Washington University in St. Louis

McKelvey School of Engineering

Spring Semester 2023

CSE463M-563M: Digital Integrated Circuit Design and Architecture

Homework #1

CSE 463M and CSE 563M students:

1) Solve problems (10 points each)

3.1, 3.2, 3.3, 3.6, 3.10, 3.12.

from Chapter 3 from the textbook. Please show all your work and be neat and circle your answers.

2) Cadence exercise (20 points):

Repeat Transistor I-V characteristics exercise from the Cadence wiki. Print out and submit Transistor schematics and I-V graph just like it is done in the Cadence wiki. To access Cadence you need to login to a Linux Machine at

https://linuxlab.engr.wustl.edu

Once you login to the Linux Machine start running Cadence exercise.

CSE 563M students only:

In addition to the problem set above, please solve the following problem from the textbook: 3.9 (10 points).

3.1

+ We know the following:

$t_{ox} = 1.6\, nm = 1.6 \cdot 10^{-7} cm$

$\Phi_{GC} = -1.04\, V$

$N_A = 2.8 \cdot 10^{18} cm^{-3}$

$Q_{ox} = q4 \cdot 10^{10} C/cm^2 = 1.6 \cdot 10^{-19} \cdot 4 \cdot 10^{10} C/cm^2 = 6.4 \cdot 10^{-9} C/cm^2$

$\varepsilon_{ox} = 3.97\varepsilon_0 = 3.97 \cdot 8.85 \cdot 10^{-14}\, F/cm \ \ (1\, Farad = 1\, \frac{C}{V})$

$\varepsilon_{si} = 11.7\varepsilon_0 = 11.7 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} (1\, Farad = 1\, \frac{C}{V})$

$n_i = 1.45 \cdot 10^{10} cm^{-3}$

$k = 1.38 \cdot 10^{-23} J/K$

$q = 1.6 \cdot 10^{-19} C$

+ To get the threshold voltage, solve the equation below:

$$V_{T0} = \Phi_{GC} - 2\phi_F(sub) - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

+ First, $\phi_F(sub)$:

$$\phi_F(sub) = \frac{kT}{q} \ln\left(\frac{n_i}{N_A}\right)$$

$$\frac{kT}{q} = \frac{1.38 \cdot 10^{-23} J/K \ \cdot 300K}{1.6 \cdot 10^{-19} C} = 0.026\ \mathrm{V}$$

$$\ln\left(\frac{n_i}{N_A}\right) = \ln\left(\frac{1.45 \cdot 10^{10} cm^{-3}}{2.8 \cdot 10^{18} cm^{-3}}\right) = -19$$

$$\phi_F(sub) = \frac{kT}{q} \ln\left(\frac{n_i}{N_A}\right) = 0.026V \cdot -19 = -0.496V$$

+ Second, Q_{B0}:

$$Q_{B0} = -\sqrt{2 \cdot q \cdot N_A \cdot \varepsilon_{si} \cdot |-2\phi_F|}$$

$$= -\sqrt{2 \cdot (1.6 \cdot 10^{-19} C) \cdot (2.8 \cdot 10^{18} cm^{-3}) \cdot (11.7 \cdot 8.85 \cdot 10^{-14}\, F/cm) \cdot |-2(-0.496V)|}$$

$$= -9.593 \cdot 10^{-7} C/cm^2$$

+ Third, C_{ox}:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.97 \cdot 8.85 \cdot 10^{-14}\, F/cm}{1.6 \cdot 10^{-7} cm} = 2.195 \cdot 10^{-6} F/cm^2$$

+ Finally, plug in all values above into equation:

$$V_{T0} = \Phi_{GC} - 2\phi_F(sub) - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

$$= -1.04 - 2(-0.496) - \frac{-9.593 \cdot 10^{-7}}{2.195 \cdot 10^{-6}} - \frac{6.4 \cdot 10^{-9}}{2.195 \cdot 10^{-6}}$$

$$= 0.386V$$

3.1.b.

+ The amount of channel implant:

$$\Delta V = 0.6 - V_{T0} = 0.6 - 0.386 = 0.214 = \frac{qN_1}{C_{ox}}$$

$$N_I = \frac{\Delta V \cdot C_{ox}}{q} = \frac{0.214 \cdot 2.195 \cdot 10^{-6}}{1.6 \cdot 10^{-19}} = 2.953 \cdot 10^{12} cm^{-2}$$

+ The type is p-type since threshold voltage should be increased.

