FIRST-ORDER LOGIC

CSE 511A: Introduction to Artificial Intelligence

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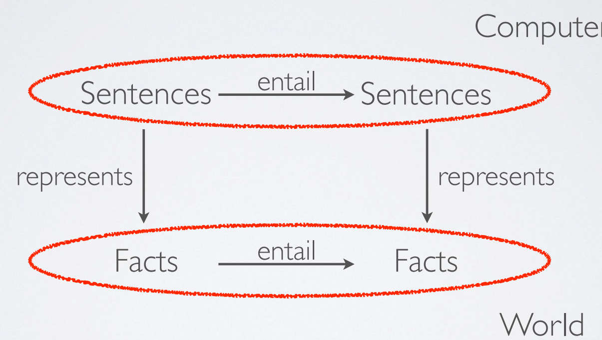
OVERVIEW



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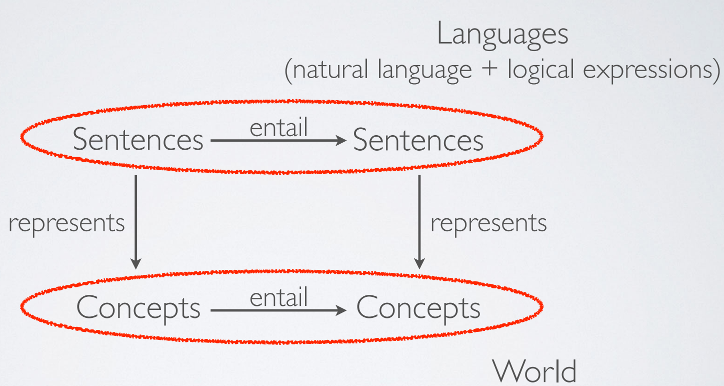
OVERVIEW



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OVERVIEW

Facts	Sentences
All students in this class are studying AI	??
Michael is a student in this class	??

entail

Michael is studying AI	??

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OVERVIEW

Facts	Sentences
Student 1 is studying AI, Student 2 is studying AI, Student 3 is studying AI,….	$S1 \Rightarrow AI \wedge S2 \Rightarrow AI \wedge S3 \Rightarrow AI \wedge \ldots$
Michael is Student 3	$M = S3$

entail

Michael is studying AI	$M \Rightarrow AI$

OVERVIEW

- Ontological commitment: what exists - facts? objects? time? belief?
- Epistemological commitment: what states of knowledge?

Language	Ontological Commitment	Epistemological Commitment
Propositional Logic	facts	true/false/unknown
First-order Logic	facts, objects, relations	true/false/unknown
Temporal Logic	facts, objects, relations, times	true/false/unknown
Probability Logic	facts	degree of belief
Fuzzy Logic	facts, degrees of truth	known interval value

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SYNTAX

- Objects (constants in the world):
  - e.g., wheel, door, body, engine, baseball, football, green, blue, cyan, 0, 5, 1.2, minutes, centuries, AI, Amy, Bob, Charles, …

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- Functions (returns an object):
  - e.g., ColorOf(car), FatherOf(Amy), AgeOf(Bob), …
- Predicates (returns true/false):
  - Relations (predicates between multiple objects):
    - e.g., IsInside(Amy, car), IsMotherOf(Amy, Bob), …
  - Properties (predicates of a single object):
    - e.g., IsBlue(car), IsOpen(door), IsOld(Bob), …

SYNTAX

- "One plus two equals tree"
  - Objects: *one, two, three, one plus two*
  - Functions: *plus*
  - Relations: *equals*
  - Properties: N/A

SYNTAX

- "One plus two equals tree"
  - Objects: *one, two, three, one plus two*
  - Functions: *plus*
  - Relations: *equals*
  - Properties: N/A
- "All mailboxes are blue, except for the ones at WashU"
  - Objects: *WashU, blue, mailbox 1, mailbox 2, …*
  - Functions: *location of mailbox, color of mailbox*
  - Relations: N/A
  - Properties: *is blue*

SYNTAX

- Objects (constants in the world)
- Functions (returns an object)
- Predicates (returns true/false)
  - Relations (predicates between multiple objects)
  - Properties (predicates of a single object)
- Connectives: $\Rightarrow$, $\Leftrightarrow$, $\wedge$, $\vee$
- Quantifiers: $\forall$, $\exists$
- Equality: =

