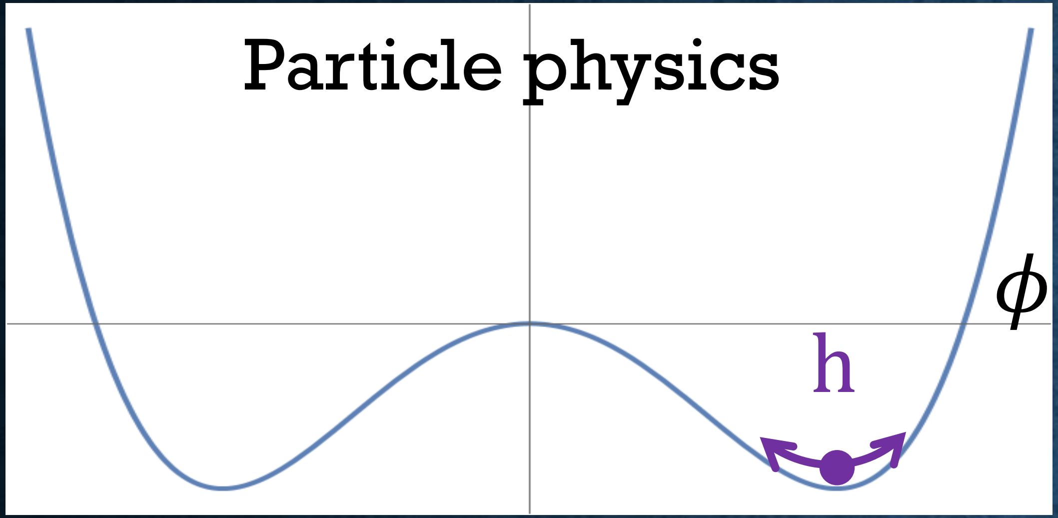
The Higgs Mode:
Ubiquitous in Superconducting Devices

High energy Accelerator Research Organization (KEK)
Innovation Center for Applied Superconducting Accelerators (iCASA)
Takayuki Kubo

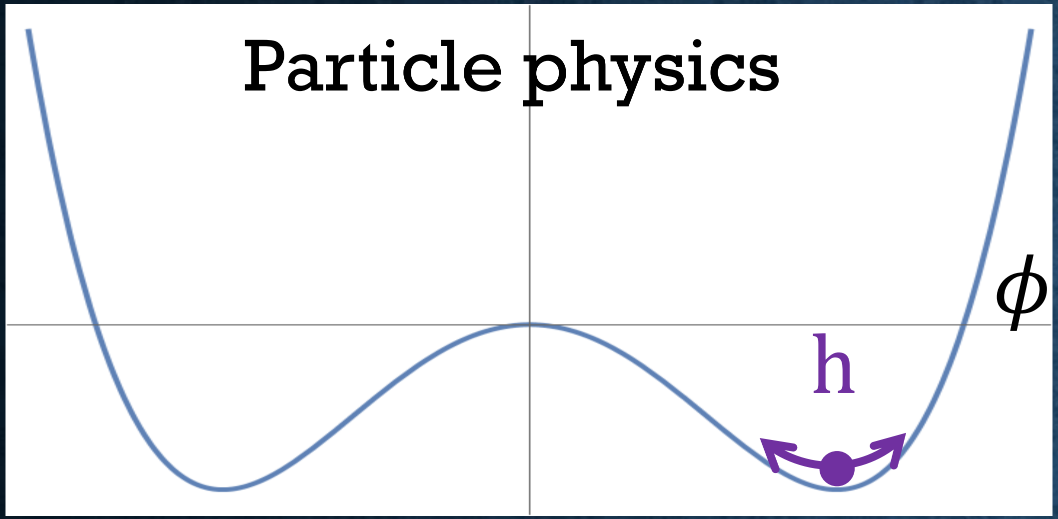
This talk is based on the following 3 papers

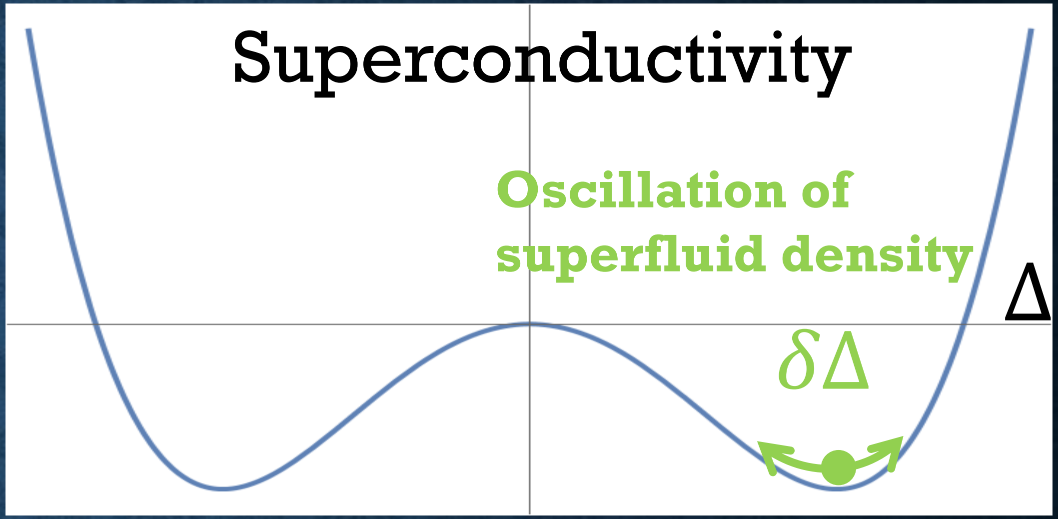
- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024) DOI: https://doi.org/10.1103/PhysRevApplied.22.044042
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025) DOI: https://doi.org/10.1103/PhysRevApplied.23.054091
- T. Kubo, arXiv:2509.09766 (Published soon in a journal) DOI: https://doi.org/10.48550/arXiv.2509.09766

Higgs?



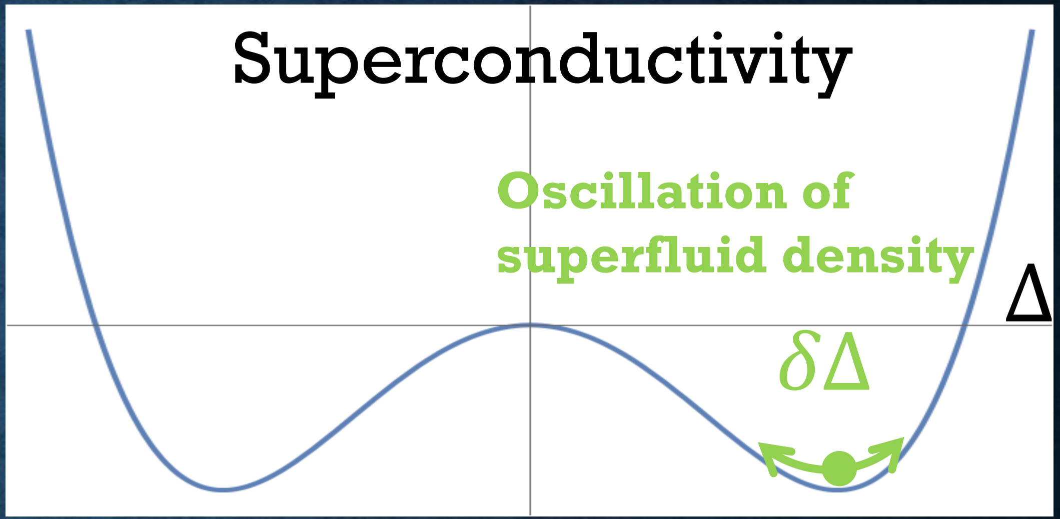
Higgs?


Particle physics
ϕ
h


Superconductivity
Oscillation of
superfluid density
Δ
$\delta\Delta$

Higgs?

- The Higgs mode in superconductivity is an $\mathcal{O}(A^2)$ effect. It does not appear in standard linear-response theories such as Mattis–Bardeen theory, but it does emerge in nonlinear response:



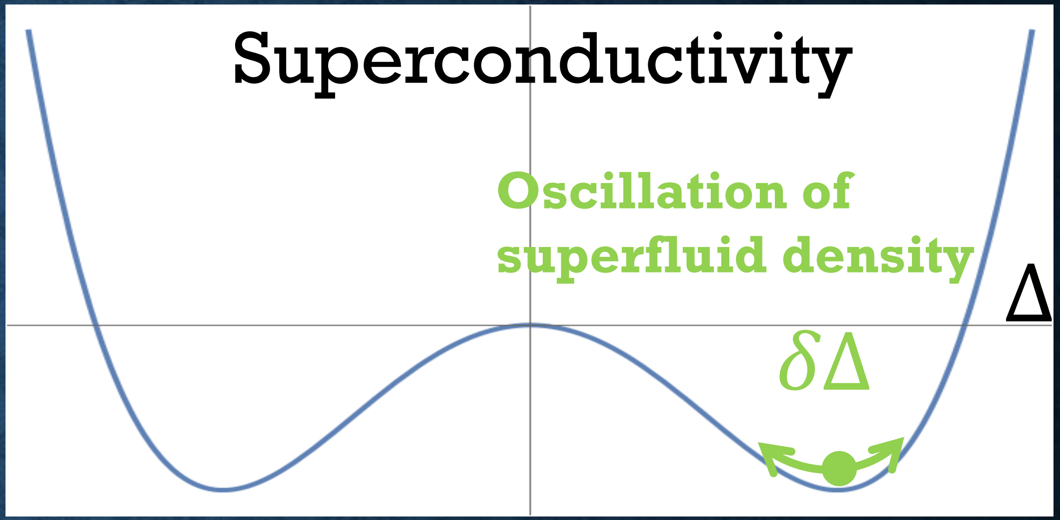
$$\delta\Delta \propto A^2$$

Nonlinear correction to the current density

$$\delta J \sim A\delta\Delta \propto A^3 = A_0^3 \cos^3 \omega t$$

Higgs?

- The Higgs mode in superconductivity is an $\mathcal{O}(A^2)$ effect. It does not appear in standard linear-response theories such as Mattis–Bardeen theory, but it does emerge in nonlinear response:



$\delta\Delta \propto A^2$

Nonlinear correction to the current density

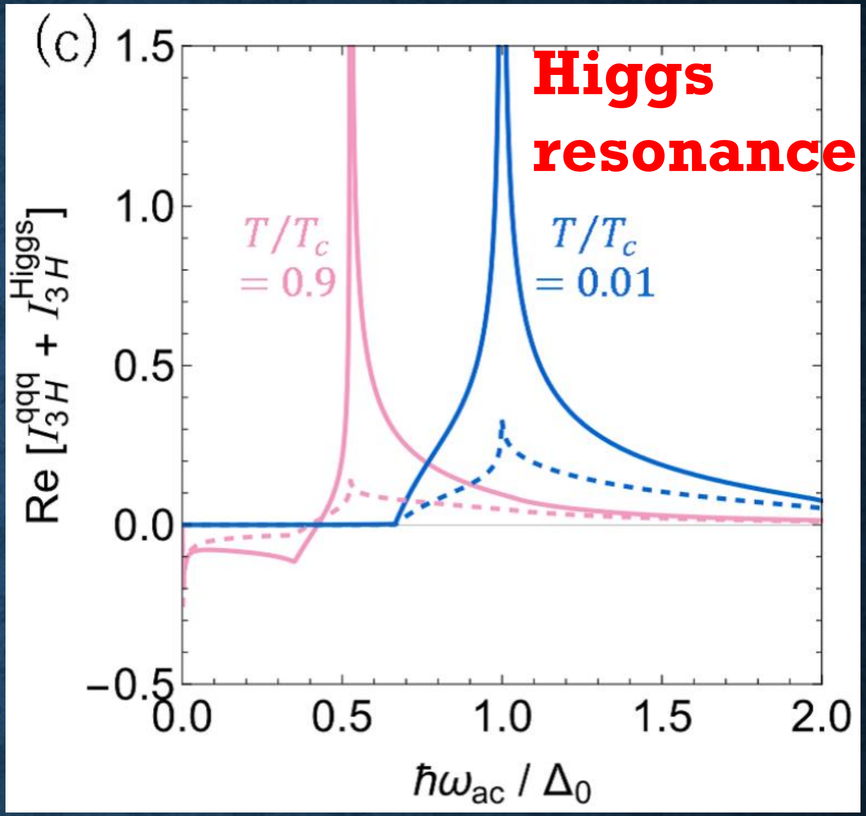
$$\delta J \sim A\delta\Delta \propto A^3 = A_0^3 \cos^3 \omega t$$

$$\sim \mathcal{O}(A_0^3) \cos \omega t + \mathcal{O}(A_0^3) \cos 3\omega t$$

$\downarrow$ First harmonic

$\downarrow$ Third harmonic

Higgs?



$$\delta J \sim A\delta\Delta \propto A^3 = A_0^3 \cos^3 \omega t$$

$$\sim \mathcal{O}(A_0^3) \cos \omega t + \mathcal{O}(A_0^3) \cos 3\omega t$$

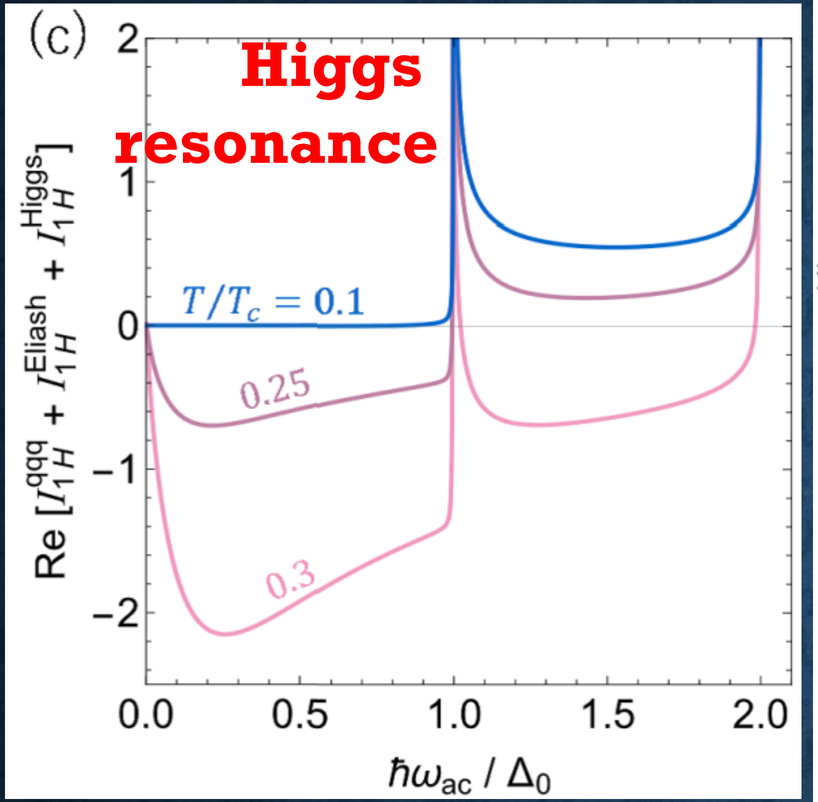
$\downarrow$ First harmonic

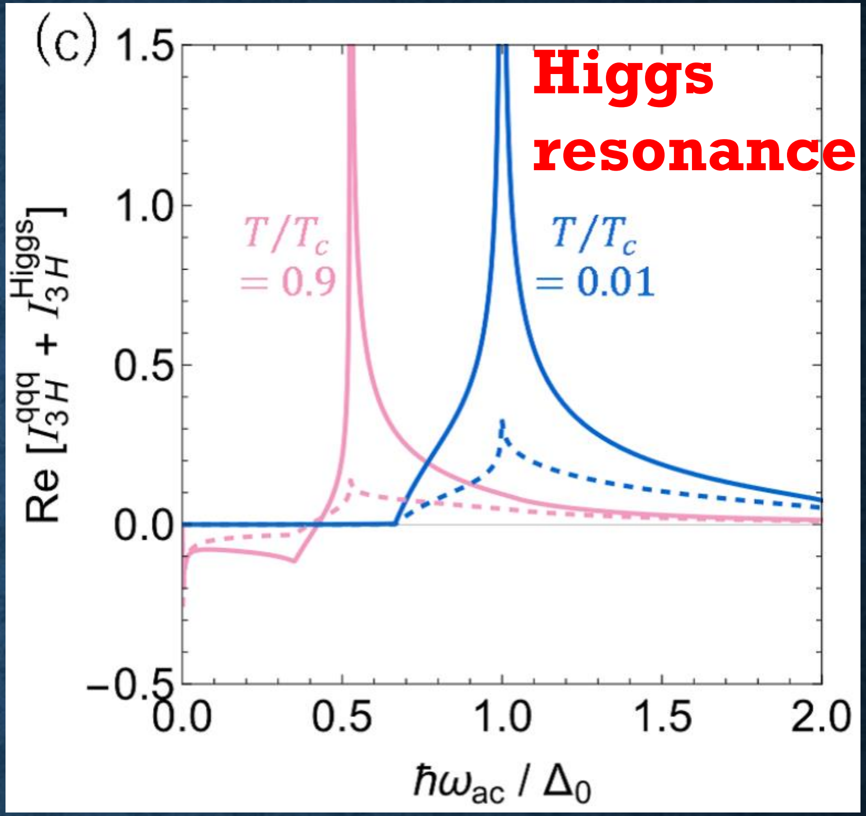
$\downarrow$ Third harmonic

As for the 3rd harmonic response, see also, M. Silaev, Phys. Rev. B **99**, 224511 (2019); P. Derendorf et al., Phys. Rev. B **109**, 024510 (2024)

T. Kubo, arXiv:2509.09766 (Published soon in a journal) DOI: https://doi.org/10.48550/arXiv.2509.09766

Higgs?





$$\delta J \sim A\delta\Delta \propto A^3 = A_0^3 \cos^3 \omega t$$

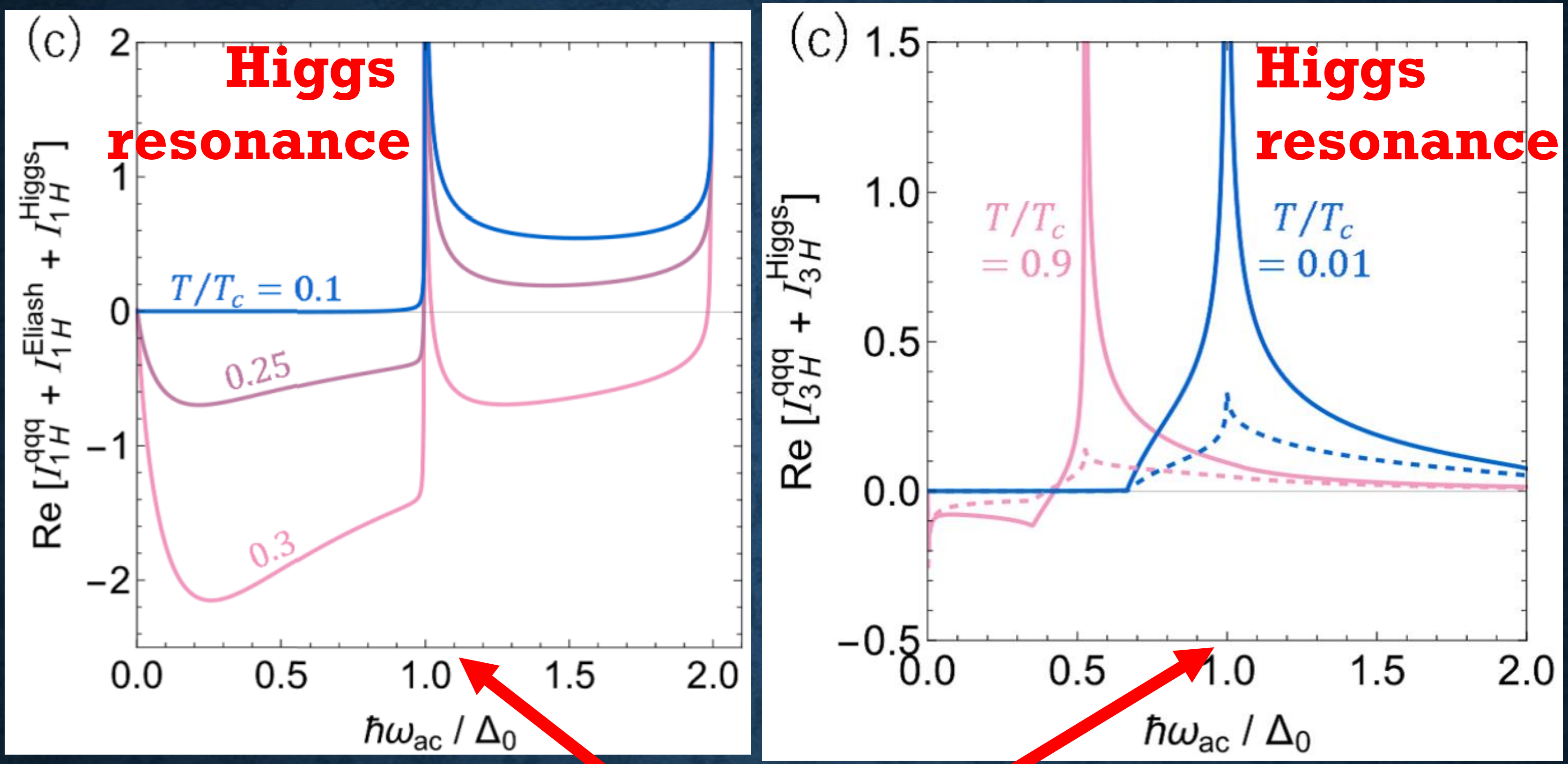
$$\sim \mathcal{O}(A_0^3) \cos \omega t + \mathcal{O}(A_0^3) \cos 3\omega t$$

$\downarrow$ First harmonic $\qquad$ $\downarrow$ Third harmonic

As for the 3rd harmonic response, see also, M. Silaev, Phys. Rev. B **99**, 224511 (2019); P. Derendorf et al., Phys. Rev. B **109**, 024510 (2024)

T. Kubo, arXiv:2509.09766 (Published soon in a journal) DOI: https://doi.org/10.48550/arXiv.2509.09766

Higgs?



