Flow solver equations:

$$\begin{pmatrix} \frac{\partial \mathbf{R}_\rho}{\partial \rho} & \frac{\partial \mathbf{R}_\rho}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_\rho}{\partial \rho E} & \frac{\partial \mathbf{R}_\rho}{\partial c_1} & \frac{\partial \mathbf{R}_\rho}{\partial c_2} & \frac{\partial \mathbf{R}_\rho}{\partial c_3} & \frac{\partial \mathbf{R}_\rho}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial c_4} \\ \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial c_4} \end{pmatrix} \begin{pmatrix} dQ_\rho \\ dQ_{\rho\mathbf{u}} \\ dQ_{\rho E} \\ dQ_{c_1} \\ dQ_{c_2} \\ dQ_{c_3} \\ dQ_{c_4} \end{pmatrix} = \begin{pmatrix} R_\rho \\ R_{\rho\mathbf{u}} \\ R_{\rho E} \\ R_{c_1} \\ R_{c_2} \\ R_{c_3} \\ R_{c_4} \end{pmatrix} \tag{1}$$

Adjoint solver equations:

$$\begin{pmatrix}
\frac{\partial \mathbf{R}_\rho}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial \rho} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial \rho} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial \rho} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial \rho} \\
\frac{\partial \mathbf{R}_\rho}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial \rho\mathbf{u}} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial \rho\mathbf{u}} \\
\frac{\partial \mathbf{R}_\rho}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho E} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial \rho E} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial \rho E} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial \rho E} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial \rho E} \\
\frac{\partial \mathbf{R}_\rho}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_1} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_1} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial c_1} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial c_1} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial c_1} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial c_1} \\
\frac{\partial \mathbf{R}_\rho}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_2} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_2} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial c_2} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial c_2} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial c_2} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial c_2} \\
\frac{\partial \mathbf{R}_\rho}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_3} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_3} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial c_3} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial c_3} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial c_3} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial c_3} \\
\frac{\partial \mathbf{R}_\rho}{\partial c_4} & \frac{\partial \mathbf{R}_{\rho\mathbf{u}}}{\partial c_4} & \frac{\partial \mathbf{R}_{\rho E}}{\partial c_4} & \frac{\partial \mathbf{R}'_{\rho_1}}{\partial c_4} & \frac{\partial \mathbf{R}'_{\rho_2}}{\partial c_4} & \frac{\partial \mathbf{R}'_{\rho_3}}{\partial c_4} & \frac{\partial \mathbf{R}'_{\rho_4}}{\partial c_4}
\end{pmatrix}
\begin{pmatrix}
\Lambda_\rho \\ \Lambda_{\rho\mathbf{u}} \\ \Lambda_{\rho E} \\ \Lambda_{c_1} \\ \Lambda_{c_2} \\ \Lambda_{c_3} \\ \Lambda_{c_4}
\end{pmatrix}
= -
\begin{pmatrix}
\frac{\partial f}{\partial \rho} \\ \frac{\partial f}{\partial \rho\mathbf{u}} \\ \frac{\partial f}{\partial \rho E} \\ \frac{\partial f}{\partial c_1} \\ \frac{\partial f}{\partial c_2} \\ \frac{\partial f}{\partial c_3} \\ \frac{\partial f}{\partial c_4}
\end{pmatrix} \qquad (2)$$

Adjoint solver equations (before contraction):

$$\begin{pmatrix} \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_1} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_2} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_3} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_4} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho \mathbf{u}} \\ \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho E} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho E} \end{pmatrix} \begin{pmatrix} \Lambda_{\rho_1} \\ \Lambda_{\rho_2} \\ \Lambda_{\rho_3} \\ \Lambda_{\rho_4} \\ \Lambda_{\rho \mathbf{u}} \\ \Lambda_{\rho E} \end{pmatrix} = - \begin{pmatrix} \frac{\partial f}{\partial \rho_1} \\ \frac{\partial f}{\partial \rho_2} \\ \frac{\partial f}{\partial \rho_3} \\ \frac{\partial f}{\partial \rho_4} \\ \frac{\partial f}{\partial \rho \mathbf{u}} \\ \frac{\partial f}{\partial \rho E} \end{pmatrix} \tag{3}$$

Flow solver equations (before contraction):

$$\begin{pmatrix} \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_1}}{\partial \rho E} \\ \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_2}}{\partial \rho E} \\ \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_3}}{\partial \rho E} \\ \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho_4}}{\partial \rho E} \\ \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho \mathbf{u}}}{\partial \rho E} \\ \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_1} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_2} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_3} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho_4} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho \mathbf{u}} & \frac{\partial \mathbf{R}_{\rho E}}{\partial \rho E} \end{pmatrix} \begin{pmatrix} dQ_{\rho_1} \\ dQ_{\rho_2} \\ dQ_{\rho_3} \\ dQ_{\rho_4} \\ dQ_{\rho \mathbf{u}} \\ dQ_{\rho E} \end{pmatrix} = - \begin{pmatrix} R_{\rho_1} \\ R_{\rho_2} \\ R_{\rho_3} \\ R_{\rho_4} \\ R_{\rho \mathbf{u}} \\ R_{\rho E} \end{pmatrix} \tag{4}$$