Honor code: I pledge on my honor that: I have completed all steps in the below quiz on my own, I have not used any unauthorized materials while completing this quiz, and I have not given anyone else access to my answers.

Name and Signature

Instructions

1. All 25 questions are mandatory.
2. Each question carries 4 marks.
3. For every question, the answer is either a number (integer, real number or fraction) or a formula (e.g., p^{k-1}, $\frac{k}{n}$, etc.). So just write the number or the formula. You can use a single A4 sheet to write down the answers (similar to what you did in Quiz 1)
4. DO NOT WRITE ANY VERBOSE/TEXTUAL ANSWERS.
5. You can work out the solutions in detail in your worksheets. You will be given ample time at the end to upload the worksheets.
6. You are allowed to use a scientific calculator.
7. You are allowed to use handwritten or printed notes after getting them verified by the invigilating TA.

Questions

1. There are 100 students in your LARP class. For the second quiz, the instructor wants to create a unique question paper for every student in the class. He uses the following strategy: He creates a total of 25 questions and then for each question he creates 5 variants. The 5 variants are created by changing the numerals in the question or some other details which change the answer but do not change the difficulty of the question. The final question paper for each student is created by uniformly sampling one of these 5 variants for each of the 25 questions. What is the probability that you and your best friend will have more than 3 questions which are exactly the same?

   ____

Solution: Probability for question getting same variant $= \frac{5}{25} = \frac{1}{5}$
Probability for question getting different variant $= \frac{20}{25} = \frac{4}{5}$
No variant being common $= (\frac{4}{5})^{25}$
1 question being common $= 25 * \frac{1}{5} * (\frac{20}{25})^{24}$
2 question being common $= \binom{25}{2} * (\frac{1}{5})^2 * (\frac{4}{5})^{23}$
3 question being common $= \binom{25}{3} * (\frac{1}{5})^3 * (\frac{4}{5})^{22}$
probability that more than 3 questions which are exactly the same =
1 - $[(\frac{4}{5})^{25} + 25 * \frac{1}{5} * (\frac{20}{25})^{24} + \binom{25}{2} * (\frac{1}{5})^2 * (\frac{4}{5})^{23} + \binom{25}{3} * (\frac{1}{5})^3 * (\frac{4}{5})^{22}]$
Therefore, Final answer is 1 - 0.233 = 0.76

2. Let n be the number of times that 3 dice are rolled. What is the minimum value of n so that the probability of getting a triple six at least one time in these n trials is greater than 0.5 ?

   ____

Solution: We need to find n such that,
1 - $(215/216)^{n} > 0.5$
$(215/216)^n < 1 - 0.5$
n = 150

3. Consider two villages, one big and one small. In the big village, 45 babies are born everyday and in the small village 25 babies are born everyday. At the end of the year, let a be the number of days on which more than 60 percent of the babies born in the large village were boys. Similarly, let b be the number of days on which more than 60 percent of the babies born in the small village were boys. Assume that a baby can be a boy or a girl with equal probability.

   $a =$ _____________, $b =$ _____________

Solution:
For 45 and 25, we have a = 24.666 (i.e., approx. 365*0.067578) and b = 41.8879 (i.e., approx. 365* 0.11476)
For 40 and 20, we have a = 28.0794 (i.e., approx. 365*0.07693) and b = 48.0296 (i.e., approx. 365* 0.131588)
For 50 and 30, we have a = 21.70298 (i.e., approx. 365*0.05946) and b = 36.5891 (i.e., approx. 365* 0.10024)
For 55 and 15, we have a = 19.12408 (i.e., approx. 365*0.05239) and b = 55.0708 (i.e., approx. 365* 0.15088)
For 60 and 35, we have a = 16.874129 (i.e., approx. 365*0.04623) and b = 32.0224 (i.e., approx. 365* 0.08773)

Steps / explanation:
Compute the probability of more than 60 percent of the babies are boys in a day. Then multiply it by 365 (to compute how many such days would occur in a year).
Therefore, we have:

$$a = 365 \times \left[\sum_{i=p}^{m} {}^{m}C_i \left(\frac{1}{2}\right)^m \right]$$
$$b = 365 \times \left[\sum_{i=q}^{n} {}^{n}C_i \left(\frac{1}{2}\right)^n \right]$$

where m = no. of babies born in big village everyday
$p = (0.6 \times m) + 1$
n = no. of babies born in small village everyday
$q = (0.6 \times n) + 1$

4. I will keep tossing a coin till I get two consecutive heads or two consecutive tails. What is the probability that my experiment will not terminate even after 10 tosses? _____________ (If you want, you can give a formula instead of computing the value)

Solution:
Let the event A be that in n tosses no two consecutive heads or tails came.
We assume the coins are fair with probability of heads(p) equals probability of tails(q) equals $\frac{1}{2}$.
Since no two consecutive tosses came up as heads or tails, it means every consecutive toss was different. For n tosses there are two outcomes in which this happens.

