Separating the magnetospheric disturbance magnetic field into external and transient internal contributions using a 1D conductivity model of the Earth

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[1] Magnetospheric fields and their induced counterparts are the largest source of error in models representing the geomagnetic field. Of particular concern is the current practice of coupling the internal induced field for convenience to the external field by a real constant, independent of the frequency content of the external inducing source. The error introduced into field models by this simplified representation is of the order of 5 nT on average. Here, we propose an accurate representation of the symmetric part of the disturbance field which is easy to implement. Using a 1D conductivity model of the Earth, we split the disturbance D_{st} index into two new indices, E_{st} and I_{st}, which track the transient evolution of the symmetric part of the external and induced disturbance field. The ensuing D_{st}-based transient correction for geomagnetic field models is in remarkable agreement with the transient effect observed in CHAMP, Ørsted and SAC-C satellite magnetic measurements. *INDEX TERMS:* 1515 Geomagnetism and Paleomagnetism: Geomagnetic induction; 1555 Geomagnetism and Paleomagnetism: Time variations—diurnal to secular; 2778 Magnetospheric Physics: Ring current. **Citation:** Maus, S., and P. Weidelt (2004), Separating the magnetospheric disturbance magnetic field into external and transient internal contributions using a 1D conductivity model of the Earth, *Geophys. Res. Lett.*, *31*, L12614, doi:10.1029/2004GL020232.

1. Introduction

[2] Charged particles trapped by the geomagnetic field in the magnetosphere drift around the Earth at a distance of $3 - 8\ R_E$ creating a westward electric ring current whose field opposes the main geomagnetic field [*Daglis and Kozyra*, 2002]. The strength of this field is of the order of tens of nT during quiet times and several hundred nT during magnetic storms. Magnetopause, tail and partial ring currents make additional contributions leading to asymmetries in the field which increase during storms. The symmetric part of this composite disturbance field is tracked by the D_{st} (disturbance storm-time) index [*Sugiura*, 1964], derived from the measurements of four low latitude observatories. In geomagnetic field modeling, the D_{st} index is used to represent the symmetric disturbance field, while its asymmetric part is commonly ignored. However, even the symmetric part of the disturbance field is currently not represented in an optimal way.

[3] The difficulty in disturbance magnetic field representations is that the time varying external source field induces electric currents in the Earth which in turn give rise to a secondary field whose strength is roughly one third of the inducing field. Hence, the disturbance field observed at the Earth's surface and quantified by the D_{st} index is actually the sum of the external source field and its induced counterpart. If the Earth were an ideal conductor, then the two fields would be exactly in phase because currents would be induced in such a way as to prevent any external field from entering into the conductor. Furthermore, for an ideal conductor the strength of the induced field would be such that its radial component would cancel the radial component of the inducing field everywhere on its surface. For the real Earth, however, the phase lag and amplitude relation between the induced internal and inducing external field depends on the frequency content of the external source field. Therefore, a correct representation of the internal field requires a Fourier decomposition of the external field. Since, generally, only a small subset of measurements at the most quiet times are used in geomagnetic field modeling, a Fourier analysis of the data is not possible. For convenience, the inducing and induced fields are therefore assumed to be in phase and their amplitude relation is assumed to be constant [*Langel and Estes*, 1985; *Olsen*, 2002; *Maus et al.*, 2004]. However, there is a simple way to arrive at an accurate disturbance field representation: Instead of Fourier transforming the measured data, one can Fourier transform the D_{st} index and directly derive the internal and external contributions. This provides a physically correct and convenient representation of the symmetric disturbance field.

