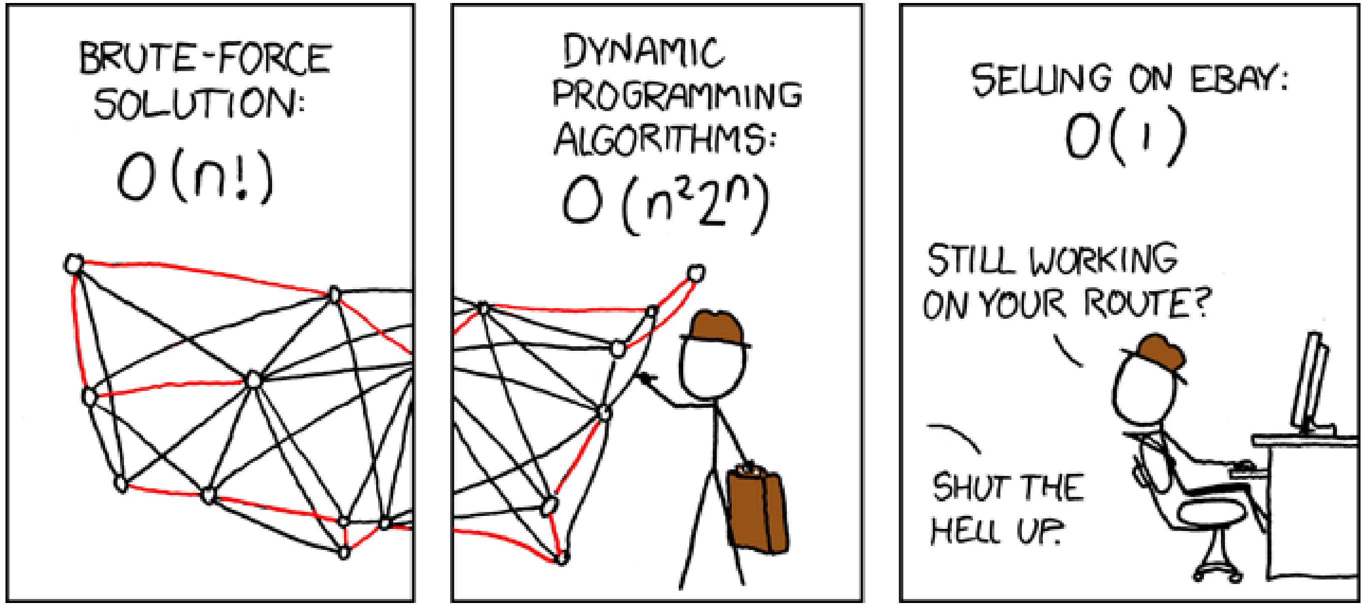
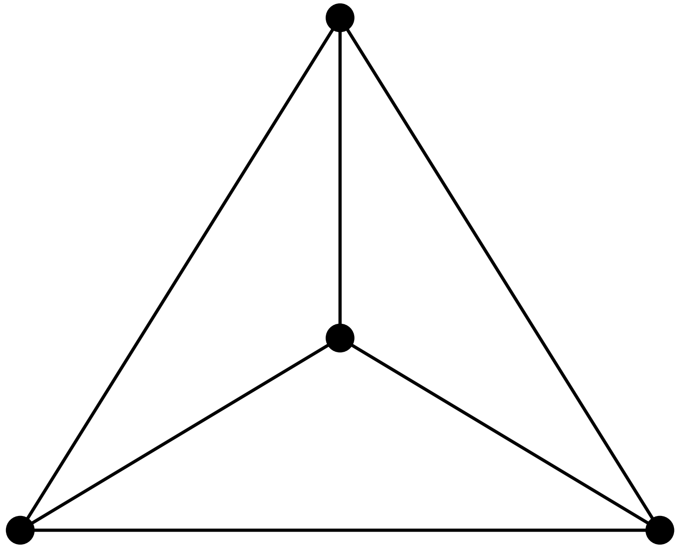
Approximation Algorithms for Steiner Tree, TSP and Multiway Cut



https://xkcd.com/399/

SteinerTree

Given: A graph G



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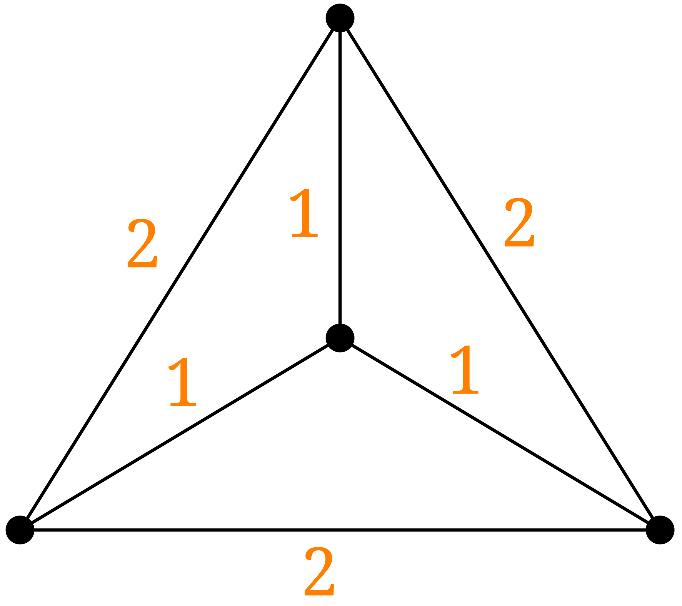
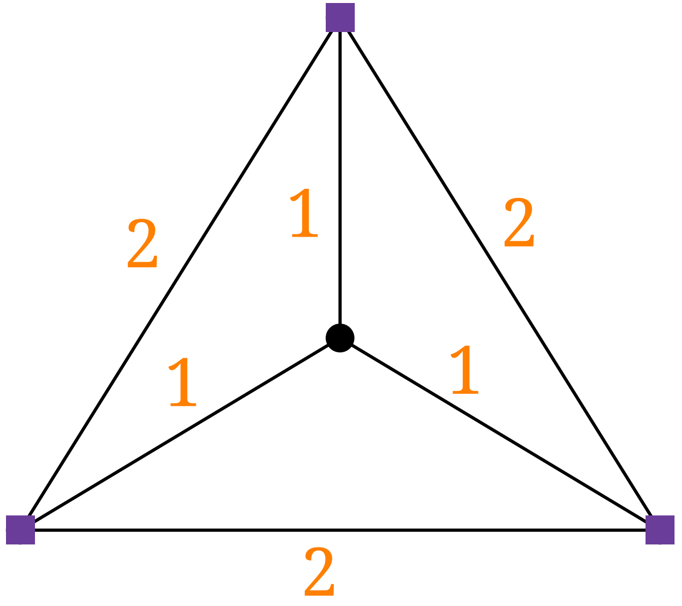
Given: A graph G with edge weights $c\colon E(G) \to \mathbb{Q}^+$



STEINERTREE

Given: A graph G with edge weights $c\colon E(G) \to \mathbb{Q}^+$
and a partition of $V(G)$ into a set T of terminals



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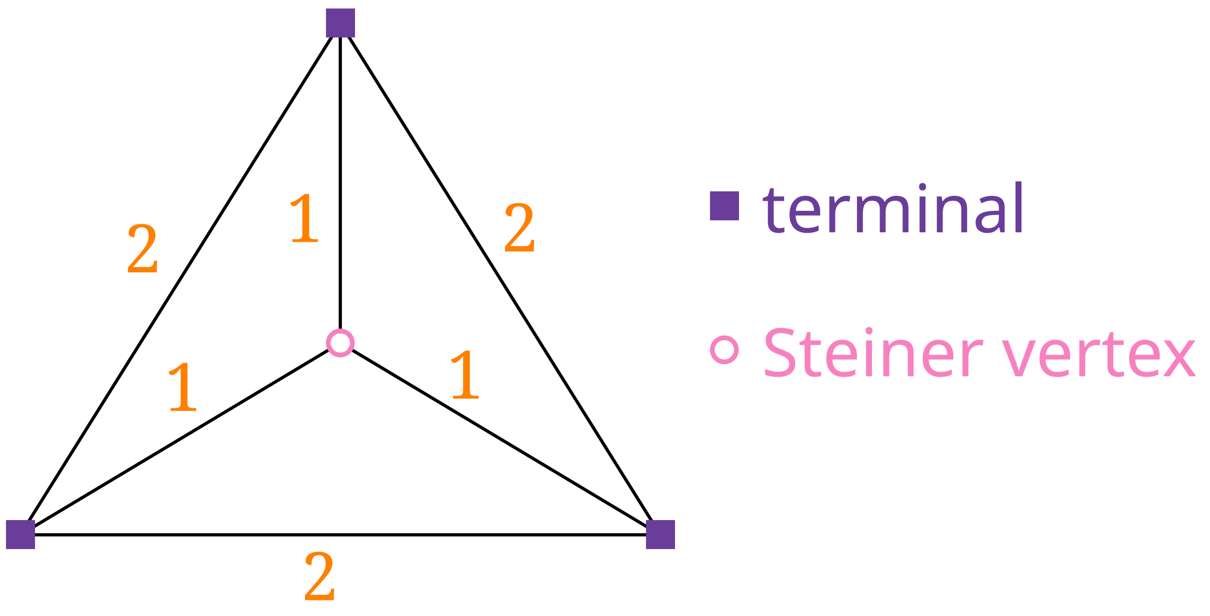

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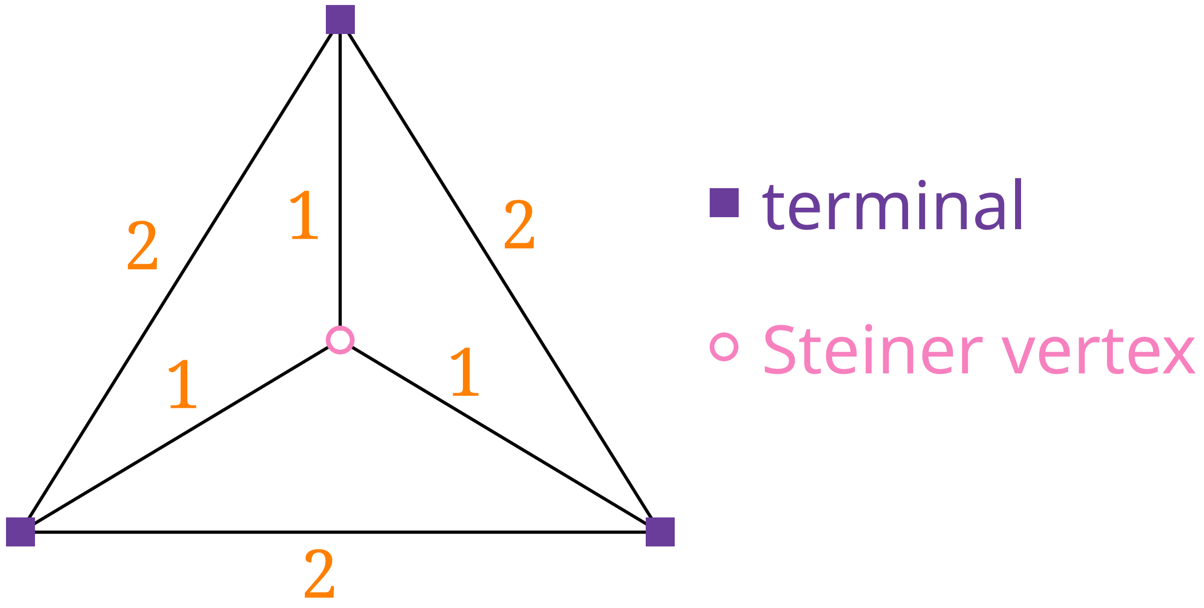

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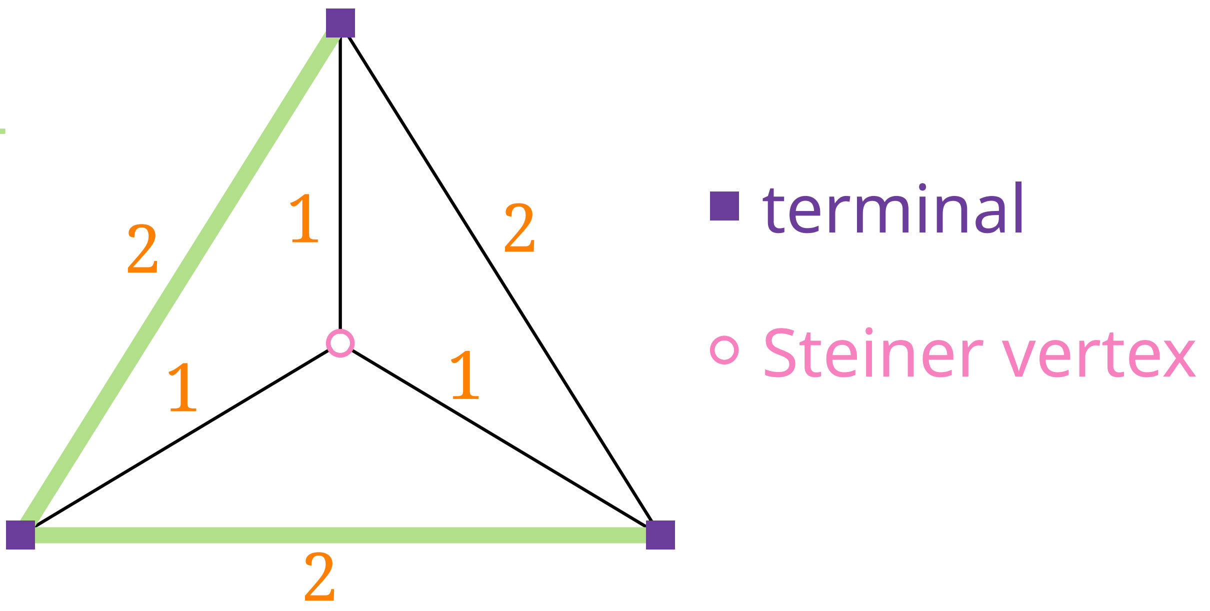
valid solution with cost 4



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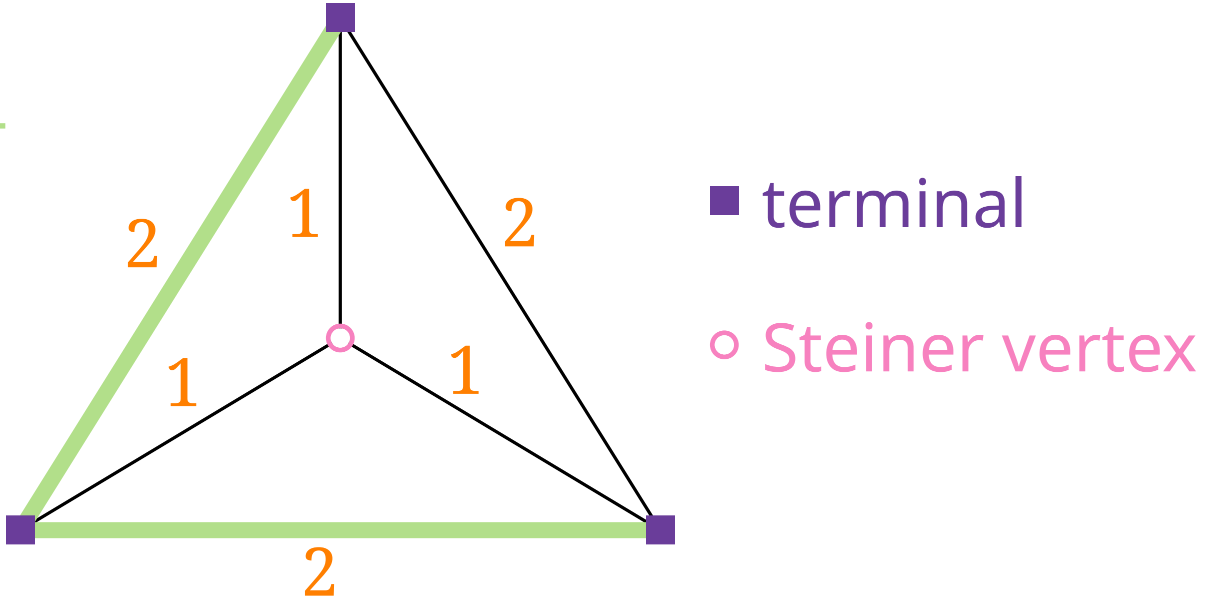

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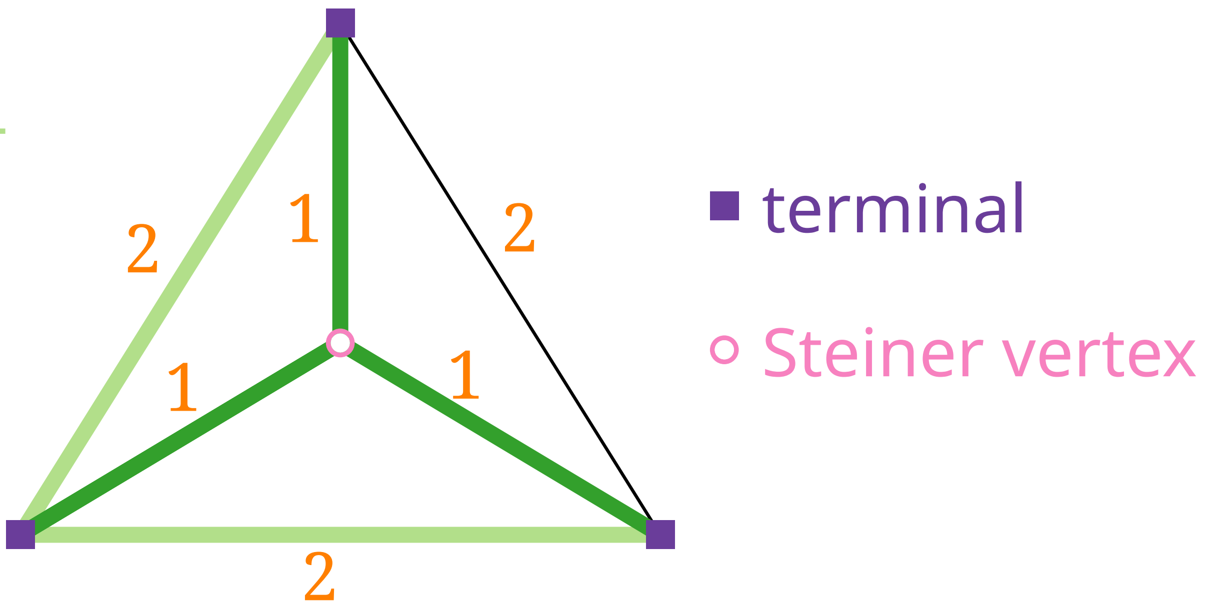

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Special cases:

$T = V$: MST in P

$T = \{s, t\}$: shortestPath in P

valid solution with cost 4

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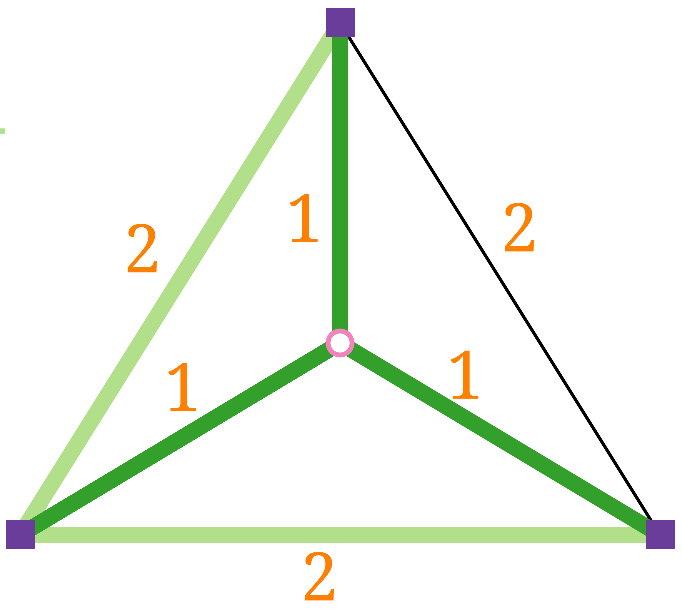

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MetricSteinerTree

Restriction of SteinerTree where the graph G is complete and the cost function is metric,

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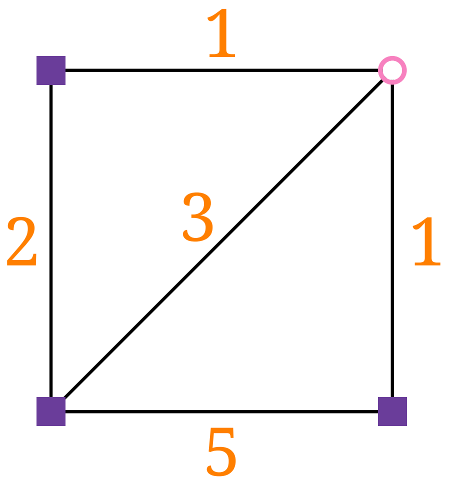

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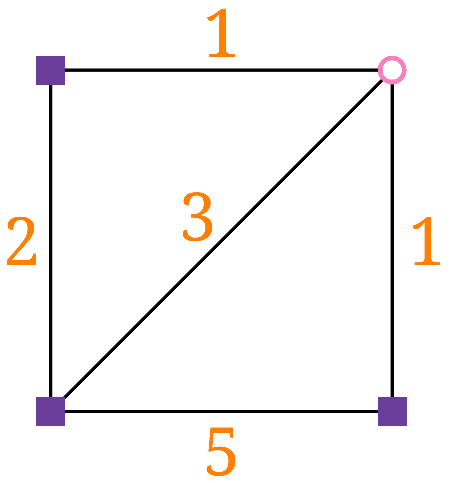

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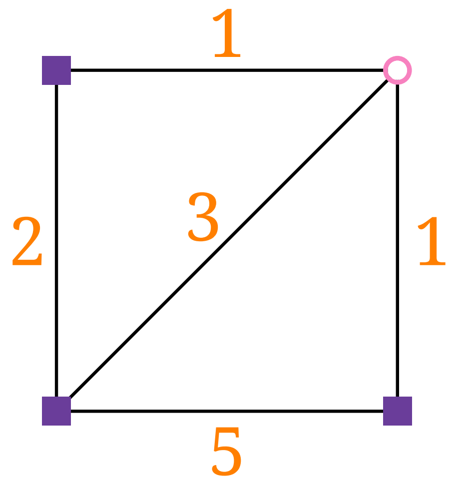

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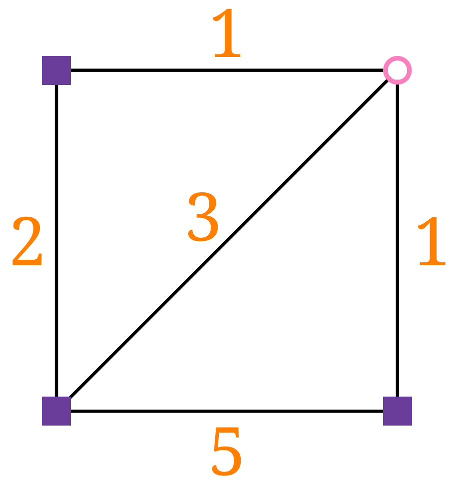

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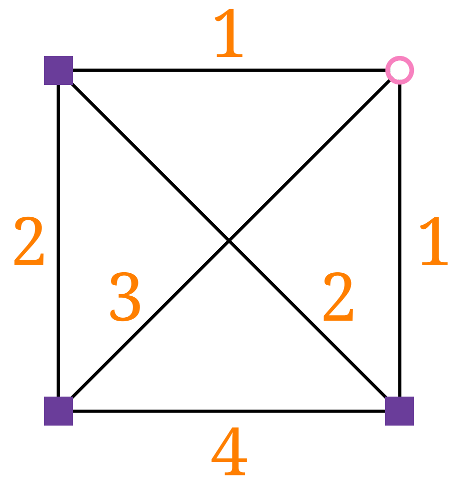

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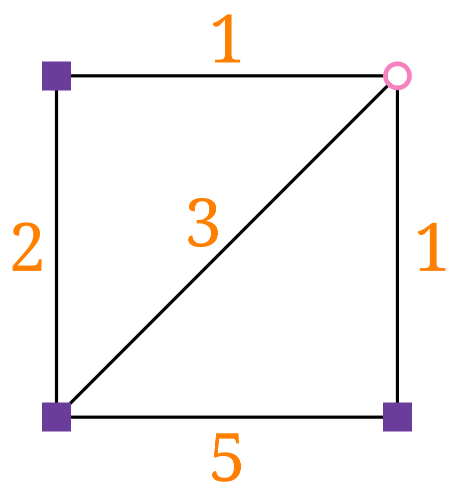

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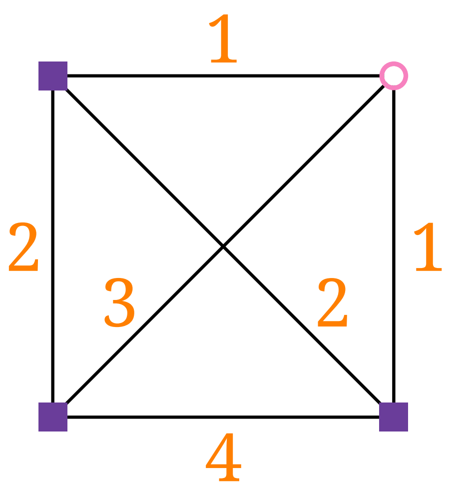

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Approximation-Preserving Reductions

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problems Π_1 Π_2

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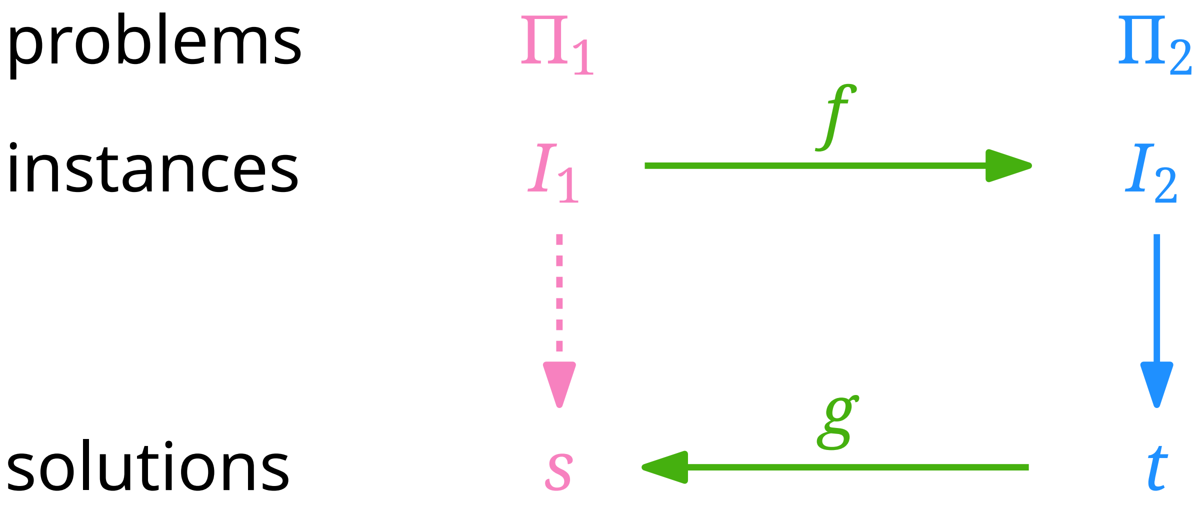

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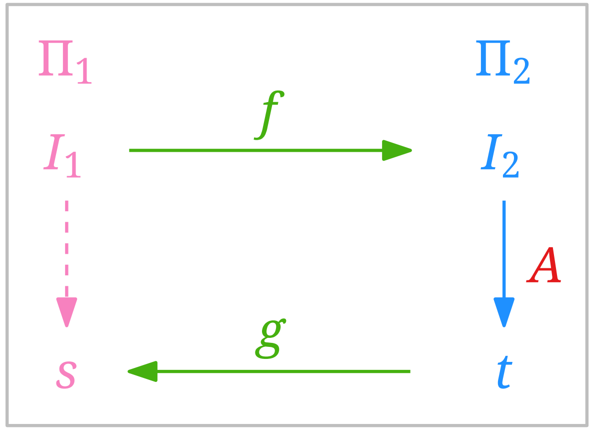

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Reduction to MetricSteinerTree

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Theorem. There is an approximation-preserving reduction from SteinerTree to MetricSteinerTree.

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Proof. (1) Mapping f $\quad I_1 \xrightarrow{f} I_2$

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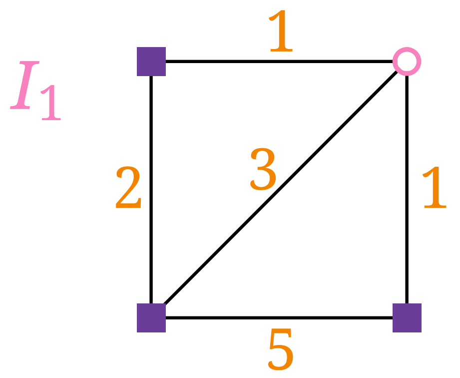

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Metric instance $I_2 := f(I_1)$:

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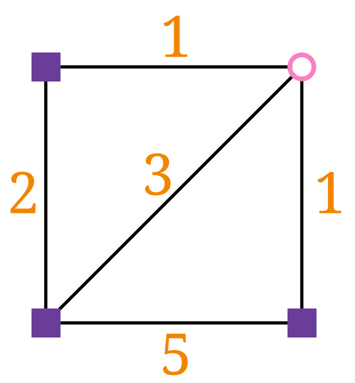
I_1



MetricSteinerTree

Theorem. There is an approximation-preserving reduction from SteinerTree to MetricSteinerTree.

Proof. (1) Mapping f $\quad I_1 \xrightarrow{f} I_2$

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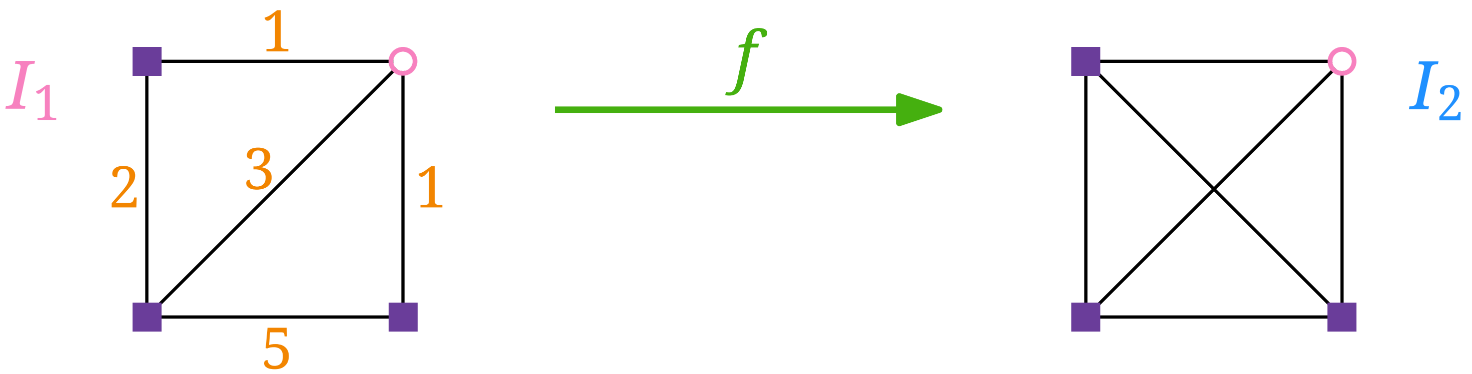

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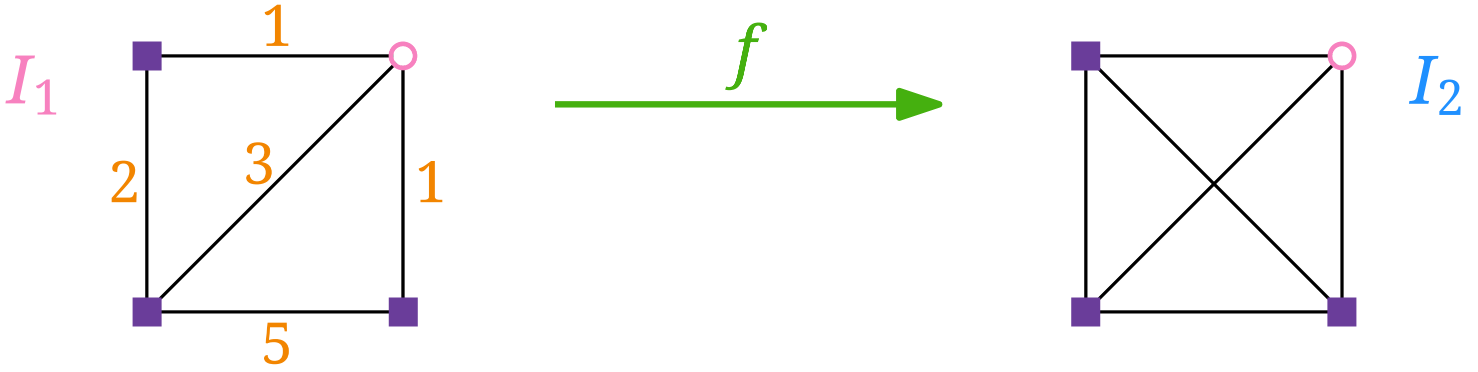
G_2 is the "metric closure" of G_1



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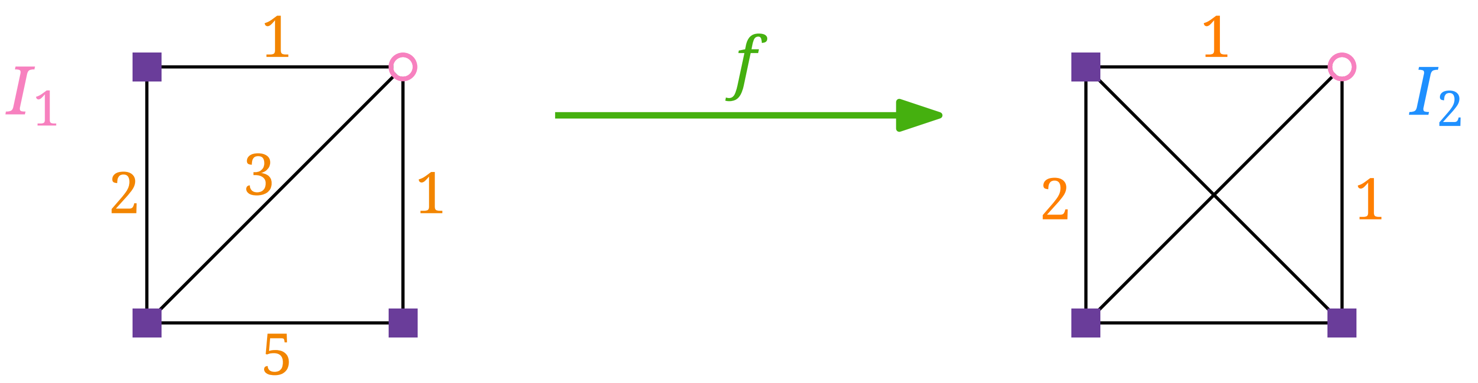

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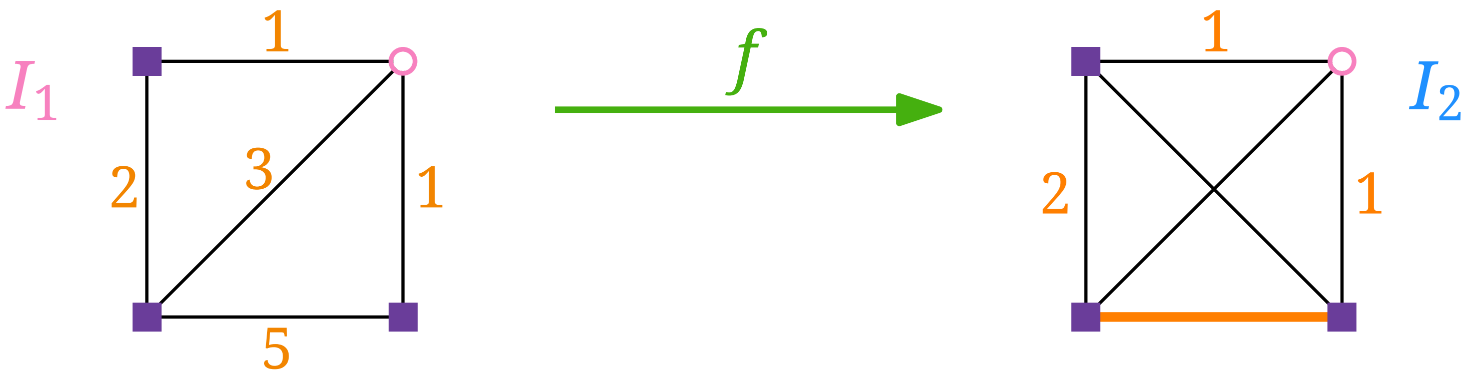

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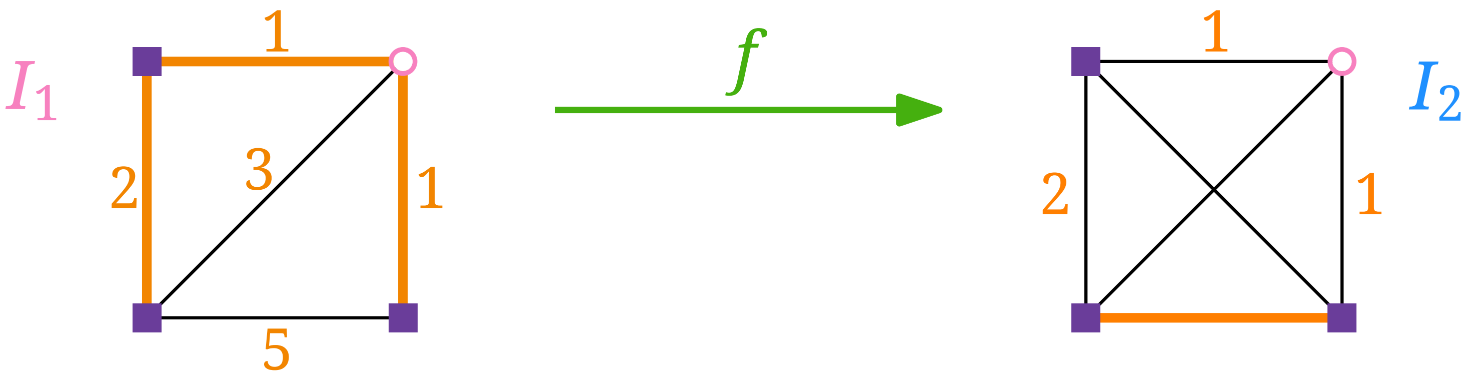

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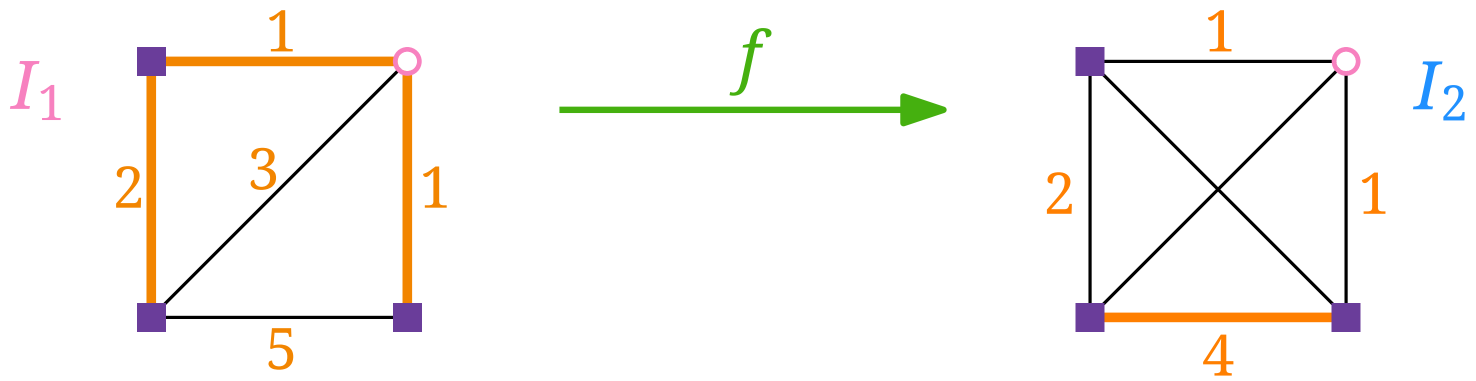

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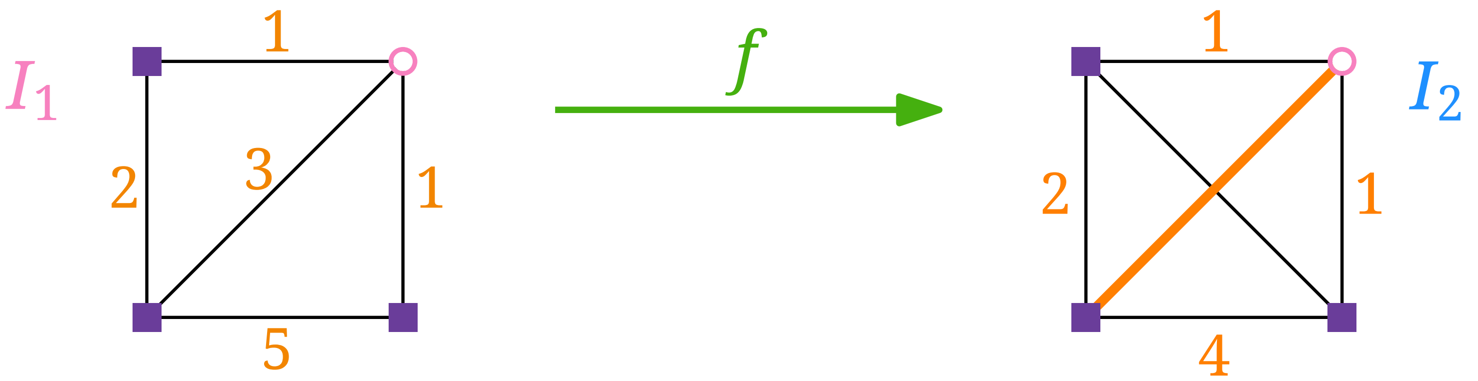

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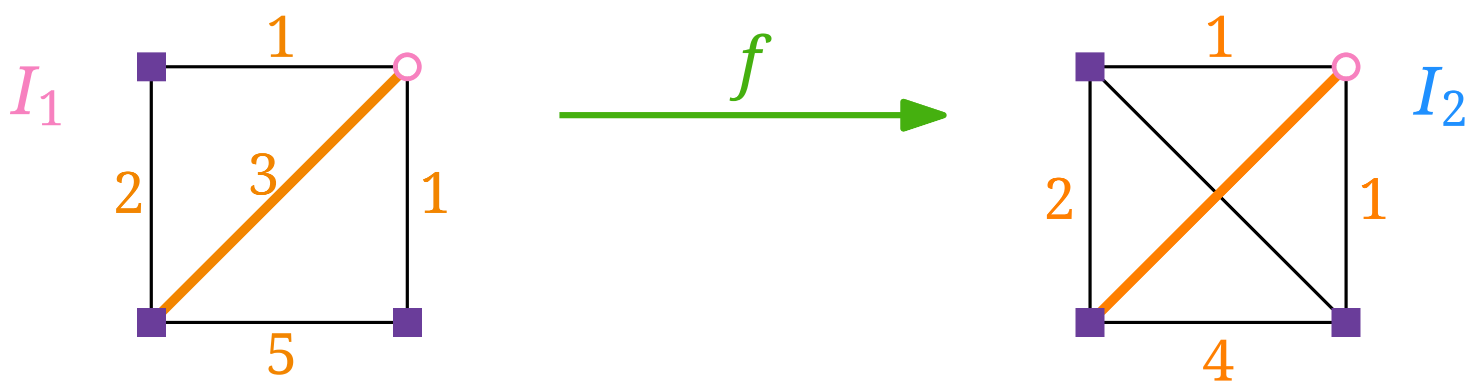

