Assignment 1

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1 Problem 1

1.1 Part (a)

Let $\Pi^* = \{\mu_0^*, \mu_1^*, \ldots, \mu_{N-1}^*\}$ denote the optimal policy.
Also, let $\Pi^k = \{\mu_k, \mu_{k+1}, \ldots, \mu_{N-1}\}$. Consider the tail sub-problem,

$$\min_{\Pi^k} \mathbb{E}\Big\{exp(g_N(x_N) + \textstyle\sum_{i=k}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\Big\}$$

Let J_k^* be the optimal cost-to-go for the tail sub-problem with state x_k and stage k. For $k = 0, 1, ...N-1$, we need to show:

$$J_k^*(x_k) = J_k(x_k)$$

For $k = N$ and $\forall\, x_N \in \mathcal{X},\ J_N^*(x_N) = exp(g_N(x_N)) = J_N(x_N)$.

Induction hypothesis: Assume for $k+1$ and $\forall\ x_k \in \mathcal{X},\ J_{k+1}^*(x_{k+1}) = J_{k+1}(x_{k+1})$. Then, we can solve our tail sub-problem in the following way:

$$
\begin{aligned}
&J_k^*(x_k) = \min_{\{\mu_k, \Pi^{k+1}\}} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_N(x_N) + \sum_{i=k}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\Big\} \\
&J_k^*(x_k) = \min_{\{\mu_k, \Pi^{k+1}\}} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_N(x_N) + g_k(x_k, \mu_k(x_k), x_{k+1}) + \sum_{i=k+1}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\Big\} \\
&J_k^*(x_k) = \min_{\{\mu_k, \Pi^{k+1}\}} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_k(x_k, \mu_k(x_k), x_{k+1})) \cdot exp(g_N(x_N)) \cdot exp(\sum_{i=k+1}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\Big\} \\
&J_k^*(x_k) = \min_{\mu_k} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_k(x_k, \mu_k(x_k), x_{k+1})) \cdot \min_{\Pi^{k+1}} \big[exp(g_N(x_N)) \cdot exp(\sum_{i=k+1}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\big]\Big\} \\
&J_k^*(x_k) = \min_{\mu_k} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_k(x_k, \mu_k(x_k), x_{k+1})) \cdot \min_{\Pi^{k+1}} \big[exp(g_N(x_N) + \sum_{i=k+1}^{N-1} g_i(x_i, \mu_i(x_i), x_{i+1}))\big]\Big\} \\
&J_k^*(x_k) = \min_{\mu_k} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_k(x_k, \mu_k(x_k), x_{k+1})) \cdot J_{k+1}^*(x_{k+1})\Big\} \\
&J_k^*(x_k) = \min_{a_k \in A(x_k)} \mathop{\mathbb{E}}_{x_{k+1}} \Big\{exp(g_k(x_k, \mu_k(x_k), x_{k+1})) \cdot J_{k+1}^*(x_{k+1})\Big\} \\
&\therefore J_k^*(x_k) = J_k(x_k)
\end{aligned}
$$

Hence, the optimal cost and policy can be obtained.

1.2 Part (b)

Here, single stage cost g_k depends only on x_k and a_k and does not depend on x_{k+1}. Also, we have $V_k(x_k) = \log\ J_k(x_k)$. From part (a), we have:

$$J_N(x_N) = exp(g_N(x_N))$$

Taking log on both sides,

$$
\begin{aligned}
\log(J_N(x_N)) &= \log(exp(g_N(x_N))) \\
V_N(x_N) &= g_N(x_N)
\end{aligned}
$$

Also, from part (a), we have:

$$J_k(x_k) = \min_{a_k \in A(x_k)} \mathop{\mathbb{E}}_{x_{k+1}} \big(exp(g_k(x_k, a_k) \cdot J_{k+1}(x_{k+1}))\big)$$

Taking log on both sides,

$$\log(J_k(x_k)) = \log\left[\min_{a_k\in A(x_k)} \mathop{\mathbb{E}}_{x_{k+1}} \big(exp(g_k(x_k,a_k))\cdot J_{k+1}(x_{k+1})\big)\right]$$
$$V_k(x_k) = \min_{a_k\in A(x_k)} \log\left[\mathop{\mathbb{E}}_{x_{k+1}} \big(exp(g_k(x_k,a_k))\cdot J_{k+1}(x_{k+1})\big)\right]$$

