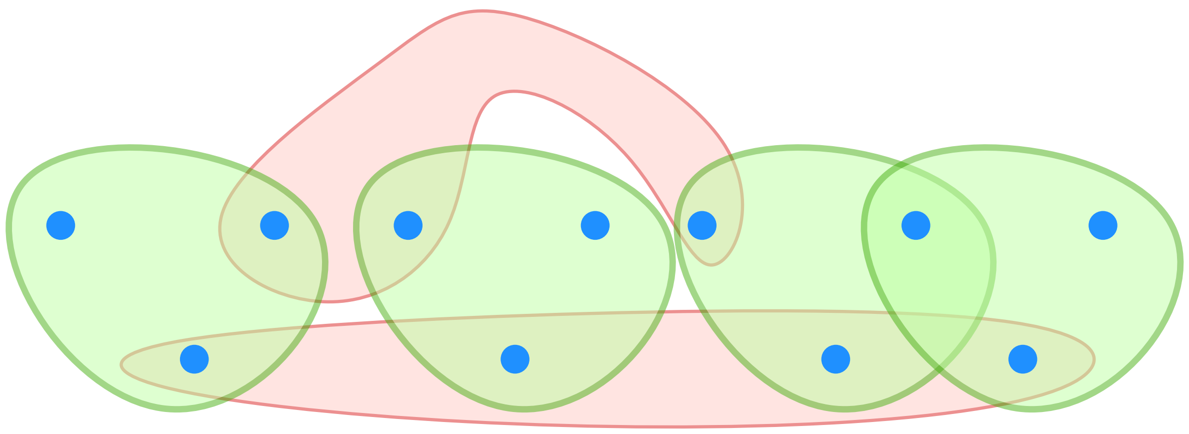

SetCover

greedy approximation algorithm

layering algorithm for weighted VertexCover

application to ShortestSuperString

SetCover (cardinality)

Let U be some ground set (universe),

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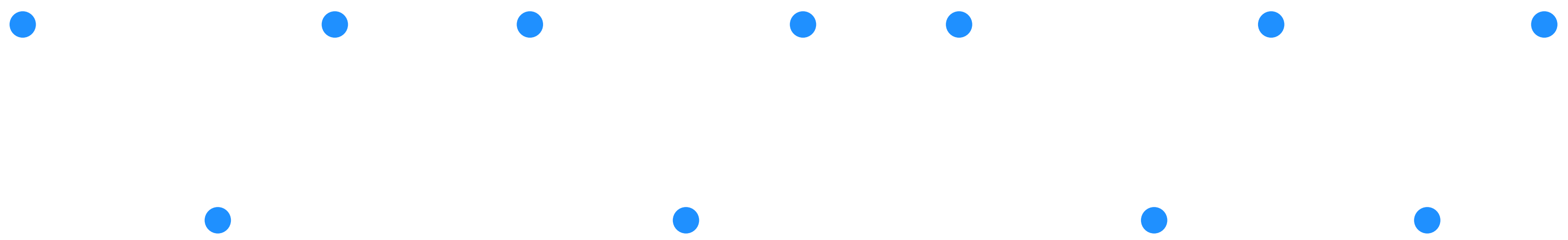

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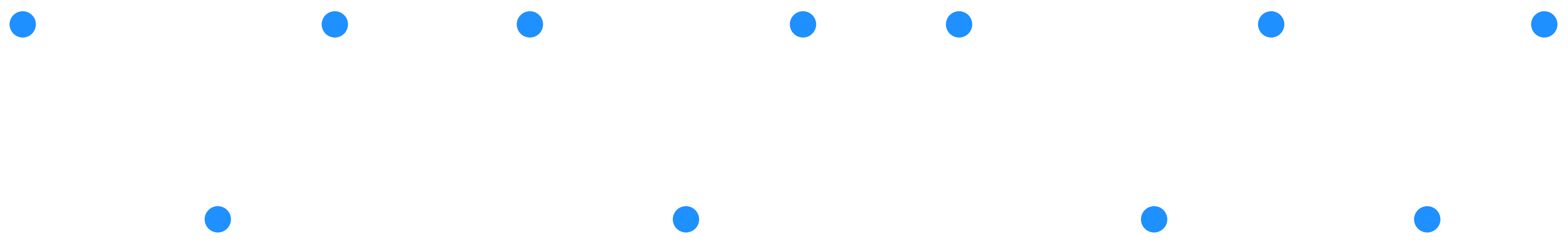

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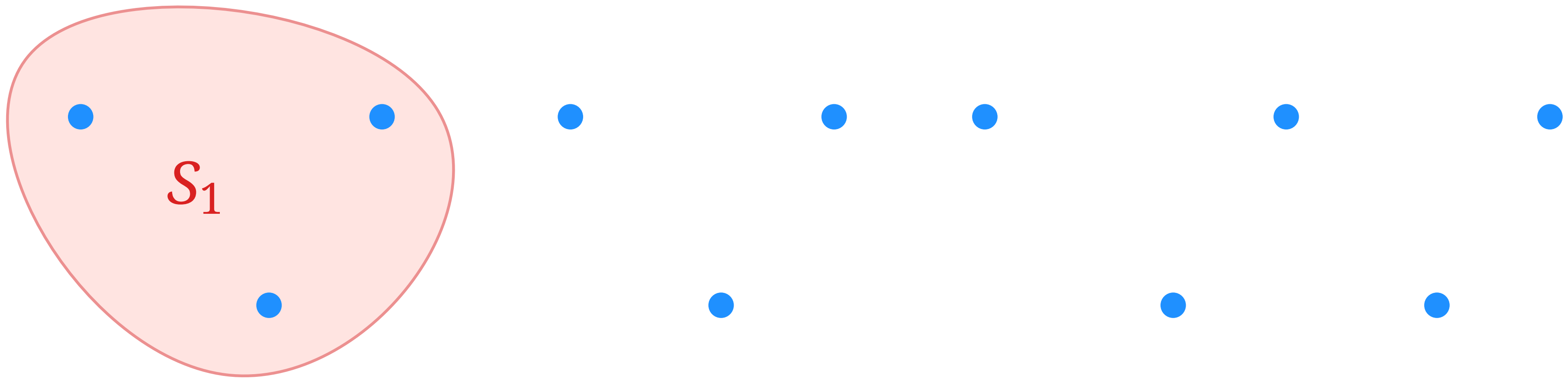

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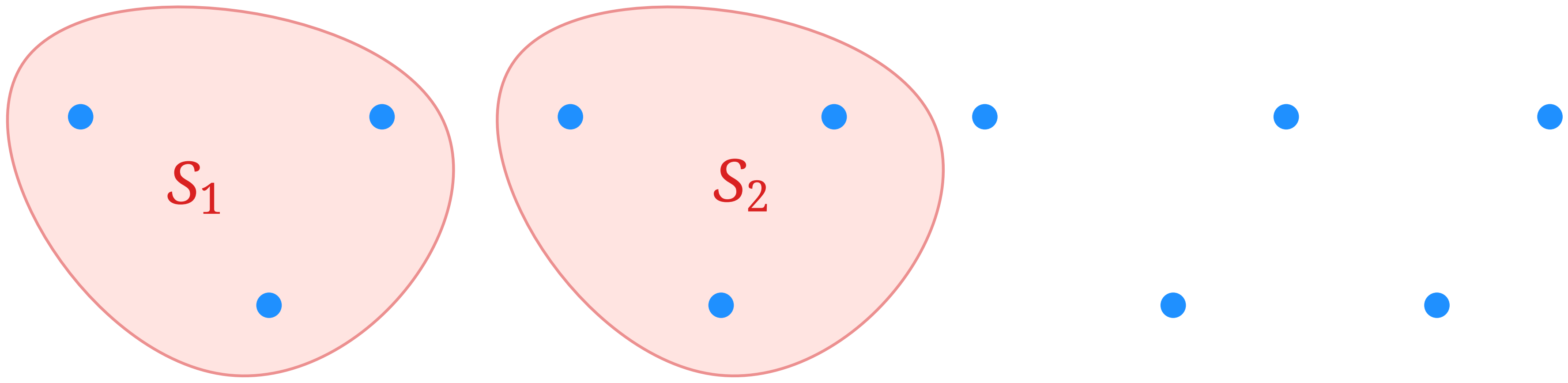

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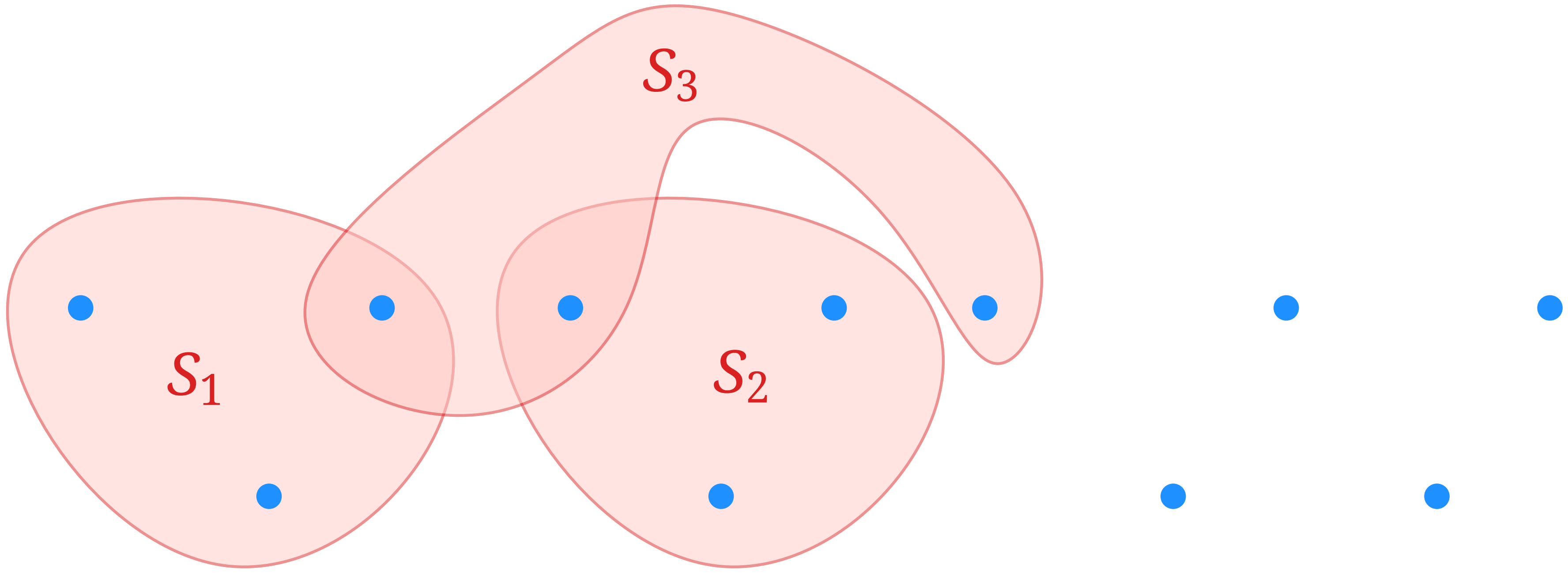

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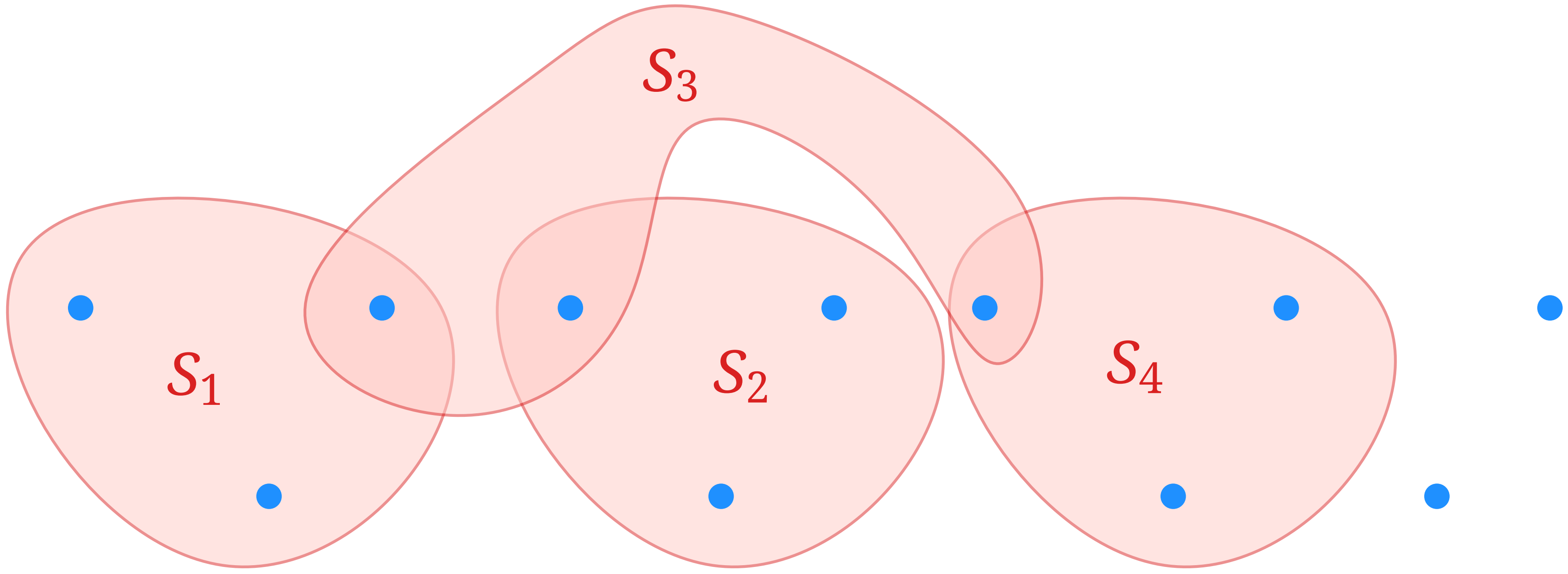

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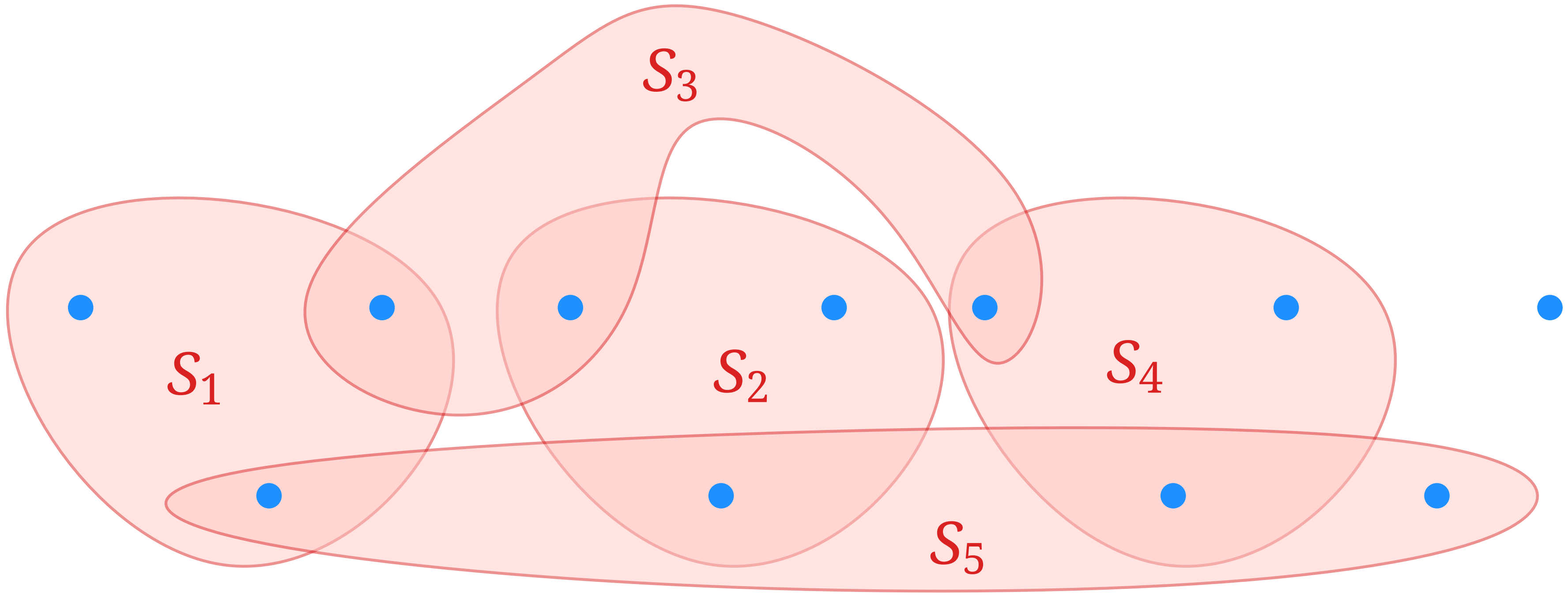