$\Delta_0/h = 360\text{GHz (Nb)}$ higher than typical frequencies of SC devices

$\Delta_0/h = 44\ \text{GHz}\ \ \text{(Al)}$

$\Delta_0/h = 18\ \text{GHz}\ \ \text{(Ti)}$ Accessible?

As for the 3rd harmonic response, see also, M. Silaev, Phys. Rev. B **99**, 224511 (2019); P. Derendorf et al., Phys. Rev. B **109**, 024510 (2024)

T. Kubo, arXiv:2509.09766 (Published soon in a journal) DOI: https://doi.org/10.48550/arXiv.2509.09766

Higgs?

Another way of exciting Higgs mode: dc + ac

A. Moor et al., Phys. Rev. Lett. **118**, 047001 (2017)
T. Jujo, J. Phys. Soc. Jpn. **91**, 074711 (2022)
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$\delta\Delta \propto A^2 \quad \Rightarrow \quad \delta\Delta \propto (A_{dc} + A)^2$$

Higgs?

Another way of exciting Higgs mode: dc + ac

A. Moor et al., Phys. Rev. Lett. **118**, 047001 (2017)
T. Jujo, J. Phys. Soc. Jpn. **91**, 074711 (2022)
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$\delta\Delta \propto A^2 \quad \Longrightarrow \quad \delta\Delta \propto (A_{dc} + A)^2 \supset A_{dc} \cdot A + \mathcal{O}(A^2)$$

Higgs?

Another way of exciting Higgs mode: dc + ac

A. Moor et al., Phys. Rev. Lett. **118**, 047001 (2017)
T. Jujo, J. Phys. Soc. Jpn. **91**, 074711 (2022)
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$\delta\Delta \propto A^2 \quad \Longrightarrow \quad \delta\Delta \propto (A_{dc} + A)^2 \supset \underbrace{A_{dc} \cdot A}_{\text{Linear response}} + \underbrace{\mathcal{O}(A^2)}_{\text{Nonlinear response}}$$

Higgs?

Another way of exciting Higgs mode: dc + ac

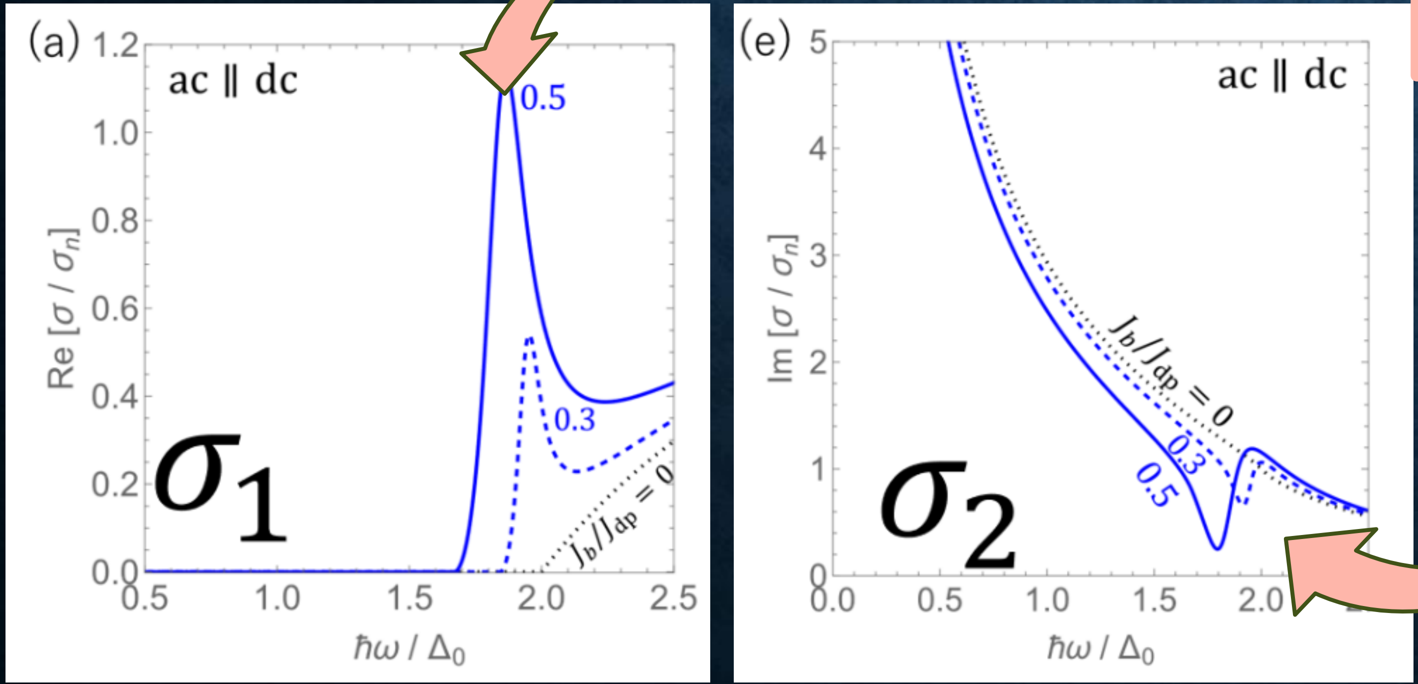
A. Moor et al., Phys. Rev. Lett. **118**, 047001 (2017)
T. Jujo, J. Phys. Soc. Jpn. **91**, 074711 (2022)
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$\delta\Delta \propto A^2 \Rightarrow \delta\Delta \propto (A_{dc} + A)^2 \supset A_{dc} \cdot A + \mathcal{O}(A^2)$$

$\boxed{\text{Linear response}}$ ($A_{dc} \cdot A$)

Nonlinear response ($\mathcal{O}(A^2)$)

T. Kubo,
Phys. Rev. Applied **23**, 054091 (2025)



$2\Delta_0/h = 720\text{GHz}$ (Nb)
$2\Delta_0/h = 88$ GHz (Al)
$2\Delta_0/h = 36$ GHz (Ti)

These are the hallmarks of the Higgs mode, which appear at frequencies around Δ, far higher than those of typical superconducting devices.

These are the hallmarks of the Higgs mode, which appear at frequencies around Δ, far higher than those of typical superconducting devices.

However, **its low-frequency tail still plays an important and nontrivial role in superconducting devices**.

These are the hallmarks of the Higgs mode, which appear at frequencies around Δ, far higher than those of typical superconducting devices.

However, **its low-frequency tail still plays an important and nontrivial role in superconducting devices**.

Some effects of the Higgs mode have, in fact, already manifested themselves, although they have not been recognized as such. A representative example is the current-dependent kinetic inductance.

Outline

(1)

Only weak ac

Outline

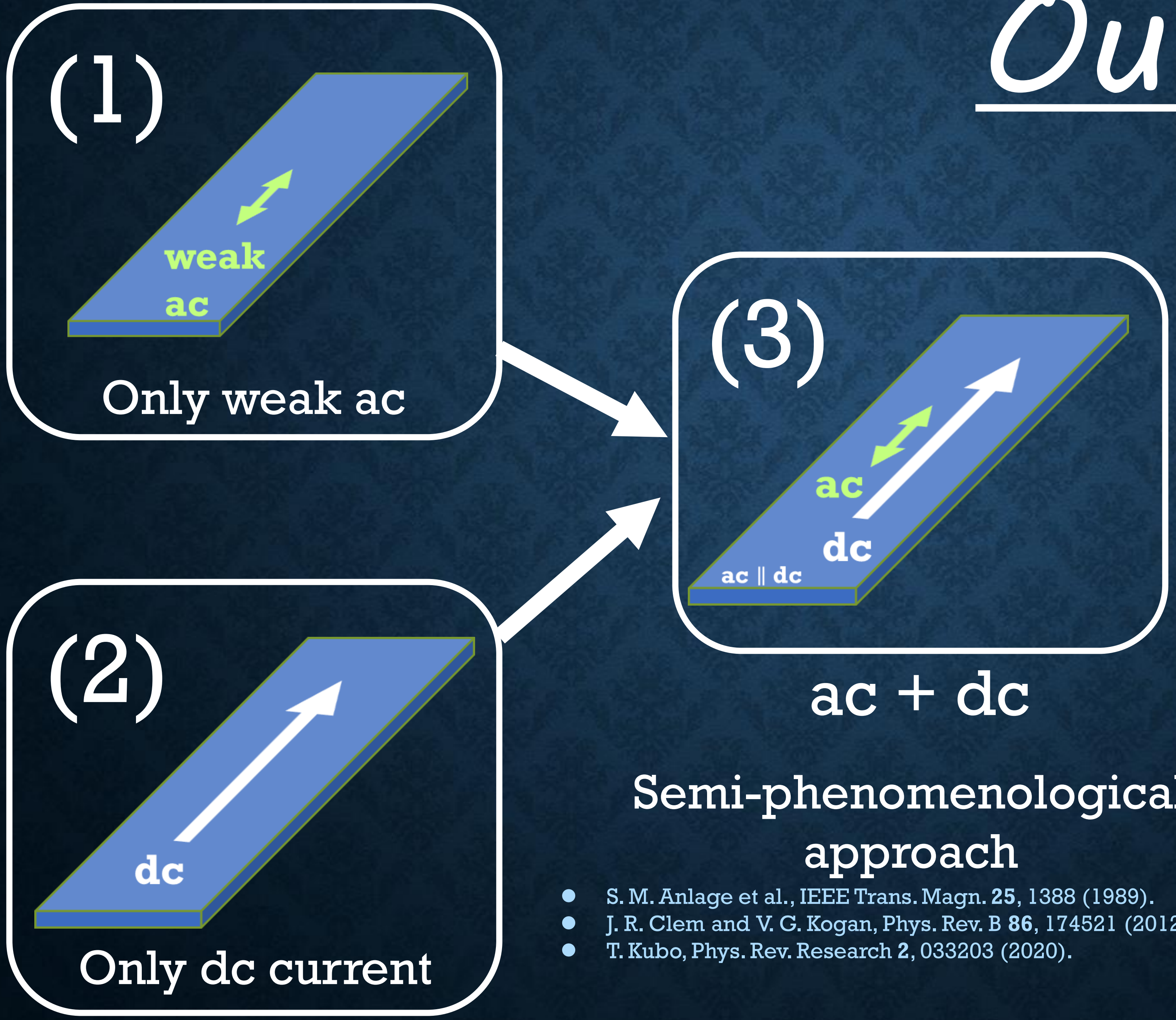
(1)

Only weak ac

(2)

Only dc current

Outline



Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).

Outline

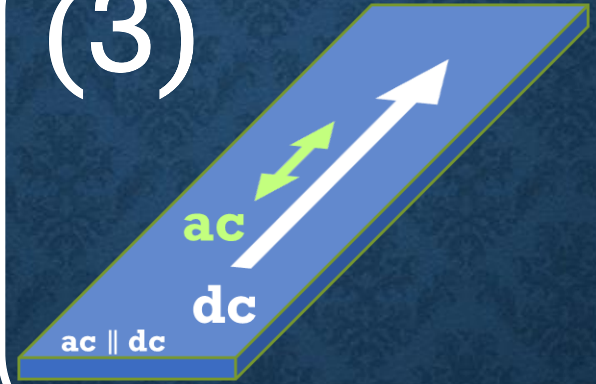
(1)

Only weak ac

(2)

Only dc current

(3)

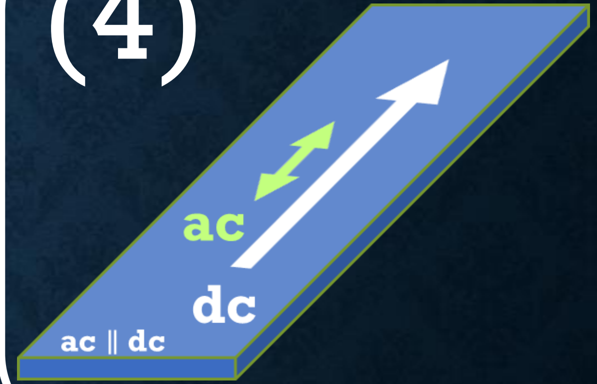


ac + dc

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).

(4)



ac + dc

Microscopic nonequilibrium superconductivity

- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

Outline

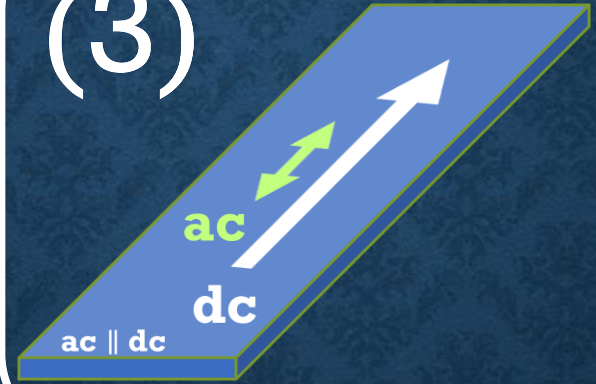
(1)

Only weak ac

(2)

Only dc current

(3)

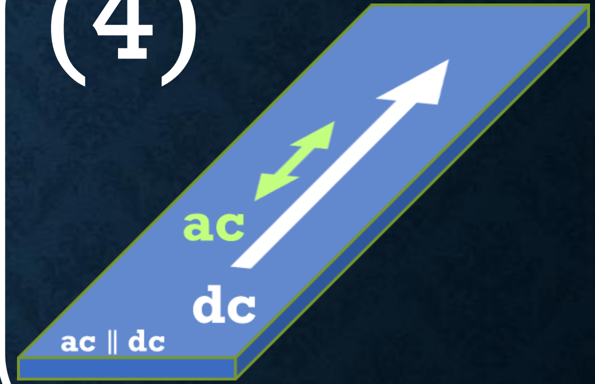


ac + dc

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).

(4)



ac + dc

Microscopic nonequilibrium superconductivity

- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

Kinetic inductance under an AC perturbation (no DC current)

The kinetic inductivity is defined by

$$\boxed{L_k \frac{dJ_s}{dt} = E}$$

Kinetic inductance under an AC perturbation (no DC current)

The kinetic inductivity is defined by

$$\boxed{L_k \frac{dJ_s}{dt} = E}$$

or

$$\boxed{L_k = \frac{E}{\dot{J}_s} = -\frac{\dot{A}}{\dot{J}_s}}$$

Kinetic inductance under an AC perturbation (no DC current)

The kinetic inductivity is defined by

$$\boxed{L_k \frac{dJ_s}{dt} = E}$$

or

$$\boxed{L_k = \frac{E}{\dot{J}_s} = -\frac{\dot{A}}{\dot{J}_s}}$$

London eq.

$$J_s = -\frac{A}{\mu_0 \lambda^2}$$

$$d\lambda/dt = 0$$

Kinetic inductance under an AC perturbation (no DC current)

The kinetic inductivity is defined by

$$L_k \frac{dJ_s}{dt} = E$$

or

$$L_k = \frac{E}{\dot{J}_s} = -\frac{\dot{A}}{\dot{J}_s}$$

London eq.

$$J_s = -\frac{A}{\mu_0 \lambda^2}$$

$$d\lambda/dt = 0$$

Very well-known result

$$L_k = \mu_0 \lambda^2 \propto \frac{1}{n_s}$$

This basic result already implies that an oscillation of the superfluid density n_s (i.e., the Higgs mode) can influence the kinetic inductance.

Outline

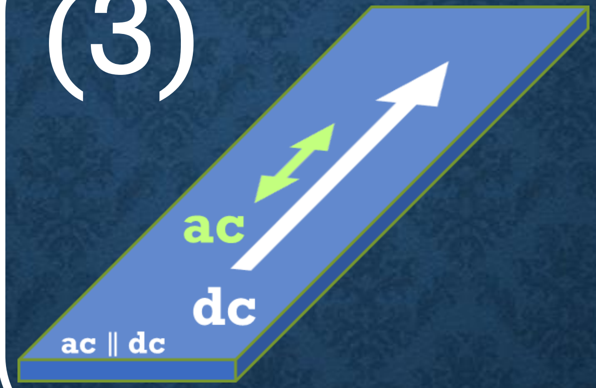
(1)

Only weak ac

(2)

Only dc current

(3)

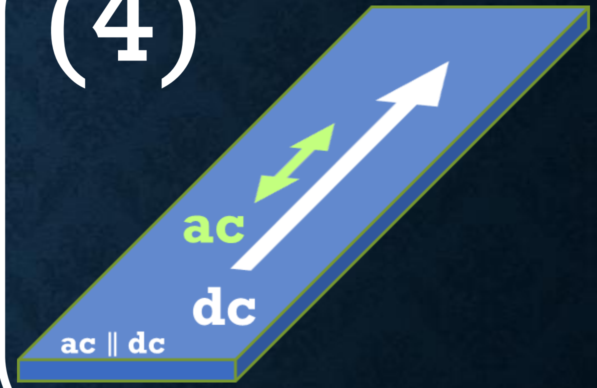


ac + dc

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).

(4)



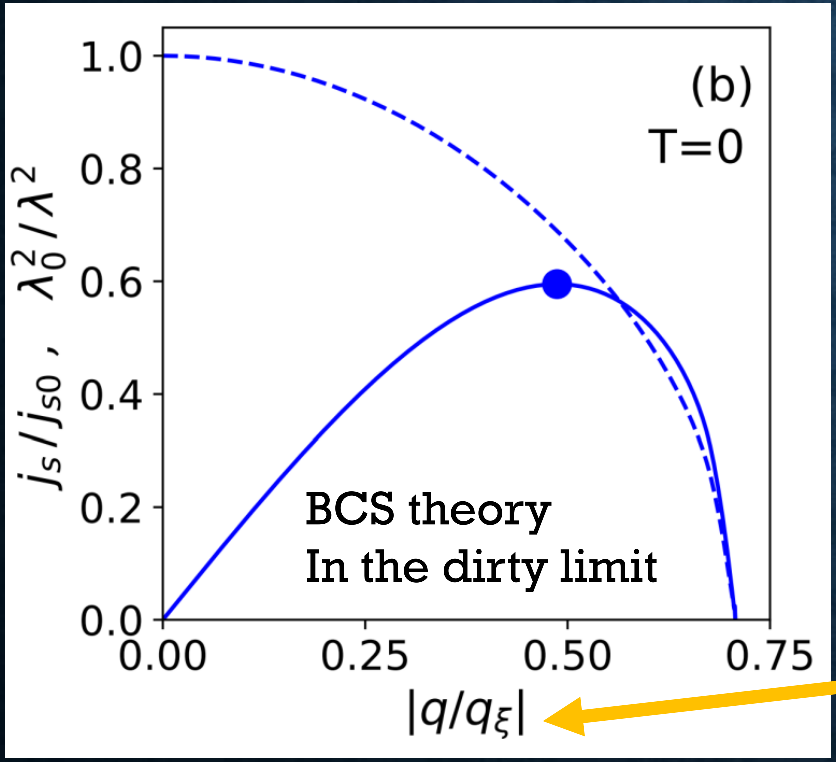
ac + dc

Microscopic nonequilibrium superconductivity

- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.