SENTENCES

- Atomic sentences (sentences that return true/false):
  - predicate(term1, term2, term3, …)
  - term1 = term2
  - term:
    - function(term1, term2, …)
    - constant
    - variable
- Examples: SchoolOf(Alex), Colleague(TeacherOf(Alex), FatherOf(Bob))

SENTENCES

- Complex sentences (atomic sentences combined together using connectives and quantifiers):
  - sentence = atomic sentence
| sentence connective sentence
| quantifier variable: sentence
| ¬sentence
- Examples:
  - FatherOf(Amy, Bob)∧ BrotherOf(Bob, Charles)⇒UncleOf(Amy,Chales)
  - ∀x: IsMailbox(x) ⇒ IsBlue(x)

QUANTIFIERS

- All mailboxes are blue
- ∀x: IsMailbox(x) ⇒ IsBlue(x), corresponds to
  (IsMailBox(object 1) ⇒ IsBlue(object 1)) ∧
  (IsMailBox(object 2) ⇒ IsBlue(object 2)) ∧ …

QUANTIFIERS

- All mailboxes are blue
- $\forall x$: IsMailbox(x) $\Rightarrow$ IsBlue(x), corresponds to

  (IsMailBox(object 1) $\Rightarrow$ IsBlue(object 1)) $\wedge$

  (IsMailBox(object 2) $\Rightarrow$ IsBlue(object 2)) $\wedge$ …

- Common mistake:
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  which means that ALL objects are blue mailboxes.

QUANTIFIERS

- There exists a blue mailbox:
- $\exists x$: IsMailbox(x) $\wedge$ IsBlue(x), which corresponds to

  (IsMailBox(object 1) $\wedge$ IsBlue(object 1)) $\vee$

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  $\exists x$: IsMailbox(x) $\Rightarrow$ IsBlue(x), which means?

QUANTIFIERS

- There exists a blue mailbox:
- ∃x: IsMailbox(x) ∧ IsBlue(x), which corresponds to

  (IsMailBox(object 1) ∧ IsBlue(object 1)) ∨

  (IsMailBox(object 2) ∧ IsBlue(object 2)) ∨ …
- Common mistake:
  ∃x: IsMailbox(x) ⇒ IsBlue(x), which corresponds

  (IsMailBox(object 1) ⇒ IsBlue(object 1)) ∨

  (IsMailBox(object 2) ⇒ IsBlue(object 2)) ∨ …

  which means can be true if there exists an object that isn't a mailbox

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QUANTIFIERS

- Relationships between quantifiers: ∀,∃
- ∀x ∀y is the same as ∀y ∀x?
- ∃x ∃y is the same as ∃y ∃x?
- ∀x ∃y is the same as ∃y ∀x?

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QUANTIFIERS

- Relationships between quantifiers: ∀,∃
- ∀x ∀y is the same as ∀y ∀x? YES
- ∃x ∃y is the same as ∃y ∃x? YES
- ∀x ∃y is the same as ∃y ∀x? NO
  - ∀x ∃y: Loves (x,y) means "everyone loves someone in the world"
  - ∃y ∀x: Loves (x,y) means "someone is loved by everyone in the world"

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  - ∀x ∃y: Loves (x,y) means "everyone loves someone in the world"
  - ∃y ∀x: Loves (x,y) means "someone is loved by everyone in the world"

  - ∀x: Likes(x, IceCream) is the same as ¬∃x: ¬Likes(x, IceCream)?
  - ∃x: Likes(x, Broccoli) is the same as ¬∀x: ¬Likes(x, Broccoli)?

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QUANTIFIERS

- Relationships between quantifiers: ∀,∃
- ∀x ∀y is the same as ∀y ∀x? YES
- ∃x ∃y is the same as ∃y ∃x? YES
- ∀x ∃y is the same as ∃y ∀x? NO
  - ∀x ∃y: Loves (x,y) means "everyone loves someone in the world"
  - ∃y ∀x: Loves (x,y) means "someone is loved by everyone in the world"
- Quantifier duality:
  - ∀x: Likes(x, IceCream) is the same as ¬∃x: ¬Likes(x, IceCream)? YES
  - ∃x: Likes(x, Broccoli) is the same as ¬∀x: ¬Likes(x, Broccoli)? YES