

Q1. [15 pts] First-Order Logic

Using *only* the following predicates and functions:

- $\text{IsInsect}(x)$, which returns true iff x is an insect.
- $\text{IsMoth}(x)$, which returns true iff x is a moth.
- $\text{IsDragonfly}(x)$, which returns true iff x is a dragonfly.
- $\text{IsSpider}(x)$, which returns true iff x is a spider.
- $\text{Eats}(x,y)$, which returns true iff x eats y .
- $\text{Wings}(x,y)$, which returns true iff x has exactly y wings.
- $\text{Order}(x)$, which returns the order that x belongs to. (Recall that zoologists classify creatures into kingdoms, phylums, classes, orders, families, etc.)

Translate the following sentences into first-order logic:

- (a) [3 pts] Not all insects have exactly 4 wings.

$$\exists x \text{ IsInsect}(x) \wedge \neg \text{Wings}(x, 4)$$

- (b) [3 pts] All insects with exactly 2 wings are in the same order.

$$\forall y \forall x (\text{IsInsect}(x) \wedge \text{IsInsect}(y) \wedge \text{Wings}(x, 2) \wedge \text{Wings}(y, 2) \Rightarrow (\text{Order}(x) = \text{Order}(y)))$$

- (c) [3 pts] Moths and dragonflies are insects but not in the same order.

$$\forall x \forall y ((\text{IsMoth}(x) \Rightarrow \text{IsInsect}(x)) \wedge (\text{IsDragonfly}(y) \Rightarrow \text{IsInsect}(y))) \wedge ((\text{IsMoth}(x) \wedge \text{IsDragonfly}(y)) \Rightarrow \neg (\text{Order}(x) = \text{Order}(y)))$$

- (d) [3 pts] All spiders eat insects.

$$\forall x \exists y \text{ IsInsect}(y) \wedge (\text{IsSpider}(x) \Rightarrow \text{Eats}(x, y))$$

- (e) [3 pts] Some spiders eat only insects.

$$\exists x \forall y \text{ IsSpider}(x) \wedge (\text{Eats}(x, y) \Rightarrow \text{IsInsect}(y))$$