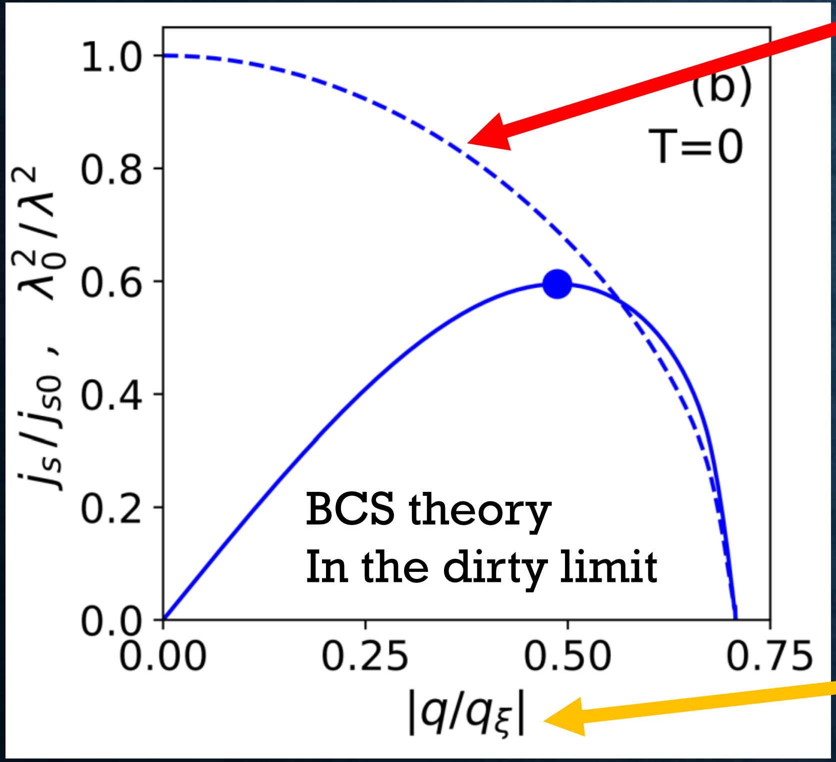


q (superfluid momentum)

See, e.g.,
T. Kubo, Phys. Rev. Research **2**, 033203 (2020)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.



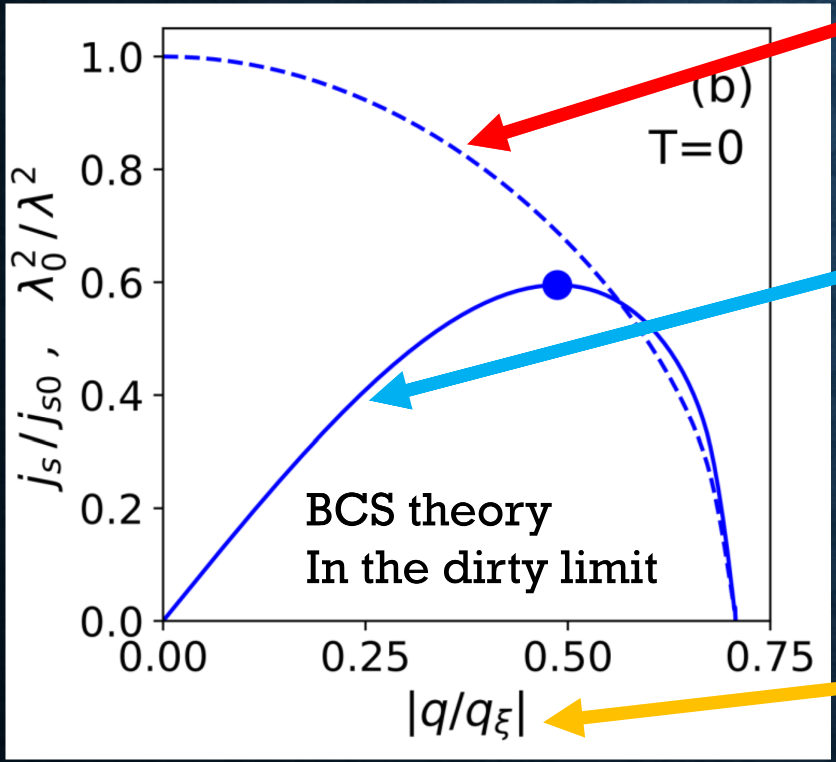
$n_s \propto \lambda^{-2}$ (superfluid density)

q (superfluid momentum)

See, e.g.,
T. Kubo, Phys. Rev. Research **2**, 033203 (2020)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.



$n_s \propto \lambda^{-2}$ (superfluid density)

$J \sim n_s q$ (current density)

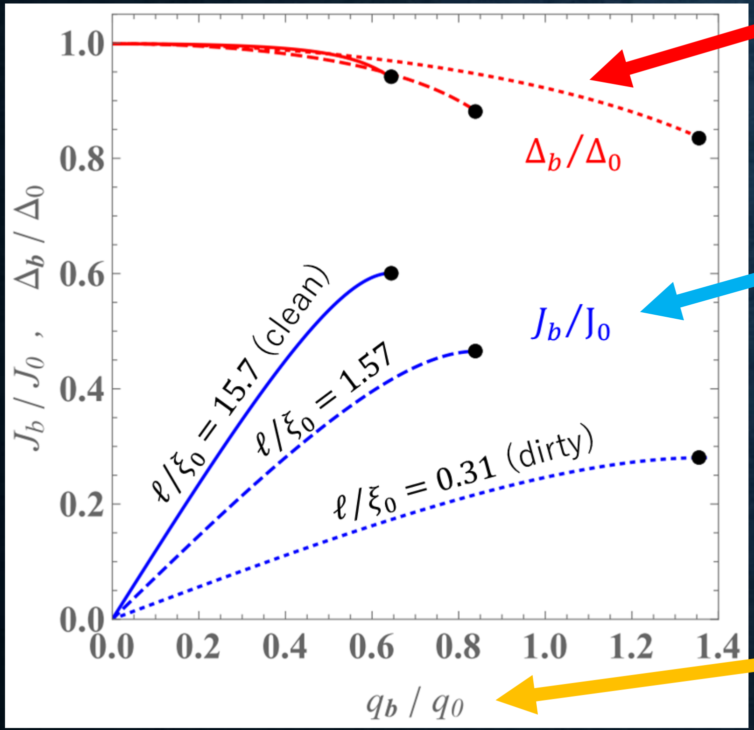
the maximum value ● is called the depairing current

q (superfluid momentum)

See, e.g.,
T. Kubo, Phys. Rev. Research **2**, 033203 (2020)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.



Δ (pair potential)

$J \sim n_s q$ (current density)

the maximum value ● is called the depairing current

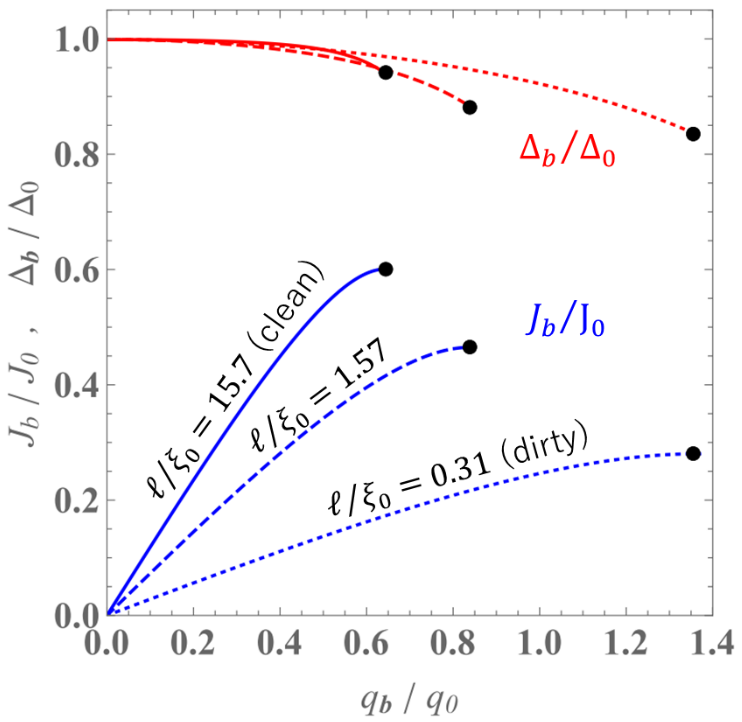
q (superfluid momentum)

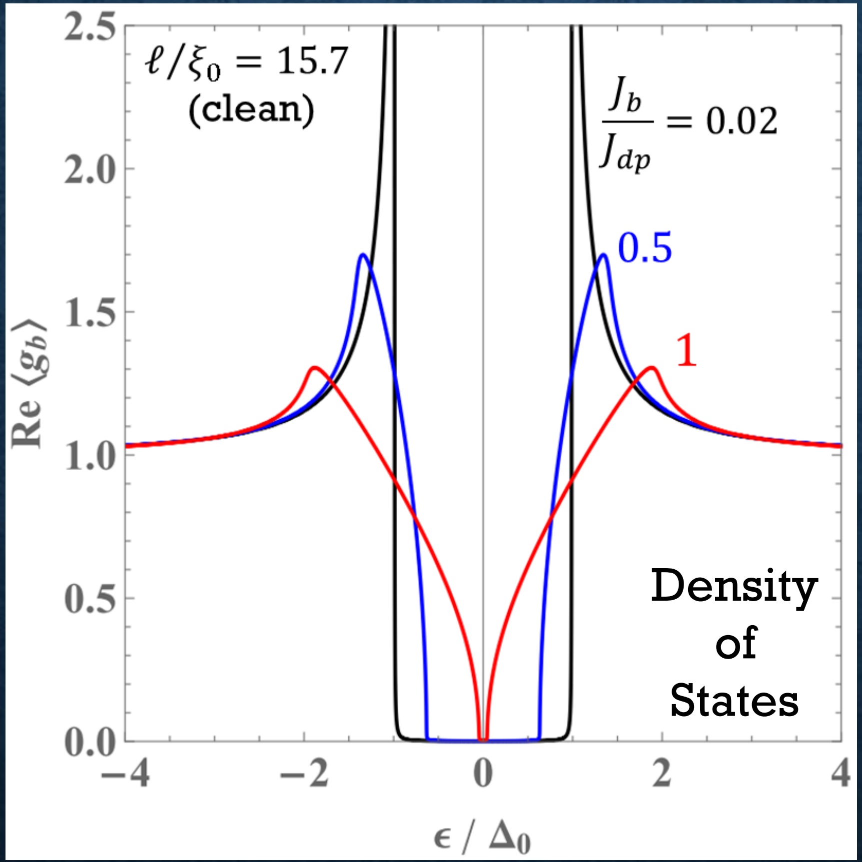
T. Kubo, Phys. Rev. Applied 22, 044042 (2024) DOI: https://doi.org/10.1103/PhysRevApplied.22.044042

See also, T. Kubo, Phys. Rev. Research **2**, 033203 (2020); F. Pei-Jen Lin and A. Gurevich, Phys. Rev. B **85**, 054513 (2012)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.



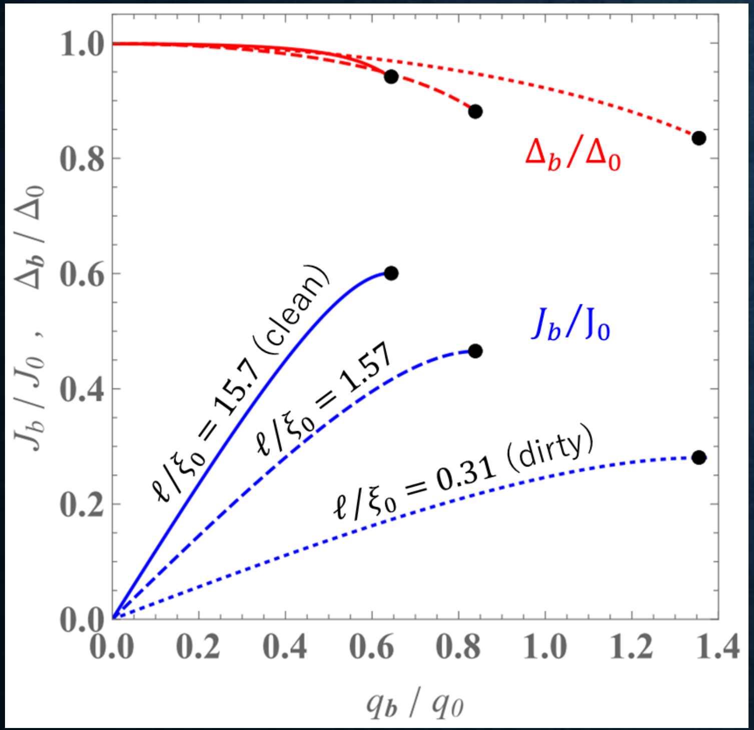


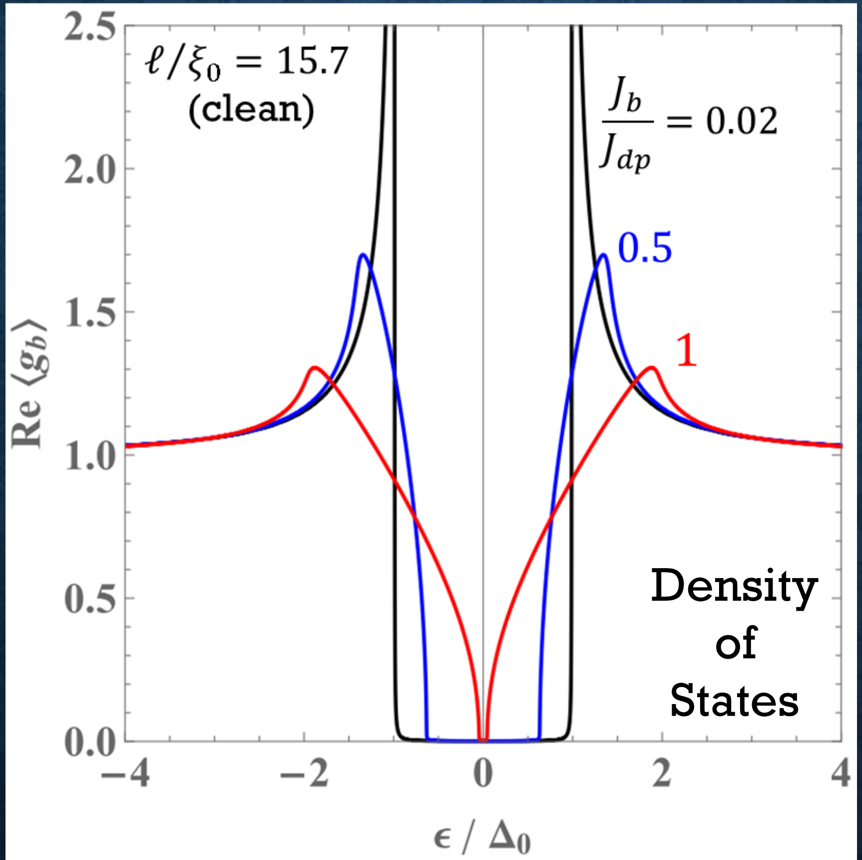
T. Kubo, Phys. Rev. Applied 22, 044042 (2024) DOI: https://doi.org/10.1103/PhysRevApplied.22.044042

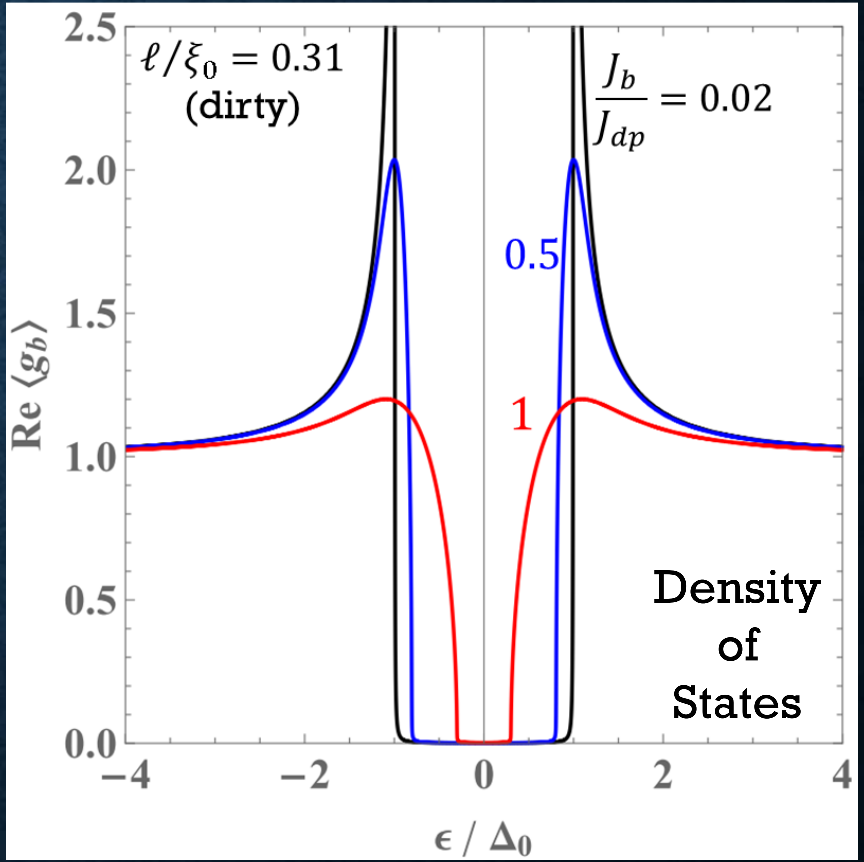
See also, T. Kubo, Phys. Rev. Research **2**, 033203 (2020); F. Pei-Jen Lin and A. Gurevich, Phys. Rev. B **85**, 054513 (2012)

Next, we consider **only a dc current**.

This case is also well understood from decades ago.







T. Kubo, Phys. Rev. Applied 22, 044042 (2024) DOI: https://doi.org/10.1103/PhysRevApplied.22.044042

See also, T. Kubo, Phys. Rev. Research **2**, 033203 (2020); F. Pei-Jen Lin and A. Gurevich, Phys. Rev. B **85**, 054513 (2012)

Outline

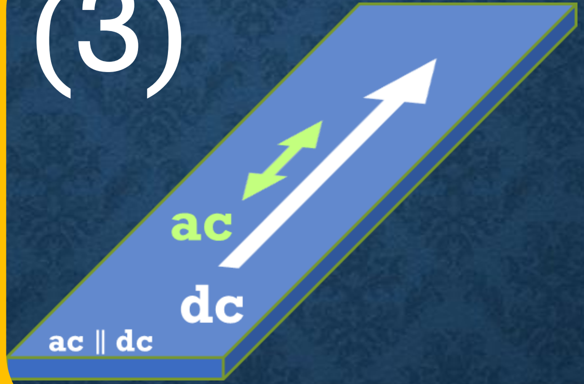
(1)

Only weak ac

(2)

Only dc

(3)

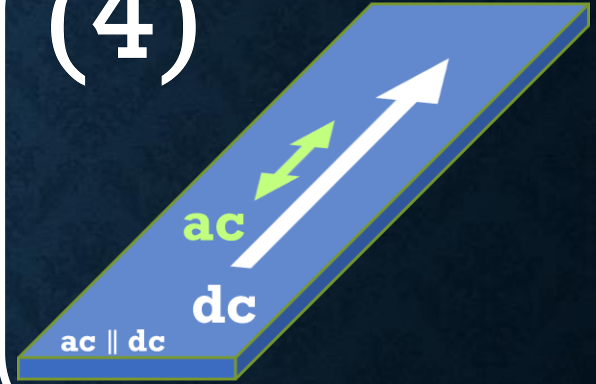


ac + dc

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).

