Honor code: I pledge on my honor that: I have completed all steps in the below quiz on my own, I have not used any unauthorized materials while completing this quiz, and I have not given anyone else access to my quiz.

Name and Signature

1. The associative law of matrix multiplication says that $(AB)C = A(BC)$. If A is a 1×2 matrix, B is a 2×5 matrix and C is a 5×12 matrix then from the point of view of computational efficiency which one would you prefer:

   A. $A(BC)$

   B. $(AB)C$

   C. doesn't matter

2. Which of the following statements are true? (select all statements that are true)

   A. If A is a $m \times p$ matrix and B is a $p \times n$ matrix then $rank(A) \leq p$ (**always**) and $rank(B) \leq p$ (**always**) but the rank of AB can be greater than p.

   B. If A and B are two rank-1 matrices then the rank of their product AB can **never** be greater than 1.

   C. Any rank-1 matrix $A(m \times n)$ can **always** be written as $\mathbf{u}\mathbf{v}^\top$ where $\mathbf{u} \in \mathbb{R}^m$ and $\mathbf{v} \in \mathbb{R}^n$.

   D. If A and B are two rank-1 matrices then the rank of their sum $A + B$ can **never** be greater than 1.

3. The columns of this matrix are always independent if $c \neq 0$ $\begin{bmatrix} c & c & c \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{bmatrix}$

   A. True

   B. False

4. Which of the following statements is/are True (select all that are true)

   A. If A is a non-zero matrix (i.e. at least one of its elements in non-zero) then $A^\top A$ is **always** non-zero

   B. If A is not symmetric then A^{-1} can **never** be symmetric

   C. If A is a non-zero matrix (i.e. at least one of its elements in non-zero) then A^2 is **always** non-zero

   D. If LDU factorisation of a square symmetric matrix A exists then $U = L^\top$ (**always**).

5. . What multiple of $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ should be subtracted from the vector $\mathbf{b} = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$ so that the resulting vector is orthogonal to $\mathbf{a}$.

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6. Let $\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be a linear combination of the vectors $\mathbf{q_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\mathbf{q_2} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ such that among all linear combinations of $\mathbf{q_1}$ and $\mathbf{q_2}$, $\mathbf{p}$ is closest to the vector $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Then

$x =$ ____________
$y =$ ____________
$z =$ ____________

7. Consider a subspace in $\mathbf{R}^3$ such that for all elements in this subspace the first element is equal to the third element. The number of vectors in the basis of this subspace is

____

8. If A is a $m \times m$ lower triangular matrix such that all its diagonal entries are 1. If you do LU factorisation of A then which of the following statements is/are true?

   a $L = A$ (**always**)

   b $U = I$ (**always**)

   A. Only a is true.

   B. Only b is true.

   C. Both a and b are true.

   D. Both a and b are false.

9. What values of c and r will lead to this system of equations having infinite solutions?

$$
\begin{aligned}
1x + 4y + 3z &= 5 \\
2x + cy + z &= 7 \\
y - 5z &= r
\end{aligned}
$$

10. The determinant of the matrix $\begin{bmatrix} 13 & 23 & 53 \\ 12 & 22 & 52 \\ 11 & 21 & 51 \end{bmatrix}$ is ____________

11. Consider a $m \times n$ matrix with rank r. If there exists a b such that $A\mathbf{x} = \mathbf{b}$ has infinite solutions then which of the following statements **cannot** be True.

    A. $r = n$ and $n < m$

    B. $r = m$ and $m < n$

    C. $r < n$ and $m = n$

    D. $r < m$ and $r < n$

12. . Consider the vectors $\mathbf{u} = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$. The sum of the elements of any linear combination of $\mathbf{u}$ and $\mathbf{v}$ will be ____________

13. Consider the planes $x + 2y - 2z = 4$ and $x + y + z = 2$. These two planes intersect in a line. Consider a point on this line whose z-coordinate is 0. Find the x and y coordinate of this point.

    $x =$ ____________
    $y =$ ____________

14. If $AB = B$ and $BA = B$ then $A = I$

    A. False

    B. True

15. Which of the following statements are True

    A. If A is a 3×3 matrix such that $A_{ij} = ij$ then determinant of A is 1.

    B. If A is a 3×3 matrix such that $A_{ij} = i + j$ then determinant of A is 0.

16. Fill in the values of a, b, c, d so that the matrix A will be a rank-1 matrix. $A = \begin{bmatrix} a & 9 & b \\ 2 & c & d \\ 2 & 6 & -3 \end{bmatrix}$

    $a =$ ____________
    $b =$ ____________
    $c =$ ____________
    $d =$ ____________