Since the single stage cost doesn't depend on future states,

$$V_k(x_k) = \min_{a_k\in A(x_k)} \log\left[exp(g_k(x_k,a_k))\cdot \mathop{\mathbb{E}}_{x_{k+1}} \big(J_{k+1}(x_{k+1})\big)\right]$$
$$V_k(x_k) = \min_{a_k\in A(x_k)} \left[g_k(x_k,a_k) + \log \mathop{\mathbb{E}}_{x_{k+1}} \big(J_{k+1}(x_{k+1})\big)\right]$$
$$\therefore V_k(x_k) = \min_{a_k\in A(x_k)} \left[g_k(x_k,a_k) + \log \mathop{\mathbb{E}}_{x_{k+1}} \big(exp(V_{k+1}(x_{k+1})\big)\right]$$

1.3 Part (c)

The 'oven problem' is a two stage problem where the exponentiated cost is:

$$J_{a_0,a_1}(x_0) = \mathbb{E}\left[exp(\theta(a_0^2 + a_1^2 + (x_2 - T)^2))\right]$$

Hence, the final stage cost for the problem is:

$$J_2(x_2) = \mathbb{E}\left[exp(\theta(x_2 - T)^2)\right]$$

Going back a stage:

$$J_1(x_1) = \min_{a_1} \mathbb{E}\left[exp(\theta(a_1^2 + J_2(x_2)))\right]$$
$$J_1(x_1) = \min_{a_1} \mathbb{E}\left[exp(\theta(a_1^2 + (x_2 - T)^2))\right]$$
$$J_1(x_1) = \min_{a_1} \mathbb{E}\left[exp(\theta(a_1^2 + ((1-\alpha)x_1 + \alpha a_1 + w_1 - T)^2))\right]$$
$$J_1(x_1) = \min_{a_1} \left[exp(\theta(a_1^2 + ((1-\alpha)x_1 + \alpha a_1 - T)^2 + 2\mathbb{E}w_1((1-\alpha)x_1 + \alpha a_1 - T) + \mathbb{E}w_1^2))\right]$$

Since $\mathbb{E}w_1 = 0$,

$$J_1(x_1) = \min_{a_1} \left[exp(\theta(a_1^2 + ((1-\alpha)x_1 + \alpha a_1 - T)^2 + \mathbb{E}w_1^2))\right]$$
$$J_1(x_1) = \min_{a_1} \left[exp(\theta(a_1^2))\cdot exp(\theta((1-\alpha)x_1 + \alpha a_1 - T)^2)\cdot exp(\theta(\mathbb{E}w_1^2)))\right]$$

Differentiating w.r.t a_1 and equating RHS to 0, we get:

$2\theta a_1 + 2\theta\alpha\left[(1-\alpha)x_1 + \alpha a_1 - T\right] = 0$

$$\implies \mu_1^*(x_1) = \frac{\alpha[T - (1-\alpha)x_1]}{1+\alpha^2} \tag{1}$$

Now, we can calculate $J_1^*(x_1)$,

$$J_1^*(x_1) = exp\left[\theta\left[\frac{\alpha^2[T-(1-\alpha)x_1]^2}{(1+\alpha^2)^2} + ((1-\alpha)x_1 + \alpha\cdot\frac{\alpha[T-(1-\alpha)x_1]}{1+\alpha^2} - T)^2\right]\right] + \mathbb{E}w_1^2$$

Simplifying RHS further, we get:

$$\implies J_1^*(x_1) = exp\left[\frac{\theta((1-\alpha)x_1 - T)^2}{1+\alpha^2}\right] + \mathbb{E}w_1^2 \tag{2}$$

Going back to the first stage:

$$J_0(x_0) = \min_{a_0} \mathbb{E}\left[exp(\theta(a_0^2 + J_1(x_1)))\right]$$
$$J_0(x_0) = \min_{a_0} \mathbb{E}\left[exp(\theta(a_0^2 + J_1((1-\alpha)x_0 + \alpha a_0 + w_0)))\right]$$