3.2

+ Constant values:

$$\varepsilon_{si} = 11.7\varepsilon_0 = 11.7 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} (1\, Farad = 1\, \frac{C}{V})$$

$$n_i = 1.45 \cdot 10^{10} cm^{-3}$$

$$q = 1.6 \cdot 10^{-19} C$$

+ The expression for the junction capacitance:

$$C_j(V) = A \sqrt{\frac{\varepsilon_{si} \cdot q}{2}\left(\frac{N_A \cdot N_D}{N_A + N_D}\right)} \cdot \frac{1}{\sqrt{\phi_0 - V}}$$

+ for ϕ_0:

$$\phi_0 = \frac{kT}{q}\ln\left(\frac{N_A \cdot N_D}{{n_i}^2}\right) = 0.026\ln\left(\frac{2\cdot 10^{20}\cdot 2\cdot 10^{20}}{(1.45\cdot 10^{10})^2}\right) = 1.214$$

+ for A:

$A = 0.2\cdot 0.4 + 2(0.2\cdot 0.032) + 2(0.4\cdot 0.032) = 1.184\cdot 10^{-9}$

+ Plug-in values:

$$C_j(V) = 1.184\ \cdot 10^{-9}\sqrt{\frac{11.7\cdot 8.85\cdot 10^{-14}\cdot 1.6\cdot 10^{-19}}{2}\left(\frac{4\cdot 10^{40}}{4\cdot 10^{20}}\right)}\cdot\frac{1}{\sqrt{1.214+1.2}} = 2.193\cdot 10^{-15}$$

3.3.

They are not identical.

The electric channel length and the mask channel length have a relationship below:

$L = L_M - 2L_D, where\ L_D\ is\ the\ lateral\ diffusion\ length.$

3.6

+ we know the following:

$N_D = 2\cdot 10^{20} cm^{-3}$

$N_A = 2\cdot 10^{20} cm^{-3}$

$X_j = 32\ nm$

$L_D = 10\ nm$

$t_{ox} = 1.6\ nm$

$V_{T0} = 0.53\ V$

$Channel\ stop\ doping = 16.0\ X\ (p\ type\ substrate\ doping)$

$\varepsilon_{si} = 11.7\varepsilon_0 = 11.7\cdot 8.85\cdot 10^{-14}\frac{F}{cm}(1\ Farad = 1\ \frac{C}{V})$

$q = 1.6\cdot 10^{-19} C$

+ Drain capacitance equation:

$C_{drain} = A\cdot C_{j0}\cdot K_{eq} + P\cdot C_{jsw}K_{eq(sw)}$

+ for ϕ_0:

$$\phi_0 = \frac{kT}{q}\ln\left(\frac{N_A \cdot N_D}{{n_i}^2}\right) = 0.026\ln\left(\frac{2\cdot 10^{20}\cdot 2\cdot 10^{20}}{(1.45\cdot 10^{10})^2}\right) = 1.21$$

+ for ϕ_{0sw}:

$$\phi_{0sw} = \frac{kT}{q}\ln\left(\frac{N_{A(sw)} \cdot N_D}{{n_i}^2}\right) = 0.026\ln\left(\frac{16\cdot 2\cdot 10^{20}\cdot 2\cdot 10^{20}}{(1.45\cdot 10^{10})^2}\right) = 1.28$$

+ for C_{j0}, C_{jsw}:

$$C_{j0} = \sqrt{\frac{\varepsilon_{si}\cdot q}{2}\left(\frac{N_A\cdot N_D}{N_A+N_D}\right)}\cdot\frac{1}{\sqrt{\phi_0}}$$

$$= \sqrt{\frac{11.7\cdot 8.85\cdot 10^{-14}\cdot 1.6\cdot 10^{-19}}{2}\left(\frac{4\cdot 10^{40}}{4\cdot 10^{20}}\right)}\cdot\frac{1}{\sqrt{1.21}} = 2.61\cdot 10^{-6}\ F/cm^2$$

$$C_{j0sw} = \sqrt{\frac{\varepsilon_{si}\cdot q}{2}\left(\frac{N_{A(sw)}\cdot N_D}{N_A+N_D}\right)}\cdot\frac{1}{\sqrt{\phi_{0sw}}}$$

$$= \sqrt{\frac{11.7\cdot 8.85\cdot 10^{-14}\cdot 1.6\cdot 10^{-19}}{2}\left(\frac{4\cdot 10^{40}}{4\cdot 10^{20}}\right)}\cdot\frac{1}{\sqrt{1.28}} = 3.49\cdot 10^{-6}\ F/cm^2$$

$$C_{jsw} = X_j C_{j0sw} = 32\cdot 10^{-9}\cdot 3.49\cdot 10^{-6} = 0.11\cdot 10^{-12}\ F/cm$$