Outcome 1(O1): The first toss is heads, and then each toss is alternated thereafter, following the pattern H, T, H, T, ... n times
The probability for this outcome is $P(O1) = (\frac{1}{2})^n$.
Outcome 2(O2): The first toss is tails, and then each toss is alternated thereafter, following the pattern T, H, T, H. .. n times
The probability for this outcome is $P(O2) = (\frac{1}{2})^n$

A = O1 + O2
Therefore, desired Probability is sum of probabilities of outcome 1 and outcome 2.

$P(A) = P(O1) + P(O2) = (\frac{1}{2})^n + (\frac{1}{2})^n = 2 * (\frac{1}{2})^n = (\frac{1}{2})^{n-1}$

Answer is $\frac{1}{2^{n-1}}$ where n is the number of tosses.

5. You select two cards from a standard deck of 52 cards. What is the probability that the rank of the second card is higher than the rank of the first card (The ranks in ascending order are 2, 3, 4, 5, 6, 7, 8, 9, 10, J, K, Q, A).
_____________ (the answer is a fraction)

Solution:
Let A be the event described in the question whose probability we want to find out.
The probability of choosing any particular card as the first card is $\frac{1}{52}$
Therefore, once the first card is chosen, the probability of picking a card of a different rank is $\frac{1}{52} * \frac{48}{51}$.
By symmetry, half of different cards as second card candidate will be of a higher rank than the first. Therefore the probability of choosing a second card of a higher rank is $\frac{1}{2} * \frac{48}{51*52} = \frac{24}{51*52} = \frac{8}{17*52}$.
Computing this probability for all the different 52 cards that could have been chosen as the first card, it gives us
$P(A) = 52 * \frac{8}{17*52} = \frac{8}{17}$

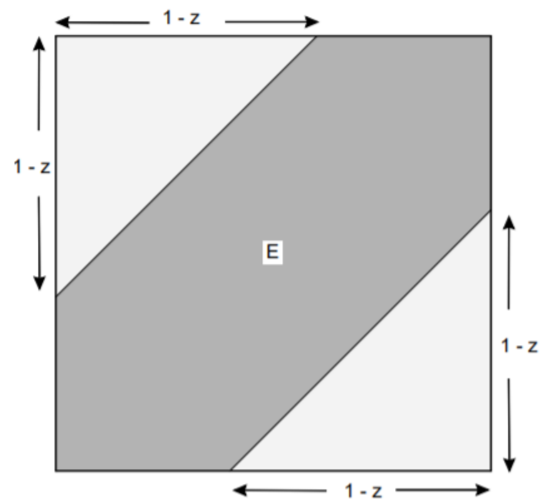
6. Suppose you repeat Buffon's needle experiment 100 times with a needle of length 3 cm and a wooden plank of length 6 cm. What is the probability that the needle will intersect with an edge of the plank at least one time? (Hint: this is a very easy question. Don't overthink.)
   ______________ (If you want, you can write an expression for the answer instead of actually computing it (e.g., $(\frac{1}{6})^{10}$)

Solution:
This is a binomial random variable with $n = 100$ and $p = \frac{2*3}{6*\pi} = \frac{1}{\pi}$
Probability that it will not intersect in n repetitions is $P(X = 0) = \left(1 - \frac{1}{\pi}\right)^n$
Hence the correct answer is $1 - P(X = 0) = 1 - (1 - \frac{1}{\pi})^n$ where n is the number of repetitions.

