Quadratic Interpolation Between Thermodynamic Curve Fits

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March 10, 2016

We seek to blend the two thermodynamic curve fits in such a way that we maintain c_0 continuity in both specific heat (C_p) and enthalpy (h). To accomplish this, a quadratic function must be used, of the form

$$aT^2 + bT + c = C_p \tag{1}$$

The coefficients a, b, and c are determined by solving the system that results from the boundary value problem

$$\begin{cases} aT_1^2 + bT_1 + c = C_{p_1} \\ aT_2^2 + bT_2 + c = C_{p_2} \\ a\frac{\left(T_2^3 - T_1^3\right)}{3} + b\frac{\left(T_2^2 - T_1^2\right)}{2} + c\left(T_2 - T_1\right) = h_2 - h_1 \end{cases} \tag{2}$$

Where the x_1 and x_2 subscripts describe the left and right states, respectively. Solving the linear system, the coefficients are

$$\begin{cases} a = \frac{3\left(C_{p_2} + C_{p_1}\right)}{(T_2 - T_1)^2} - \frac{6(h_2 - h_1)}{(T_2 - T_1)^3} \\\\ b = -\frac{2\left[(C_{p_2} + 2C_{p_1})T_2 + (2C_{p_2} + C_{p_1})T_1\right]}{(T_2 - T_1)^2} + \frac{6(T_2 + T_1)(h_2 - h_1)}{(T_2 - T_1)^3} \\\\ c = \frac{C_{p_1}T_2(T_2 + 2T_1) + C_{p_2}T_1(T_1 + 2T_2)}{(T_2 - T_1)^2} - \frac{6T_1 T_2(h_2 - h_1)}{(T_2 - T_1)^3} \end{cases} \tag{3}$$

This can be simplified to

$$\begin{cases} a = 3B - A \\\\ b = \frac{-2(C_{p_1}T_2 + C_{p_2}T_2)}{(T_2 - T_1)^2} + (T_2 + T_1)(A - 2B) \\\\ c = \frac{C_{p_1}T_2^2 + C_{p_2}T_1^2}{(T_2 - T_1)^2} + T_1 T_2(2B - A) \end{cases} \tag{4}$$

$$A = \frac{6(h_2 - h_1)}{(T_2 - T_1)^3} \tag{5}$$

$$B = \frac{C_{p_2} + C_{p_1}}{(T_2 - T_1)^2} \tag{6}$$