Without loss of generality, we will substitute $\mathbb{E}w_1^2$ later, as it is a constant.

$$J_0(x_0) = \min_{a_0} \mathbb{E}\left[exp(\theta(a_0^2 + \frac{\left[(1-\alpha)((1-\alpha)x_0 + \alpha a_0 + w_0) - T\right]^2}{1+\alpha^2}))\right]$$
$$J_0(x_0) = \min_{a_0} \mathbb{E}\left[exp(\theta(a_0^2 + \frac{\left[(1-\alpha)^2 x_0 + \alpha(1-\alpha)a_0 + (1-\alpha)w_0 - T\right]^2}{1+\alpha^2}))\right]$$

For clarity of the below equation, let $Z = \big[(1-\alpha)^2 x_0 + \alpha(1-\alpha)a_0 - T\big]$.

$$J_0(x_0) = \min_{a_0} exp\left[\theta\left[a_0^2 + \frac{Z^2 + (1-\alpha)^2\mathbb{E}w_0^2 + 2\cdot\mathbb{E}w_0(1-\alpha)Z}{1+\alpha^2}\right]\right]$$

Since $\mathbb{E}w_0 = 0$,

$$J_0(x_0) = \min_{a_0} exp\left[\theta\left[a_0^2 + \frac{\big[(1-\alpha)^2 x_0 + \alpha(1-\alpha)a_0 - T\big]^2 + (1-\alpha)^2\mathbb{E}w_0^2}{1+\alpha^2}\right]\right]$$

Differentiating w.r.t a_0 and equating RHS to 0, we get:

$$\implies \mu_0^*(x_0) = \frac{\alpha(1-\alpha)[T-(1-\alpha)^2 x_0]}{1+\alpha^2(1+(1-\alpha))^2} \tag{3}$$

This gives us $J_0^*(x_0)$,

$$\implies J_1^*(x_1) = exp\left[\frac{\theta((1-\alpha)^2 x_0 - T)^2}{1+\alpha^2(1+(1-\alpha))^2}\right] + \mathbb{E}w_0^2 + \mathbb{E}w_1^2 \tag{4}$$

2 Problem 2

2.1 Part (a)

1. Actions: $\{B, H\}$; B = buy, H = hold
2. State space: $x_{k+1} = \begin{cases} T: & \text{if } x_k \neq T, a_k = B \text{ or } x_k = T \\ \bar{T}: & \text{otherwise} \end{cases}$
3. Terminal Cost: $g_N(x_N) = J_N(\overline{T}) = \begin{cases} \frac{1}{1-p}: \text{ if } x_N \neq T \\ 0: \text{ otherwise} \end{cases}$
4. Single stage cost: $g_k(x_k, a_k, x_{k+1}) = \begin{cases} (N-k): \text{ if } x_k \neq T \text{ and } a_k = B \\ 0: \text{ otherwise} \end{cases}$

2.2 Part (b)

Assumption: We can say that penalizing buying a food one doesn't like is the same as not buying a food one likes. We go with the former narrative.

The DP algorithm for the problem is:

$$\begin{aligned}
J_k(x_k) &= \min_{a_k \in \{B,H\}} \mathop{\mathbb{E}}_{x_{k+1}} \big((g_k(x_k, a_k, x_{k+1}) + J_{k+1}(x_{k+1}))\big) \\
J_k(x_k) &= \begin{cases} \min\left[p\cdot(N-k) + (1-p)\cdot\mathbb{E}J_{k+1}(x_{k+1}),\ \mathbb{E}J_{k+1}(x_{k+1})\right], \text{ if } x_k \neq T \\ 0, \text{otherwise} \end{cases} \\
J_k(\bar{T}) &= \min\big[p\cdot(N-k) + q\cdot J_{k+1}(\bar{T}),\ J_{k+1}(\bar{T})\big] \\
J_k(\bar{T}) &= p\cdot\min\big[N-k,\ J_{k+1}(\bar{T})\big] + q\cdot J_{k+1}(\bar{T})
\end{aligned} \tag{1}$$

