PROPOSITIONAL LOGIC

CSE 511A: Introduction to Artificial Intelligence

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WHAT IS AN AGENT?

Intelligent Agents

think like humans	think rationally
act like humans	act rationally

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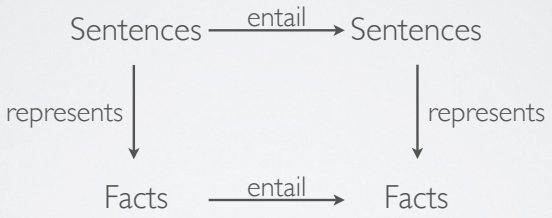
OVERVIEW



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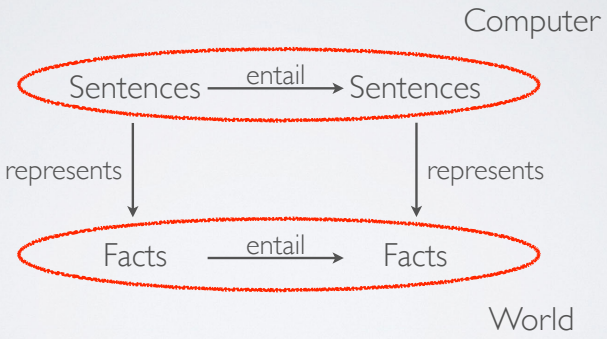
OVERVIEW



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OVERVIEW

Facts	Sentences
If the head comes up in a coin toss, I win	H⇒IW
If the tail comes up in a coin toss, you lose	T⇒UL
Either the head or the tail must come up in a coin toss	H⇔¬T
You losing is equivalent to me winning	IW⇔UL

entail

I win	IW

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OVERVIEW

- Ontological commitment: what exists - facts? objects? time? belief?
- Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional Logic	facts	true/false/unknown
First-order Logic	facts, objects, relations	true/false/unknown
Temporal Logic	facts, objects, relations, times	true/false/unknown
Probability Logic	facts	degree of belief
Fuzzy Logic	facts, degrees of truth	known interval value

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OVERVIEW

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PROPOSITIONAL LOGIC

Expression	Type	English interpretation
P		P is true
¬P	negation	P is false
P∨Q	disjunction	either P is true or Q is true or both are true
P∧Q	conjunction	both P and Q are true
P⇒Q	implication	if P is true, then Q is true
P⇔Q	equivalence	P and Q are either both true or both false

PROPOSITIONAL LOGIC

Expression	is true iff
P	P is true
¬P	P is false
P∨Q	P is true **or** Q is true
P∧Q	P is true **and** Q is true
P⇒Q	P is false **or** Q is true (easier: false iff P is true **and** Q is false)
P⇔Q	P⇒Q is true **and** Q⇒P is true

PROPOSITIONAL LOGIC

P	Q	¬P	P∨Q	P∧Q	P⇒Q	P⇔Q
TRUE	TRUE					
TRUE	FALSE					
FALSE	TRUE					
FALSE	FALSE					

called a *truth table*

PROPOSITIONAL LOGIC

P	Q	¬P	P∨Q	P∧Q	P⇒Q	P⇔Q
TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE

called a *truth table*
each row of the table is called a *model*

PROPOSITIONAL LOGIC

$$\neg(\neg P) \equiv P$$
$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$
$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$
$$P \Rightarrow Q \equiv \neg P \vee Q$$
$$P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$$

PROPOSITIONAL LOGIC

- A sentence is *valid* if it is true in all models
  (all rows of the truth table are true)
  - e.g., A ∨ ¬A (sentence is true irregardless of whether A is true or false)
- A sentence is *satisfiable* if it is true in some model
  (at least one row of the truth table is true)
  - e.g., A ∨ B (sentence is true if A or B is true)
- A sentence is *unsatisfiable* if it is true in no model
  (all rows of the truth table are false)
  - e.g., A ∧ ¬A (sentence is false irregardless of whether A is true or false)