7. Suppose you and your friend decide to meet at the Marina beach some time between 7:00 pm and 8:00 pm. Each of you arrives independently at a time between 7:00 pm and 8:00 pm chosen randomly with uniform probability. Let Z be the random variable indicating the time that the first person has to wait before the second person arrives (Z can take on values in the interval $[0, 1]$ hours. For example, the person arriving first could have to wait for 0.25 hours). The density function of Z can be written as:

$$f_z(z) = \begin{cases} 0 \quad if \quad z < 0, \\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_ \quad if \quad 0 \leq z \leq 1, \\ 0 \quad if \quad z > 1. \end{cases}$$

Solution: Here we can take the unit square to represent the sample space, and (X, Y) as the arrival times (after 7:00 P.M.). Let $Z = |X - Y|$. Then we have $F_X(x) = x$ and $F_Y(y) = y$.



$$F_Z(z) = P(Z \le z) = P(|X - Y| \le z) = \text{Area of E} \tag{1}$$

$$F_Z(z) = \begin{cases} 0 & \text{if z} < 0 \\ 1\text{ - }(1\text{ - z})^2 & \text{if } 0 \le \text{z} \le 1 \\ 1 & \text{if z} > 1 \end{cases} \tag{2}$$

$$f_Z(z) = \begin{cases} 0 & \text{if z} < 0 \\ \mathbf{2(1 - z)} & \text{if } 0 \le \text{z} \le 1 \\ 0 & \text{if z} > 1 \end{cases} \tag{3}$$

8. Suppose we choose two numbers A and B at random from the interval [0,1].
   (a) (3 points) What is the probability that $AB < \frac{1}{2}$? _____________
   (b) (1 point) What is the probability that $A^2 + B^2 \le \frac{1}{2}$? _____________

Solution: (a) Let z = ab, then

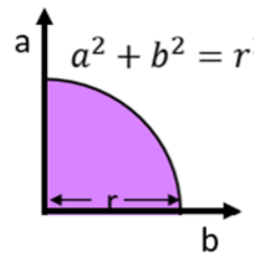
$$F_Z(z) = z - zlogz$$

[See derivation here]

$$F_Z(0.5) = P(X < 0.5) = 0.5 - log(0.5) = \frac{1 + log(2)}{2}$$

(b) Area of quarter circle =

$$\frac{\pi}{4}r^2 = \frac{\pi}{4} * \frac{1}{2}$$



$$P(A^2 + B^2 \leq \frac{1}{2}) = \frac{\pi}{8}$$

9. A casino owner calls you to design the bets for the following game. A player throws a 1 rupee coin on a large chess board. The board is set up in a way that some part of the coin will always fall on it (*i.e.*, the coin will never be completely outside the chess board). The length of each square on the chess board is twice the diameter of the coin. If the coin lands in a way such that it intersects with an edge of any square then the player pays 1 rupee to the casino. However, if the coin lands completely inside a square then the casino pays k rupees to the player. What should the value of k be so that this is a fair game (*i.e.*, in the long run neither the player nor the casino will have an advantage over the other)? [Hint: The game of roulette as discussed in the class is not a fair game.]
$k =$ _____________

Solution: "player pays y rupees". The answer will be $3y$.

10. You are playing a game of cards with 6 other players. The dealer shuffles a standard

deck of 52 cards and deals one card to each of the 7 players. If you are the last player to be dealt a card what is the probability that you will get a king?

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Solution: Let's assume you get a king (no matter where you sit).
The number of ways in which rest of the 51 cards can be distributed to 6 other players : ${}^{51}P_6 = \frac{51!}{45!}$
The total number of ways of distributing 52 cards among 7 players : ${}^{52}P_{6+1} = \frac{52!}{45!}$
As there are 4 kings, the final probability of you getting a king is: $4 * \frac{51! * 45!}{52! * 45!} = 1/13$

Note that the answer doesn't change even if number of players changes.

11. Given 6 friends, what is the probability that at least two of them will have the same birth month?

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Solution: Total number of combinations of birth months among 6 friends: 12^6
Number of ways in which each friend will have a different birth month : ${}^{12}P_6$
In the rest of the scenarios at least two friends will always have same birth month. Therefore the answer is : $1 - \frac{{}^{12}P_6}{12^6} = 0.778$

For other number of friends, the answer can be found by replacing 6 with the corresponding number.