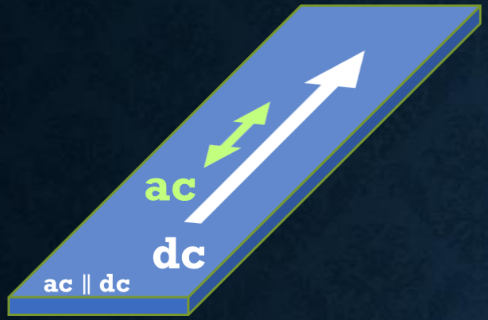
(4)



ac + dc

Microscopic nonequilibrium superconductivity

- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)



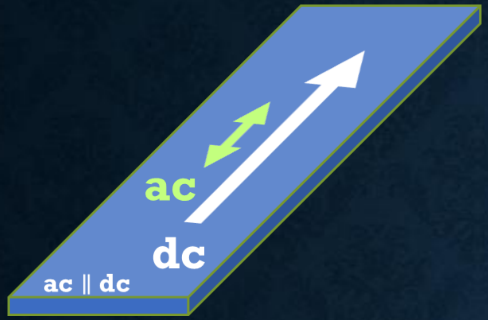
Let us go back to the definition of L_k.

We soon find that it is more complicated than expected.

$$L_k \frac{dJ_s}{dt} = E$$

$$\boldsymbol{J_s \propto n_s q}$$

$$\frac{dJ_s}{dt} \propto \dot{n_s} q + n_s \dot{q}$$



Let us go back to the definition of L_k.
We soon find that it is more complicated than expected.

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

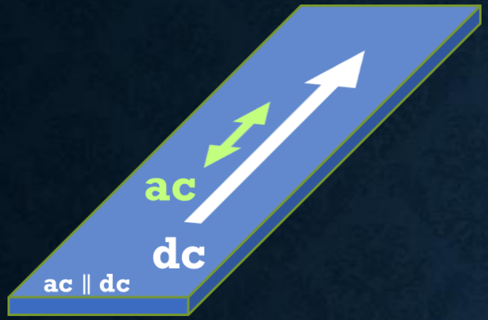
$$L_k \frac{dJ_s}{dt} = E$$

$$\boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}}}$$

$$\boldsymbol{J_s \propto n_s q}$$

$$\frac{dJ_s}{dt} \propto \dot{n}_s q + n_s \dot{q}$$

$$n_{s0} := n_s(q = 0, T = 0)$$



Let us go back to the definition of L_k.
We soon find that it is more complicated than expected.

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

$$L_k \frac{dJ_s}{dt} = E$$

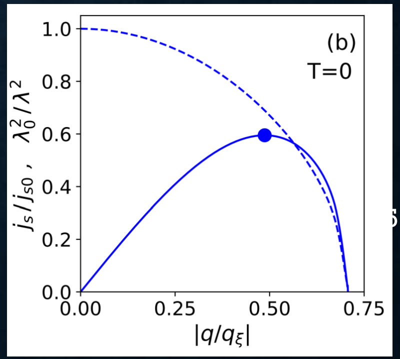
$$\boldsymbol{J_s \propto n_s q}$$

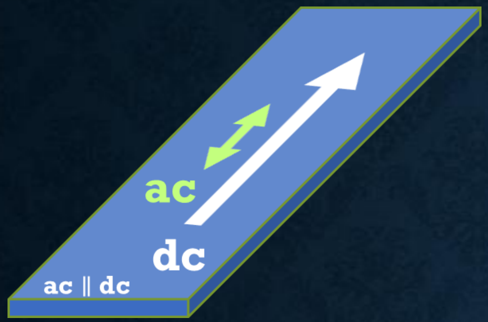
$$\frac{dJ_s}{dt} \propto \dot{n_s} q + n_s \dot{q}$$

$$n_{s0} := n_s(q=0, T=0)$$

$$\boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n_s}}{n_{s0}} q + \boxed{\frac{n_s}{n_{s0}}} \dot{q}}}$$

This part can be calculated from the equilibrium theory: the BCS theory





Let us go back to the definition of L_k.
We soon find that it is more complicated than expected.

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

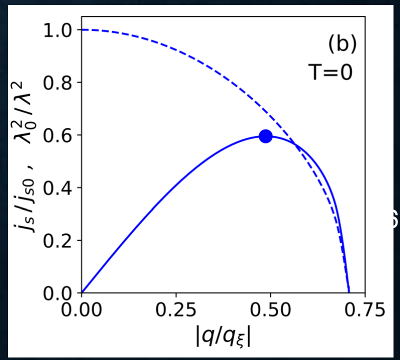
$$L_k \frac{dJ_s}{dt} = E \quad \Rightarrow \quad \boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\boxed{\frac{\dot{n}_s}{n_{s0}}} q + \boxed{\frac{n_s}{n_{s0}}} \dot{q}}}$$

$$\boldsymbol{J_s \propto n_s q}$$
$$\frac{dJ_s}{dt} \propto \dot{n}_s q + n_s \dot{q}$$

$$n_{s0} := n_s(q = 0, T = 0)$$

Dynamics of superfluid density!

This part can be calculated from the equilibrium theory: the BCS theory



This looks very much like a nonequilibrium situation.

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

$$\boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}}}$$

To circumvent the complexity of the nonequilibrium problem,
previous studies introduced the following simplifying assumptions.

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

$$\boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}}}$$

To circumvent the complexity of the nonequilibrium problem, previous studies introduced the following simplifying assumptions.

Assumption about n_s dynamics

Fast experiment (Frozen $\boldsymbol{n_s}$): $\dot{n}_s = 0$

$$\boxed{L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}} \to \mu_0 \lambda_0^2 \frac{1}{\frac{n_s(q)}{n_{s0}}} = \mu_0 \lambda^2(q)}$$

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

$$L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}}$$

To circumvent the complexity of the nonequilibrium problem, previous studies introduced the following simplifying assumptions.

Assumption about n_s dynamics

Fast experiment (Frozen $\boldsymbol{n_s}$): $\dot{n}_s = 0$

$$L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}} \rightarrow \mu_0 \lambda_0^2 \frac{1}{\frac{n_s(q)}{n_{s0}}} = \mu_0 \lambda^2(q)$$

Calculate L_k

GL theory
or
BCS theory

Semi-phenomenological approach

- S. M. Anlage et al., IEEE Trans. Magn. **25**, 1388 (1989).
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Physical Review Research **2**, 033203 (2020).

To circumvent the complexity of the nonequilibrium problem, previous studies introduced the following simplifying assumptions.

$$L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}}$$

Assumption about n_s dynamics

Fast experiment (Frozen $\boldsymbol{n_s}$): $\dot{n}_s = 0$

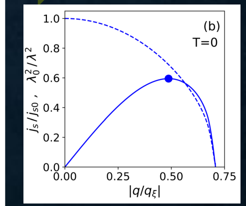
$$L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}} \to \mu_0 \lambda_0^2 \frac{1}{\frac{n_s(q)}{n_{s0}}} = \mu_0 \lambda^2(q)$$

Calculate L_k

GL theory

or

BCS theory



Slow experiment (oscillating $\boldsymbol{n_s}$): $\dot{n}_s = (dn_s/dq)\dot{q}$

$$L_k = \mu_0 \lambda_0^2 \frac{\dot{q}}{\frac{\dot{n}_s}{n_{s0}} q + \frac{n_s}{n_{s0}} \dot{q}} \to \mu_0 \lambda_0^2 \frac{1}{(1 + q\partial_q) \frac{n_s(q)}{n_{s0}}}$$

Calculate L_k

We can calculate L_k for **any dc bias current**.

We can calculate L_k for **any dc bias current**. **However**, here we focus on the small-bias regime ($J_b/J_{dp} \ll 1$), where the coefficient C in the following expansion can be determined analytically.

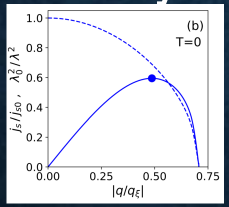
$$L_k(J_b) = L_k(0)\left\{1 + C\left(\frac{J_b}{J_{dp}}\right)^2 + \cdots\right\}$$

We can calculate L_k for **any dc bias current**. **However**, here we focus on the small-bias regime ($J_b/J_{dp} \ll 1$), where the coefficient C in the following expansion can be determined analytically.

$$L_k(J_b) = L_k(0)\left\{1 + C\left(\frac{J_b}{J_{dp}}\right)^2 + \cdots\right\}$$

Fast experiment (Frozen n_s)

BCS theory in the dirty limit



$$C(T \to 0) = \frac{(3\pi^2 + 16)s_d}{12\pi}\left(\Delta_d - \frac{4s_d}{3\pi}\right)^2 = \boxed{0.136}$$

T. Kubo, Physical Review Research **2**, 033203 (2020).
Physical Review Research **6**, 039002 (2024) [Erratum].

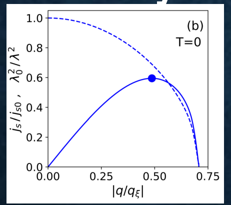
We can calculate L_k for **any dc bias current**. **However**, here we focus on the small-bias regime ($J_b/J_{dp} \ll 1$), where the coefficient C in the following expansion can be determined analytically.

$$L_k(J_b) = L_k(0)\left\{1 + C\left(\frac{J_b}{J_{dp}}\right)^2 + \cdots\right\}$$

Fast experiment (Frozen $\boldsymbol{n_s}$)

BCS theory in the dirty limit

$$C(T \to 0) = \frac{(3\pi^2 + 16)s_d}{12\pi}\left(\Delta_d - \frac{4s_d}{3\pi}\right)^2 = \boxed{0.136}$$



Slow experiment (Oscillating $\boldsymbol{n_s}$)

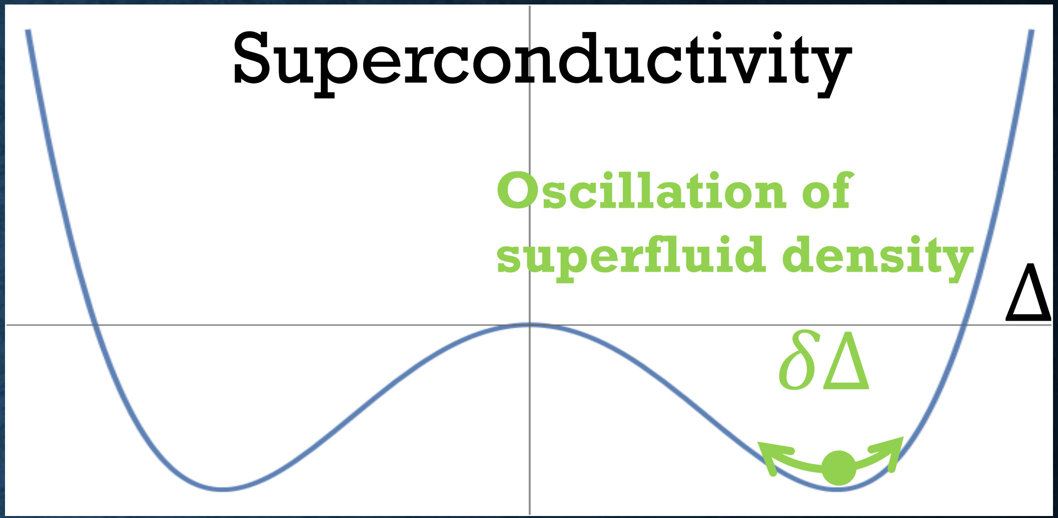
BCS theory in the dirty limit

$$C(T \to 0) = \frac{(3\pi^2 + 16)s_d}{4\pi}\left(\Delta_d - \frac{4s_d}{3\pi}\right)^2 = \boxed{0.409}$$

$$\Delta_d = e^{-\pi\zeta_d/4} \qquad s_d = \Delta_d\zeta_d \qquad \zeta_d = \frac{2}{\pi} + \frac{3\pi}{8} - \sqrt{\left(\frac{2}{\pi} + \frac{3\pi}{8}\right)^2 - 1} \simeq 0.300$$

T. Kubo, Physical Review Research **2**, 033203 (2020).
Physical Review Research **6**, 039002 (2024) [Erratum].

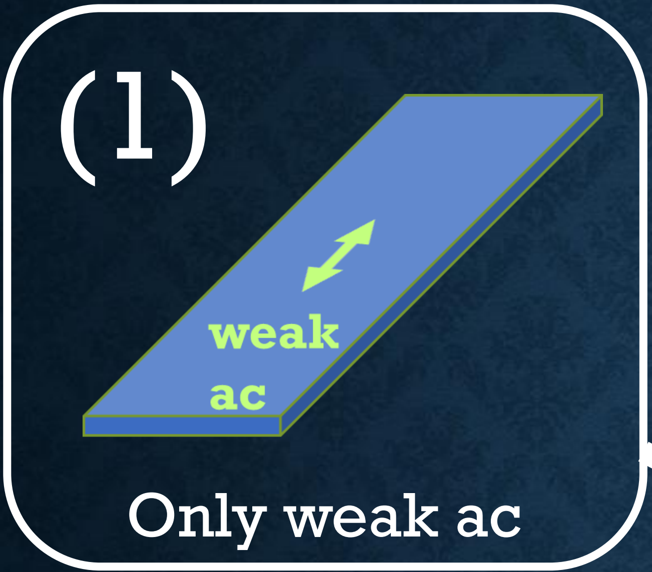
Here, let us recall that
the Higgs mode essentially represents
an oscillation of the superfluid density n_s.

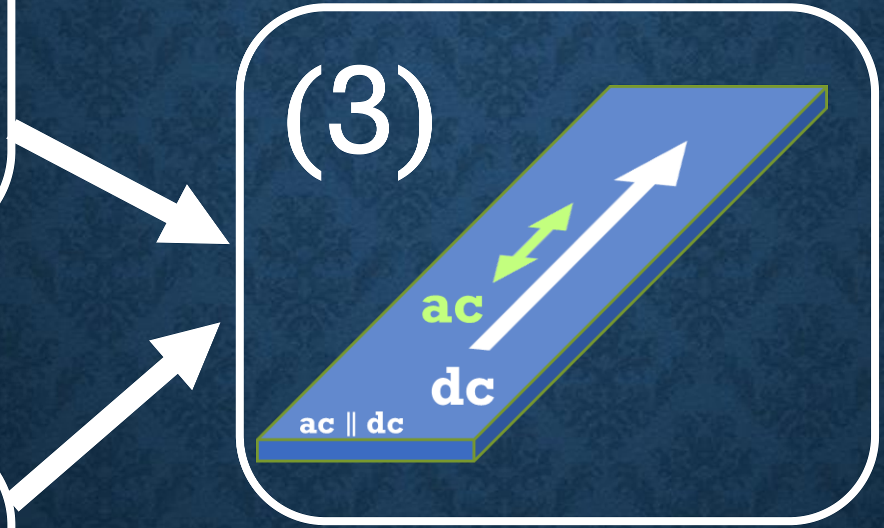


Then,
does the decades-old assumption of "frozen" and "**oscillating**" n_s correspond to calculations that **neglect** and **include** the **Higgs-mode** effect, respectively?

Does the difference between **C=0.136** and **C=0.409** arise from the Higgs mode?

Outline

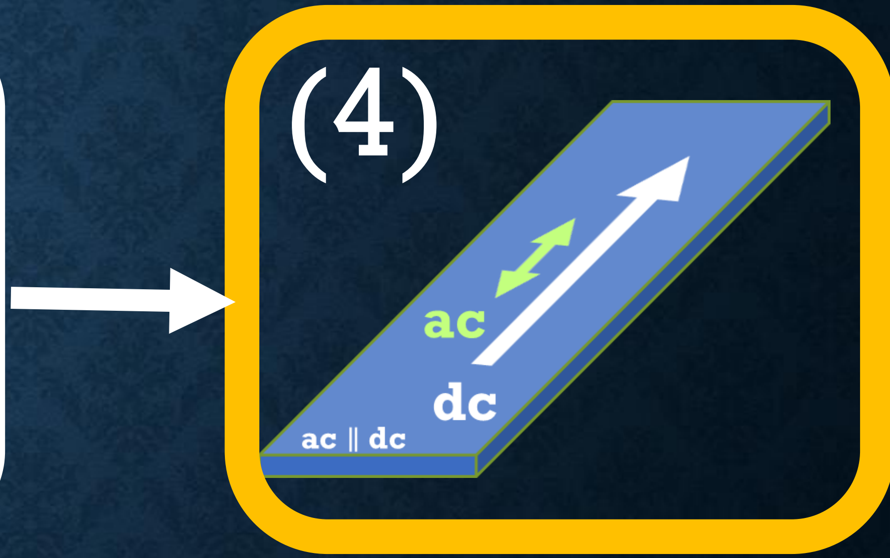




ac + dc

Semi-phenomenological approach

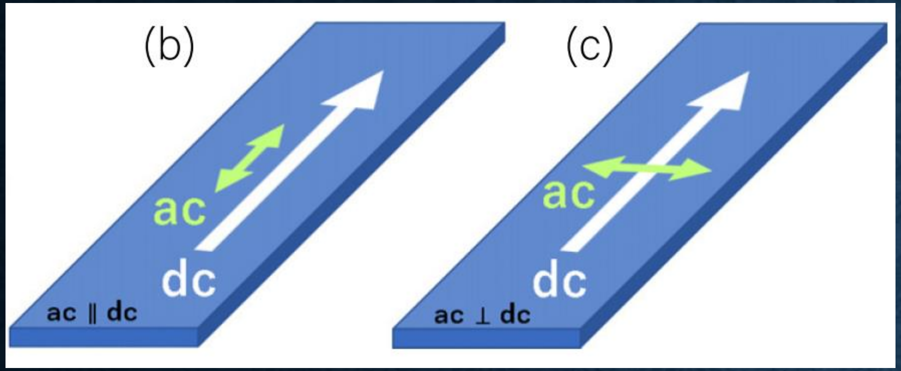
- J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
- T. Kubo, Phys. Rev. Research **2**, 033203 (2020).



ac + dc

Microscopic nonequilibrium superconductivity

- T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
- T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)



In this case, we need to use the microscopic theory of **nonequilibrium** superconductivity.

A. Moor et al., Phys. Rev. Lett. **118**, 047001 (2017)
T. Jujo, J. Phys. Soc. Jpn. **91**, 074711 (2022)
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

- **The Keldysh-Eilenberger theory** is a microscopic theory of **nonequilibrium** superconductivity. It is applicable at any temperature ($0 \le T \le T_c$) and for arbitrary mean free path. In this sense, it serves as the "*theory of everything for conventional superconductivity*".

T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

- **The Keldysh–Usadel theory** represents the dirty-limit reduction of the Keldysh–Eilenberger theory of nonequilibrium superconductivity, applicable at any T ($0 \le T \le T_c$).

T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

Keldysh–Usadel Equation: DC Current with AC Perturbation

T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$
\begin{aligned}
&-i(s/2)\left[\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)-\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\right.\\
&\quad+\hat{\tau}_3\delta\hat{g}^R(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)-\delta\hat{g}^R(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\\
&\quad+\hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\delta\hat{g}^A(\epsilon,\omega)-\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\delta\hat{g}^A(\epsilon,\omega)\hat{\tau}_3\\
&\quad\left.+\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)-\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\hat{\tau}_3\right]\\
&\quad-i(\delta W/2)\left[\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)-\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\right.\\
&\quad+\hat{\tau}_3\hat{g}_b^R(\epsilon_-)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)-\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\\
&\quad+\hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)-\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\hat{\tau}_3\\
&\quad\left.+\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)-\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_+)\hat{\tau}_3\right]\\
&\quad=\epsilon_+\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)-\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\epsilon_-\\
&\quad+[\hat{\Delta}_b,\delta\hat{g}^K(\epsilon,\omega)]+\delta\hat{\Delta}(\omega)\hat{g}_b^K(\epsilon_-)-\hat{g}_b^K(\epsilon_+)\delta\hat{\Delta}(\omega). \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
&-i(s/2)\left\{\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)-\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\right.\\
&\quad\left.+\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)-\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3\right\}\\
&\quad-i(\delta W/2)\left\{\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\right.\\
&\quad-\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3+\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\\
&\quad\left.-\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\right\}=\epsilon_+\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)-\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\epsilon_-\\
&\quad+[\hat{\Delta}_b,\delta\hat{g}^r(\epsilon,\omega)]+\delta\hat{\Delta}(\omega)\hat{g}_b^r(\epsilon_-)-\hat{g}_b^r(\epsilon_+)\delta\hat{\Delta}(\omega), \qquad (13)
\end{aligned}
$$

$$
\delta\Delta(\omega)=-\frac{\mathcal{G}}{8}\int d\epsilon\,\mathrm{Tr}[(-i\tau_2)\delta\hat{g}^K(\epsilon,\omega)]. \qquad (17)
$$

Higgs

↓

We solve these equations to obtain the AC-induced **nonequilibrium corrections** ($\delta\hat{g}^{R,A,K}$ and $\delta\Delta$).