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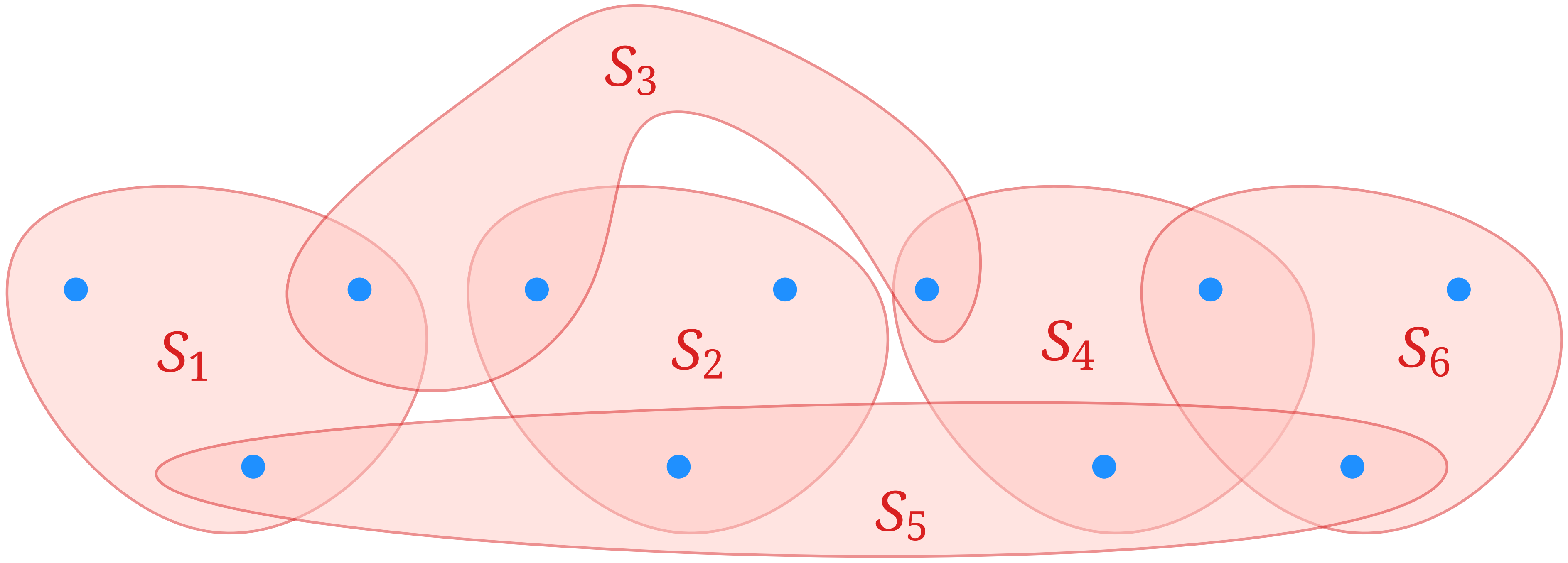

SETCOVER (cardinality)

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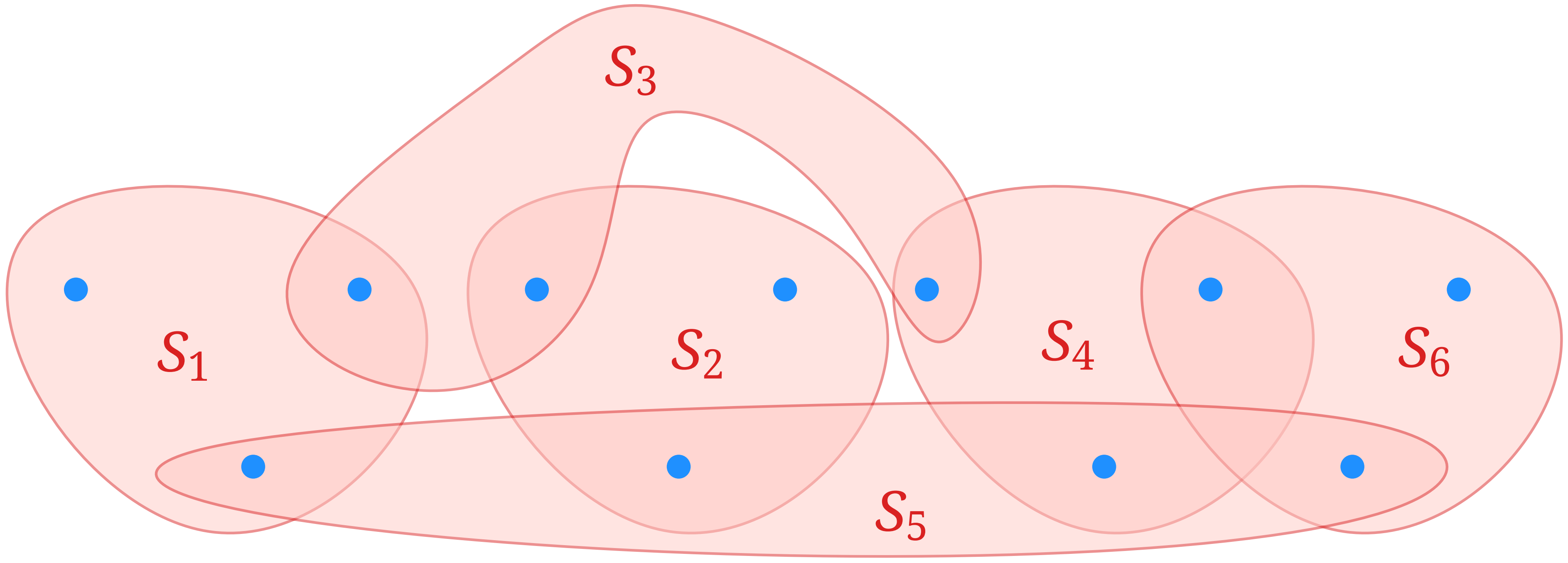
Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum cardinality.



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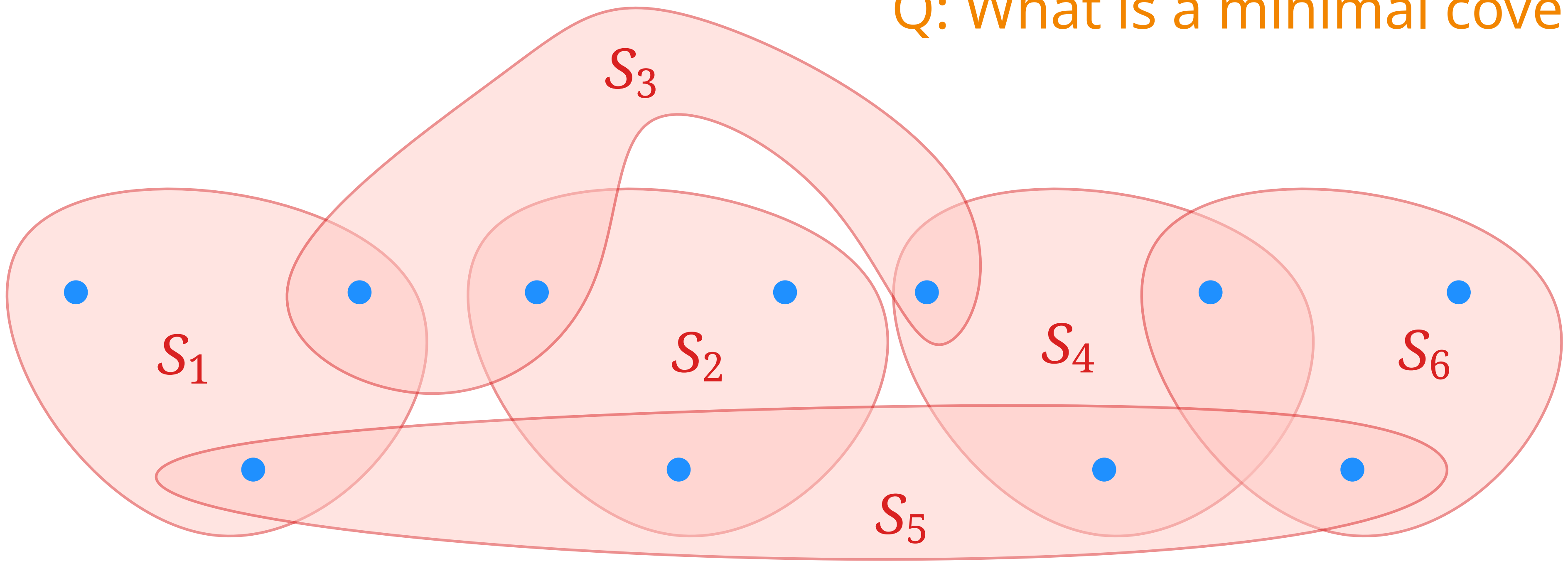
Q: What is a minimal cover here?



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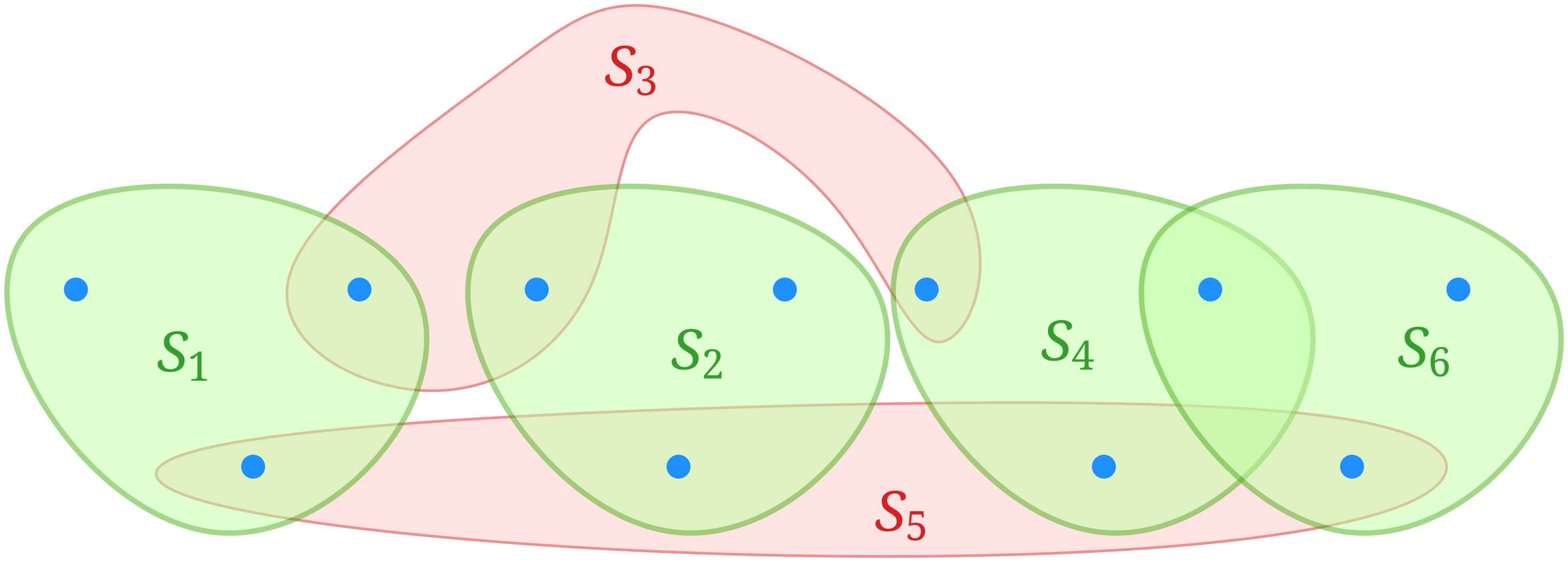

SetCover (general)

Let U be some ground set (universe),
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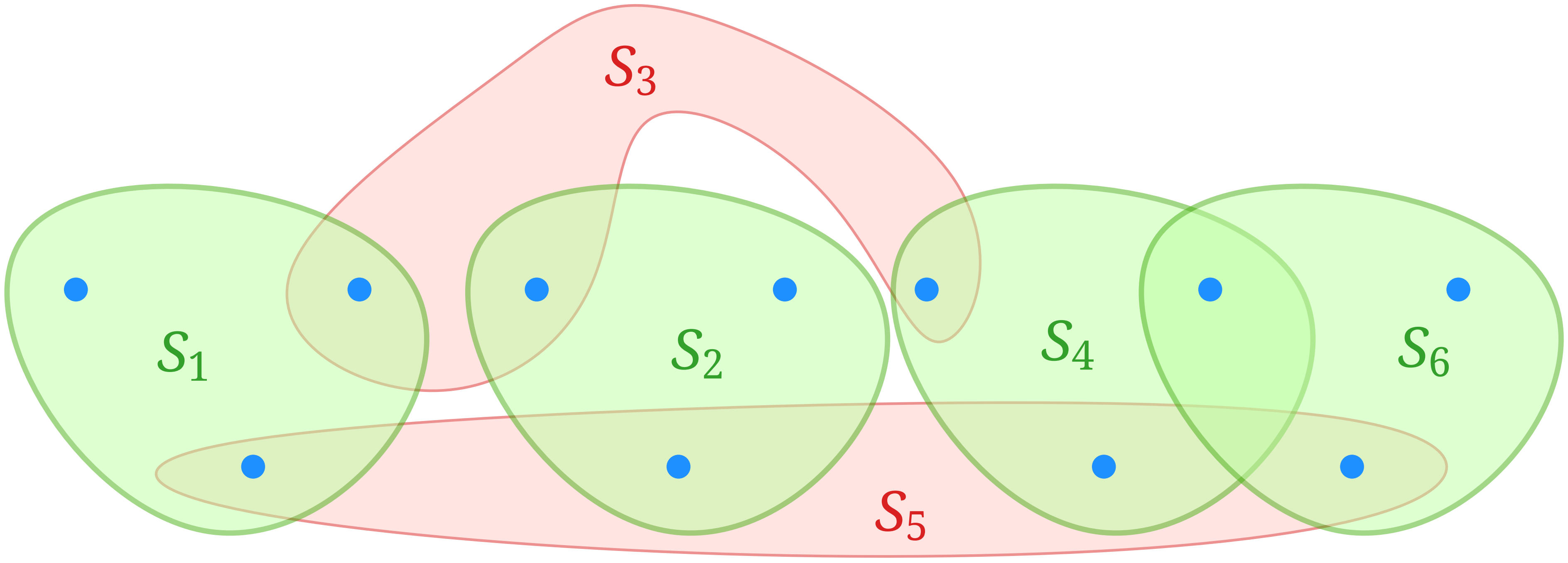

SetCover (general)

Let U be some ground set (universe),
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Each $S \in \mathcal{S}$ has cost $c(S) > 0$.