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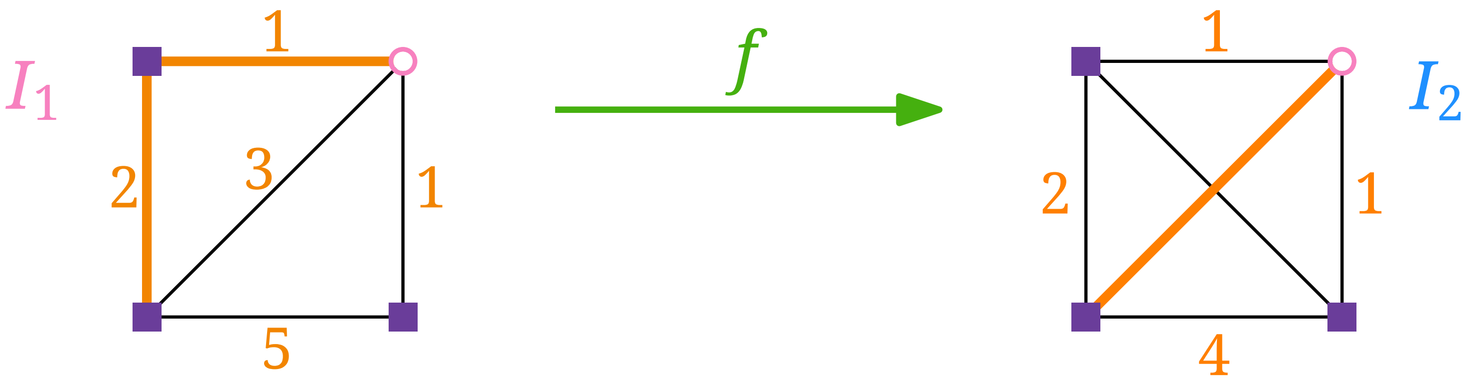

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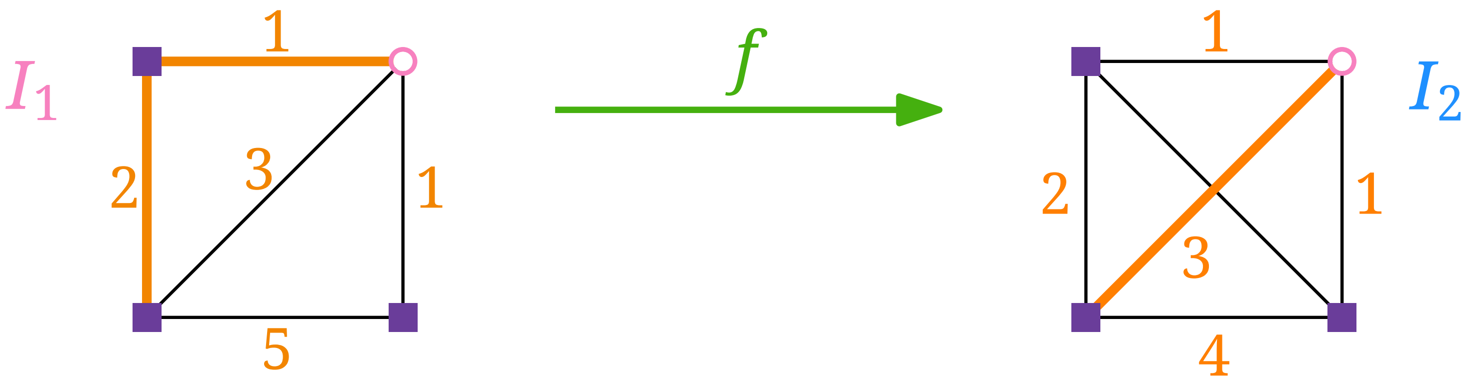

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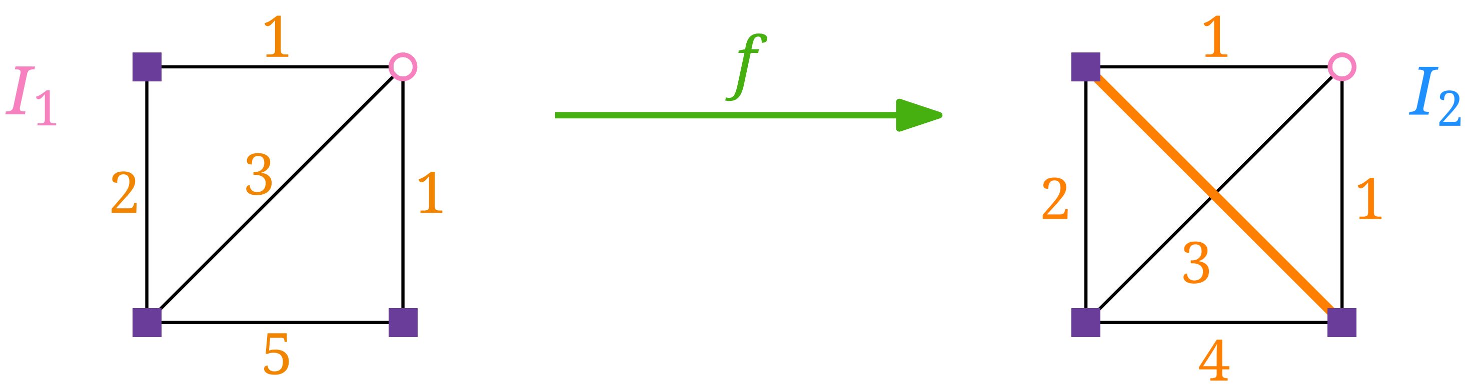

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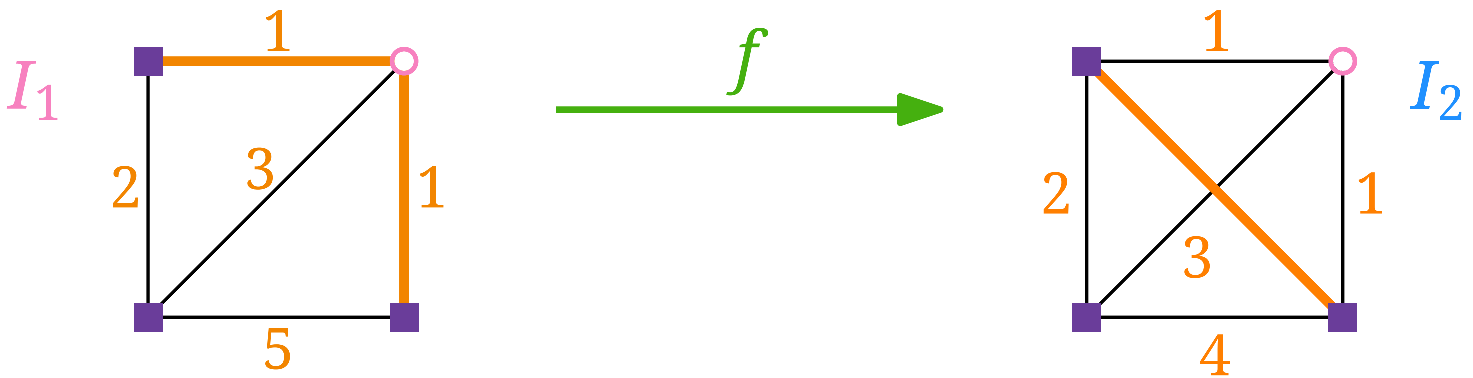

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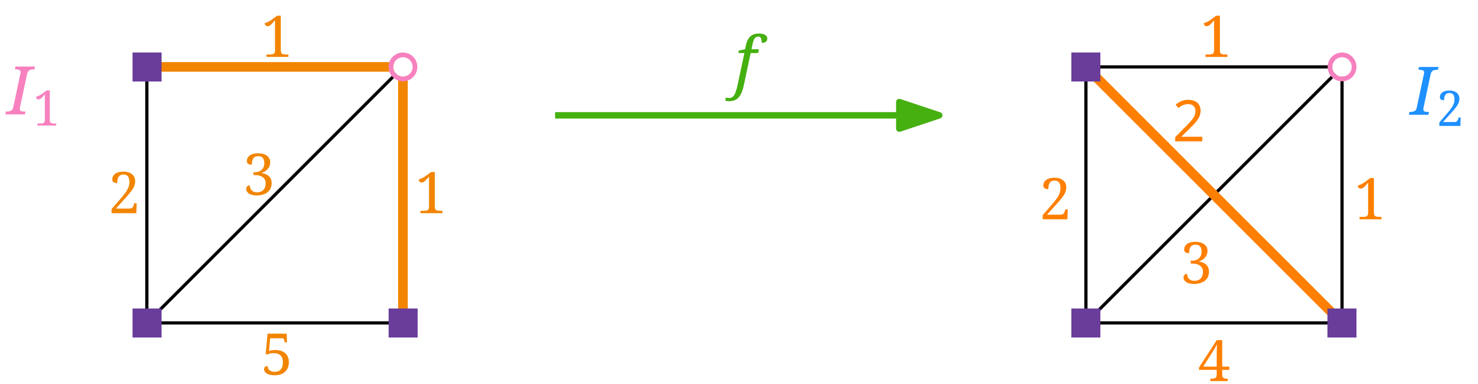

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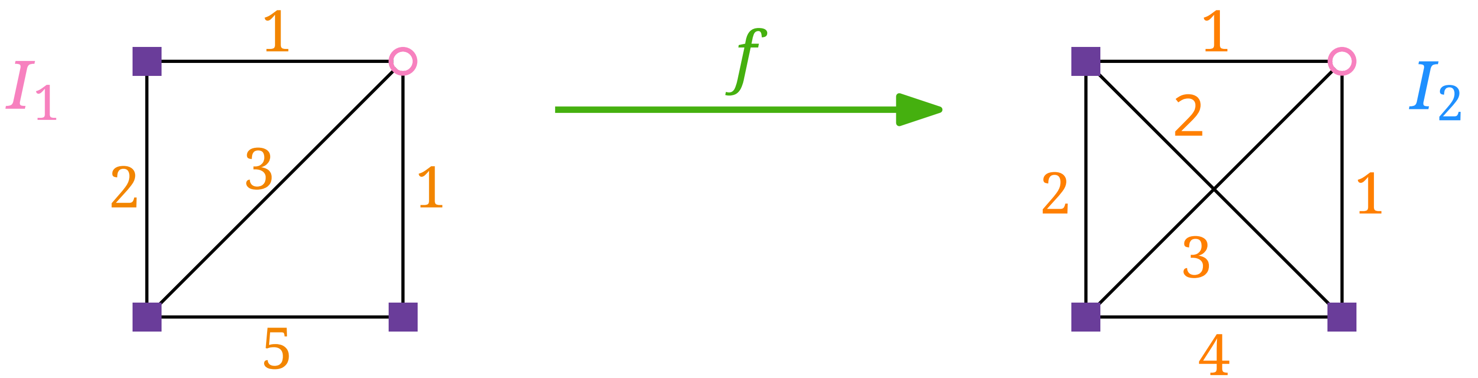

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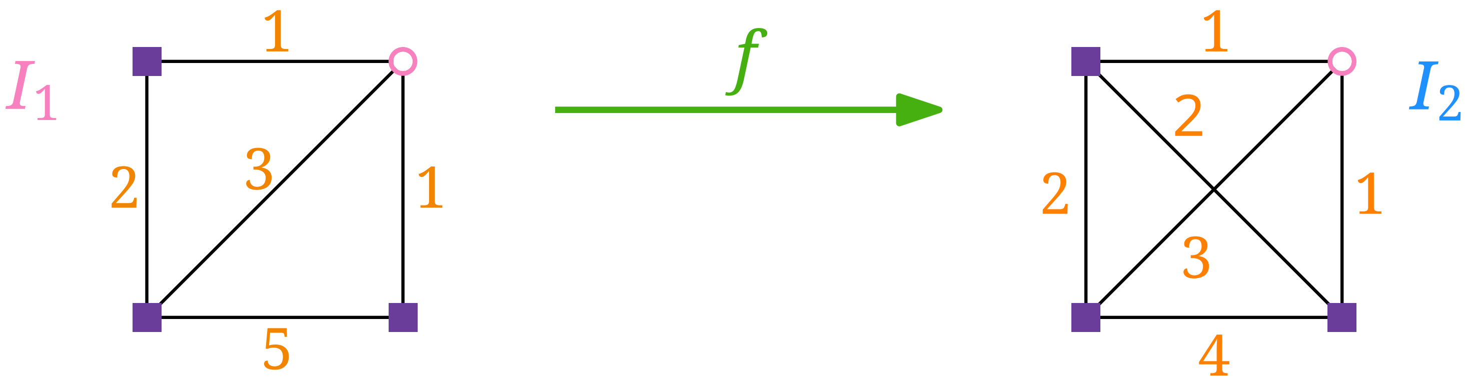
$c_2(u, v) :=$ Length of a shortest u–v path in G_1.

$c_2(u, v) \leq c_1(u, v)$ for every edge $(u, v) \in E_1$.

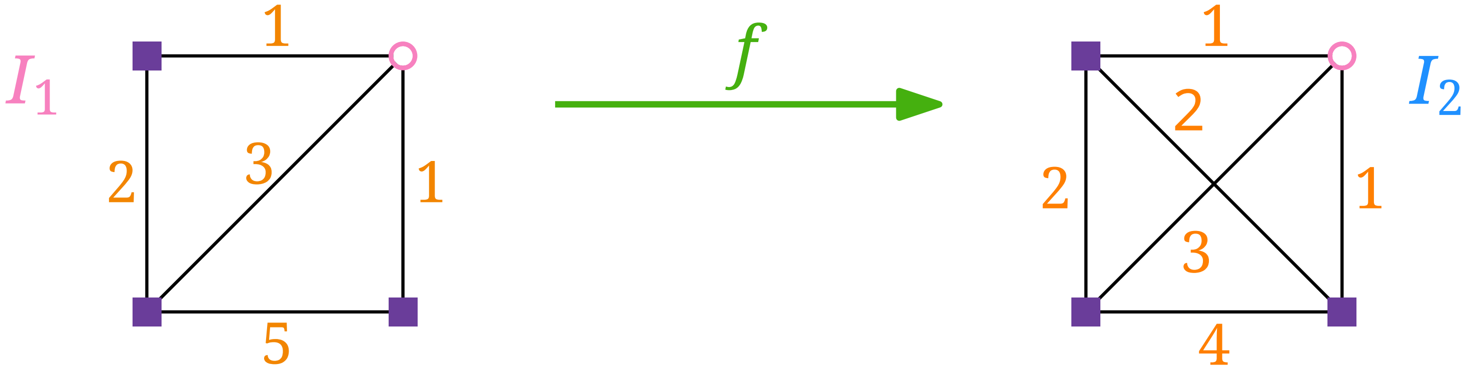
G_2 is the "metric closure" of G_1



MetricSteinerTree

Theorem. There is an approximation-preserving reduction from SteinerTree to MetricSteinerTree.

Proof. $\mathrm{OPT}(I_2) \leq \mathrm{OPT}(I_1)$



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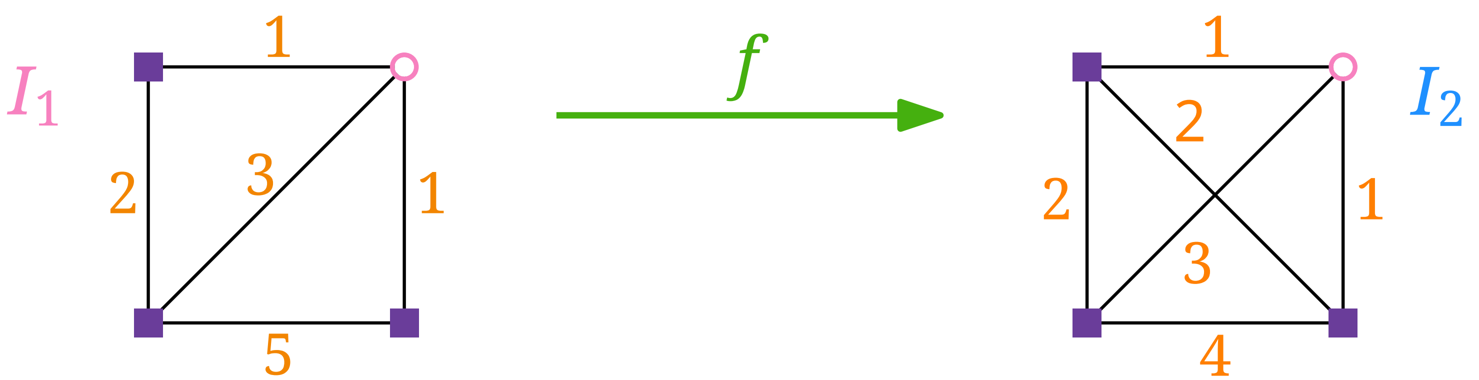
Theorem. There is an approximation-preserving reduction from STEINERTREE to METRICSTEINERTREE.

Proof. $\mathrm{OPT}(I_2) \leq \mathrm{OPT}(I_1)$

Let B^* be an optimal Steiner tree for I_1.



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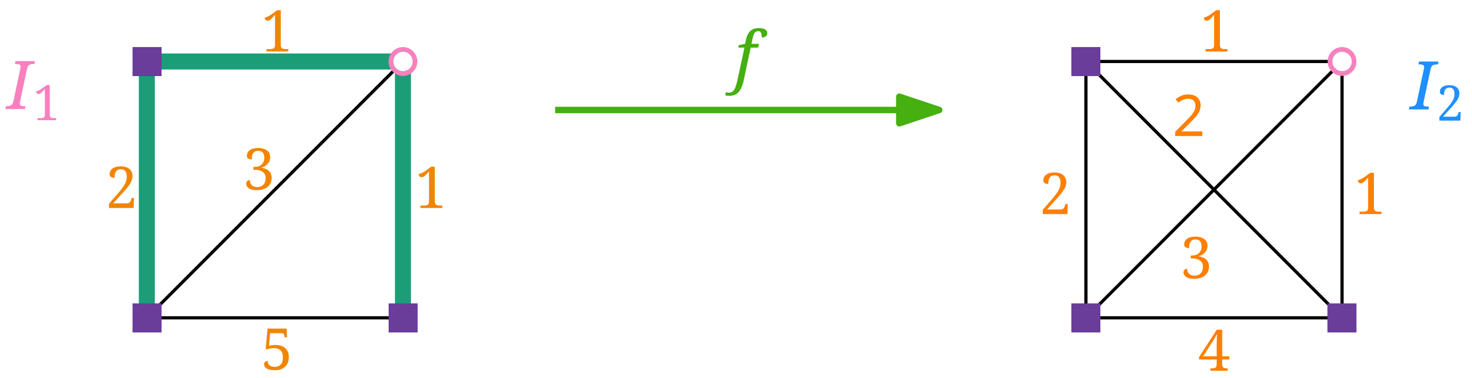

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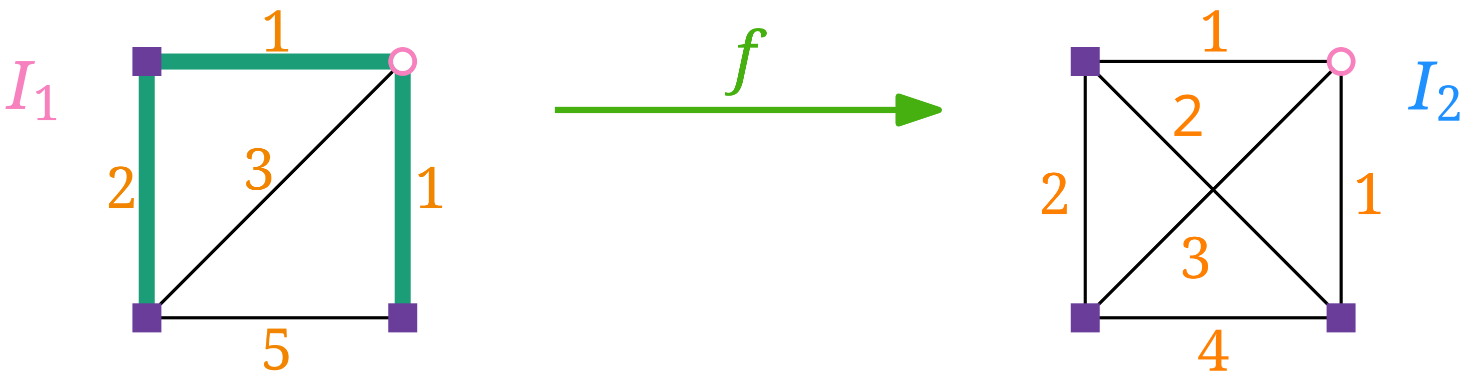
Let B^* be an optimal Steiner tree for I_1.

Note that B^* is also a feasible solution for I_2:

$E_1 \subseteq E_2$ and the vertex sets V, T, S are the same.



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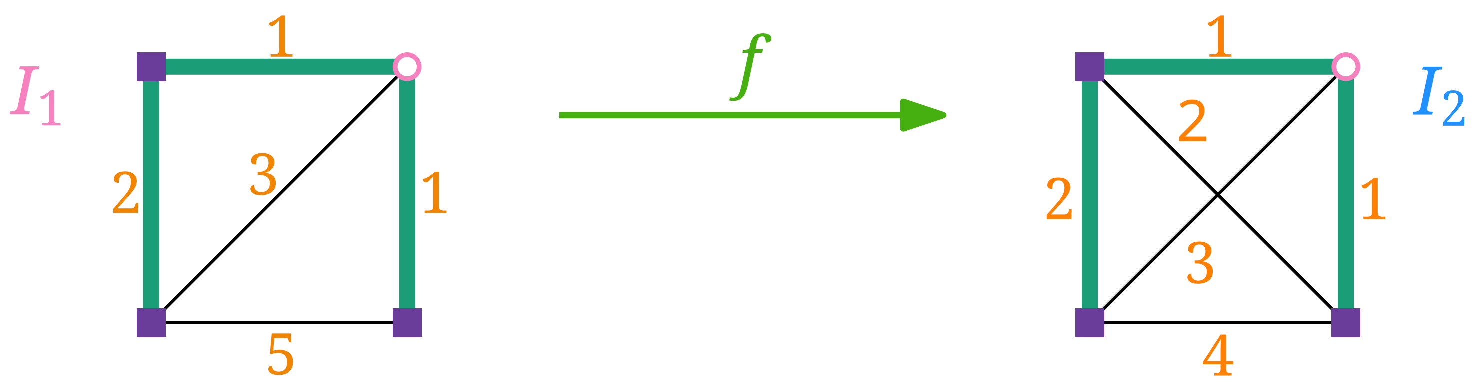

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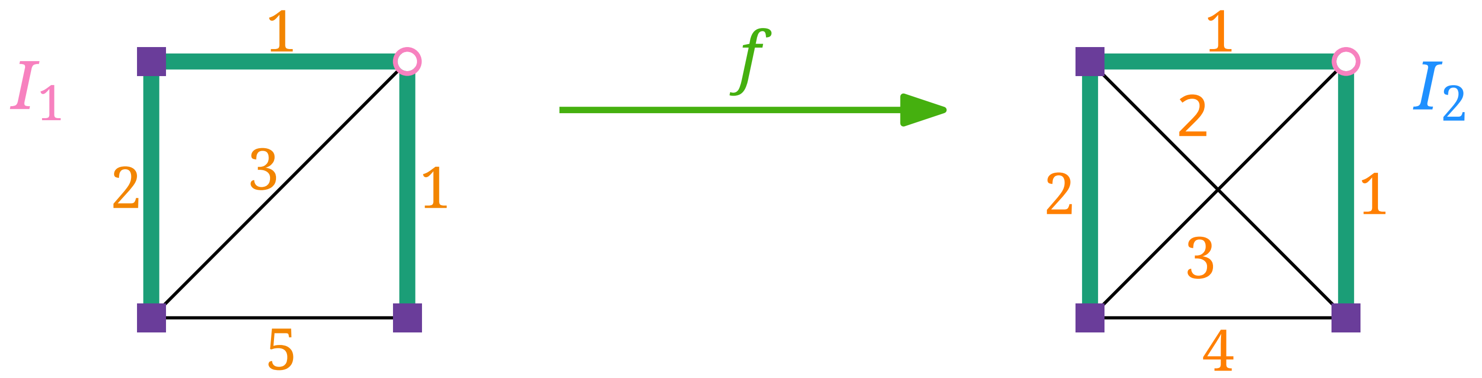
$\mathrm{OPT}(I_2)$



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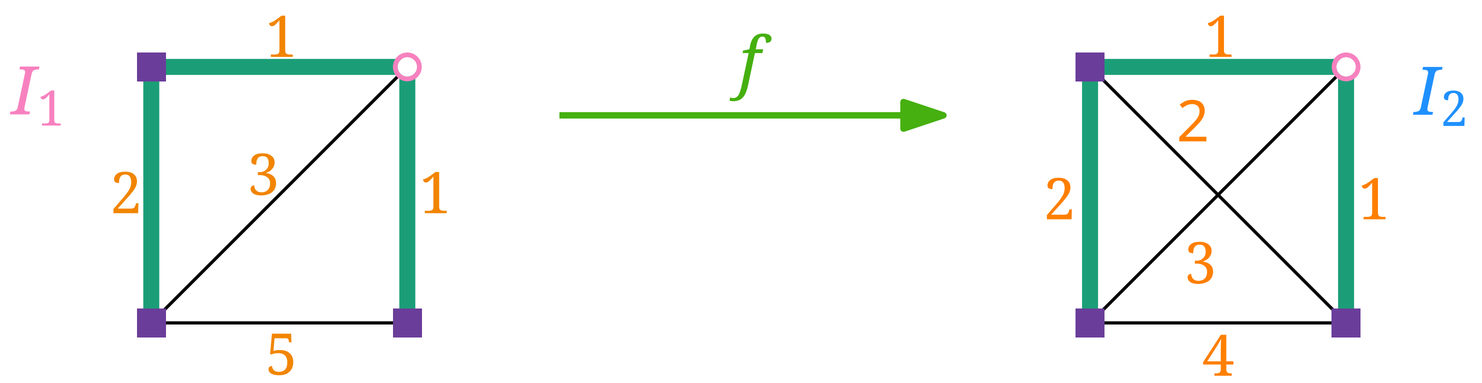

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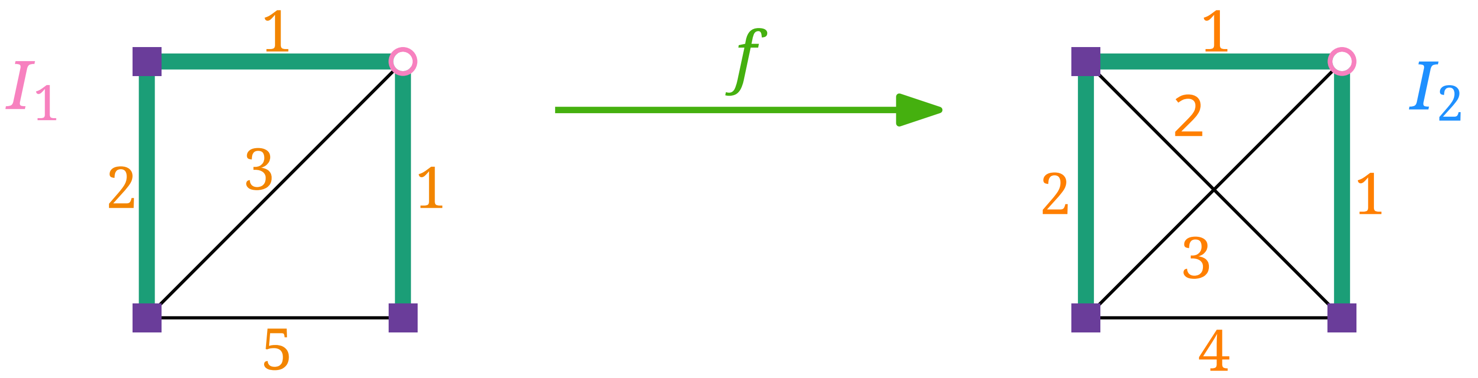

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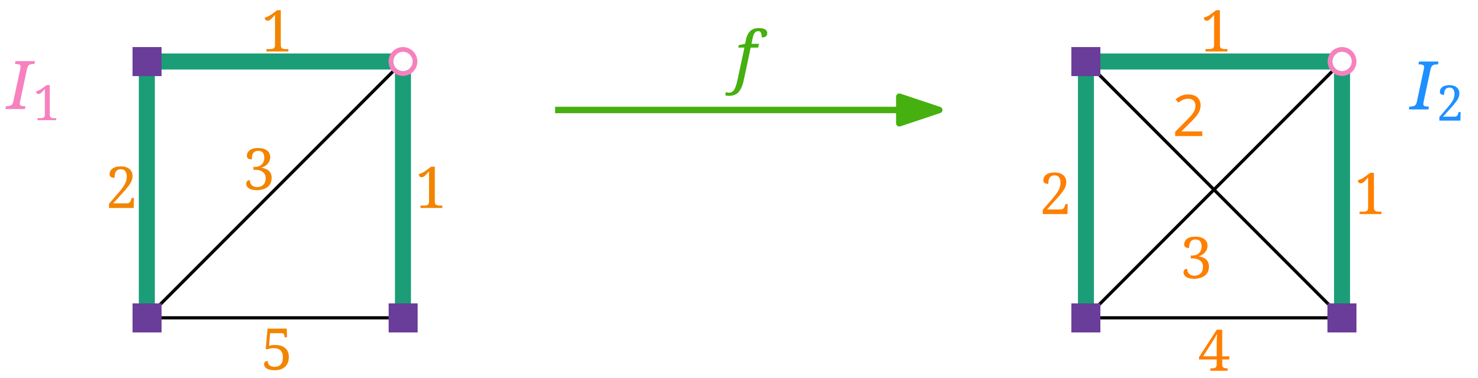

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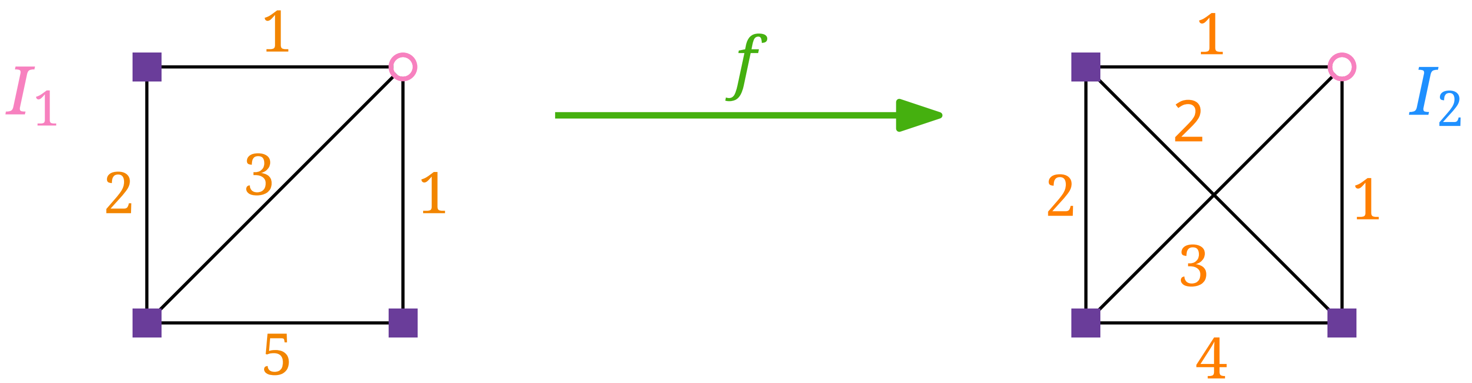
Proof. (2) Mapping g $\quad s \xleftarrow{g} t$



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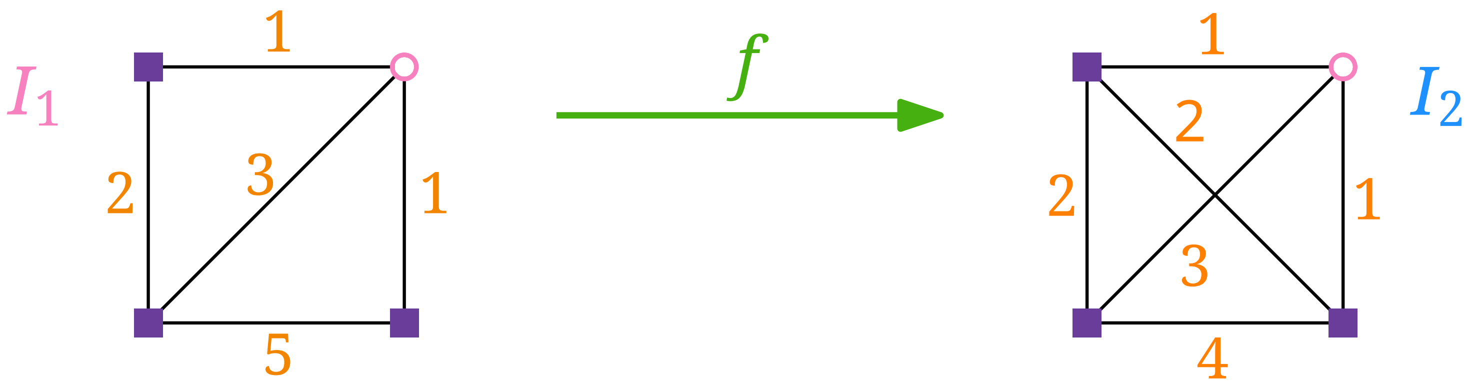

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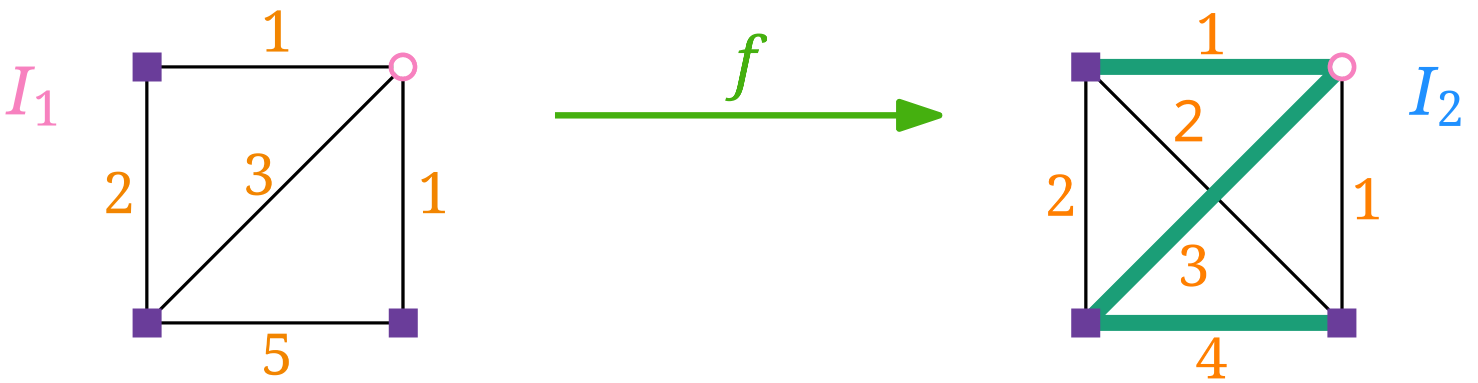

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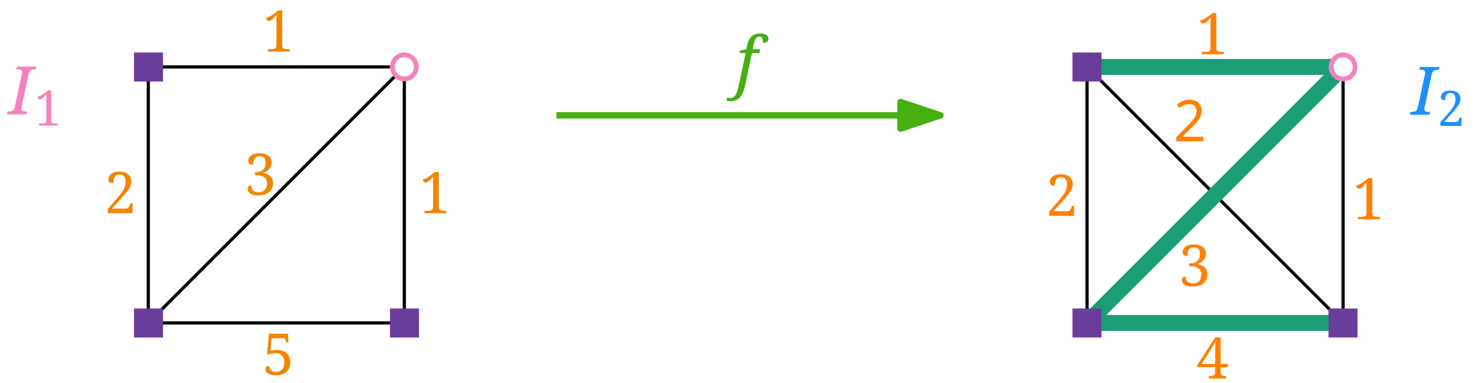

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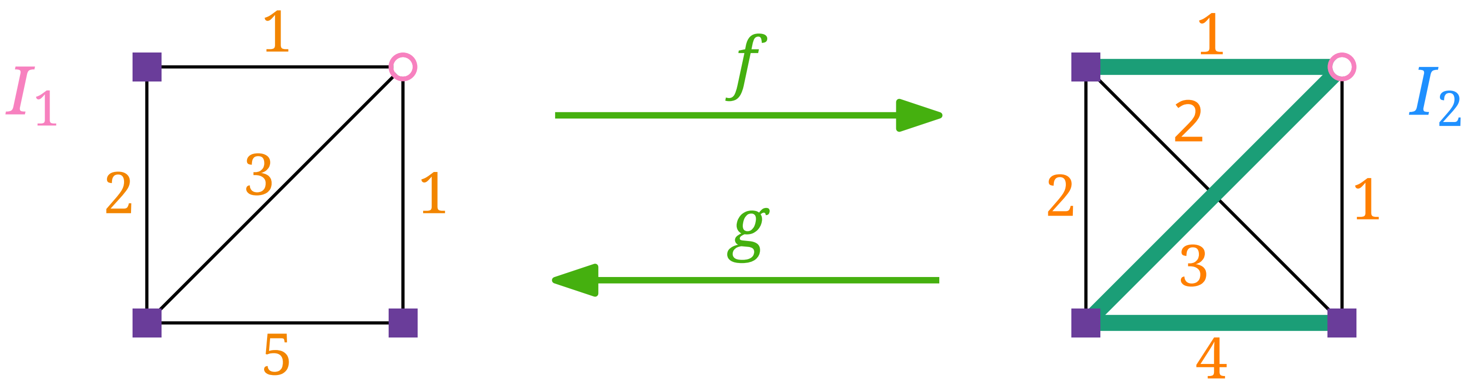

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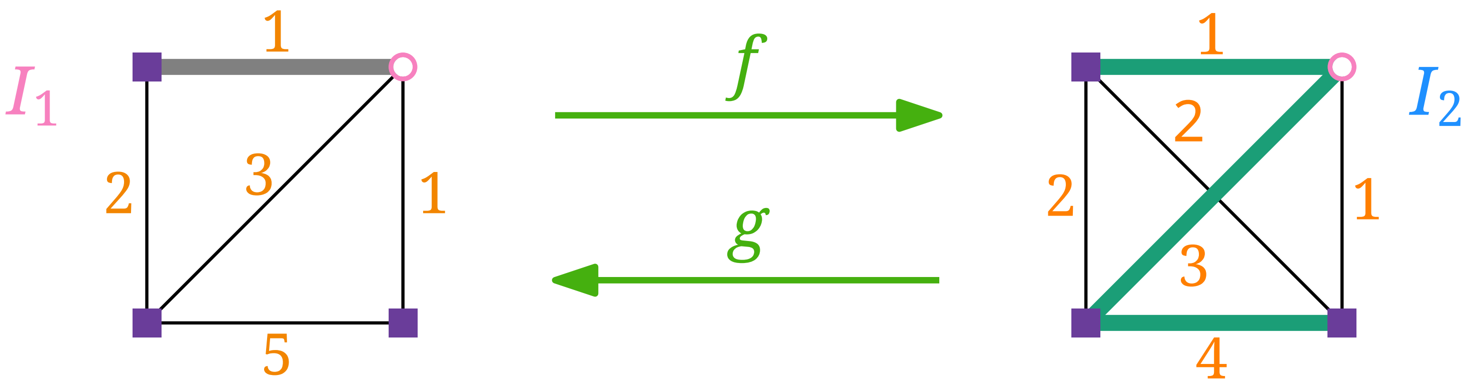

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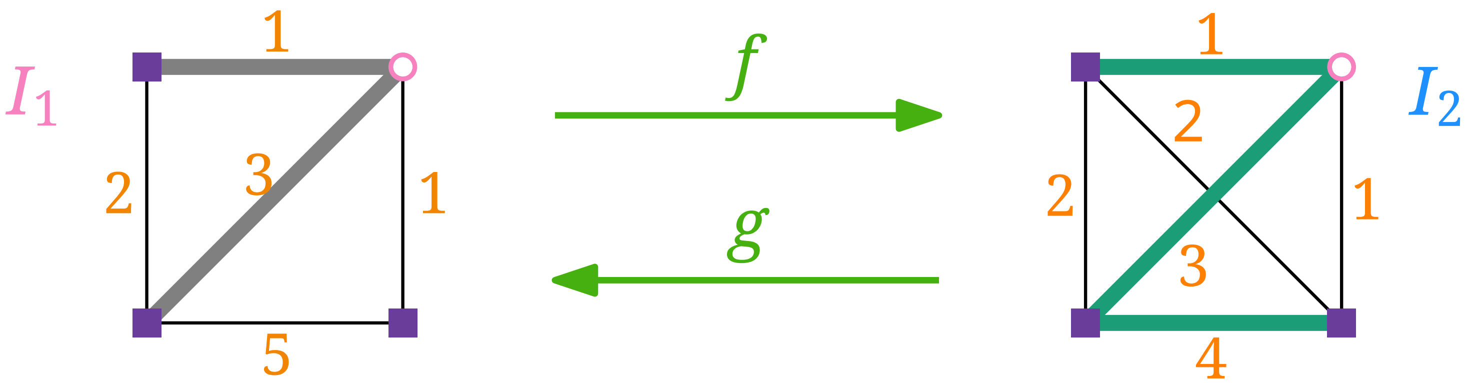

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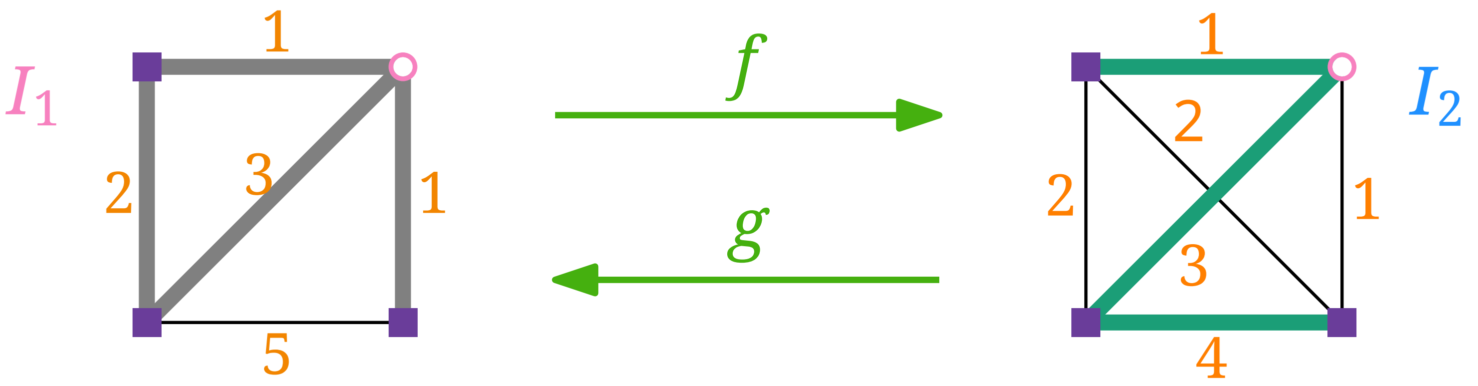

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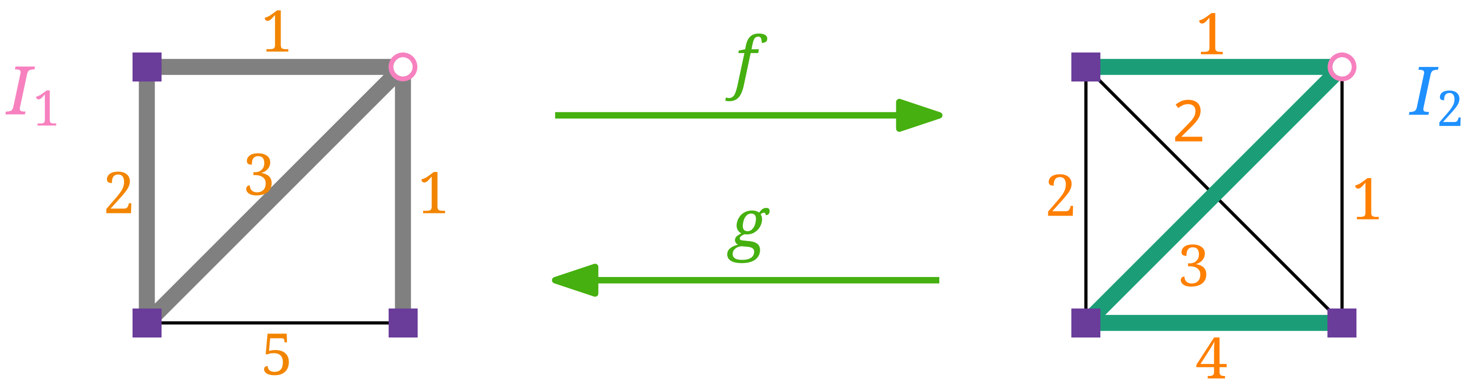

