MSA 8190 Statistical Foundations

Fall 2016

Assignment 1

Issued: August 30, 2016

Due: September 13, 2016

Problem 1

1. Prove the validity of associativity and distribution properties for set. Hint: Use the definitions of equality, union and intersection for sets.
2. Illustrate the above results by Venn diagram.

Problem 2

1. Prove $A \backslash B = A \cap \bar{B} = A \backslash (A \cap B)$.
2. Show that if $A \subset B$, then $\bar{B} \subset \bar{A}$.
3. Illustrate the above results by Venn diagram.

Problem 3
Show that if events E and F are independent, then

1. $P(E \cap F) = P(E)P(F)$
2. $\bar{E}$ and F are independent
3. E and $\bar{F}$ are independent
4. $\bar{E}$ and $\bar{F}$ are independent

Problem 4
Let μ be a measure on the measure space $(\Omega, \mathcal{F}, \mu)$. Prove that

$$\mu(\cup_i A_i) \leq \sum_i \mu(A_i)$$

.

Problem 5
Let

$$h(x) = \begin{cases} 0 & \text{if } x < -1, \\ 2 & \text{if } -1 \le x < 1, \\ 5 & \text{if } 1 \le x < 2, \\ 6 & \text{if } 2 \le x. \end{cases}$$

Let $f : \mathbb{R} \to \mathbb{R}$ be any Borel measurable function. Derive an expression for $\int_{\mathbb{R}} f(x) d\mu_h$, where μ_h is the measure on $(\mathbb{R}, \mathcal{B})$ induced by h.

Problem 6
Let X be a random variable with cdf

$$F(x) = \begin{cases} 0 & \text{if } x < -1, \\ 0.2 & \text{if } -1 \le x < 1, \\ 0.5 & \text{if } 1 \le x < 2, \\ 1 - 0.5e^{2-x} & \text{if } 2 \le x. \end{cases}$$

Find $\mathbb{E}(X)$.

Problem 7 (OPTIONAL- Bonus points)
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Let X be a random variable on the probability space and $f : \mathbb{R} \to \mathbb{R}$ be a Borel measurable function. Prove that $f(X)$ is a random variable.

Note: Use Excel for computations in the following questions and include the R code in your solution sheet.

Problem 8
The sample space of a random experiment is $\{a, b, c, d, e, f\}$, and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	c	f
x	0	1	1.5	1.5	2	3

1. Determine the pmf and cdf of X.
2. Calculate $\mathbb{E}(X)$, σ^2, and σ.

Problem 9
Probability mass function of X is as follows:

$$f(x) = \frac{x+1}{2\theta}, \quad x = 0, 1, 2, 3, 4, 5$$

where θ is a constant.

1. Find θ such that $f(\cdot)$ is a valid pmf.
2. Calculate $\mathbb{E}(X)$, σ^2, and σ.