Keldysh–Usadel Equation: DC Current with AC Perturbation

T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$$
\begin{aligned}
&- i(s/2)\left[\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega) - \hat{g}_b^R(\epsilon_+)\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\right.\\
&\quad + \hat{\tau}_3\delta\hat{g}^R(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^K(\epsilon_-) - \delta\hat{g}^R(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\\
&\quad + \hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\delta\hat{g}^A(\epsilon,\omega) - \hat{g}_b^K(\epsilon_+)\hat{\tau}_3\delta\hat{g}^A(\epsilon,\omega)\hat{\tau}_3\\
&\quad \left.+\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^A(\epsilon_-) - \delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\hat{\tau}_3\right]\\
&\quad - i(\delta W/2)\left[\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_-) - \hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\right.\\
&\quad + \hat{\tau}_3\hat{g}_b^R(\epsilon_-)\hat{\tau}_3\hat{g}_b^K(\epsilon_-) - \hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\\
&\quad + \hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_-) - \hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\hat{\tau}_3\\
&\quad \left.+\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\hat{\tau}_3\hat{g}_b^A(\epsilon_-) - \hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_+)\hat{\tau}_3\right]\\
&= \epsilon_+\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega) - \delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\epsilon_-\\
&\quad + [\hat{\Delta}_b, \delta\hat{g}^K(\epsilon,\omega)] + \delta\hat{\Delta}(\omega)\hat{g}_b^K(\epsilon_-) - \hat{g}_b^K(\epsilon_+)\delta\hat{\Delta}(\omega). \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
&- i(s/2)\left\{\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega) - \hat{g}_b^r(\epsilon_+)\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\right.\\
&\quad \left.+ \hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^r(\epsilon_-) - \delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3\right\}\\
&\quad - i(\delta W/2)\left\{\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\right.\\
&\quad - \hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3 + \hat{\tau}_3\hat{g}_b^r(\epsilon_-)\hat{\tau}_3\hat{g}_b^r(\epsilon_-)\\
&\quad \left.- \hat{g}_b^r(\epsilon_+)\hat{\tau}_3\hat{g}_b^r(\epsilon_+)\hat{\tau}_3\right\} = \epsilon_+\hat{\tau}_3\delta\hat{g}^r(\epsilon,\omega) - \delta\hat{g}^r(\epsilon,\omega)\hat{\tau}_3\epsilon_-\\
&\quad + [\hat{\Delta}_b, \delta\hat{g}^r(\epsilon,\omega)] + \delta\hat{\Delta}(\omega)\hat{g}_b^r(\epsilon_-) - \hat{g}_b^r(\epsilon_+)\delta\hat{\Delta}(\omega), \qquad (13)
\end{aligned}
$$

$$
\delta\Delta(\omega) = -\frac{\mathscr{G}}{8}\int d\epsilon\, \mathrm{Tr}[(-i\tau_2)\delta\hat{g}^K(\epsilon,\omega)]. \qquad (17)
$$

Higgs ↓

We solve these equations to obtain the AC-induced **nonequilibrium corrections** ($\delta\hat{g}^{R,A,K}$ and $\delta\Delta$).

To obtain the ac response,
we substitute the solutions ($\delta\hat{g}^{R,A,K}$, $\delta\Delta$) into

$$
\delta\mathbf{J}(\omega) = -i\frac{\sigma_n}{e}\int d\epsilon\,\delta\mathbf{S}(\epsilon,\omega), \qquad (18)
$$

$$
\begin{aligned}
\delta\mathbf{S}(\epsilon,\omega) &= (i/16)\mathrm{Tr}\left[i\mathbf{q}_b\right.\\
&\times\left\{\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega) + \hat{\tau}_3\delta\hat{g}^R(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^K(\epsilon_-)\right.\\
&\left.+ \hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\delta\hat{g}^A(\epsilon,\omega) + \hat{\tau}_3\delta\hat{g}^K(\epsilon,\omega)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\right\}\\
&\left.+ i\delta\mathbf{q}_\omega\left\{\hat{\tau}_3\hat{g}_b^R(\epsilon_+)\hat{\tau}_3\hat{g}_b^K(\epsilon_-) + \hat{\tau}_3\hat{g}_b^K(\epsilon_+)\hat{\tau}_3\hat{g}_b^A(\epsilon_-)\right\}\right]. \qquad (19)
\end{aligned}
$$

Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

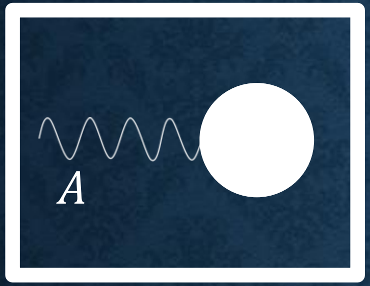
$$J \sim Agg + A_{dc} g \delta g$$

$$\delta \mathbf{J}(\omega) = -i \frac{\sigma_n}{e} \int d\epsilon \, \delta \mathbf{S}(\epsilon, \omega), \tag{18}$$

$$\begin{aligned} \delta \mathbf{S}(\epsilon, \omega) = {} & (i/16) \mathrm{Tr} \Big[i \mathbf{q}_b \\ & \times \left\{ \hat{\tau}_3 \hat{g}_b^R(\epsilon_+) \hat{\tau}_3 \delta \hat{g}^K(\epsilon, \omega) + \hat{\tau}_3 \delta \hat{g}^R(\epsilon, \omega) \hat{\tau}_3 \hat{g}_b^K(\epsilon_-) \right. \\ & \left. + \hat{\tau}_3 \hat{g}_b^K(\epsilon_+) \hat{\tau}_3 \delta \hat{g}^A(\epsilon, \omega) + \hat{\tau}_3 \delta \hat{g}^K(\epsilon, \omega) \hat{\tau}_3 \hat{g}_b^A(\epsilon_-) \right\} \\ & + i \delta \mathbf{q}_\omega \left\{ \hat{\tau}_3 \hat{g}_b^R(\epsilon_+) \hat{\tau}_3 \hat{g}_b^K(\epsilon_-) + \hat{\tau}_3 \hat{g}_b^K(\epsilon_+) \hat{\tau}_3 \hat{g}_b^A(\epsilon_-) \right\} \Big] . \end{aligned} \tag{19}$$

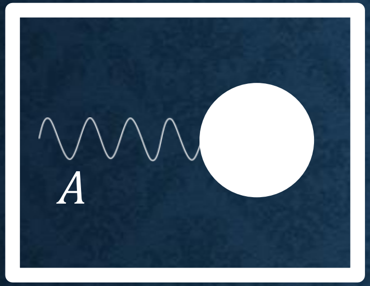
Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

$$J \sim \boxed{Agg} + A_{dc} g \delta g$$



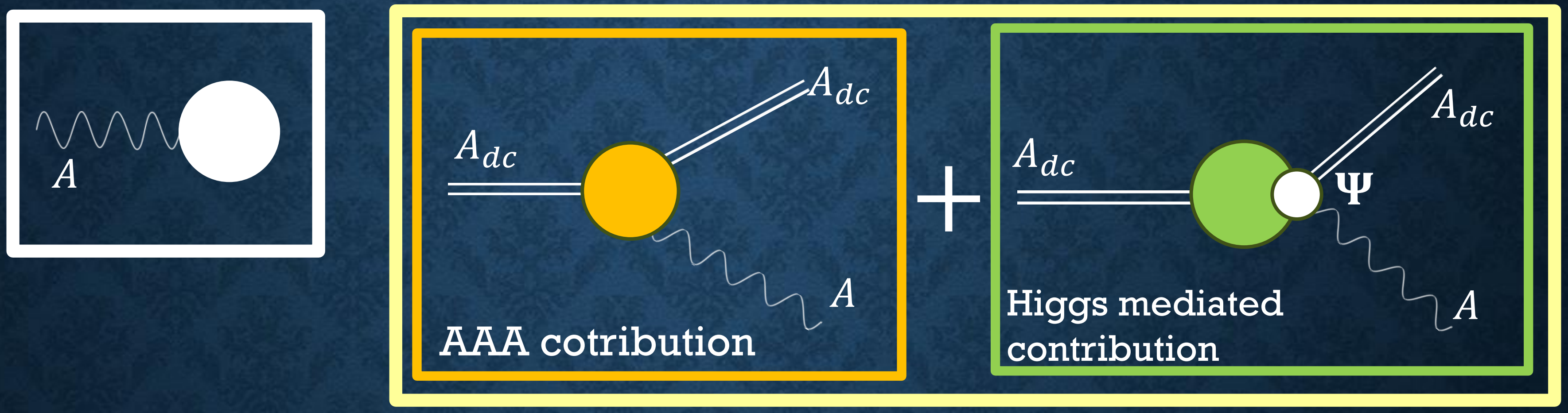
Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

$$J \sim \boxed{Agg} + A_{dc} g \delta g \quad \delta g \sim \boldsymbol{A}_{dc} \cdot \boldsymbol{A} + \delta\Delta \quad \text{and} \quad \delta\Delta \sim \Psi\, \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$$



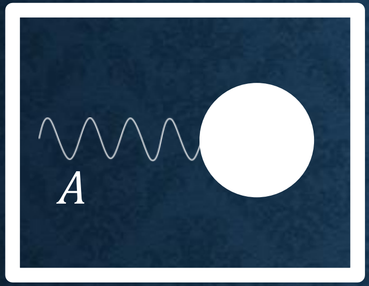
Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

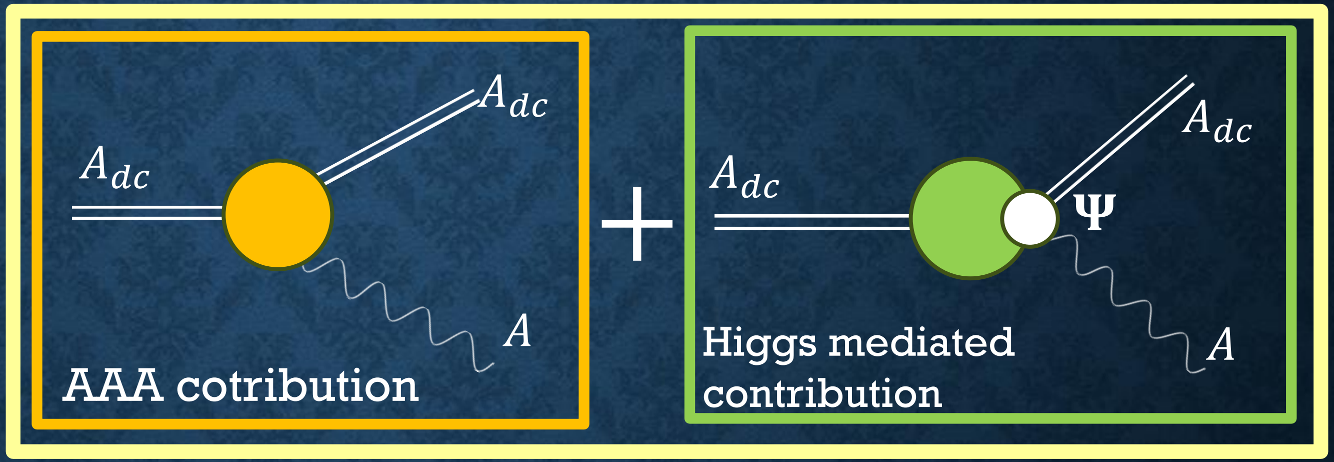
$$J \sim \boxed{Agg} + \boxed{A_{dc} g \delta g} \quad \delta g \sim \boldsymbol{A}_{dc} \cdot \boldsymbol{A} + \delta\Delta \quad \text{and} \quad \delta\Delta \sim \Psi\, \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$$



Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

$$J \sim \boxed{Agg} + \boxed{A_{dc} g \delta g} \qquad \delta g \sim \boldsymbol{A}_{dc} \cdot \boldsymbol{A} + \delta\Delta \quad \text{and} \quad \delta\Delta \sim \Psi\, \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$$



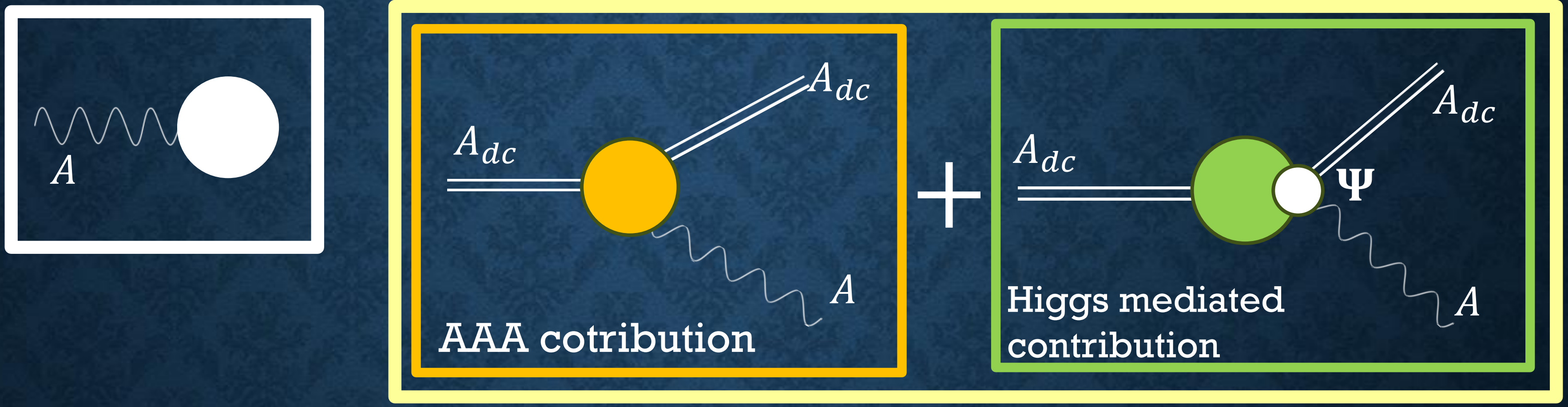


$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$ $\qquad$ $\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

Nonequilibrium corrections due to the Doppler fluctuation of flow $\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

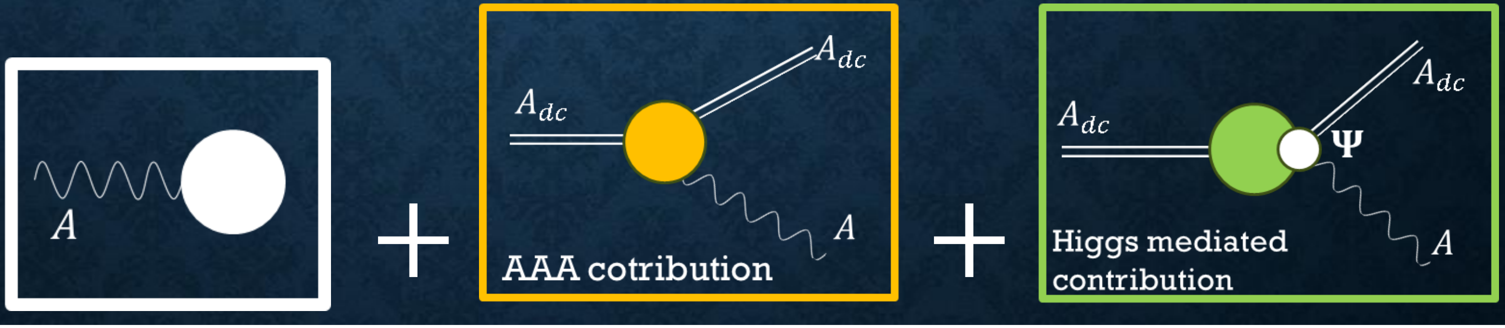
Instead of showing the rigorous algebraic calculations, we will look at schematic illustrations that represent the final results.

$$J \sim \boxed{Agg} + \boxed{A_{dc} g \delta g} \quad \delta g \sim \boldsymbol{A}_{dc} \cdot \boldsymbol{A} + \delta\Delta \quad \text{and} \quad \delta\Delta \sim \Psi\, \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$$



Then, the complex conductivity is given by

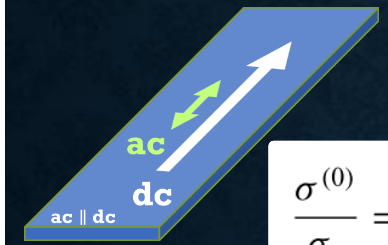
$$\sigma = \frac{J}{E} \sim \frac{\text{(diagrams)}}{A}$$



Complex conductivity formula

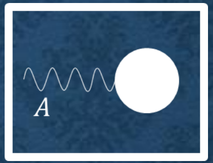
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$ac \parallel dc$ case



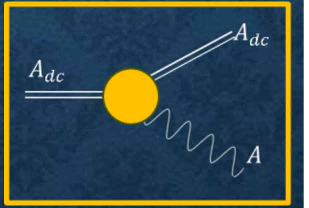
$$\sigma = \sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)}$$

$$\frac{\sigma^{(0)}}{\sigma_n} = \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b \,\mathrm{Re}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Re}\, F_b')(f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b \,\mathrm{Im}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Im}\, F_b')(2f_{\mathrm{FD}} - 1), \tag{41}$$



AAA contribution

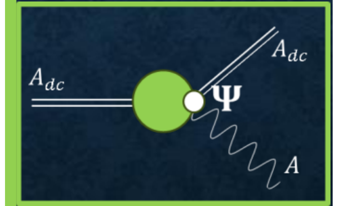
$$\frac{\sigma^{(1)}}{\sigma_n} = \frac{8s}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \mathrm{Re}\, F_b \,\mathrm{Im}\, F_b \,\mathrm{Re}\, G_b' (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \frac{2s}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \left[2\,\mathrm{Re}\, F_b \,\mathrm{Im}\, F_b \,\mathrm{Im}\left\{G_b + G_b'\right\} + \left\{(\mathrm{Re}\, F_b')^2 - (\mathrm{Re}\, F_b)^2 + (\mathrm{Im}\, F_b)^2 - (\mathrm{Im}\, F_b')^2\right\} \mathrm{Re}\, G_b \right] (2f_{\mathrm{FD}} - 1), \tag{42}$$



$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

Higgs mediated contribution

$$\frac{\sigma^{(2)}}{\sigma_n} = \frac{2s\Psi}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, F_b \,\mathrm{Re}\, G_b' - \mathrm{Re}\, G_b \,\mathrm{Re}\, F_b') \times (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \frac{2s\Psi}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \left\{\mathrm{Re}\, G_b \,\mathrm{Im}(F_b - F_b') + \mathrm{Re}\, F_b \,\mathrm{Im}(G_b + G_b')\right\} (2f_{\mathrm{FD}} - 1). \tag{43}$$



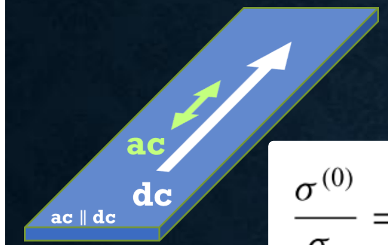
$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

$$\left.\begin{array}{l} \sigma^{(1)} \\ \sigma^{(2)} \end{array}\right\}$$ Nonequilibrium corrections due to the Doppler fluctuation of flow $\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

Complex conductivity formula

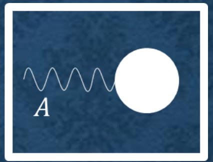
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$ac \parallel dc$ case



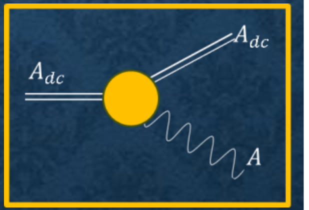
$$\sigma = \sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)}$$

$$\frac{\sigma^{(0)}}{\sigma_n} = \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b\, \mathrm{Re}\, G_b' + \mathrm{Re}\, F_b\, \mathrm{Re}\, F_b')(f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b\, \mathrm{Im}\, G_b' + \mathrm{Re}\, F_b\, \mathrm{Im}\, F_b')(2f_{\mathrm{FD}} - 1), \quad (41)$$