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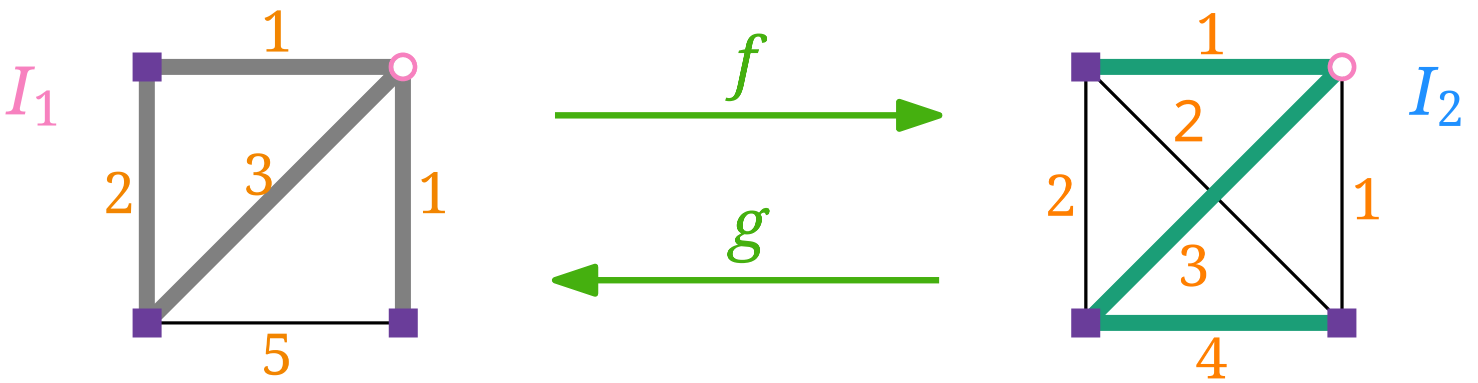

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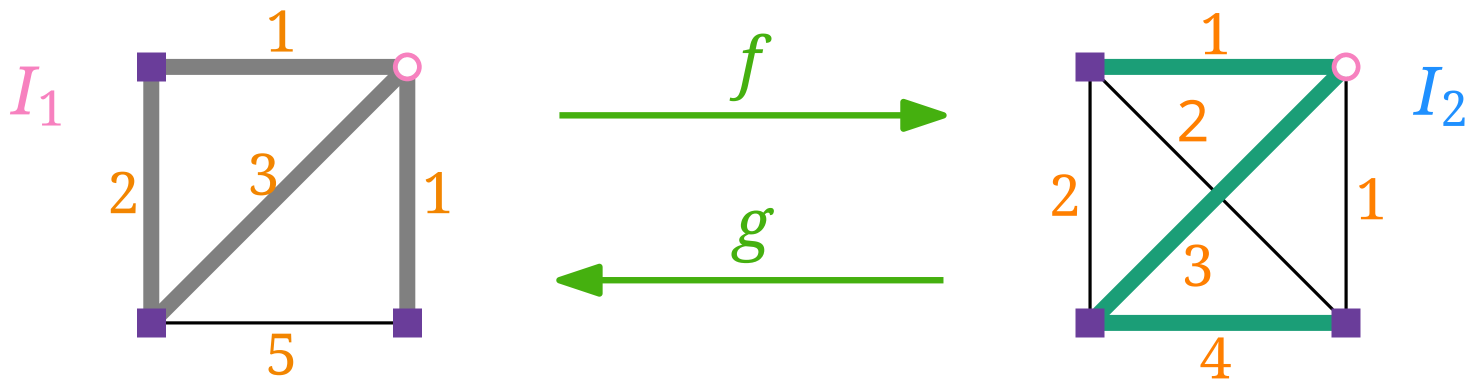

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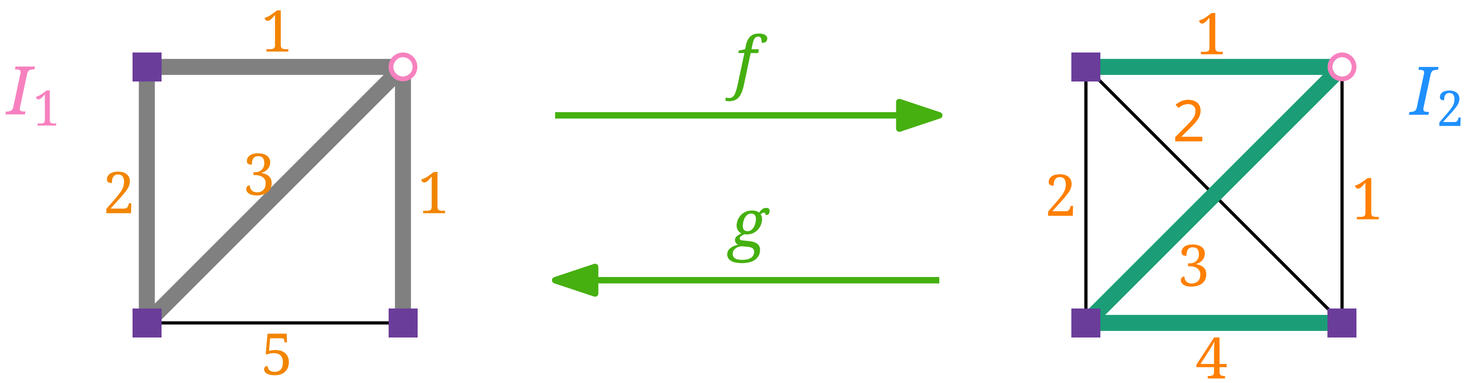

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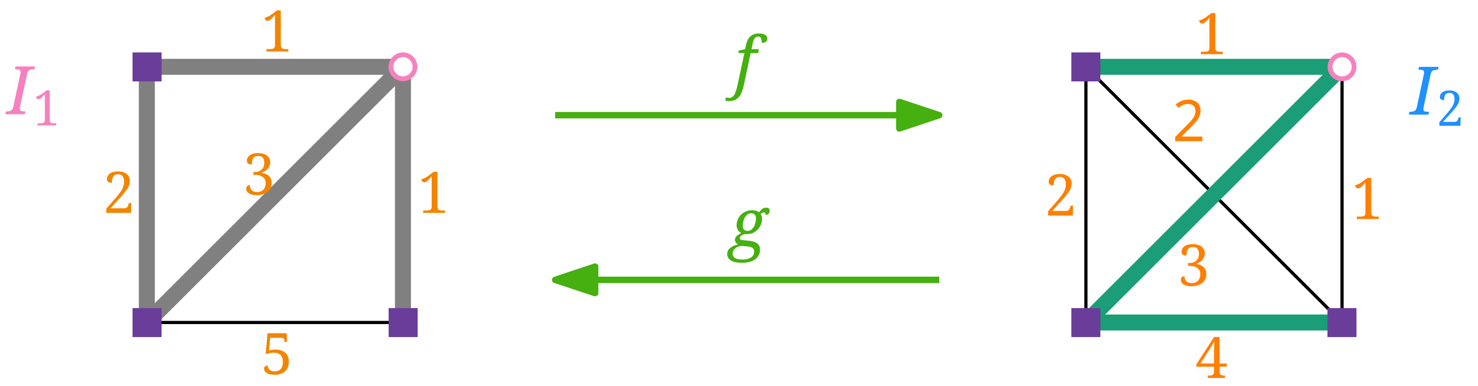

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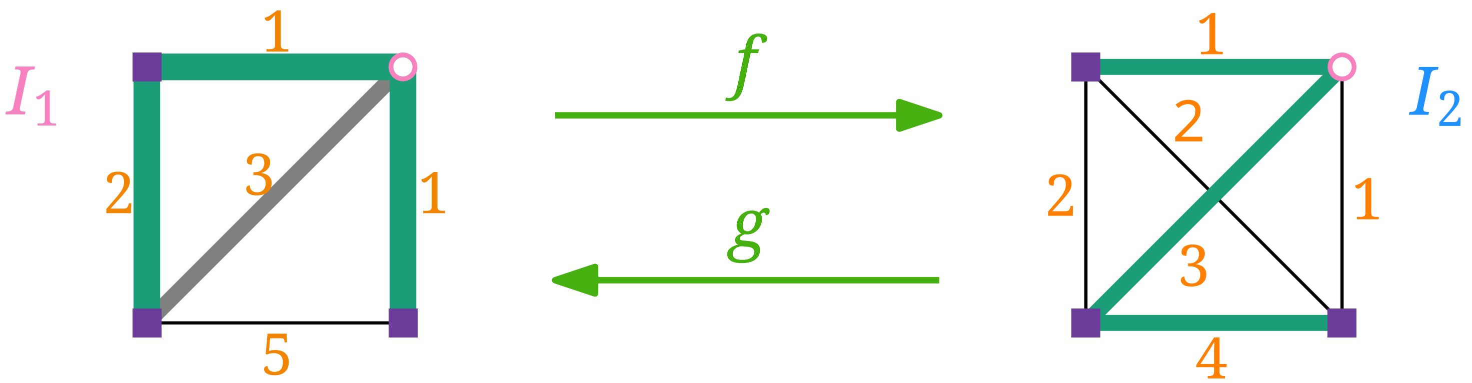

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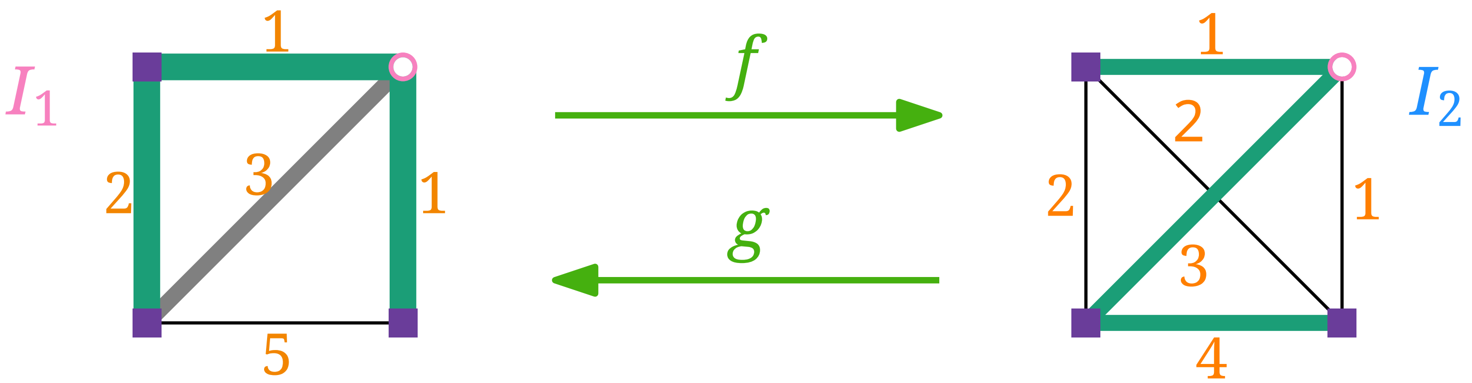

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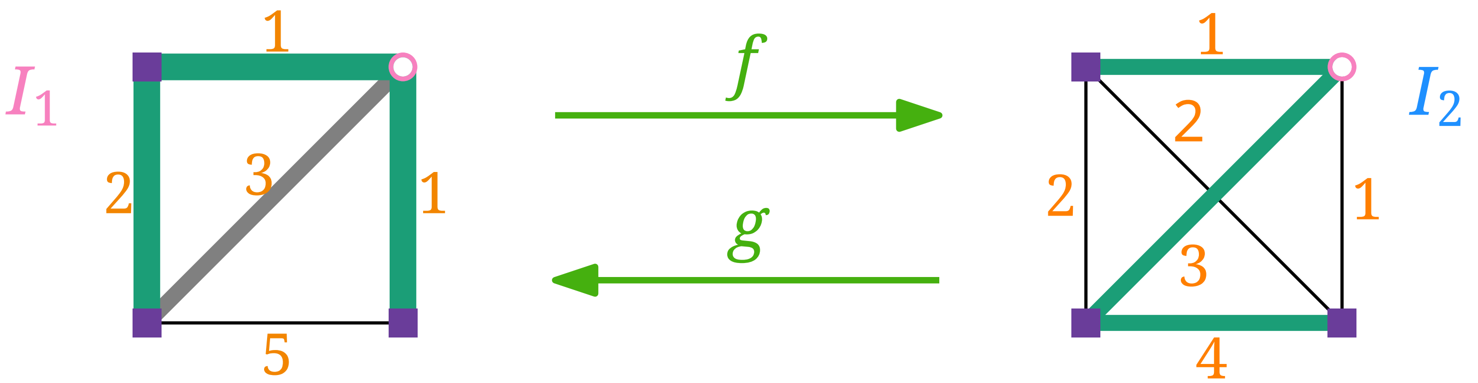
Thus $c_1(B_1) \leq c_1(G_1') \leq c_2(B_2)$, i.e. $\mathrm{obj}_{\Pi_1}(I_1, s) \leq \mathrm{obj}_{\Pi_2}(I_2, t)$



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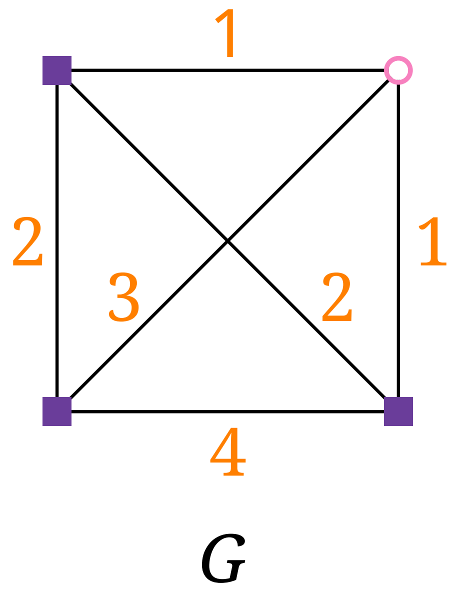

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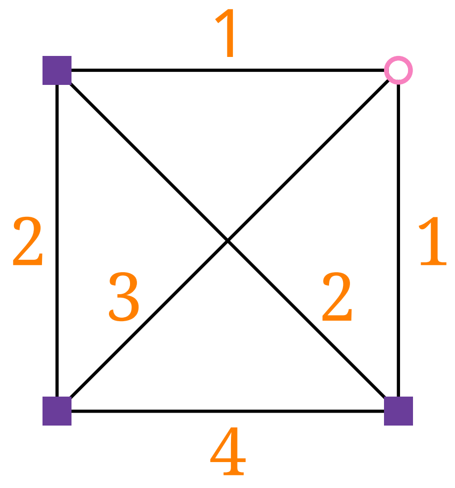

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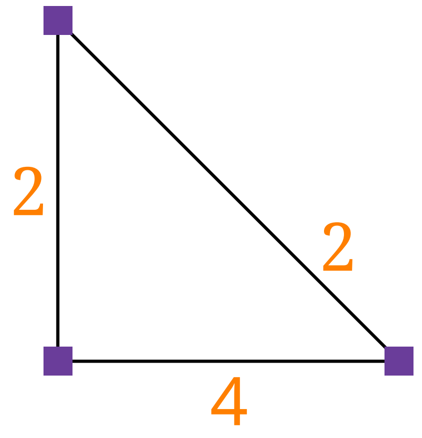

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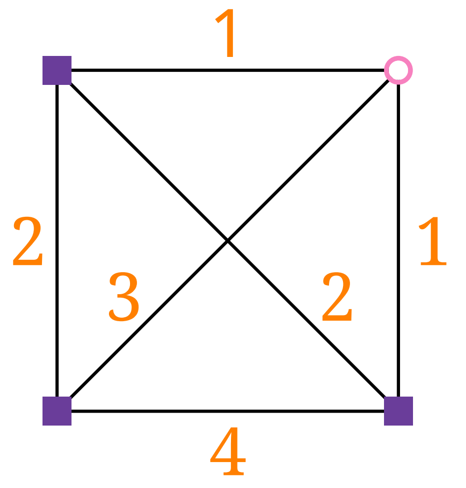

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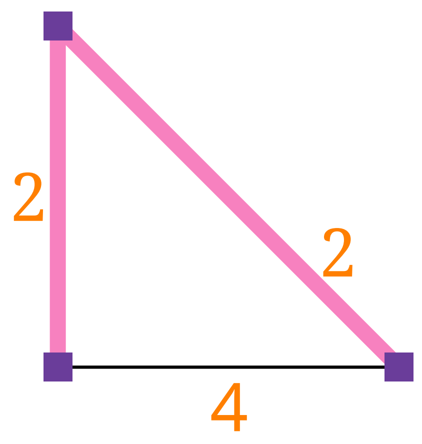

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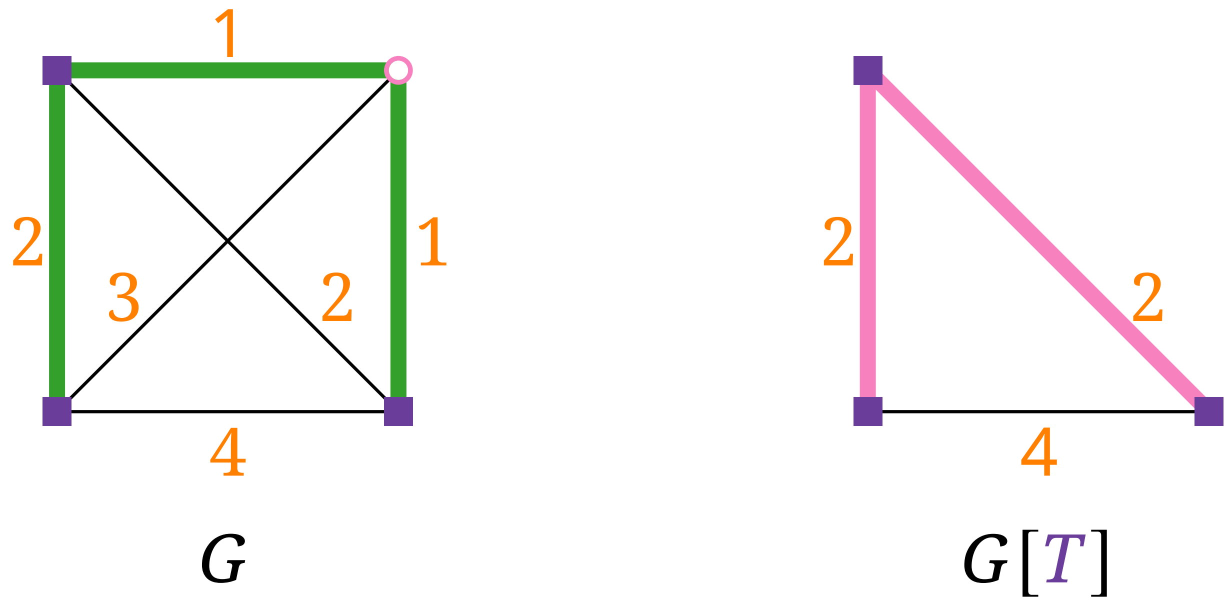

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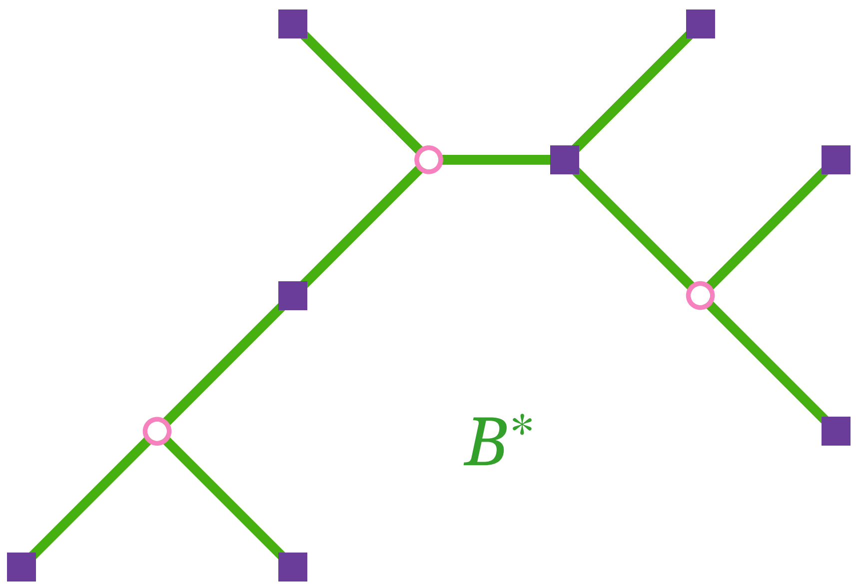

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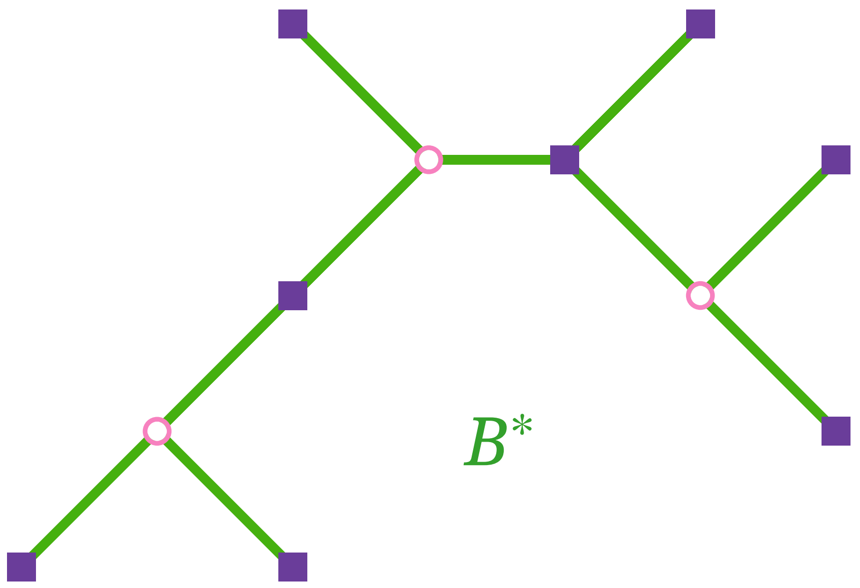

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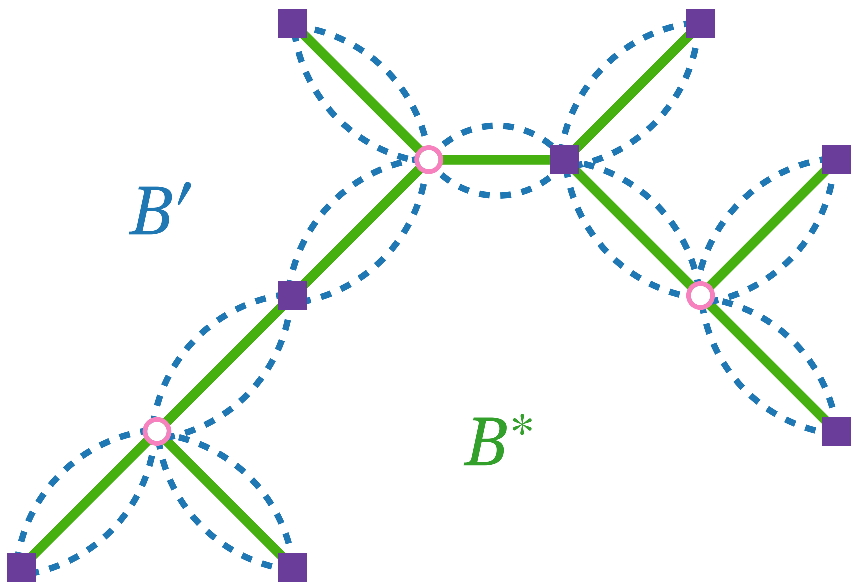

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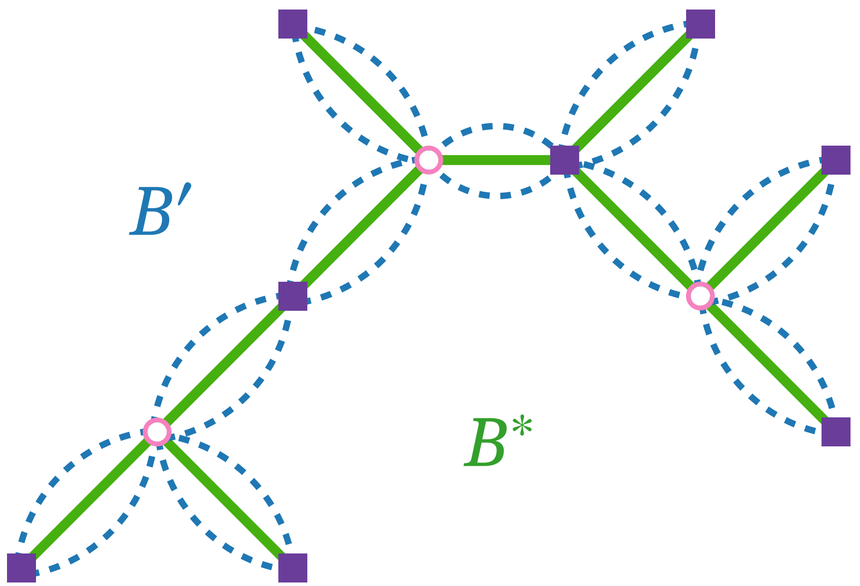

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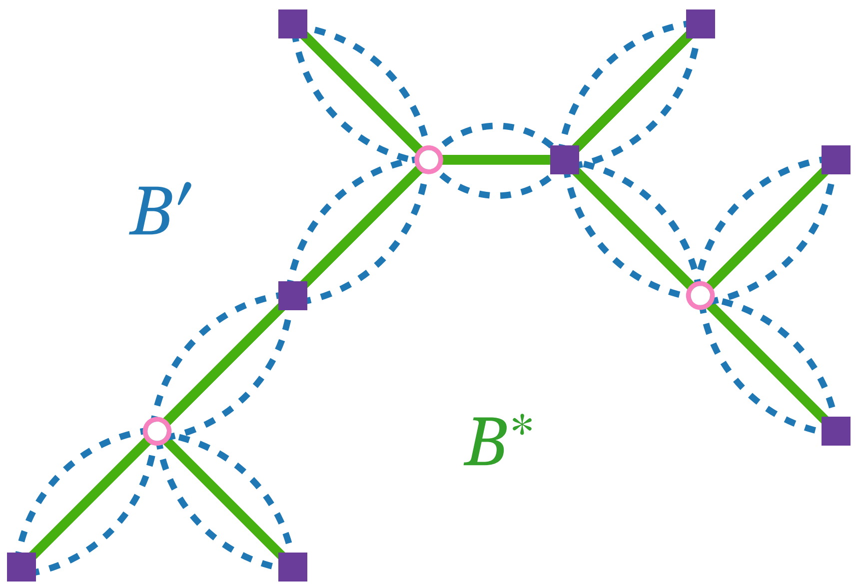

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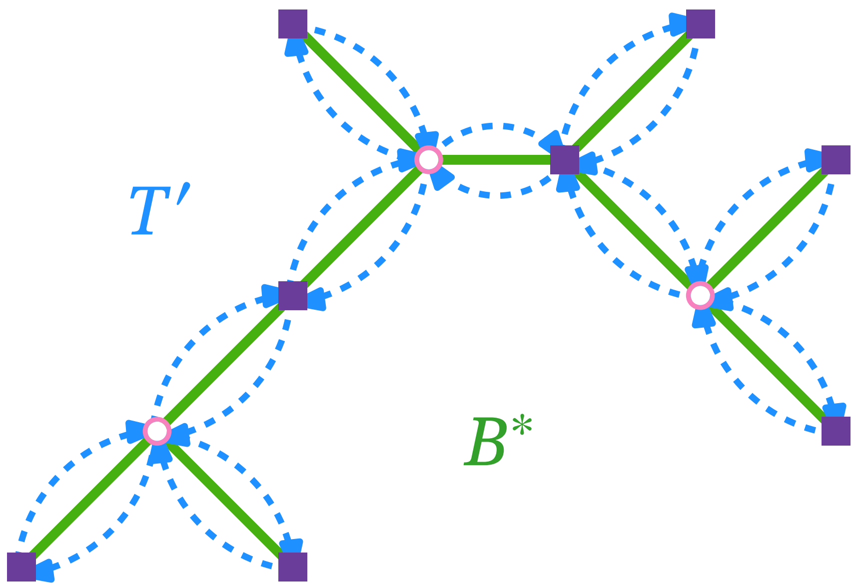

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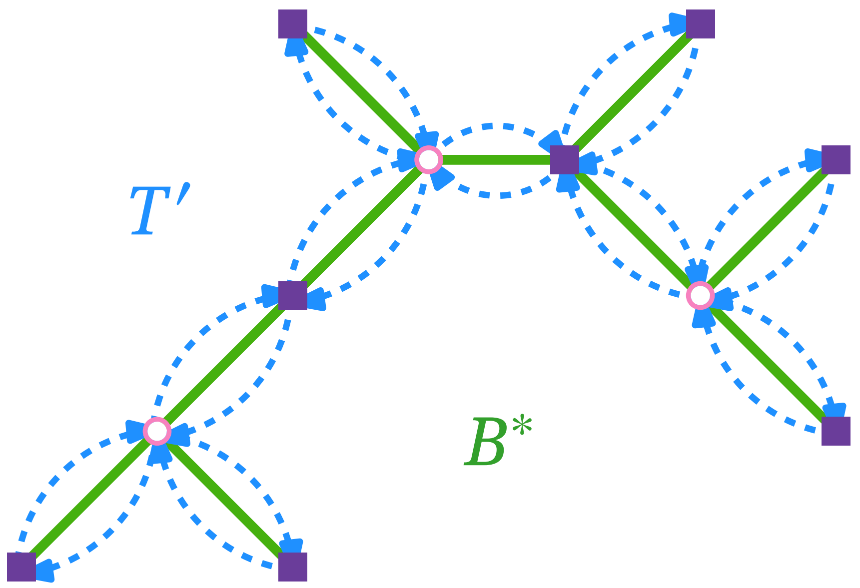

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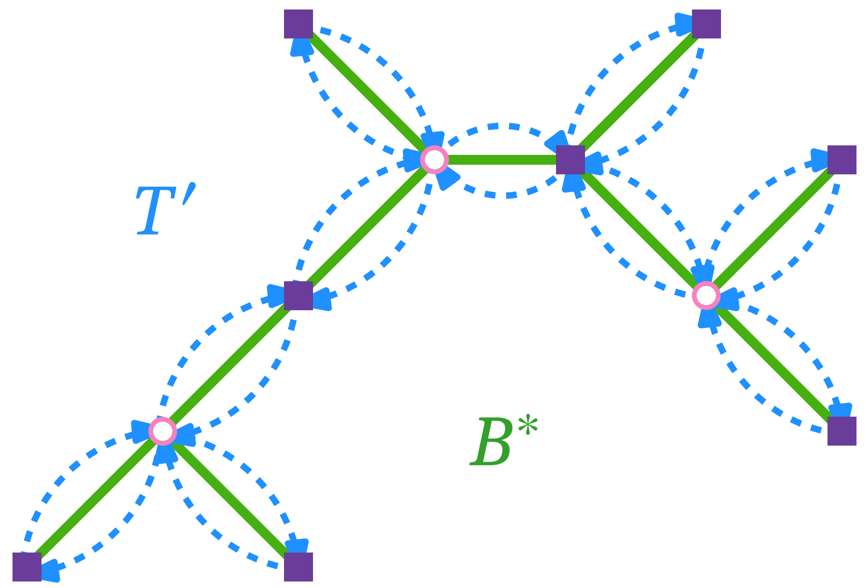

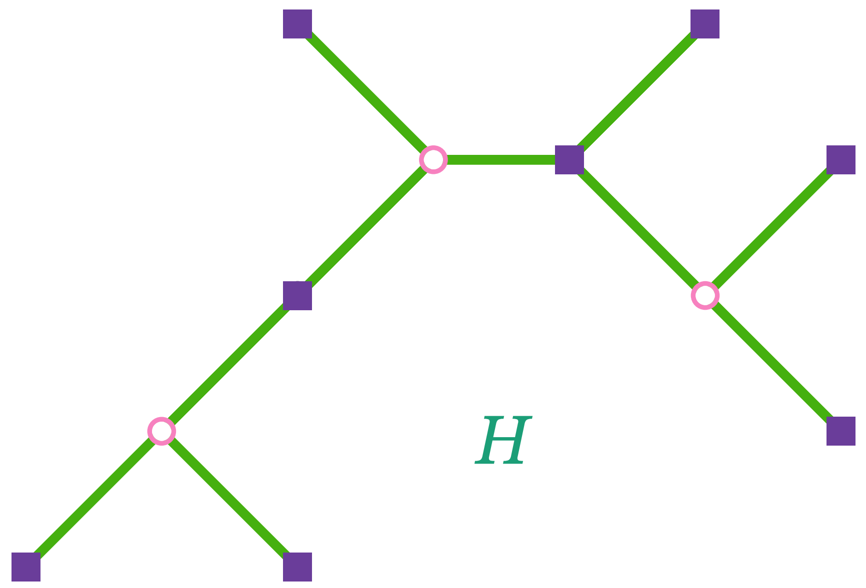

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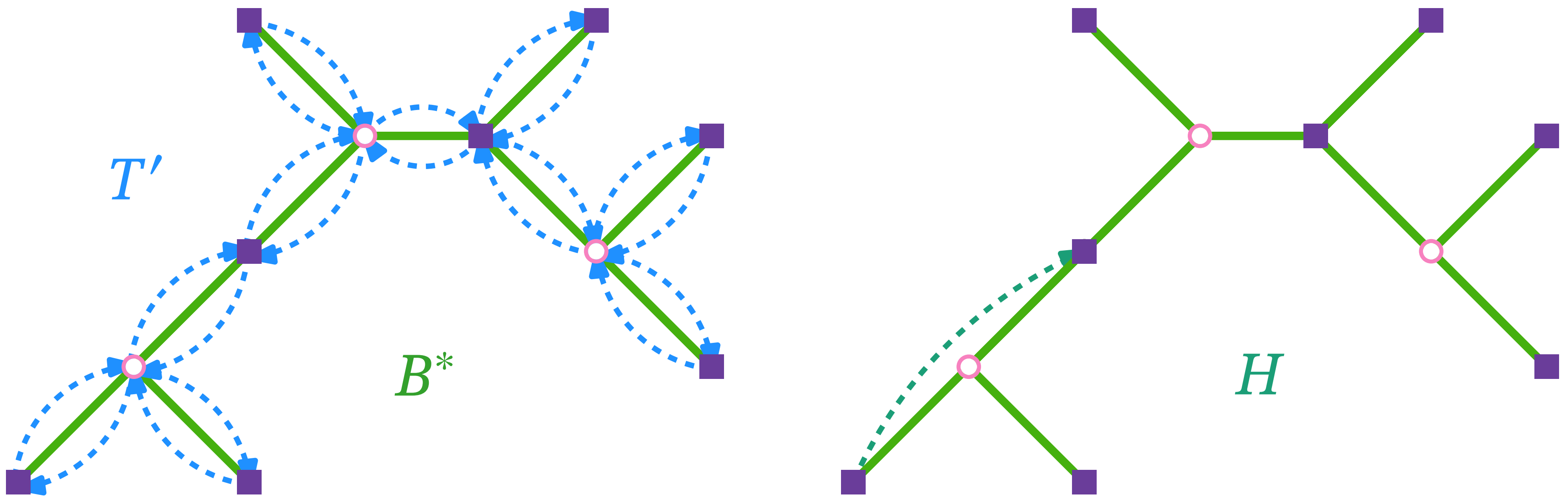

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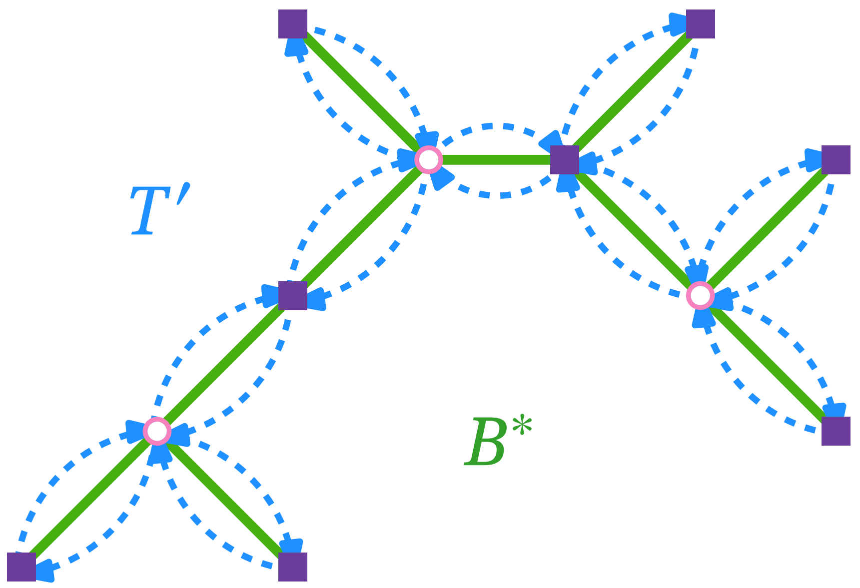

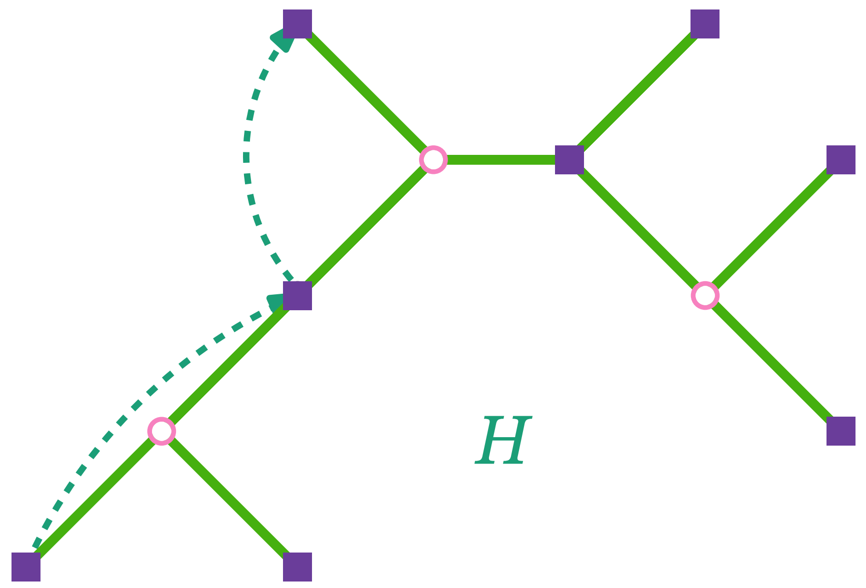

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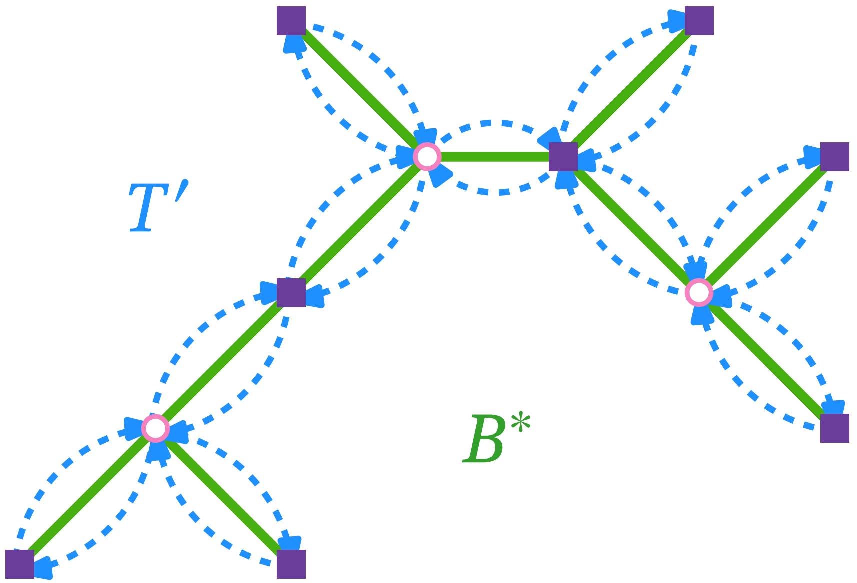

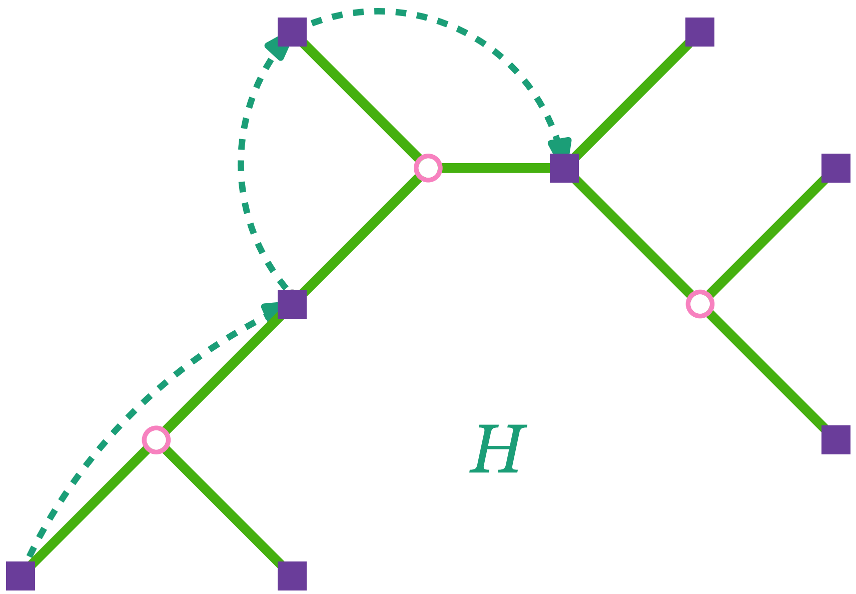

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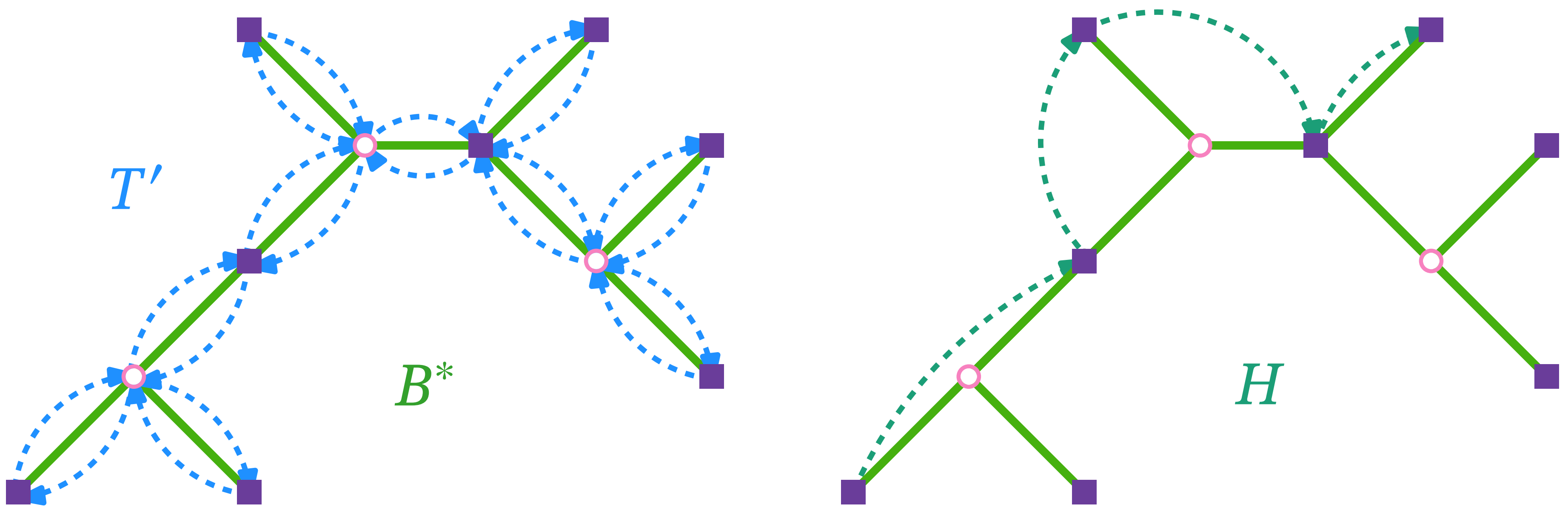