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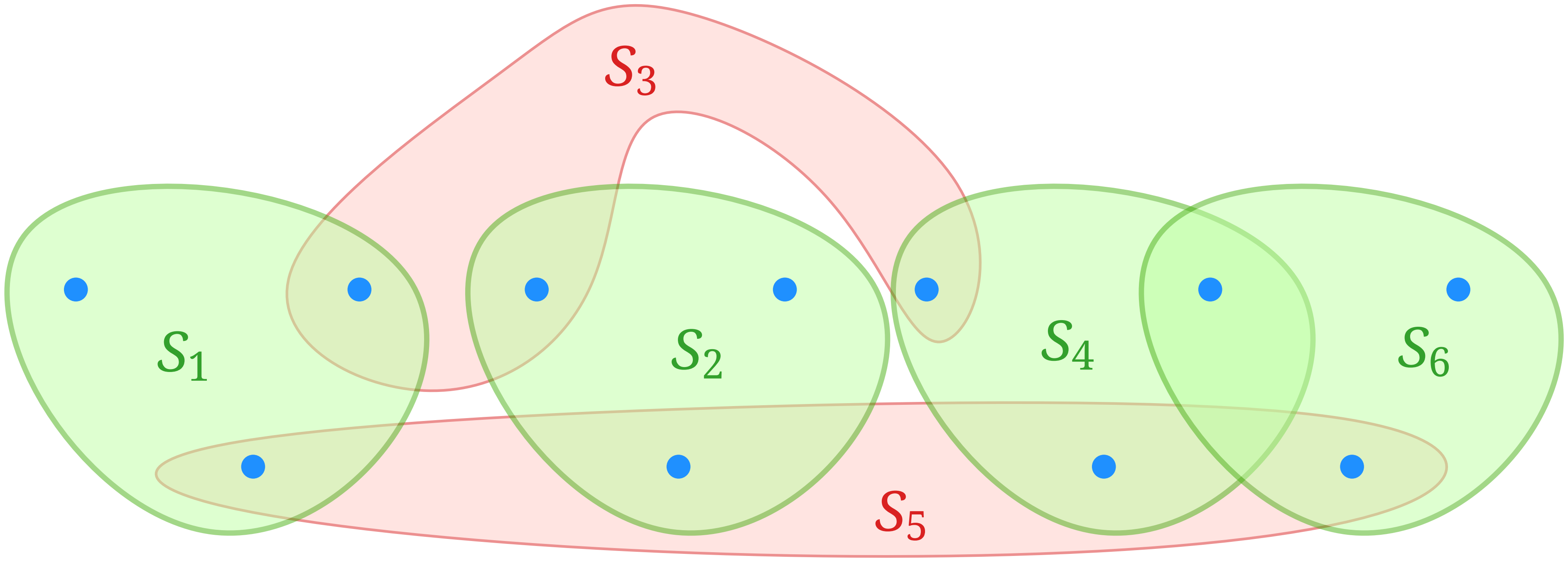

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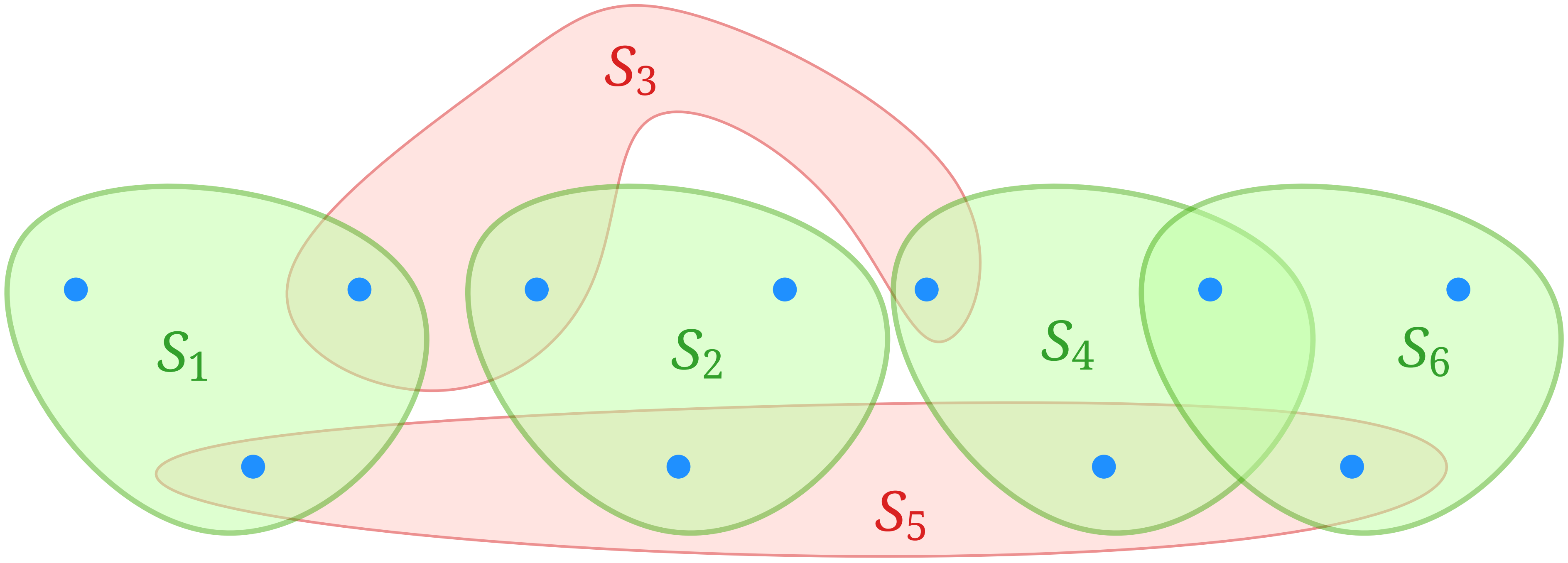

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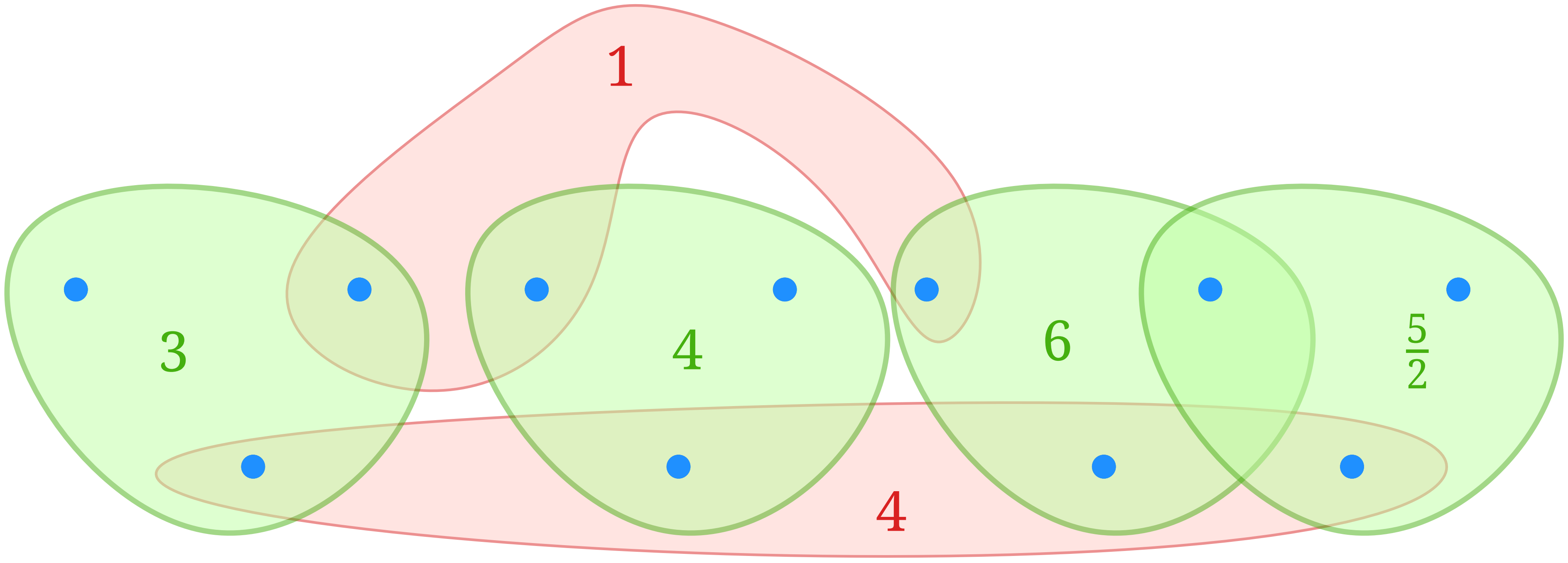

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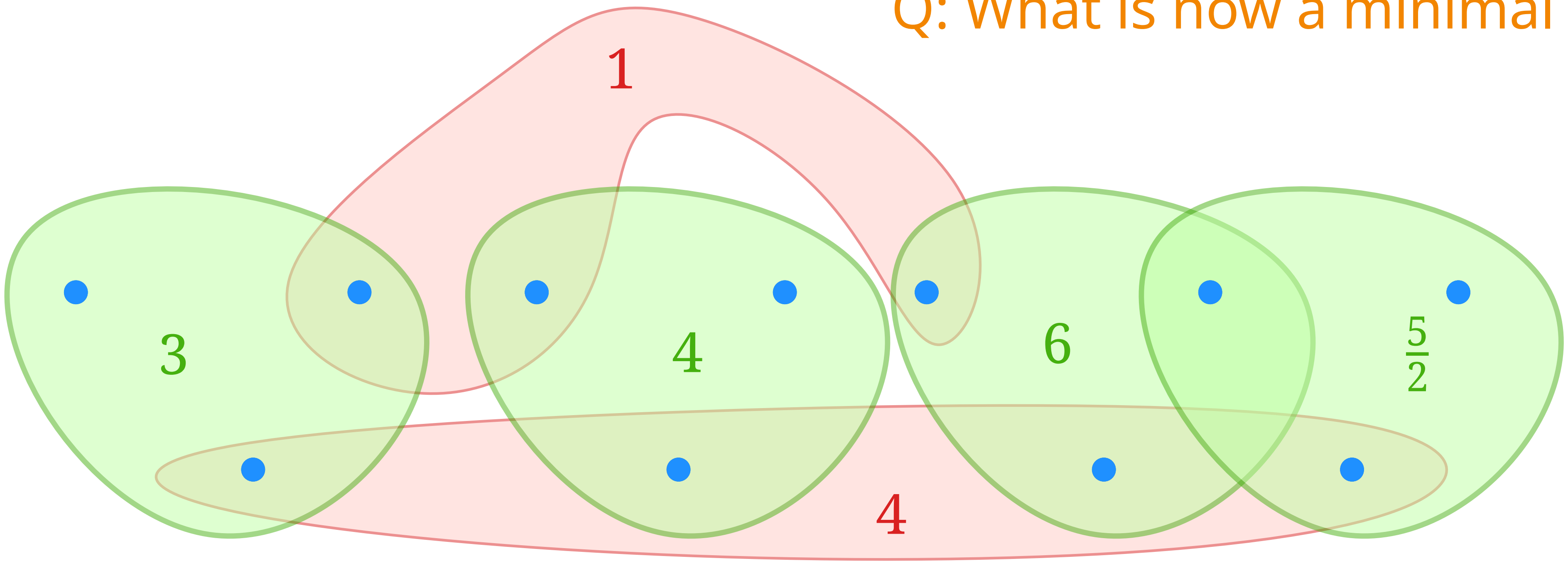
Q: What is now a minimal cover?



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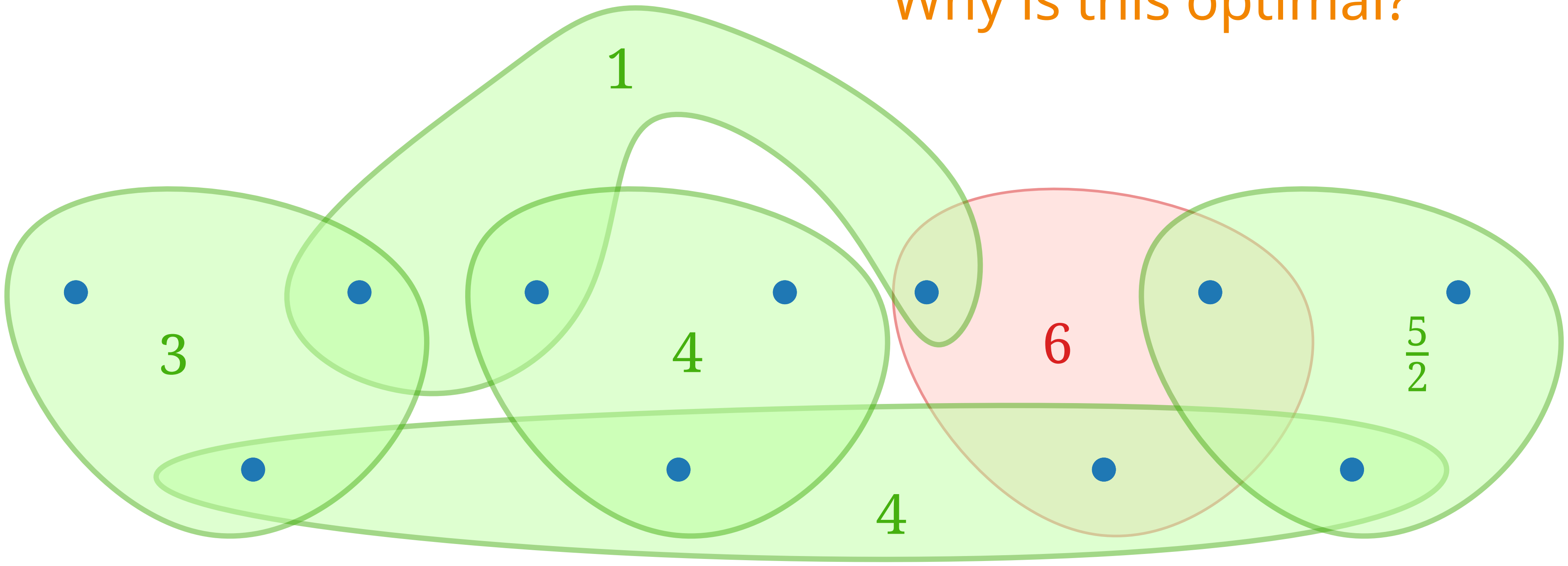
Why is this optimal?