2. Separating Internal and External Parts of the Symmetric Disturbance Field

[4] The D_{st} index tracks variations in the strength of the disturbance field at the magnetic equator. To be precise, D_{st} is the equatorial northward component of the symmetric part of the disturbance field at the Earth's surface in magnetic dipole coordinates. Here, symmetric means rotational symmetry about the Earth's magnetic dipole axis. Neglecting higher order zonal harmonics, D_{st} is the sum of an external uniform source field (external dipole) and a corresponding dipolar induced field. This assertion is confirmed by the agreement of the D_{st} index with the sum of the external and induced dipole strengths inferred from

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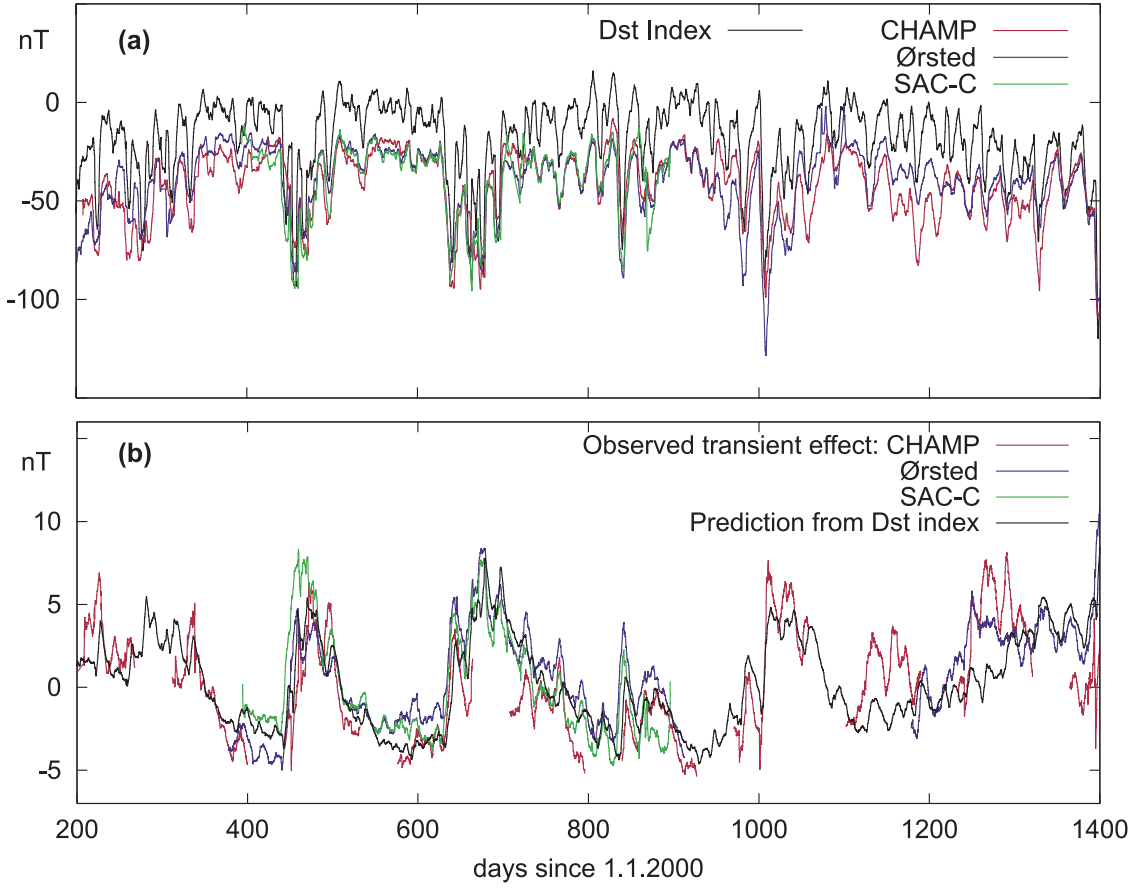
Figure 1. (a) Comparison of the D_{st} index (black) with the sum of the external and induced dipole strengths of the geomagnetic field, inferred from night-side satellite magnetic measurements (color). All curves are smoothed by a 10 day running mean. The satellite estimates show a negative offset of about 20 nT against D_{st} which is due to the stable quiet time ring current, not represented in the D_{st} index. Disagreements between the satellites are caused by their sampling of the asymmetric field at different local times. (b) Displayed in the lower graph are four independent estimates of the average transient error in current geomagnetic field models as a function of time. All curves are based on equations (19) and (14), taking $f_1 = 0.76$ and $f_2 = 0.32$, and are smoothed using a 10 day running mean. On one hand, time series of the external and internal dipole field ($E_{st}(t)$ and $I_{st}(t)$) can be estimated by splitting the D_{st} index (black). On the other hand, corresponding time series can be estimated directly from satellite magnetic measurements giving an independent observation of the transient effect (color). The agreement between the predicted and observed curves validates the proposed approach and illustrates the magnitude of the corresponding field modeling error, which is of the order of 5 nT at the Earth's surface. Interestingly, the transient effect persists over long time spans into quiet magnetic periods.

