

Written Preliminary Exam #1
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1. Constant density flow and incompressible flow are two different things. Constant density implies that a fluid(s) density is constant, but capable of changing (through compression or expansion), whereas incompressible implies the fluid(s) density is incapable of changing but that the density in the flow is not constant. A good example from the MAE 560 course to distinguish the two is stagnant air in a room, versus an oil/water mixture. Stagnant air in a room is at constant density, but is certainly capable of both compression or expansion; therefore, it is a constant density flow, but not incompressible. An oil/water mixture is not constant density. As illustrated in figure 1, the water and oil are different fluids with different densities, but the fluids are incompressible because both the water and oil are not capable of compression or expansion (at least in a practical sense); therefore, it is not a constant density flow, but is incompressible.

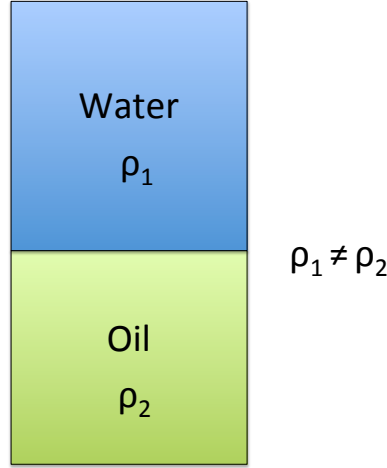


Figure 1: Incompressible Fluid Flow Example

2. The Pohlhausen method is typically defined as a velocity profile polynomial

$$\frac{u}{U_e} = a + b\eta + c\eta^2 + d\eta^3 + e\eta^4 \quad (1)$$

$$\eta = \left(\frac{y}{\delta}\right)$$

where u is the velocity, U_e is the velocity at the edge of the boundary layer, and δ is the boundary thickness. The polynomial coefficients are defined by the boundary conditions for $u = u(x, y)$, which Pohlhausen specified as

$$u(x, 0) = 0 \quad (2)$$

$$u(x, \delta) = U_e \quad (3)$$

$$\frac{\partial u}{\partial y}(x, \delta) = 0 \quad (4)$$

$$\frac{\partial^2 u}{\partial y^2}(x, 0) = -\frac{U_e}{\nu} \frac{\partial U_e}{\partial x} \quad (5)$$

$$\frac{\partial^2 u}{\partial y^2}(x, \delta) = 0 \quad (6)$$

As discussed in MAE 589, the last boundary condition (Eq. (6)) was a poor choice, because the fact that the first derivative of velocity, $\partial u / \partial y$, is zero at a point does not necessarily mean that the curvature at that point is also zero. A suggested improvement would be to

substitute that boundary condition for

$$\frac{\partial^3 u}{\partial y^3}(x, 0) = 0 \quad (7)$$

which comes from differentiating the momentum equation

$$\begin{aligned} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \frac{\partial}{\partial y} \left(-\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \right) \\ \frac{\partial^3 u}{\partial y^3}(x, 0) &= 0 \end{aligned} \quad (8)$$

with this new boundary condition, the polynomial coefficients can be derived as

$$a = 0 \quad (9)$$

$$b = \frac{4 + \lambda}{3} \quad (10)$$

$$c = -\frac{\lambda}{2} \quad (11)$$

$$d = 0 \quad (12)$$

$$e = \frac{\lambda - 2}{6} \quad (13)$$

where $\lambda = \frac{\delta^2}{\nu} \frac{\partial U_e}{\partial x}$ is the Pohlhausen pressure gradient parameter. The separation condition $\tau_w = \mu(\partial u / \partial y)_{y=0} = 0$ is used to determine the value of λ at separation

$$\begin{aligned} \left(\frac{\partial u}{\partial y} \right)_{y=0} &= 0 = \frac{b}{\delta} \\ 0 &= \frac{4 + \lambda}{3\delta} \\ \lambda &= -4 \end{aligned} \quad (14)$$

The same procedure can be used with Eq. (6) in place of Eq. (7) to find that the original Pohlhausen method yields $\lambda = -12$ at separation. The Falkner-Skan equations can be used to verify that Eq. (7) does better at predicting separation for wedge flows, where $U_e = U_1 x^m$. In the literature it is reported that flow separates at $m = -0.0904$, and

$$\frac{\delta}{x} \sqrt{Re_x} = 7.1 \quad (15)$$

at separation. This can be rearranged to give the exact value of the Pohlhausen pressure gradient, λ , at separation

$$\begin{aligned} \left(\frac{\delta}{x} \sqrt{Re_x} \right)^2 &= \frac{\delta^2}{\nu} \frac{U_e}{x} \\ &= \frac{\delta^2}{\nu} \frac{U_e}{x} \\ &= \frac{\delta^2}{\nu} \frac{\partial U_e}{\partial x} \frac{1}{m} \\ &= \frac{\lambda}{m} \end{aligned} \quad (16)$$

Thus, the exact value of λ at separation is

$$\begin{aligned} \lambda_{exact} &= m \left(\frac{\delta}{x} \sqrt{Re_x} \right)^2 \\ &= (-0.0904)(7.1)^2 \\ &= -4.56 \end{aligned} \quad (17)$$

It is therefore clear the Pohlhausen method with Eq. (7), yielding $\lambda = -4$ instead of $\lambda = -12$ at separation, is an improvement over using the original boundary condition (Eq. (6)).