2.3 Part (c)

We buy the food at the kth shop if $(N-k) < J_{k+1}(\bar{T})$. Ideally, we would like to buy the likeable food if it available at the $(k+1)$th. Let's assume that it is optimal to buy the likeable food at the $(k+1)$th shop. Then,

$$\begin{aligned}
& N-(k+1) < N-k \\
\implies & N-(k+1) < N-k < J_{k+1}(\bar{T}) \\
\implies & N-k-1 < N-k < J_{k+1}(\bar{T}) \\
\implies & N-k-1 < N-k < J_k(\bar{T}) \leq J_{k+1}(\bar{T}) && \text{(From (1))}
\end{aligned}$$

This tells us that, so long that we find a likeable food at $(k+1)$th shop, given that we already found the likeable food at the kth shop, it would be optimal to buy the food at the $(k+1)$th shop to minimize the cost.

3 Problem 3

3.1 Part (a)

1. State space $x_k = \{R, B\}$; R = machine running, B = machine broken

2. Action space $\mu_k(x_k)$:
   (a) $\mu_k(R) = \{m, nm\}$; m = maintenance, nm = no maintenance
   (b) $\mu_k(B) = \{n, r\}$; n = replace/new, r = repair
3. Transition probabilities
   (a) $P(x_{k+1} = B | x_k = R, a_k = nm) = p_{BRnm} = 0.7$
   (b) $P(x_{k+1} = B | x_k = R, a_k = m) = p_{BRm} = 0.4$
   (c) $P(x_{k+1} = R | x_k = R, a_k = nm) = 1 - p_{BRnm} = 0.3$
   (d) $P(x_{k+1} = R | x_k = R, a_k = m) = 1 - p_{BRm} = 0.6$
   (e) $P(x_{k+1} = B | x_k = B, a_k = r) = p_{BBr} = 0.4$
   (f) $P(x_{k+1} = R | x_k = B, a_k = r) = 1 - p_{BBr} = 0.6$
   (g) $P(x_{k+1} = R | x_k = B, a_k = n) = p_{RBn} = 1$
4. Action cost
   (a) $a_k = m \rightarrow C_m = 200$
   (b) $a_k = r \rightarrow C_r = 400$
   (c) $a_k = n \rightarrow C_n = 1500$
   (d) $a_k = nm \rightarrow C_{nm} = 0$
5. Single stage cost
   (a) $g_N(R) = g_N(B) = 0$; Terminal cost is 0
   (b) $g_k(x_{k+1} = R) = 1000 - C_{a_k}$
   (c) $g_k(x_{k+1} = B) = C_{a_k}$

3.2 Part (b)

We have to find optimal policy for $N = 4$ stage problem. We know that $J_4(R) = g_4(R) = J_4(B) = g_4(B) = 0$.

For the 3rd week, we have:

$$
\begin{aligned}
J_3(R) &= \max_{a_3} \mathbb{E}_{x_4} \left[g_3(R, a_3, x_4) + J_4(x_4) \right] \\
J_3(R) &= \max \left(\mathbb{E}_{x_4} \left[g_3(R, m, x_4) \right], \mathbb{E}_{x_4} \left[g_3(R, nm, x_4) \right] \right) \\
J_3(R) &= \max((1 - p_{BRm}) * 1000 - C_m, (1 - p_{BRnm}) * 1000) \\
J_3(R) &= \max(0.6 * 1000 - 200, 0.3 * 1000) \\
J_3(R) &= 400
\end{aligned}
$$

Similarly, for $J_3(B)$,

$$
\begin{aligned}
J_3(B) &= \max_{a_3} \mathbb{E}_{x_4} \left[g_3(B, a_3, x_4) + J_4(x_4) \right] \\
J_3(B) &= \max \left(\mathbb{E}_{x_4} \left[g_3(B, n, x_4) \right], \mathbb{E}_{x_4} \left[g_3(B, r, x_4) \right] \right) \\
J_3(B) &= \max(1000 - C_n, (1 - p_{BBr}) * 1000 - C_r) \\
J_3(B) &= \max(1000 - 1500, 0.6 * 1000 - 400) \\
J_3(B) &= 200
\end{aligned}
$$

Therefore, **optimal policies after week 3**: $\mu_3^*(R) = m$ and $\mu_3^*(B) = r$.

