Naïve Bayes

Following Applications:

- Spam Classification – Given an email, predict whether it is spam or not .

  (Spam or Ham)

- Medical Diagnosis – Given a list of symptoms, predict whether a patient has disease X or not

- Weather – Based on temperature, humidity, etc… predict if it will rain tomorrow

Characteristics

- The relationship between attribute set and the class variable is non-deterministic.
- Even if the attributes are same, the class label may differ in training set even and hence can not be predicted with certainty.
- Reason: noisy data, certain other attributes are not included in the data.

Prediction with limited attributes

- Task of predicting whether a person is at risk for heart disease based on the person’s diet and workout frequency.
- So need an approach to model probabilistic relationship between attribute set and the class variable.

Problem statement:

– Given features X1,X2,…,Xn – Predict a label Y

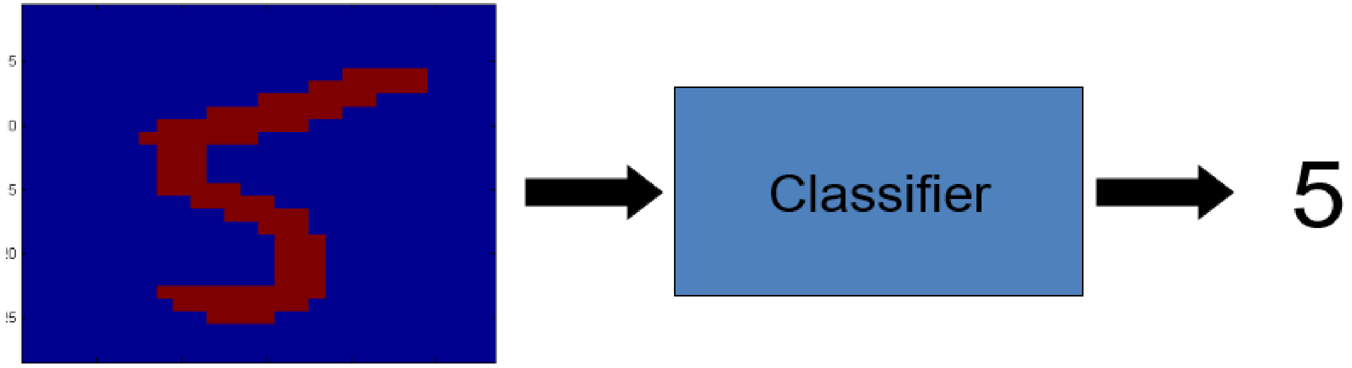
Bayesian Classifiers

- compute the posterior probability P(C | A1, A2, …, An) for all values of C using the Bayes theorem

- Choose value of C that maximizes   P(C | A1, A2, …, An)

Another Application

- Digit Recognition

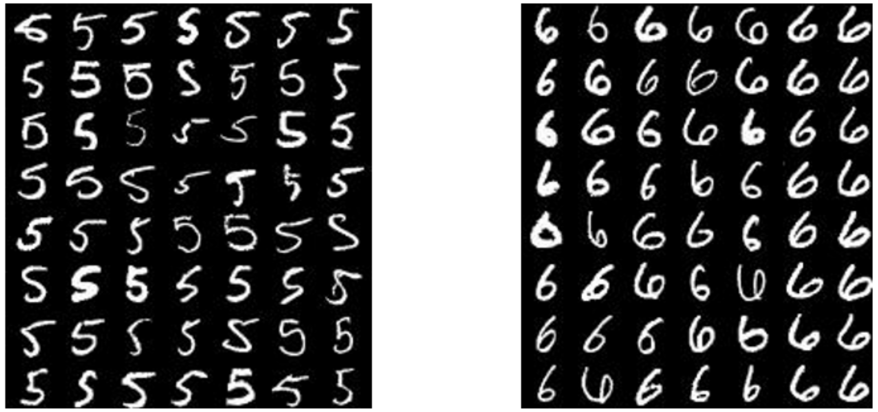


- $X1,\ldots,Xn \in \{0,1\}$ (Black vs. White pixels) • $Y \in \{5,6\}$ (predict whether a digit is a 5 or a 6)
- Cycle thru all the image cells (pixels) predicting a 5 or 6
- Compute P(5, X1,….Xn) and P(6,X1,……Xn) and decide if it is 5 or 6.