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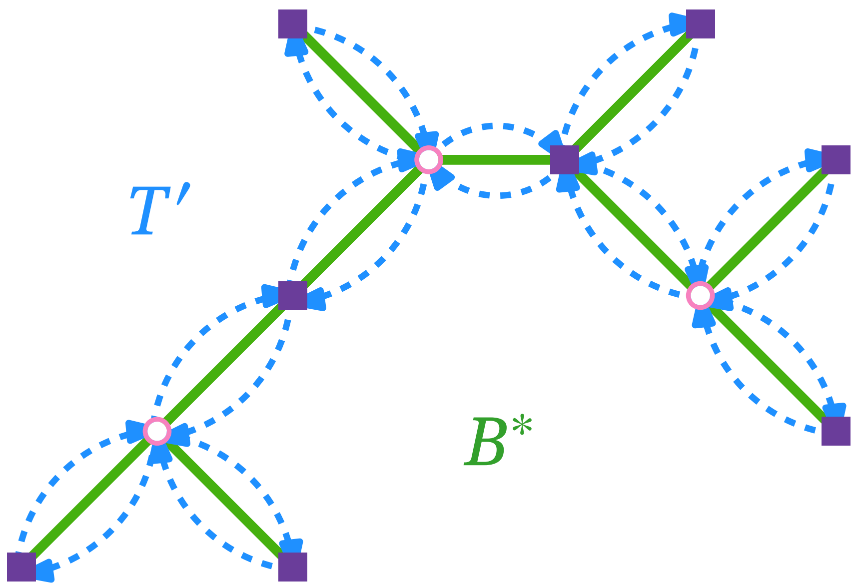

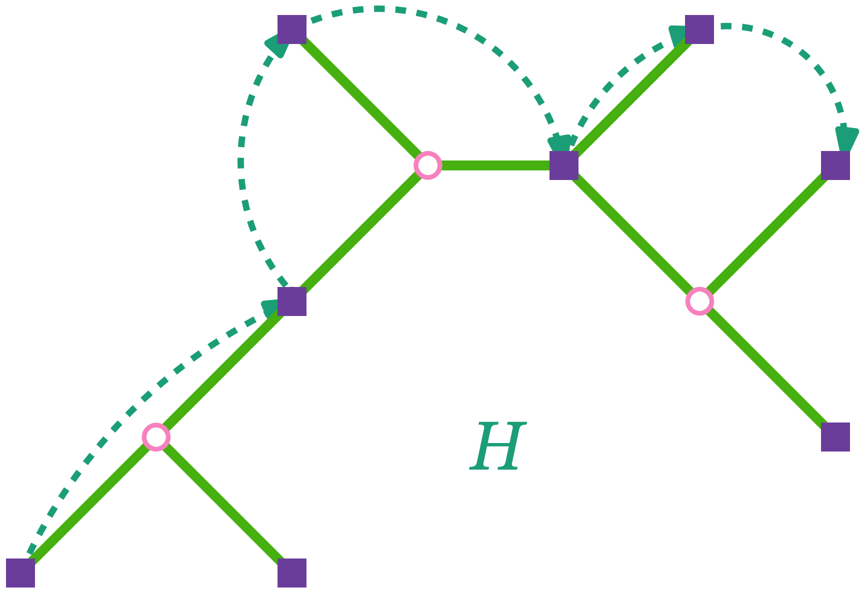

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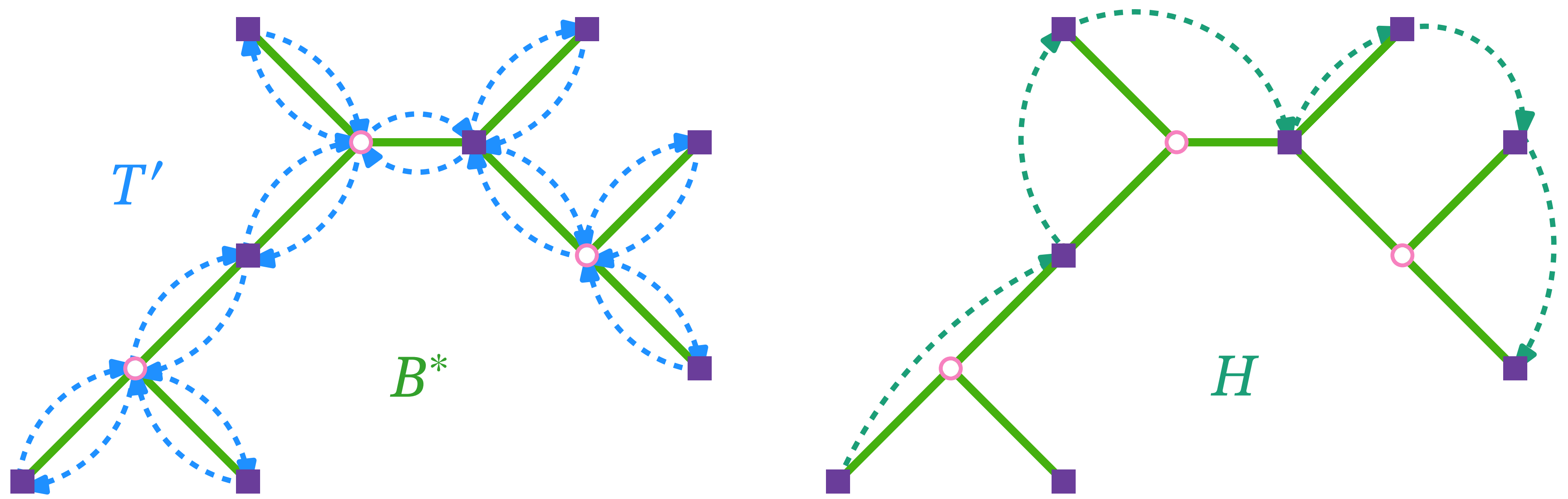

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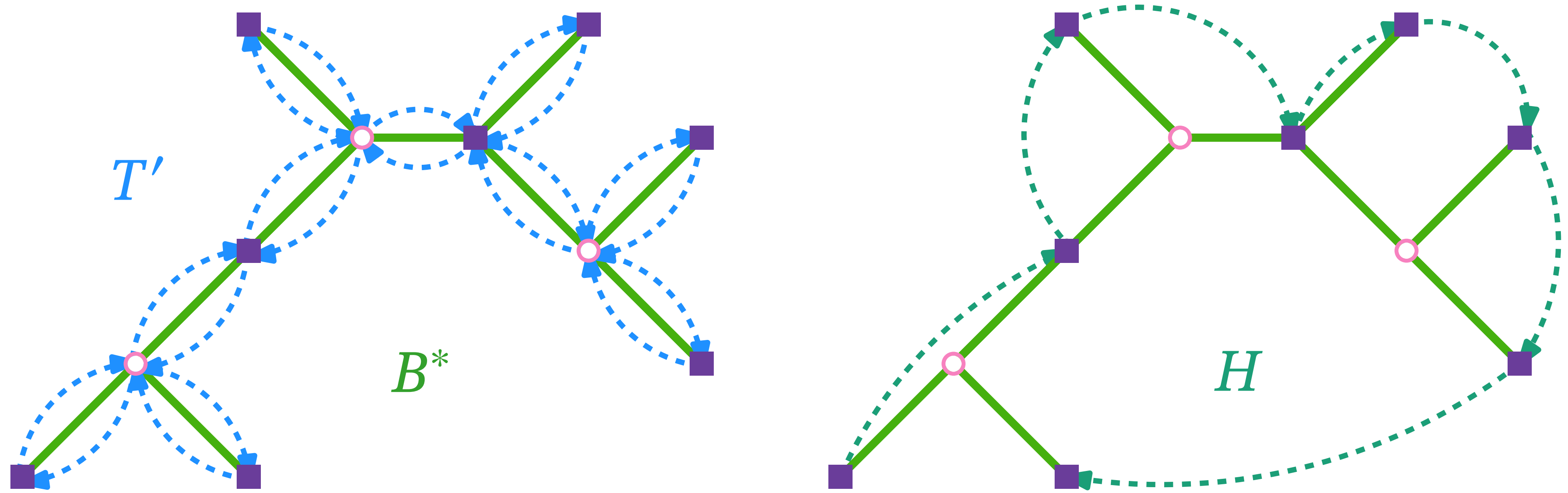

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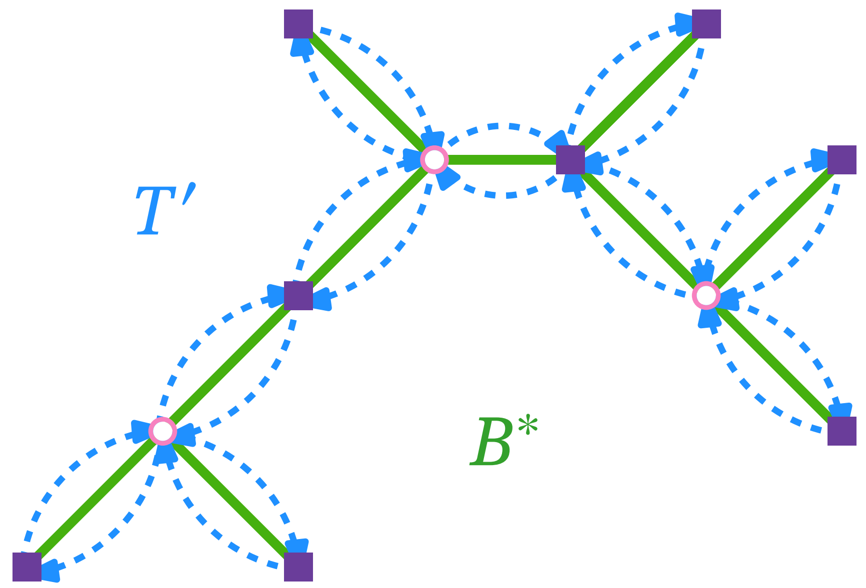

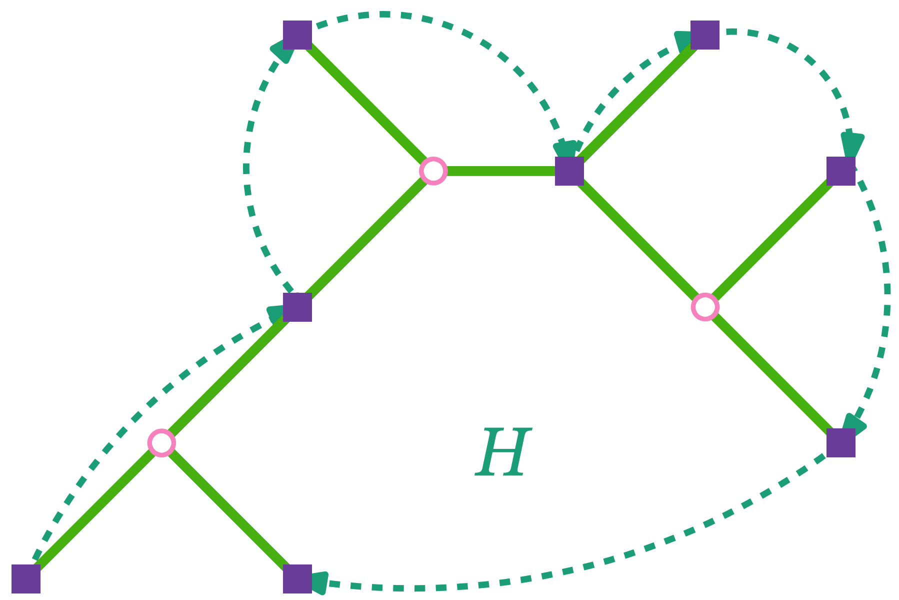

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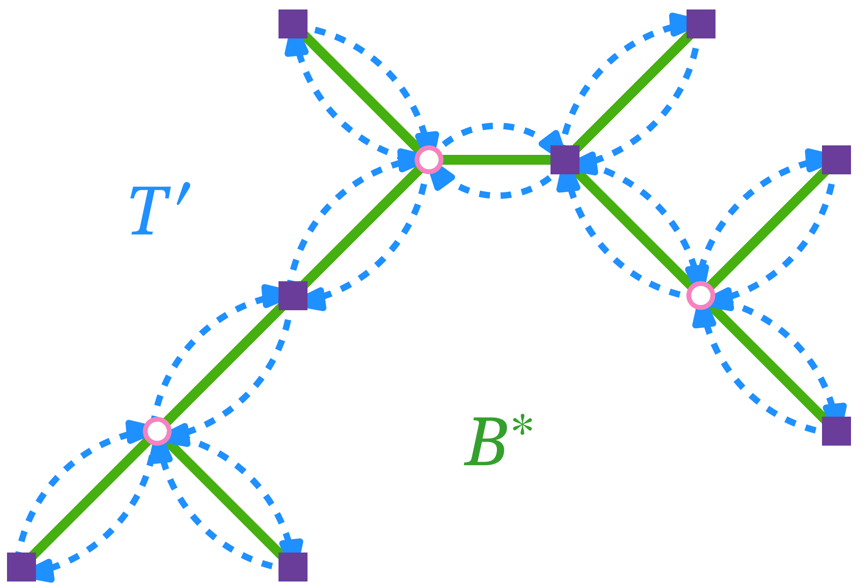

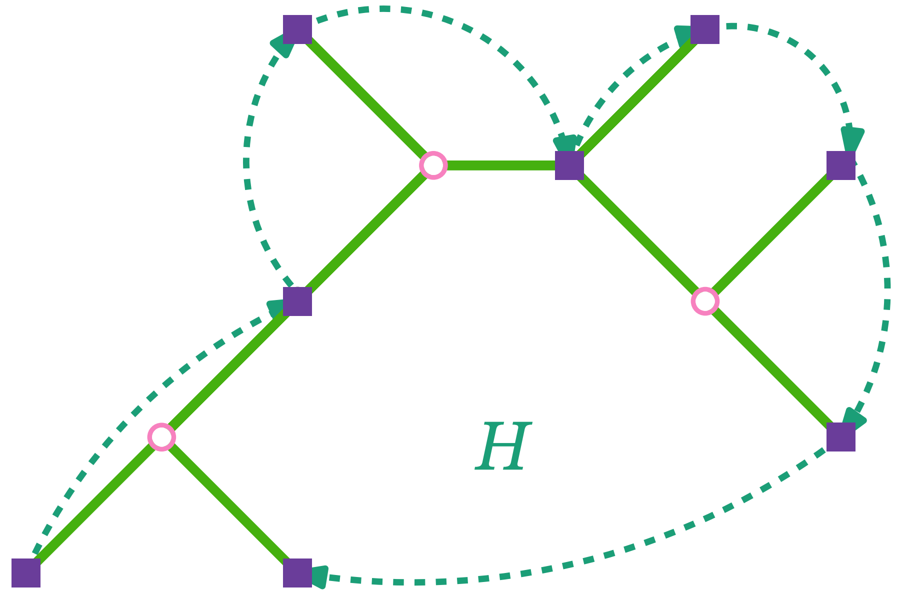

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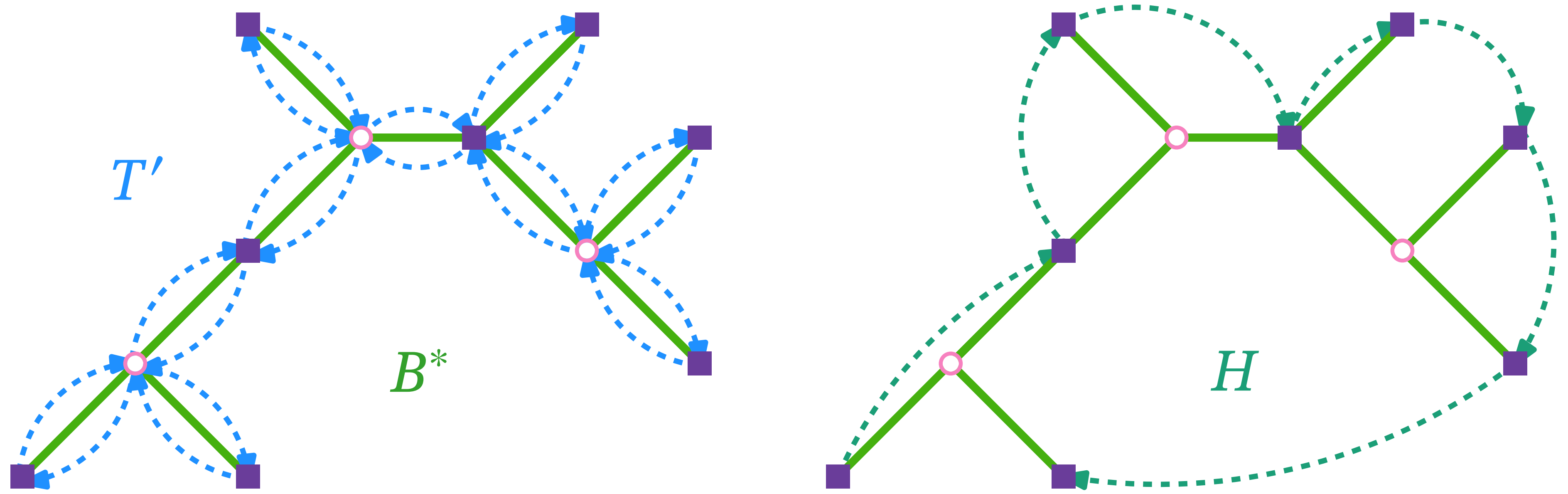
MST B of $G[T]$ has cost $c(B) \leq c(H) \leq 2 \cdot \mathrm{OPT}$ since H is a spanning tree of $G[T]$.



Analysis Tight?

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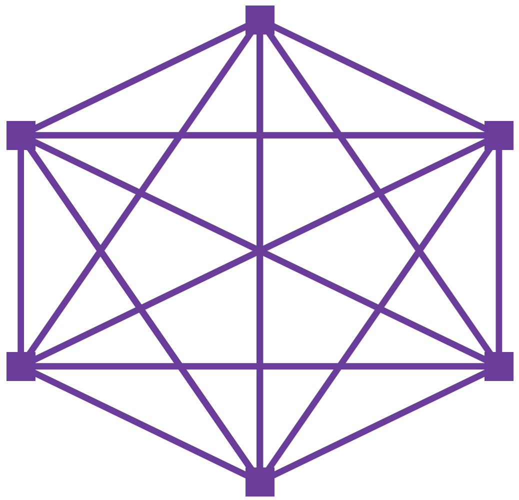
K_n



■ terminal

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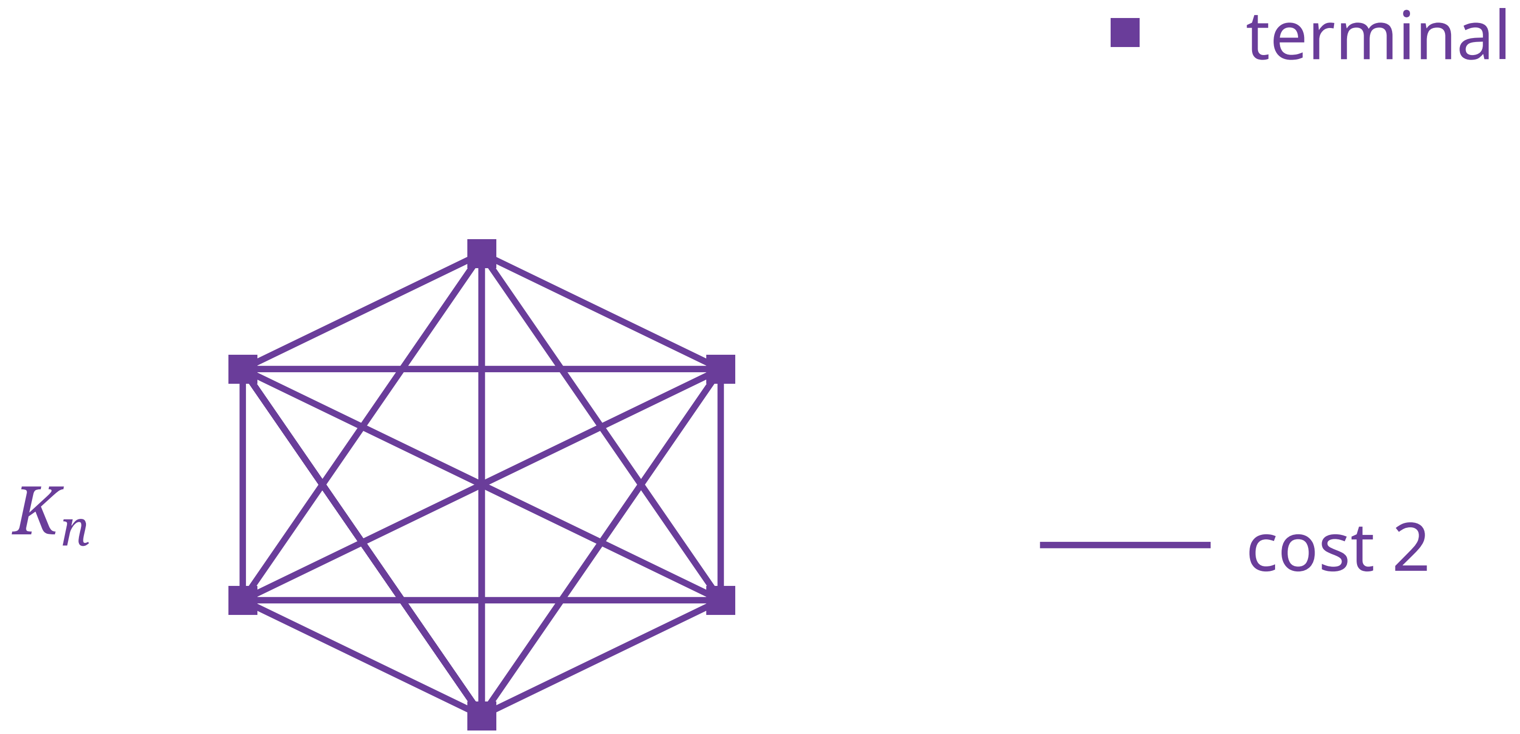

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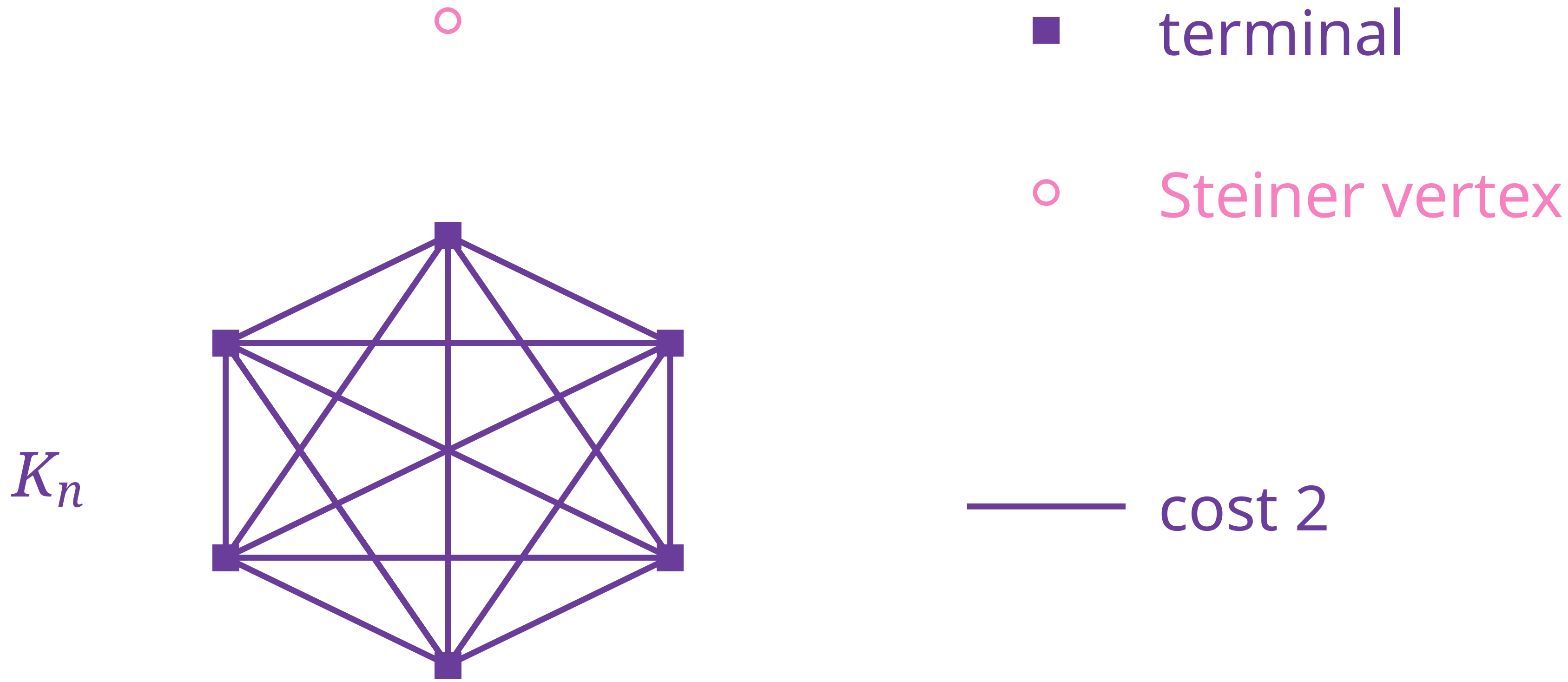

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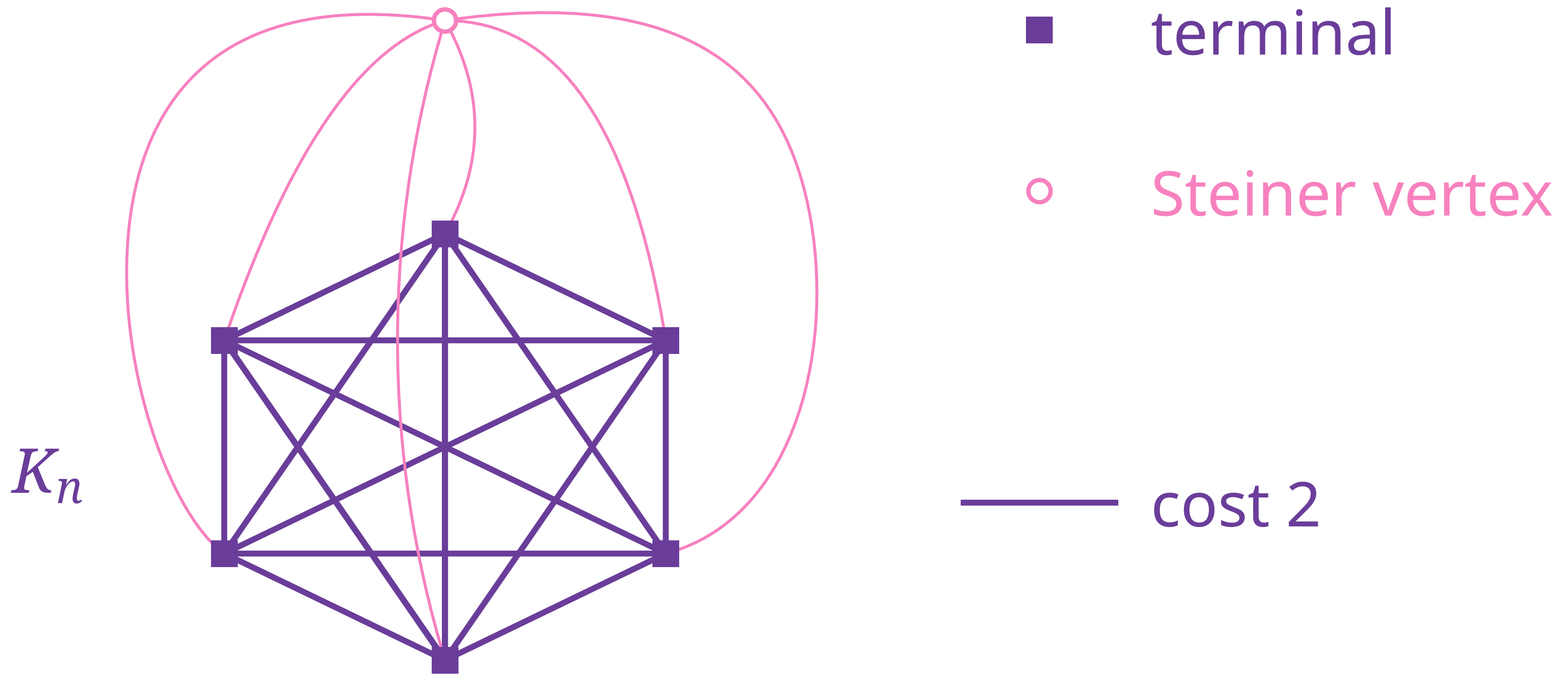

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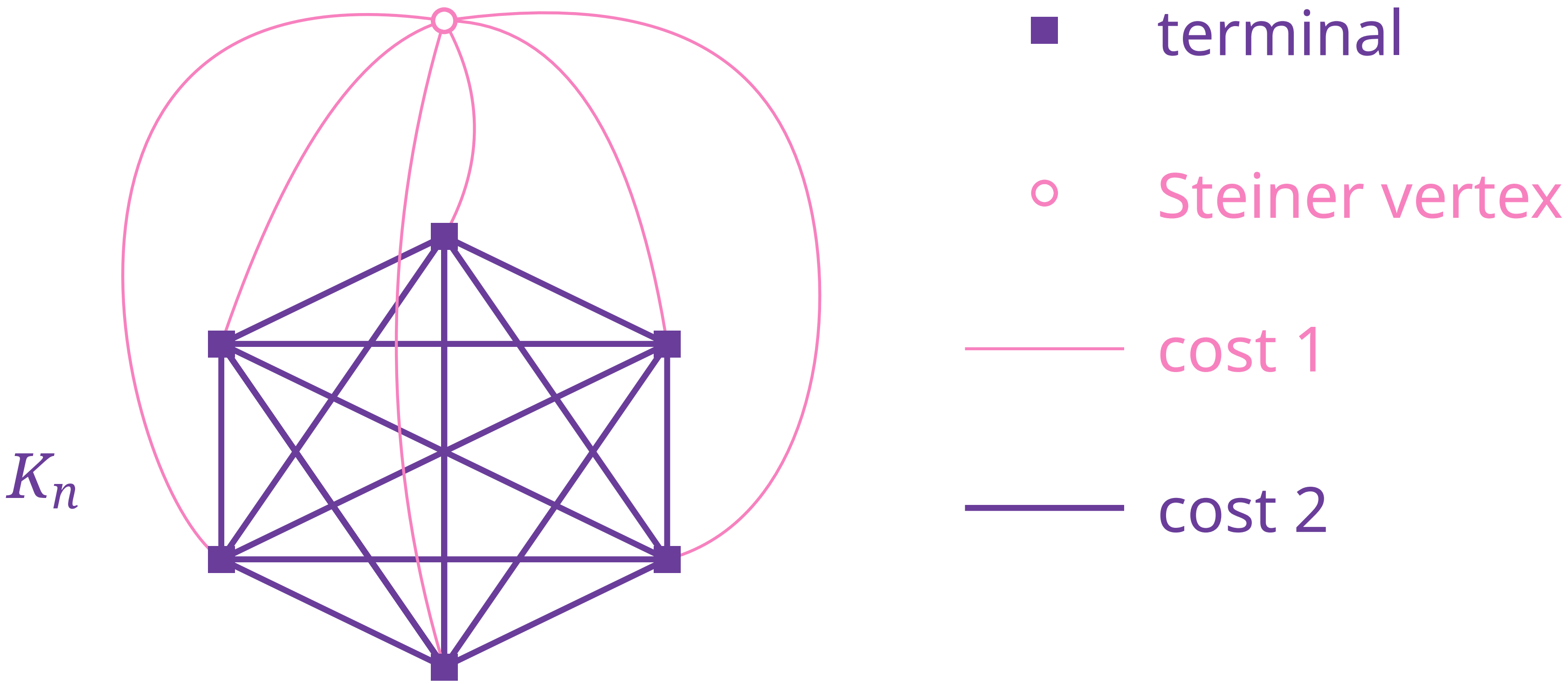

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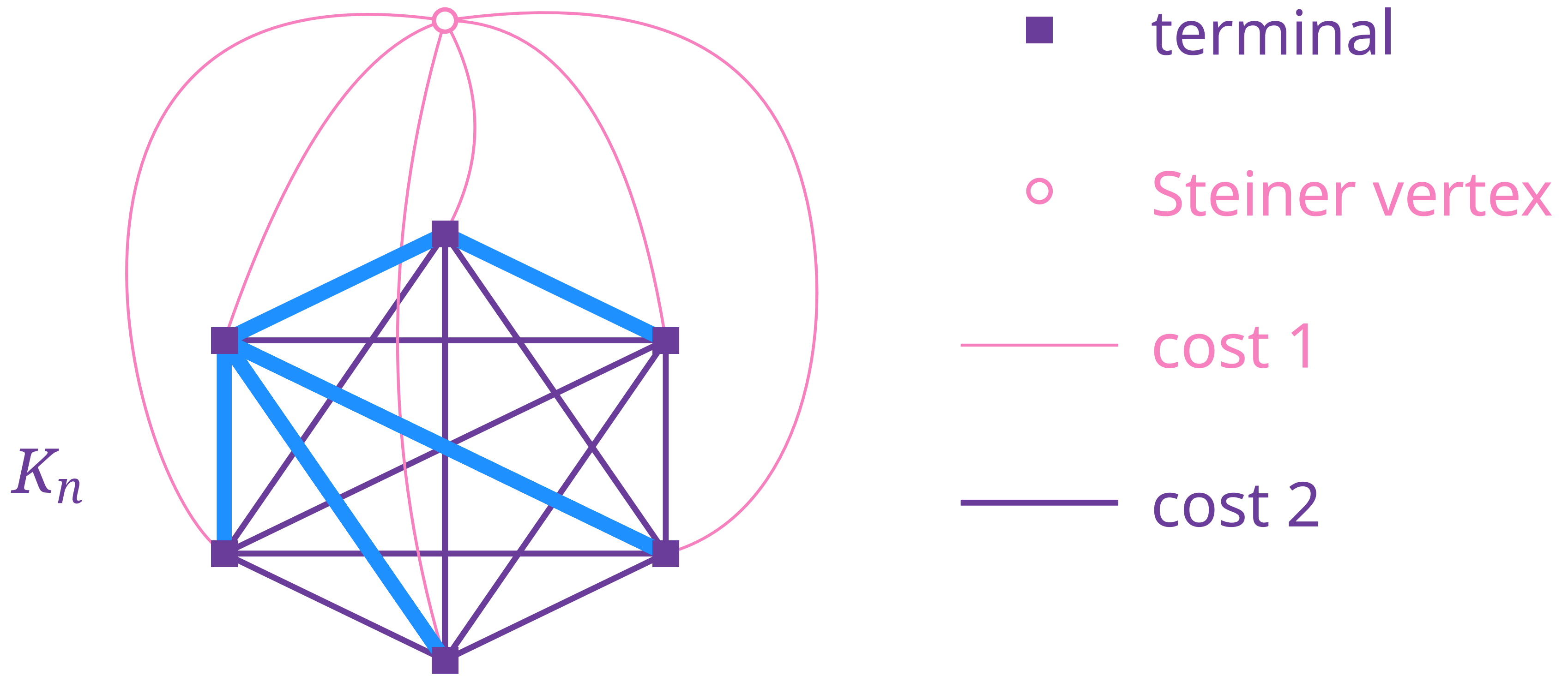
MST of $G[T]$ with cost $2(n-1)$



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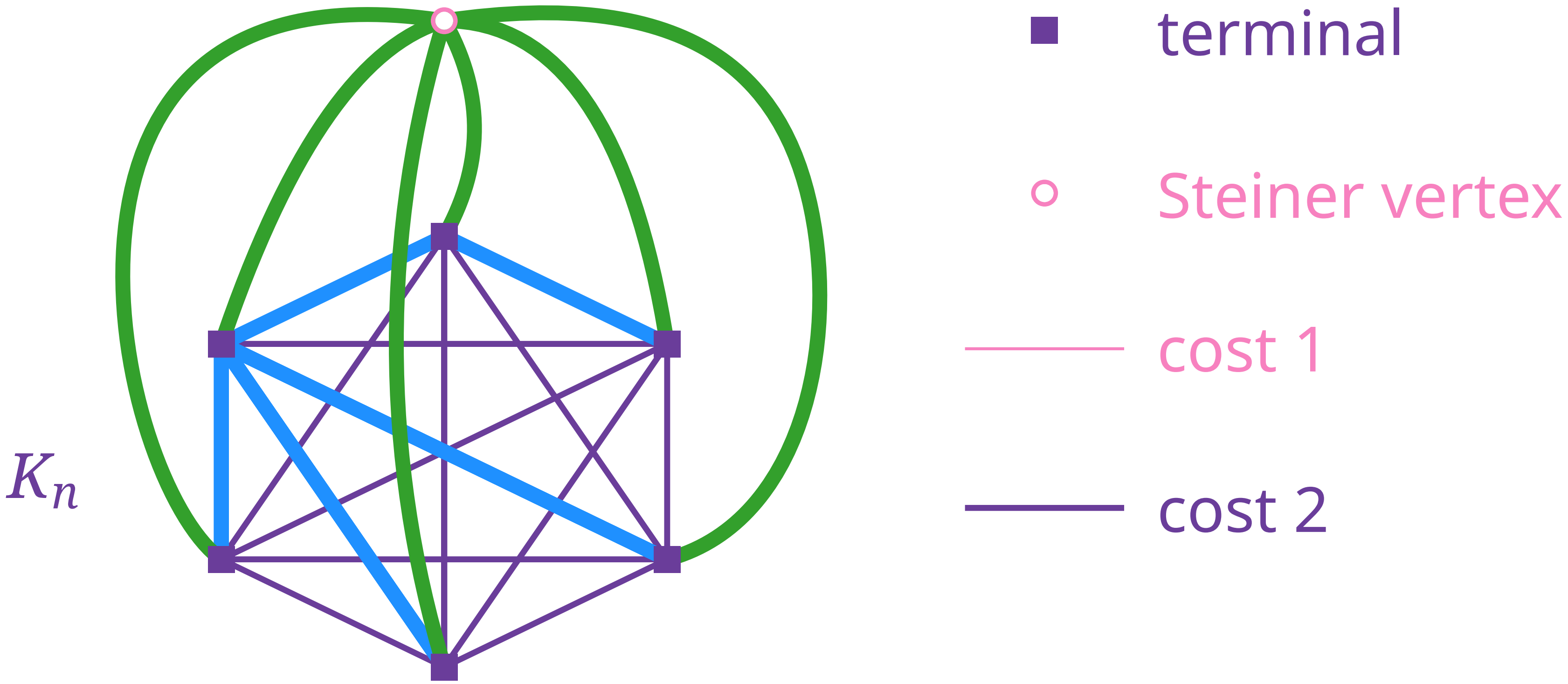

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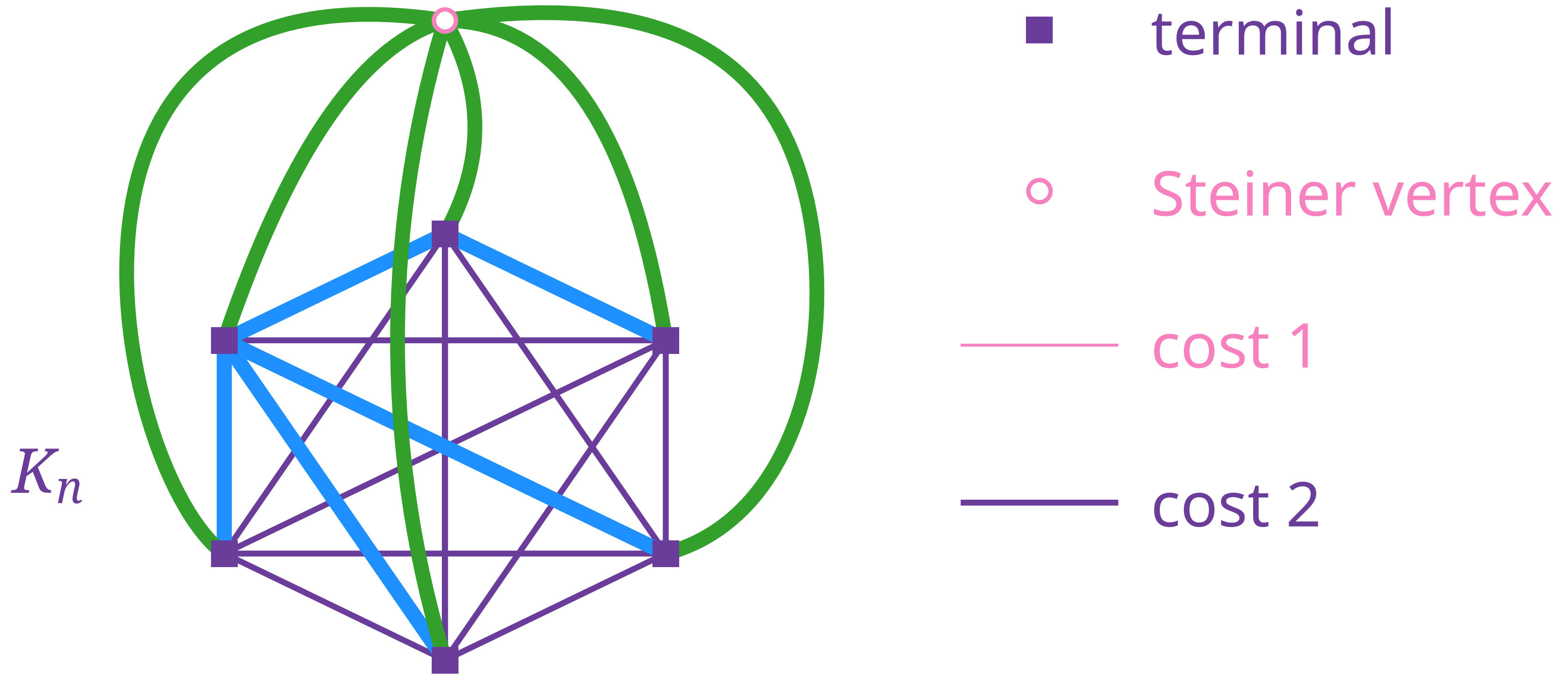
$$\frac{2(n-1)}{n} \rightarrow 2$$



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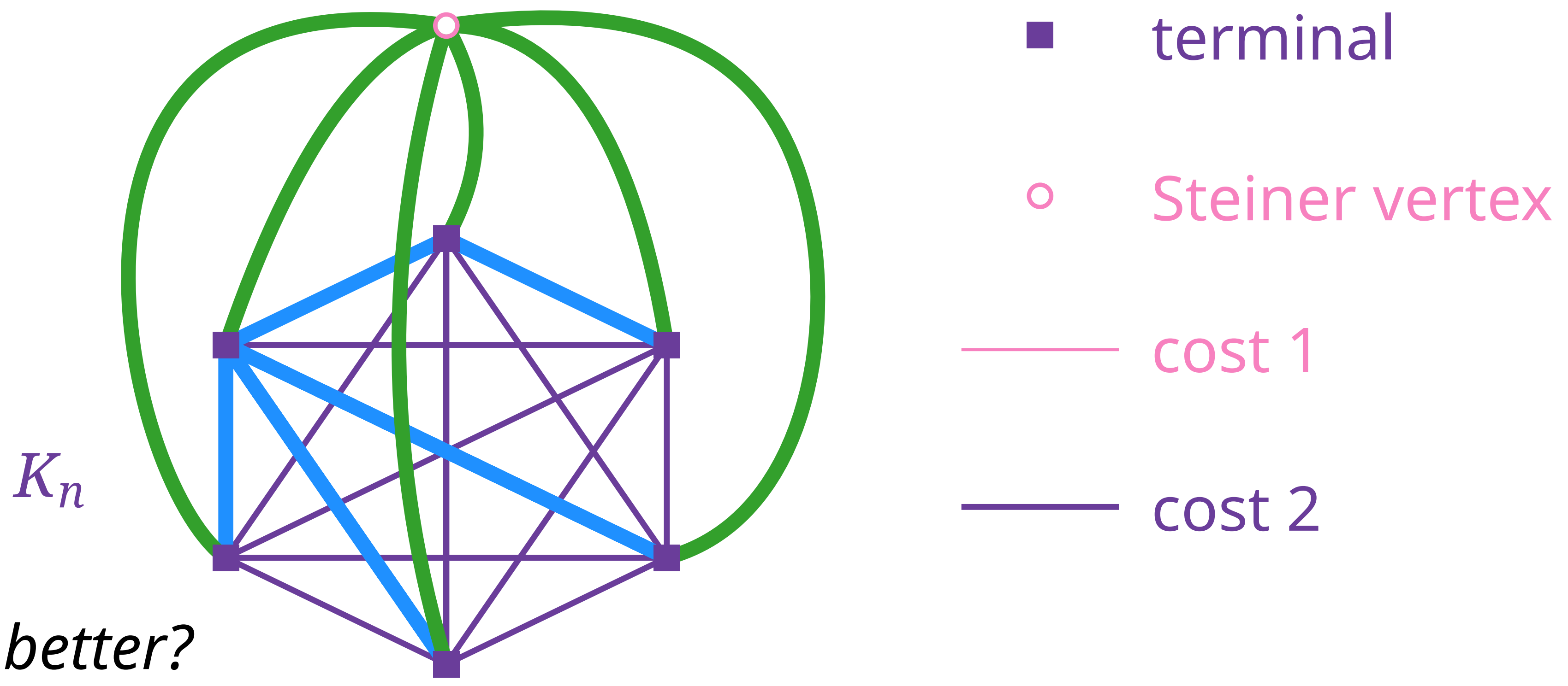

Can we do better?

