Genotype probabilities at intermediate generations in the construction of recombinant inbred lines

SUPPLEMENT

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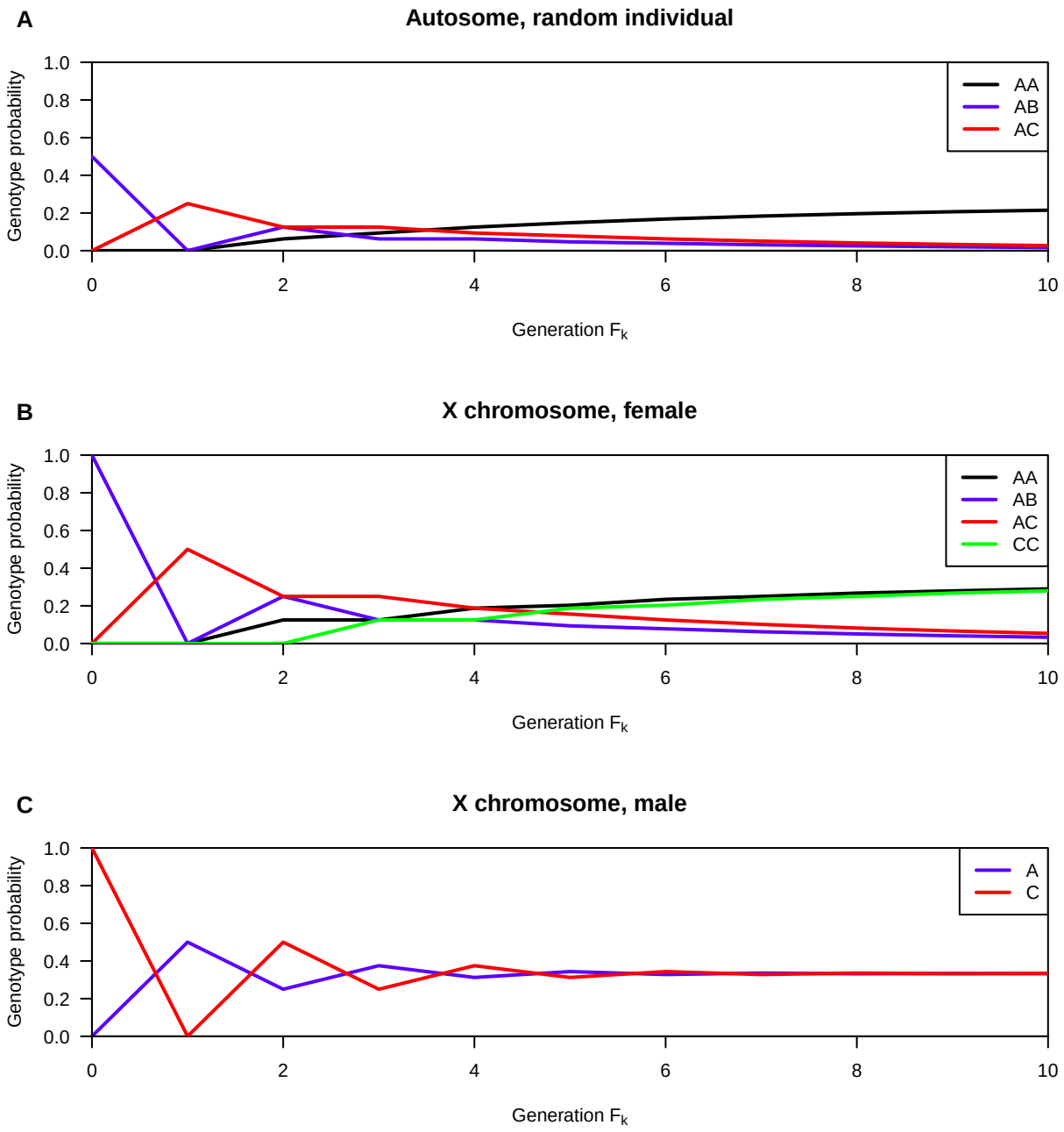


Figure S1 One-locus genotype probabilities for a random individual on the autosome **(A)**, the female on the X chromosome **(B)**, and the male on the X chromosome **(C)**, at generation F_k in the production of four-way RIL by sibling mating, as a function of k.