3. There are many ways to derive an expression for change in entropy. From a classical thermodynamics perspective, this comes from the definition of Gibbs free energy, G

$$G = H - TS \quad (18)$$

Where H is enthalpy, T is temperature, and S is entropy. It is straightforward to see that the first and second laws of thermodynamics can be used to determine the change of entropy

$$dQ = dH - VdP \quad (19)$$

$$dS = \frac{dQ}{T} + dS_i = \frac{dH - Vdp}{T} + dS_i \quad (20)$$

Where dS_i is entropy generated from irreversible processes. Using the definition of the enthalpy

$$H = E + pV \quad (21)$$

Eq. (20) can be expanded as

$$\begin{aligned} dS &= \frac{dE + pdV + Vdp - Vdp}{T} + dS_i \\ &= \frac{dE + pdV}{T} + dS_i \end{aligned} \quad (22)$$

While this is useful, this derivation is deficient as it does not account for multiple species in chemical non-equilibrium. Revisiting Gibbs free energy for multiple species, if a system is considered for each isolated species then

$$G_s = G_s(p, T, N_s) \quad (23)$$

and differentiating yields

$$dG_s = \left(\frac{\partial G_s}{\partial T} \right)_{(p, N_s)} dT + \left(\frac{\partial G_s}{\partial p} \right)_{(T, N_s)} dp + \left(\frac{\partial G_s}{\partial N_s} \right)_{(p, T)} dN_s \quad (24)$$

Where G_s is the Gibbs free energy of the isolated species s . Because $G = \sum_{s=1}^{N_{ns}} G_s$, Eq. (24) can be written as

$$dG = \left(\frac{\partial G}{\partial T} \right)_{(p, N_s)} dT + \left(\frac{\partial G}{\partial p} \right)_{(T, N_s)} dp + \sum_{s=1}^{N_{ns}} \left(\left(\frac{\partial G}{\partial N_s} \right)_{(p, T)} dN_s \right) \quad (25)$$

Introducing the concept of chemical potential

$$\mu_s = \left(\frac{\partial G}{\partial N_s} \right)_{p, T} \quad (26)$$

and writing Eq. (18) in differential form and substituting Eq. (22) yields

$$\begin{aligned} dG &= dH - SdT - TdS \\ &= dH - SdT - T \left(\frac{dH - Vdp}{T} + dS_i \right) \\ &= Vdp - SdT - TdS_i \end{aligned} \quad (27)$$

It is clear that in comparing Eq. (27) and Eq. (25) that

$$\left(\frac{\partial G}{\partial T}\right)_{(p, N_s)} = V, \quad \left(\frac{\partial G}{\partial p}\right)_{(T, N_s)} = S, \quad (28)$$

and most importantly

$$dS_i = -\frac{1}{T} \sum_{s=1}^{N_{ns}} \mu_s dN_s \quad (29)$$

Thus, it is now clear that the change of entropy is defined in two components: the entropy generated by reversible processes, S_r , and the entropy generated by irreversible processes, S_i . If Eq. (27) is expanded with $dS_i = 0$ and the definition of enthalpy in Eq. (21) is used, the entropy from reversible processes is given by

$$dS_r = \frac{dE}{T} + \frac{pdV}{T} \quad (30)$$

combining Eq.s (29-30) the change of entropy expression is obtained

$$dS = \frac{dE}{T} + \frac{pdV}{T} - \frac{1}{T} \sum_{s=1}^{N_{ns}} \mu_s dN_s \quad (31)$$

If two gases with the same pressure and temperature are mixed, then Eq. (31) shows that entropy must be generated, regardless of whether chemical reactions occur. Since entropy of a system of gases is additive,

$$dS_{mix} = dS_1 + dS_2 \quad (32)$$

where dS_{mix} is the change of entropy in the mixture, and $dS_{(1,2)}$ are the changes in entropy of species being mixed. There is no change in temperature or pressure, so $dE = 0$ for both species; however, the volume has changed, so for inert gases mixing there will be production of entropy

$$dS_{mix} = \frac{p}{T} (dV_1 + dV_2) \quad (33)$$

Where dV_1 and dV_2 are the changes in volume for species 1 and 2, respectively. The production of entropy is to be expected, intuitively, since the diffusion of one gas into the other happens spontaneously. Each gas will expand into the new volume available after mixing naturally, and it takes effort to separate them; hence $dS_{mix} > 0$. Likewise, if chemical reactions occur, entropy will also be generated as dictated by Eq. (29).