12. An urn contains r red balls and g green balls. You draw one ball from the urn. If the color of the ball is red then you replace the ball and add one more red ball in the urn (so the urn now has $r + 1$ red balls and g green balls. Similarly, if the color of the ball is green then you replace the ball and add 2 more green balls in the urn (so the urn now has r red balls and $g + 1$ green balls). You repeat this experiment forever. What is the probability that the ball drawn in the i-th trial will be a red ball? _____________

Solution:
The answer will always be $\frac{r}{r+g}$ (for any trial) where r is the number of red balls and $r+g$ is the total number of balls.
For the first draw, we know the probability would be: $\frac{r}{r+g}$
For the second draw, we have $\frac{r}{r+g}.\frac{r+1}{r+g+1} + \frac{g}{r+g}.\frac{r}{r+g+1}$
$= \frac{r^2+r+gr}{(r+g)(r+g+1)} = \frac{r(r+g+1)}{(r+g)(r+g+1)} = \frac{r}{(r+g)}$
In every trial of this process, the probability is only dependent on the initial ratio of the balls.
(This question is based on "Polya Urn process". Please refer to that for more details)

13. There are 33 coins in a bag out of which one coin has heads on both the faces and the remaining coins are fair (i.e., $P(H) = P(T)$). You randomly select one of the coins and toss it 5 times. If you get 5 heads in a row what is the probability that the coin that you selected was one of the fair coins? _____________

Solution:
Using Bayes rule, we compute the $P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|A^c)P(A^c)}$
In this case,
event A denotes the coin selected was fair coin,
event B is seeing 't' heads in a row,
event A^c denotes the coin selected was the one coin with heads on both sides.
Therefore, the formula used for total 'c' coins tossed 't' times: $\frac{(1/2)^t((c-1)/c)}{(1/2)^t((c-1)/c)+(1/c)}$
Correct answer is 1/2 or 0.5

14. The dealer in a casino gives you 25 white balls and 25 black balls. You are asked to place these balls in two urns in any way you want (e.g., all 50 balls in one urn or the all white balls in one urn and all black balls in the other or 10 black balls in one urn and the remaining 40 balls in the other or ...). The dealer will be blindfolded and will not see how you place these balls in the urn. Once you have done this, the dealer will randomly select one urn and then randomly select one ball from that urn (assume he is still blindfolded). You will win 1 million INR if the ball is white. Let w_1, b_1 be the number of white and black balls that you place in urn 1 respectively. Similarly, let w_2, b_2 be the number of white and black balls that you place in urn 2 respectively (of course, $w_1 + w_2 = 25$ and $b_1 + b_2 = 25$). How should you choose the value of w_1, w_2, b_1, b_2 so that you maximize your probability of winning?
$w_1 =$ _____________
$b_1 =$ _____________

$w_2 =$ _____________
$b_2 =$ _____________
(If you think there are multiple correct answers to this problem then you can write any one of those correct answers.)

Solution: CorrectAnswer is only one white ball in one urn, and the rest of the balls in the second urn.

15. You start observing a radioactive element at time $t = 0$. Let T be the random variable indicating the number of seconds after which a radioactive element emits an α-particle (T is a positive real number). Based on past data, we know that the density of this random variable is given by $f(t) = 0.1e^{-0.1t}$.

    (a) (2 points) What is the probability that a particle will be emitted in the first 10 seconds given that no particle has been emitted in the first 1 second?

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    (b) (2 points) What is the probability that a particle will be emitted in the first 10 seconds given that a particle has been emitted in the first 20 seconds?

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Solution: (a)

$$1 - e^{-\lambda(10-1)} = 1 - e^{-0.9} = 0.593$$

(b)

$$\frac{1 - e^{-10\lambda}}{1 - e^{-20\lambda}} = \frac{1 - e^{-1}}{1 - e^{-2}} = \frac{0.632}{0.864} = 0.731$$

16. I choose two real numbers x and y independently and randomly from the interval $[0, 1]$. I do not tell you what these numbers are but I tell you that their sum lies in the interval $[0, 1]$.

    (a) (2 points) What is the probability that $x > y$?_____________

    (b) (2 points) What is the probability that $xy < \frac{1}{2}$?_____________

Solution: Part (a) answer is 1/2
Part (b) answer is 1 for both $xy < 1/2$ and $xy < 1/3$ (if x + y = 1 then the max value of xy is when (x=0.5) and (y=0.5) = 0.25, so xy is always <= 0.25)

17. I have a biased coin with $P(H) = p$. I toss the coin n times.

    (a) (2 points) What is the conditional probability of getting a head in the i-th trial ($i < n$), given that there were a total of k heads in the n trials?
    _____________ (I am expecting an expression/formula as the answer)

    (b) (2 points) What is the conditional probability of getting a head in the j-th trial ($j > n$), given that there were a total of k heads in the n trials?
    _____________ (I am expecting an expression/formula as the answer)

Solution:
(a) Answer is $\frac{k}{n}$

(b) Given P(H) = p,
A = Getting head in j-th trial where $j > n$
B = Total of k heads in the n-trials.
$P(A) = p$
A and B are independent events thus,
P(A|B) = P(A) = p

18. A fair coin is tossed many times. What is the probability that the first head will appear after the 10th toss given that it has not appeared in the first 2 tosses?