+ for A:

$$A = Y\cdot W = 6\cdot 10 = 60\ \mu m^2 = 60\ \cdot 10^{-8} cm^2$$

+ for P:

$$P = 2(Y+W) = 2(6+10) = 32\mu m = 32\cdot 10^{-4}\ cm$$

+ for $K_{eq}, K_{eq(sw)}$:

$$K_{eq} = -\frac{2\sqrt{\phi_0}}{V_2-V_1}\left(\sqrt{\phi_0 - V_2} - \sqrt{\phi_0 - V_1}\right) = 0.759$$

$$K_{eq(sw)} = -\frac{2\sqrt{\phi_{0sw}}}{V_2-V_1}\left(\sqrt{\phi_{0sw} - V_2} - \sqrt{\phi_{0sw} - V_1}\right) = 0.768$$

+ Now, we can plugin values into Drain capacitance equation:

$$C_{drain} = A\cdot C_{j0}\cdot K_{eq} + P\cdot C_{jsw}\cdot K_{eq(sw)}$$

$$= 60\ \cdot 10^{-8}\cdot 2.61\cdot 10^{-6}\cdot 0.759 + 32\cdot 10^{-4}\cdot 0.11\cdot 10^{-12}\ \cdot 0.768$$

$$= 1.188\ pF$$

3.10

+ we know the following:

$k' = 168\, \mu A/V^2$

$V_{T0} = 0.48\, V$

$\gamma = 0.52\, V^{1/2}$

$|2\phi_F| = 1.01\, V$

+ Gate voltage is high and the Vx is expected to low. The load is in saturation and the driver is in linear region.

+ for V_T:

$V_T = V_{T0} + \gamma\left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|}\right)$

$V_T = 0.48 + 0.52\left(\sqrt{1.01 + V_x} - \sqrt{1.01}\right)$

+ From KCL:

$I_D = I_{D,Driver} = I_{D,Load}$

$\frac{1}{2}k'\frac{W}{L}(1 - V_x - V_{T,L}(V_x))^2 = \frac{1}{2}k'\frac{W}{L}(2(1 - V_{T0})V_x - {V_x}^2)$

$(1 - V_x - V_{T,L}(V_x))^2 = (2(1 - 0.48)V_x - {V_x}^2)$

$(1 - V_x - V_{T,L}(V_x))^2 = 1.04V_x - {V_x}^2$

+ To get Vx, plug in V_T into KCL equation:

$(1 - V_x - V_{T,L}(V_x))^2 = 1.04V_x - {V_x}^2$

$(1 - V_x - (0.48 + 0.52(\sqrt{1.01 + V_x} - \sqrt{1.01})))^2 = 1.04V_x - {V_x}^2$

$V_x = 0.1357$

+ To get Id, plug in values into KCL equation:

$I_D = I_{D,Driver} = I_{D,Load} = \frac{1}{2}k'\frac{W}{L}\left(1.04V_x - {V_x}^2\right) = \frac{1}{2} \cdot 168 \cdot 10 \cdot (1.04 \cdot 0.1357 - 0.1357^2) = 103.1\, \mu A$

3.12

+ For λ:

Choose row2 and row4 from the table.

$\lambda = \frac{I_{D1} - I_{D2}}{V_{DS1}I_{D2} - V_{DS2}I_{D1}}$

$\lambda = \frac{59 - 60}{0.8 \cdot 60 - 1.0 \cdot 59} = 0.09\, V^{-1}$

+ For V_{T0}:

Choose row1 and row2 from the table.

$$\frac{I_{D1}}{I_{D2}} = \frac{(V_{GS1} - V_{T0})^2}{(V_{GS2} - V_{T0})^2}$$

$$V_{T0} = \frac{\sqrt{\frac{I_{D1}}{I_{D2}}}V_{GS2} - V_{GS1}}{\sqrt{\frac{I_{D1}}{I_{D2}}} - 1}$$

$$V_{T0} = \frac{\sqrt{\frac{8}{59}} \cdot 0.8 - 0.6}{\sqrt{\frac{8}{59}} - 1} = 0.48\, V$$