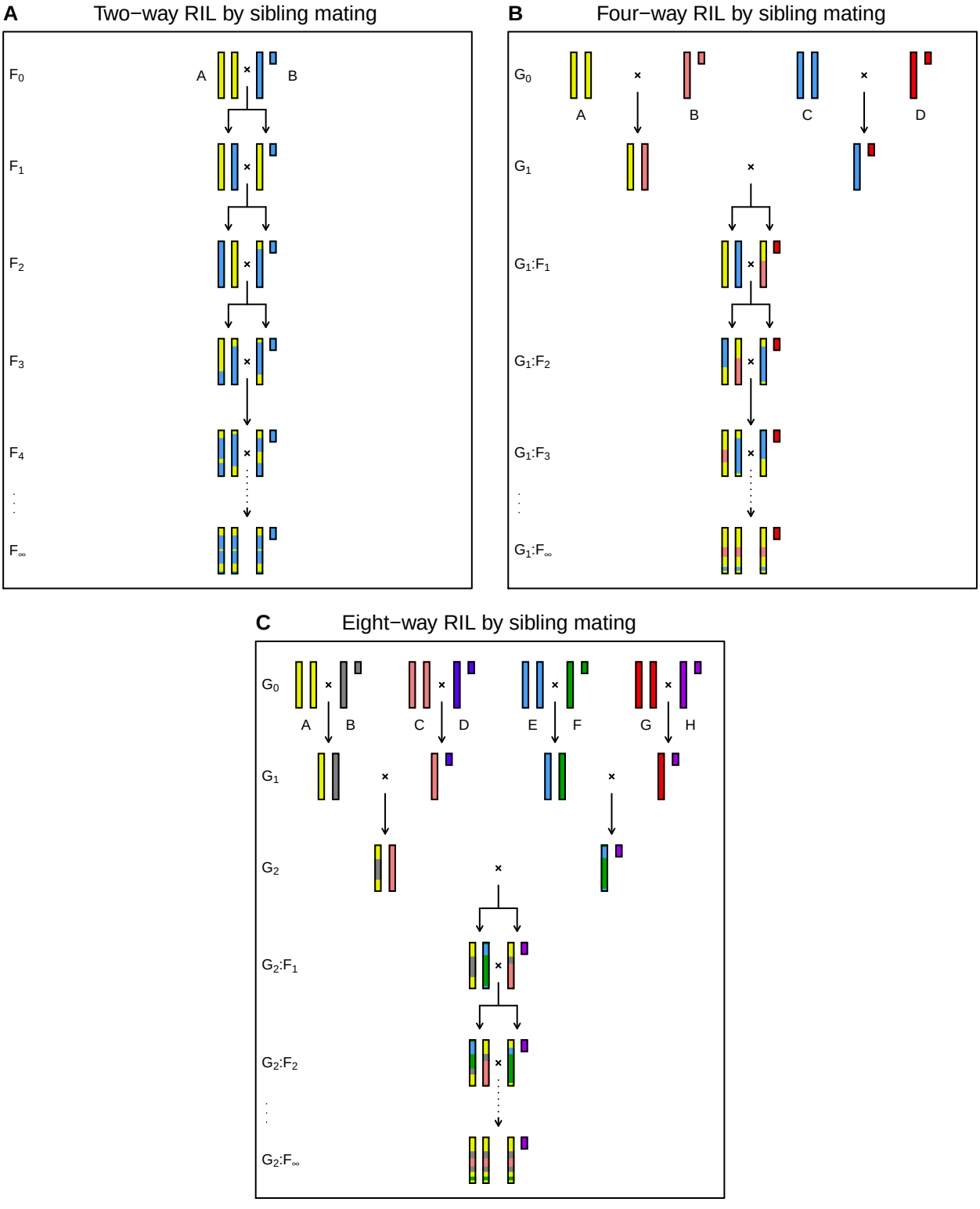


Figure S2 The X chromosome in the generation of two-way (A), four-way (B), and eight-way (C) RIL by sibling mating.

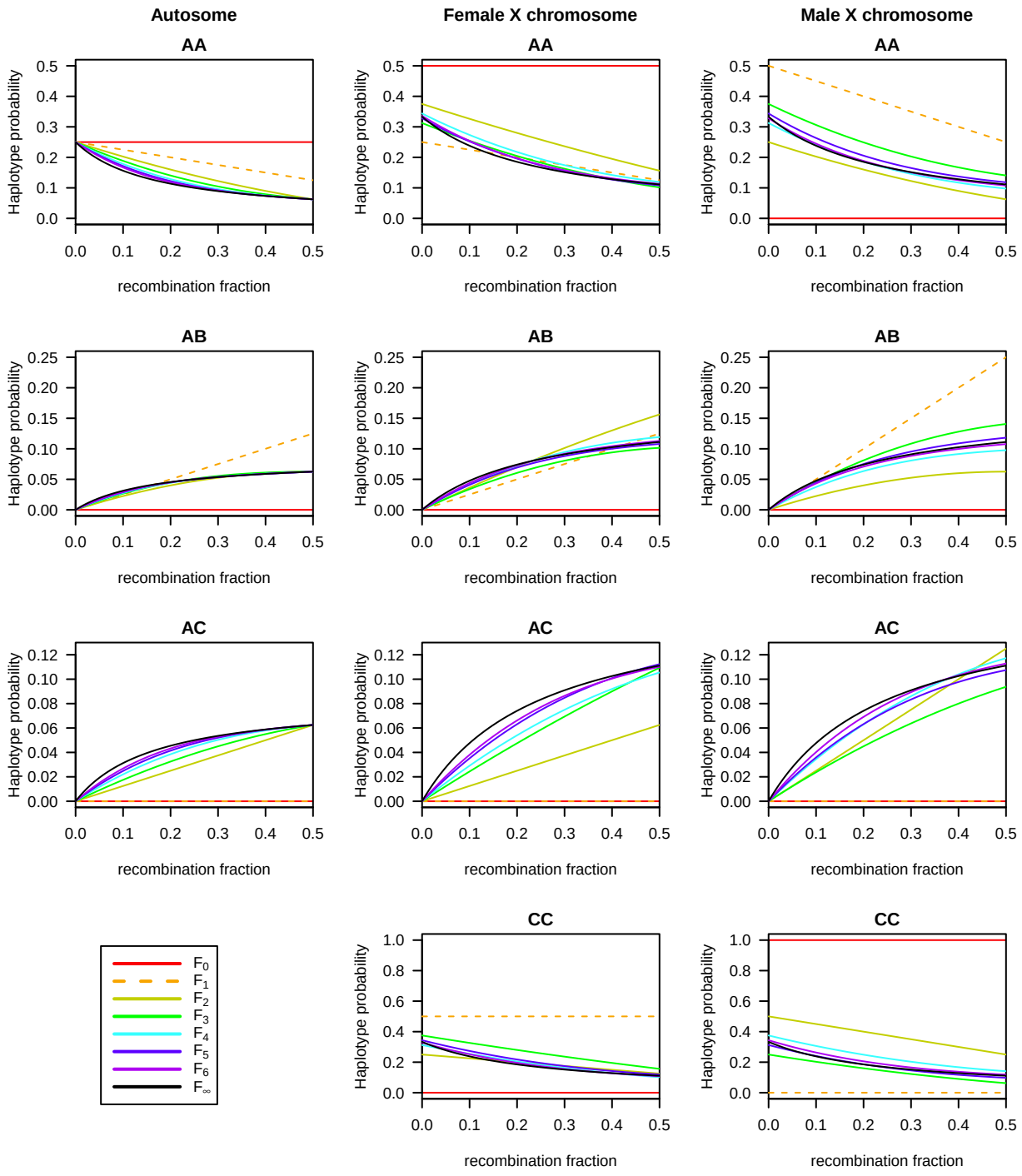


Figure S3 Two-locus haplotype probabilities, as a function of recombination fraction, for a random autosome haplotype (left column), a random X chromosome haplotype from the female (middle column), and the male X chromosome haplotype (right column) at generation F_k in the production of four-way RIL by sibling mating, with the individual curves corresponding to different values of k.