satellite magnetic measurements, as shown in Figure 1a. Let us therefore split the D_{st} index and introduce an E_{st} index tracking the external dipole source and an I_{st} index tracking the internal induced dipolar response of the Earth. Then

$$D_{st}(t) = E_{st}(t) + I_{st}(t) \qquad (1)$$

at any given instant of time, and the corresponding external and internal dipole fields can be represented as

$$\mathbf{B}_{ext}(t) = -E_{st}(t)\left(\sin\vartheta\,\hat{\boldsymbol{\vartheta}} - \cos\vartheta\,\hat{\boldsymbol{r}}\right) \qquad (2)$$

$$\mathbf{B}_{int}(t) = -I_{st}(t)\left(\sin\vartheta\,\hat{\boldsymbol{\vartheta}} + 2\cos\vartheta\,\hat{\boldsymbol{r}}\right) \qquad (3)$$

where $\hat{\boldsymbol{\vartheta}}$ and $\hat{\boldsymbol{r}}$ are the local southward and outward unit vectors, and the negative signs arise because D_{st} represents

the northward field while $\hat{\boldsymbol{\vartheta}}$ points southward. Property (1) must also hold in the frequency domain, as

$$\tilde{D}_{st}(\omega) = \tilde{E}_{st}(\omega) + \tilde{I}_{st}(\omega). \qquad (4)$$

Let us introduce the complex transfer function $q_1(\omega)$ as

$$q_1(\omega) = \frac{\tilde{I}_{st}(\omega)}{\tilde{E}_{st}(\omega)}, \qquad (5)$$

where the index "1" refers to the assumed spherical harmonic degree $n = 1$ geometry of the external and internal fields. Then we can rewrite equation (4) as

$$\tilde{D}_{st}(\omega) = \tilde{E}_{st}(\omega) + q_1(\omega)\tilde{E}_{st}(\omega) = (1 + q_1(\omega))\tilde{E}_{st}(\omega). \qquad (6)$$

Thus, the external and internal contributions to the disturbance field are given as

$$\tilde{E}_{st}(\omega) = \frac{1}{1 + q_1(\omega)} \tilde{D}_{st}(\omega) \qquad (7)$$

$$\tilde{I}_{st}(\omega) = \frac{q_1(\omega)}{1 + q_1(\omega)} \tilde{D}_{st}(\omega). \qquad (8)$$

Assuming that we know the transfer function $q_1(\omega)$, which is a function of Earth conductivity alone, we can separate the D_{st} index into two new indices, representing the external and internal parts of the disturbance field by the following scheme:

1. Fourier transform $D_{st}(t)$ to obtain $\tilde{D}_{st}(\omega)$
2. Apply the transfer function $q_1(\omega)$ using equations (7) and (8) to obtain $\tilde{E}_{st}(\omega)$ and $\tilde{I}_{st}(\omega)$
3. Transform $\tilde{E}_{st}(\omega)$ and $\tilde{I}_{st}(\omega)$ back to the time domain to obtain $E_{st}(t)$ and $I_{st}(t)$

The same operation can be performed as a convolution in the time domain. Since D_{st} only provides an accurate representation of the disturbance field for periods significantly shorter than a year, we use a convolution time window of one year to process the entire D_{st} time series from 1957 to present.