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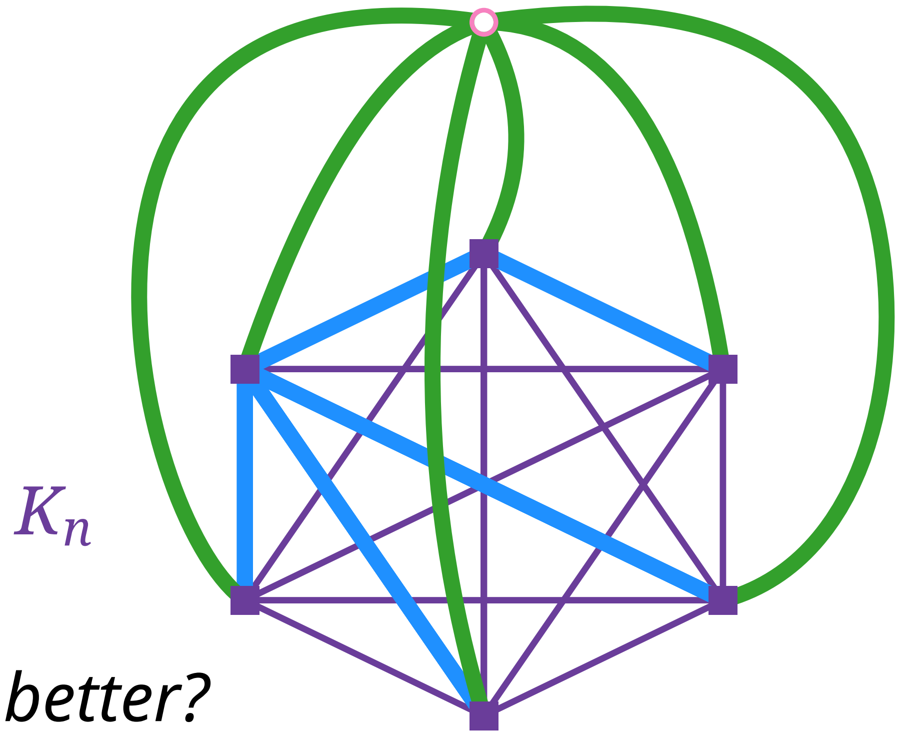

terminal

Steiner vertex

cost 1

cost 2

Can we do better?

The best known approximation factor for STEINERTREE is $\ln(4) + \varepsilon \approx 1.39$

[Byrka, Grandoni, Rothvoß & Sanità, J. ACM'13]

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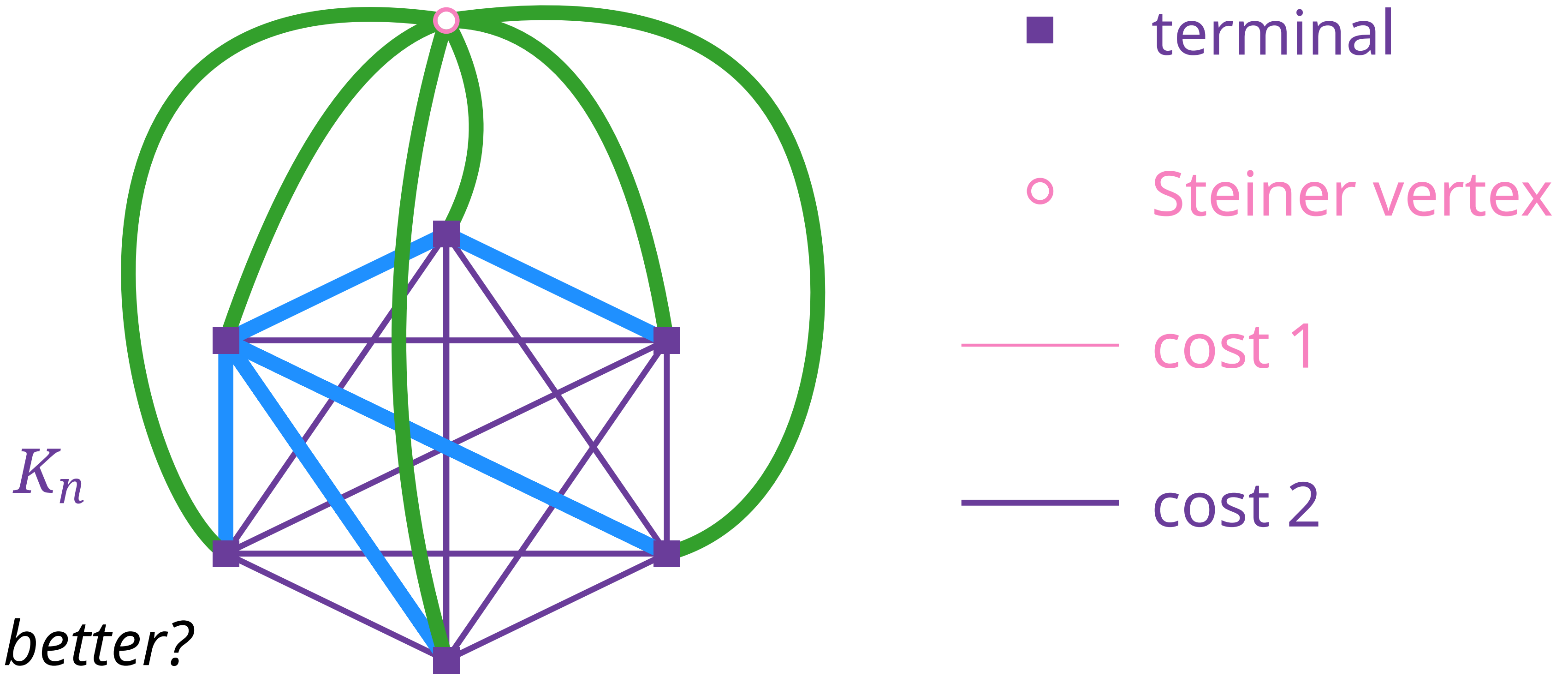

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[Byrka, Grandoni, Rothvoß & Sanità, J. ACM'13]

STEINERTREE cannot be approximated within factor $\frac{96}{95} \approx 1.0105$ (unless P=NP)

[Chlebík & Chlebíková, TCS'08]

Traveling Salesperson Problem

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Given: A complete graph G with edge weights $c \colon E(G) \to \mathbb{Q}^+$

Find: A Hamilton cycle C of G of minimum cost $c(C) := \sum_{e \in E(C)} c(e)$.

Can we use similar ideas for TSP?

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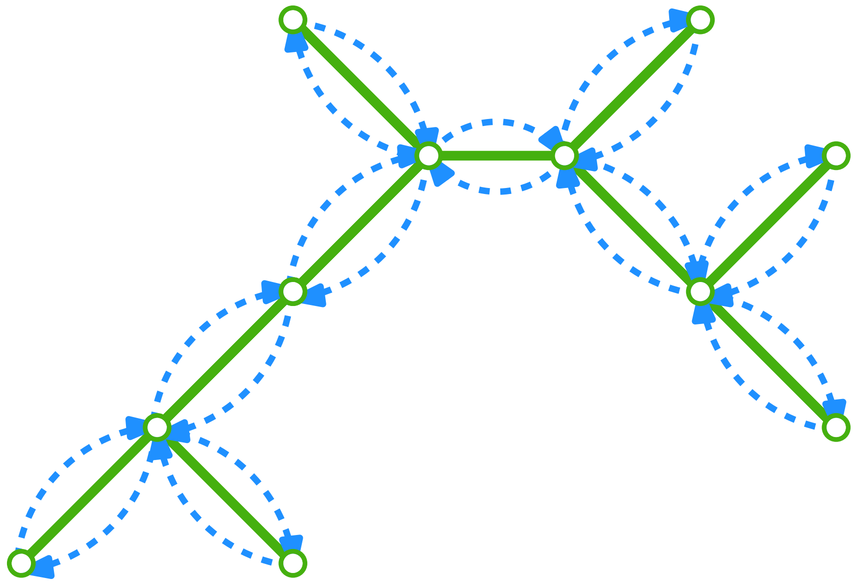

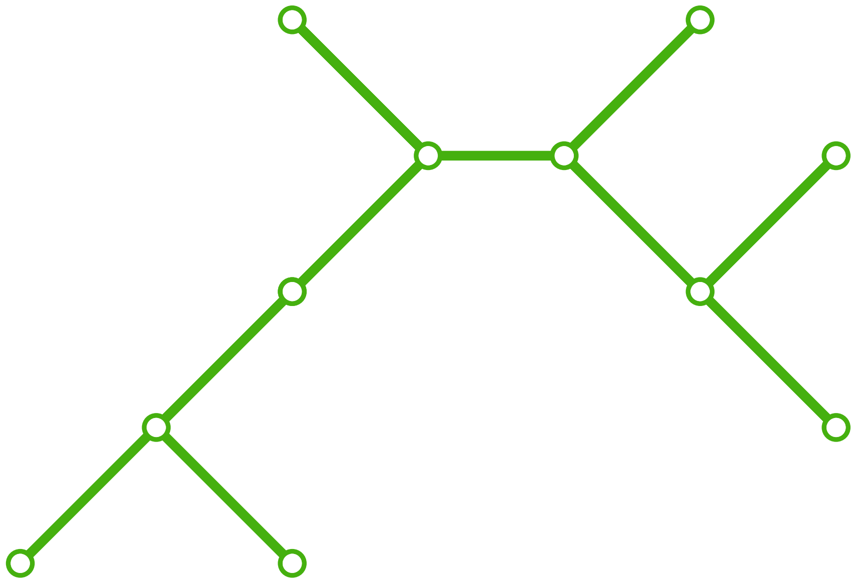

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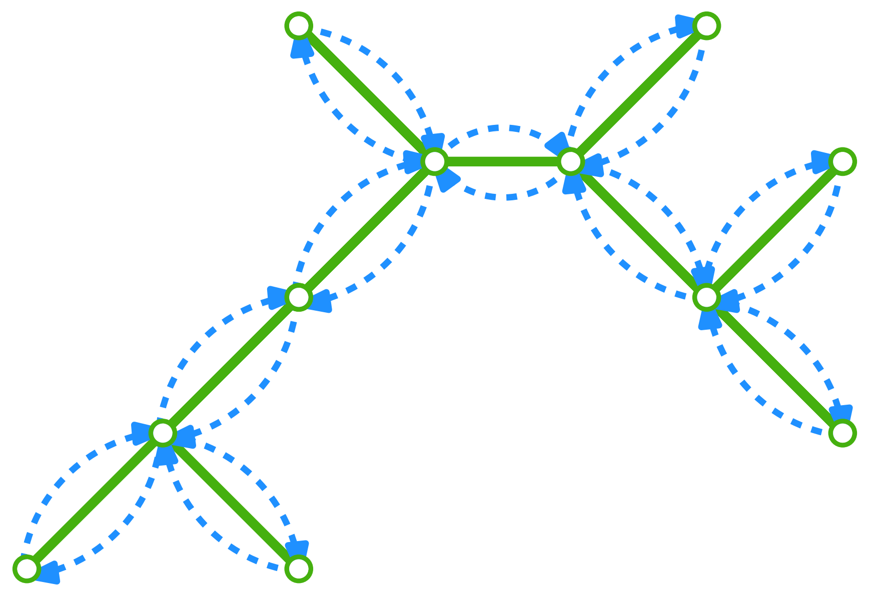

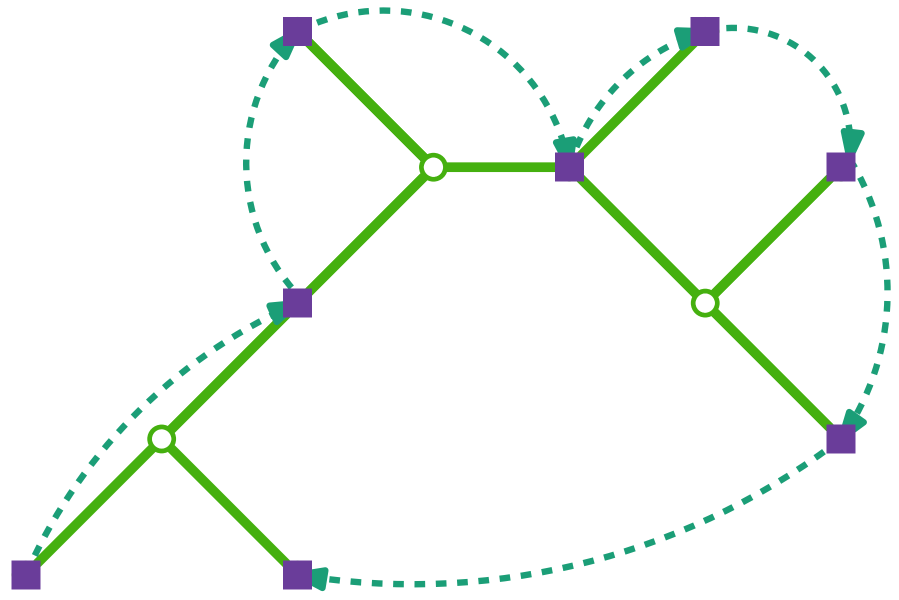

For SteinerTree:
skip non-terminals
and repeated vertices

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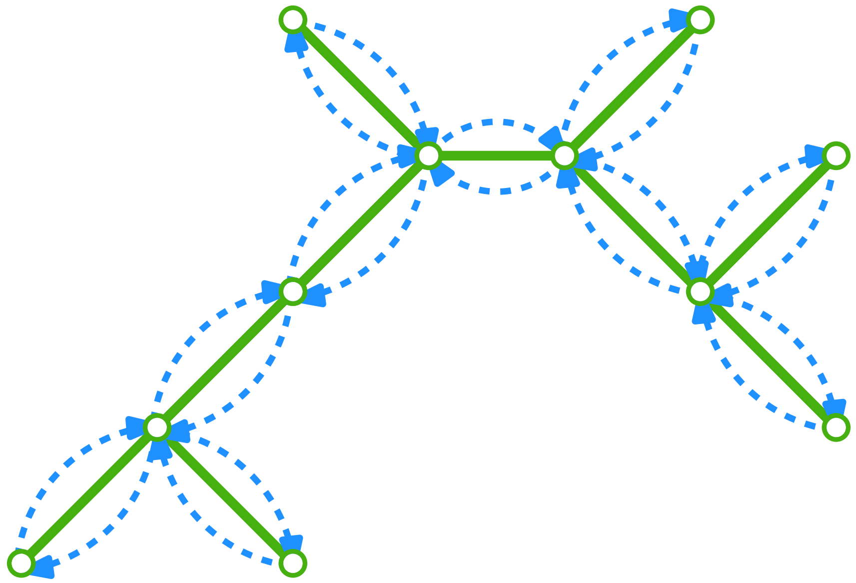

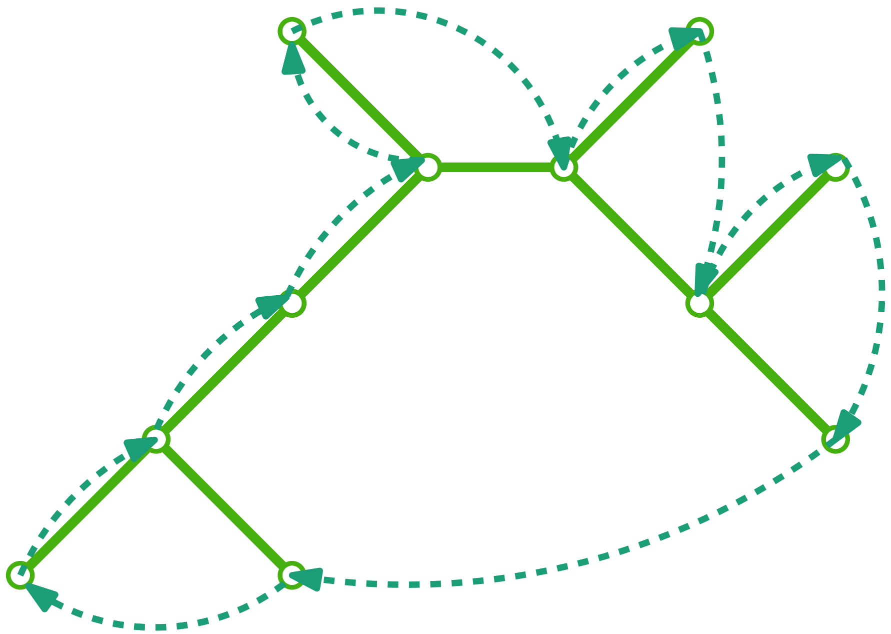

For TSP:
skip repeated vertices
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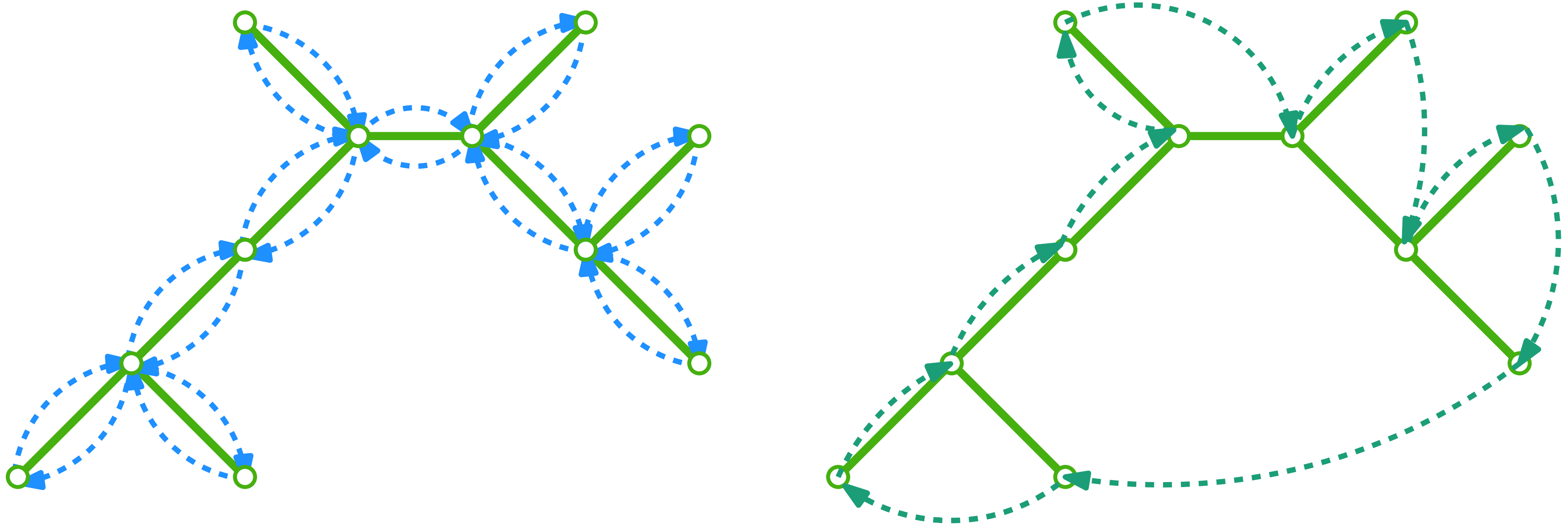

For TSP:
skip repeated vertices
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But this will only work for metric TSP (non-metric TSP is hard to approximate)

Metric Traveling Salesperson Problem (Metric TSP)

Algorithm:

- Compute a minimum spanning tree (MST) T in G,
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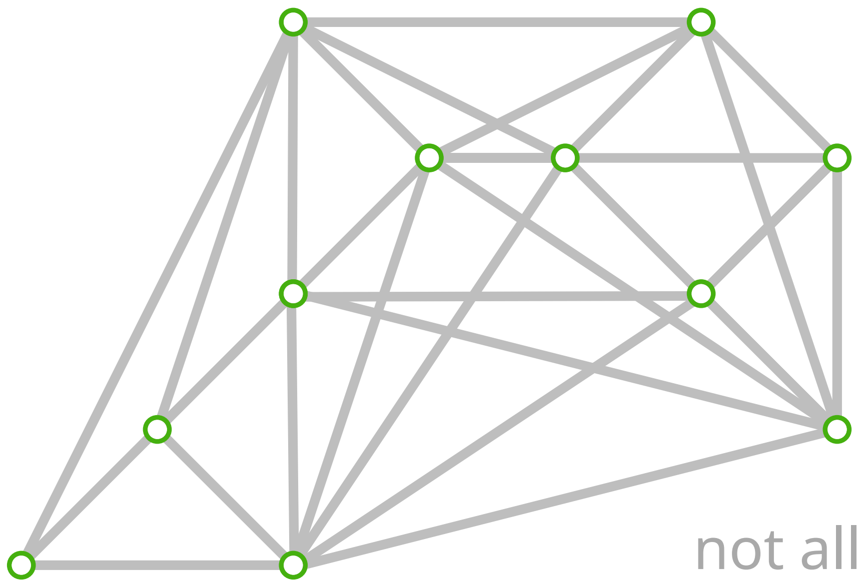

not all edges in complete graph shown

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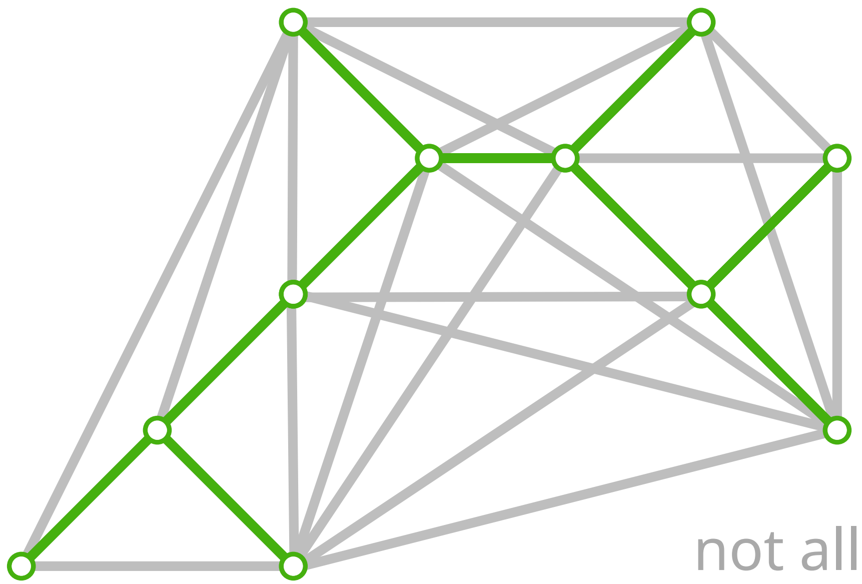

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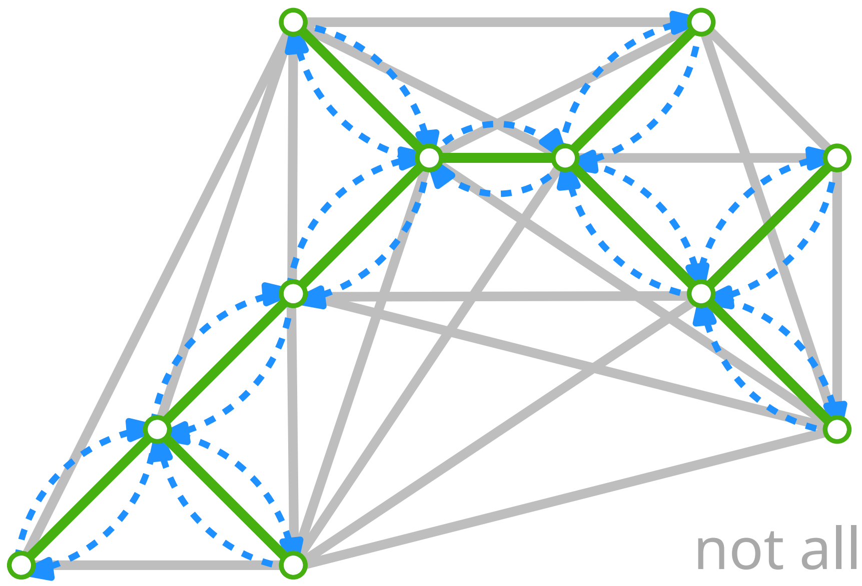

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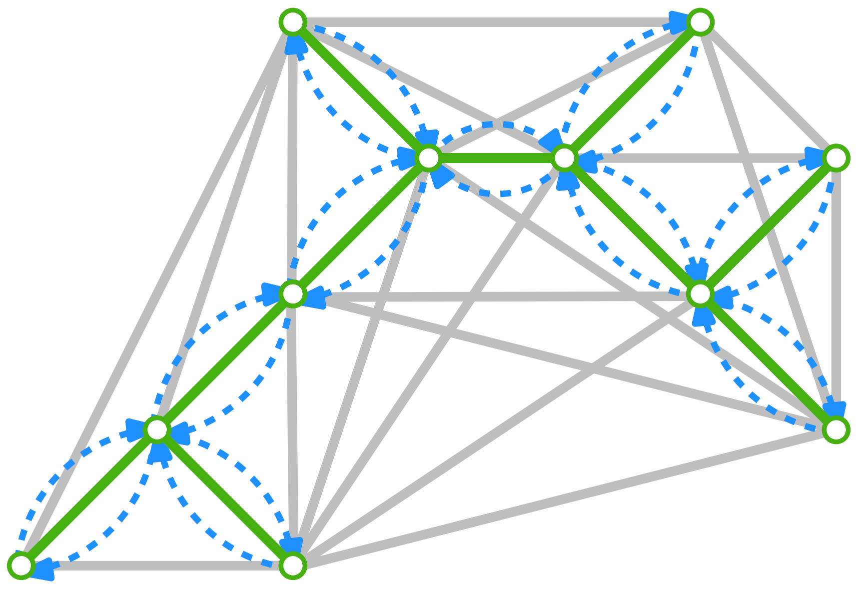

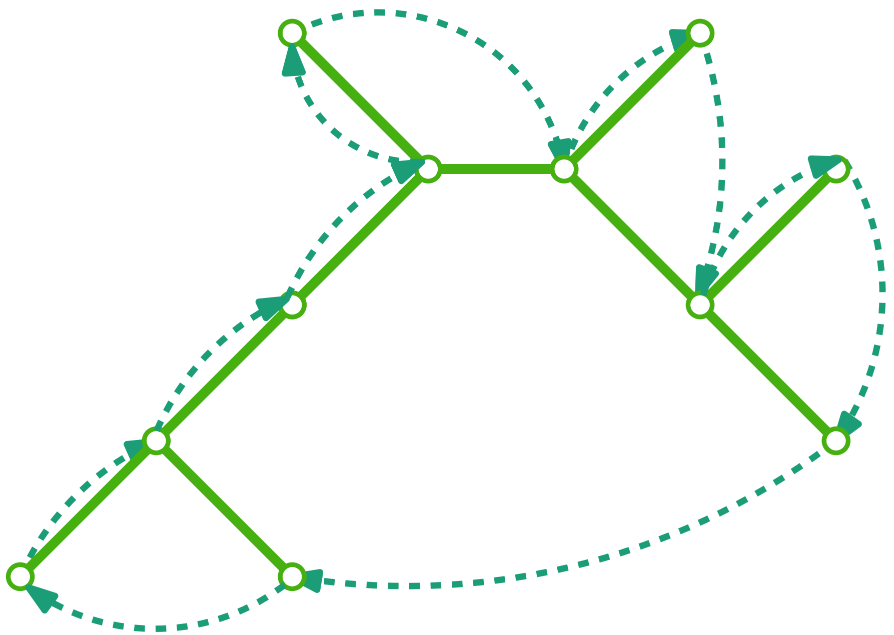

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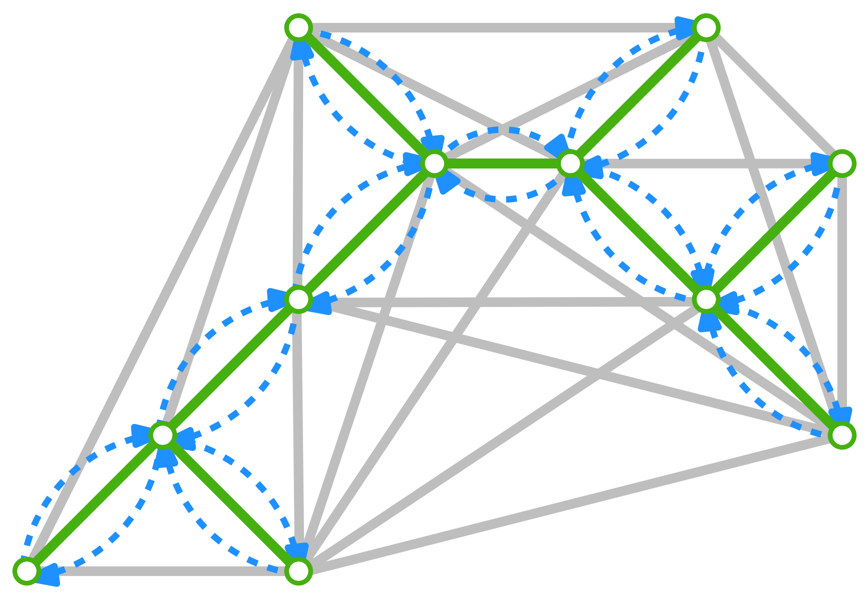

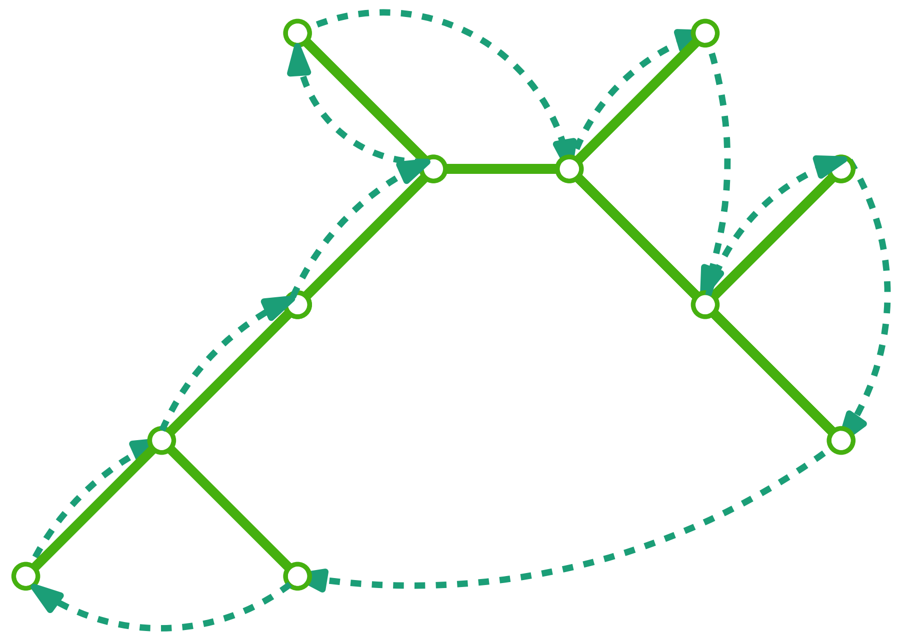

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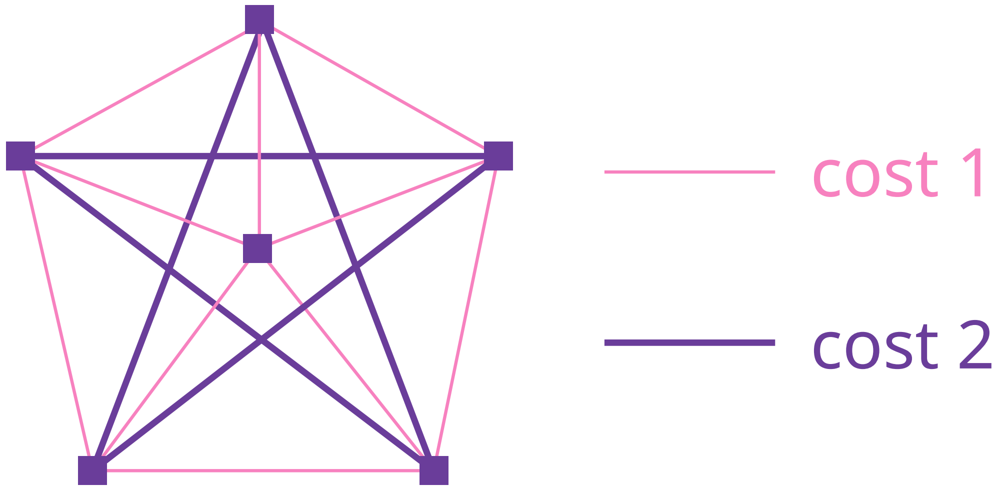
K_n



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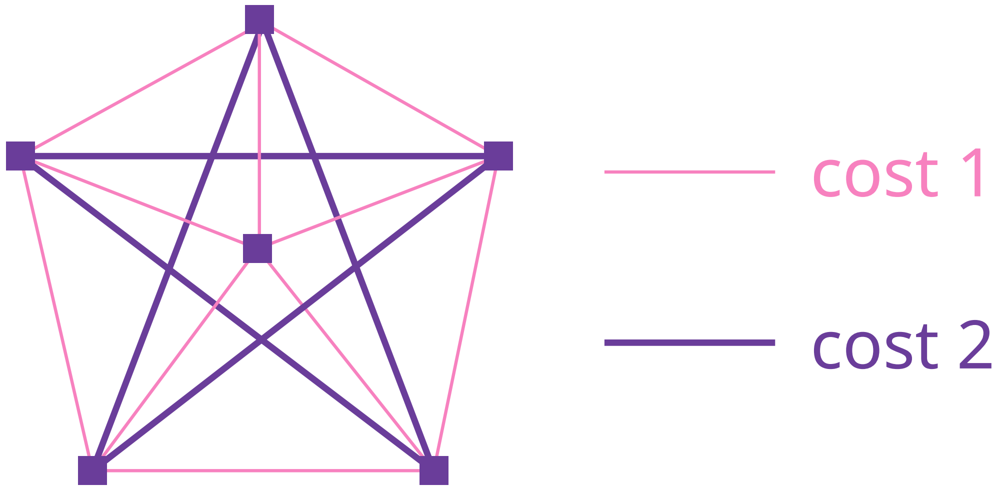

What is OPT?

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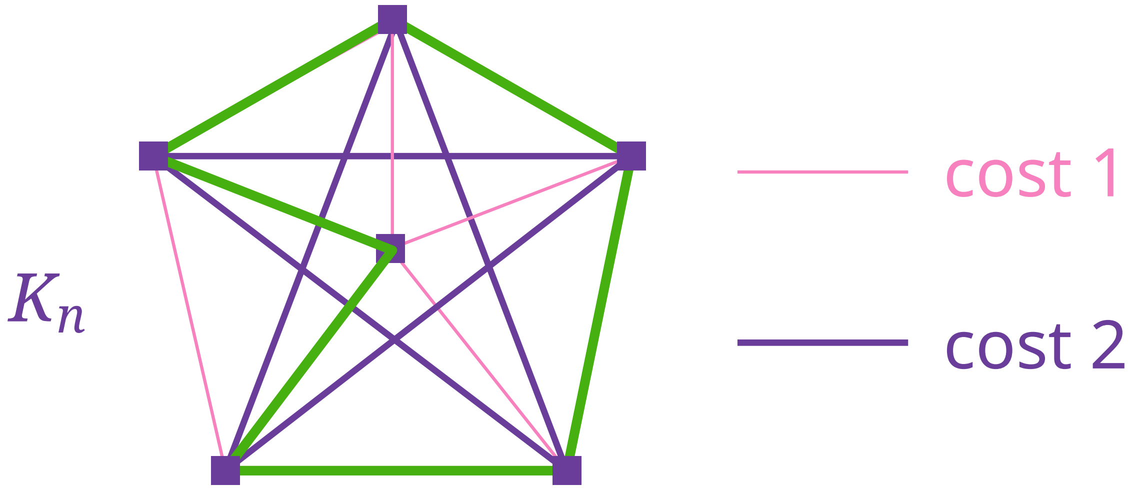

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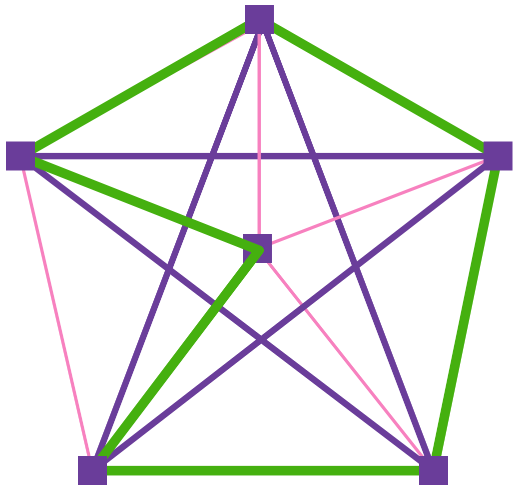

cost 1

cost 2

What is OPT? n

What does the algorithm compute?

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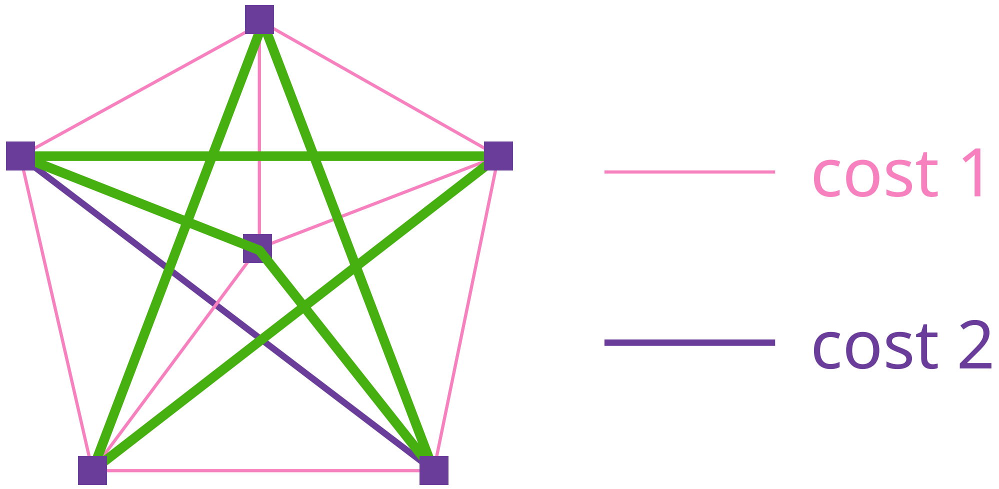

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depends on Eulertour
but can be $2n - 1$

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Christofides Algorithm:

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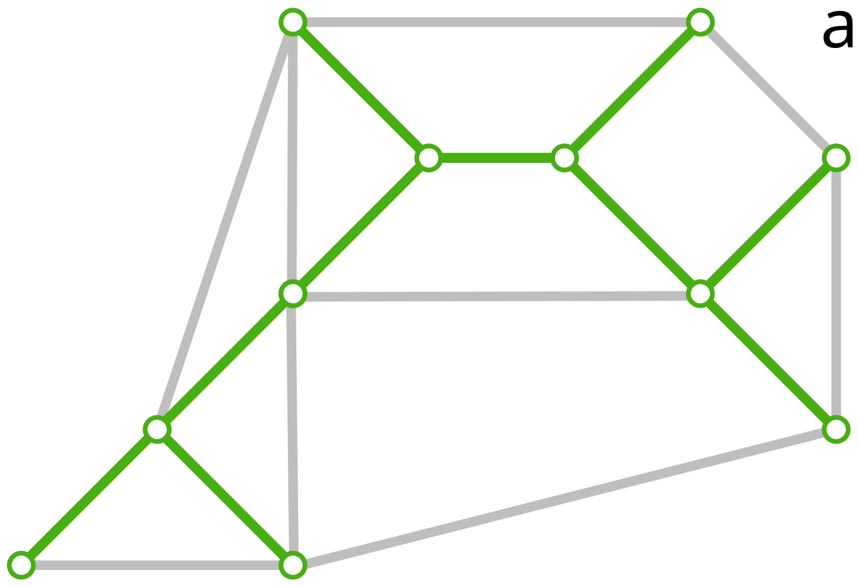
Can we improve this algorithm? Yes! instead of doubling the edges in T add a mincost perfect matching of all odd degree vertices



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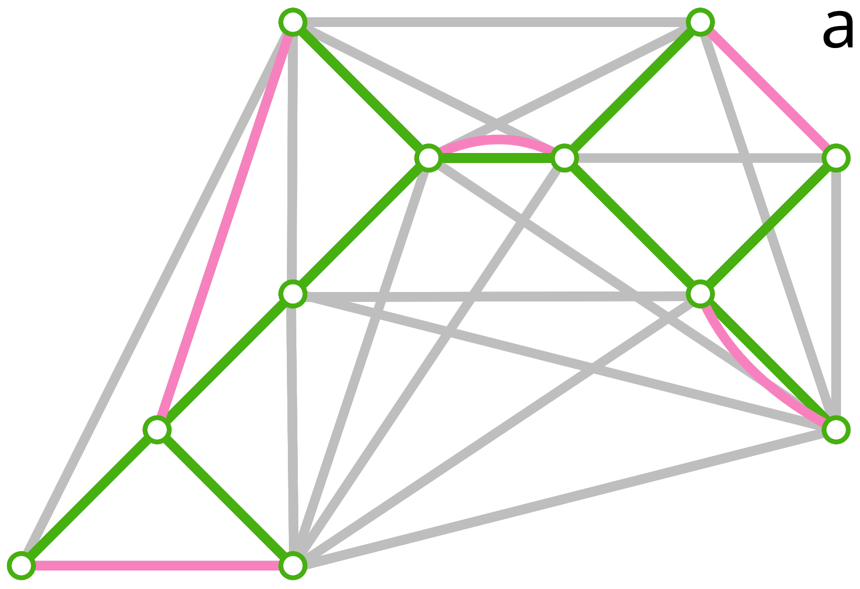

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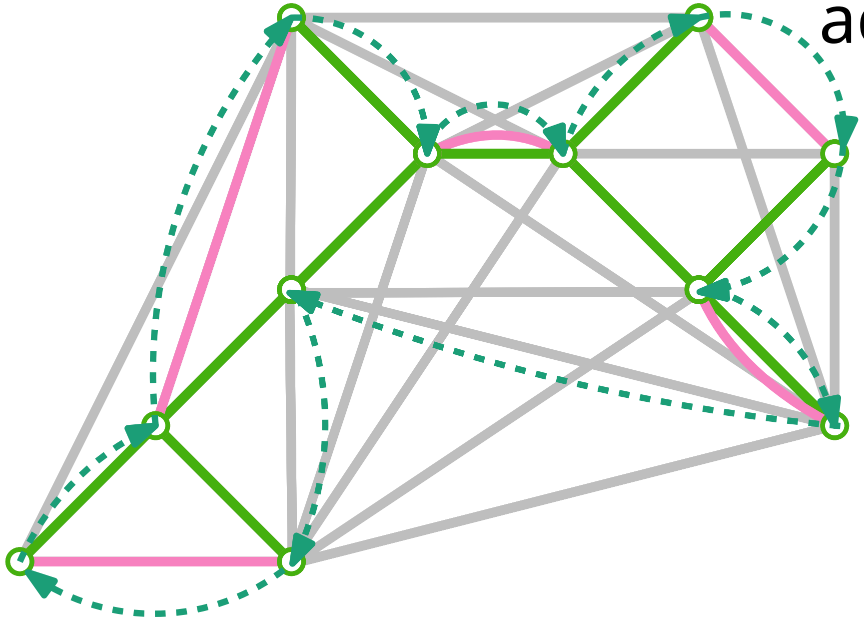

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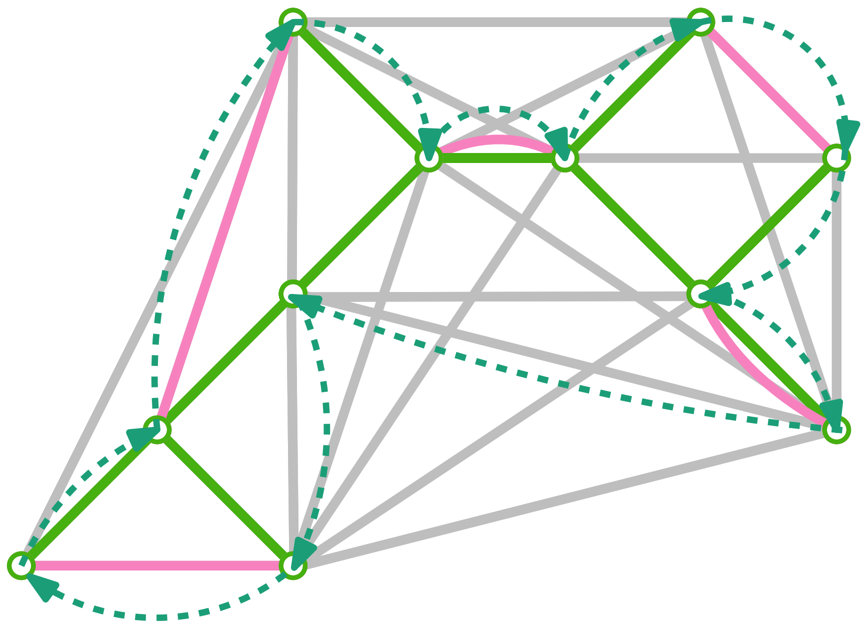

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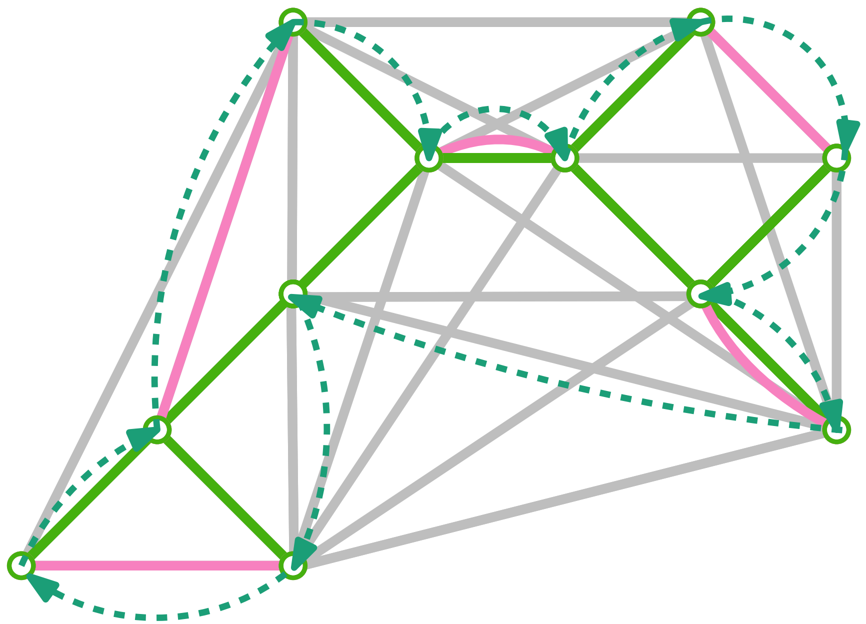

Why does a perfect matching always exist?