AAA contribution

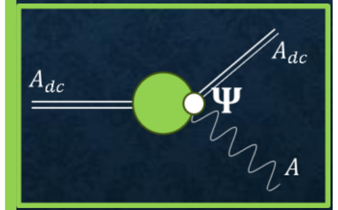
$$\frac{\sigma^{(1)}}{\sigma_n} = \frac{8s}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \mathrm{Re}\, F_b\, \mathrm{Im}\, F_b\, \mathrm{Re}\, G_b' (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \frac{2s}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \left[2\, \mathrm{Re}\, F_b\, \mathrm{Im}\, F_b\, \mathrm{Im} \{G_b + G_b'\} + \{(\mathrm{Re}\, F_b')^2 - (\mathrm{Re}\, F_b)^2 + (\mathrm{Im}\, F_b)^2 - (\mathrm{Im}\, F_b')^2\} \mathrm{Re}\, G_b \right] (2f_{\mathrm{FD}} - 1), \quad (42)$$



$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

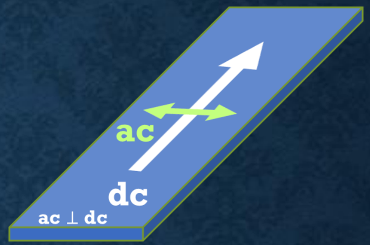
Higgs mediated contribution

$$\frac{\sigma^{(2)}}{\sigma_n} = \frac{2s\Psi}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, F_b\, \mathrm{Re}\, G_b' - \mathrm{Re}\, G_b\, \mathrm{Re}\, F_b') \times (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \frac{2s\Psi}{\hbar\omega} \int \frac{d\epsilon}{\hbar\omega} \{\mathrm{Re}\, G_b\, \mathrm{Im}(F_b - F_b') + \mathrm{Re}\, F_b\, \mathrm{Im}(G_b + G_b')\} (2f_{\mathrm{FD}} - 1). \quad (43)$$

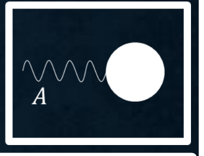


$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

$ac \perp dc$ case



$$\sigma = \sigma^{(0)}$$

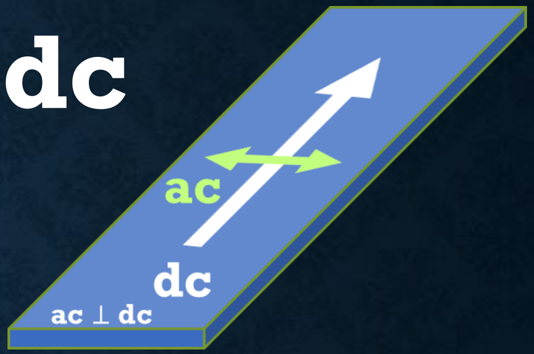


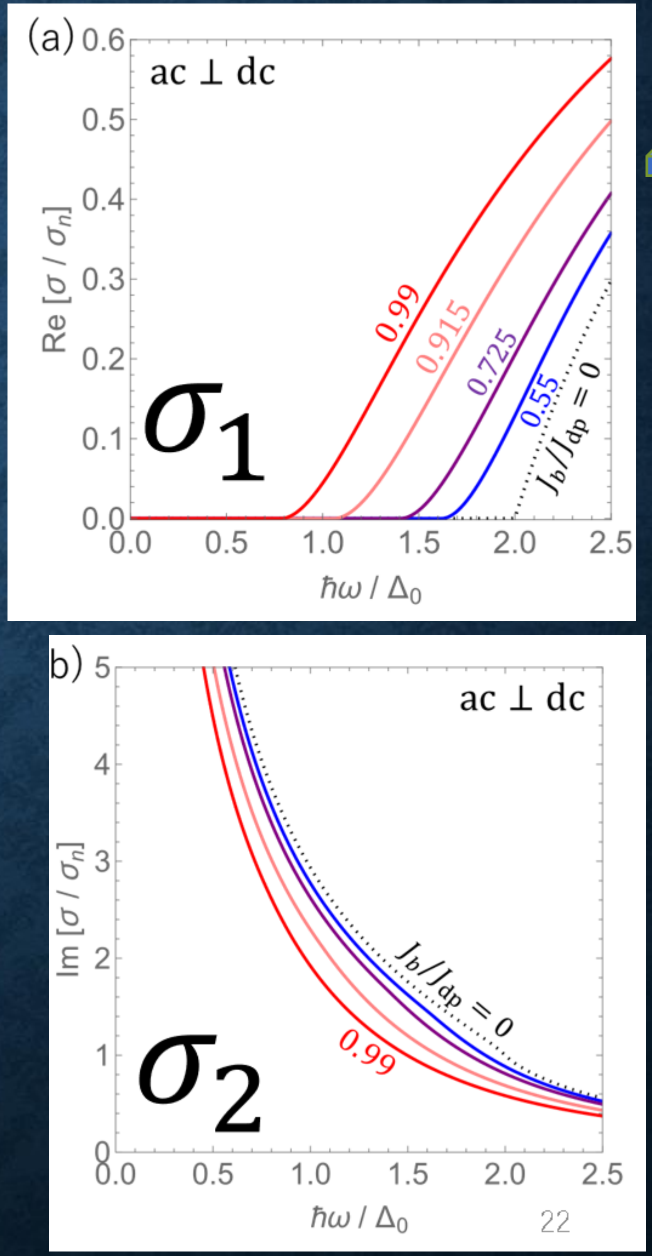
$$\frac{\sigma^{(0)}}{\sigma_n} = \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b\, \mathrm{Re}\, G_b' + \mathrm{Re}\, F_b\, \mathrm{Re}\, F_b')(f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \int \frac{d\epsilon}{\hbar\omega} (\mathrm{Re}\, G_b\, \mathrm{Im}\, G_b' + \mathrm{Re}\, F_b\, \mathrm{Im}\, F_b')(2f_{\mathrm{FD}} - 1), \quad (41)$$

No Doppler fluctuation

Nonequilibrium corrections due to the Doppler fluctuation of flow $\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

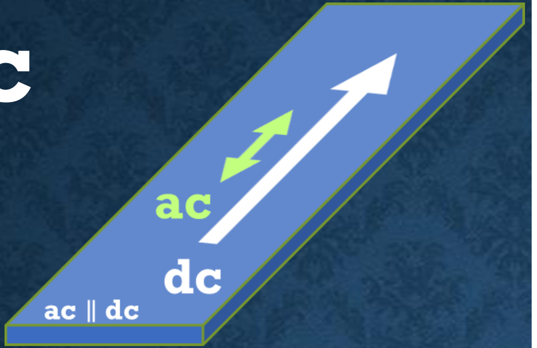
ac ⊥ dc

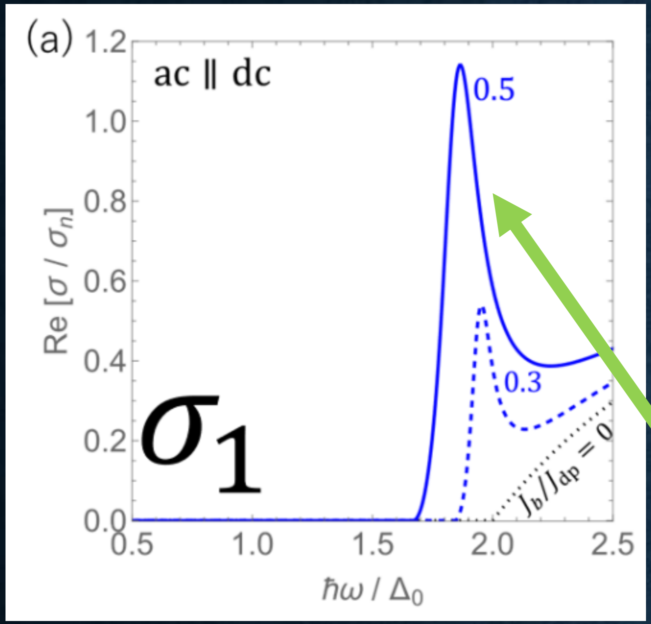


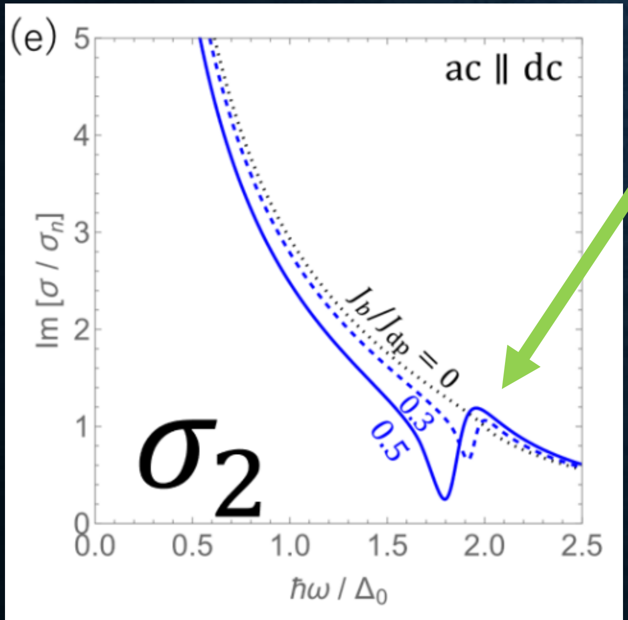


T. Kubo,
Phys. Rev. Applied **23**, 054091 (2025)

ac ∥ dc





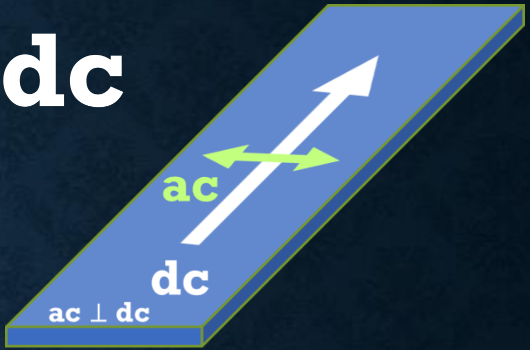


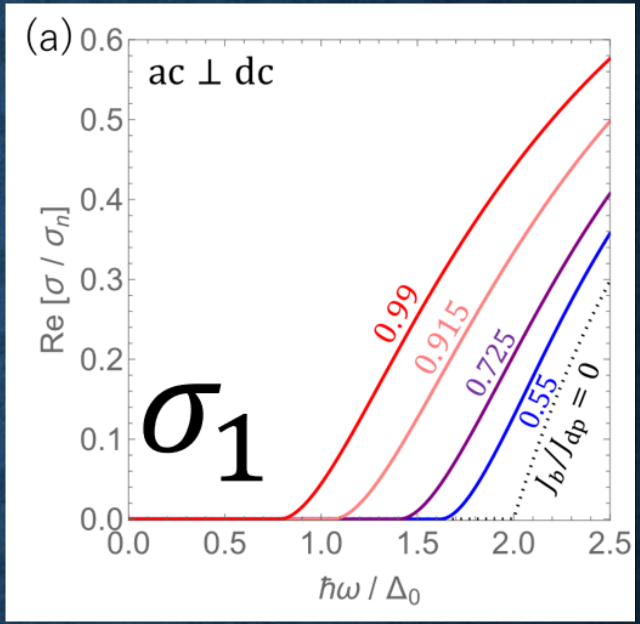
Resonance peak due to the **Higgs** mode.

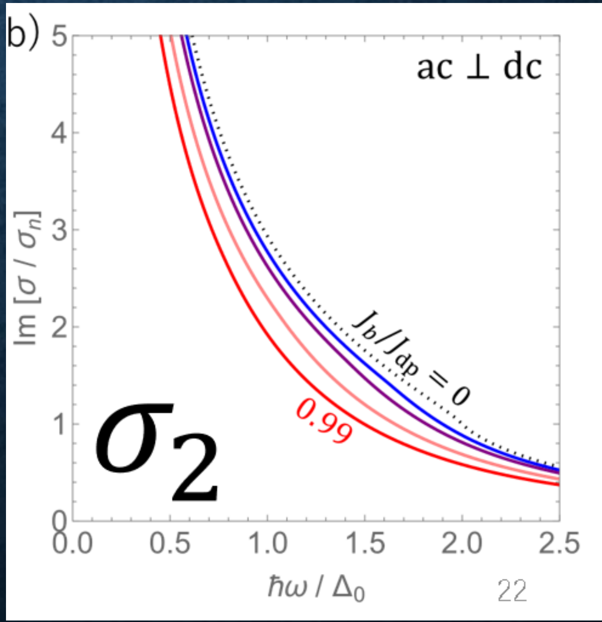
Already observed in experiments!
S. Nakamura et al., PRL **122**, 257001 (2019)

T. Kubo,
Phys. Rev. Applied **23**, 054091 (2025)

ac ⊥ dc







T. Kubo,
Phys. Rev. Applied **23**, 054091 (2025)

To calculate the kinetic inductance, we just go back to its definition.

$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega \sigma_2} \Rightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

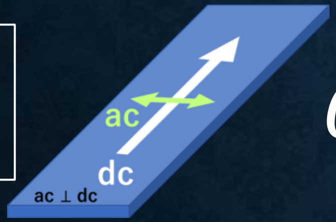
For $(T, \omega) \to (0, 0)$, we can analytically calculate the coefficient C

To calculate the kinetic inductance, we just go back to its definition.

$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xRightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega\sigma_2} \Rightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

For $(T, \omega) \to (0, 0)$, we can analytically calculate the coefficient C

$ac \perp dc$
case



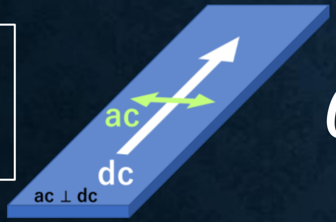
$$C = C^{(0)} \simeq 0.136$$

To calculate the kinetic inductance, we just go back to its definition.

$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega\sigma_2} \Rightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

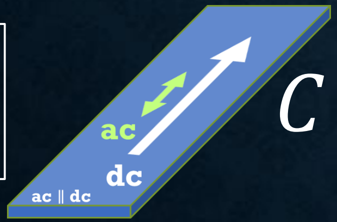
For $(T,\omega) \to (0,0)$, we can analytically calculate the coefficient C

$$\boxed{\begin{array}{c} ac \perp dc \\ \text{case} \end{array}}$$



$$C = C^{(0)} \simeq 0.136$$

$$\boxed{\begin{array}{c} ac \parallel dc \\ \text{case} \end{array}}$$



$$C = C^{(0)} + C^{(1)} + C^{(2)} \simeq 0.409$$

$C^{(0)} \simeq 0.136$

$C^{(1)}$: AAA $\simeq 0.0956$ **(23%)**

$C^{(2)}$: Higgs $\simeq 0.177$ **(43%)**

To calculate the kinetic inductance, we just go back to its definition.

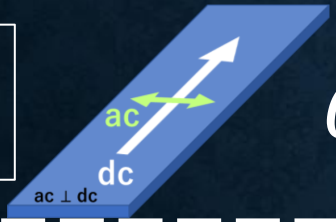
$$L_k = \frac{E}{\dot{J}_s} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega\sigma_2} \longrightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

For $(T,\omega) \to (0,0)$, we can analytically calculate the coefficient C

Microscopic theory of nonequilibrium superconductivity

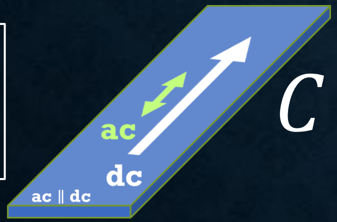
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$ac \perp dc$ case



$$C = C^{(0)} \simeq 0.136$$

$ac \parallel dc$ case



$$C = C^{(0)} + C^{(1)} + C^{(2)} \simeq 0.409$$

$C^{(0)} \simeq 0.136$

$C^{(1)}$: AAA $\simeq 0.0956$ **(23%)**

$C^{(2)}$: Higgs $\simeq 0.177$ **(43%)**

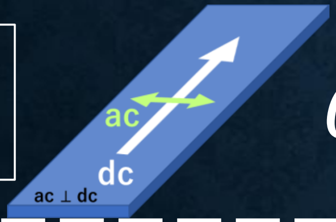
To calculate the kinetic inductance, we just go back to its definition.

$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega \sigma_2} \Longrightarrow L_k(J) = L_k(0) \left\{ 1 + \boxed{C} \left(\frac{J}{J_{dp}} \right)^2 + \cdots \right\}$$

For $(T, \omega) \to (0, 0)$, we can analytically calculate the coefficient C

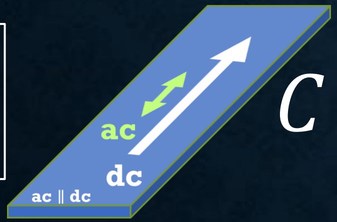
Microscopic theory of nonequilibrium superconductivity	Semi-phenomenological
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024) T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)	J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012). T. Kubo, Physical Review Research **2**, 033203 (2020).

$ac \perp dc$ case



$$C = C^{(0)} \simeq 0.136$$

$ac \parallel dc$ case



$$C = \underbrace{C^{(0)}}_{\simeq 0.136} + \underbrace{C^{(1)}}_{\text{AAA} \simeq 0.0956\ (23\%)} + \underbrace{C^{(2)}}_{\text{Higgs} \simeq 0.177\ (43\%)} \simeq 0.409$$

To calculate the kinetic inductance, we just go back to its definition.

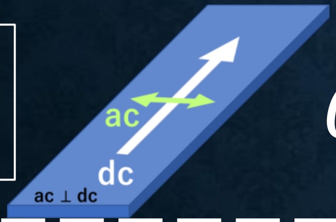
$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega \sigma_2} \longrightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

For $(T, \omega) \to (0, 0)$, we can analytically calculate the coefficient C

Microscopic theory of nonequilibrium superconductivity

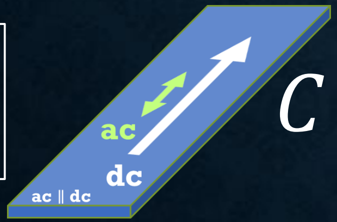
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)
T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$ac \perp dc$ case



$$C = C^{(0)} \simeq 0.136$$

$ac \parallel dc$ case



$$C = C^{(0)} + C^{(1)} + C^{(2)} \simeq 0.409$$

$C^{(0)} \simeq 0.136$

$C^{(1)}$: AAA $\simeq 0.0956$ (23%)

$C^{(2)}$: Higgs $\simeq 0.177$ (43%)

Semi-phenomenological

J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012).
T. Kubo, Physical Review Research **2**, 033203 (2020).

Fast experiment (Frozen n_s)

$$C \simeq 0.136$$

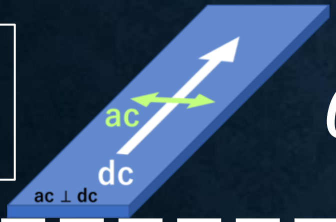
To calculate the kinetic inductance, we just go back to its definition.

$$\boxed{L_k = \frac{E}{\dot{J}_s}} \xrightarrow{J_s \propto e^{-i\omega t}} L_k = \frac{1}{\omega\sigma_2} \longrightarrow L_k(J) = L_k(0)\left\{1 + \boxed{C}\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

For $(T, \omega) \to (0, 0)$, we can analytically calculate the coefficient C

Microscopic theory of nonequilibrium superconductivity	Semi-phenomenological
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024) T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)	J. R. Clem and V. G. Kogan, Phys. Rev. B **86**, 174521 (2012). T. Kubo, Physical Review Research **2**, 033203 (2020).

