MODELING CONSTRAINT SATISFACTION PROBLEMS

CSE 511A: Introduction to Artificial Intelligence

MAP COLORING

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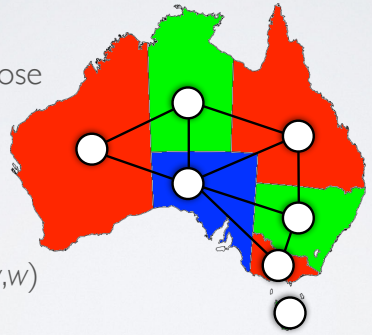
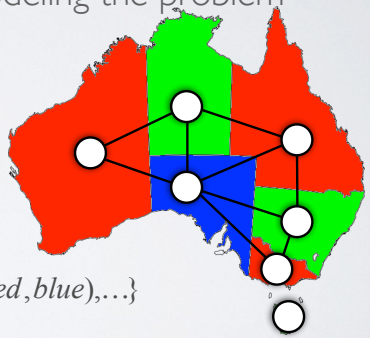
Each node can choose a color from {red, green, blue}

Each constraint c(*v*,*w*) is satisfied if color(*v*) ≠ color(*w*)



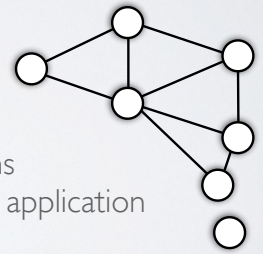
MAP COLORING

- Main challenge in CSPs is in modeling the problem
- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $\{red, green, blue\}$
- Constraints: adjacent regions must have different colors
  - Implicit: $WA \neq NT$
  - Explicit: $(WA, NT) \in \{(red, green), (red, blue), \ldots\}$



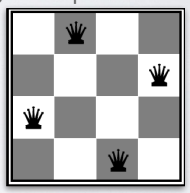
CONSTRAINT GRAPHS

- Constraint (hyper-)graph
  - Nodes: Variables of the CSP
  - (Hyper-)edges: Constraints of the CSP
- Allows CSPs to model applications abstractly
- CSP solvers operate on constraint graphs without needing to know the underlying application



MODELING EXERCISES

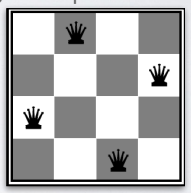
- Main challenge in CSPs is in modeling the problem
- N-Queens:
  - Variables: ?
  - Domains: ?
  - Constraints: ?



MODELING EXERCISES

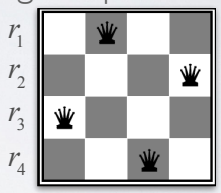
- Main challenge in CSPs is in modeling the problem
- N-Queens:
  - Variables: $x_{i,j}$
  - Domains: $\{0,1\}$
  - Constraints:

$$\forall i,j,k : (x_{i,j}, x_{i,k}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k : (x_{i,j}, x_{k,j}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k : (x_{i,j}, x_{i+k,j+k}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k : (x_{i,j}, x_{i+k,j-k}) \in \{(0,0),(0,1),(1,0)\}$$



MODELING EXERCISES

- Main challenge in CSPs is in modeling the problem
- N-Queens:
  - Variables: r_k
  - Domains: $\{1,2,3,\ldots,N\}$
  - Constraints:



  - Implicit representation: $\forall i,j$ non-threatening(r_i, r_j)
  - Explicit representation: $(r_1, r_2) \in \{(1,3),(1,4),(1,5),\ldots\}$
    $(r_1, r_3) \in \{(1,2),(1,4),(1,5),\ldots\}$
    …

MODELING EXERCISES

- Sudoku:
  - Variables: ?
  - Domains: ?
  - Constraints: ?

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					

MODELING EXERCISES

- Sudoku:
  - Variables: Cells in the grid
  - Domains: {1,2,…,9}
  - Constraints:
    - Unary constraint for each filled cell
    - 9-way alldiff constraint for each row
    - 9-way alldiff constraint for each column
    - 9-way alldiff constraint for each subgrid
      - alldiff constraint = all variables in the constraint must take different values

					8			4
	8	4		1	6			
			5			1		
1		3	8			9		
6		8				4		3
		2			9	5		1
		7			2			
			7	8		2	6	
2			3					