+ For k:

Choose row2 from the table.

$$I_{DSAT} = \frac{k}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

$$59 = \frac{k}{2}(0.8 - 0.48)^2(1 + 0.09 \cdot 0.8)$$

$$k = 1.08\, mA/V^2$$

$$2I_{DSAT}/\,((V_{GS} - V_T)^2(1 + \lambda V_{DS})) = k$$

+ For γ:

Choose row3 from the table.

$$37 = \frac{1075}{2}(0.8 - V_T(V_{BS} = -0.3))^2(1 + 0.09 \cdot 0.8)$$

$$V_T(V_{BS} = -0.3) = 0.55\, V$$

$$V_T = V_{T0} + \gamma\left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|}\right)$$

$$0.55 = 0.48 + \gamma\left(\sqrt{|\,1.1 + 0.3|} - \sqrt{|1.1|}\right)$$

$$\gamma = 0.52\, V^{1/2}$$

3.9(For 563M)

+ We know the following:

$$t_{ox} = 1.6\, nm = 1.6 \cdot 10^{-7} cm$$

$\Phi_{GC} = -1.04\,V$

$N_{D,sub} = 2 \cdot 10^{16} cm^{-3}$

$N_{D,poly} = 10^{20} cm^{-3}$

$Q_{ox} = q4 \cdot 10^{10} C/cm^2 = 1.6 \cdot 10^{-19} \cdot 4 \cdot 10^{10} C/cm^2 = 6.4 \cdot 10^{-9} C/cm^2$

$\varepsilon_{ox} = 3.97\varepsilon_0 = 3.97 \cdot 8.85 \cdot 10^{-14}\, F/cm \;\; (1\, Farad = 1\, \frac{C}{V})$

$\varepsilon_{si} = 11.7\varepsilon_0 = 11.7 \cdot 8.85 \cdot 10^{-14} \frac{F}{cm} (1\, Farad = 1\, \frac{C}{V})$

$n_i = 1.45 \cdot 10^{10} cm^{-3}$

$k = 1.38 \cdot 10^{-23} J/K$

$q = 1.6 \cdot 10^{-19} C$

$$\frac{kT}{q} = \frac{1.38 \cdot 10^{-23} J/K \cdot 300K}{1.6 \cdot 10^{-19} C} = 0.026\text{ V}$$

+ To get the Φ_{GC}:

$\Phi_{GC} = \phi_F(sub) - \phi_F(gate)$

$$\phi_F(sub) = \frac{kT}{q} \ln\left(\frac{N_{D,sub}}{n_i}\right) = 0.026 \cdot \ln\left(\frac{2 \cdot 10^{16}}{1.45 \cdot 10^{10}}\right) = 0.367\, V$$

$$\phi_F(gate) = \frac{kT}{q} \ln\left(\frac{N_{D,poly}}{n_i}\right) = 0.026 \cdot \ln\left(\frac{10^{20}}{1.45 \cdot 10^{10}}\right) = 0.589\; V$$

$\Phi_{GC} = 0.367 - 0.589 = -0.221\, V$

+ Second, Q_{B0}:

$Q_{B0} = \sqrt{2 \cdot q \cdot N_{D,sub} \cdot \varepsilon_{si} \cdot |2\phi_F|}$

$= \sqrt{2 \cdot (1.6 \cdot 10^{-19} C) \cdot (2 \cdot 10^{16} cm^{-3}) \cdot (11.7 \cdot 8.85 \cdot 10^{-14}\, F/cm) \cdot |2 \cdot (0.374V)|}$

$= 6.974 \cdot 10^{-8} C/cm^2$

+ Third, C_{ox}:

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.97 \cdot 8.85 \cdot 10^{-14}\, F/cm}{1.6 \cdot 10^{-7} cm} = 2.195 \cdot 10^{-6} F/cm^2$$

+ Finally, plug in all values above into equation:

$$V_{T0} = \Phi_{GC} - 2\phi_F(sub) - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

$$= -0.221 - 2(0.367) - \frac{6.974 \cdot 10^{-8}}{2.195 \cdot 10^{-6}} - \frac{6.4 \cdot 10^{-9}}{2.195 \cdot 10^{-6}}$$

$$= -0.9896\, V$$