# MSA 8190 Statistical Foundations

## Fall 2016

## Assignment 4

## Issued: September 27, 2016

## Due: October 4, 2016

**Note**: Use R for computations in the following questions and include the R code in your solution sheet. Moreover, upload the R script files in iCollege.

**Problem 1**
Consider the following function:

| $x$ | $y$ | $f_{XY}(x, y)$ |
|---|---|---|
| 1 | 1 | 1/4 |
| 1.5 | 2 | 1/8 |
| 1.5 | 3 | 1/4 |
| 2.5 | 4 | 1/4 |
| 3 | 5 | 1/8 |

(a) Show that $f_{XY}(x, y)$ satisfies the properties of a joint probability mass function.

Determine the following:

(b) $\mathbb{P}(X < 2.5, Y < 3)$

(c) $\mathbb{P}(X < 2.5)$

(d) $\mathbb{P}(Y < 3)$

(e) $\mathbb{P}(X > 1.8, Y > 4.7)$

(f) The marginal probability distribution of the random variable $X$.

(g) The conditional probability distribution of $Y$ given that $X = 1.5$.

(h) The conditional probability distribution of $X$ given that $Y = 2$.

(i) $\mathbb{E}(Y|X = 1.5)$

(j) Are X and Y independent?

(k) $\text{Cov}(x, y)$

(l) $\rho_{x,y}$

**Problem 2**
Consider the joint probability mass function $f_{XY}(x, y) = c(x + y)$ over the nine points with $x = 1, 2, 3$ and $y = 1, 2, 3$. Determine

(a) $c$

(b) $\mathbb{P}(X = 1, Y < 4)$

(c) $\mathbb{P}(X = 1)$

(d) $\mathbb{P}(Y = 2)$

(e) $\mathbb{P}(X \leq 2, Y \leq 2)$

(f) $\mathbb{E}(X)$

(g) $\mathbb{E}(Y)$

(h) $\mathrm{Var}(X)$

(i) $\mathrm{Var}(Y)$

(j) The marginal probability distribution of the random variable $X$.

(k) The conditional probability distribution of $Y$ given that $X = 1$.

(l) The conditional probability distribution of $X$ given that $Y = 2$.

(m) $\mathbb{E}(Y|X = 1)$

(n) Are $X$ and $Y$ independent?

(o) $\mathrm{Cov}(x, y)$

(p) $\rho_{x,y}$

**Problem 3**
A manufacturing company employs two inspecting devices to sample a fraction of their output for quality control purposes. The first inspection monitor is able to accurately detect 99.3% of the defective items it receives, whereas the second is able to do so in 99.7% of the cases. Assume that four defective items are produced and sent out for inspection. Let $X$ and $Y$ denote the number of items that will be identified as defective by inspecting devices 1 and 2, respectively. Assume the devices are independent. Determine

(a) $f_{XY}(x, y)$

(b) $f_X(x)$

(c) $\mathbb{E}(X)$

(d) $f_{Y|x=2}(y)$

(e) $\mathbb{E}(Y|X=2)$

(f) $\text{Var}(Y|X=2)$

(g) Are $X$ and $Y$ independent?

(h) $\text{Cov}(x,y)$

(i) $\rho_{x,y}$

**Problem 4**

Suppose the random variables $X$, $Y$, and $Z$ have the following joint probability distribution:

| $x$ | $y$ | $z$ | $f_{XYZ}(x,y,z)$ |
|---|---|---|---|
| 1 | 1 | 1 | 0.05 |
| 1 | 1 | 2 | 0.10 |
| 1 | 2 | 1 | 0.15 |
| 1 | 2 | 2 | 0.20 |
| 2 | 1 | 1 | 0.20 |
| 2 | 1 | 2 | 0.15 |
| 2 | 2 | 1 | 0.10 |
| 2 | 2 | 2 | 0.05 |

Determine the following:

(a) $\mathbb{P}(X=2)$

(b) $\mathbb{P}(X=1, Y=2)$

(c) $\mathbb{P}(Z<1.5)$

(d) $\mathbb{P}(X=1 \text{ or } Z=2)$

(e) $\mathbb{E}(X)$

(f) $\mathbb{P}(X=1|Y=1)$

(g) $\mathbb{P}(X=1, Y=1|Z=2)$

(h) $\mathbb{P}(X=1|Y=1, Z=2)$

(i) Conditional probability distribution of $X$ given that $Y=1$ and $Z=2$.