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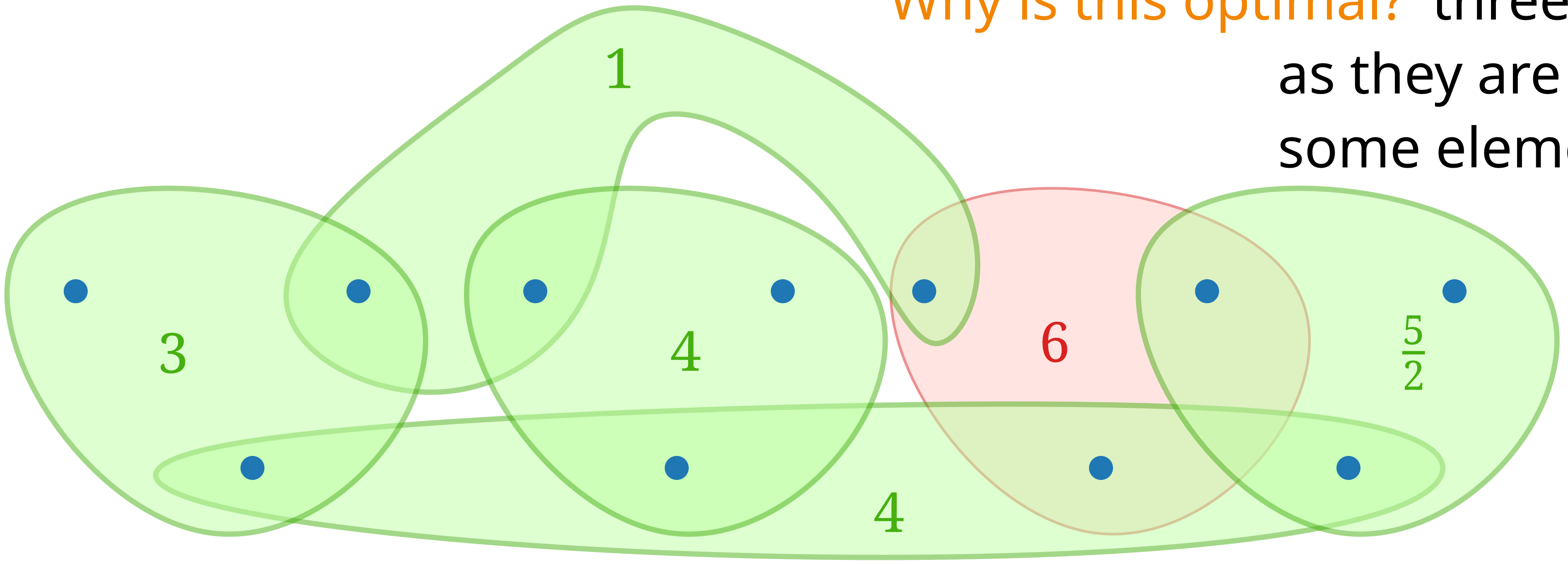
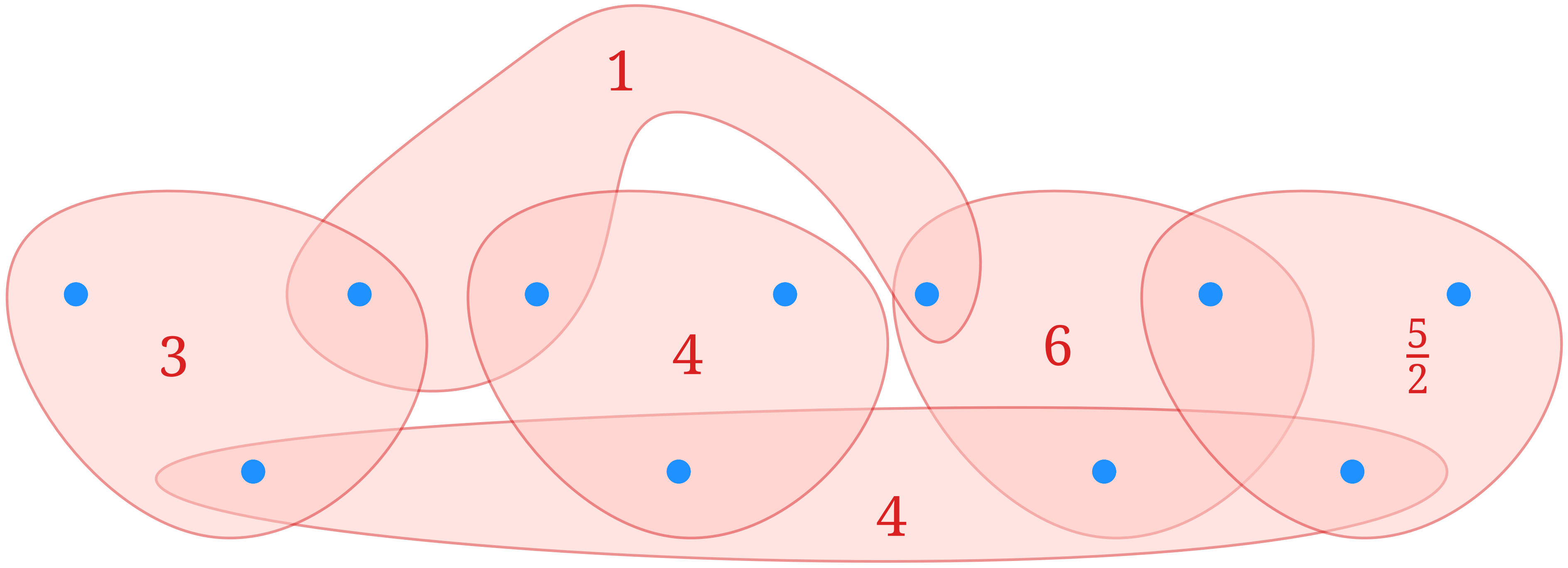
Why is this optimal? three sets need to be chosen as they are the only ones to cover some elements; then $1 + 4 < 6$



Greedy Approximation Algorithm for SetCover

Iterative "Buying" of Elements

What is the real cost of picking a set?



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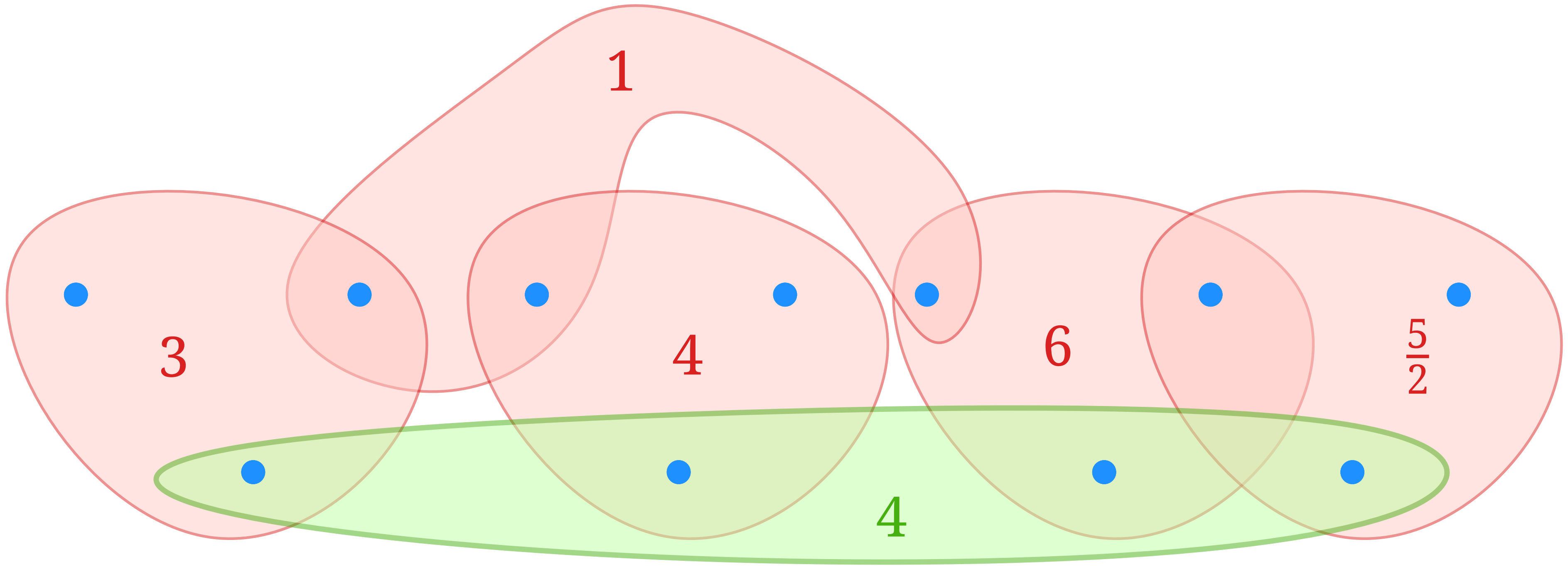

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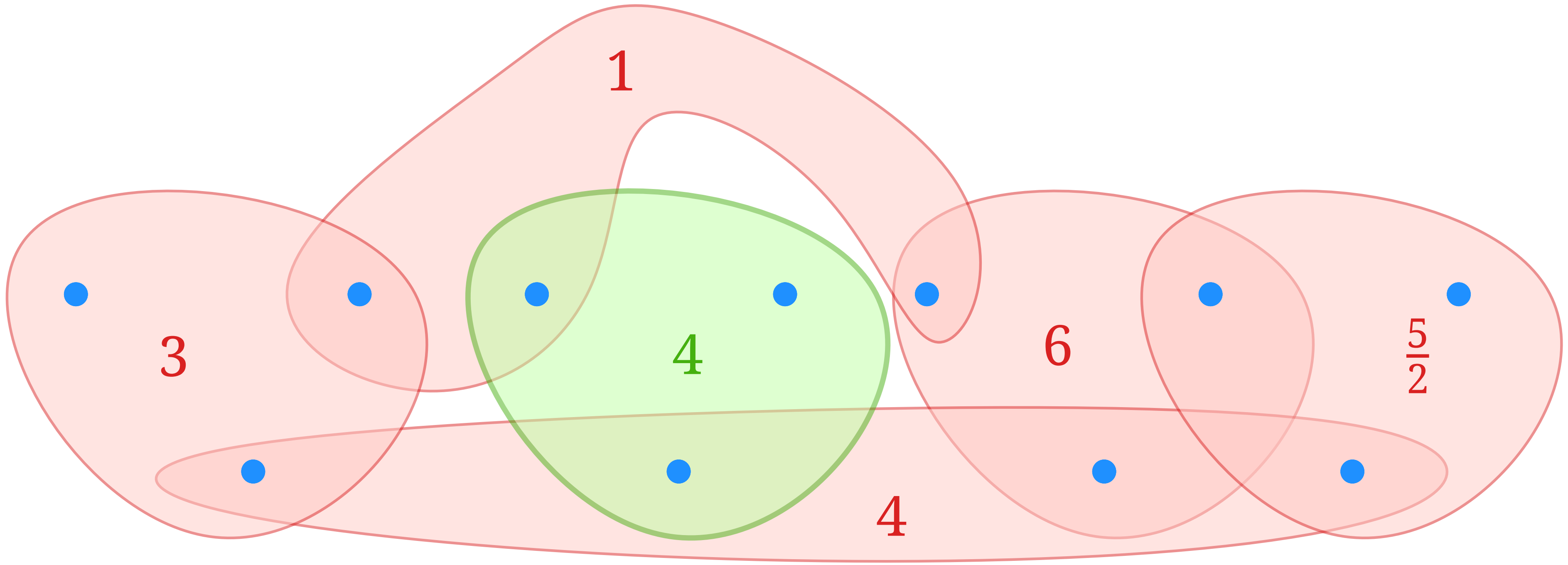

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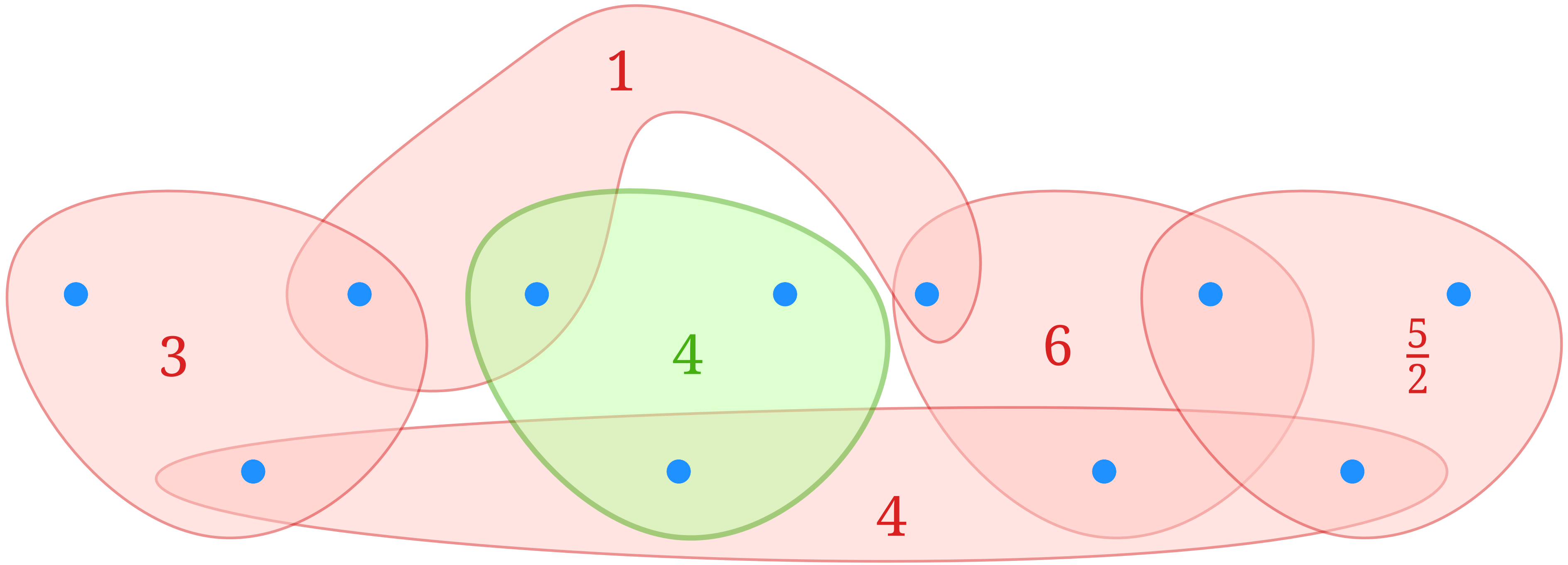
What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.