Table S1 Recursion matrix for calculating two-locus autosomal haplotype probabilities in the generation of four-way RIL by sibling mating

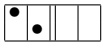
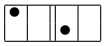
State at k		State at $k+1$: 1	2	3
1		$1-r$	0	$1/4$
2		r	0	$1/4$
3		0	1	$1/2$

Table S2 Starting states for calculating two-locus autosomal haplotype probabilities in the generation of four-way RIL by sibling mating

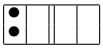
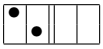
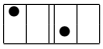
Prototype	No. states	Initial pattern	Initial probability
AA	4	 (1)	$1/4$
AB	4	 (2)	$1/4$
AC	8	 (3)	$1/8$

Table S3 Transition matrix for two loci in the generation of two-way RIL by selfing

	g_{k+1}				
g_k	$AA\|AA$	$AB\|AB$	$AA\|AB$	$AA\|BB$	$AB\|BA$
$AA\|AA$	1	0	0	0	0
$AB\|AB$	0	1	0	0	0
$AA\|AB$	$1/4$	$1/4$	$1/2$	0	0
$AA\|BB$	$(1-r)^2/2$	$r^2/2$	$2r(1-r)$	$(1-r)^2/2$	$r^2/2$
$AB\|BA$	$r^2/2$	$(1-r)^2/2$	$2r(1-r)$	$r^2/2$	$(1-r)^2/2$

Table S4 Transition matrix for one autosomal locus in the generation of four-way RIL by sibling mating

		g_{k+1}												
	g_k	1	2	3	4	5	6	7	8	9	10	11	12	13
1:	$AA \times AA$	1	0	0	0	0	0	0	0	0	0	0	0	0
2:	$AA \times AB$	1/4	1/2	0	0	0	0	0	1/4	0	0	0	0	0
3:	$AA \times AC$	1/4	0	1/2	0	0	0	0	0	0	0	1/4	0	0
4:	$AA \times BB$	0	0	0	0	0	0	0	1	0	0	0	0	0
5:	$AA \times BC$	0	0	0	0	0	0	0	1/4	1/2	0	1/4	0	0
6:	$AA \times CC$	0	0	0	0	0	0	0	0	0	0	1	0	0
7:	$AA \times CD$	0	0	0	0	0	0	0	0	0	0	1/2	1/2	0
8:	$AB \times AB$	1/8	1/2	0	1/8	0	0	0	1/4	0	0	0	0	0
9:	$AB \times AC$	1/16	1/8	1/8	0	1/8	0	0	1/16	1/4	0	1/8	1/8	0
10:	$AB \times CD$	0	0	0	0	0	0	0	0	0	0	1/4	1/2	1/4
11:	$AC \times AC$	1/8	0	1/2	0	0	1/8	0	0	0	0	1/4	0	0
12:	$AC \times AD$	1/16	0	1/4	0	0	0	1/8	1/16	1/4	0	1/8	1/8	0
13:	$AC \times BD$	0	0	0	0	0	0	0	1/8	1/2	1/8	1/8	0	1/8

Table S5 Probabilities for the genotypes of the pair of individuals at a single autosomal locus, at generation F_k in the formation of four-way RIL by sibling mating

Prototype	No. states	Probability of each
$AA \times AA$	4	$\frac{1}{4}+\frac{1}{4}\left(\frac{1}{2}\right)^k-\frac{1}{20}\left(\frac{1}{4}\right)^k-\left(\frac{9+4\sqrt{5}}{40}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{9-4\sqrt{5}}{40}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times AB$	4	$\frac{1}{6}\left(-\frac{1}{4}\right)^k+\frac{1}{10}\left(\frac{1}{4}\right)^k-\frac{1}{6}\left(\frac{1}{2}\right)^k-\left(\frac{1-\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{1+\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times AC$	8	$-\frac{1}{12}\left(-\frac{1}{4}\right)^k+\frac{1}{20}\left(\frac{1}{4}\right)^k-\frac{1}{6}\left(\frac{1}{2}\right)^k+\frac{1}{10}\left[\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{1-\sqrt{5}}{4}\right)^k\right]$
$AA \times BB$	2	$\frac{1}{3}\left(-\frac{1}{4}\right)^k-\frac{2}{15}\left(-\frac{1}{8}\right)^k+\frac{1}{30}\left(\frac{1}{4}\right)^k-\frac{1}{30}\left(\frac{1}{2}\right)^k-\left(\frac{2-\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{2+\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times BC$	8	$-\frac{1}{12}\left(-\frac{1}{4}\right)^k+\frac{2}{15}\left(-\frac{1}{8}\right)^k-\frac{1}{12}\left(\frac{1}{4}\right)^k+\frac{1}{30}\left(\frac{1}{2}\right)^k$
$AA \times CC$	4	$-\frac{1}{6}\left(-\frac{1}{4}\right)^k+\frac{1}{30}\left(-\frac{1}{8}\right)^k+\frac{1}{60}\left(\frac{1}{4}\right)^k-\frac{1}{30}\left(\frac{1}{2}\right)^k+\left(\frac{3-\sqrt{5}}{40}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{3+\sqrt{5}}{40}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times CD$	4	$\frac{1}{6}\left(-\frac{1}{4}\right)^k-\frac{1}{5}\left(-\frac{1}{8}\right)^k+\frac{1}{30}\left(\frac{1}{2}\right)^k$
$AB \times AB$	2	$-\frac{2}{3}\left(-\frac{1}{4}\right)^k+\frac{2}{15}\left(-\frac{1}{8}\right)^k+\frac{1}{15}\left(\frac{1}{4}\right)^k-\frac{2}{15}\left(\frac{1}{2}\right)^k+\left(\frac{3-\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{3+\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AB \times AC$	8	$\frac{1}{6}\left(-\frac{1}{4}\right)^k-\frac{2}{15}\left(-\frac{1}{8}\right)^k-\frac{1}{6}\left(\frac{1}{4}\right)^k+\frac{2}{15}\left(\frac{1}{2}\right)^k$
$AB \times CD$	1	$\frac{2}{3}\left(-\frac{1}{8}\right)^k+\frac{1}{3}\left(\frac{1}{4}\right)^k$
$AC \times AC$	4	$\frac{1}{3}\left(-\frac{1}{4}\right)^k-\frac{1}{30}\left(-\frac{1}{8}\right)^k+\frac{1}{30}\left(\frac{1}{4}\right)^k-\frac{2}{15}\left(\frac{1}{2}\right)^k-\left(\frac{2-2\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{2+2\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AC \times AD$	4	$-\frac{1}{3}\left(-\frac{1}{4}\right)^k+\frac{1}{5}\left(-\frac{1}{8}\right)^k+\frac{2}{15}\left(\frac{1}{2}\right)^k$
$AC \times BD$	2	$-\frac{1}{3}\left(-\frac{1}{8}\right)^k+\frac{1}{3}\left(\frac{1}{4}\right)^k$