(j) $\text{Cov}(x,y)$

(k) $\text{Cov}(x,z)$

(l) $\mathrm{Cov}(y, z)$

**Problem 5**

A marketing company performed a risk analysis for a manufacturer of synthetic fibers and concluded that new competitors present no risk 13% of the time (due mostly to the diversity of fibers manufactured), moderate risk 72% of the time (some overlapping of products), and very high risk (competitor manufactures the exact same products) 15% of the time. It is known that 12 international companies are planning to open new facilities for the manufacture of synthetic fibers within the next three years. Assume the companies are independent. Let $X$, $Y$, and $Z$ denote the number of new competitors that will pose no, moderate, and very high risk for the interested company, respectively.

(a) What is the range of the joint probability distribution of $X$, $Y$, and $Z$?

(b) Determine $\mathbb{P}(X = 1, Y = 3, Z = 1)$.

(c) Determine $\mathbb{P}(Z \leq 2)$.

**Problem 6**

Consider the joint probability density function $f_{XY}(x, y) = c(x+y)$ over the range $0 < x < 3$ and $x < y < x + 2$. Determine

(a) $c$

(b) $\mathbb{P}(X < 1, Y < 2)$

(c) $\mathbb{P}(1 < X < 2)$

(d) $\mathbb{P}(Y > 1)$

(e) $\mathbb{P}(X < 2, Y < 2)$

(f) $\mathbb{E}(X)$

(g) Marginal probability distribution of $X$

(h) Conditional probability distribution of $Y$ given that $X = 1$

(i) $\mathbb{E}(Y|X = 1)$

(j) $\mathbb{P}(Y > 2|X = 1)$

(k) Conditional probability distribution of $X$ given that $Y = 2$

(l) $\mathrm{Cov}(x, y)$

(m) $\rho_{x,y}$

**Problem 7**
Suppose the random variables $X$, $Y$, and $Z$ have the joint probability density function $f_{XYZ}(x, y, z) = c$ over the cylinder $x^2 + y^2 < 4$ and $0 < z < 4$. Determine the following.

(a) $c$

(b) $\mathbb{P}(X^2 + Y^2 < 2)$

(c) $\mathbb{P}(Z < 2)$

(d) $\mathbb{E}(X)$

(e) $\mathbb{P}(X < 1|Y = 1)$

(f) $\mathbb{P}(X^2 + Y^2 < 1|Z = 1)$

(g) Conditional probability distribution of $Z$ given that $X = 1$ and $Y = 1$.

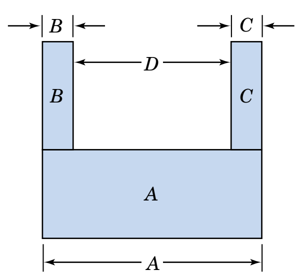
(h) $\text{Cov}(x, y)$

(i) $\text{Cov}(x, z)$

(j) $\text{Cov}(y, z)$

**Problem 8**
Let $X$ and $Y$ represent two dimensions of an injection molded part. Suppose $X$ and $Y$ have a bivariate normal distribution with $\sigma_X = 0.04$, $\sigma_Y = 0.08$, $\mu_X = 3.00$, $\mu_Y = 7.70$, and $\rho = 0$. Determine $\mathbb{P}(2.95 < X < 3.05, 7.60 < Y < 7.80)$.

**Problem 9**
A U-shaped component is to be formed from the three parts A, B, and C. The picture is shown in Fig. 5-20. The length of A is normally distributed with a mean of 10 millimeters and a standard deviation of 0.1 millimeter. The thickness of parts B and C is normally distributed with a mean of 2 millimeters and a standard deviation of 0.05 millimeter. Assume all dimensions are independent.




(a) Determine the mean and standard deviation of the length of the gap D.

(b) What is the probability that the gap D is less than 5.9 millimeters?