$ac \perp dc$ case

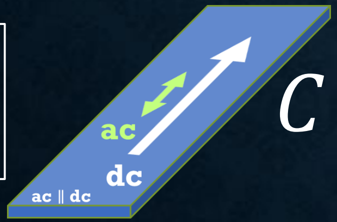


$$C = C^{(0)} \simeq 0.136$$

Fast experiment (Frozen n_s)

$$C \simeq 0.136$$

$ac \parallel dc$ case



$$C = C^{(0)} + C^{(1)} + C^{(2)} \simeq 0.409$$

$\simeq 0.136$

AAA
$\simeq 0.0956$
(23%)

Higgs
$\simeq 0.177$
(43%)

Slow experiment (Oscillating n_s)

$$C \simeq 0.409$$

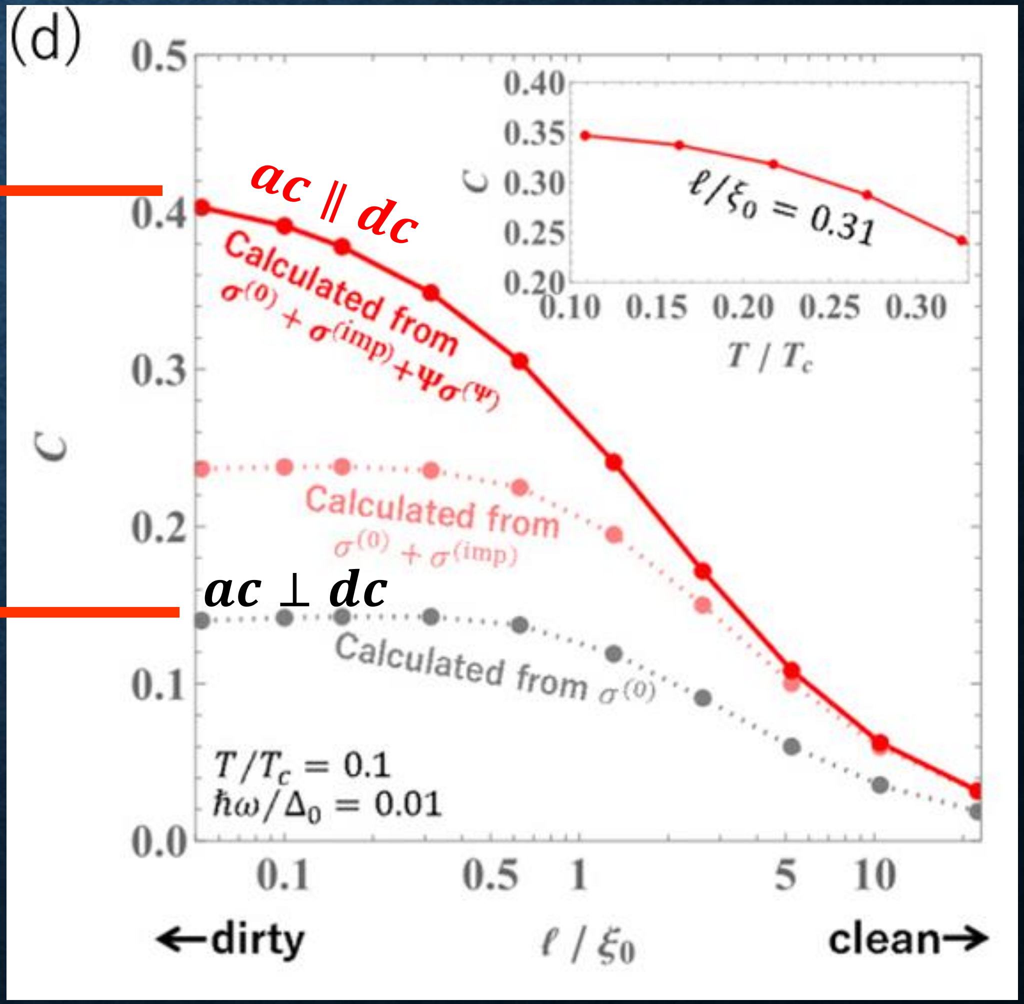
"C" from the Keldysh-Eilenberger theory, which is a microscopic theory of **nonequilibrium** superconductivity and is applicable at any temperature ($0 \leq T \leq T_c$) and for arbitrary mean free path.

$$L_k(J_b) = L_k(0)\left\{1 + C\left(\frac{J_b}{J_{dp}}\right)^2 + \cdots\right\}$$

$C \simeq 0.4$

Consistent with the dirty limit result obtained from the Keldysh-Usadel

$C \simeq 0.14$



We have finally come to understand the ***<u>true meaning</u>*** of the semi-phenomenological approach: frozen and oscillating n_s

	[When Does it happen?] What has long been believed
Frozen n_s	The ac frequency is so **<u>fast</u>** that the superfluid density cannot follow it. (Fast experiment)

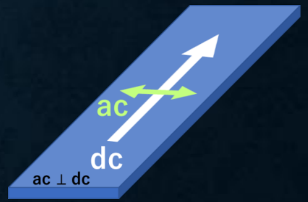
We have finally come to understand the ***true meaning*** of the semi-phenomenological approach: frozen and oscillating n_s

	[When Does it happen?] What has long been believed
Frozen n_s	The ac frequency is so **fast** that the superfluid density cannot follow it. (Fast experiment)
Oscillating n_s	The ac frequency is so **slow** that the superfluid density can follow it. (Slow experiment)

We have finally come to understand the ***true meaning*** of the semi-phenomenological approach: frozen and oscillating n_s

	[When Does it happen?] What has long been believed	[When Does it happen?] What has been revealed by the microscopic theory of nonequilibrum superconductivity
Frozen n_s	The ac frequency is so **fast** that the superfluid density cannot follow it. (Fast experiment)	
Oscillating n_s	The ac frequency is so **slow** that the superfluid density can follow it. (Slow experiment)	

We have finally come to understand the ***true meaning*** of the semi-phenomenological approach: frozen and oscillating n_s

	[When Does it happen?] What has long been believed	[When Does it happen?] What has been revealed by the microscopic theory of nonequilibrium superconductivity
Frozen n_s	~~The ac frequency is so **fast** that the superfluid density cannot follow it. (Fast experiment)~~	$ac \perp dc$ 
Oscillating n_s	~~The ac frequency is so **slow** that the superfluid density can follow it. (Slow experiment)~~ wrong	$ac \parallel dc$  **(AAA and Higgs contribution)**

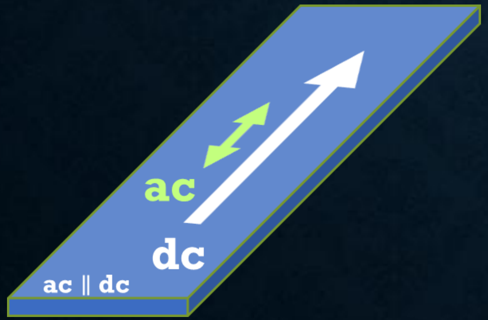
Summary

The Higgs mode is ubiquitous in superconducting devices. The current dependence of the kinetic inductance is a representative example.

$$L_k(J) = L_k(0)\left\{1 + C\left(\frac{J}{J_{dp}}\right)^2 + \cdots\right\}$$

$\boxed{\begin{array}{c} ac \perp dc \\ \text{case} \end{array}}$

$$C = C^{(0)} \simeq 0.136$$



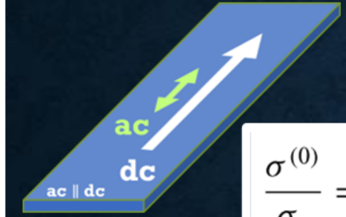
$\boxed{\begin{array}{c} ac \parallel dc \\ \text{case} \end{array}}$

$$C = \underbrace{C^{(0)}}_{\simeq 0.136} + \underbrace{C^{(1)}}_{\substack{\text{AAA} \\ \simeq 0.0956 \\ (23\%)}} + \underbrace{C^{(2)}}_{\substack{\text{Higgs} \\ \simeq 0.177 \\ (43\%)}} \simeq 0.409$$

Complex conductivity formula

T. Kubo, Phys. Rev. Applied **23**, 054091 (2025)

$ac \parallel dc$ case

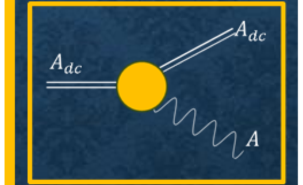


$$\sigma = \sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)}$$

$$\frac{\sigma^{(0)}}{\sigma_n} = \int \frac{d\epsilon}{\hbar\omega}(\mathrm{Re}\, G_b \,\mathrm{Re}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Re}\, F_b')(f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \int \frac{d\epsilon}{\hbar\omega}(\mathrm{Re}\, G_b \,\mathrm{Im}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Im}\, F_b')(2f_{\mathrm{FD}} - 1), \tag{41}$$

AAA contribution

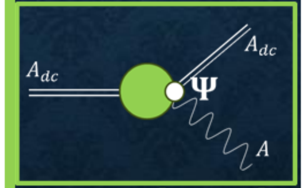
$$\frac{\sigma^{(1)}}{\sigma_n} = \frac{8s}{\hbar\omega}\int \frac{d\epsilon}{\hbar\omega} \mathrm{Re}\, F_b \,\mathrm{Im}\, F_b \,\mathrm{Re}\, G_b' (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i\frac{2s}{\hbar\omega}\int \frac{d\epsilon}{\hbar\omega}\left[2\,\mathrm{Re}\, F_b \,\mathrm{Im}\, F_b \,\mathrm{Im}\left\{G_b + G_b'\right\} + \left\{(\mathrm{Re}\, F_b')^2 - (\mathrm{Re}\, F_b)^2 + (\mathrm{Im}\, F_b)^2 - (\mathrm{Im}\, F_b')^2\right\}\mathrm{Re}\, G_b\right](2f_{\mathrm{FD}} - 1), \tag{42}$$



$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

Higgs mediated contribution

$$\frac{\sigma^{(2)}}{\sigma_n} = \frac{2s\Psi}{\hbar\omega}\int \frac{d\epsilon}{\hbar\omega}(\mathrm{Re}\, F_b \,\mathrm{Re}\, G_b' - \mathrm{Re}\, G_b \,\mathrm{Re}\, F_b') \times (f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i\frac{2s\Psi}{\hbar\omega}\int \frac{d\epsilon}{\hbar\omega}\left\{\mathrm{Re}\, G_b \,\mathrm{Im}(F_b - F_b') + \mathrm{Re}\, F_b \,\mathrm{Im}(G_b + G_b')\right\}(2f_{\mathrm{FD}} - 1). \tag{43}$$



$\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

$ac \perp dc$ case

$$\sigma = \sigma^{(0)}$$

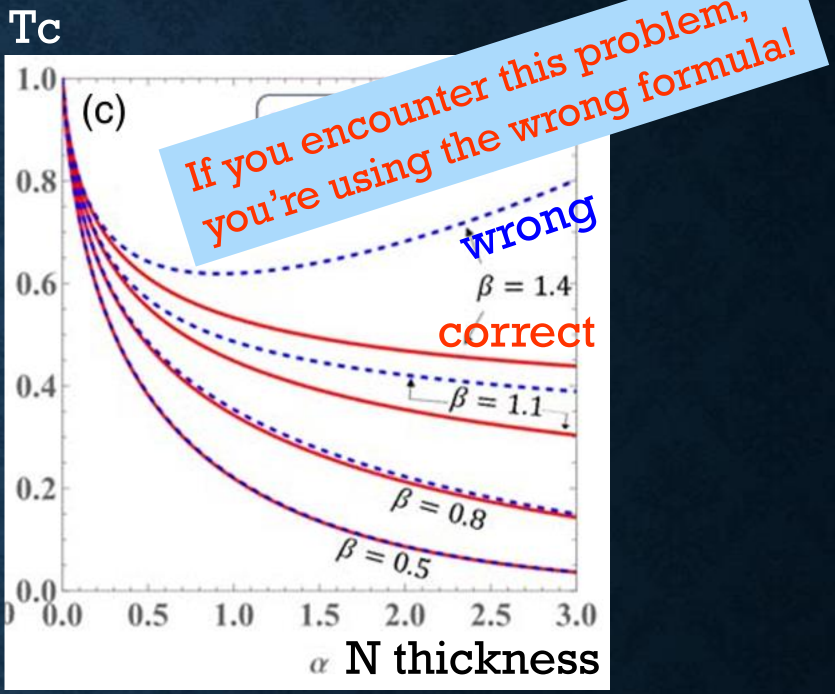
$$\frac{\sigma^{(0)}}{\sigma_n} = \int \frac{d\epsilon}{\hbar\omega}(\mathrm{Re}\, G_b \,\mathrm{Re}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Re}\, F_b')(f_{\mathrm{FD}} - f_{\mathrm{FD}}') + i \int \frac{d\epsilon}{\hbar\omega}(\mathrm{Re}\, G_b \,\mathrm{Im}\, G_b' + \mathrm{Re}\, F_b \,\mathrm{Im}\, F_b')(2f_{\mathrm{FD}} - 1), \tag{41}$$

No Doppler fluctuation

Nonequilibrium corrections due to the Doppler fluctuation of flow $\propto \boldsymbol{A}_{dc} \cdot \boldsymbol{A}$

Other works of mine that may interest you:

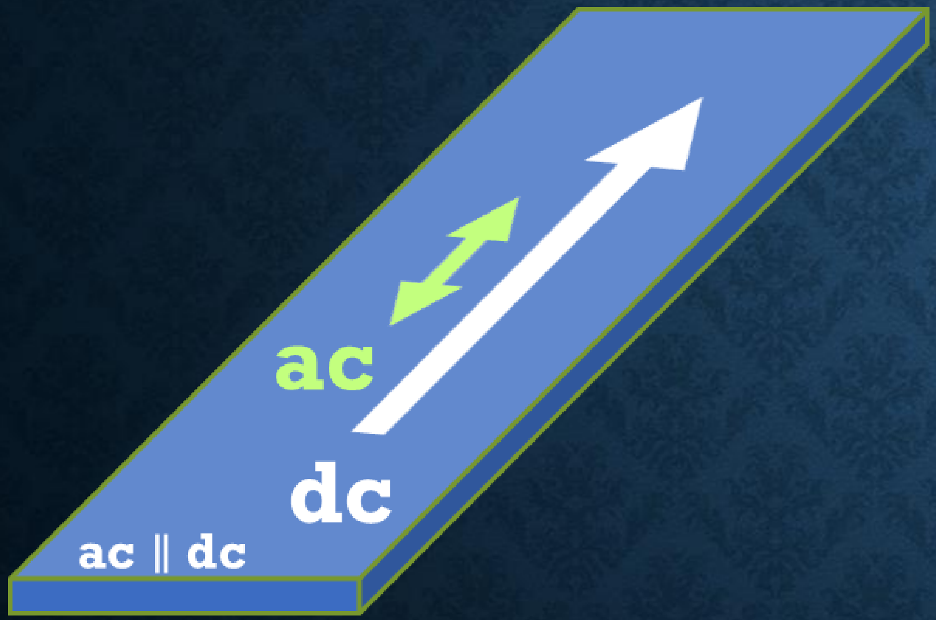
- T. Kubo, On the applicability ranges of Tc formulas for proximity-coupled thin SN and SS bilayers, Japanese Journal of Applied Physics 64, 018001 (2025).
  This is **a brief review of Tc formulas** of SN and SS bilayers. Because several distinct formulas exist, misuse is common. Please verify that your choice is appropriate. For instance, if a chosen formula yields a Tc that increases with the N-layer thickness, that formula is being applied outside its valid range.
  The solution is here.



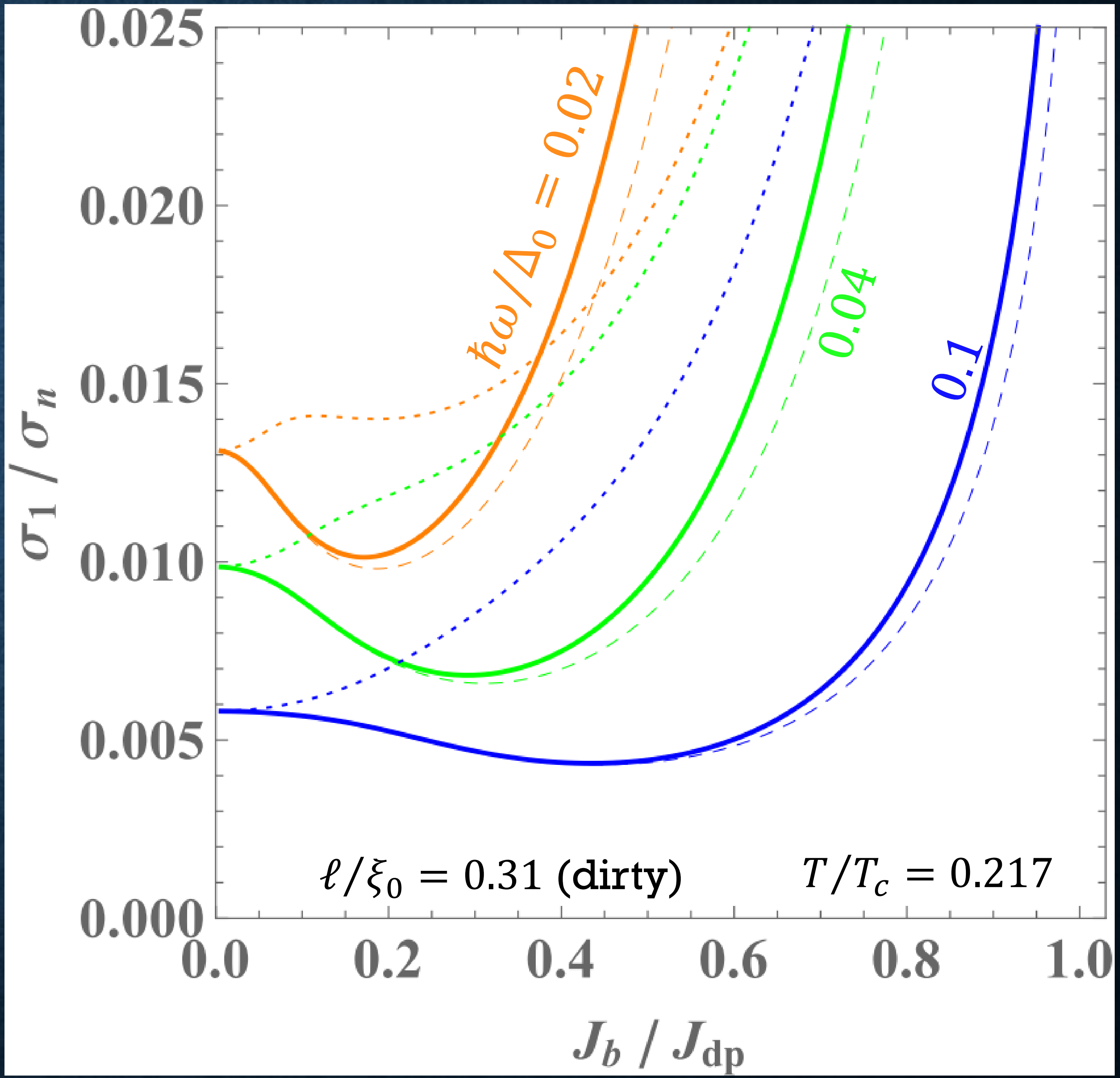
- T. Takenaka, T. Kubo et al., Three-Dimensional Niobium Coaxial Cavity with ~0.1 second Lifetime, arXiv:2510.01819.
  In this work, we applied surface-treatment technologies developed for accelerator cavities and achieved **a world-record quality factor** in this type of quantum-memory cavity for quantum computing applications.