Table S6 Transition matrix for one X chromosome locus in the generation of four-way RIL by sibling mating

		g_{k+1}									
	g_k	1	2	3	4	5	6	7	8	9	10
1:	$AA \times A$	1	0	0	0	0	0	0	0	0	0
2:	$AA \times B$	0	0	0	1	0	0	0	0	0	0
3:	$AA \times C$	0	0	0	0	0	1	0	0	0	0
4:	$AB \times A$	1/4	1/4	0	1/2	0	0	0	0	0	0
5:	$AB \times C$	0	0	0	0	0	1/2	1/2	0	0	0
6:	$AC \times A$	1/4	0	1/4	0	0	1/4	0	1/4	0	0
7:	$AC \times B$	0	0	0	1/4	1/4	0	1/4	1/4	0	0
8:	$AC \times C$	0	0	0	0	0	1/4	0	1/4	1/4	1/4
9:	$CC \times A$	0	0	0	0	0	0	0	1	0	0
10:	$CC \times C$	0	0	0	0	0	0	0	0	0	1

Table S7 Probabilities for the genotypes of the pair of individuals at a single X chromosome locus, at generation F_k in the formation of four-way RIL by sibling mating

Prototype	No. states	Probability of each
$AA \times A$	2	$\frac{1}{3}+\frac{1}{24}\left(-\frac{1}{2}\right)^k+\frac{1}{8}\left(\frac{1}{2}\right)^k-\left(\frac{5+2\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{5-2\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times B$	2	$\frac{1}{3}\left(-\frac{1}{4}\right)^k-\frac{1}{12}\left(\frac{1}{2}\right)^k-\left(\frac{5-3\sqrt{5}}{40}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{5+3\sqrt{5}}{40}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AA \times C$	2	$\frac{1}{8}\left(-\frac{1}{2}\right)^k-\frac{1}{24}\left(\frac{1}{2}\right)^k-\frac{1}{3}\left(-\frac{1}{4}\right)^k+\left(\frac{5-\sqrt{5}}{40}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{5+\sqrt{5}}{40}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AB \times A$	2	$-\frac{1}{6}\left(\frac{1}{2}\right)^k-\frac{1}{3}\left(-\frac{1}{4}\right)^k+\left(\frac{5-\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{5+\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$AB \times C$	1	$\frac{1}{3}\left(\frac{1}{2}\right)^k+\frac{2}{3}\left(-\frac{1}{4}\right)^k$
$AC \times A$	2	$-\frac{1}{4}\left(-\frac{1}{2}\right)^k-\frac{1}{12}\left(\frac{1}{2}\right)^k+\frac{1}{3}\left(-\frac{1}{4}\right)^k+\frac{\sqrt{5}}{10}\left[\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{1-\sqrt{5}}{4}\right)^k\right]$
$AC \times B$	2	$\frac{1}{3}\left(\frac{1}{2}\right)^k-\frac{1}{3}\left(-\frac{1}{4}\right)^k$
$AC \times C$	2	$\frac{1}{4}\left(-\frac{1}{2}\right)^k-\frac{1}{4}\left(\frac{1}{2}\right)^k+\frac{\sqrt{5}}{10}\left[\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{1-\sqrt{5}}{4}\right)^k\right]$
$CC \times A$	2	$-\frac{1}{8}\left(-\frac{1}{2}\right)^k-\frac{1}{8}\left(\frac{1}{2}\right)^k+\left(\frac{5-\sqrt{5}}{40}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k+\left(\frac{5+\sqrt{5}}{40}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$
$CC \times C$	1	$\frac{1}{3}-\frac{1}{12}\left(-\frac{1}{2}\right)^k+\frac{1}{4}\left(\frac{1}{2}\right)^k-\left(\frac{5+3\sqrt{5}}{20}\right)\left(\frac{1+\sqrt{5}}{4}\right)^k-\left(\frac{5-3\sqrt{5}}{20}\right)\left(\frac{1-\sqrt{5}}{4}\right)^k$

Table S8 Recursion matrix for calculating two-locus X chromosome haplotype probabilities in the generation of four-way RIL by sibling mating

		State at $k+1$			
State at k		1	2	3	4
1		$(1-r)/2$	0	$1-r$	$1/4$
2		$r/2$	0	r	$1/4$
3		$1/2$	0	0	0
4		0	1	0	$1/2$

Table S9 Starting states for calculating two-locus X chromosome haplotype probabilities in the generation of four-way RIL by sibling mating

Prototype	No. states	Initial pattern		Initial probability
AA	2		(1)	$1/2$
AB	2		(2)	$1/2$
AC	4		(4)	$1/4$
CC	1		(3)	1

Table S10 Transpose of the recursion matrix for calculating probabilities of two-locus autosomal diplotypes of the form $AA|AA$, in the generation of four-way RIL by sibling mating. Only the non-zero entries are shown

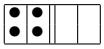
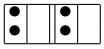
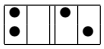
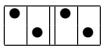
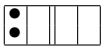
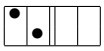
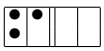
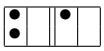
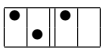
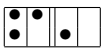
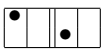
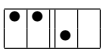
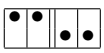
State at $k+1$		State at k						
1		2: $(1-r)^2$	3: $2r(1-r)$	4: r^2				
2		1: $\frac{[r^2+(1-r)^2]}{4}$	2: $\frac{(1-r)^2}{2}$	3: $r(1-r)$	4: $\frac{r^2}{2}$	5: $\frac{(1-r)^2}{4}$	6: $\frac{r^2}{4}$	7: $r(1-r)$
3		8: $\frac{1-r}{2}$	9: $\frac{r}{2}$	10: $\frac{1}{2}$				
4		2: $\frac{1}{8}$	3: $\frac{1}{4}$	4: $\frac{1}{8}$	11: $\frac{1}{8}$	12: $\frac{1}{4}$	13: $\frac{1}{8}$	
5		5: $1-r$	6: r					
6		11: 1						
7		8: $1-r$	9: r					
8		5: $\frac{1-r}{4}$	6: $\frac{r}{4}$	7: $\frac{1}{4}$	8: $\frac{1-r}{2}$	9: $\frac{r}{2}$		
9		8: $\frac{1}{4}$	9: $\frac{1}{4}$	11: $\frac{1}{4}$	12: $\frac{1}{4}$			
10		2: $\frac{1-r}{4}$	3: $\frac{1}{4}$	4: $\frac{r}{4}$	8: $\frac{1-r}{4}$	9: $\frac{r}{4}$	10: $\frac{1}{4}$	
11		5: $\frac{1}{4}$	6: $\frac{1}{4}$	11: $\frac{1}{2}$				
12		8: $\frac{1}{2}$	9: $\frac{1}{2}$					
13		2: $\frac{1}{4}$	3: $\frac{1}{2}$	4: $\frac{1}{4}$				