For the 2nd week, we have:

$$
\begin{aligned}
J_2(R) &= \max_{a_2} \mathbb{E}_{x_3} \left[g_2(R, a_2, x_3) + J_3(x_3) \right] \\
J_2(R) &= \max \left(\mathbb{E}_{x_3} \left[g_2(R, m, x_3) + J_3(x_3) \right], \mathbb{E}_{x_3} \left[g_2(R, nm, x_3) + J_3(x_3) \right] \right) \\
J_2(R) &= \max((1 - p_{BRm}) * (1000 + J_3(R)) + p_{BRm} * J_3(B) - C_m, (1 - p_{BRnm}) * (1000 + J_3(R)) + p_{BRnm} * J_3(B)) \\
J_2(R) &= \max(0.6 * 1400 + 0.4 * 200 - 200, 0.3 * 1400 + 0.7 * 200) \\
J_2(R) &= 720
\end{aligned}
$$

Similarly, for $J_2(B)$,

$$
\begin{aligned}
J_2(B) &= \max_{a_2} \mathbb{E}_{x_3} \left[g_2(B, a_2, x_3) + J_3(x_3) \right] \\
J_2(B) &= \max \left(\mathbb{E}_{x_3} \left[g_2(B, n, x_3) + J_3(x_3) \right], \mathbb{E}_{x_3} \left[g_2(B, r, x_3) + J_3(x_3) \right] \right) \\
J_2(B) &= \max(1000 - C_n + J_3(R), (1 - p_{BBr}) * (1000 + J_3(R)) + p_{BBr} * J_3(B) - C_r) \\
J_2(B) &= \max(1000 - 1500 + 400, 0.6 * 1400 + 0.4 * 200 - 400) \\
J_2(B) &= 520
\end{aligned}
$$

Therefore, **optimal policies after week 2**: $\mu_2^*(R) = m$ and $\mu_2^*(B) = r$.

Finally, for the 1st week, we have:

$$J_1(R) = \max_{a_1} \mathbb{E}_{x_2}\left[g_1(R, a_1, x_2) + J_2(x_2)\right]$$
$$J_1(R) = \max\left(\mathbb{E}_{x_2}\left[g_1(R, m, x_2) + J_2(x_2)\right], \mathbb{E}_{x_2}\left[g_1(R, nm, x_2) + J_2(x_2)\right]\right)$$
$$J_1(R) = \max((1 - p_{BRm}) * (1000 + J_2(R)) + p_{BRm} * J_2(B) - C_m, (1 - p_{BRnm}) * (1000 + J_2(R)) + p_{BRnm} * J_2(B))$$
$$J_1(R) = \max(0.6 * 1720 + 0.4 * 520 - 200, 0.3 * 1720 + 0.7 * 520)$$
$$J_1(R) = 1040$$

Similarly, for $J_1(B)$,

$$J_1(B) = \max_{a_1} \mathbb{E}_{x_2}\left[g_1(B, a_1, x_2) + J_2(x_2)\right]$$
$$J_1(B) = \max\left(\mathbb{E}_{x_2}\left[g_1(B, n, x_2) + J_2(x_2)\right], \mathbb{E}_{x_2}\left[g_1(B, r, x_2) + J_2(x_2)\right]\right)$$
$$J_1(B) = \max(1000 - C_n + J_2(R), (1 - p_{BBr}) * (1000 + J_2(R)) + p_{BBr} * J_2(B) - C_r)$$
$$J_1(B) = \max(1000 - 1500 + 720, 0.6 * 1720 + 0.4 * 520 - 400)$$
$$J_1(B) = 840$$

Therefore, **optimal policies after week 1**: $\mu_1^*(R) = m$ and $\mu_1^*(B) = r$.

Since the machine is guaranteed to run in its first week 0, $J_0(x_0) = 1000 + J_1(R) = 2040$. Hence, expected profit is 2040 rupees.