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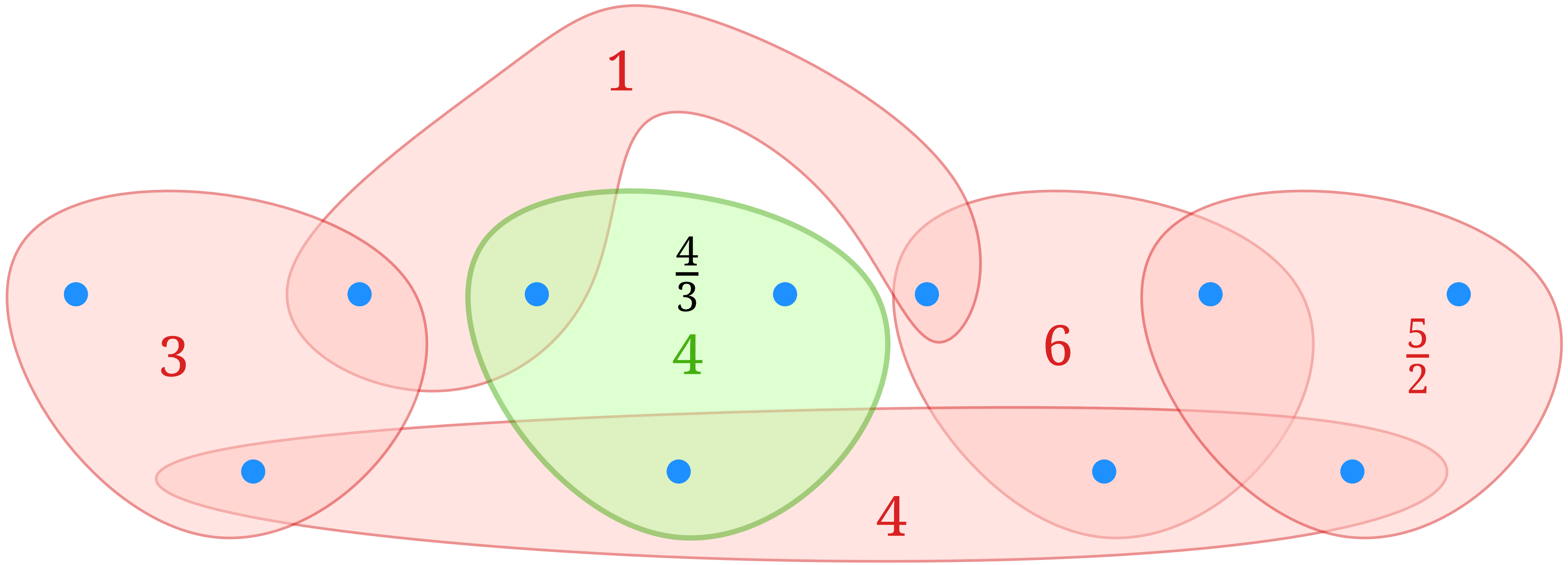

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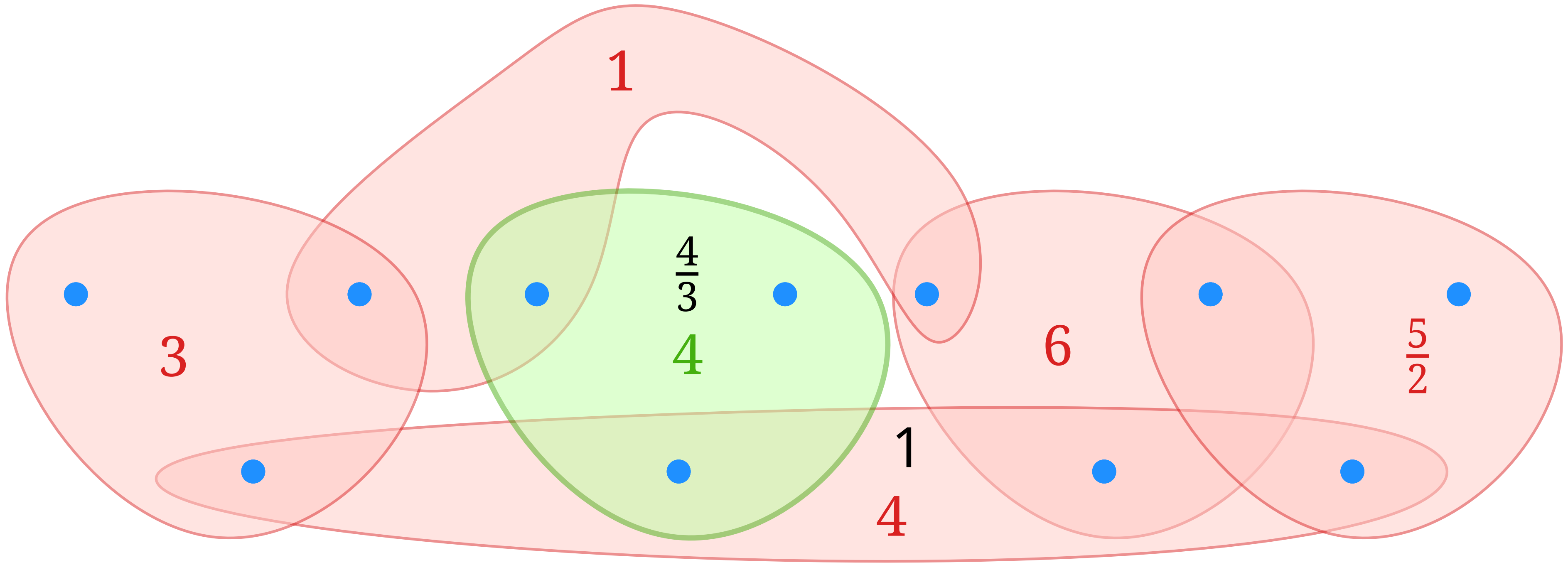

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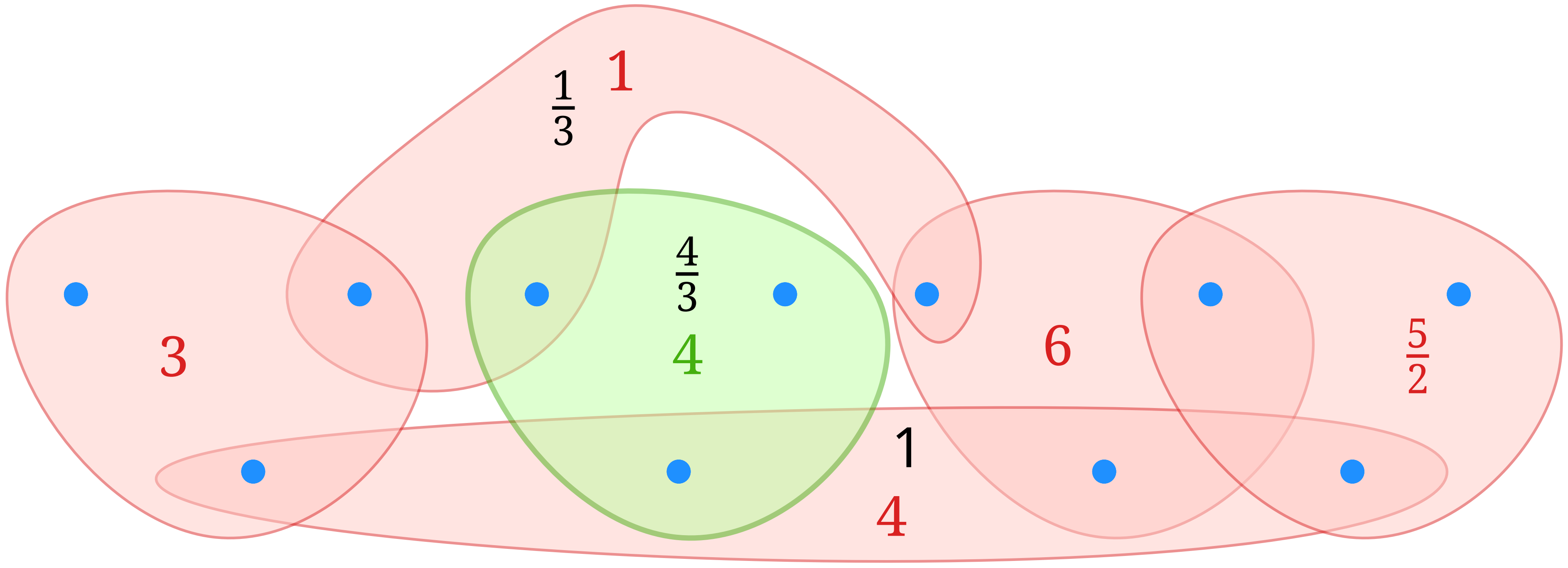

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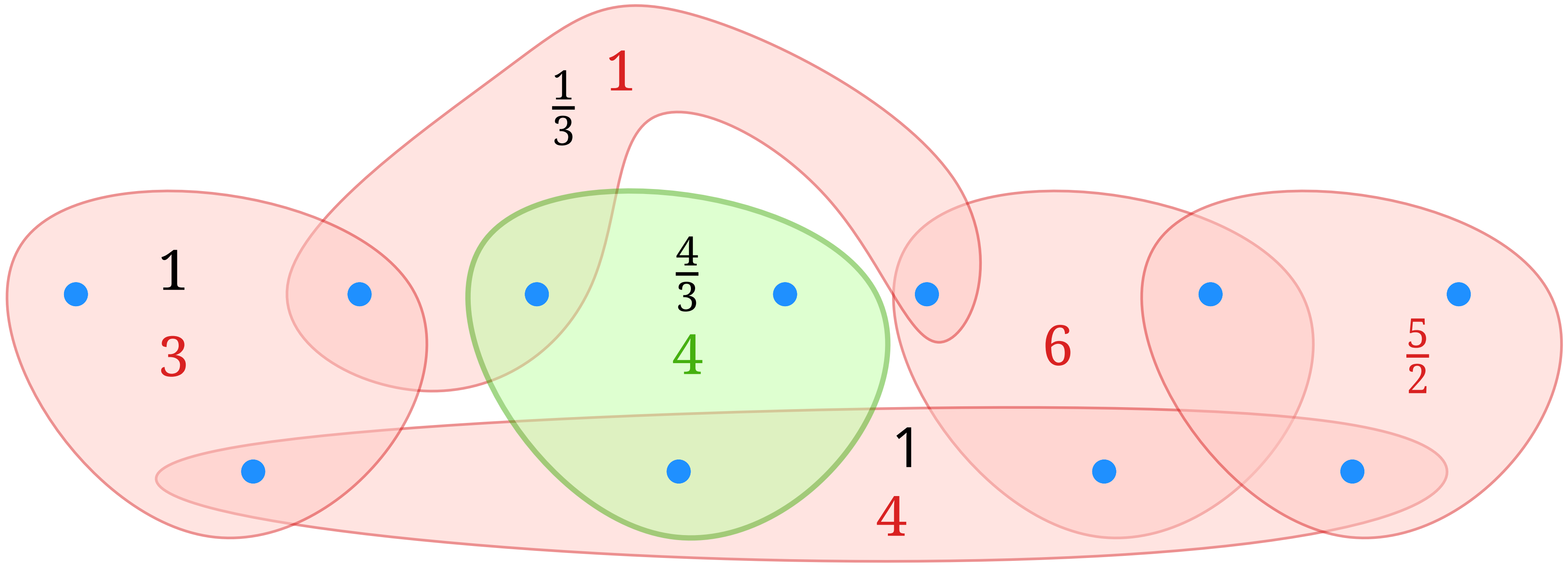

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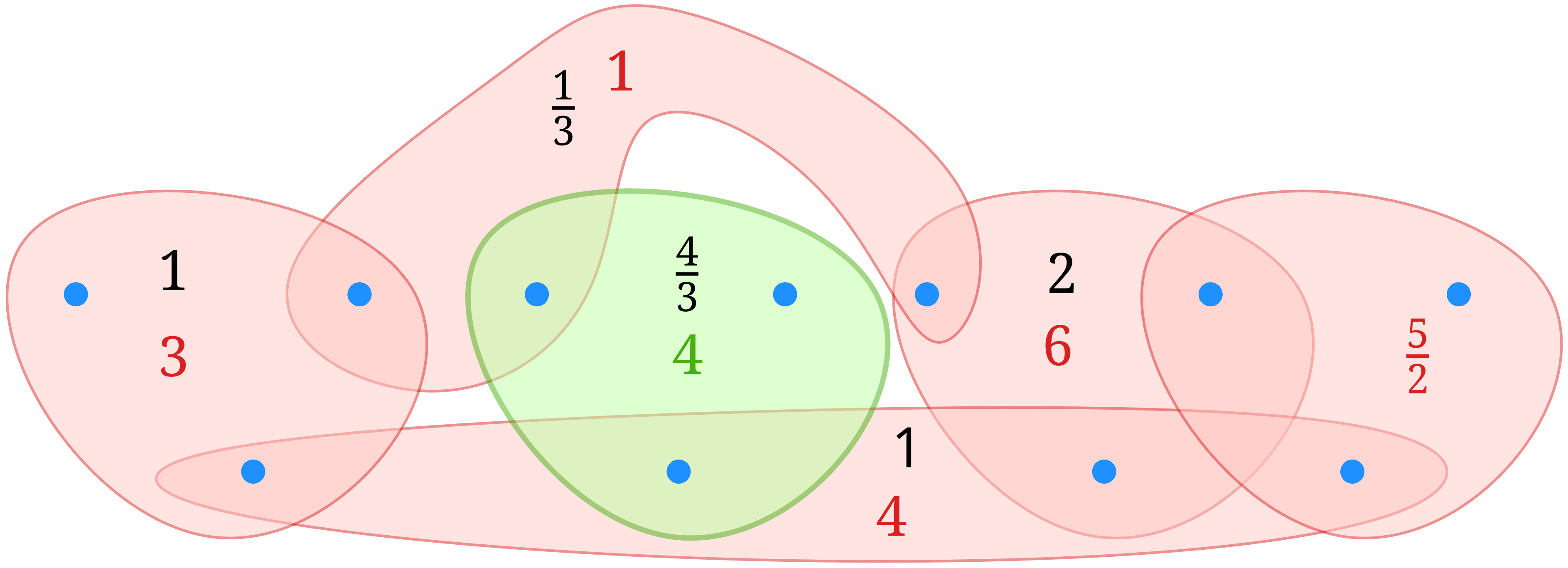

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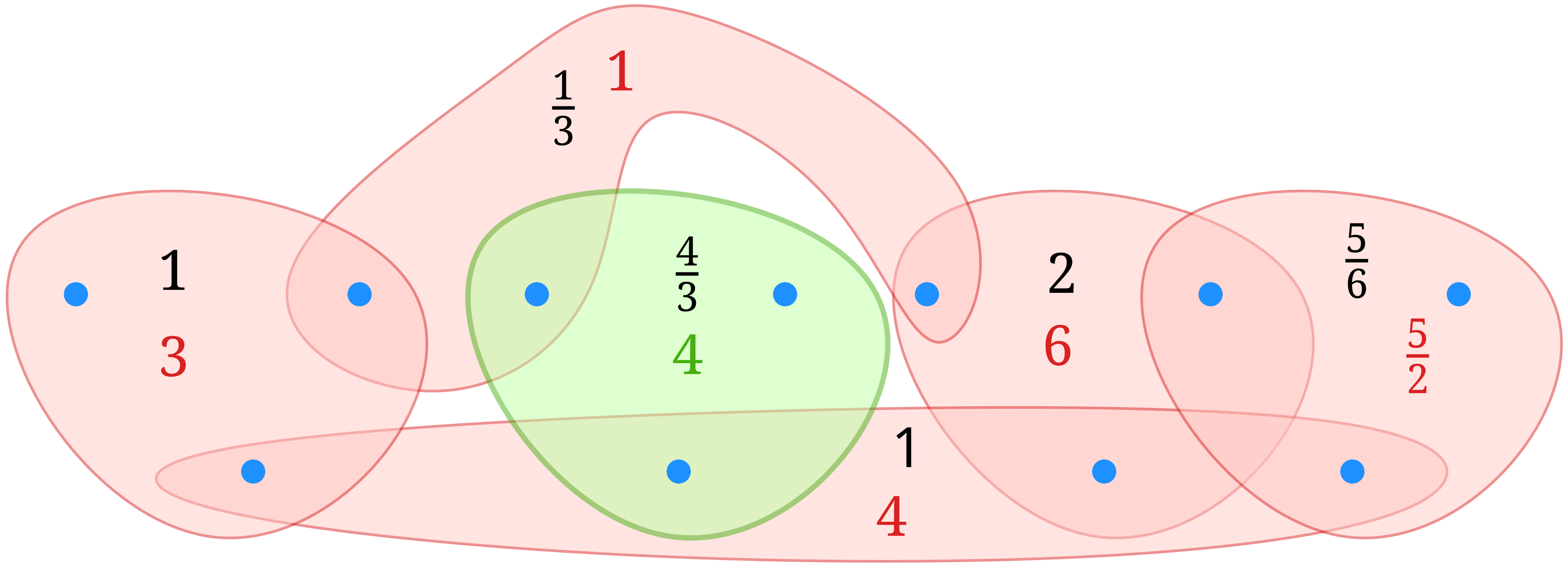