Table S11 Starting states for the calculation of probabilities of two-locus autosomal diplotypes of the form $AA|AA$, in the generation of four-way RIL by sibling mating

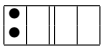
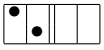
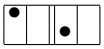
Prototype	No. states	Initial pattern	Initial probability
$AA\|AA$	4	 (5)	$1/4$
$AB\|AB$	4	 (6)	$1/4$
$AC\|AC$	8	 (11)	$1/8$

Table S12 Transpose of the recursion matrix for calculating probabilities of two-locus autosomal diplotypes of the form $AA|AB$, in the generation of four-way RIL by sibling mating

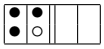
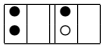
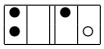
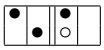
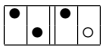
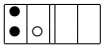
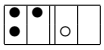
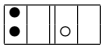
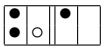
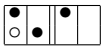
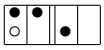
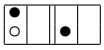
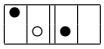
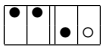
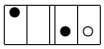
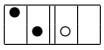
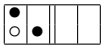
State at $k+1$		State at k						
1		2: $(1-r)^2$	3: $r(1-r)$	4: $r(1-r)$	5: r^2			
2		1: $\frac{r^2+(1-r)^2}{4}$	2: $\frac{(1-r)^2}{2}$	3: $\frac{r(1-r)}{2}$	4: $\frac{r(1-r)}{2}$	5: $\frac{r^2}{2}$	6: $\frac{r(1-r)}{4}$	17: $\frac{r(1-r)}{4}$
3		7: $\frac{1}{4}$	8: $\frac{1-r}{4}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$	16: $\frac{r}{4}$		
4		9: $\frac{r}{4}$	10: $\frac{1-r}{4}$	11: $\frac{1}{4}$	12: $\frac{1-r}{4}$	13: $\frac{r}{4}$		
5		2: $\frac{1}{8}$	3: $\frac{1}{8}$	4: $\frac{1}{8}$	5: $\frac{1}{8}$	14: $\frac{1}{8}$	15: $\frac{1}{8}$	
6		8: $(1-r)$	16: r					
7		2: $\frac{1-r}{4}$	3: $\frac{1-r}{4}$	4: $\frac{r}{4}$	5: $\frac{r}{4}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$	
8		6: $\frac{1-r}{4}$	8: $\frac{1-r}{2}$	16: $\frac{r}{2}$	17: $\frac{r}{4}$			
9		2: $\frac{1-r}{4}$	3: $\frac{1-r}{4}$	4: $\frac{r}{4}$	5: $\frac{r}{4}$	7: $\frac{1}{4}$	8: $\frac{1-r}{4}$	16: $\frac{r}{4}$
10		2: $\frac{1-r}{4}$	3: $\frac{r}{4}$	4: $\frac{1-r}{4}$	5: $\frac{r}{4}$	11: $\frac{1}{4}$	12: $\frac{1-r}{4}$	13: $\frac{r}{4}$
11		2: $\frac{1-r}{4}$	3: $\frac{r}{4}$	4: $\frac{1-r}{4}$	5: $\frac{r}{4}$	9: $\frac{r}{4}$	10: $\frac{1-r}{4}$	
12		6: $\frac{r}{4}$	12: $\frac{1-r}{2}$	13: $\frac{r}{2}$	17: $\frac{1-r}{4}$			
13		8: $\frac{1}{4}$	15: $\frac{1}{4}$	16: $\frac{1}{4}$				
14		2: $\frac{1}{4}$	3: $\frac{1}{4}$	4: $\frac{1}{4}$	5: $\frac{1}{4}$			
15		8: $\frac{1}{4}$	12: $\frac{1}{4}$	13: $\frac{1}{4}$	16: $\frac{1}{4}$			
16		12: $\frac{1}{4}$	13: $\frac{1}{4}$	15: $\frac{1}{4}$				
17		12: $(1-r)$	13: r					

Table S13 Starting states for the calculation of probabilities of two-locus autosomal diplotypes of the form $AA|AB$, in the generation of four-way RIL by sibling mating

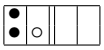
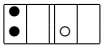
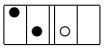
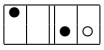
Prototype	No. states	Initial pattern	Initial probability
$AA\|AB$	8	 (6)	$1/2$
$AA\|AC$	16	 (8)	$1/4$
$AB\|AC$	16	 (16)	$1/4$
$AC\|AD$	8	 (15)	$1/4$

Table S14 Transpose of the recursion matrix for calculating probabilities of two-locus autosomal diplotypes of the form $AA|BB$, in the generation of four-way RIL by sibling mating