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Solution: Given in the first two tosses head did not appear i.e first two tosses(b) are tails and the head should appear after 10th toss(a) thus the probability is $(\frac{1}{2})^{10-2} = \frac{1}{2^8}$
For other variants, answer = $(\frac{1}{2})^{a-b}$

19. You are sitting on a beach waiting to see a shooting star. The probability that you will see a shooting star during each second is 0.01. You have been waiting for long and are bored. You decide to take a break and check your emails over your phone. If it takes

you 5 minutes to do so what is the probability that you will miss at least one shooting star while you check your emails? _____________

Solution:
Poisson Distribution
λ = 5 x 60 x 0.01 = 3
Ans = $1 - \frac{3^0 e^{-3}}{0!}$ = 1-0.05 = 0.95

20. You stay in Andheri (a suburb in Mumbai) and have to travel to Churchgate 100 times a year for work related purposes. You use the local train where the return ticket from Andheri-Churchgate-Andheri costs INR 20. You know that the fine for traveling without a ticket is INR 250 but given the rush at railway stations and in trains, the probability of you getting caught each time you travel without a ticket is 0.01 (you can think of the to and fro journey as a single travel - so you make 100 such travels in a year). If you buy a ticket every time then the total cost of 100 travels is of course 2000 INR. What is the expected cost of traveling without a ticket 100 times? (Hint: $\sum_{i=0}^{\infty} \frac{x^i}{i!} = e^x$)
____

Solution:
Poisson Distribution
λ = 0.01 x 100 = 1
$P(X = i) = \frac{\lambda^i e^{-\lambda}}{i!}$, where X is a random variable denoting the number of times you get caught.

$$E[penalty] = \sum_{i=0}^{100} i * 250 * P(X = i)$$

$$= \frac{250}{e} \sum_{i=0}^{100} \frac{\lambda^i * i}{i!}$$

$$= \frac{250 * \lambda}{e} \sum_{i=0}^{100} \frac{\lambda^{(i-1)}}{(i-1)!}$$

$$= \frac{250 * e^{\lambda}}{e}$$

$$Ans = 250$$

21. Consider a uniform continuous random variable X which can take on any real value in

the interval $[0, 1]$. If $Y = X^3$ then it is easy to see that Y can also take on values in the interval $[0, 1]$. Write down an expression for the cumulative distribution function and the density function of Y.
$F_Y(y) =$ _____________
$f_Y(y) =$ _____________

Solution: In general, the CDF of uniform distribution for a rv x in the interval [a, b] is:

$$F_X(x) = \begin{cases} 0 & \text{if x < a} \\ \frac{x-a}{b-a} & \text{if a} \leq \text{x} \leq \text{b} \\ 1 & \text{if x > 1} \end{cases}$$

Here, we see:

$$F_Y(y) = P(Y \leq y) = P(X^3 \leq y) = P(X \leq y^{1/3}) = y^{1/3}$$

$$\mathbf{F_Y(y) = y^{1/3}}$$

$$\mathbf{f_Y(y) = F'_Y(y) = \frac{1}{3}y^{-2/3}}$$

22. In the canonical form (i.e., using natural parameters), the density function of an exponential family is written as:

$$f_X(x) = h(x)exp[\eta.T(x) - A(\eta)]$$

Now consider the following density function:

$$f_X(x) = \frac{1}{2^{\frac{k}{2}}\Gamma(\frac{k}{2})} x^{(\frac{k}{2}-1)} e^{-\frac{x}{2}}$$

Express this in the canonical form shown above and write down the following:

$h(x) =$ _____________
$T(x) =$ _____________
$\eta =$ _____________
$A(\eta) =$ _____________

Solution:

$$h(x) = e^{-\frac{x}{2}}, T(x) = log(x), \eta = \frac{k}{2} - 1, A(\eta) = log\Gamma(\eta + 1) + (\eta + 1)log2 \qquad (3)$$

Refer to chi-square distribution in the table given on this link.