2.1. Quantifying the Transient Effect

[5] Current practice in geomagnetic field modelling is to assume for convenience that the transfer function $q_1(\omega)$ is real and independent of ω. Under this simplifying assumption the external and internal disturbance fields can be represented as

$$E(t) = f_1 D_{st}(t) \qquad (9)$$

$$I(t) = f_2 f_1 D_{st}(t) \qquad (10)$$

where $E(t)$ and $I(t)$ are the northward strengths of external and internal dipole fields at the magnetic equator on the Earth's surface, and the factors f_1 and f_2 are real factors which are further assumed to be independent of the frequency content of the inducing field. By a linear regression analysis of 3 years of CHAMP measurements, *Maus et al.* [2004] found optimum values of $f_1 = 0.76$ and $f_2 = 0.32$. In order to fulfill equations (1) and (4), the real factors f_1 and $f_1 f_2$ must add to unity,

$$f_1 + f_1 f_2 = 1 \qquad (11)$$

which is indeed the case with the above pair of values, but does not hold for other geomagnetic field models [e.g., *Holme et al.*, 2002; *Olsen*, 2002], where the sum of f_1 and $f_1 f_2$ falls significantly short of unity.

[6] Let us now analyze the error made in current geomagnetic field models when assuming a real, constant transfer function. From equation (9), the error in the external field is

$$\delta_{ext}(t) = f_1 D_{st}(t) - E_{st}(t) \qquad (12)$$

where $f_1 D_{st}(t)$ is the approximate and $E_{st}(t)$ is the true external field. Since D_{st} is the sum of the external and internal field, we can substitute

$$\delta_{ext}(t) = f_1(E_{st}(t) + I_{st}(t)) - E_{st}(t) \qquad (13)$$

$$= f_1 I_{st}(t) - (1 - f_1) E_{st}(t) \qquad (14)$$

The same line of reasoning can be applied to the internal field, yielding $\delta_{int}(t) = -\delta_{ext}(t)$ for those models which fulfill equation (11), since any part of the horizontal field at the equator which is erroneously attributed to the external field must be missing in the internal induced field. If we look at the ensuing error in the representation given by equations (2) and (3), it depends on the location of the observer. While it vanishes at the magnetic equator, the error in the radial component reaches $3\delta_{ext}(t)$ at the magnetic poles, which may explain why current D_{st} corrections do not work well at high latitudes [*Sillanpaa et al.*, 2004]. On average, the erroneous field **B** at the Earth's surface is

$$\langle B^2 \rangle = \frac{1}{4\pi R_E^2} \int_S \left(B_r^2 + B_\vartheta^2 \right) dS. \qquad (15)$$

Substituting E_{st} with δ_{ext} and I_{st} with δ_{int} in equations (2) and (3) and using $\delta_{int} = -\delta_{ext}$ gives

$$B_r = (\delta_{ext} - 2\delta_{int}) \cos \vartheta = 3\delta_{ext} \cos \vartheta \qquad (16)$$

$$B_\vartheta = (-\delta_{ext} - \delta_{int}) \sin \vartheta = 0 \qquad (17)$$

Hence, the error made in ignoring the transient effect is entirely limited to the vertical component of the disturbance field. This is not surprising because at the equator the internal and external horizontal field are forced to add up to the observed D_{st} field. Since D_{st} is assumed to be the sum of a single external and internal spherical harmonic, the error in the horizontal component must vanish at all latitudes. On average,

$$\langle B^2 \rangle = \frac{1}{4\pi R_E^2} \delta_{ext}^2 \int_S 9 \cos^2 \vartheta dS = 3\delta_{ext}^2 \qquad (18)$$

and the RMS error made at the Earth's surface at a particular moment in time is

$$RMS(t) = \sqrt{3} |\delta_{ext}(t)| \qquad (19)$$

which shall be referred to in the following as the average transient effect or average transient error.