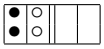
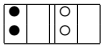
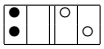
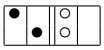
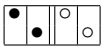
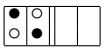
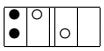
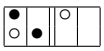
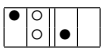
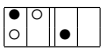
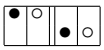
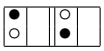
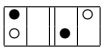
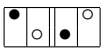
State at $k+1$		State at k						
1		2: $\frac{(1-r)^2}{2}$	3: $\frac{r(1-r)}{2}$	4: $\frac{r(1-r)}{2}$	5: $\frac{r^2}{2}$			
2		1: $\frac{(1-r)^2}{2}$	2: $\frac{(1-r)^2}{2}$	3: $\frac{r(1-r)}{2}$	4: $\frac{r(1-r)}{2}$	5: $\frac{r^2}{2}$	6: $\frac{r^2}{2}$	
3		7: $\frac{1-r}{4}$	8: $\frac{r}{4}$					
4		9: $\frac{1-r}{4}$	10: $\frac{r}{4}$					
5		11: $\frac{1}{8}$	12: $\frac{1}{8}$	13: $\frac{1}{8}$	14: $\frac{1}{8}$			
6		12: $\frac{(1-r)^2}{2}$	13: $\frac{r(1-r)}{2}$	14: $\frac{r^2}{2}$				
7		2: $\frac{1-r}{2}$	3: $\frac{1-r}{2}$	4: $\frac{r}{2}$	5: $\frac{r}{2}$	7: $\frac{1-r}{4}$	8: $\frac{r}{4}$	
8		9: $\frac{r}{4}$	10: $\frac{1-r}{4}$	12: $\frac{1-r}{2}$	13: $\frac{1}{4}$	14: $\frac{r}{2}$		
9		2: $\frac{1-r}{2}$	3: $\frac{r}{2}$	4: $\frac{1-r}{2}$	5: $\frac{r}{2}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$	
10		7: $\frac{r}{4}$	8: $\frac{1-r}{4}$	12: $\frac{1-r}{2}$	13: $\frac{1}{4}$	14: $\frac{r}{2}$		
11		2: $\frac{1}{8}$	3: $\frac{1}{8}$	4: $\frac{1}{8}$	5: $\frac{1}{8}$	12: $\frac{1}{8}$	13: $\frac{1}{8}$	14: $\frac{1}{8}$
12		1: $\frac{r^2}{2}$	6: $\frac{(1-r)^2}{2}$	12: $\frac{(1-r)^2}{2}$	13: $\frac{r(1-r)}{2}$	14: $\frac{r^2}{2}$		
13		7: $\frac{r}{4}$	8: $\frac{1-r}{4}$	9: $\frac{r}{4}$	10: $\frac{1-r}{4}$			
14		2: $\frac{1}{8}$	3: $\frac{1}{8}$	4: $\frac{1}{8}$	5: $\frac{1}{8}$	11: $\frac{1}{8}$		

Table S15 Starting states for the calculation of probabilities of two-locus autosomal diplotypes of the form $AA|BB$, in the generation of four-way RIL by sibling mating

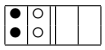
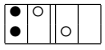
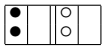
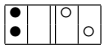
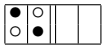
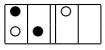
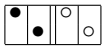
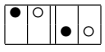
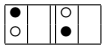
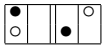
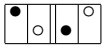
Prototype	No. states	Initial pattern		Initial probability
$AA\|BB$	2		(1)	$1/2$
$AA\|BC$	16		(7)	$1/2$
$AA\|CC$	4		(2)	$1/2$
$AA\|CD$	8		(3)	$1/2$
$AB\|BA$	2		(6)	$1/2$
$AB\|BC$	16		(8)	$1/2$
$AB\|CD$	4		(5)	$1/2$
$AC\|BD$	4		(11)	$1/2$
$AC\|CA$	4		(12)	$1/2$
$AC\|CB$	8		(13)	$1/2$
$AC\|DB$	4		(14)	$1/2$

Table S16 Transpose of the recursion matrix for calculating probabilities of the two-locus X chromosome female diplotype of the form $AA|AA$, in the generation of four-way RIL by sibling mating

State at $k+1$		State at k					
1		2: $(1-r)$	3: r				
2		1: $\frac{r^2+(1-r)^2}{4}$	2: $\frac{1-r}{2}$	3: $\frac{r}{2}$	4: $\frac{(1-r)^2}{4}$	5: $r(1-r)$	9: $\frac{r^2}{4}$
3		6: $\frac{1-r}{2}$	7: $\frac{r}{2}$	11: $\frac{1}{2}$			
4		4: $\frac{1-r}{2}$	9: $\frac{r}{2}$	10: $\frac{1}{2}$			
5		6: $\frac{1-r}{2}$	7: $\frac{r}{2}$	12: $\frac{1}{2}$			
6		4: $\frac{1-r}{4}$	5: $\frac{1}{4}$	9: $\frac{r}{4}$	12: $\frac{1}{2}$		
7		6: $\frac{1}{4}$	7: $\frac{1}{4}$	8: $\frac{1}{4}$	13: $\frac{1}{4}$		
8		4: $\frac{1}{4}$	8: $\frac{1}{2}$	9: $\frac{1}{4}$			
9		8: 1					
10		4: $(1-r)$	9: r				
11		2: $\frac{1}{4}$	3: $\frac{1}{4}$	6: $\frac{1-r}{4}$	7: $\frac{r}{4}$	11: $\frac{1}{4}$	
12		4: $\frac{1-r}{4}$	5: $\frac{1}{4}$	6: $\frac{1-r}{2}$	7: $\frac{r}{2}$	9: $\frac{r}{4}$	
13		6: $\frac{1}{2}$	7: $\frac{1}{2}$				

Table S17 Starting states for the calculation of probabilities of the two-locus X chromosome female diplotype of the form $AA|AA$, in the generation of four-way RIL by sibling mating

Prototype	No. states	Initial pattern		Initial probability
$AA\|AA$	2		(4)	$1/2$
$AB\|AB$	2		(9)	$1/2$
$AC\|AC$	4		(8)	$1/4$
$CC\|CC$	1		(10)	1

Table S18 Transpose of the recursion matrix for calculating probabilities of the two-locus X chromosome female diplotype of the form $AA|AB$, in the generation of four-way RIL by sibling mating

