

$$\frac{\Theta_L}{V_{in}} = \frac{\left(\frac{K_g K_m}{J R_m}\right)}{s\left(s + \frac{K_g^2 K_m^2}{J R_m}\right)} \dots \text{(eqn. 13 in lab document)}$$

Solve for  $V_{in}$ :

$$V_{in} = \Theta_L \frac{s\left(s + \frac{K_g^2 K_m^2}{J R_m}\right)}{\left(\frac{K_g K_m}{J R_m}\right)}$$

Set equal to eqn. 16 (Laplace transform of eqn. 15):

$$\Theta_L \frac{s\left(s + \frac{K_g^2 K_m^2}{J R_m}\right)}{\left(\frac{K_g K_m}{J R_m}\right)} = K_p(\Theta_0 - \Theta_L) - s K_d \Theta_L$$

Pull  $\Theta_L$  terms to one side:

$$\Theta_L \left[ \frac{s\left(s + \frac{K_g^2 K_m^2}{J R_m}\right)}{\left(\frac{K_g K_m}{J R_m}\right)} + K_p + s K_d \right] = K_p \Theta_0$$

Rearrange:

$$\frac{\Theta_L}{\Theta_0} = \frac{K_p}{s^2 \left(\frac{J R_m}{K_g K_m}\right) + s K_g K_m + K_p + s K_d}$$

Multiply through by  $\frac{K_g K_m}{K_g K_m}$ :

$$\frac{\Theta_L}{\Theta_0} = \frac{K_p K_g K_m}{s^2 J R_m + s K_g^2 K_m^2 + K_p K_g K_m + s K_d K_g K_m}$$

Multiply through by  $\frac{J R_m}{J R_m}$ :

$$\frac{\Theta_L}{\Theta_0} = \frac{\left(\frac{K_p K_g K_m}{J R_m}\right)}{s^2 + s \left(\frac{K_g^2 K_m^2}{J R_m}\right) + \left(\frac{K_p K_g K_m}{J R_m}\right) + s \left(\frac{K_d K_g K_m}{J R_m}\right)}$$

Rearrange in Transfer Function form:

$$\frac{\Theta_L}{\Theta_0} = \frac{\left(\frac{K_p K_g K_m}{J R_m}\right)}{s^2 + s \left(\frac{K_g^2 K_m^2}{J R_m} + \frac{K_d K_g K_m}{J R_m}\right) + \left(\frac{K_p K_g K_m}{J R_m}\right)}$$