 2 possible values: r and b

MARBLES AND BOXES



- Goal: Want to quantify the likelihood of different events such that we can analyze them.
- Examples:
  - If I chose a red marble, what is the likelihood that I chose from the yellow box?
  - If I chose the yellow box, what is the likelihood that I choose a blue marble?

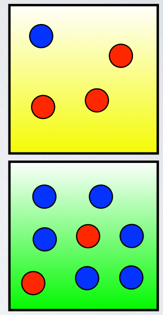
BASIC DEFINITIONS



- The *probability* (or likelihood) of an event is the fraction of times that event occurs out of *n* trials as *n* approaches infinity
- P(Random variable = value)
  - P(B=y), P(B=y|M=r), ...
- Probabilities lie in the range [0,1]
- Sum of the likelihood of all mutually exclusive events must equal 1
  - P(B=y) + P(B=g) = 1

PROBABILITIES

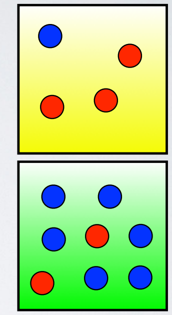
- A *probability* is the likelihood of one event occurring
  - P(B=y), P(B=g), ...
- In our example, the probability of picking either box is equal
  - P(B=y) = P(B=g) = 1/2



5

CONDITIONAL PROBABILITY

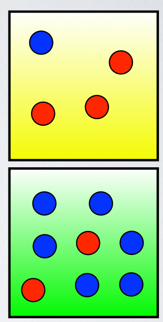
- The probability of an event occurring under the assumption that another event is occurring is called a *conditional probability*
  - P(M=r | B=y), P(B=y | M=r), ...
- In our example,
  - P(M=r | B=y)
    = 3 red marbles in yellow box out of 4 marbles in yellow box
    = 3/4
  - P(B=y | M=r) = 3/4?



6

JOINT PROBABILITY

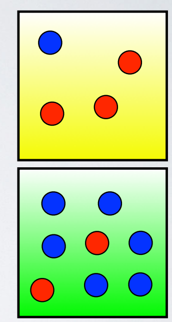
- A *joint probability* is the probability of two or more events occurring
  - P(M=r, B=y), ...
- Bayes' Rule: $\boxed{P(X,Y) = P(X \mid Y) * P(Y)}$
- In our example,
  - P(M=r, B=y) = ?
  - P(B=y, M=r) = ?
  - P(M=r, B=g) = ?



7

MARGINALIZATION

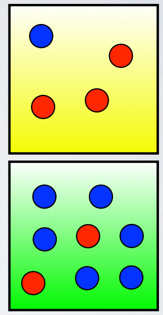
- We *marginalize* (or *sum over*) a random variable Y when we want to consider the probability of X irrespective of Y
- In our example,
  - P(M=r, B=y) = 3/8
    P(M=r, B=g) = 1/8
  - P(M=r) = P(M=r, B=y) + P(M=r, B=g)
    = 1/2



8

EXERCISE

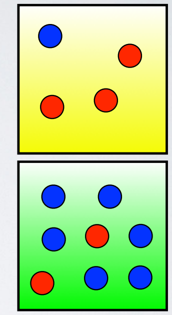
- What is the probability that the box chosen is yellow given that you picked a red marble?



9

EXERCISE

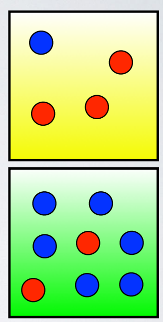
- What is the probability that the box chosen is yellow given that you picked a red marble?
  - $P(B=y \mid M=r)$
    $= P(B=y, M=r) / P(M=r)$
    $= 3/8 \,/\, 1/2$
    $= 3/4$



10

EXERCISE

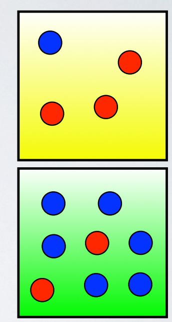
- Assume that I am inherently more likely to choose the yellow box (66.6%) than the green box (33.3%)
- If I chose a red marble, what is the likelihood that I chose the green box?



11

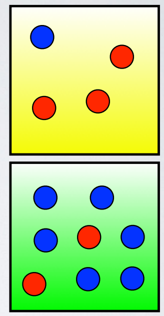
EXERCISE

- Assume that I am inherently more likely to choose the yellow box (66.6%) than the green box (33.3%)
- If I chose a red marble, what is the likelihood that I chose the green box?
  - $P(B=g \mid M=r) = P(B=g, M=r) / P(M=r)$
  - $P(B=g, M=r) = P(M=r \mid B=g) * P(B=g) = 1/4 * 1/3 = 1/12$
  - $P(B=y, M=r) = P(M=r \mid B=y) * P(B=y) = 3/4 * 2/3 = 1/2$
  - $P(M=r) = P(B=g, M=r) + P(B=y, M=r) = 1/12 + 1/2 = 7/12$



12

NORMALIZATION CONSTANT



- It is often that we need to normalize probabilities so that they sum up to 1
- In our example,
  - P(B=y | M=r) = P(M=r | B=y) P(B=y) / P(M=r)
    = 3/4 * 1/2 / P(M=r)
    = 3/8 / P(M=r)
  - P(B=g | M=r) = P(M=r | B=g) P(B=g) / P(M=r)
    = 2/8 * 1/2 / P(M=r)
    = 1/8 / P(M=r)
  - P(B=y | M=r) + P(B=g | M=r) = 1. Therefore, P(M=r) = 3/8 + 1/8 = 1/2.
  - **Don't need to explicitly calculate P(M=r)!**

APPLICATIONS IN AI

P(X | Y) = P(Y | X) * P(X) / P(Y)

P(model | data) = P(data | model) * P(model) / P(data)

Speech Recognition
P(word | utterance)
= P(utterance | word) * P(word) / P(utterance)

Medical Diagnosis
P(disease | symptoms)
= P(symptoms | disease) * P(disease) / P(symptoms)

EXERCISE

You are a witness of a night time hit-and-run accident involving a taxi in Athens. All taxis in Athens are either blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that under the dim lighting conditions, discrimination between blue and green is 75% reliable. Calculate the most likely color for the taxi, given than 90% of Athenian taxis are green.

EXERCISE

You are a witness of a night time hit-and-run accident involving a taxi in Athens. All taxis in Athens are either blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that under the dim lighting conditions, discrimination between blue and green is 75% reliable. Calculate the most likely color for the taxi, given than 90% of Athenian taxis are green.

- P(S=g | T=g) = P(S=b | T=b) = 3/4,   P(T=g) = 9/10
- P(S=b | T=g) = 1-3/4 = 1/4
- P(S=b) = P(S=b | T=b) * P(T=b) + P(S=b | T=g) * P(T=g)
  = 3/4 * 1/10 + 1/4 * 9/10 = 12/40
- P(T=g | S=b) = P(S=b|T=g) * P(T=g) / P(S=b) = 1/4 * 9/10 / 12/40 = 9/12
- P(T=b | S=b) = P(S=b|T=b) * P(T=b) / P(S=b) = 3/4 * 1/10 / 12/40 = 3/12