State at $k+1$		State at k				
1		2: $(1-r)$	3: r	4: $(1-r)$	5: r	
2		1: $\frac{r^2+(1-r)^2}{8}$	4: $\frac{1-r}{2}$	5: $\frac{r}{2}$	6: $\frac{r(1-r)}{4}$	7: $\frac{r(1-r)}{4}$
3		8: $\frac{1}{4}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$	14: $\frac{r}{4}$	15: $\frac{1-r}{4}$
4		1: $\frac{r^2+(1-r)^2}{8}$	2: $\frac{1-r}{2}$	3: $\frac{r}{2}$	6: $\frac{r(1-r)}{4}$	7: $\frac{r(1-r)}{4}$
5		11: $\frac{1}{4}$	12: $\frac{1-r}{4}$	13: $\frac{r}{4}$	14: $\frac{1-r}{4}$	15: $\frac{r}{4}$
6		12: $\frac{1-r}{2}$	13: $\frac{r}{2}$	16: $\frac{1}{2}$		
7		9: $\frac{1-r}{2}$	10: $\frac{r}{2}$	17: $\frac{1}{2}$		
8		2: $\frac{1}{4}$	3: $\frac{1}{4}$	14: $\frac{r}{4}$	15: $\frac{1-r}{4}$	
9		6: $\frac{r}{4}$	7: $\frac{1-r}{4}$	17: $\frac{1}{2}$		
10		12: $\frac{1}{4}$	13: $\frac{1}{4}$	18: $\frac{1}{8}$		
11		4: $\frac{1}{4}$	5: $\frac{1}{4}$	14: $\frac{1-r}{4}$	15: $\frac{r}{4}$	
12		6: $\frac{1-r}{4}$	7: $\frac{r}{4}$	16: $\frac{1}{2}$		
13		9: $\frac{1}{4}$	10: $\frac{1}{4}$	18: $\frac{1}{8}$		
14		4: $\frac{1}{4}$	5: $\frac{1}{4}$	11: $\frac{1}{4}$	12: $\frac{1-r}{4}$	13: $\frac{r}{4}$
15		2: $\frac{1}{4}$	3: $\frac{1}{4}$	8: $\frac{1}{4}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$
16		6: $\frac{1-r}{4}$	7: $\frac{r}{4}$	12: $\frac{1-r}{2}$	13: $\frac{r}{2}$	
17		6: $\frac{r}{4}$	7: $\frac{1-r}{4}$	9: $\frac{1-r}{2}$	10: $\frac{r}{2}$	
18		9: $\frac{1}{2}$	10: $\frac{1}{2}$	12: $\frac{1}{2}$	13: $\frac{1}{2}$	

Table S19 Starting states for the calculation of probabilities of the two-locus X chromosome female diplotype of the form $AA|AB$, in the generation of four-way RIL by sibling mating

Prototype	No. states	Initial pattern		Initial probability
$AA\|AB$	4		(6)	$1/2$
$AA\|AC$	4		(12)	$1/2$
$AB\|AC$	4		(13)	$1/2$
$AC\|BC$	2		(18)	1
$AC\|CC$	4		(16)	$1/2$

Table S20 Transpose of the recursion matrix for calculating probabilities of the two-locus X chromosome female diplotype of the form $AA|BB$, in the generation of four-way RIL by sibling mating

State at $k+1$		State at k			
1		2: $\frac{1-r}{2}$	3: $\frac{r}{2}$	4: $\frac{1-r}{2}$	5: $\frac{r}{2}$
2		1: $\frac{(1-r)^2}{4}$	4: $\frac{1-r}{2}$	5: $\frac{r}{2}$	6: $\frac{r^2}{4}$
3		7: $\frac{1-r}{4}$	8: $\frac{r}{4}$		
4		1: $\frac{(1-r)^2}{4}$	2: $\frac{1-r}{2}$	3: $\frac{r}{2}$	6: $\frac{r^2}{4}$
5		9: $\frac{1-r}{4}$	10: $\frac{r}{4}$		
6		11: $\frac{1-r}{2}$	12: $\frac{r}{2}$		
7		2: $\frac{1}{2}$	3: $\frac{1}{2}$	7: $\frac{1-r}{4}$	8: $\frac{r}{4}$
8		9: $\frac{r}{4}$	10: $\frac{1-r}{4}$	11: $\frac{1}{4}$	12: $\frac{1}{4}$
9		4: $\frac{1}{2}$	5: $\frac{1}{2}$	9: $\frac{1-r}{4}$	10: $\frac{r}{4}$
10		7: $\frac{r}{4}$	8: $\frac{1-r}{4}$	11: $\frac{1}{4}$	12: $\frac{1}{4}$
11		1: $\frac{r^2}{2}$	6: $\frac{(1-r)^2}{2}$	11: $\frac{1-r}{2}$	12: $\frac{r}{2}$
12		7: $\frac{r}{4}$	8: $\frac{1-r}{4}$	9: $\frac{r}{4}$	10: $\frac{1-r}{4}$

Table S21 Starting states for the calculation of probabilities of the two-locus X chromosome female diplotype of the form $AA|BB$, in the generation four-way RIL by sibling mating

Prototype	No. states	Initial pattern		Initial probability
$AA\|BB$	1		(1)	1
$AA\|BC$	4		(9)	1
$AA\|CC$	2		(2)	1
$AB\|BA$	1		(6)	1
$AB\|BC$	4		(10)	1
$AB\|CC$	2		(3)	1
$AC\|CA$	2		(11)	1
$AC\|CB$	2		(12)	1

Table S22 Prescription for the calculation of two-locus autosomal diplotype probabilities at intermediate generations in the construction of 8-way RIL, from the corresponding probabilities for 4-way RIL

Prototype	No. states	4-way state	Probability multiplier
$aa\|aa$	8	$AA\|AA$	$\frac{1-r}{2}$
$aa\|ab$	16	$AA\|AA$	0
$aa\|bb$	4	$AA\|AA$	0
$ab\|ab$	8	$AA\|AA$	$\frac{r}{2}$
$ab\|ba$	4	$AA\|AA$	0
$aa\|ac$	32	$AA\|AB$	$\frac{1-r}{4}$
$aa\|bc$	32	$AA\|AB$	0
$ab\|ac$	32	$AA\|AB$	$\frac{r}{4}$
$ab\|bc$	32	$AA\|AB$	0
$aa\|ae$	64	$AA\|AC$	$\frac{1-r}{4}$
$aa\|be$	64	$AA\|AC$	0
$ab\|ae$	64	$AA\|AC$	$\frac{r}{4}$
$ab\|be$	64	$AA\|AC$	0
$aa\|cc$	8	$AA\|BB$	$\frac{(1-r)^2}{4}$
$aa\|cd$	16	$AA\|BB$	$\frac{r(1-r)}{4}$
$ab\|cd$	8	$AA\|BB$	$\frac{r^2}{4}$
$aa\|ce$	128	$AA\|BC$	$\frac{1-r}{8}$
$ab\|ce$	128	$AA\|BC$	$\frac{r}{8}$
$aa\|ee$	16	$AA\|CC$	$\frac{(1-r)^2}{4}$
$aa\|ef$	32	$AA\|CC$	$\frac{r(1-r)}{4}$
$ab\|ef$	16	$AA\|CC$	$\frac{r^2}{4}$
$aa\|eg$	64	$AA\|CD$	$\frac{1-r}{8}$
$ab\|eg$	64	$AA\|CD$	$\frac{r}{8}$