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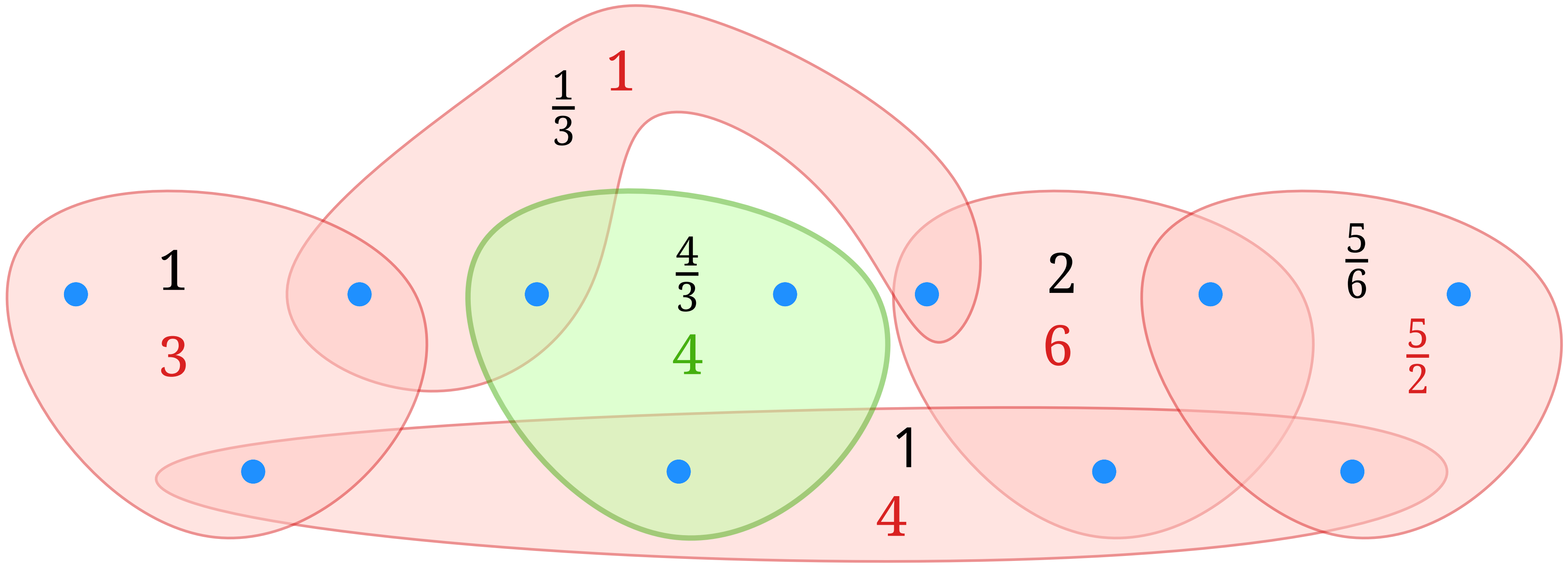

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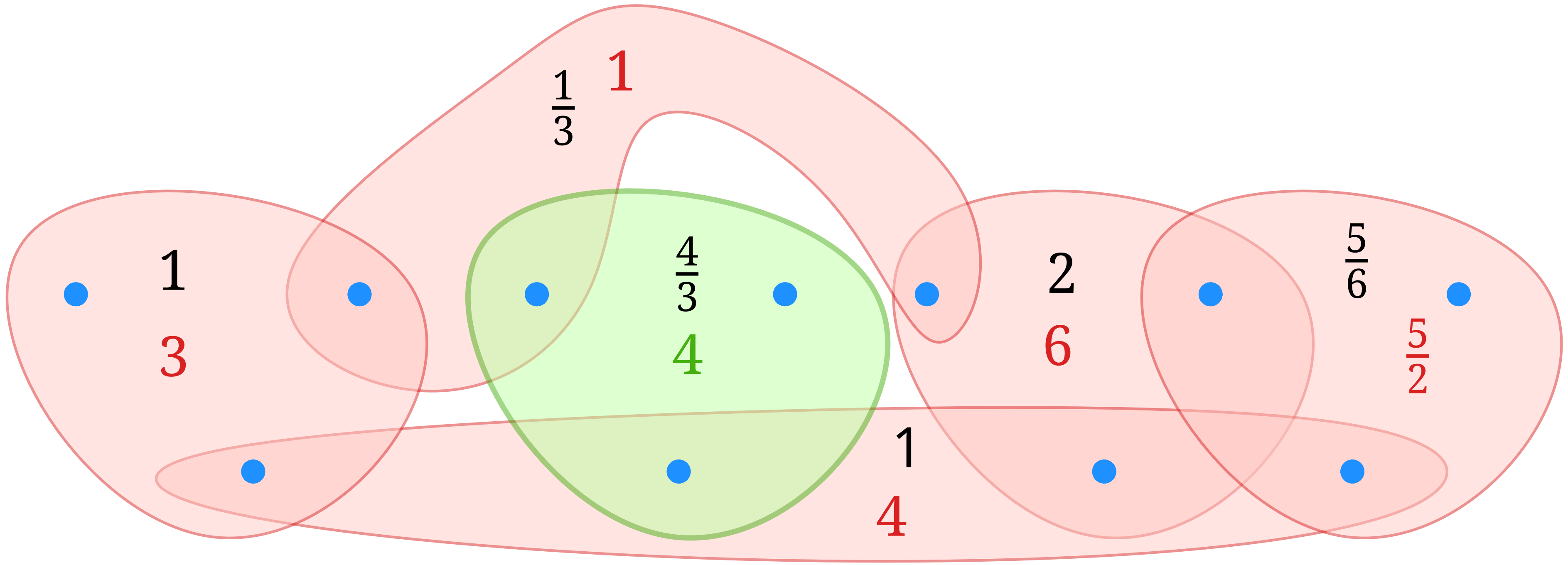
Fix price of elements bought and recompute per-element cost.



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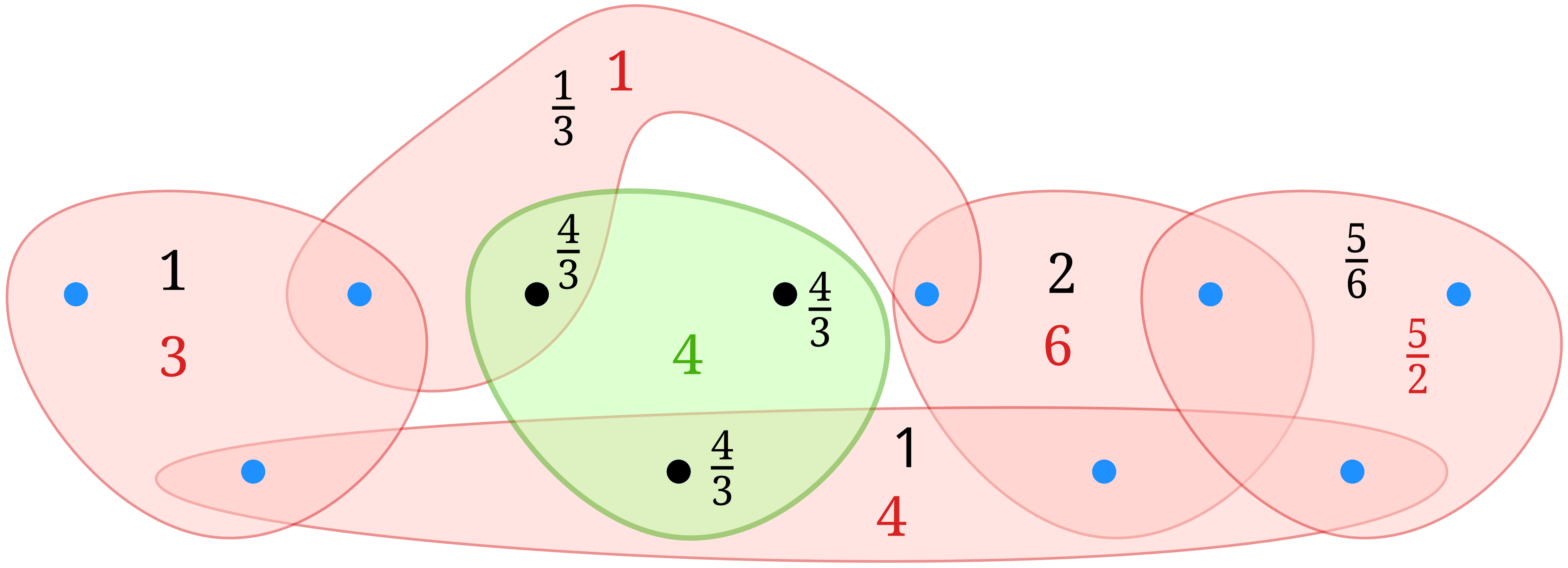

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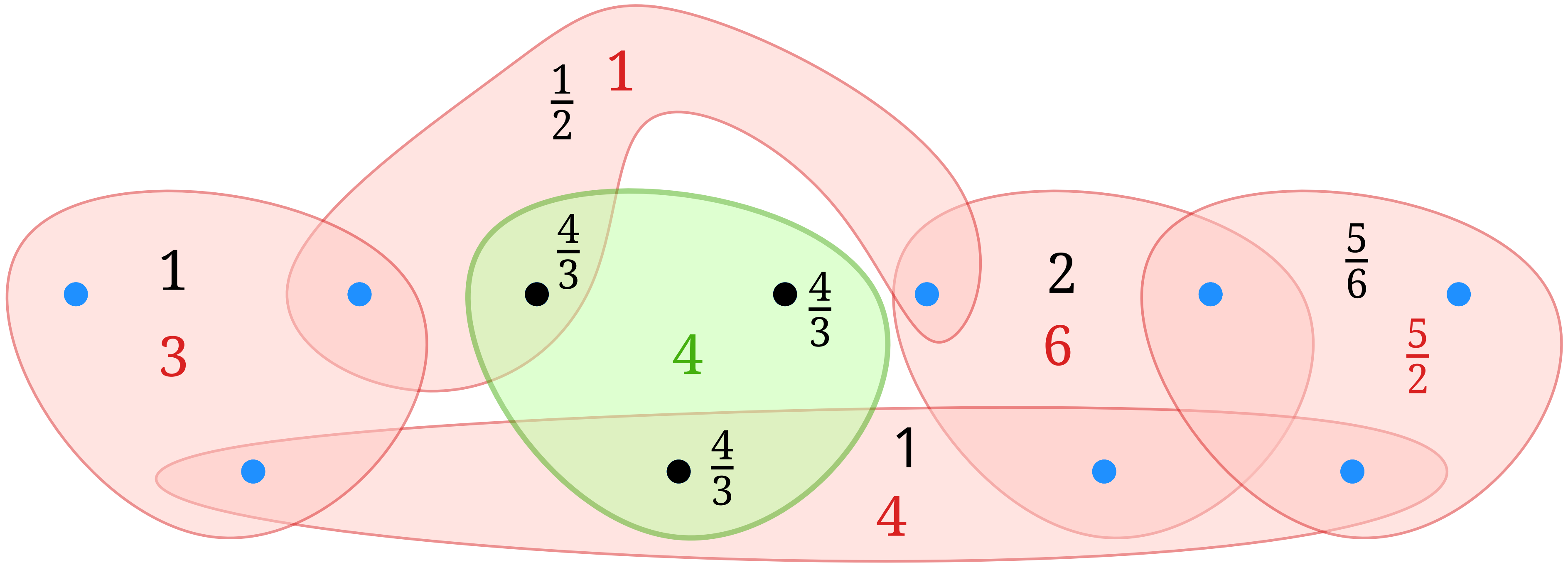

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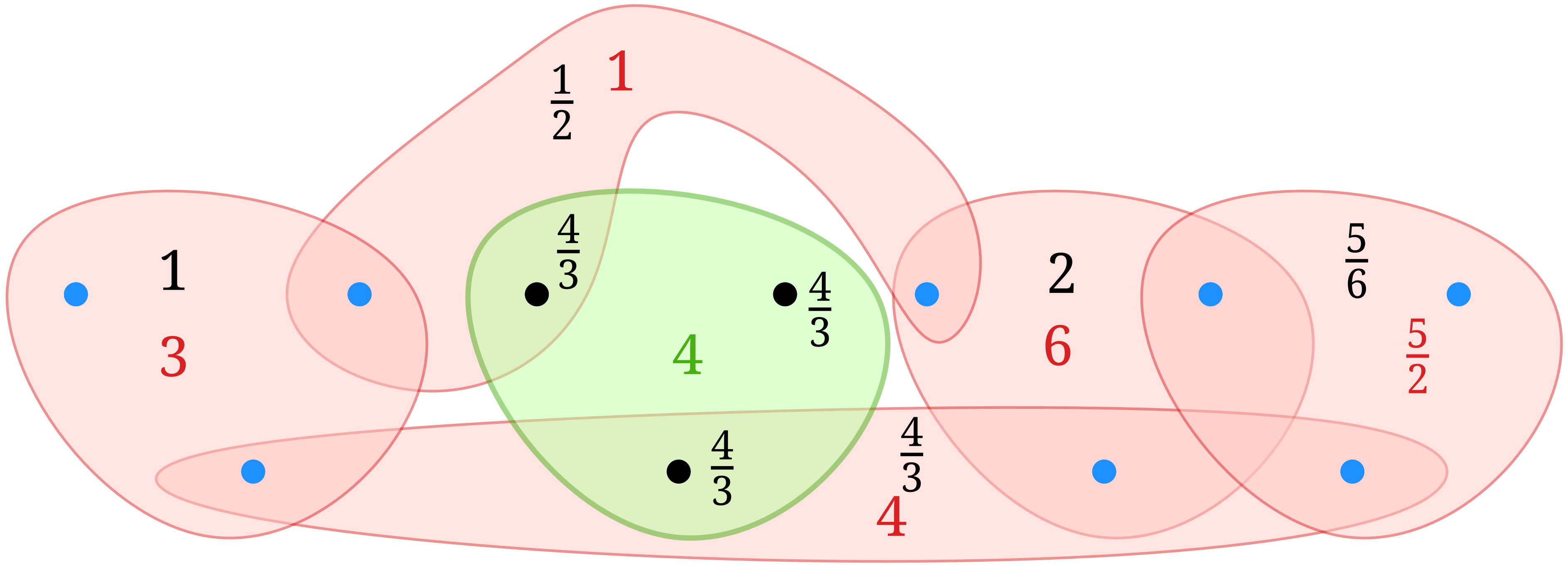

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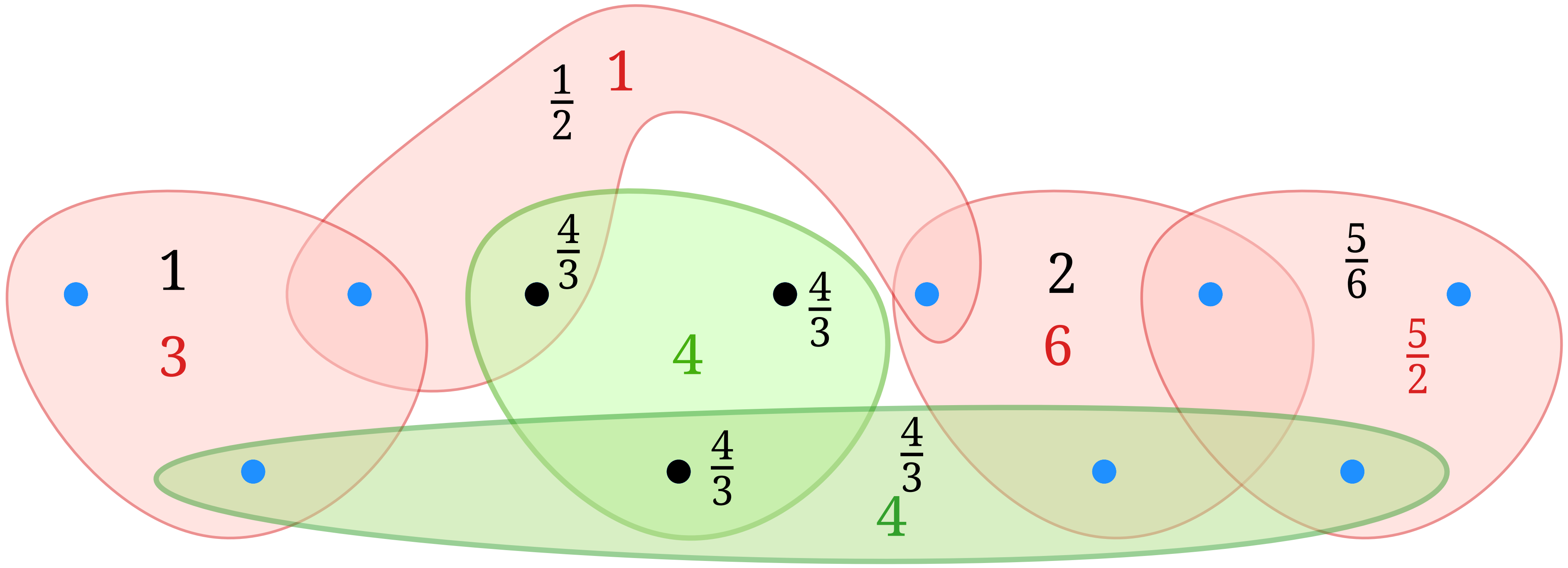

