Honor code: I pledge on my honor that: I have completed all steps in the below homework on my own, I have not used any unauthorized materials while completing this homework, and I have not given anyone else access to my homework.

Name and Signature

1. If A is a $m \times m$ lower triangular matrix such that all its diagonal entries are 1. If you do LU factorisation of A then which of the following statements is/are true?

   a $L = A$ (**always**)

   b $U = I$ (**always**)

   A. Only a is true.

   B. Only b is true.

   C. Both a and b are true.

   D. Both a and b are false.

   Solution: There is ambiguity in the Q. Requesting TAs to give 4 marks to everyone who attempted this Q.

2. Which of the following statements is/are True (select all that are true)

   A. If A is a non-zero matrix (i.e. at least one of its elements in non-zero) then A^2 is **always** non-zero

   B. If A is a non-zero matrix (i.e. at least one of its elements in non-zero) then $A^\top A$ is always non-zero

   C. If A is not symmetric then A^{-1} can never be symmetric

   D. If LDU factorisation of a square symmetric matrix A exists then $U = L^\top$ (always).

   Solution: B, C, D (earlier it was wrongly mentioned that only B and D are correct). Requesting TAs to re-evaluate this Q.

3. Let A be any $m \times n$ matrix. Let U be the matrix obtained in its LU factorisation and let R be the reduced row echelon form of U. Which of the following statements is/are true (select all that are true)?

   A. The nullspace of U is always the same as the nullspace of R

B. The nullspace of A is always the same as the nullspace of R

C. The column space of U is always the same as the column space of R

D. The column space of A is **always** the same as the column space of U

Solution: A, B, C (earlier it was wrongly mentioned that only A and B are correct). Requesting TAs to re-evaluate this Q.

4. Which of the following statements are true? (select all statements that are true)

A. Any rank-1 matrix $A(m \times n)$ can always be written as $\mathbf{uv}^\top$ where $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{v} \in \mathbb{R}^n$.

B. If A and B are two rank-1 matrices then the rank of their product AB can never be greater than 1.

C. If A and B are two rank-1 matrices then the rank of their sum $A + B$ can **never** be greater than 1.

D. If A is a $m \times p$ matrix and B is a $p \times n$ matrix then $rank(A) \leq p$ (**always**) and $rank(B) \leq p$ (**always**) but the rank of AB can be greater than p.

Solution: A, B (earlier it was wrongly mentioned that A, B and D are correct). Requesting TAs to re-evaluate this Q.