4 Problem 4

4.1 Part (a)

1. Since the intermediate point I could be anywhere between S and D, the state space: $\{S, D, I_1, \ldots, I_m\}$.
2. Action space $\mu_k(x_k)$ for $i = \{1, \ldots, m\}$:
   (a) $\mu_k(S) = \{destination, intermediate\ point(ip)\}$
   (b) $\mu_k(I_i) = \{start, destination\}$
3. Transition probabilities
   (a) $P(x_{k+1} = I_i | x_k = S, a_k = ip) = p_i$
   (b) $P(x_{k+1} = D | x_k = S, a_k = destination) = 1$
   (c) $P(x_{k+1} = D | x_k = I_i, a_k = destination) = 1$
   (d) $P(x_{k+1} = S | x_k = I_i, a_k = start) = 1$
4. Single stage cost
   (a) $g(S, destination, D) = \alpha$
   (b) $g(S, ip, I_i) = \beta$
   (c) $g(I_i, destination, D) = c_i$
   (d) $g(I_i, start, S) = \beta$

   Therefore, the Bellman's equation will be:

$$J^*(S) = 1 + \min(\alpha, \beta + \sum_{i=1}^{m} p_{I_i} J^*(I_i))$$
$$J^*(I_i) = 1 + \min(c_i, \beta + J^*(S))$$

   Hence, the optimal policies are:

$$\mu^*(S) = \begin{cases} destination, & \text{if } \alpha < \beta + \sum_{i=1}^{m} p_{I_i} J^*(I_i) \\ ip, & \text{otherwise} \end{cases} \tag{1}$$

$$\mu^*(I_i) = \begin{cases} destination, & \text{if } c_i < \beta + J^*(S) \\ start, & \text{otherwise} \end{cases} \tag{2}$$

4.2 Part (b)

No, not all policies are proper. However, since assumption 1 (i.e. There must be at least one proper policy) and assumption 2 (improper policies have infinite cost) hold, Bellman equation holds. One example of such a policy is when $\mu^*(S) = ip$ and $\mu^*(I_i) = destination$.

4.3 Part (c)

We are given $\alpha = 2, \beta = 1, c_1 = 0, c_2 = 5$, and $p_1 = p_2 = 0.5$.

Since, $c_1 = 0$, $J^*(I_1) = c_1 = 0$. Now,

$$J^*(S) = \min(2, 1 + 0.5 * J^*(I_2))$$
$$J^*(I_2) = \min(5, 1 + J^*(S))$$

Let $J^*(S) = \alpha = 2$. Then, $J^*(I_2) = 3$. This gives us our optimal policy: $\mu^*(S) = destination$, $\mu^*(I_1) = destination$, and $\mu^*(I_2) = start$

4.4 Part (d)

Since we include waiting time, our new actions $\mu_k(I_i) = \{start, destination, wait\}$. Then, $g(I_i, wait, I_j) = d$ with transition probability p_j, the new Bellman equation will be:

$$J^*(S) = 1 + \min(\alpha, \beta + \sum_{i=1}^{m} p_{I_i} J^*(I_i))$$
$$J^*(I_i) = 1 + \min(c_i, \beta + J^*(S), d + \sum_{j=1}^{m} p_{I_j} J^*(I_j))$$

And, new optimal policies are:

$$\mu^*(S) = \begin{cases} destination, & \text{if } \alpha < \beta + \sum_{i=1}^{m} p_{I_i} J^*(I_i) \\ ip, & \text{otherwise} \end{cases} \tag{3}$$

$$\mu^*(I_i) = \begin{cases} destination, & \text{if } c_i < \min(\beta + J^*(S),\ d + \sum_{j=1}^{m} p_{I_j} J^*(I_j)) \\ start, & \text{if } \beta + J^*(S) < \min(c_i,\ d + \sum_{j=1}^{m} p_{I_j} J^*(I_j)) \\ wait, & \text{if } d + \sum_{j=1}^{m} p_{I_j} J^*(I_j) < \min(c_i,\ \beta + J^*(S)) \end{cases} \tag{4}$$

References

- **Problem 1**: Lecture notes; DPOC Vol.1, Chapter 1.
- **Problem 2**: Discussed with Richa Verma (CS20D020); DPOC Vol.1, Problem 4.19; Class notes on asset selling problem.
- **Problem 3**: Lecture Notes
- **Problem 4**: Lecture Notes (Spider and Fly problem)
- **Programming Question 1**: Lecture notes
- **Programming Question 2**: Discussed with Richa Verma (CS20D020)