Naiive Bayes Training

Naïve Bayes Training

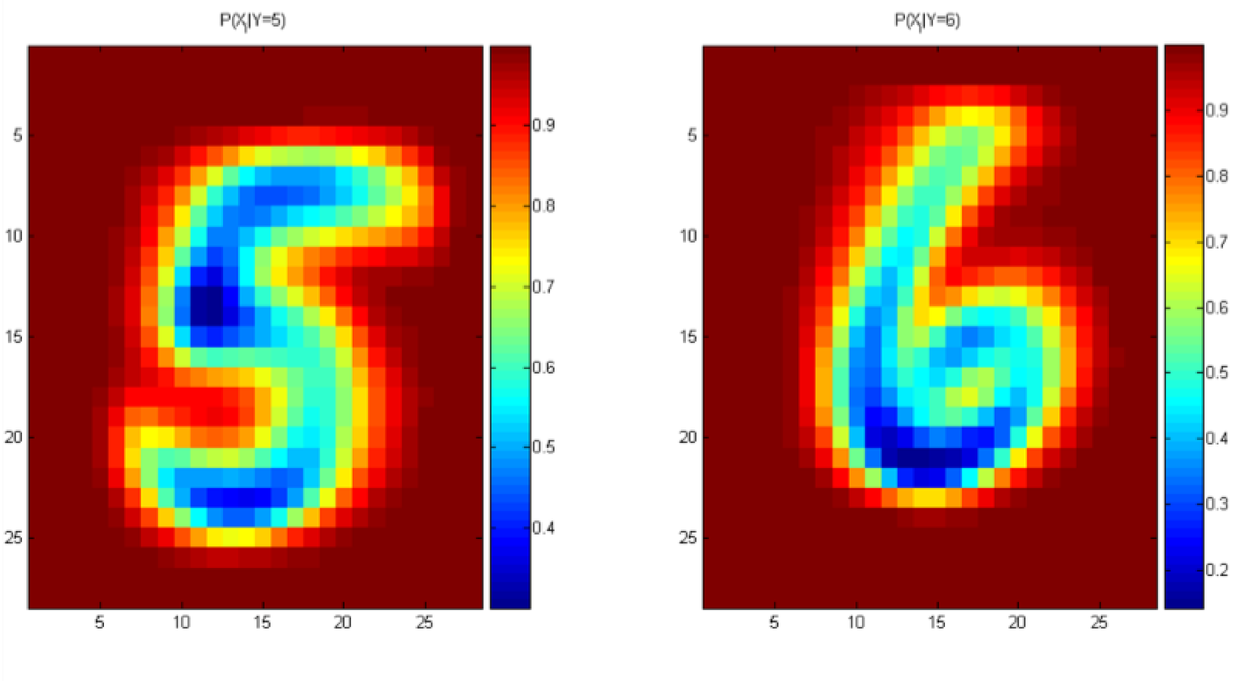
Now that we've decided to use a Naïve Bayes classifier, we need to train it with some data:



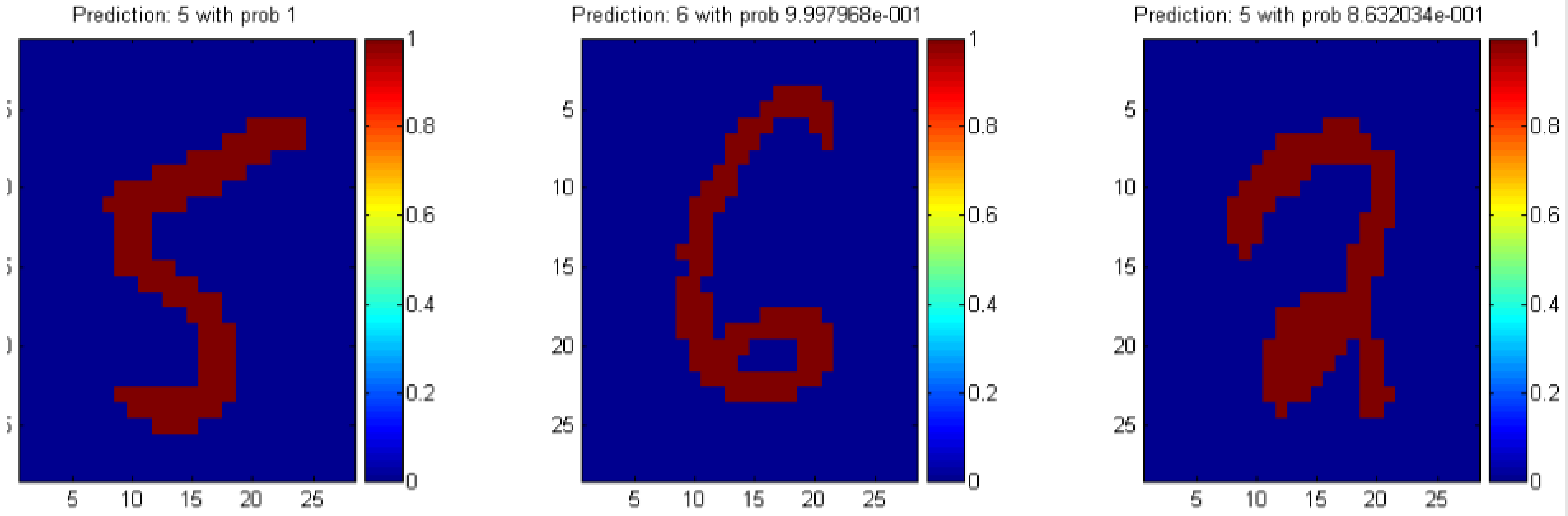
MNIST Training Data

Naïve Bayes Training

- For binary digits, training amounts to averaging all of the training fives together and all of the training sixes together.