2.2. Conductivity Model and Transfer Function

[7] As a conductivity model we use the 1D semi-global reference model of *Utada et al.* [2003, Model B], derived from a joint inversion of observatory magnetic field measurements and electric potential variations observed in submarine cables across the Pacific Ocean. For the given spherical conductivity model consisting of uniform layers and for a prescribed external magnetic field of degree n and

angular frequency ω the modified impedance is defined as $C_n := -E_{n\varphi}/(i\omega B_{n\vartheta})$ [*Schmucker*, 1985], where $E_{n\varphi}$ and $B_{n\vartheta}$ denote the eastward and southward components of the degree n electric and magnetic fields, respectively. The impedance is recursively continued upwards from the core mantle boundary (where $C_n = 0$) to the Earth's surface $r = a$. From $C_n(a, \omega)$ the q-ratio is then obtained via

$$q_n(\omega) = \frac{n}{n+1} \cdot \frac{a - (n+1)C_n(a,\omega)}{a + nC_n(a,\omega)}. \tag{20}$$

For the present work we assume that the inducing disturbance field is uniform, corresponding to an external dipole with $n = 1$.

3. Results

[8] From the D_{st} index of the years 1957–2003 we have computed the proposed E_{st} and I_{st} indices using the transfer function corresponding to model B of *Utada et al.* [2003]. With equations (19) and (14), we can then predict the average transient error of a geomagnetic field model which has a disturbance field representation based on a real, constant transfer function. This predicted transient error is displayed as a black curve in Figure 1b.

3.1. Comparison With Observed Transient Effect

[9] Instead of predicting the external and internal disturbance field variations from the D_{st} index, it is also possible to directly observe a time series of the external and internal dipole field from satellite magnetic measurements. Note, however, that a direct comparison of predicted and observed E_{st} and I_{st} time series is not feasible because their amplitudes are tens to hundreds of nT, while the transient effect to be studied here is only of the order of single nT. This is why the transient effect has to be isolated first. Directly comparing the predicted transient effect with the independently observed transient effect then provides a means to verify the proposed disturbance field representation.

[10] To obtain observed time series of the external and internal magnetic field, we analyze the latest total field measurements from the CHAMP, Ørsted and SAC-C satellites. Tracks ranging from $-65°$ to $65°$ in magnetic latitude are selected for the 20:00 to 4:00 local time interval. In order to obtain a time series with few missing values, data are used for all levels of magnetic activity. After subtracting the recent model Ørsted-10b-03 of the OVSM series [*Olsen*, 2002], omitting the external and induced coefficients, we individually fit to each track a 5 parameter model (S. Maus et al., Earth's crustal magnetic field determined to spherical harmonic degree 90 from CHAMP satellite measurements, submitted to *Geophysical Journal International*) consisting of (1) an external dipole in the direction of the Earth's main field dipole, (2) an external dipole perpendicular to the main field dipole and aligned with the orbital plane, (3) an internal dipole in the direction of the main field dipole, and (4) one northern and (5) one southern polar electrojet current (PEJ) with their current axis outside of the range of the tracks. These PEJ corrections are important, particular during disturbed times. The reason is that the internal and external dipoles point in the same direction at the magnetic equator. Hence, their separation (which is the essence of this paper) depends on the magnetic field at higher latitudes. There, on the other hand, the field is strongly influenced by the polar electrojets.

[11] From the time series of the observed external and internal dipole field, the average transient effect is calculated using equations (19) and (14), in the same way as for the prediction from the D_{st} index. The comparison shows a remarkable agreement (Figure 1b). Remaining disagreements can be attributed to local time effects due to the asymmetry of the disturbance field.

4. Conclusions

[12] From the D_{st} index a new pair of indices E_{st} and I_{st} has been derived. The indices track the transient behavior of the symmetric part of the external and the induced internal disturbance field. The accuracy of the separation was demonstrated by a comparison with independently derived magnetic field variations from CHAMP, Ørsted, and SAC-C satellite measurements. These indices provide an improved disturbance field representation for geomagnetic models. Using the new indices requires minimal adjustment of current field modeling practice and should yield a significant gain of the order of 5 nT in overall model accuracy. The indices are available from the National Geophysical Data Center at http://www.ngdc.noaa.gov/seg/geomag/est_ist.shtml.

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