MSA 8190 Statistical Foundations

Fall 2016

Assignment 5

Issued: October 4, 2016

Due: October 11, 2016

Note: Excel file for Problems 1,2 and 9 can be found in D2L. Use R for computations in the following questions and include the R code in your solution sheet. Moreover, upload the R script
les in iCollege.

Problem 1
The percentage of cotton in material used to manufacture mens shirts follows.

(a) Compute the sample mean, variance and median.

(b) Construct a stem-and-leaf display for the data.

(c) Construct a frequency distribution and histogram for the cotton content.

(d) Construct a box plot of the data and comment on the information in this display.

34.2	37.8	33.6	32.6	33.8	35.8	34.7	34.6
33.1	36.6	34.7	33.1	34.2	37.6	33.6	33.6
34.5	35.4	35.0	34.6	33.4	37.3	32.5	34.1
35.6	34.6	35.4	35.9	34.7	34.6	34.1	34.7
36.3	33.8	36.2	34.7	34.6	35.5	35.1	35.7
35.1	37.1	36.8	33.6	35.2	32.8	36.8	36.8
34.7	34.0	35.1	32.9	35.0	32.1	37.9	34.3
33.6	34.1	35.3	33.5	34.9	34.5	36.4	32.7

Problem 2
The following data are the viscosity measurements for a chemical product observed hourly (read down, then left to right).

(a) Construct and interpret either a digidot plot or a separate stem-and-leaf and time series plot of these data.

(b) Specifications on product viscosity are at 48 ± 2. What conclusions can you make about process performance?

47.9	48.6	48.0	48.1	43.0	43.2
47.9	48.8	47.5	48.0	42.9	43.6
48.6	48.1	48.6	48.3	43.6	43.2
48.0	48.3	48.0	43.2	43.3	43.5
48.4	47.2	47.9	43.0	43.0	43.0
48.1	48.9	48.3	43.5	42.8	
48.0	48.6	48.5	43.1	43.1	

Problem 3

Suppose the mean of a population is known (μ), show that the following estimator is a biased estimator for the variance

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(X_i - \mu)^2}{n-1}$$

Modify this estimator such that it becomes an unbiased estimator.

Problem 4

Suppose X has Bernoulli distribution with parameter p. Show that the sample mean $\bar{X}$ is a MVUE of p.

Problem 5

Use the method of moments to build estimators for the parameters of the following distributions:

(a) Bernoulli

(b) Exponential

(c) Lognormal

Problem 6

Use the MLE method to build estimators for the parameters of the following distributions:

(a) Bernoulli

(b) Exponential

(c) Lognormal

Problem 7
Let X be a random variable with the following probability distribution:

$$f(x) = \begin{cases} (\theta+1)x^{\theta}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Use the method of moments and MLE to estimate θ based on a sample of size n.

Problem 8
Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature (x) and pavement deflection (y). Summary quantities were $n = 20$, $\sum_{i=1}^{n} y_i = 12.7$, $\sum_{i=1}^{n} y_i^2 = 8.8$, $\sum_{i=1}^{n} x_i = 1487$, $\sum_{i=1}^{n} x_i^2 = 143,215$, $\sum_{i=1}^{n} x_i y_i = 1083$.

(a) Calculate the least squares estimates of the slope and intercept. Graph the regression line.

(b) Use the equation of the fitted line to predict what pavement deflection would be observed when the surface temperature is 85 °F.

(c) What is the mean pavement deflection when the surface temperature is 90 °F?

(d) What change in mean pavement deflection would be expected for a 1 °F change in surface temperature?

Problem 9
A rocket motor is manufactured by bonding together two types of propellants, an igniter and a sustainer. The shear strength of the bond y is thought to be a linear function of the age of the propellant x when the motor is cast. Twenty observations are shown in the table below (Excel file also uploaded in D2L).

(a) Draw a scatter diagram of the data. Does the straight-line regression model seem to be plausible?

(b) Find the least squares estimates of the slope and intercept in the simple linear regression model. Find an estimate of σ^2.

(c) Estimate the mean shear strength of a motor made from propellant that is 20 weeks old.

Observation Number	Strength y (psi)	Age x (weeks)	Observation Number	Strength y (psi)	Age x (weeks)
1	2158.70	15.50	11	2165.20	13.00
2	1678.15	23.75	12	2399.55	3.75
3	2316.00	8.00	13	1779.80	25.00
4	2061.30	17.00	14	2336.75	9.75
5	2207.50	5.00	15	1765.30	22.00
6	1708.30	19.00	16	2053.50	18.00
7	1784.70	24.00	17	2414.40	6.00
8	2575.00	2.50	18	2200.50	12.50
9	2357.90	7.50	19	2654.20	2.00
10	2277.70	11.00	20	1753.70	21.50