23. Consider the numbers 1 to n. There are $n!$ permutations of these n numbers. A given permutation is said to have k fixed points if k numbers in this permutation are in their original position. For example, the table below shows the number of fixed points in all the 6 permutations when $n = 3$.

			k	**Remarks**
1	2	3	3	all the 3 numbers are in their original position
1	3	2	1	only the number 1 is in its original position
2	1	3	1	only the number 3 is in its original position
2	3	1	0	none of the numbers are in their original position
3	1	2	0	none of the numbers are in their original position
3	2	1	1	only the number 2 is in its original position

Let Y be the random variable indicating the number of fixed points in a random permutation of the n numbers. If all the permutations are equally likely then what is $E[Y]$? (Hint: How does it change as n increases?)

Solution: Answer is 1
We need to find the probability of k positions being fixed.
1 2 3 4 ......n

Let Y be the number of fixed points in a random permutation of the set {a, b, c}. To find the expected value of Y , it is helpful to consider the basic random variable associated with this experiment, namely the random variable X which represents the random permutation. There are six possible outcomes of X,and we assign to each of them the probability $\frac{1}{6}$ (See Table). Then we can calculate E(Y) as :

$$3(\frac{1}{6}) + 1(\frac{1}{6}) + 1(\frac{1}{6}) + 0(\frac{1}{6}) + 0(\frac{1}{6}) + 1(\frac{1}{6}) = 1$$

Now consider a very quick way to calculate the average number of fixed points in a random permutation of the set {1, 2, 3, . . . , n}. Let Z denote the random permutation. For each i, $1 \leq i \leq n$, let X_i equal 1 if Z fixes i, and 0 otherwise. So if we let F denote the number of fixed points in Z, then

$$F = X_1 + X_2 + ... + X_n$$

$$E(F) = E(X_1) + E(X_2) + ... + E(X_n)$$

For each i,

$$E(X_i) = \frac{1}{n}$$

So,

$$E(F) = 1$$

24. The height of the students in the 7th grade of a school follows a normal distribution with mean as 4.5 feet and variance as 1 foot. Similarly, the height of the students in the 8th grade follows a normal distribution with mean as 5 feet and variance as 1 foot. Lastly, the height of the students in the 9th grade follows a normal distribution with mean as 5.5 feet and variance as 1 foot. There are 100 students in each grade and they have assembled in the school playground. You pick a student at random and measure the student's height. If the student's height is 5.25 feet what is the probability that the student is from the 8th grade.

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Solution: You need to use the following formula for computing the answer:

$$P(8th\ grade|height = 5.25) = \frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{(5.25-5)^2}{2}} \cdot \frac{1}{3}}{\frac{1}{\sqrt{2\pi}}e^{-\frac{(5.25-4.5)^2}{2}} \cdot \frac{1}{3} + \frac{1}{\sqrt{2\pi}}e^{-\frac{(5.25-5)^2}{2}} \cdot \frac{1}{3} + \frac{1}{\sqrt{2\pi}}e^{-\frac{(5.25-5.5)^2}{2}} \cdot}$$

There are different ways of computing this expression.

25. When the internet connection is down, we impatiently hit the refresh button every 2 seconds to see if the connection has been restored. This can be very frustrating. Instead I do the following when the internet connection goes down. I toss a fair coin 3 times and note down the number of heads k. I then pick a random number x which follows the Poisson distribution with $\lambda = k$ (assume that I have a program which allows me to generate samples from the Poisson distribution). I then wait for x minutes before hitting the refresh button (I have found that this helps me cope up with the stress of waiting). After following this process, if I had to wait for 4 minutes before hitting the refresh button then what is the probability that I had 2 heads in the 3 tosses?

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Solution:
Find $P(2\ heads|x = 4\ minutes)$
Let,
$A = 0\ head$
$B = 1\ head$
$C = 2\ heads$
$D = 3\ heads$
$E = 4\ minutes$
$P(A) = \frac{1}{8},\ P(B) = \frac{3}{8}, P(C) = \frac{3}{8}, P(D) = \frac{1}{8}$
$P(2\ heads|x = 4\ minutes) = P(C|E)$

$$P(C|E) = \frac{P(E|C)P(C)}{P(E|A)P(A) + P(E|B)P(B) + P(E|C)P(C) + P(E|D)P(D)}$$

$$= \frac{\frac{2^4}{4!}e^{-2} * \frac{3}{8}}{\frac{0^4}{4!}e^{-0} * \frac{1}{8} + \frac{1^4}{4!}e^{-1} * \frac{3}{8} + \frac{2^4}{4!}e^{-2} * \frac{3}{8} + \frac{3^4}{4!}e^{-3} * \frac{1}{8}} = 0.558$$