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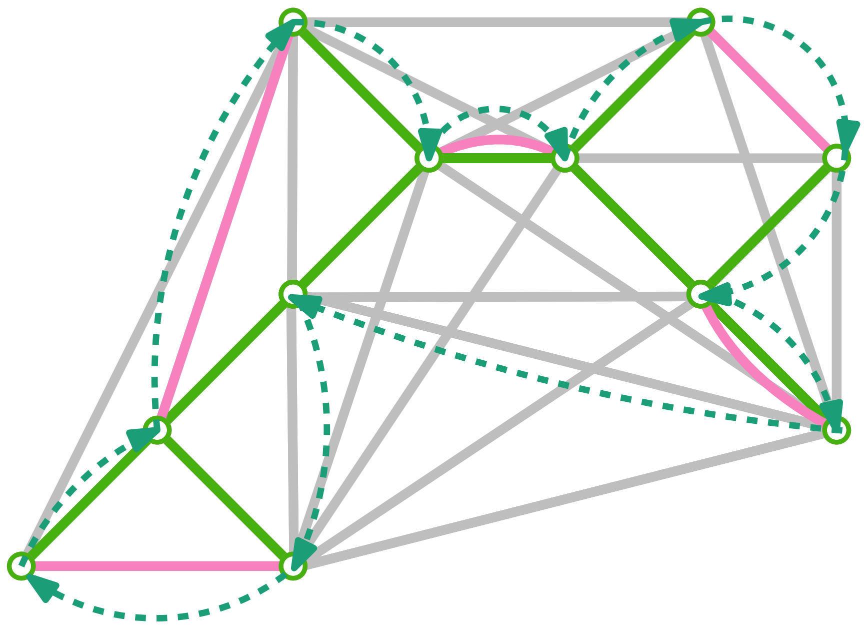

Why does a perfect matching always exist?

By handshaking lemma the total sum of degrees is even, hence the number of odd degree vertices must be even .

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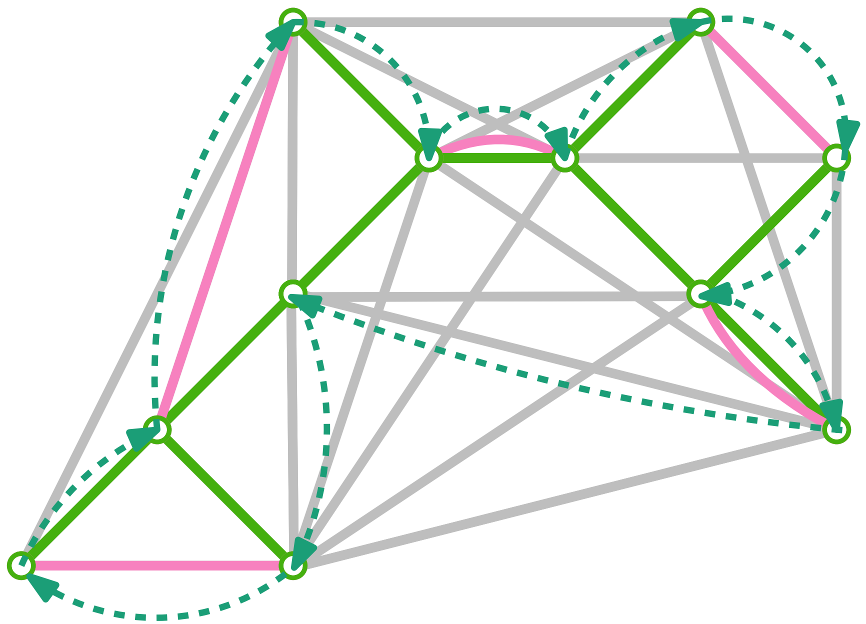

What is the cost of such a mincost matching?

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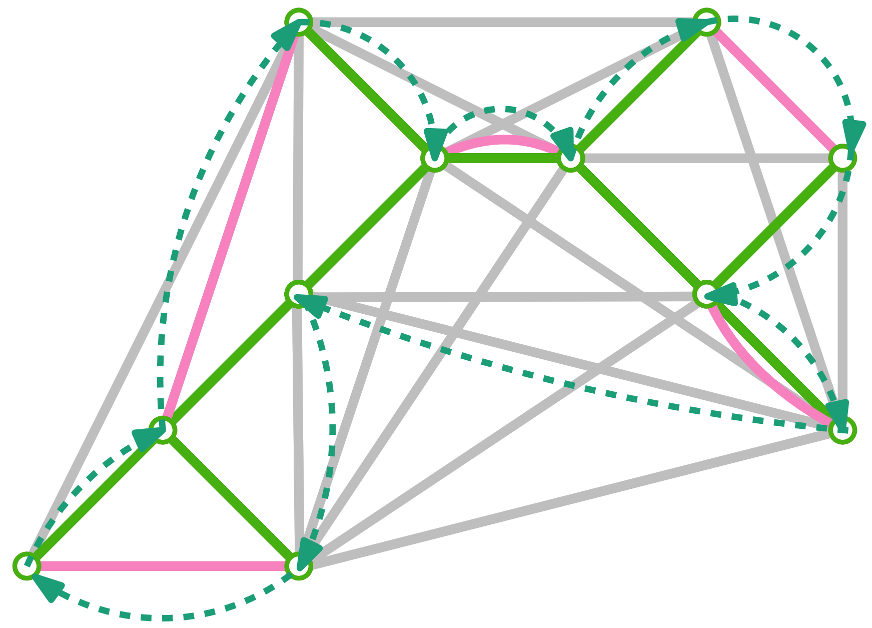

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$\text{cost}(M) \leq \text{OPT}/2$ half the cost of a cycle

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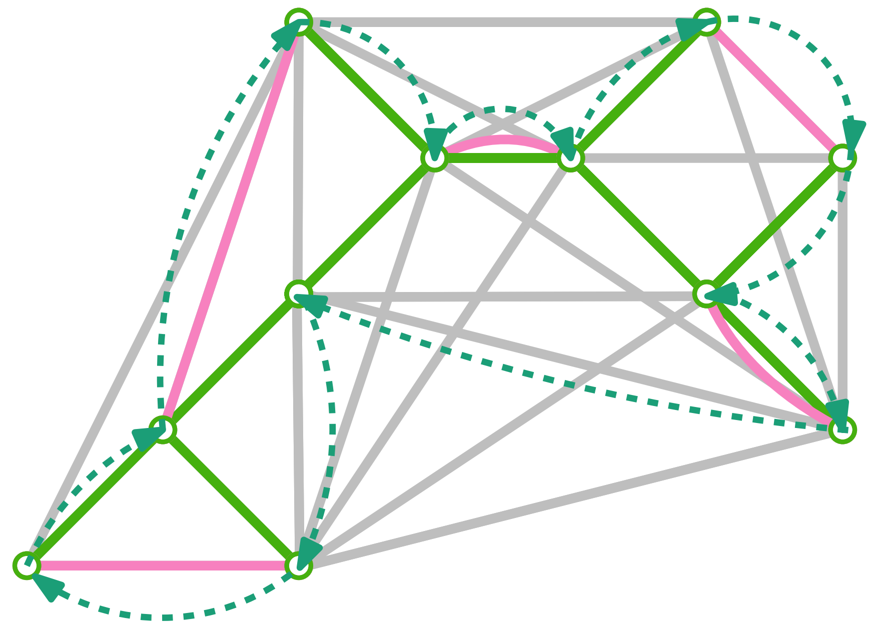

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Hence the total cost is

$\text{cost}(C) \leq \text{cost}(E) = \text{cost}(T) + \text{cost}(M) \leq 3/2\ \text{OPT}$

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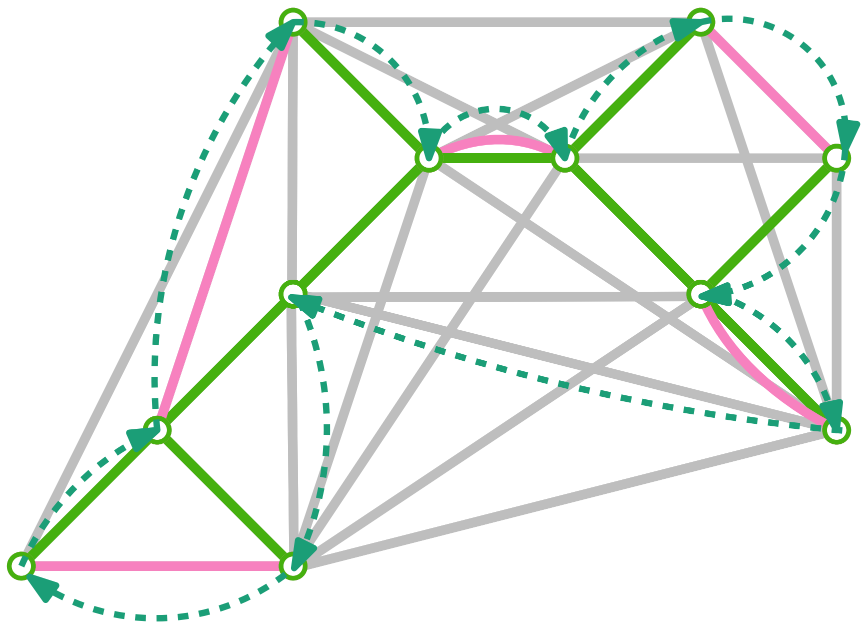

Is the analysis tight?

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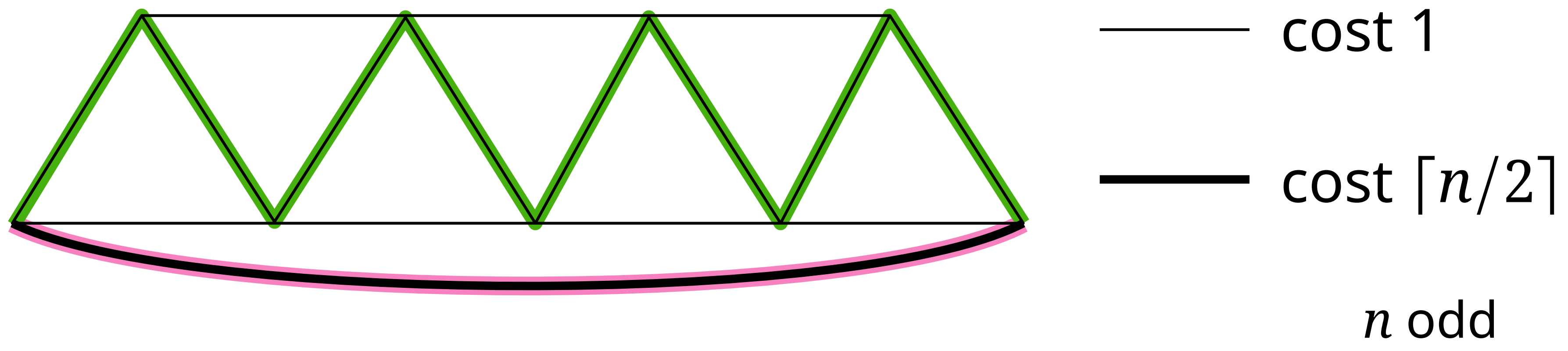
What does the algorithm compute?



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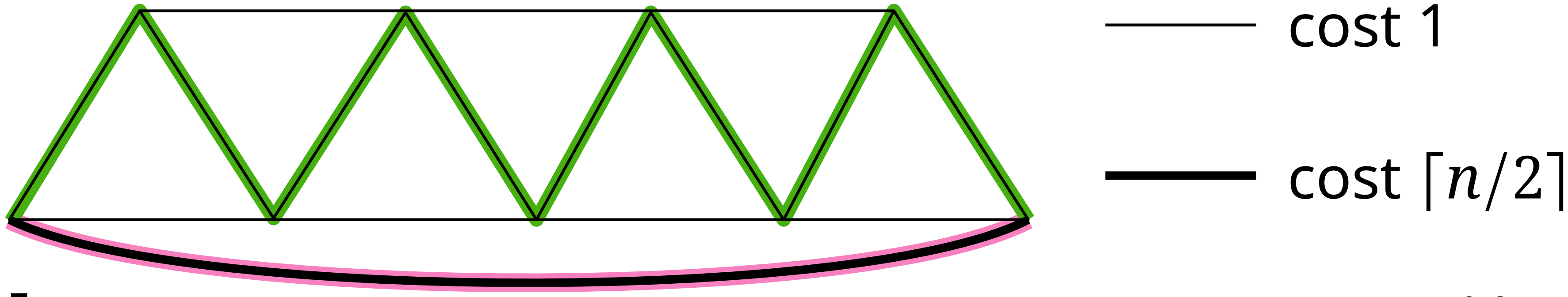
What is OPT? n

What does the algorithm compute?

$n - 1 + \lceil n/2 \rceil$



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Theorem. The algorithm is a $3/2$-approximation for TRAVELINGSALESPERSONPROBLEM.

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ALGORITHMS

Computer Scientists Break Traveling Salesperson Record

24

After 44 years, there's finally a better way to find approximate solutions to the notoriously difficult traveling salesperson problem.

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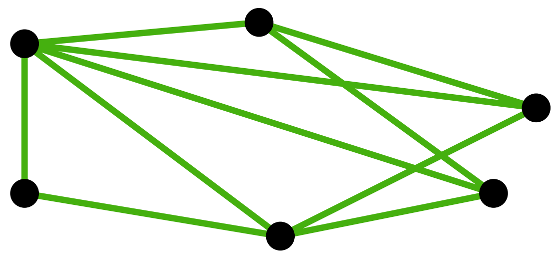

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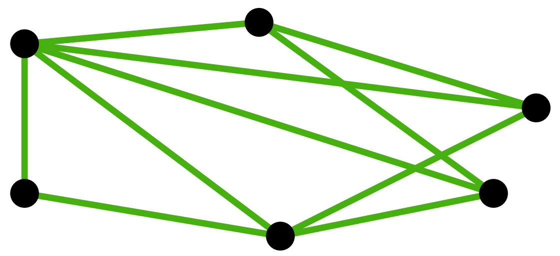

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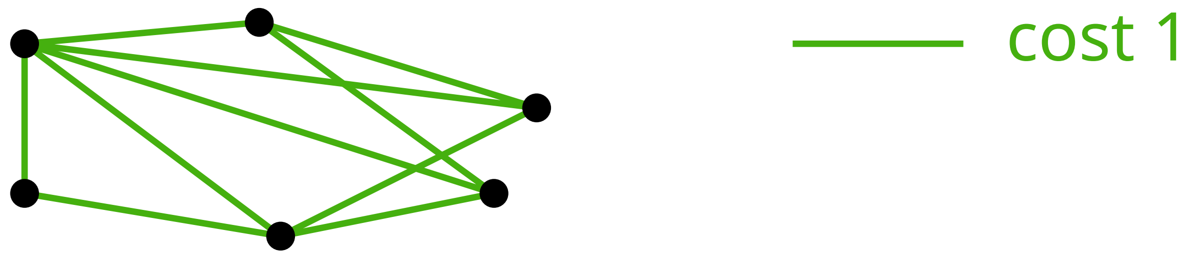

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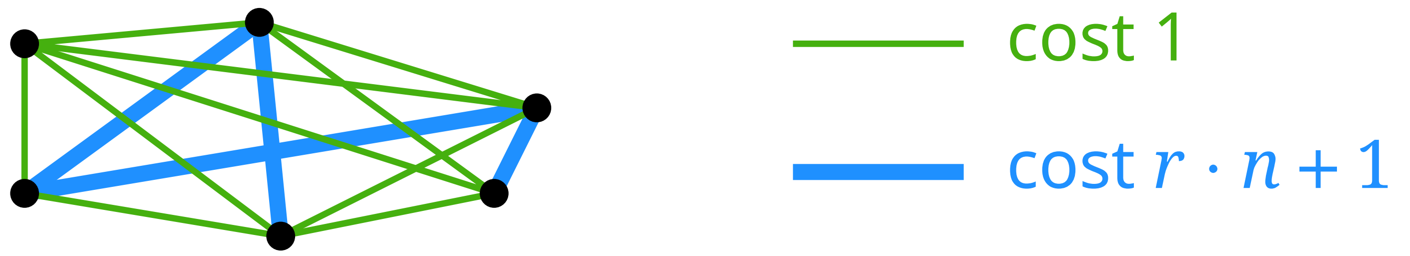

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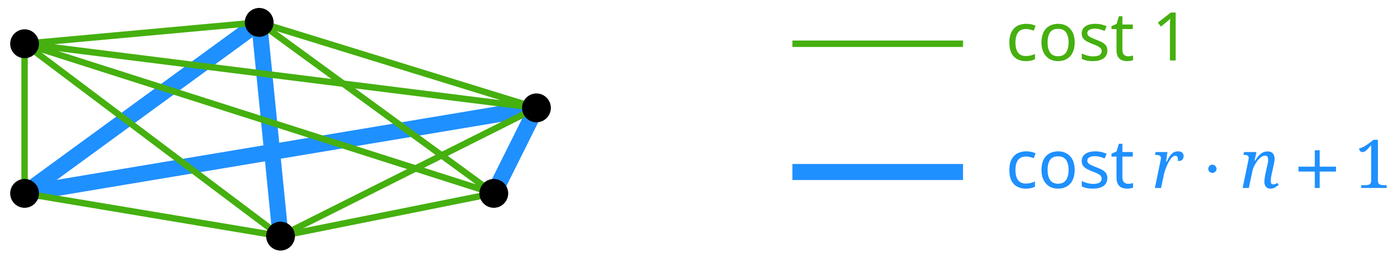

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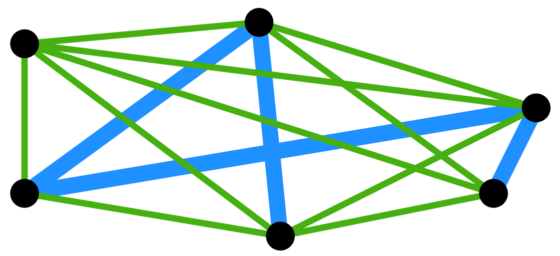

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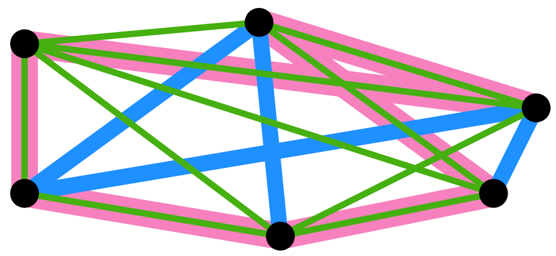

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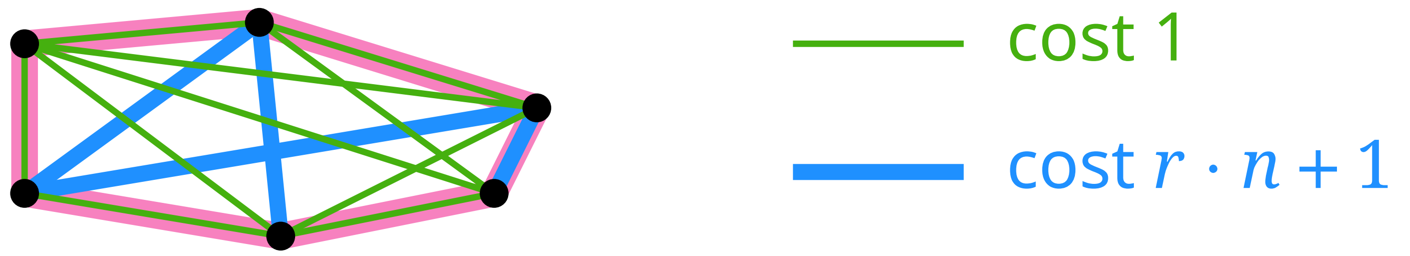

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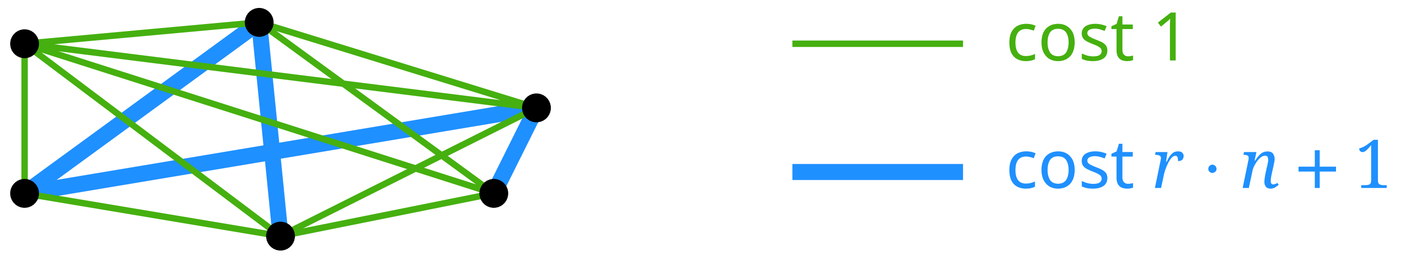

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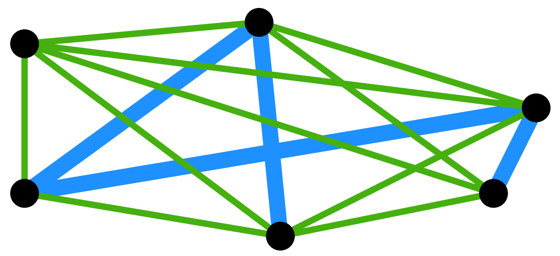

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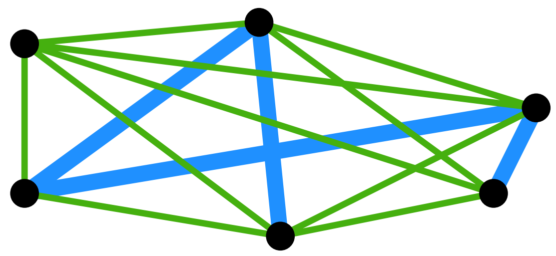

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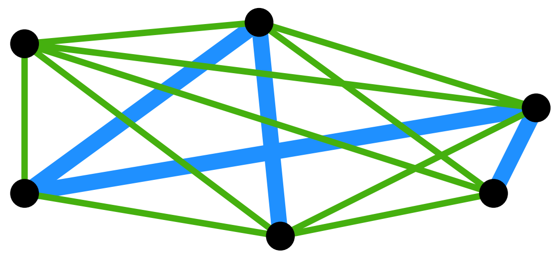

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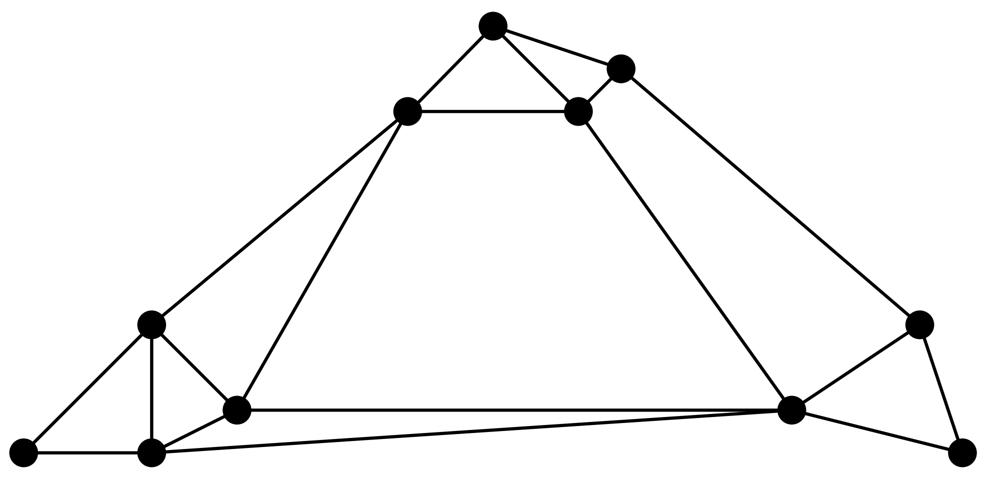

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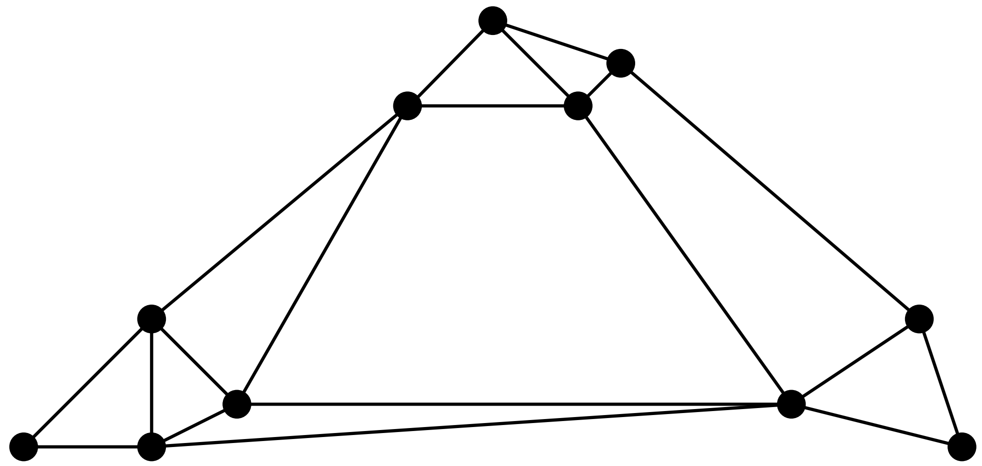

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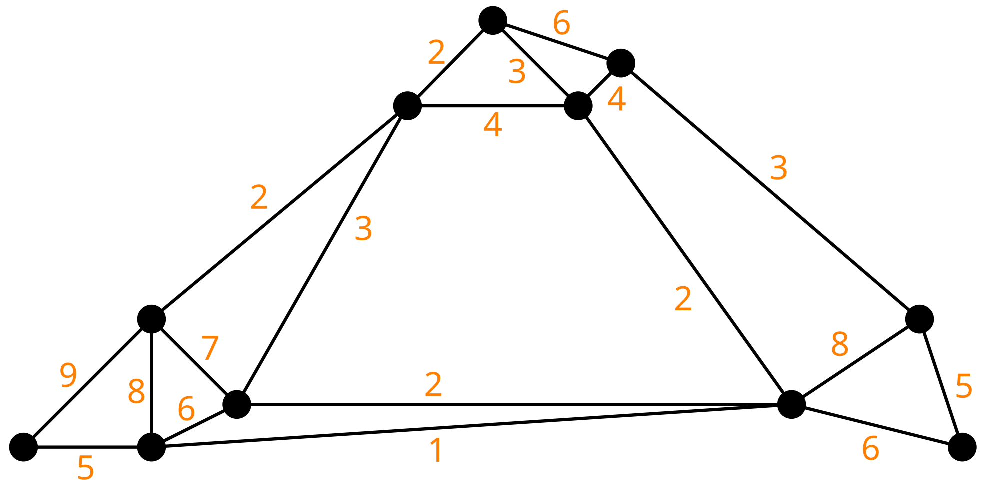

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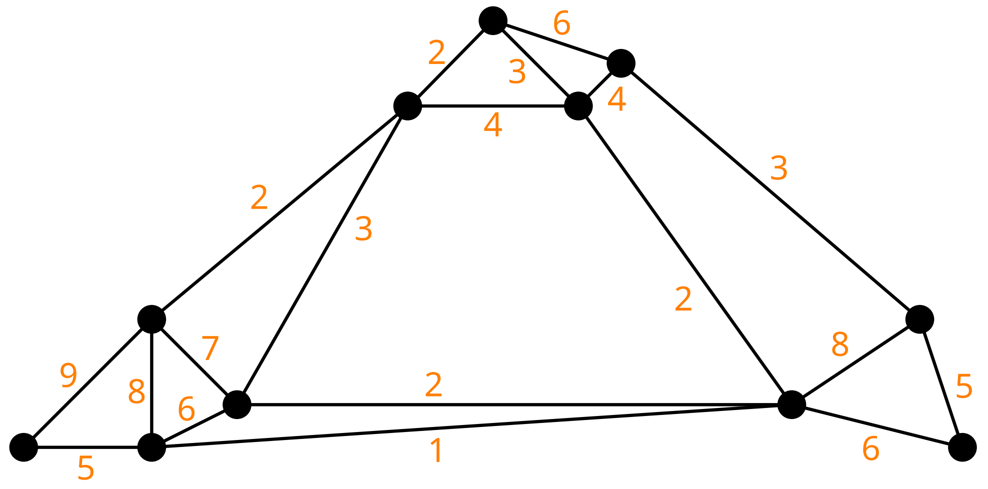

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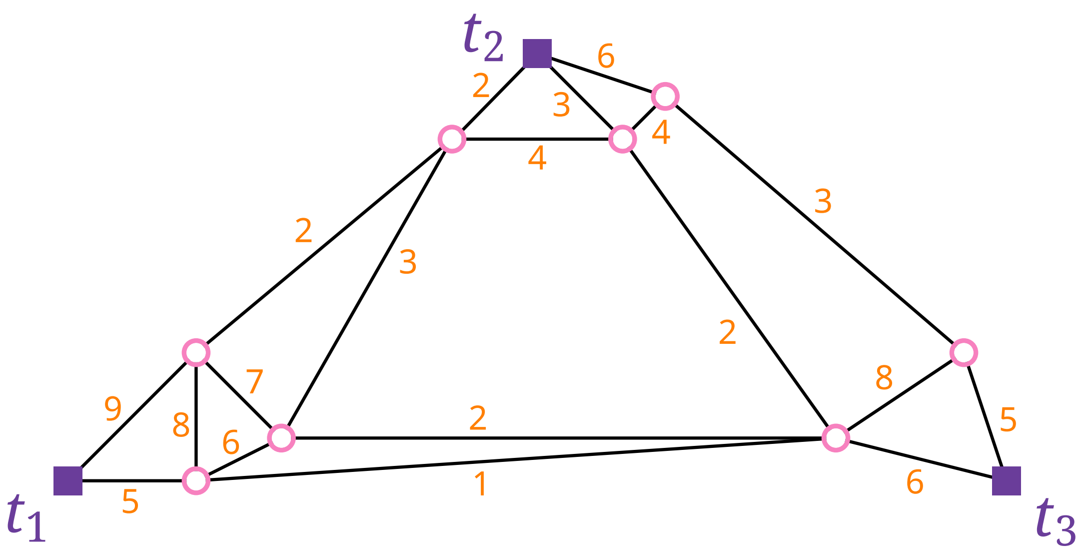

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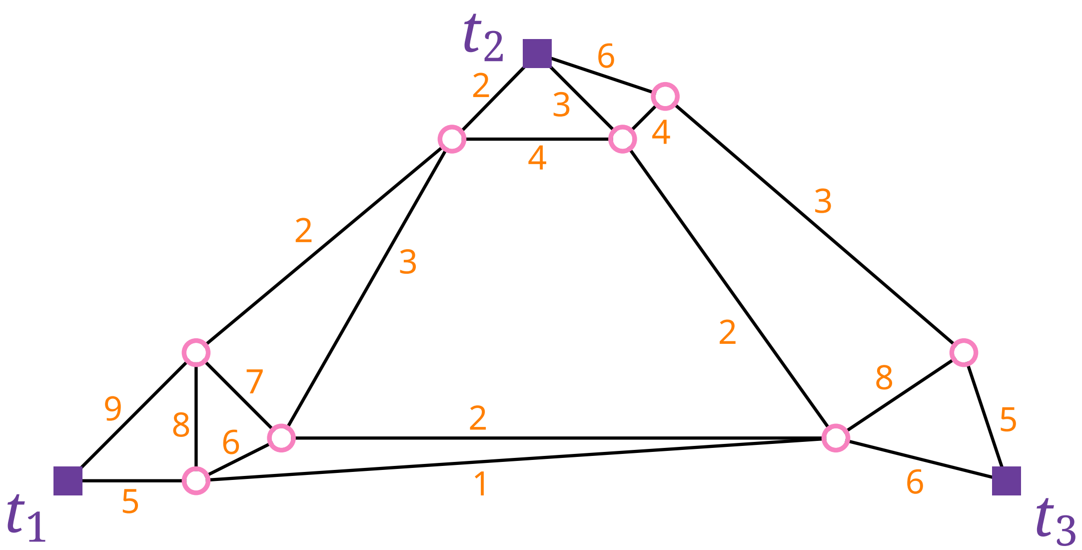

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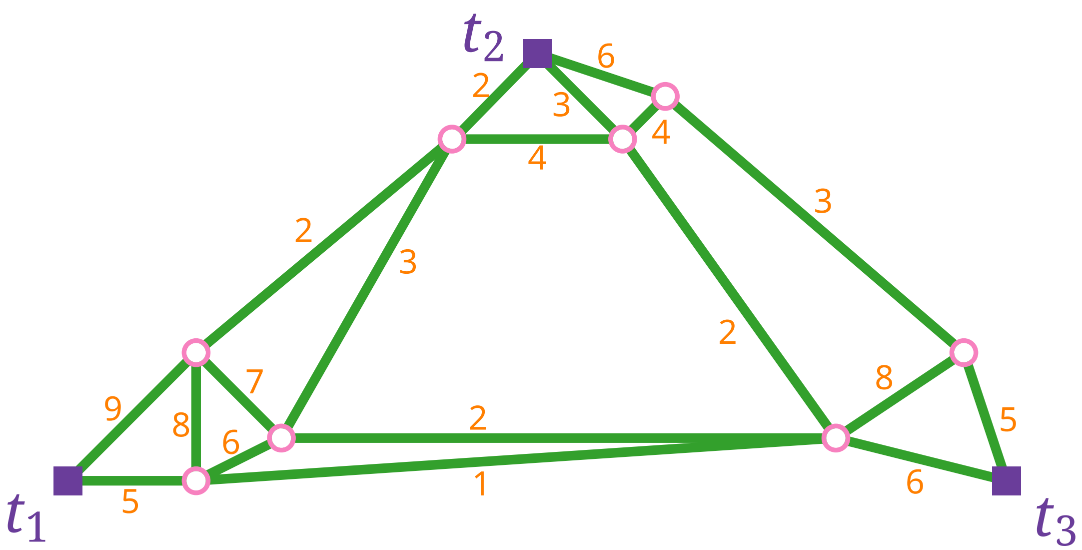

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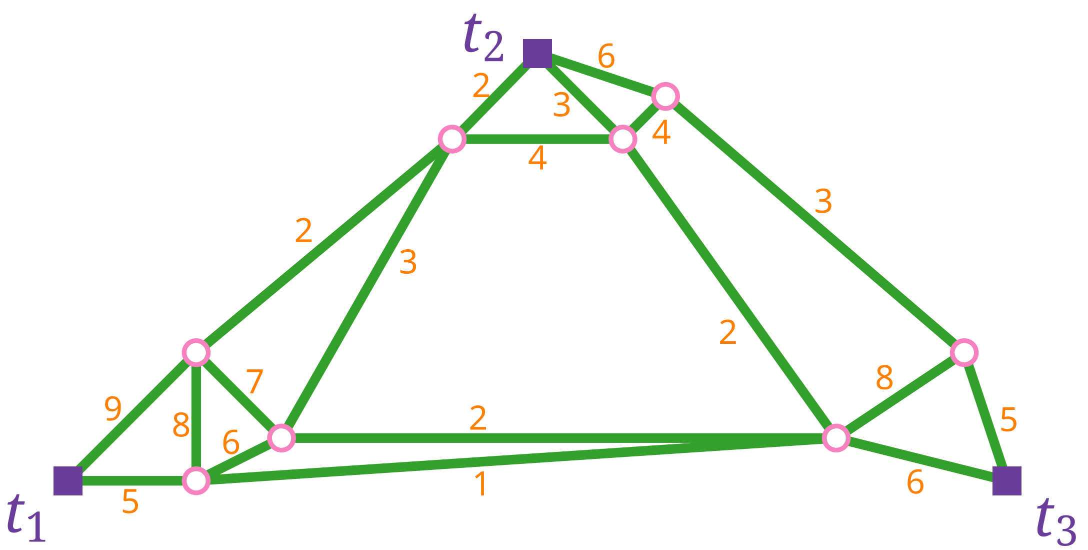

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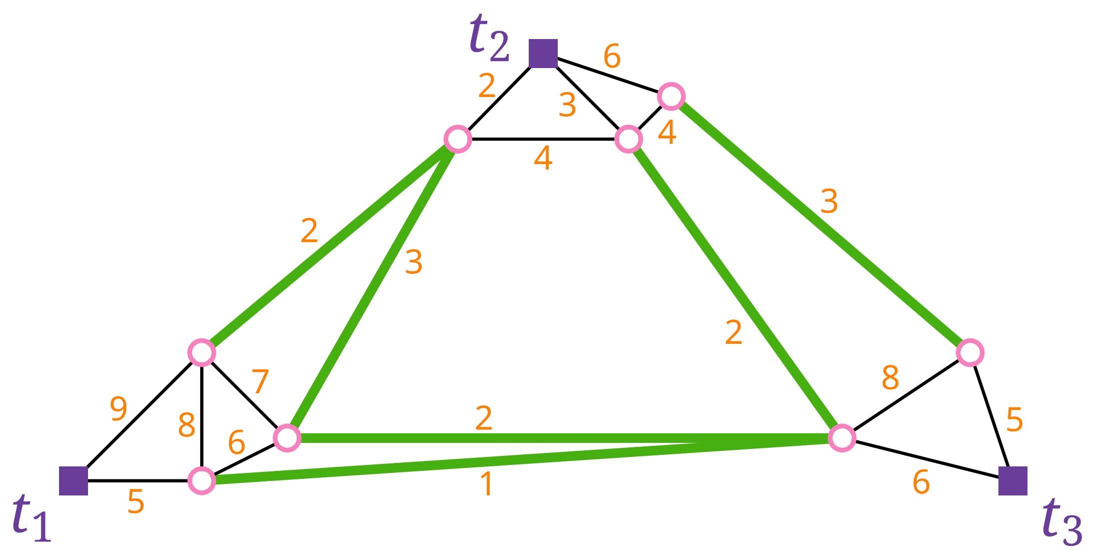

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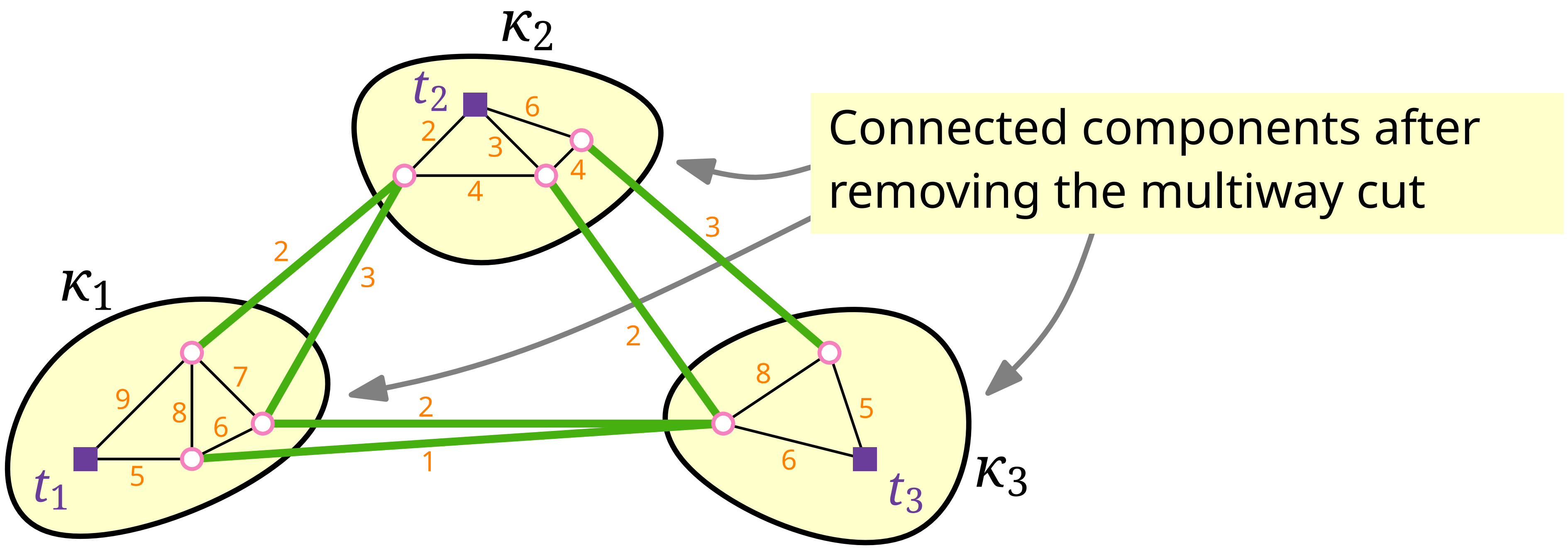

