Computer Exercises 1

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Problem 1

Implement the division algorithm for multivariate polynomials in Macaulay 2.

Proof

See code (mDivAlg)

Problem 2

In each of (1) and (2), use the Macaulay2 function you wrote for Problem 1 to compute the remainder on division of the polynomial f by the ordered set F.

(1) $f = x^2y + xy^2 + y^2, f_1 = y^2 - 1, f_2 = xy - 1 \in \mathbb{Q}[x, y]$ and $F = f_1, f_2$. Use lex with $x > y$.

(2) $f = y^2x + 4yx = 3x^2, g = 2y + x + 1 \in \mathbb{Q}[x, y]$ and $F + g$. Use grlex with $y > x$.

(1) Proof

mDivAlg(f,F) $\to \{\{1, x + 1\}, x + 2\}$ so,

$$f = f_1 + (x + 1)f_2 + (x + 2)$$

(2) Proof

mDivAlg(f,F) $\to \{\{\frac{1}{2}yx - \frac{1}{4}x^2 + \frac{7}{4}x\}, \frac{1}{4}x^3 - \frac{9}{2}x^2 - \frac{7}{4}x\}$ so,

$$f = (\frac{1}{2}yx - \frac{1}{4}x^2 + \frac{7}{4}x)g + (\frac{1}{4}x^3 - \frac{9}{2}x^2 - \frac{7}{4}x)$$

Problem 3

Write a Macaulay2 function that computes the least common multiple of two monomials.

Proof

See code (monomialLCM)

Problem 4

Write a Macaulay2 funciton that computes the S-polynomial of two non-zero polynomials.

Proof

See code (sPoly)

Problem 5

Implement Buchberger's algorithm in Macaulay2.

Proof

See code (buchberger)

Problem 6

Use the Macaulay2 function you wrote for Problem 5 to compute a Groebner basis for each of the following ideals.

(1) $I = \langle x^3 - 2xy, x^2y - 2y^2 + x \rangle \subset \mathbb{Q}[x, y]$. Use grlex with $x > y$.

(2) $I = \langle x + y + z, y^2 + yz + z^2, Z^3 - 1 \rangle \subseteq \mathbb{Q}[x, y]$. Use lex with $x > y$.

(1) Proof

buchberger $F \to \{x^3 - 2xy, x^2y - 2y^2 + x, -x^2, -2xy, -2y^2 + x\}$

(2) Proof

buchberger $F \to \{x + y + z, y + yz + z, z - 1\}$