MSA 8190 Statistical Foundations

Fall 2016

Assignment 3

Issued: September 20, 2016

Due: September 27, 2016

Note: Use R for computations in the following questions and include the R code in your solution sheet.

Problem 1
Suppose the cumulative distribution function of the random variable X is

$$F(X) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

Determine the following:

(a) $\mathbb{P}(X < 1.8)$

(b) $\mathbb{P}(X > -1.4)$

(c) $\mathbb{P}(X < -2)$

(d) $\mathbb{P}(-1 < X < 1)$

Problem 2
Determine the probability density function for each of the following cumulative distribution functions.

(a) $F(X) = 1 - e^{-2x}, x > 0$

(b)

$$F(X) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 4 \\ 0.04x + 0.64 & 4 \leq x < 9 \\ 1 & 9 \leq x \end{cases}$$

Problem 3
Suppose $f(X) = 1.5x^2$ for $-1 < x < 1$. Determine the cumulative distribution function, mean and variance of X.

Problem 4
Suppose the time it takes a data collection operator to fill out an electronic form for a database is uniformly between 1.5 and 2.2 minutes.

(a) What is the mean and variance of the time it takes an operator to fill out the form?

(b) What is the probability that it will take less than two minutes to fill out the form?

(c) Determine the cumulative distribution function of the time it takes to fill out the form.

Problem 5
Assume Z has a standard normal distribution. Determine the value for z that solves each of the following:

(a) $\mathbb{P}(Z < z) = 0.9$

(b) $\mathbb{P}(Z < z) = 0.5$

(c) $\mathbb{P}(Z > z) = 0.1$

(d) $\mathbb{P}(Z > z) = 0.9$

(e) $\mathbb{P}(-1.1 < Z < z) = 0.8$

Problem 6
Assume Z has a standard normal distribution. Determine the value for z that solves each of the following:

(a) $\mathbb{P}(-z < Z < z) = 0.9$

(b) $\mathbb{P}(-z < Z < z) = 0.99$

(c) $\mathbb{P}(-z < Z < z) = 0.68$

(d) $\mathbb{P}(-z < Z < z) = 0.9973$

Problem 7
The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.

(a) What is the probability a fill volume is less than 12 fluid ounces?

(b) If all cans less than 12.1 or greater than 12.6 ounces are scrapped, what proportion of cans is scrapped?

(c) Determine specifications that are symmetric about the mean that include 99% of all cans.

Problem 8

An electronic office product contains 5000 electronic components. Assume that the probability that each component operates without failure during the useful life of the product is 0.999, and assume that the components fail independently. Approximate the probability that 10 or more of the original 5000 components fail during the useful life of the product.

Problem 9

A high-volume printer produces minor print-quality errors on a test pattern of 1000 pages of text according to a Poisson distribution with a mean of 0.4 per page.

(a) What is the mean number of pages with errors (one or more)?

(b) Approximate the probability that more than 350 pages contain errors (one or more).

Problem 10

The time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

Problem 11

The time between calls to a plumbing supply business is exponentially distributed with a mean time between calls of 15 minutes.

(a) What is the probability that there are no calls within a 30-minute interval?

(b) What is the probability that at least one call arrives within a 10-minute interval?

(c) What is the probability that the first call arrives within 5 and 10 minutes after opening?

(d) Determine the length of an interval of time such that the probability of at least one call in the interval is 0.90.

Problem 12
In a data communication system, several messages that arrive at a node are bundled into a packet before they are transmitted over the network. Assume the messages arrive at the node according to a Poisson process with $\lambda = 30$ messages per minute. Five messages are used to form a packet.

(a) What is the mean and standard deviation of the time until a packet is formed?

(b) What is the probability that a packet is formed in less than 10 seconds?

(c) What is the probability that a packet is formed in less than 5 seconds?

Problem 13
Calls to the help line of a large computer distributor follow a Poisson distribution with a mean of 20 calls per minute.

(a) What is the mean time until the one-hundredth call?

(b) What is the mean time between call numbers 50 and 80?

(c) What is the probability that three or more calls occur within 15 seconds?

Problem 14
The life of a recirculating pump follows a Weibull distribution with parameters $\beta = 2$, and $\delta = 700$ hours.

(a) Determine the mean life of a pump.

(b) Determine the variance of the life of a pump.

(c) What is the probability that a pump will last longer than its mean?

Problem 15
The length of time (in seconds) that a user views a page on a Web site before moving to another page is a lognormal random variable with parameters $\theta = 0.5$ and $\omega^2 = 1$

(a) What is the probability that a page is viewed for more than 10 seconds?

(b) What is the length of time that 50% of users view the page?

(c) What is the mean and standard deviation of the time until a user moves from the page?