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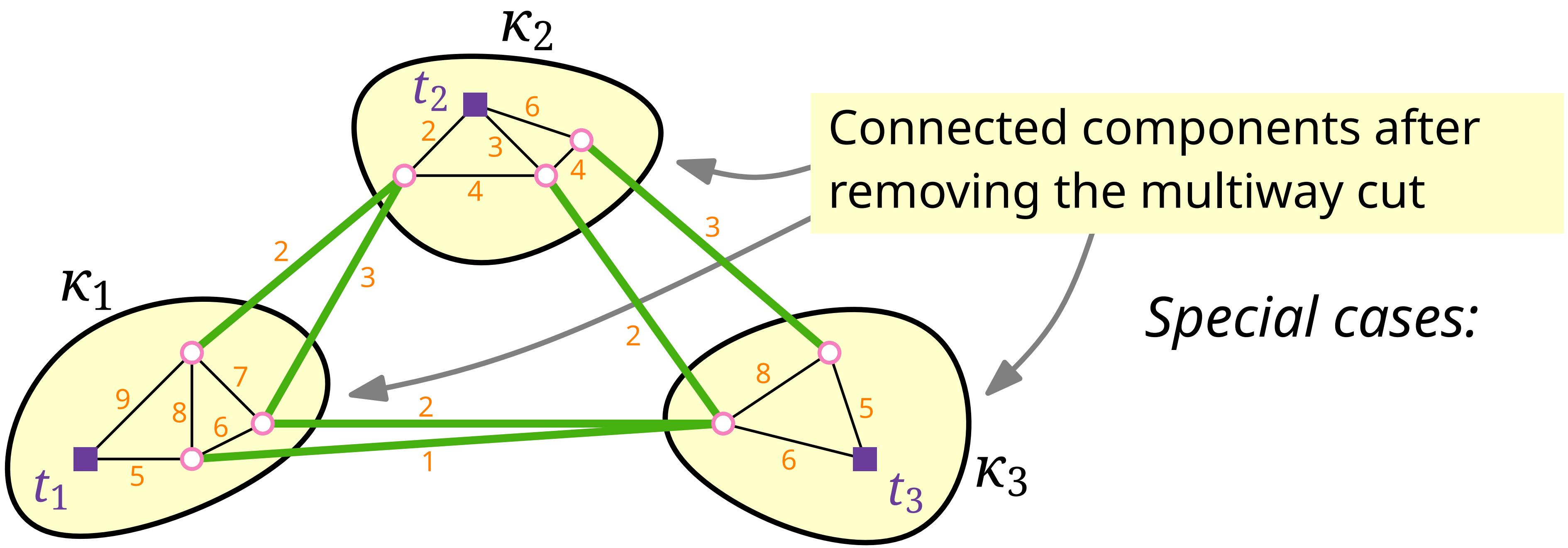

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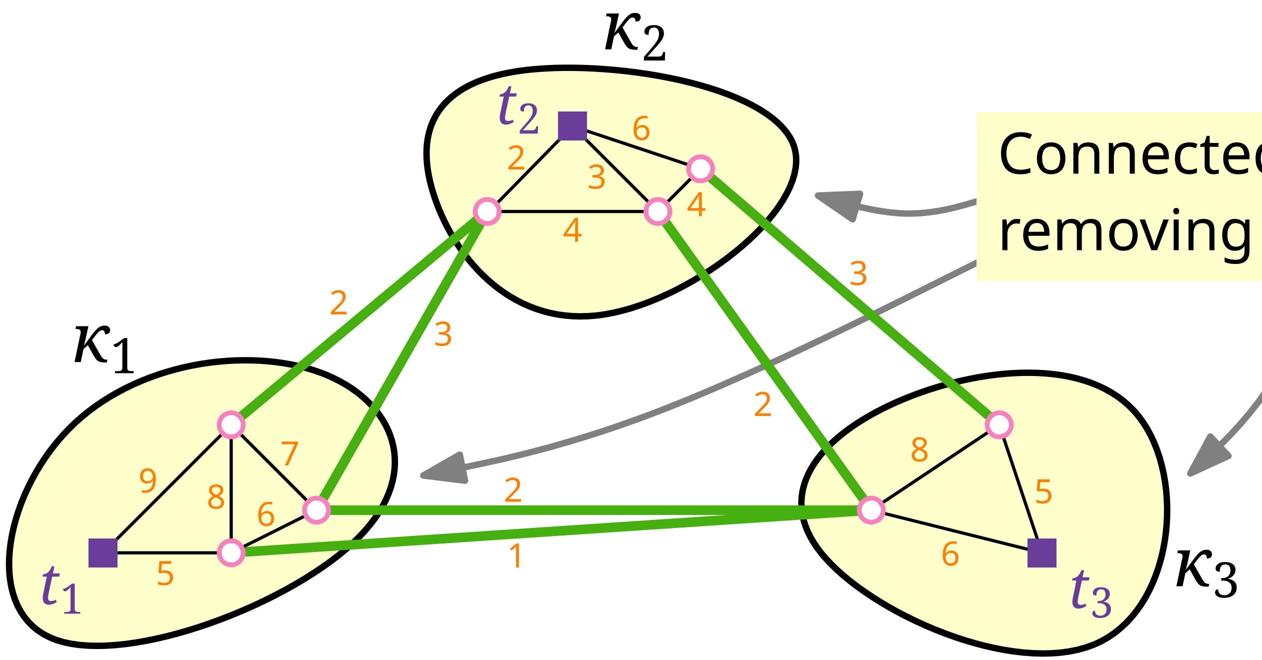

Connected components after removing the multiway cut

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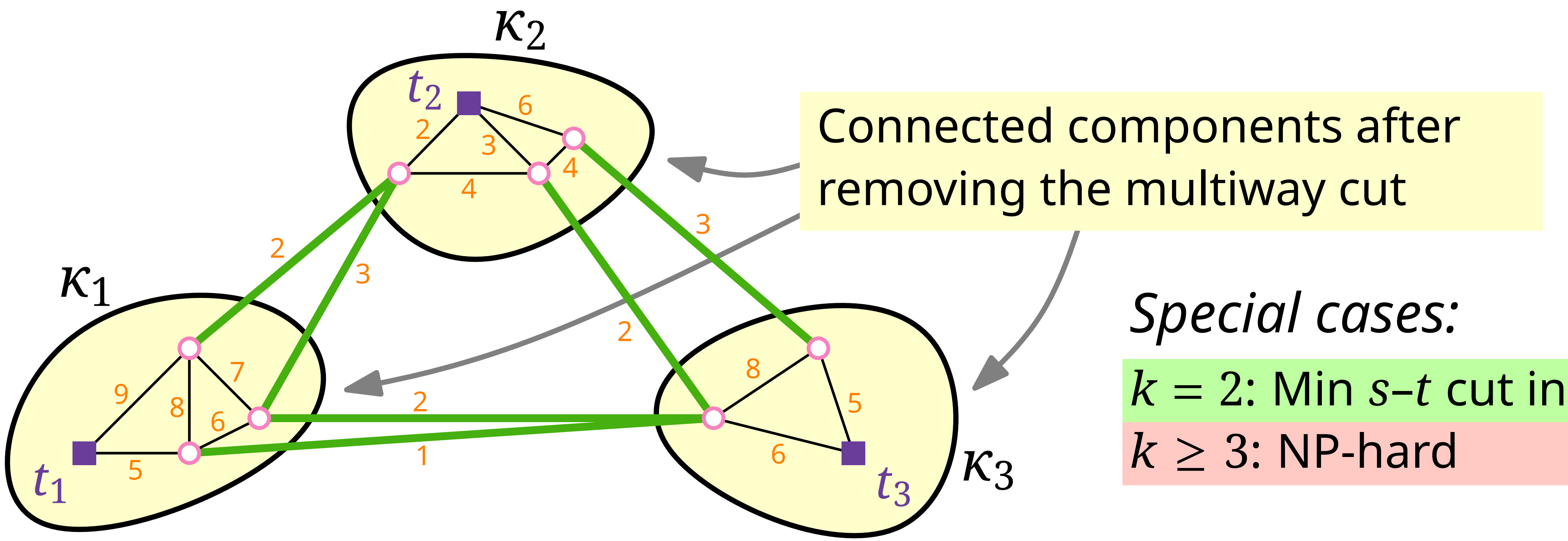

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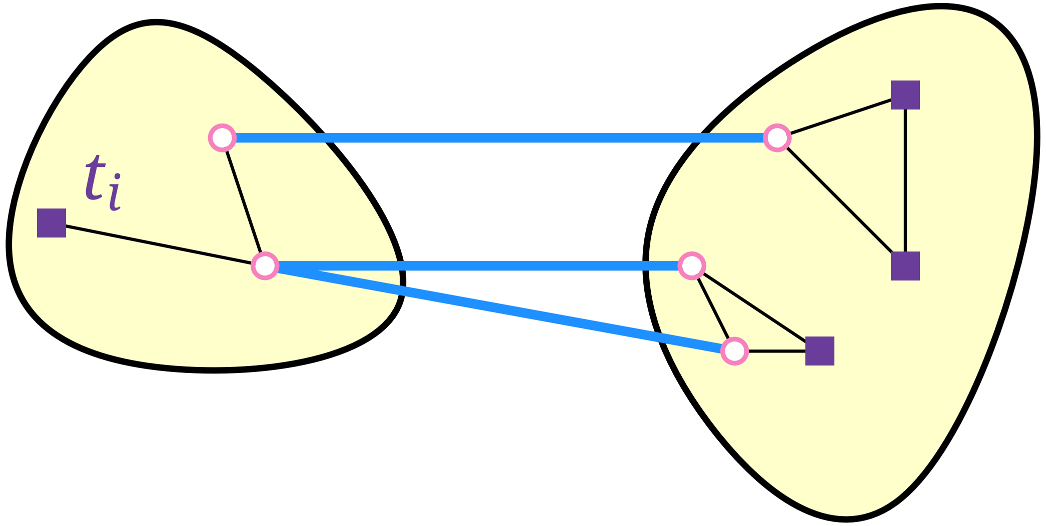

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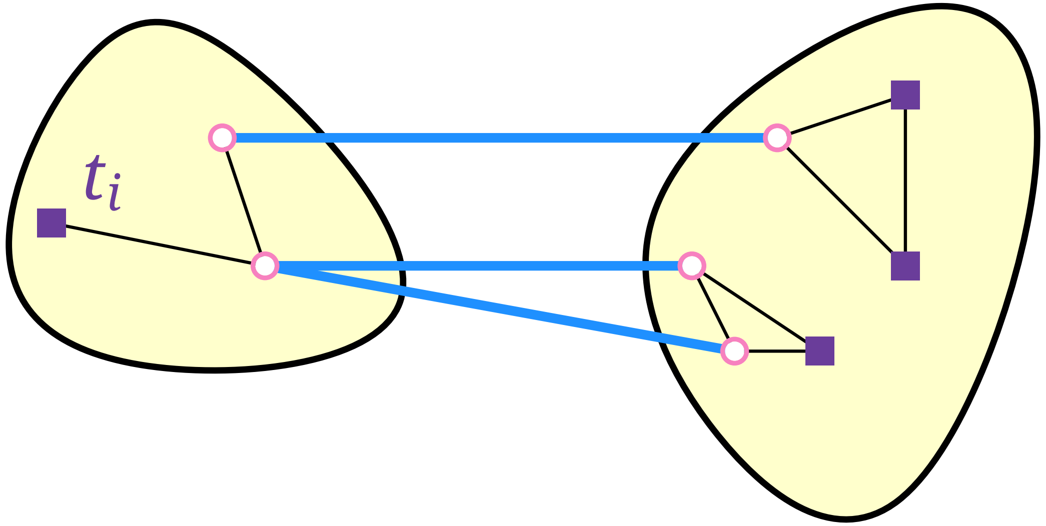

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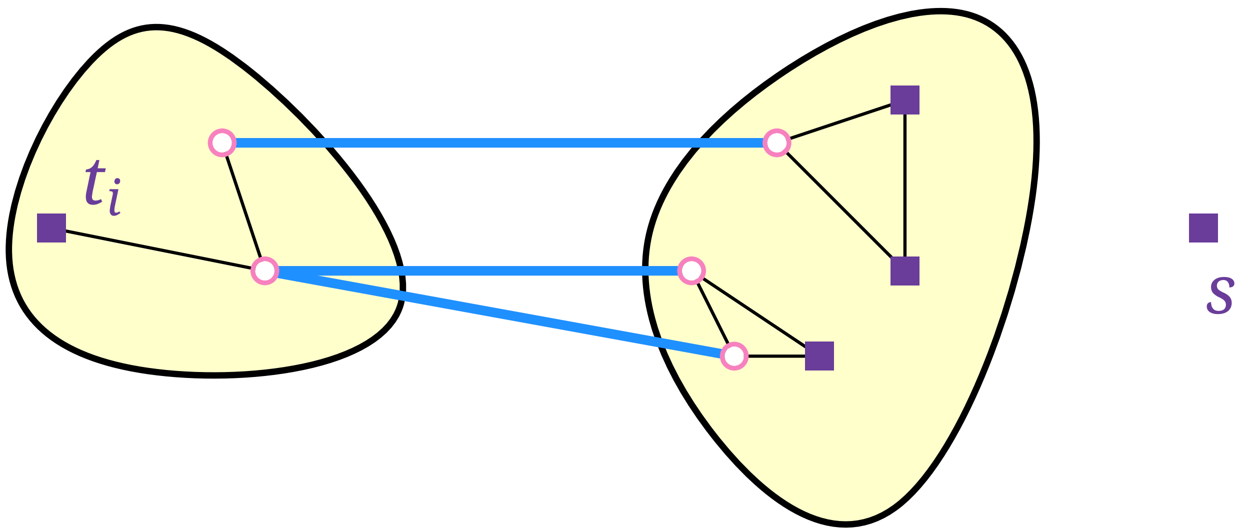

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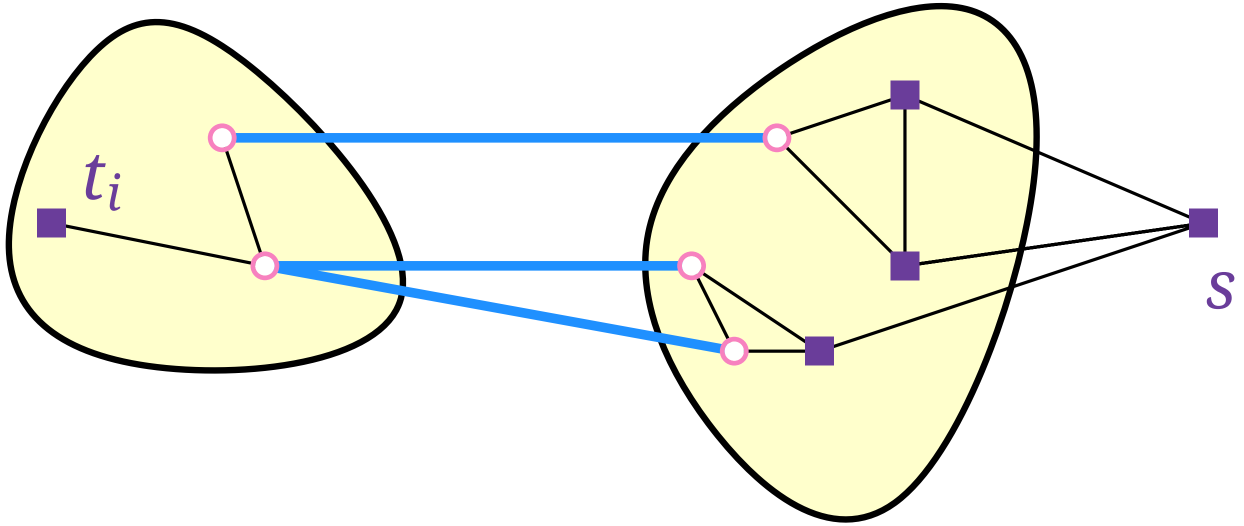

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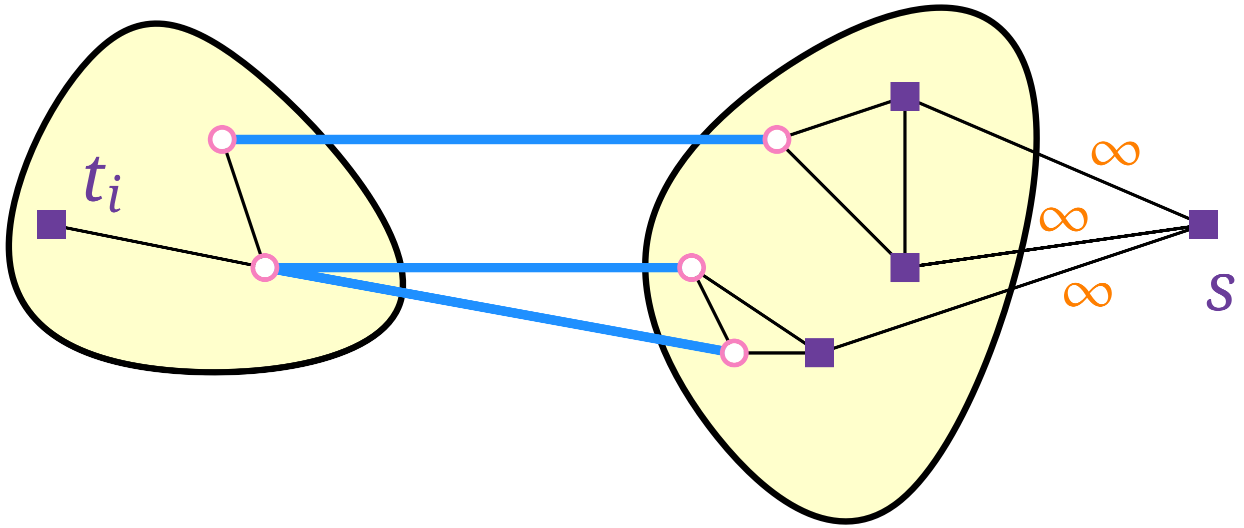

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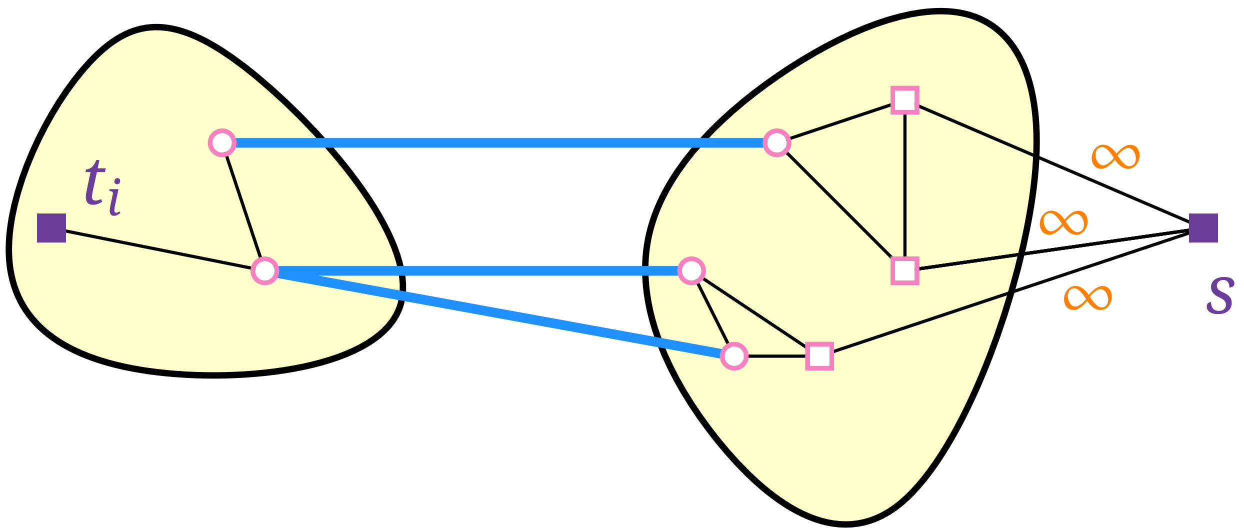

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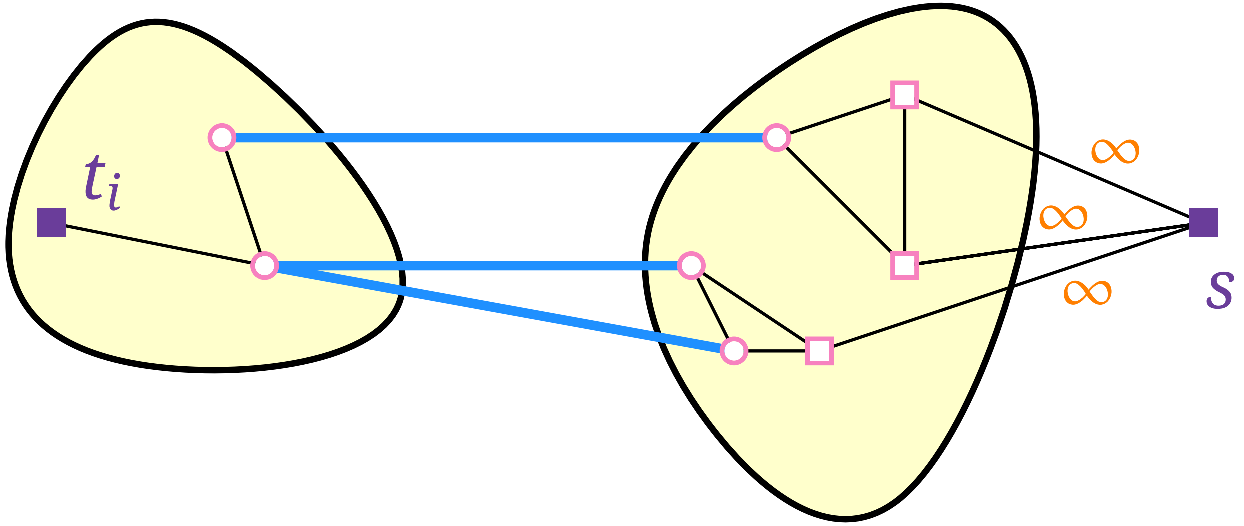

Add dummy terminal s and find a minimum-cost s–t_i cut.

Algorithm for MULTIWAYCUT

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For $i = 1, \ldots, k$:

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In other words:
Ignore the most expensive one of the isolating cuts $C_1, \ldots, C_k$.

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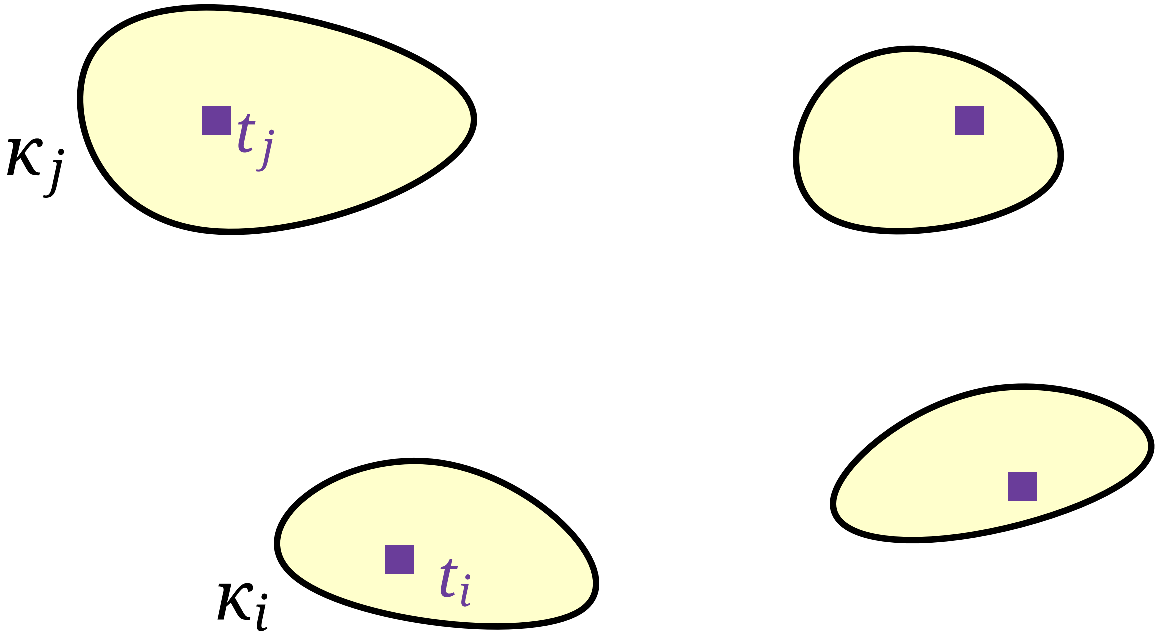

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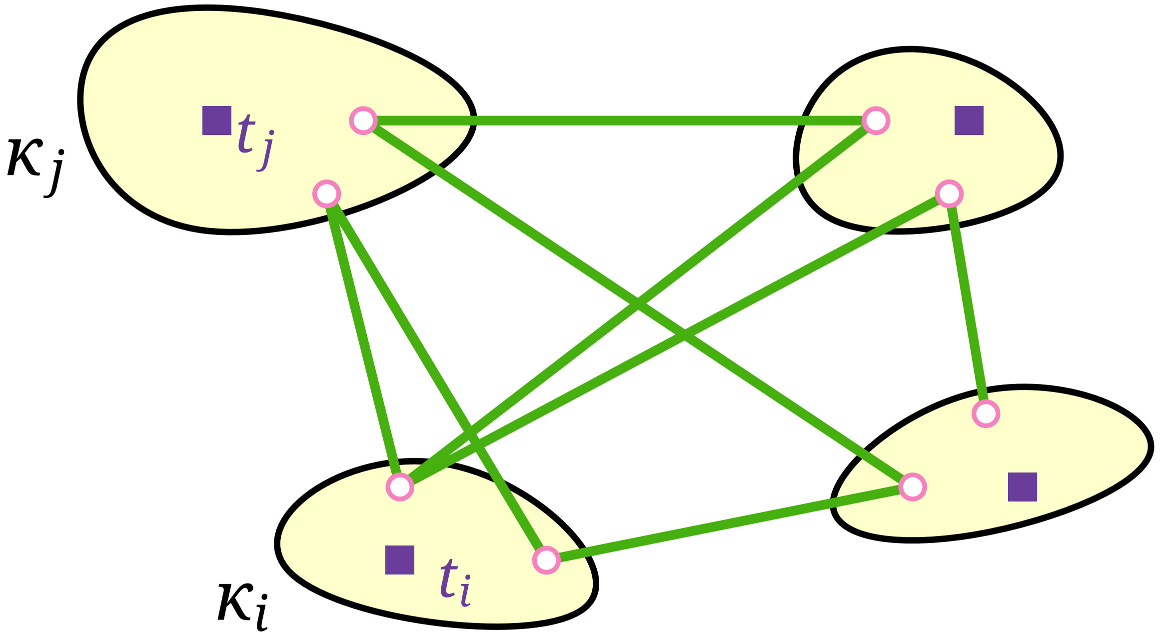

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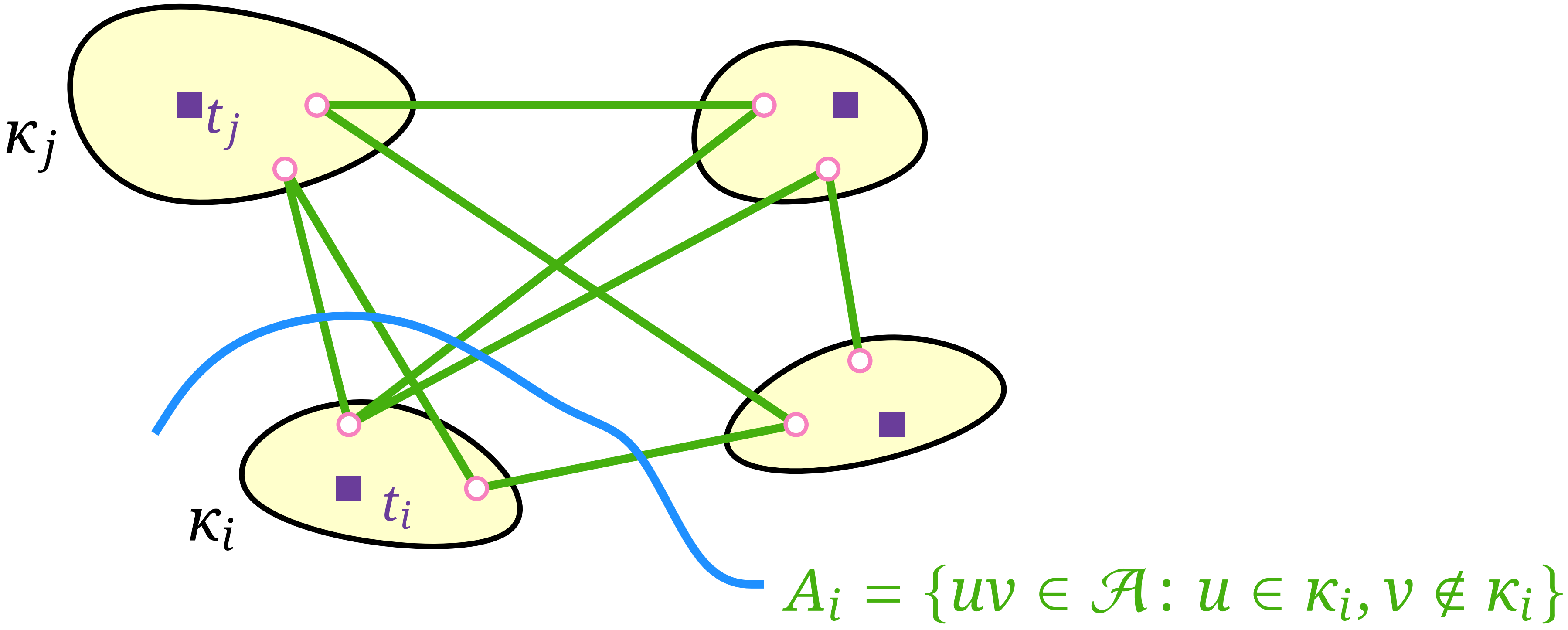

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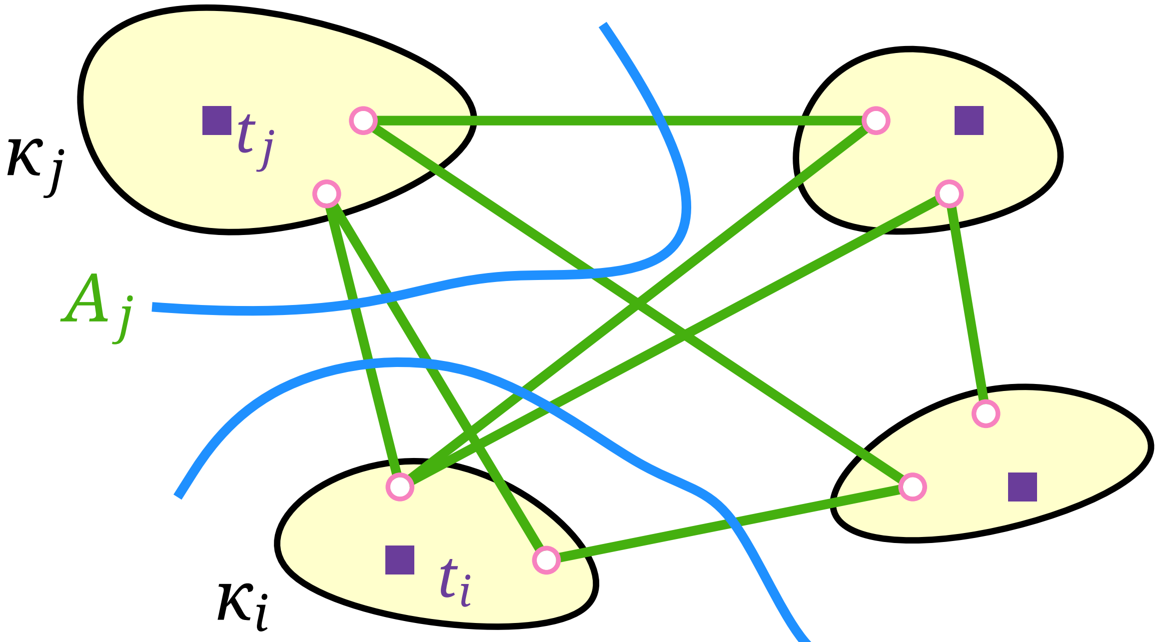

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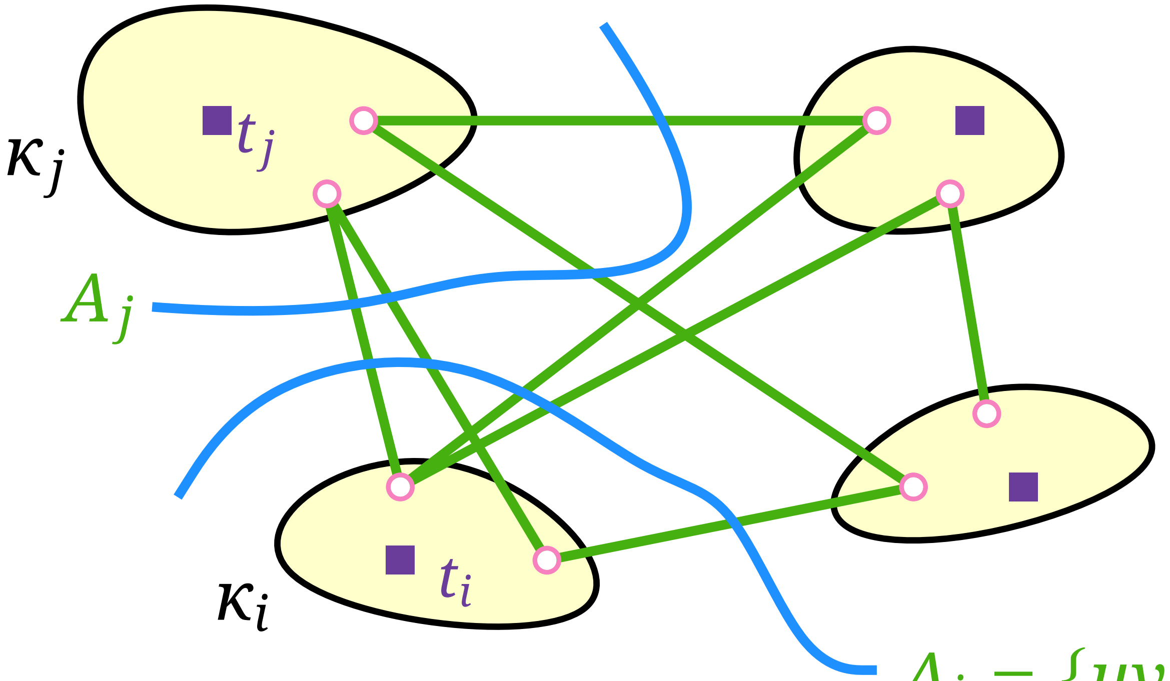

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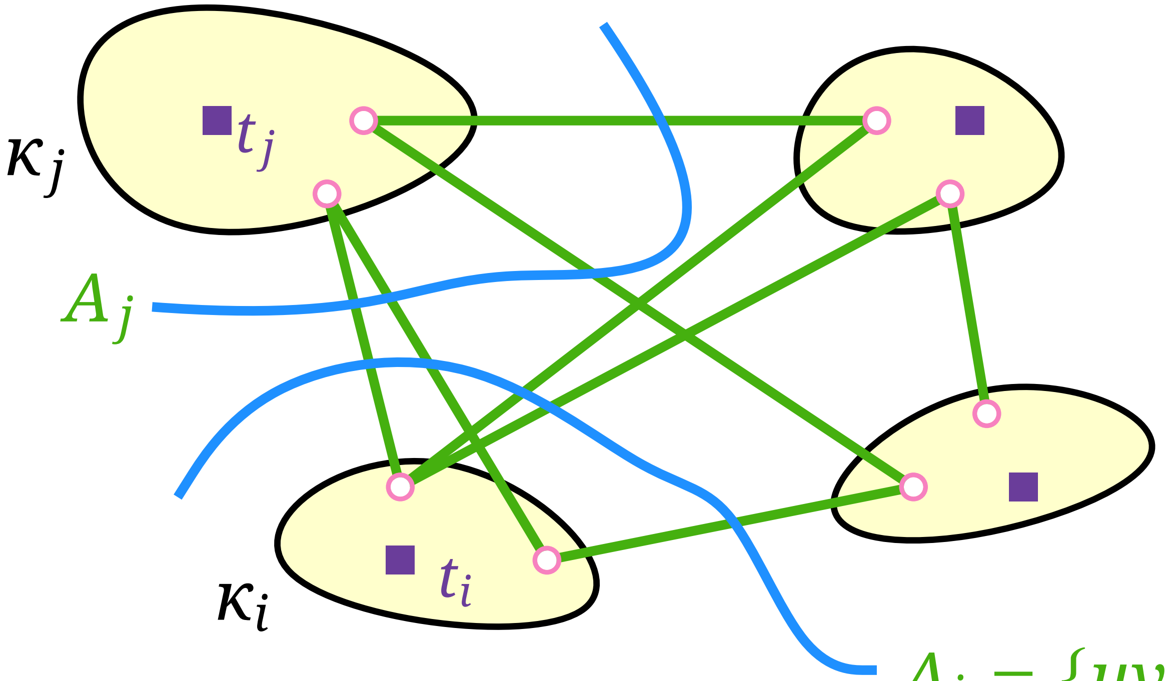

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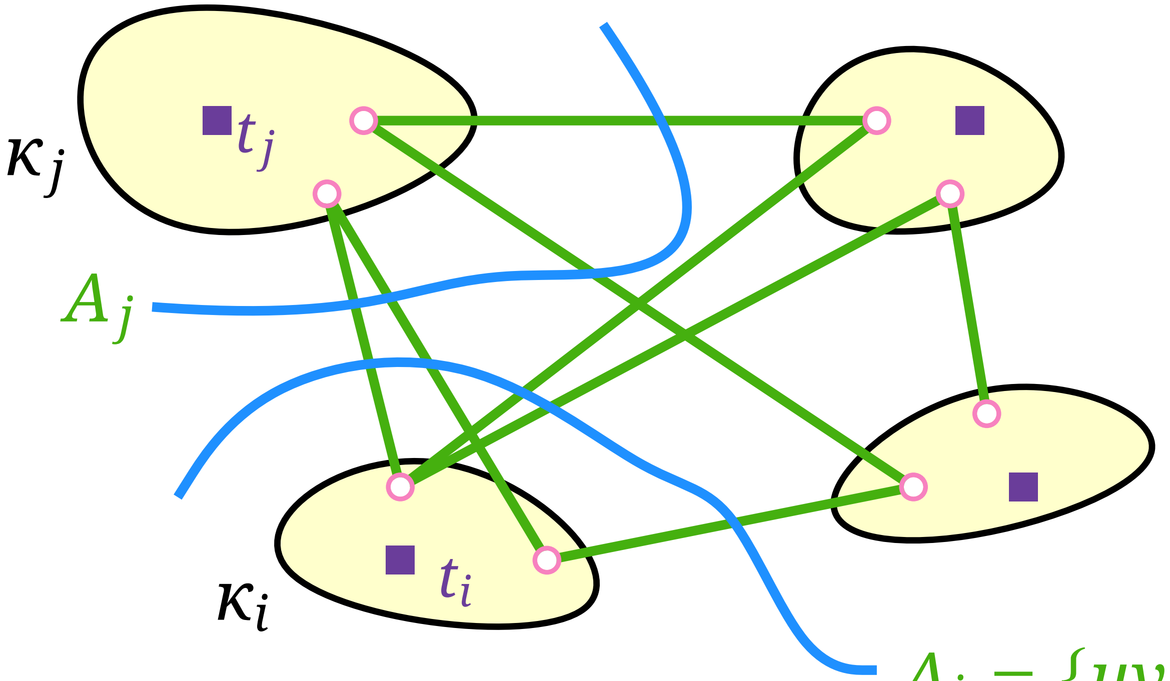

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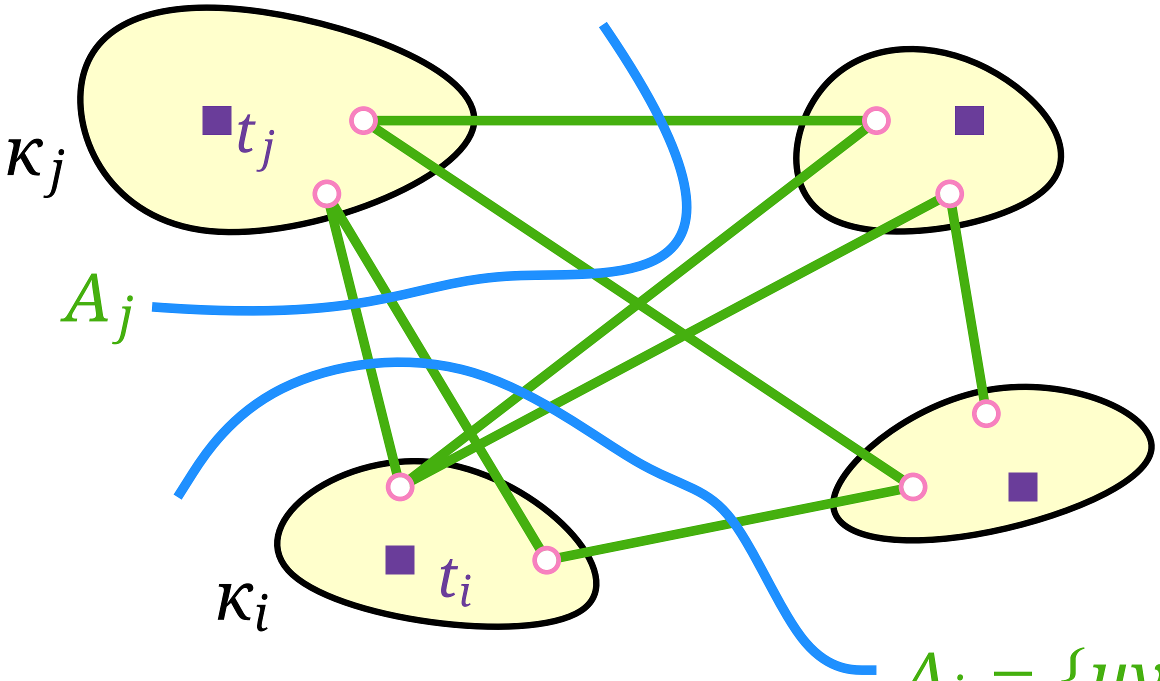

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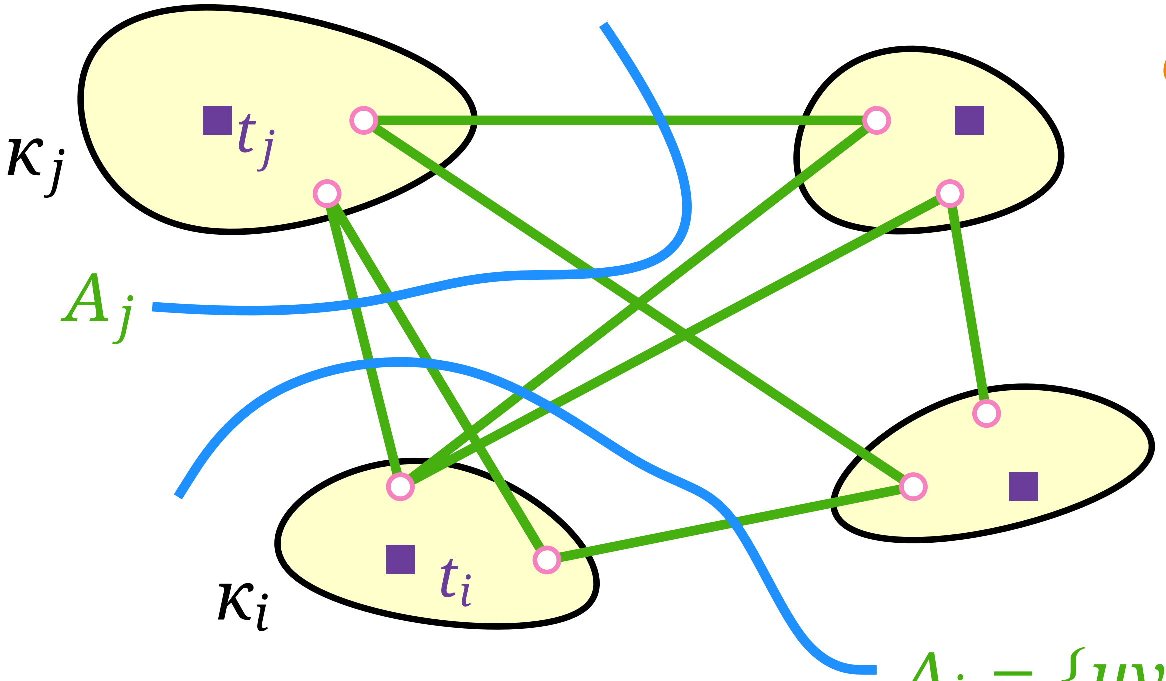