Backup (σ_1)

Analogue $R_s(E)$ curve in ac+dc system

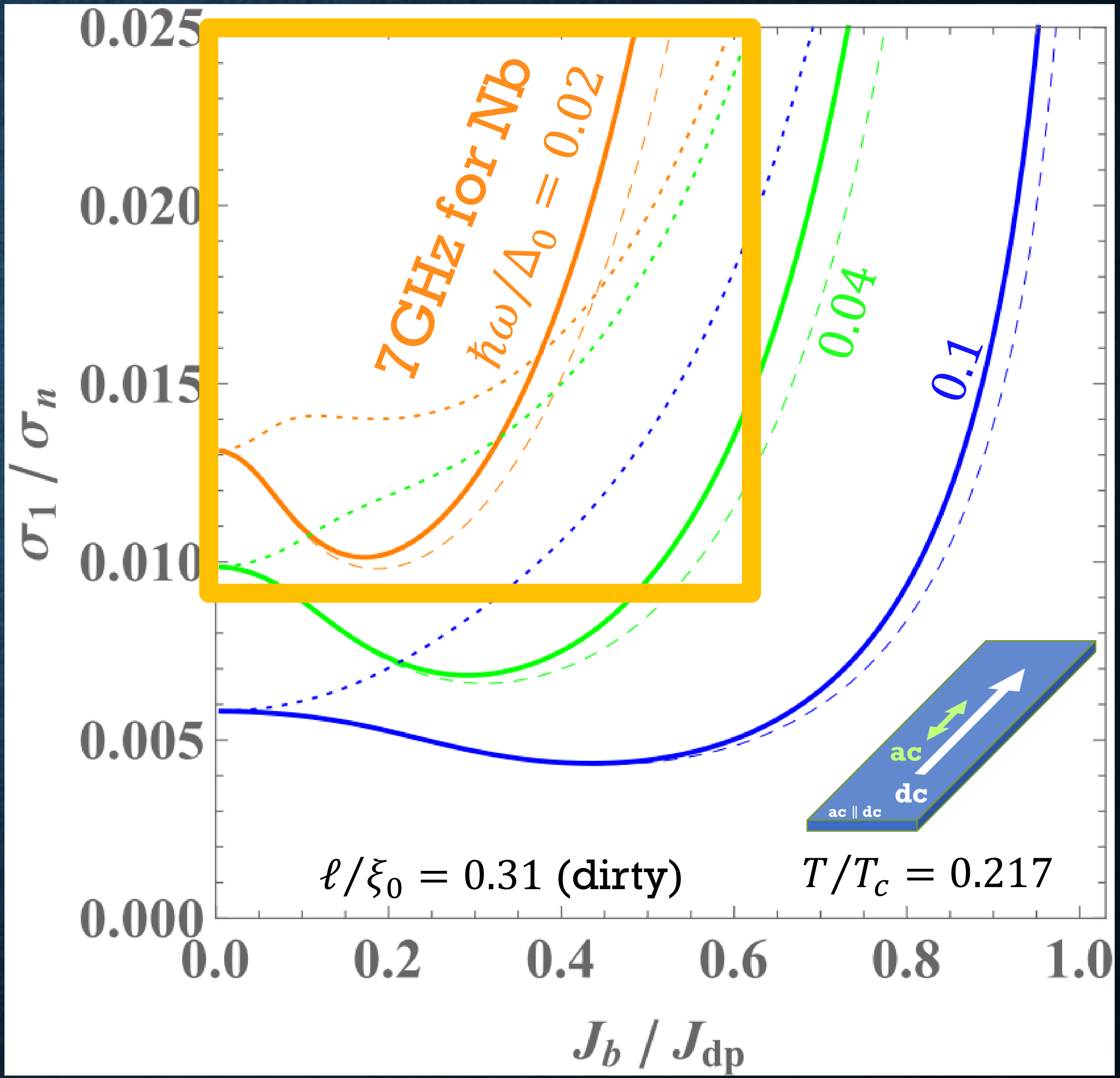


dc: arbitrary strength
ac: perturbation



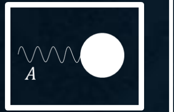
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

Let's see the orange curves ($\hbar\omega/\Delta_0 = 0.02$),which correspond to 7GHz for Nb.

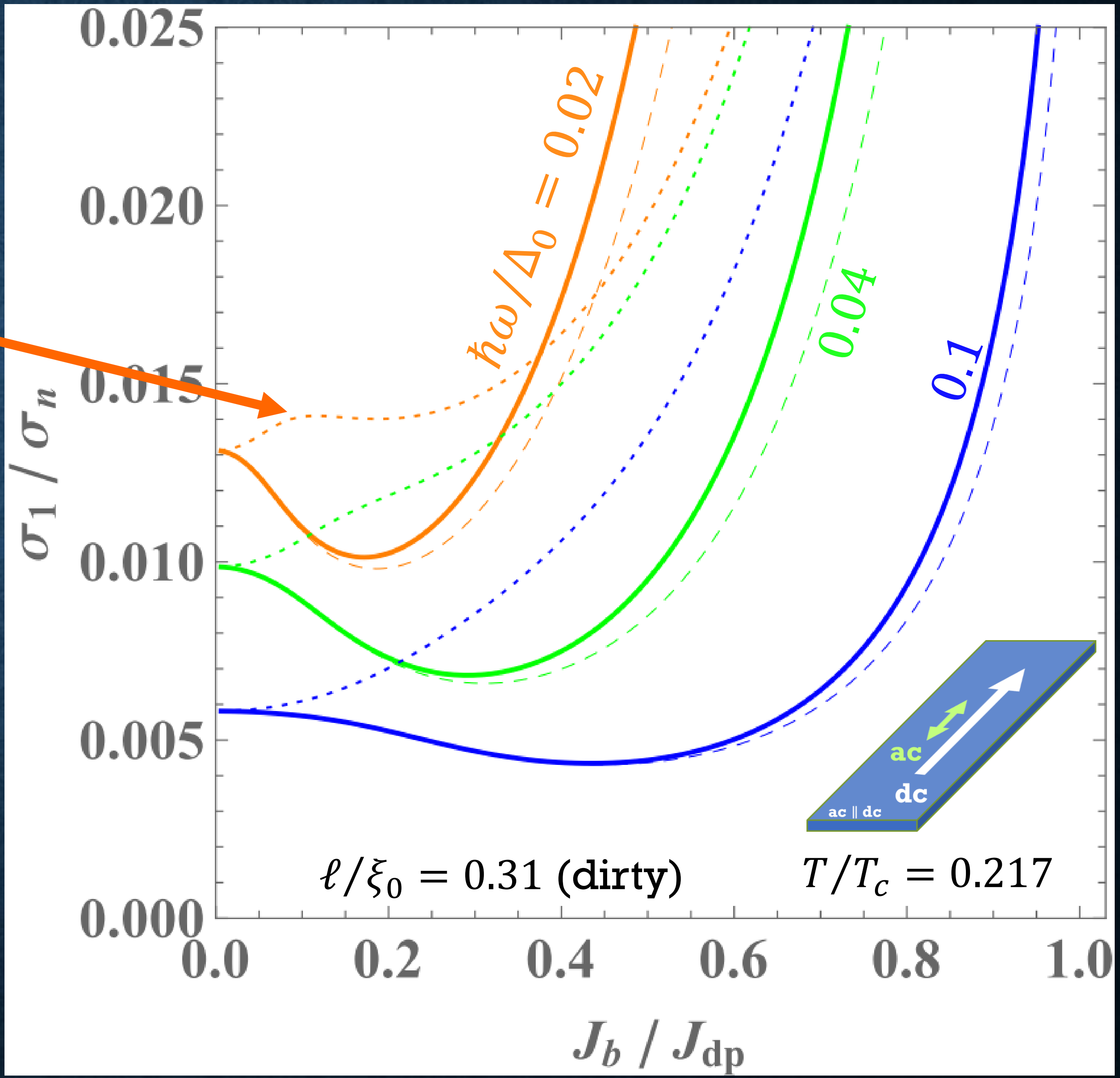


T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

Let's see the orange curves ($\hbar\omega/\Delta_0 = 0.02$),which correspond to 7GHz for Nb.

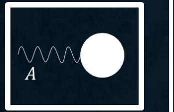


Dotted curve:
$\mathrm{Re}[\sigma^{(0)}]$

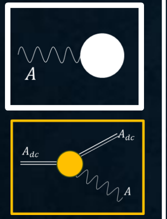


T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

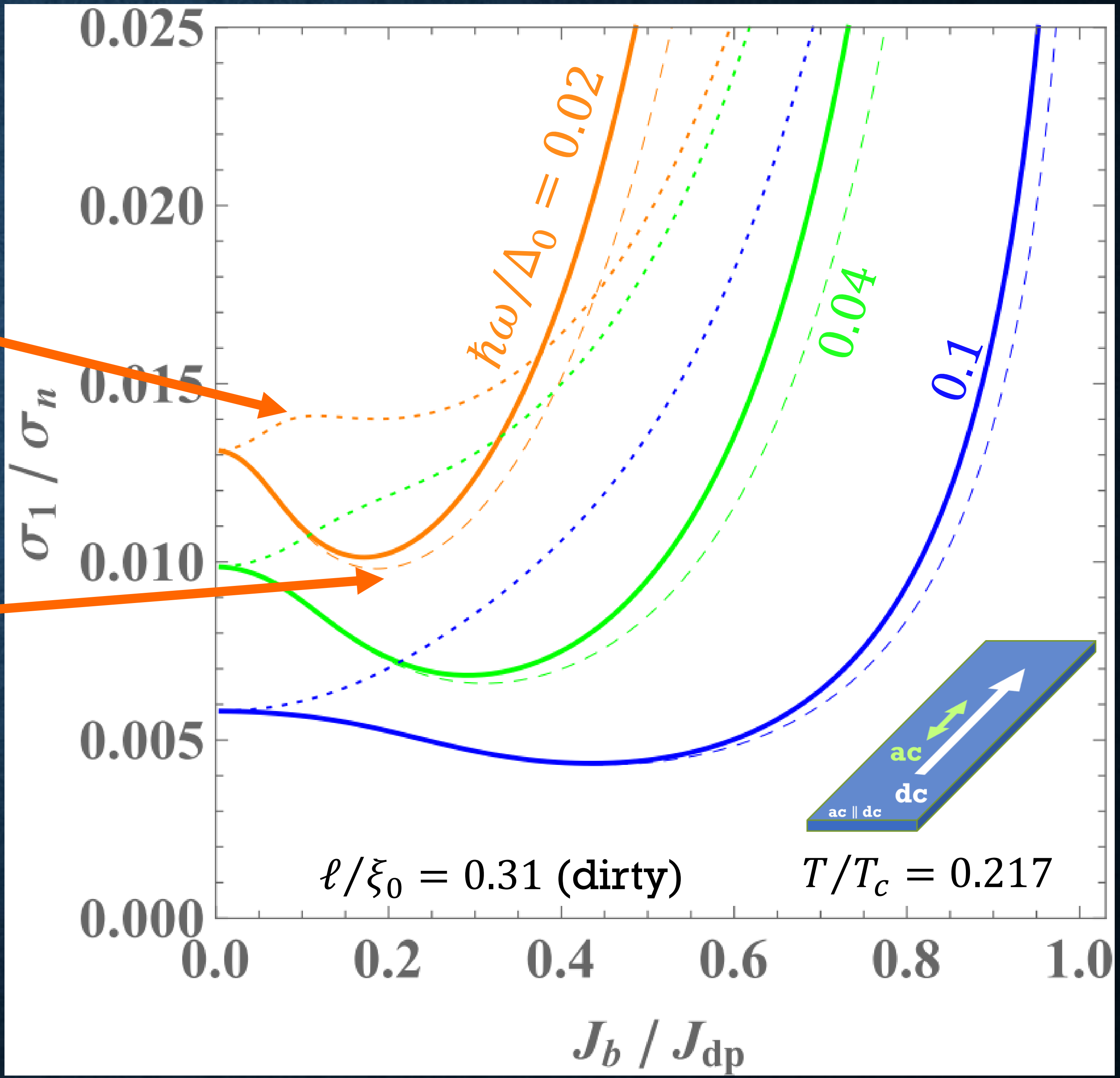
Let's see the orange curves ($\hbar\omega/\Delta_0 = 0.02$),which correspond to 7GHz for Nb.



Dotted curve:
$\mathrm{Re}[\sigma^{(0)}]$



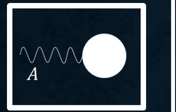
Dashed curve:
$\mathrm{Re}[\sigma^{(0)} + \sigma^{(1)}]$



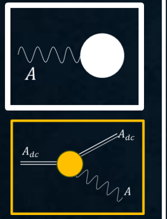
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

Significant contribution from the direct AAA action term $\sigma^{(1)}$

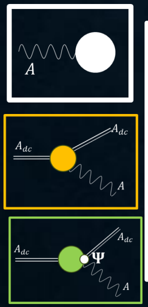
Let's see the orange curves ($\hbar\omega/\Delta_0 = 0.02$),which correspond to 7GHz for Nb.



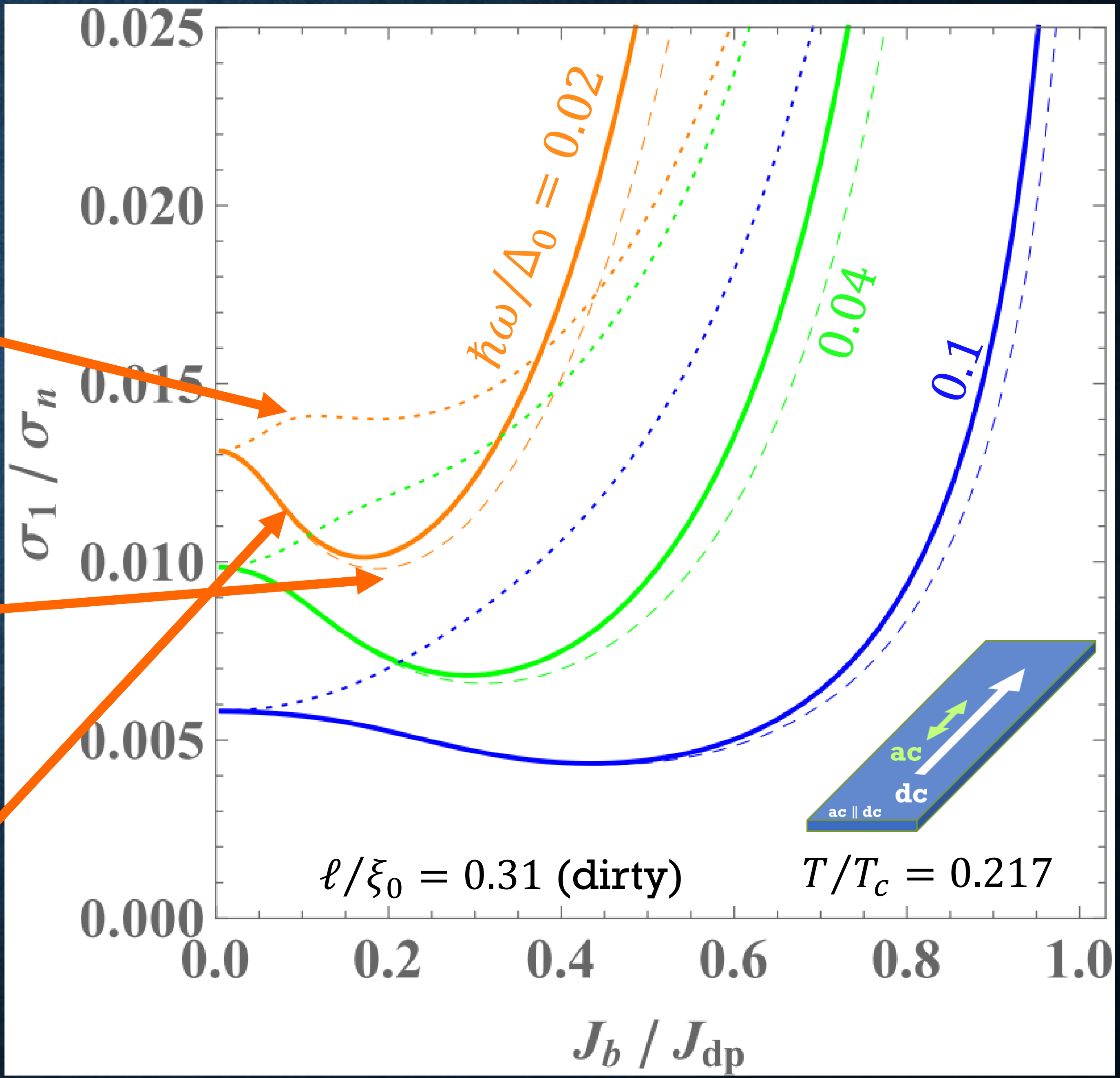
Dotted curve: $\mathrm{Re}[\sigma^{(0)}]$



Dashed curve: $\mathrm{Re}[\sigma^{(0)} + \sigma^{(1)}]$



Solid curve: $\mathrm{Re}[\sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)}]$

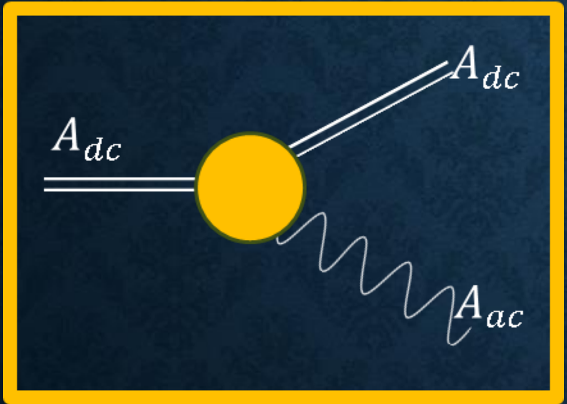


Significant contribution from the direct AAA action term $\sigma^{(1)}$

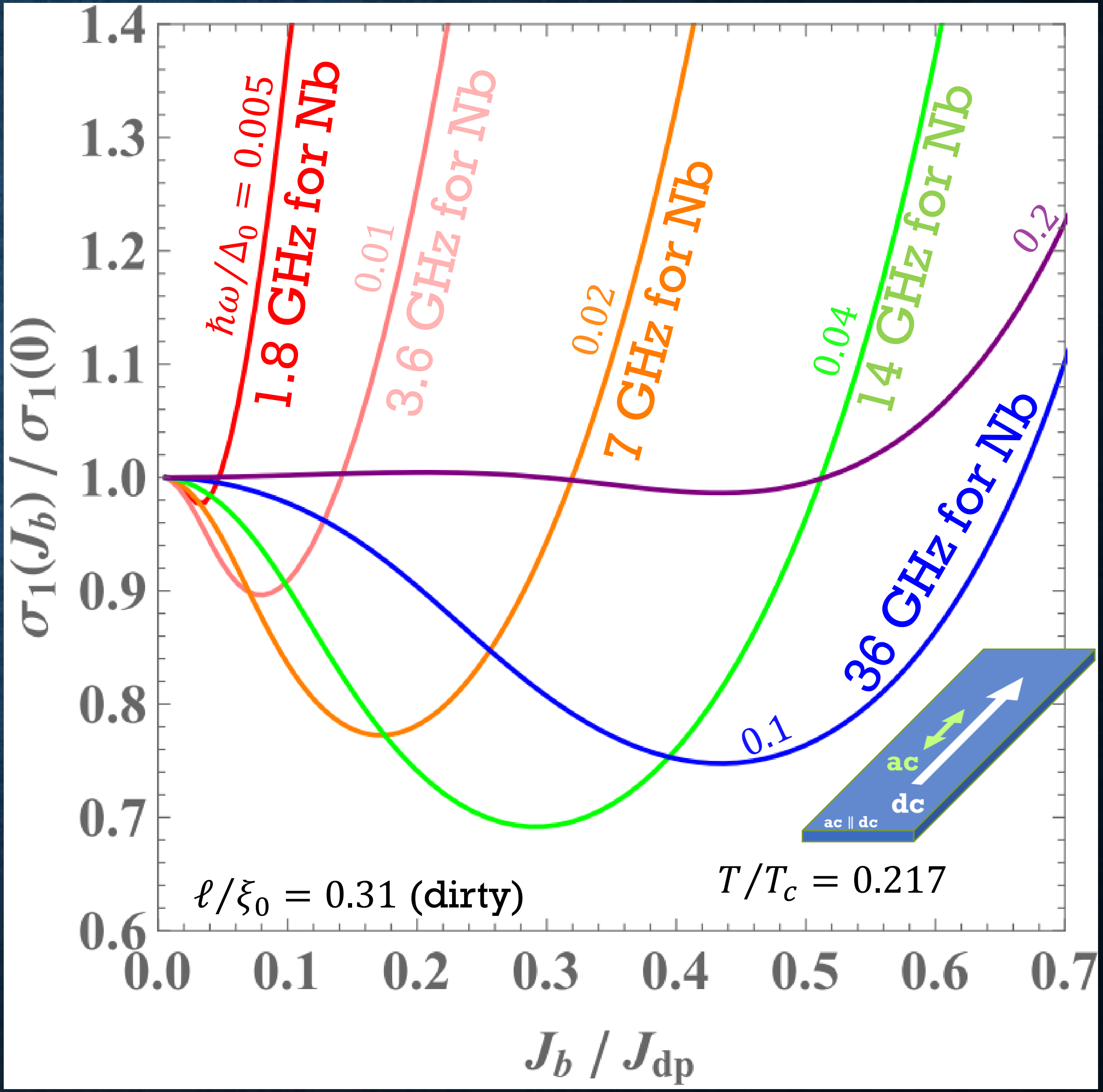
T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

The *analogue anti Q-slope* is pronounced as the frequency increases

The key player is the direct AAA photon action



$$\sigma = \sigma^{(0)} + \boxed{\sigma^{(1)}} + \sigma^{(2)}$$



T. Kubo, Phys. Rev. Applied **22**, 044042 (2024)

Big clue to understand the pronounced anti Q-slope with increasing frequency