Prototype	No. states	4-way state	Probability multiplier
$ac\|ac$	16	$AB\|AB$	$\frac{1}{4}$
$ac\|ad$	16	$AB\|AB$	0
$ac\|bd$	8	$AB\|AB$	0
$ac\|ae$	128	$AB\|AC$	$\frac{1}{8}$
$ac\|be$	128	$AB\|AC$	0
$ac\|ca$	8	$AB\|BA$	$\frac{(1-r)^2}{4}$
$ac\|cb$	16	$AB\|BA$	$\frac{r(1-r)}{4}$
$ac\|db$	8	$AB\|BA$	$\frac{r^2}{4}$
$ac\|ce$	128	$AB\|BC$	$\frac{1-r}{8}$
$ac\|de$	128	$AB\|BC$	$\frac{r}{8}$
$ac\|eg$	64	$AB\|CD$	$\frac{1}{16}$
$ae\|ae$	32	$AC\|AC$	$\frac{1}{4}$
$ae\|af$	32	$AC\|AC$	0
$ae\|bf$	16	$AC\|AC$	0
$ae\|ag$	64	$AC\|AD$	$\frac{1}{8}$
$ae\|bg$	64	$AC\|AD$	0
$ae\|cg$	64	$AC\|BD$	$\frac{1}{16}$
$ae\|ea$	16	$AC\|CA$	$\frac{(1-r)^2}{4}$
$ae\|eb$	32	$AC\|CA$	$\frac{r(1-r)}{4}$
$ae\|fb$	16	$AC\|CA$	$\frac{r^2}{4}$
$ae\|ec$	64	$AC\|CB$	$\frac{1-r}{8}$
$ae\|fc$	64	$AC\|CB$	$\frac{r}{8}$
$ae\|gc$	64	$AC\|DB$	$\frac{1}{16}$

Table S23 Prescription for the calculation of two-locus X chromosome female diplotype probabilities at intermediate generations in the construction of 8-way RIL, from the corresponding probabilities for 4-way RIL. Only the states with non-zero probability are shown.

Prototype	No. states	4-way state	Probability multiplier
$aa\|aa$	2	$AA\|AA$	$\frac{1-r}{2}$
$ab\|ab$	2	$AA\|AA$	$\frac{r}{2}$
$aa\|ac$	4	$AA\|AB$	$\frac{1-r}{2}$
$ab\|ac$	4	$AA\|AB$	$\frac{r}{2}$
$aa\|ae$	8	$AA\|AC$	$\frac{1-r}{4}$
$ab\|ae$	8	$AA\|AC$	$\frac{r}{4}$
$aa\|cc$	2	$AA\|BB$	$\frac{1-r}{2}$
$ab\|cc$	2	$AA\|BB$	$\frac{r}{2}$
$aa\|ce$	8	$AA\|BC$	$\frac{1-r}{4}$
$ab\|ce$	8	$AA\|BC$	$\frac{r}{4}$
$aa\|ee$	4	$AA\|CC$	$\frac{(1-r)^2}{4}$
$aa\|ef$	4	$AA\|CC$	$\frac{r(1-r)}{4}$
$ab\|ee$	4	$AA\|CC$	$\frac{r(1-r)}{4}$
$ab\|ef$	4	$AA\|CC$	$\frac{r^2}{4}$
$ac\|ac$	4	$AB\|AB$	$\frac{1}{2}$
$ac\|ae$	8	$AB\|AC$	$\frac{1}{4}$
$ac\|ca$	2	$AB\|BA$	$\frac{1-r}{2}$
$ac\|cb$	2	$AB\|BA$	$\frac{r}{2}$
$ac\|cc$	4	$AB\|BB$	$\frac{1}{2}$
$ac\|ce$	8	$AB\|BC$	$\frac{1}{4}$
$ac\|ea$	8	$AB\|CA$	$\frac{1-r}{4}$
$ac\|eb$	8	$AB\|CA$	$\frac{r}{4}$
$ac\|ec$	8	$AB\|CB$	$\frac{1}{4}$
$ac\|ee$	8	$AB\|CC$	$\frac{1-r}{4}$
$ac\|ef$	8	$AB\|CC$	$\frac{r}{4}$
$ae\|ae$	8	$AC\|AC$	$\frac{1}{4}$
$ae\|cc$	8	$AC\|BB$	$\frac{1}{4}$
$ae\|ce$	8	$AC\|BC$	$\frac{1}{4}$
$ae\|ea$	4	$AC\|CA$	$\frac{(1-r)^2}{4}$
$ae\|eb$	4	$AC\|CA$	$\frac{r(1-r)}{4}$
$ae\|fa$	4	$AC\|CA$	$\frac{r(1-r)}{4}$
$ae\|fb$	4	$AC\|CA$	$\frac{r^2}{4}$
$ae\|ec$	8	$AC\|CB$	$\frac{1-r}{4}$
$ae\|fc$	8	$AC\|CB$	$\frac{r}{4}$
$ae\|ee$	8	$AC\|CC$	$\frac{1-r}{4}$
$ae\|fe$	8	$AC\|CC$	$\frac{r}{4}$
$cc\|cc$	1	$BB\|BB$	1
$cc\|ce$	4	$BB\|BC$	$\frac{1}{2}$
$cc\|ee$	2	$BB\|CC$	$\frac{1-r}{2}$
$cc\|ef$	2	$BB\|CC$	$\frac{r}{2}$
$ce\|ce$	4	$BC\|BC$	$\frac{1}{2}$
$ce\|ec$	2	$BC\|CB$	$\frac{1-r}{2}$
$ce\|fc$	2	$BC\|CB$	$\frac{r}{2}$
$ce\|ee$	4	$BC\|CC$	$\frac{1-r}{2}$
$ce\|fe$	4	$BC\|CC$	$\frac{r}{2}$
$ee\|ee$	2	$CC\|CC$	$\frac{1-r}{2}$
$ef\|ef$	2	$CC\|CC$	$\frac{r}{2}$