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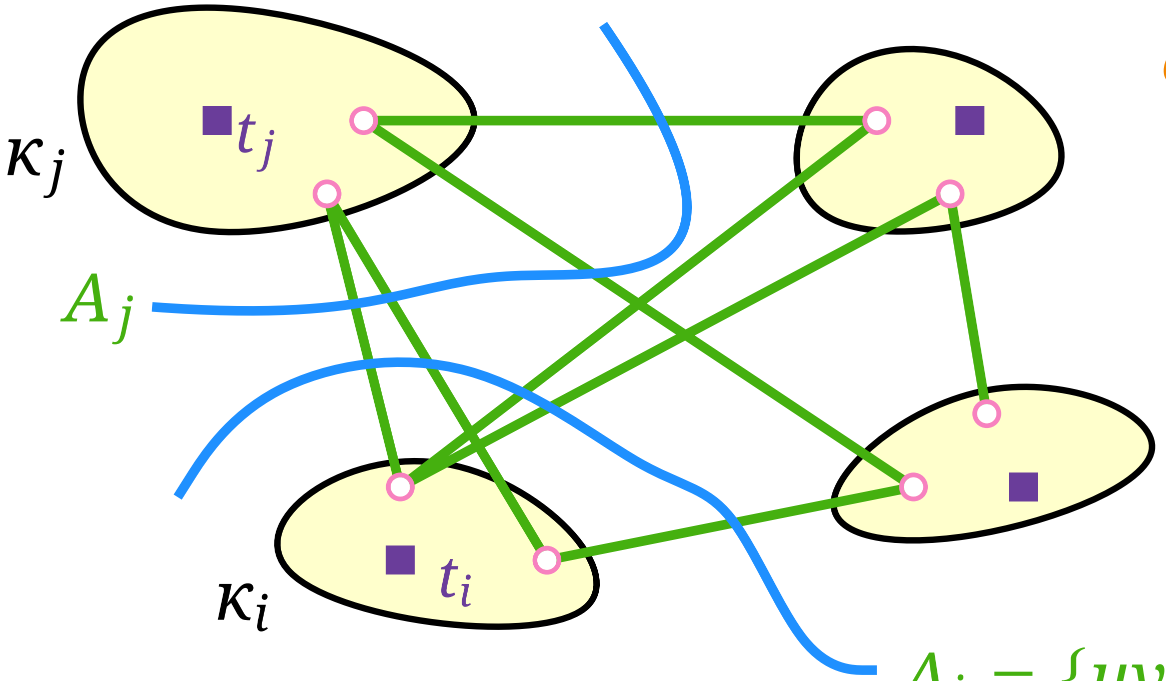

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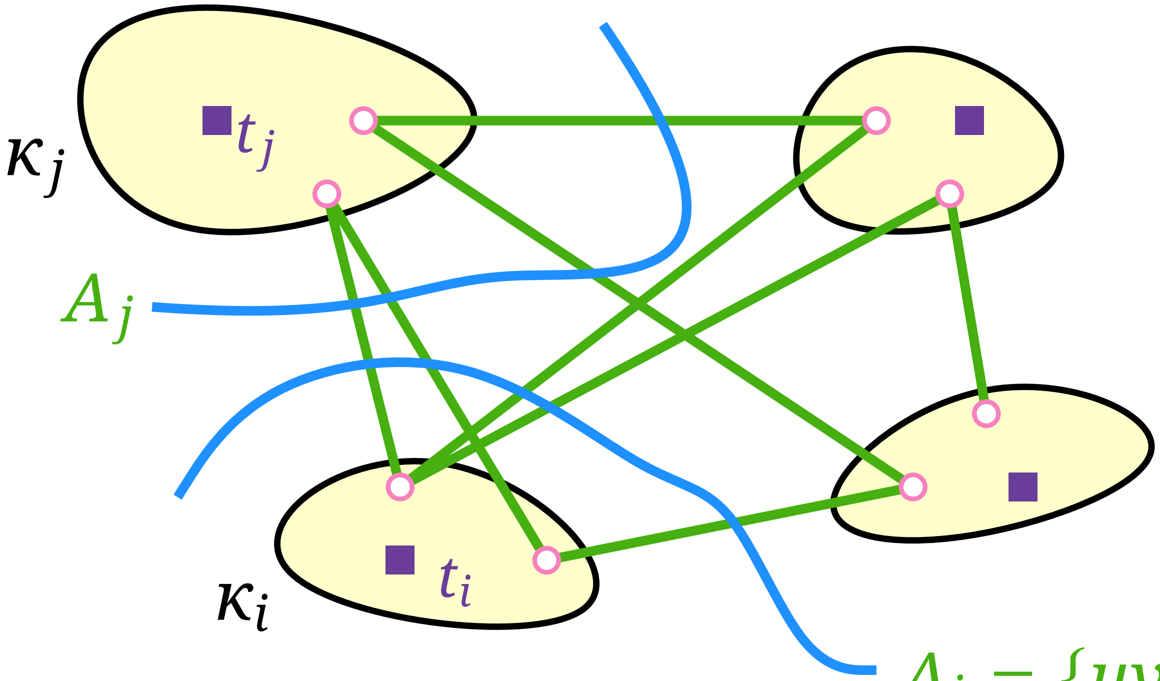

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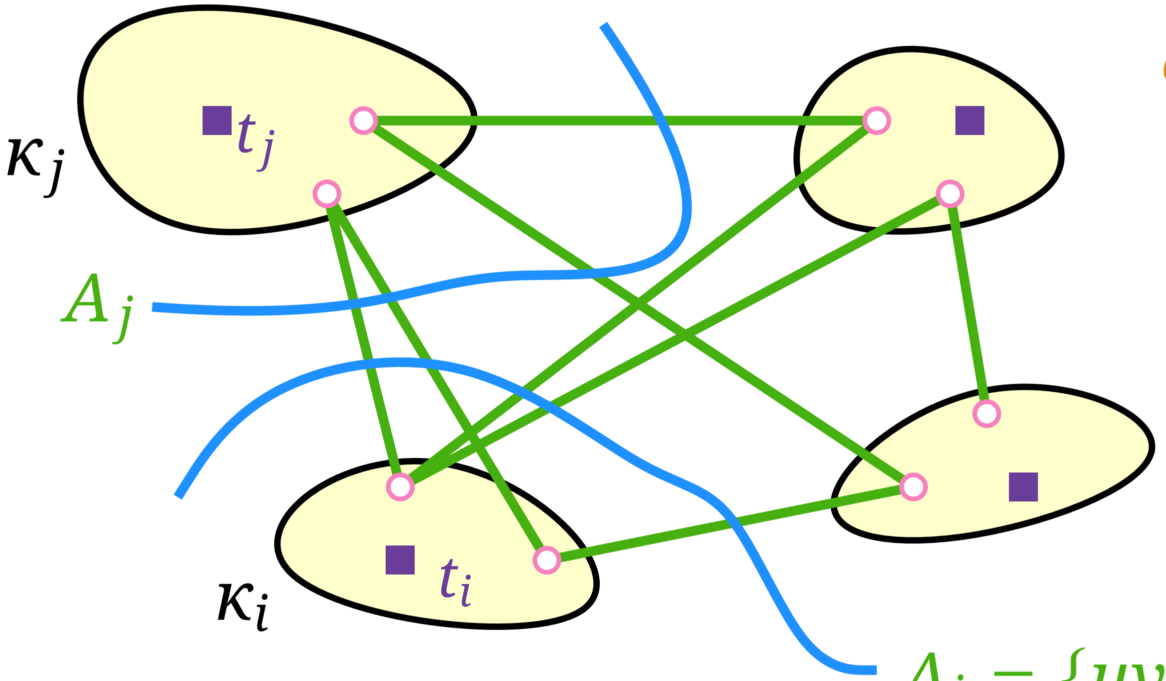

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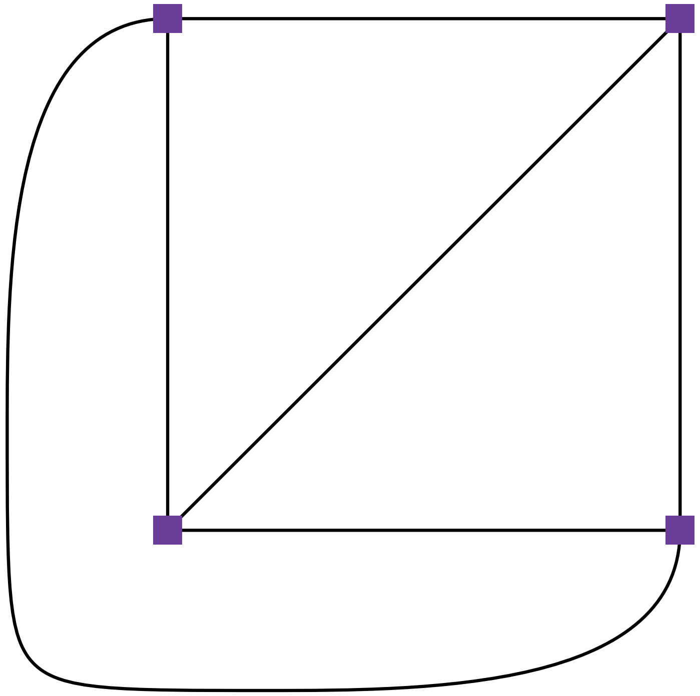

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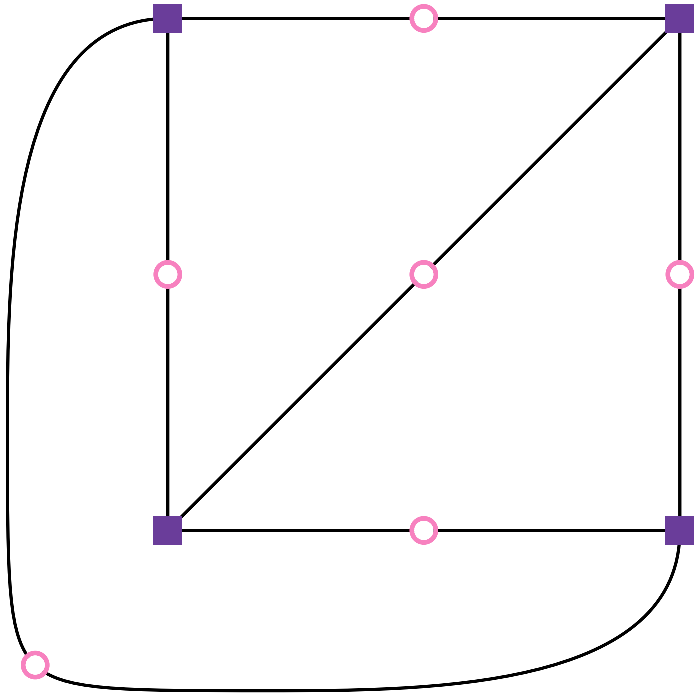

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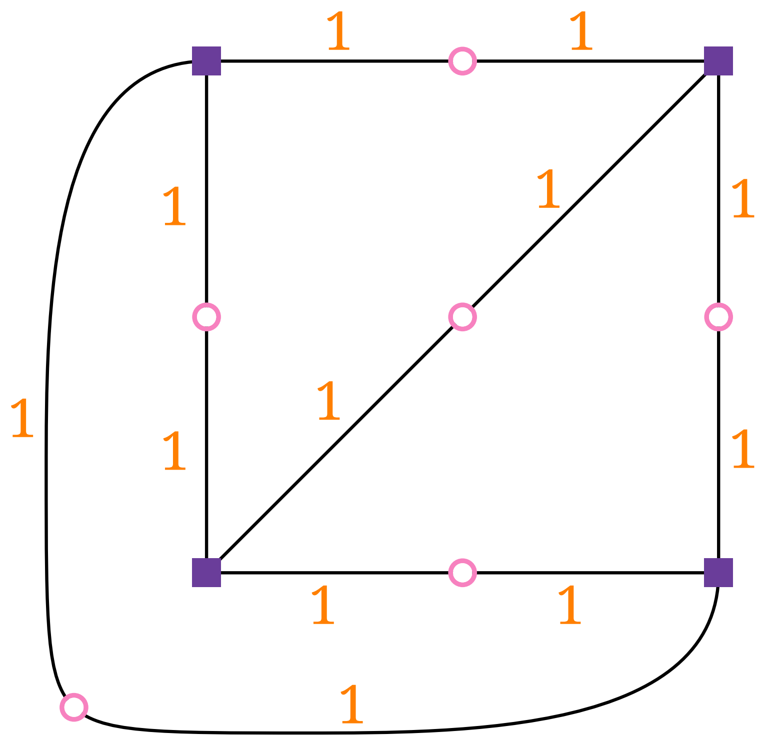

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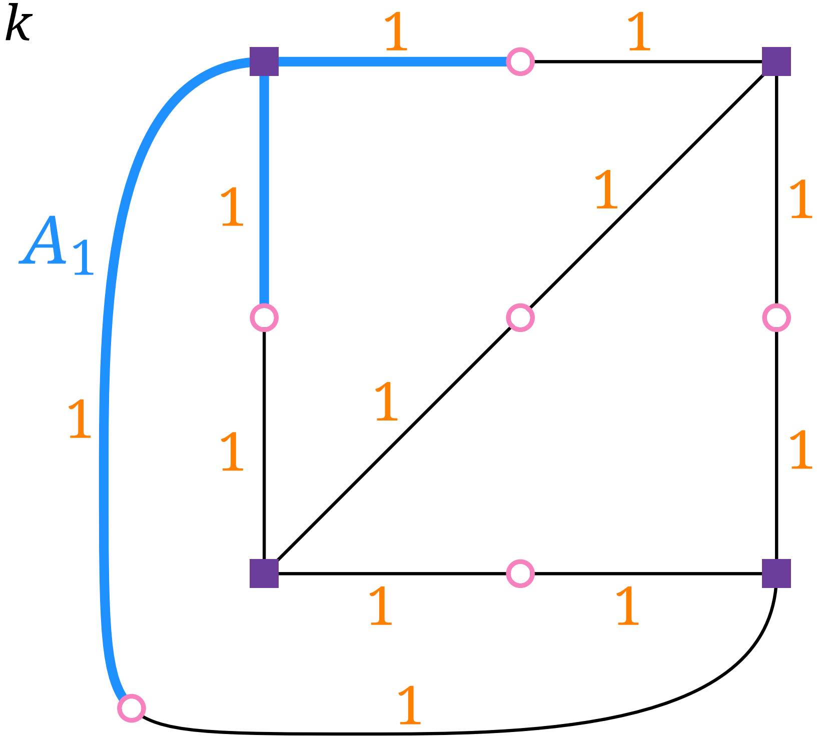

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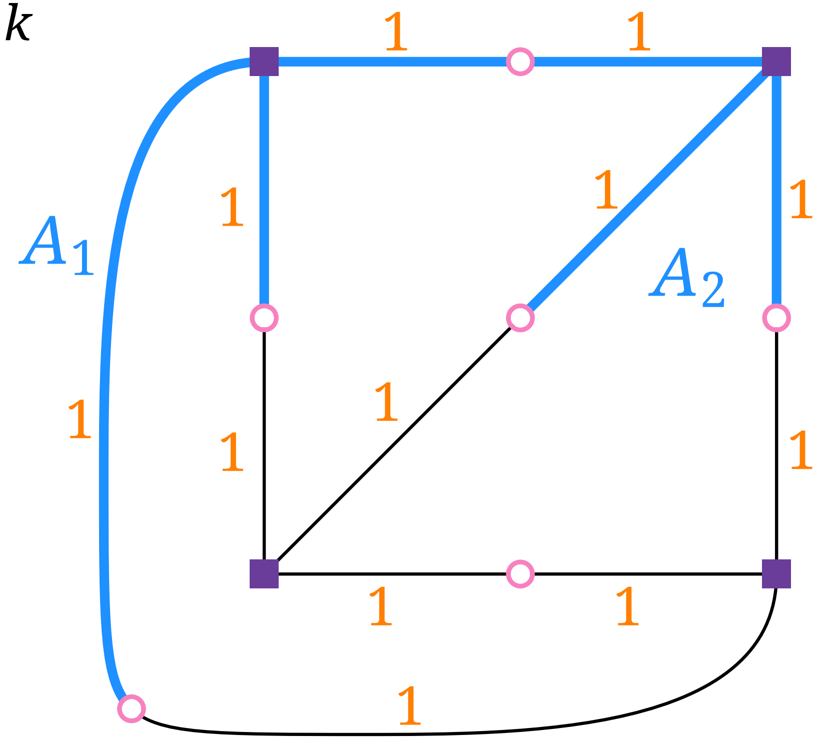

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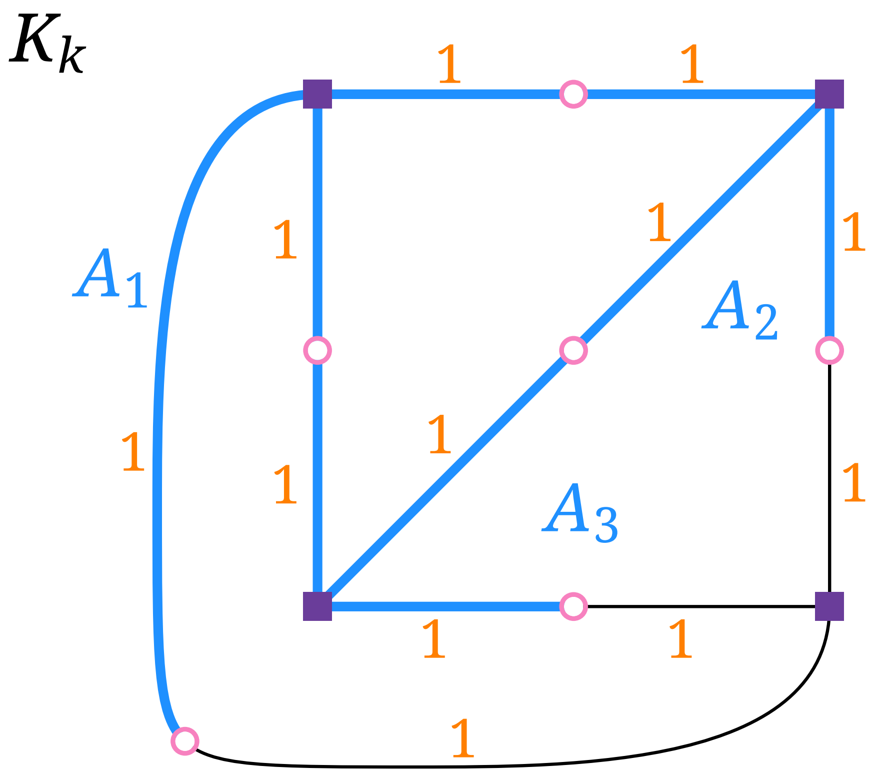

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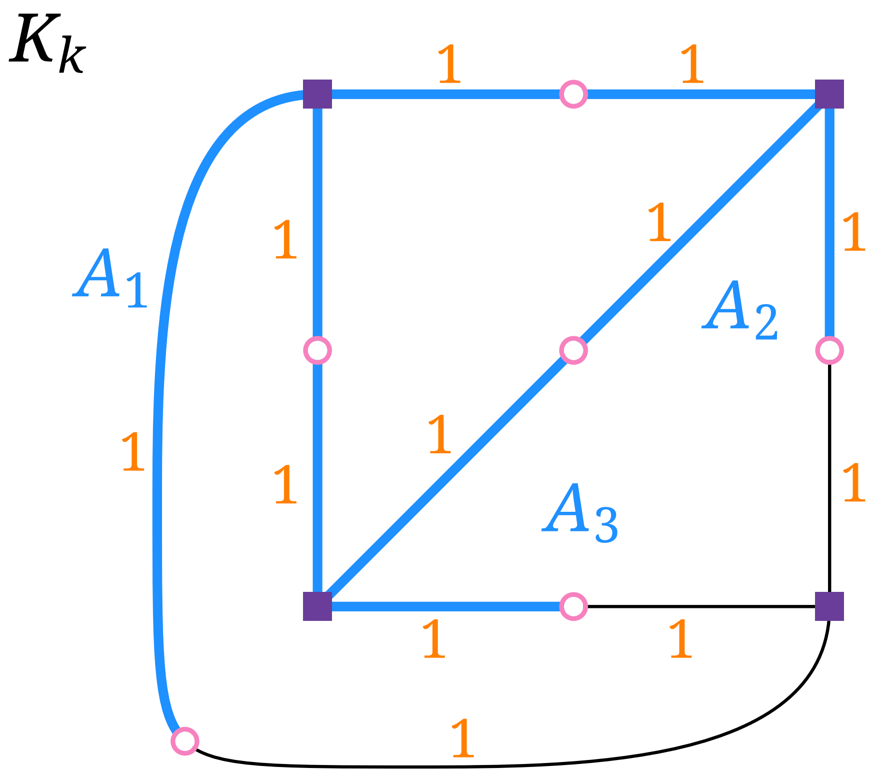

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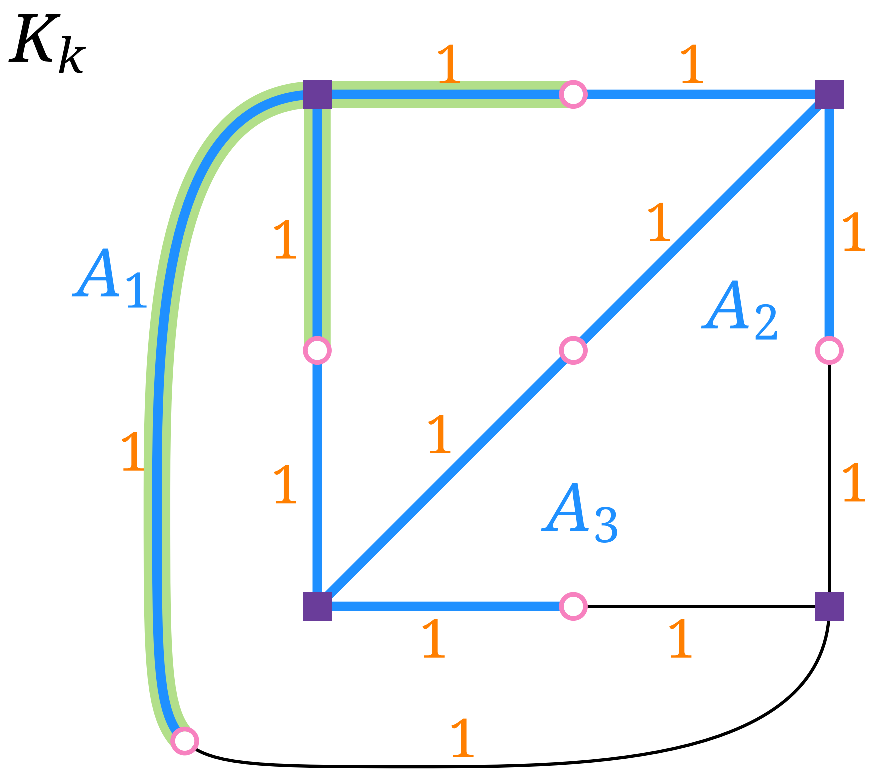

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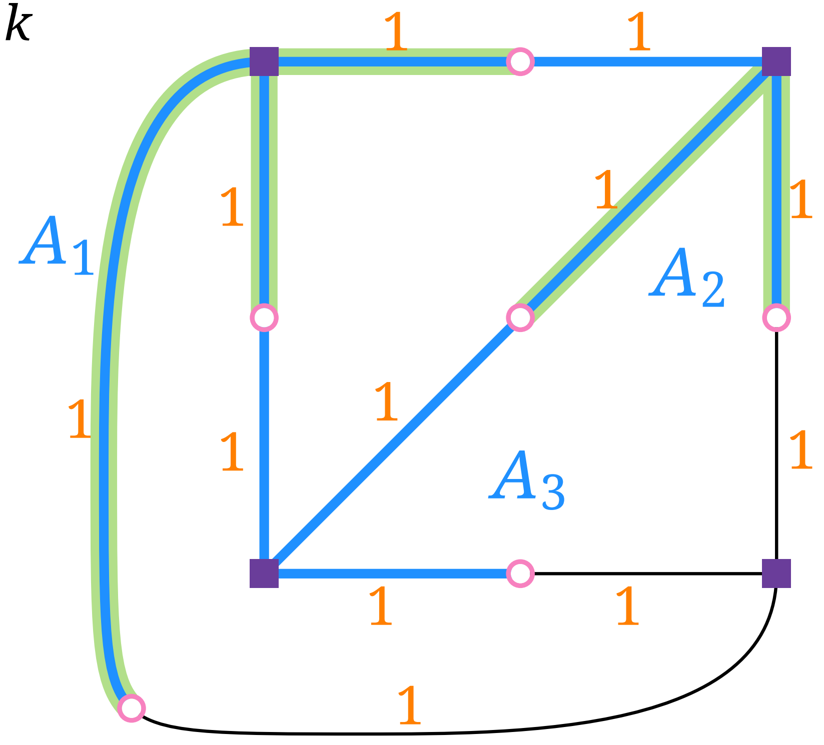

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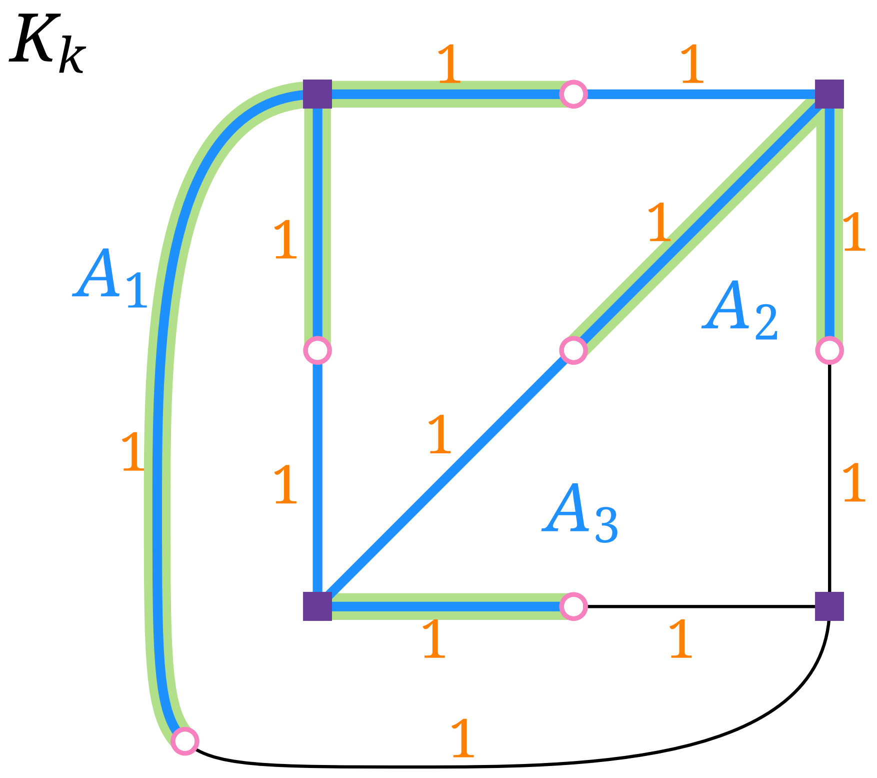

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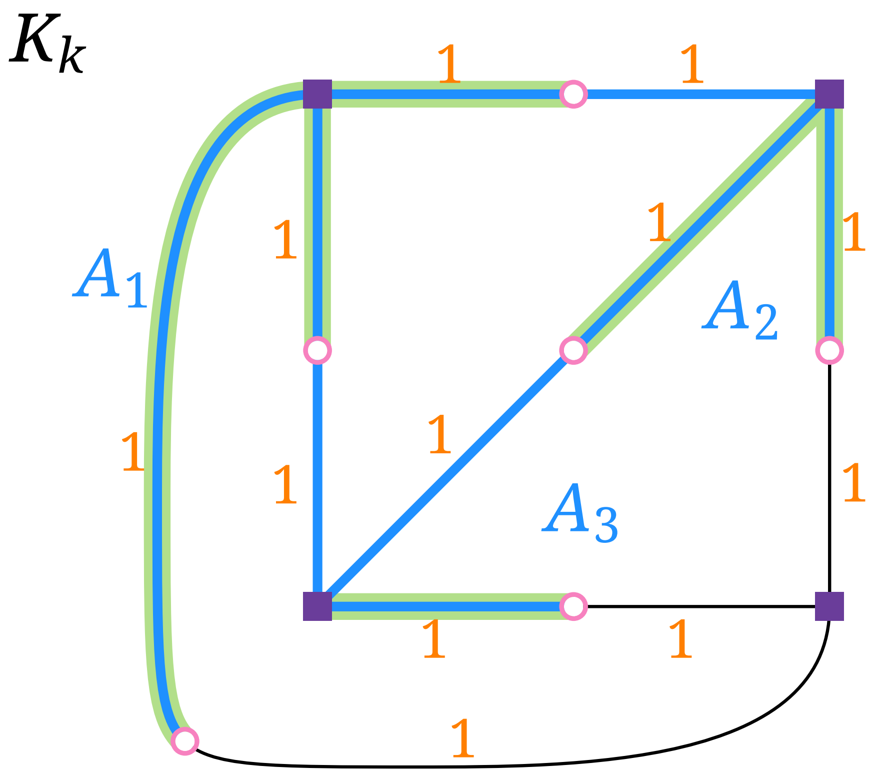

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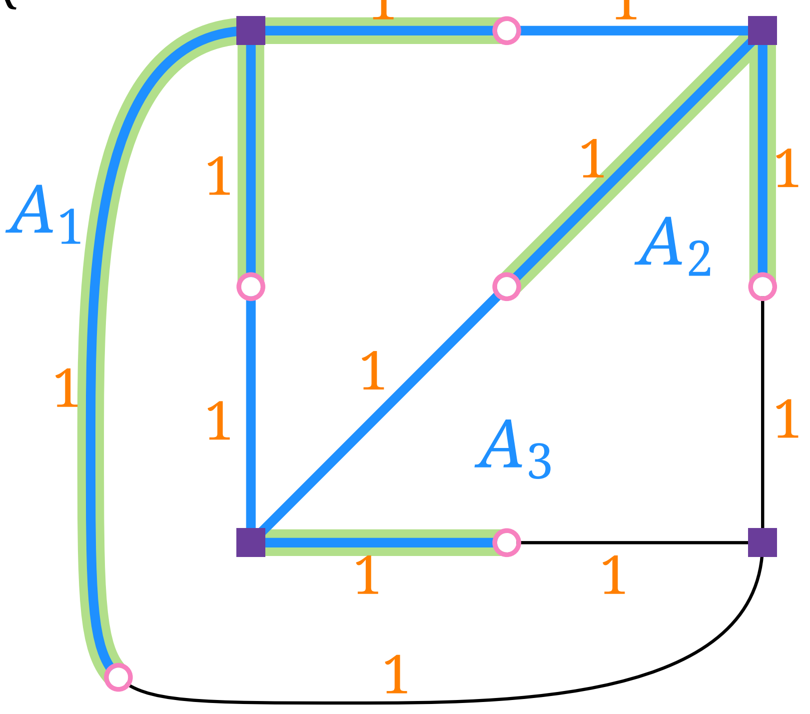

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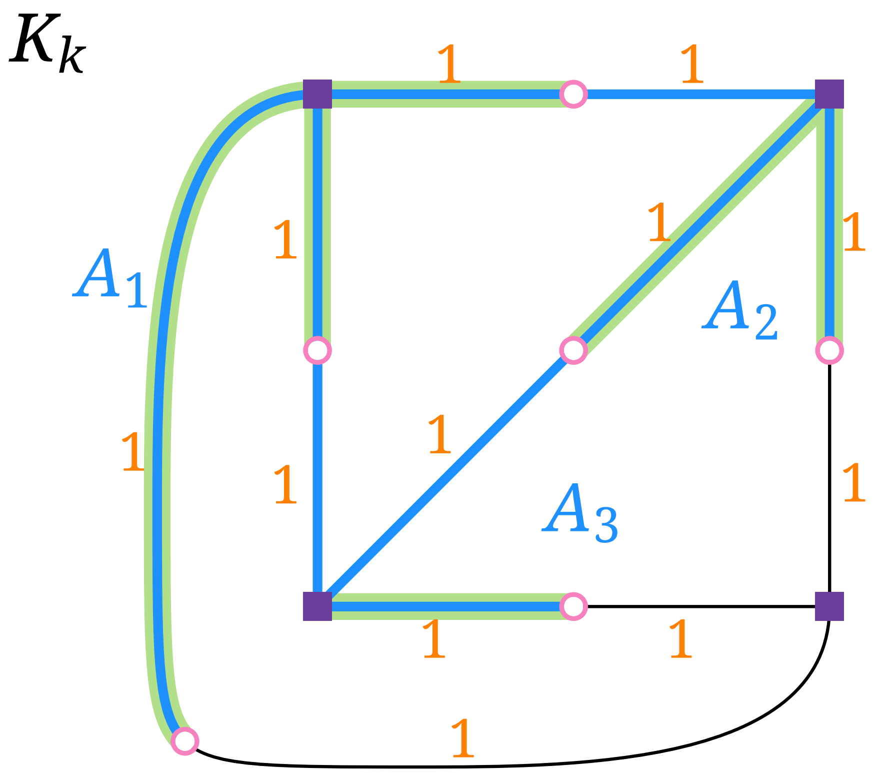

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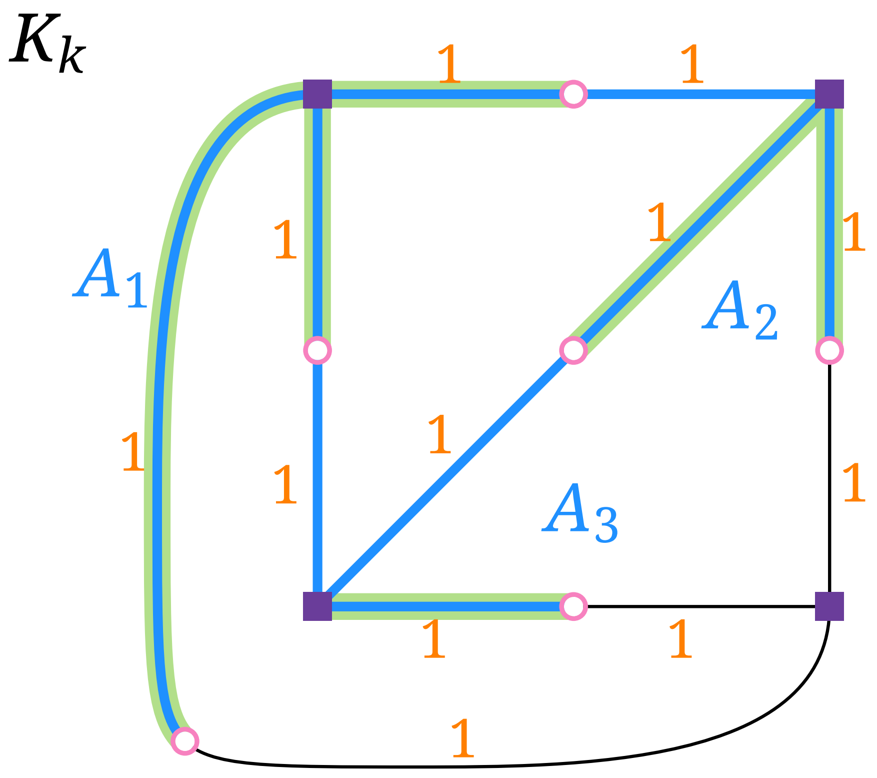

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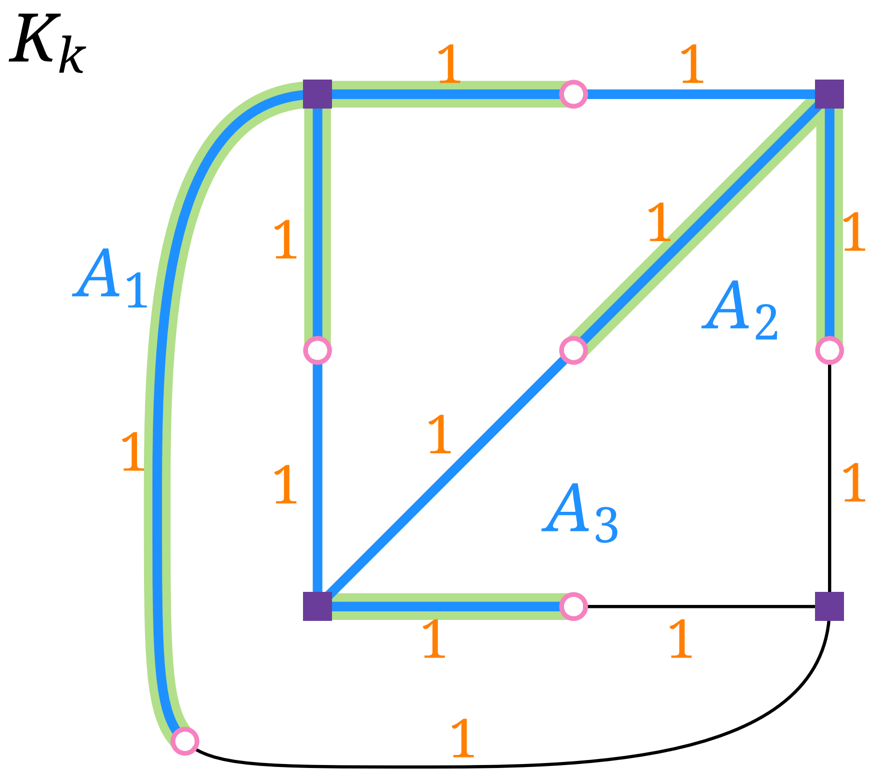

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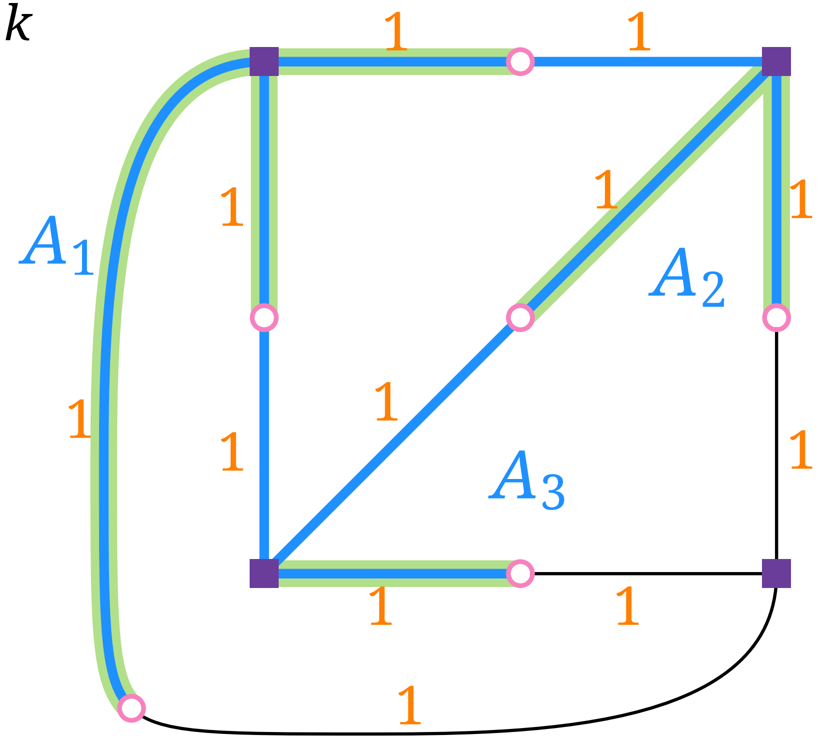

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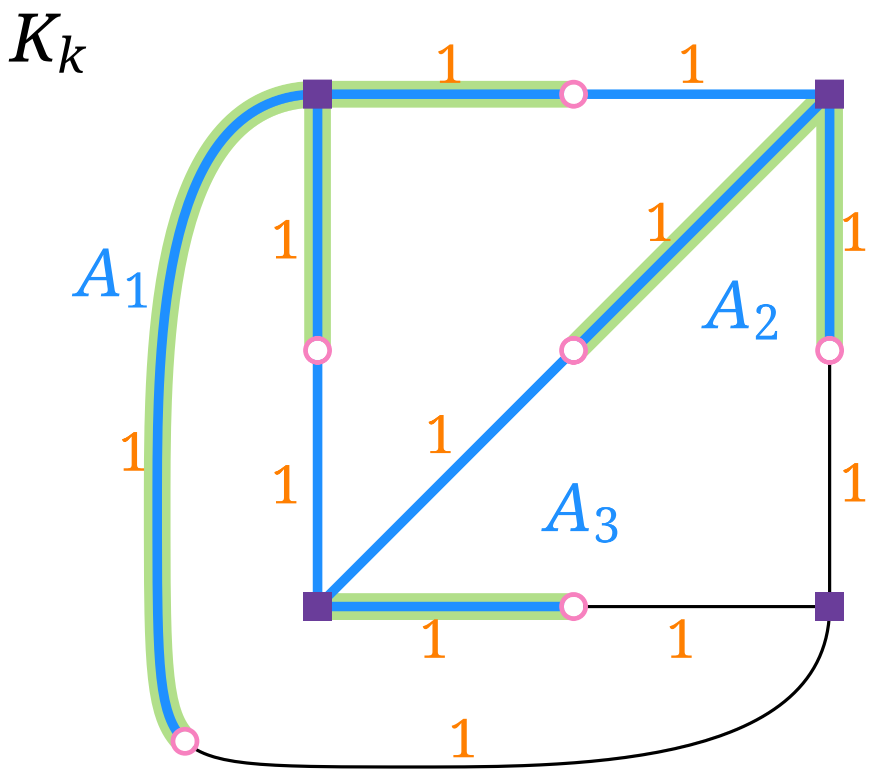

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[Sharma & Vondrák, STOC'14]

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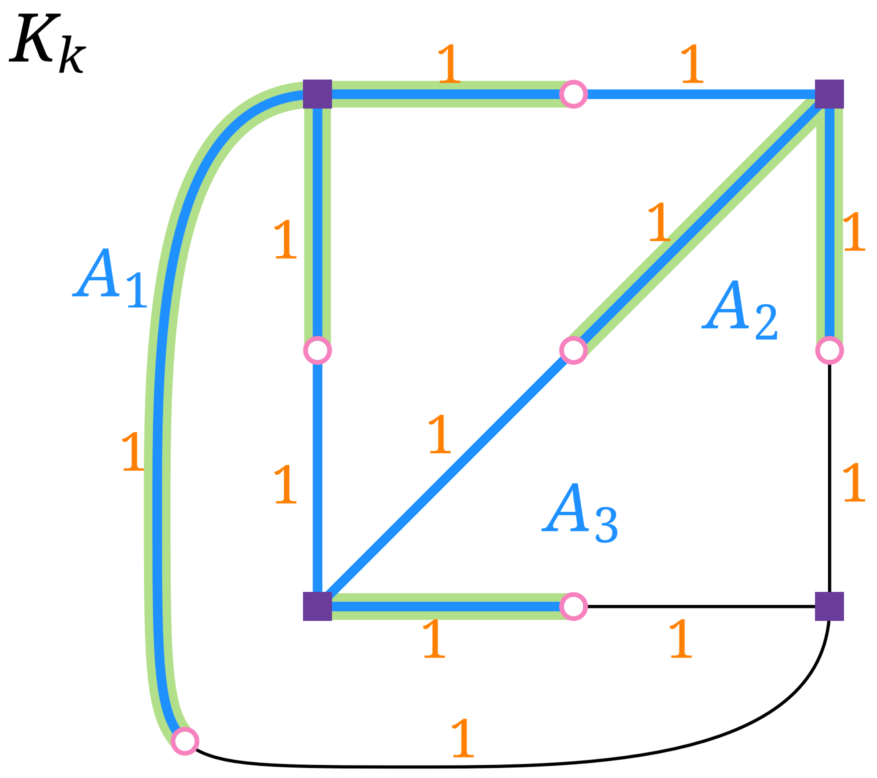

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MULTIWAYCUT cannot be approximated within factor $1.20016 - O(1/k)$ (unless P = NP).

[Bérczi, Chandrasekaran, Király & Madan, MP'18]

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$(2 - 2/k)$-approximation algorithm for MultiwayCut

based on $k - 1$ isolating cuts realized by $s - t$-cuts