Naiive Bayes Classification



Bayes Theorem

- Abstractedly stated:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}.$$

- Stated as conditional probability:
- (B|A) – can be interchanged as hypothesis and event.
- It allows us to invert the probability (B|A) to (A|B).

Bayes Cont’d

- (A|B) – B is the event , A is the hypothesis.
- B – you have tested positive for a disease
- A – Do you have the disease.
- Inversion:
- (B|A) – A you have the disease
- B – Did you test positive

Bayes – re-casted as inference, Prior and Posterior

$$P(\mathcal{H}\,|\,\mathcal{D}) = \frac{P(\mathcal{D}\,|\,\mathcal{H})P(\mathcal{H})}{P(\mathcal{D})}.$$

- The prior $P(\mathcal{H})$ is the probability that $\mathcal{H}$ is true before the data is considered.
- The posterior $P(\mathcal{H}\,|\,\mathcal{D})$ is the probability that $\mathcal{H}$ is true after the data is considered.
- The likelihood $P(\mathcal{D}\,|\,\mathcal{H})$ is the evidence about $\mathcal{H}$ provided by the data $\mathcal{D}$.
- $P(\mathcal{D})$ is the total probability of the data taking into account all possible hypotheses.

Bayes and the Base rate Fallacy

- A screening test for a disease is both sensitive and specific.

- Positive when testing a person with the disease
- Negative when testing someone without the disease.

- True positive rate is 99%.
- False positive rate is 2%.

- Suppose the prevalence of the disease in the general population is 0.5%.
- If a random person tests positive, what is the probability that they have the disease?
- Most people would guess that the answer is 99%.......

Using the notation established above for hypotheses and data:

Let:

- H+ be the hypothesis that the person has the disease
- H− be the hypothesis they do not.
- T+ and represent the data of a positive
- T− negative screening test respectively.

- We are asked to compute P(H+|T+).
- Given P(T+|H+) = 0.99, P(T+|H−) = 0.02, P(H+) = 0.005.

Expressing these facts in a tree

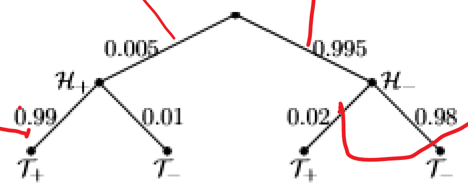
- Completing the missing part
- $P(T-|H+) = 0.01$, $P(T-|H-) = 0.98$

- $P(T+|H+) = 0.99$

$P(T+|H-) = 0.02$

-

P(H+) P(H-)



Calculation by Bayes Theorem

Bayes' theorem yields

$$P(\mathcal{H}_+|\mathcal{T}_+) = \frac{P(\mathcal{T}_+|\mathcal{H}_+)P(\mathcal{H}_+)}{P(\mathcal{T}_+)} = \frac{0.99 \cdot 0.005}{0.99 \cdot 0.005 + 0.02 \cdot 0.995} = 0.19920 \approx 20\%$$

Computing with tables

- Construct the table as follows. Pick a number, say 10000 people, and place it as the grand total in the lower right. Using P(D+) = .005 we compute that 50 out of the 10000 people are sick (D+).
- Likewise 9950 people are healthy (D−). At this point the table looks like:

	$D+$	$D-$	total
$T+$			
$T-$			
total	50	9950	10000

Computing with tables....

- Using P(T +|D+) = .9 we can compute that the number of sick people who tested positive as 90% of 50 or 45. The other entries are similar. At this point the table looks like the table below on the left.
- Finally we sum the T+ and T− rows to get the completed table on the right.

	$D+$	$D-$	total
$T+$	45	498	
$T-$	5	9452	
total	50	9950	10000

	$D+$	$D-$	total
$T+$	45	498	543
$T-$	5	9452	9457
total	50	9950	10000

Computing with tables....

Using the complete table we can compute

$$P(D+|T+) = \frac{|D+\cap T+|}{|T+|} = \frac{45}{543} = 8.3\%.$$

Computing with tables

Symbols: For completeness, we show how the solution looks when written out directly in symbols.

$$\begin{aligned}
P(D+|T+) &= \frac{P(T+|D+)\cdot P(D+)}{P(T+)} \\
&= \frac{P(T+|D+)\cdot P(D+)}{P(T+|D+)\cdot P(D+) + P(T+|D-)\cdot P(D-)} \\
&= \frac{.9 \times .005}{.9 \times .005 + .05 \times .995} \\
&= 8.3\%
\end{aligned}$$

Naïve Bayes - Acknowledgement

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