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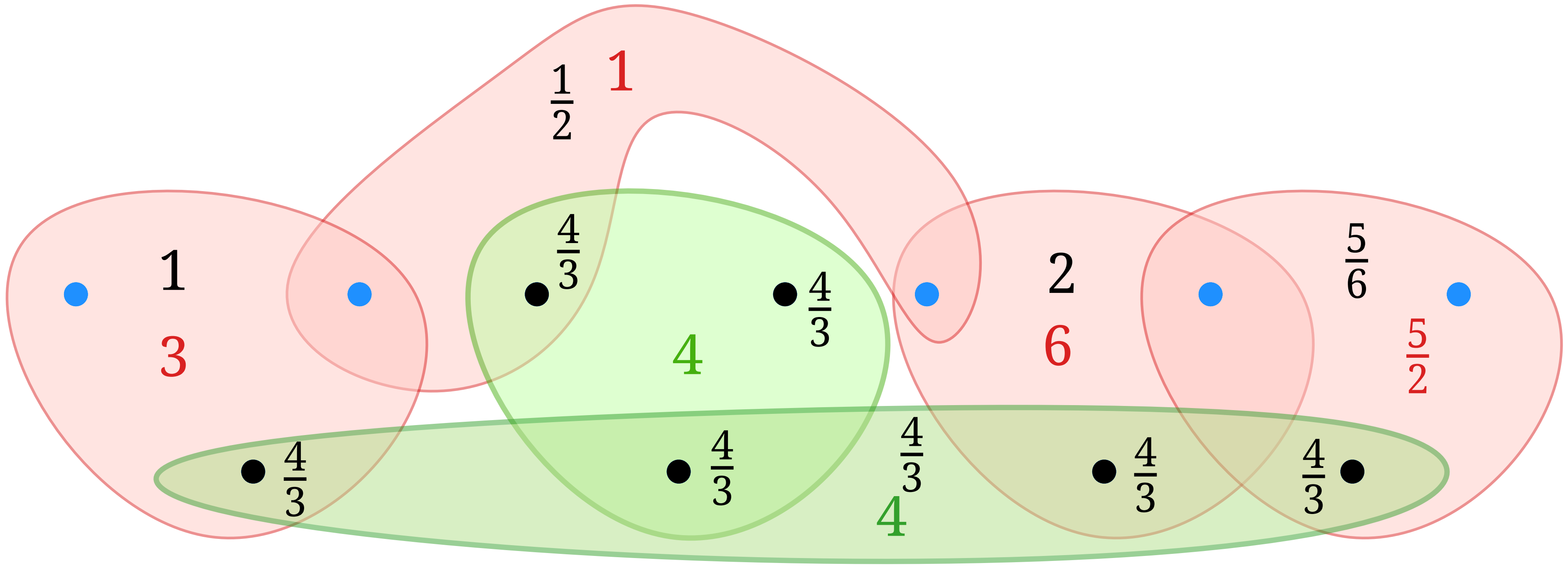

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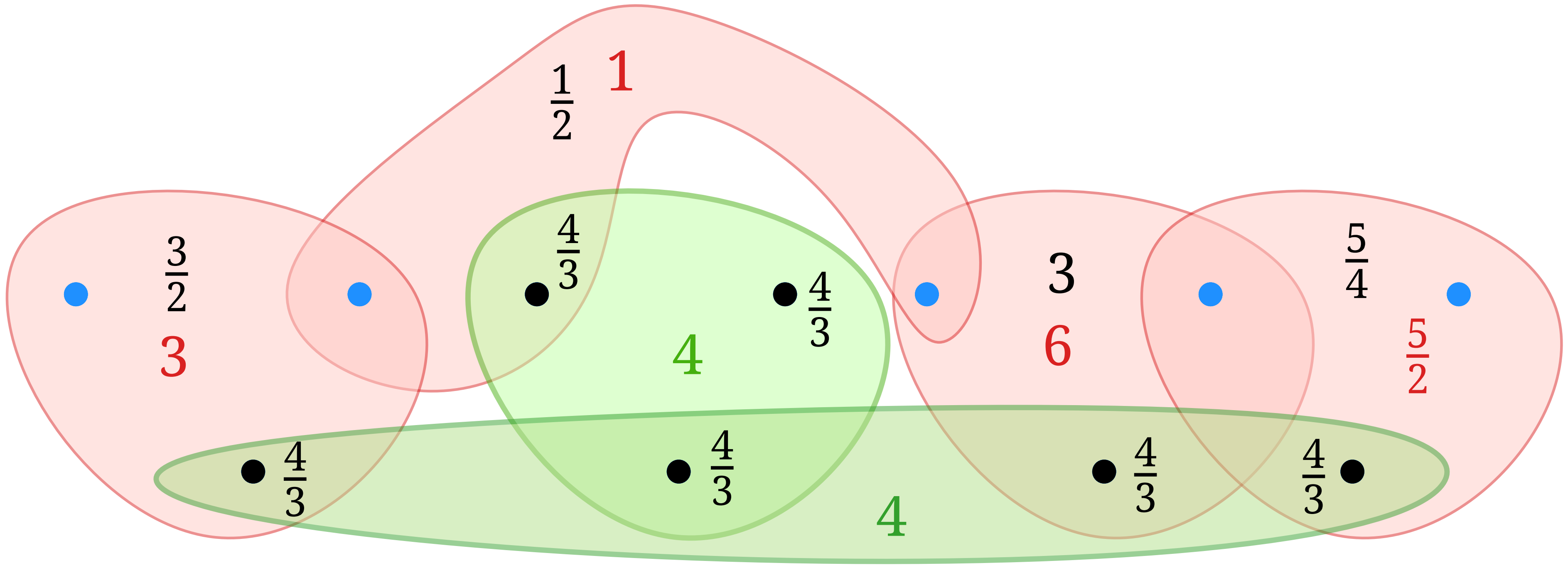

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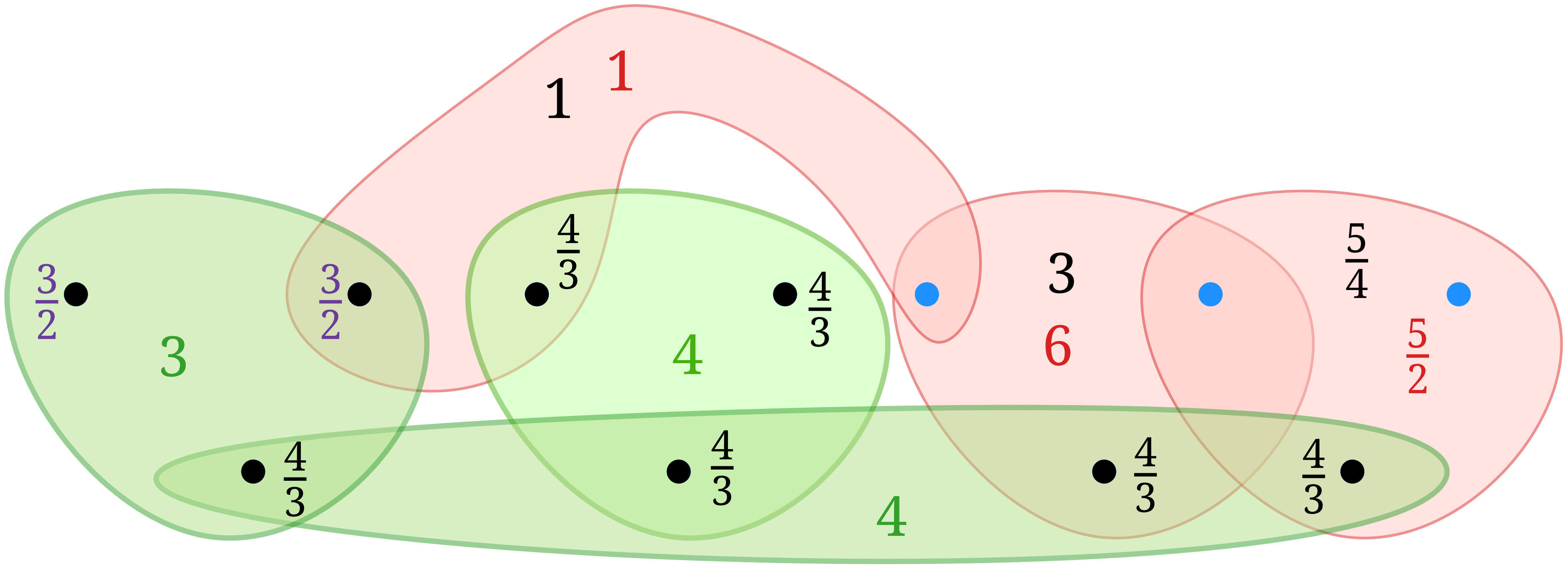

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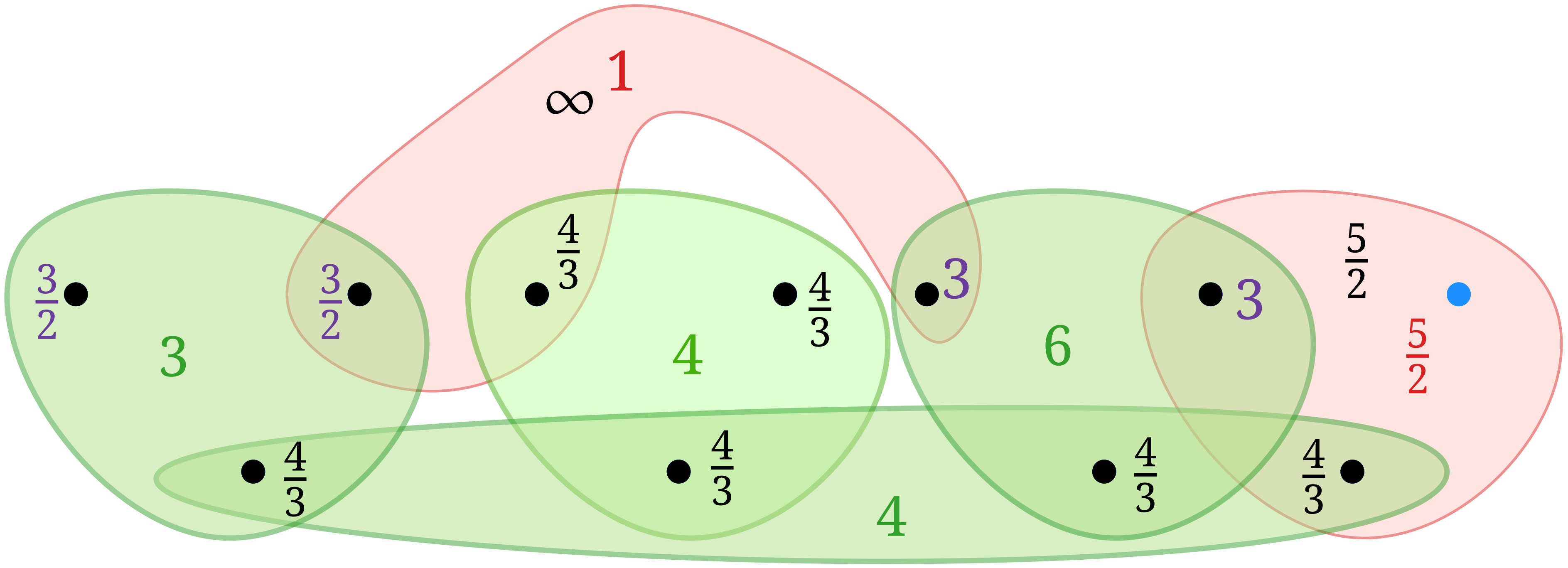

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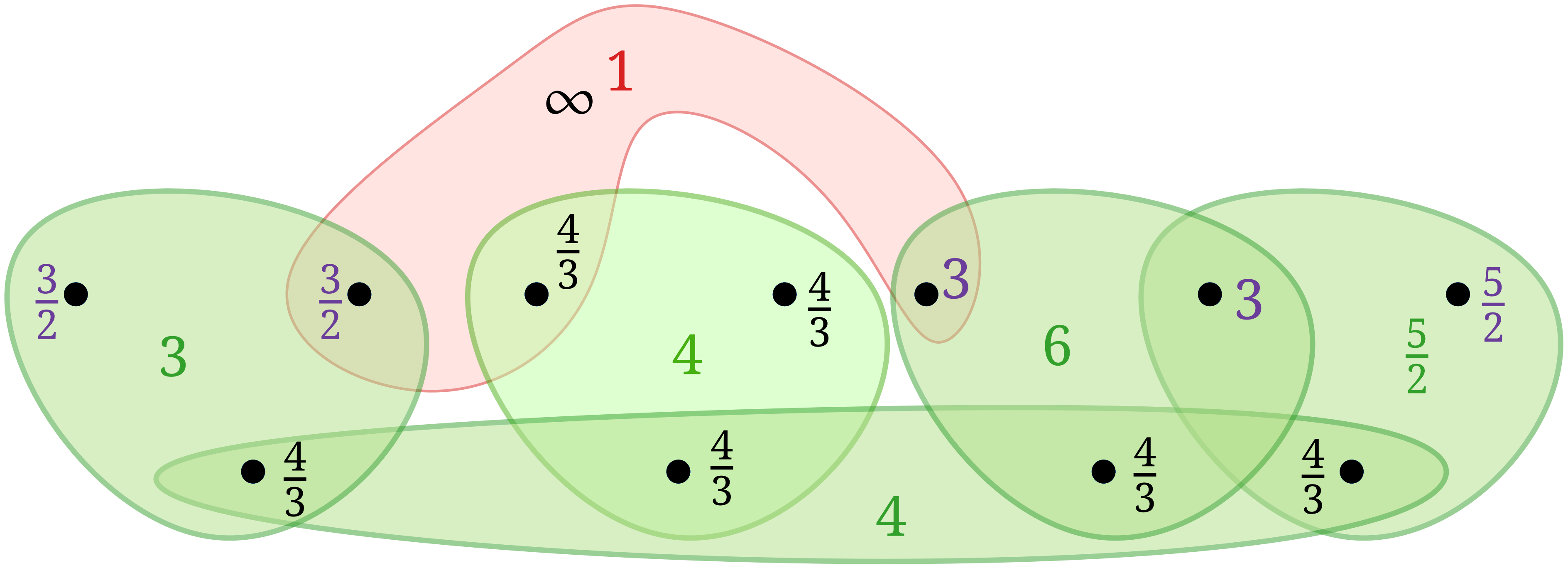