17. If $\mathbf{u}$ and $\mathbf{v}$ are **orthonormal** vectors then (select all options that are correct)

    A. $||\mathbf{u} + \mathbf{v}|| = 1$

    B. $||\mathbf{u} - \mathbf{v}|| = 0$

    C. $||\mathbf{u} + \mathbf{v}|| = \sqrt{2}$

    D. $||\mathbf{u} - \mathbf{v}|| = \sqrt{2}$

18. Consider a 5×5 matrix, such that $a_{55} = 23$ (i.e., the last entry or the last diagonal element is 23). Suppose A is invertible (i.e., it has 5 pivots) and the last pivot of A is 16. If you were asked to make A not invertible by changing a_{55} what value would you set a_{55} to? In other words, what should a_{55} be so that A is not invertible (all other entries in the matrix will remain the same)

19. . Which of the following is true? (Note that $|\mathbf{v}^\top \mathbf{w}|$ is the absolute value of the dot product of the two vectors and $||\mathbf{v}||$ is the L_2 norm of $\mathbf{v}$.)

    A. $||\mathbf{v}|| \cdot ||\mathbf{w}|| \geq |\mathbf{v}^\top \mathbf{w}|$

    B. $||\mathbf{v}|| \cdot ||\mathbf{w}|| \leq |\mathbf{v}^\top \mathbf{w}|$

20. Let A be any $m \times n$ matrix. Let U be the matrix obtained in its LU factorisation and let R be the reduced row echelon form of U. Which of the following statements is/are true (select all that are true)?

    A. The nullspace of A is **always** the same as the nullspace of R

    B. The nullspace of U is **always** the same as the nullspace of R

    C. The column space of U is **always** the same as the column space of R

    D. The column space of A is **always** the same as the column space of U

21. Which of the following subsets of $\mathbb{R}^3$ is/are subspaces ? (select all that are correct)

    A. All vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_3 = b_1/b_2 + b_1$

    B. All vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1 = 0$ and $b_1 + b_2 = 5$

    C. All vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1 + b2 - b_3 = 0$

    D. All vectors $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1 + b_2 \geq b_2 + b_3$

22. How many free variables does this matrix have? $\begin{bmatrix} 4 & 8 & 2 & 8 & 2 \\ 2 & 4 & 1 & 4 & 2 \\ 0 & 0 & 3 & 12 & 3 \end{bmatrix}$ ____________

23. If B is obtained by exchanging the second and third rows of a 4×4 *invertible* matrix A then B is *always* invertible.

    A. True

    B. False

24. Which of the following statements is/are True (select all that are true)

    A. The intersection of two $n-2$ dimensional planes passing through the origin in $\mathbb{R}^n$ **can** be a $n-4$ dimensional plane ($n > 4$)

    B. If S and T are subspaces then $S \cup T$ is also a subspace

    C. If S is a two dimensional plane passing through the origin and T is a line passing through the origin then the smallest subspace containing both S and T will **always** be a 3 dimensional plane

    D. If S and T are subspaces then $S \cap T$ is also a subspace

25. Consider a square invertible matrix $A = \begin{bmatrix} w & x \\ z & y \end{bmatrix}$ ($w \neq 0$. The first pivot of this matrix is of course w. The second pivot would be $\frac{----}{w}$ (Fill in the numerator).

26. . If three corners of a parallelogram are (1,1), (4,2) and (1,3), how many options do you have for choosing the fourth corner?

    A. 1

    B. 2

    C. 3

    D. 4

27. If $||\mathbf{u}|| = 8$ and $||\mathbf{v}|| = 9$, then the smallest possible value for $||\mathbf{u} - \mathbf{v}||$ is ____________

28. Consider a 1×3 system of linear equations (one equation, 3 variables). There exists a b such that all solutions to $A\mathbf{x} = \mathbf{b}$ have $x_{particular} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $x_{nullspace} = c \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$.

    A. True

    B. False

29. Consider a matrix A. The intersection of the null space and the column space of A. (Select all statements that are true.)

    A. will **always only** contain the $\mathbf{0}$ vector

    B. **can** contain more vectors other than the $\mathbf{0}$ vector **only** if A is a square matrix

    C. **can** contain more vectors other than the $\mathbf{0}$ vector **only** if A is a square symmetric matrix

    D. will **always** contain more vectors other than the $\mathbf{0}$ vector

30. If A and B are square symmetric matrices then $(A+B)^2$ is always equal to $A^2+2AB+B^2$ if

A. both A and B are symmetric as well as orthogonal matrices

B. both A and B are symmetric matrices

C. both A and B are orthogonal matrices

D. None of the above.