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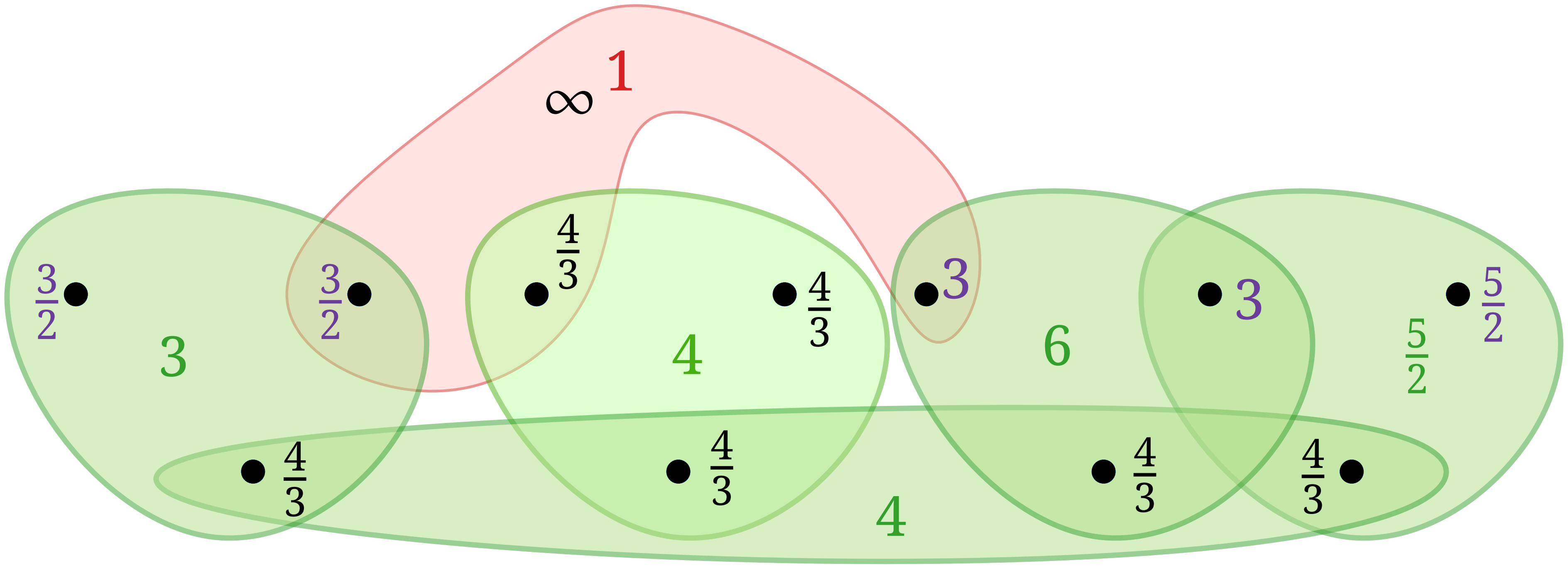
total cost: $\sum_{u \in U} \mathrm{price}(u)$



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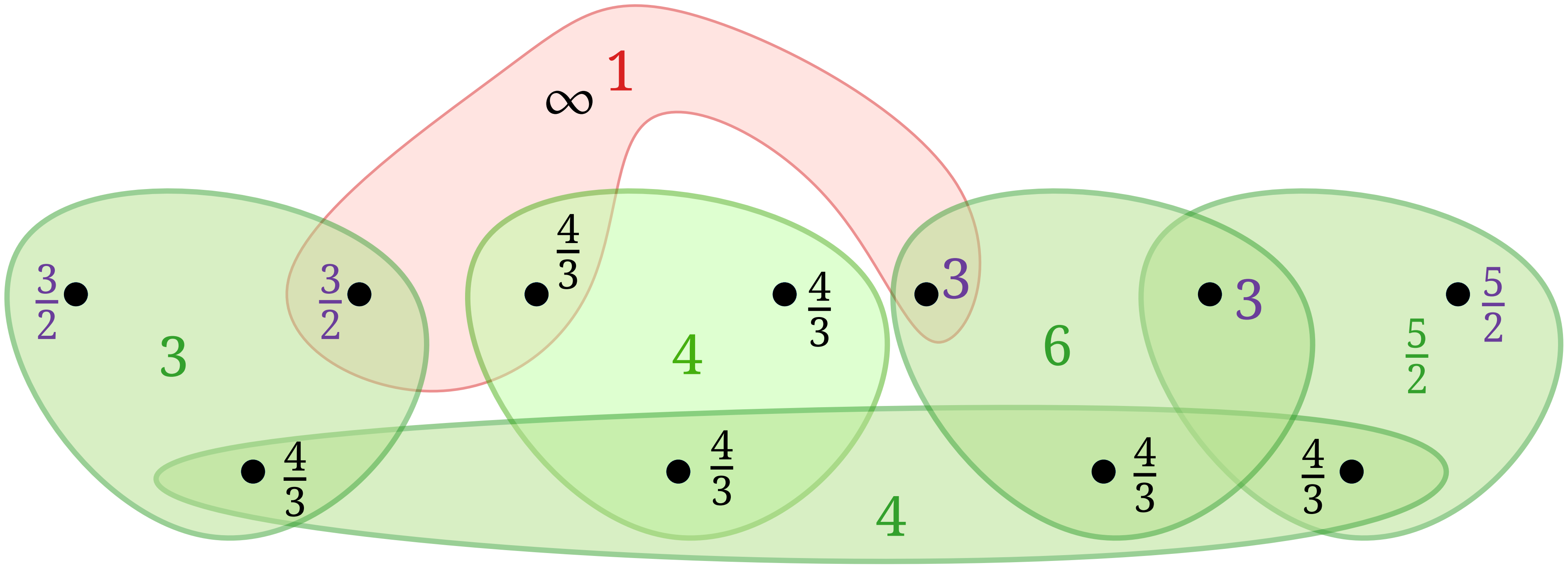
Greedy: Always choose the set with minimum per-element cost.



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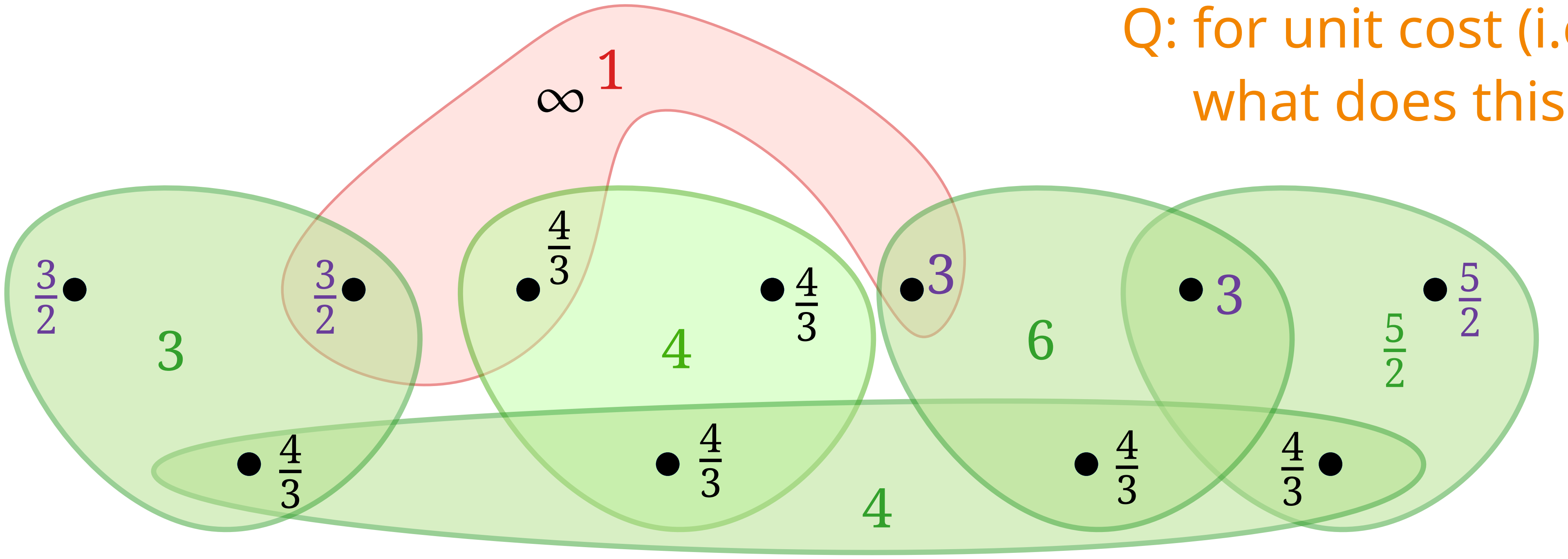

Q: for unit cost (i.e., cardinality set cover), what does this correspond to?

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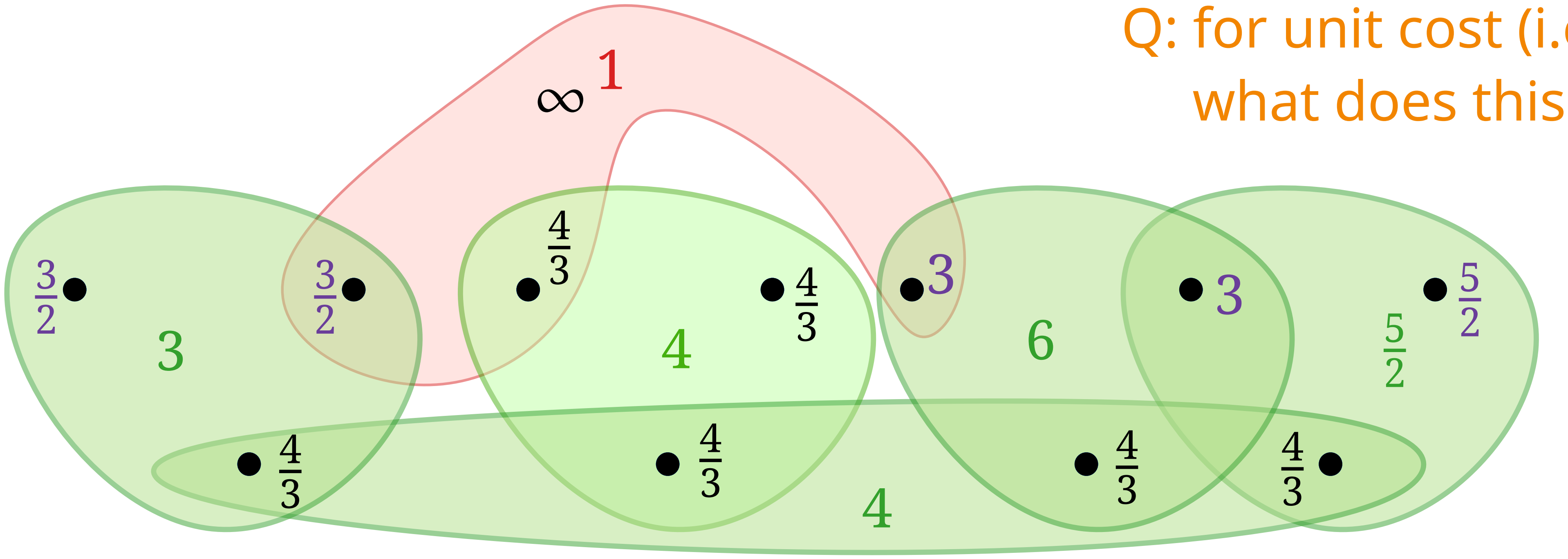


Q: for unit cost (i.e., cardinality set cover), what does this correspond to?

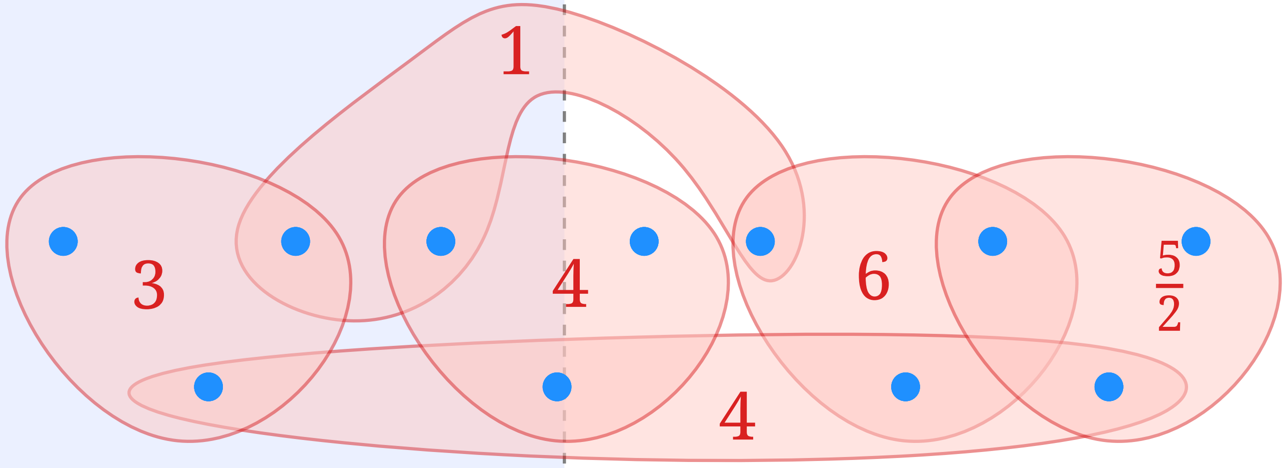
always pick the set that covers most of the remaining elements

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$ // covered elements

$\mathcal{S}' \leftarrow \emptyset$ // picked sets



return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

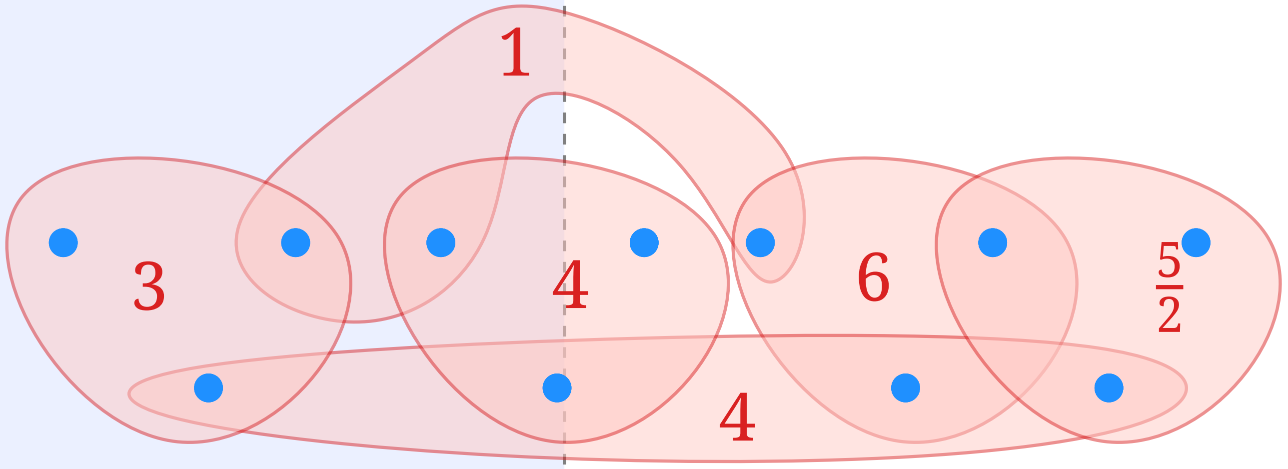
GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$ `// covered elements`

$\mathcal{S}' \leftarrow \emptyset$ `// picked sets`

while $C \neq U$ **do**

return $\mathcal{S}'$ `// Cover of` U



Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

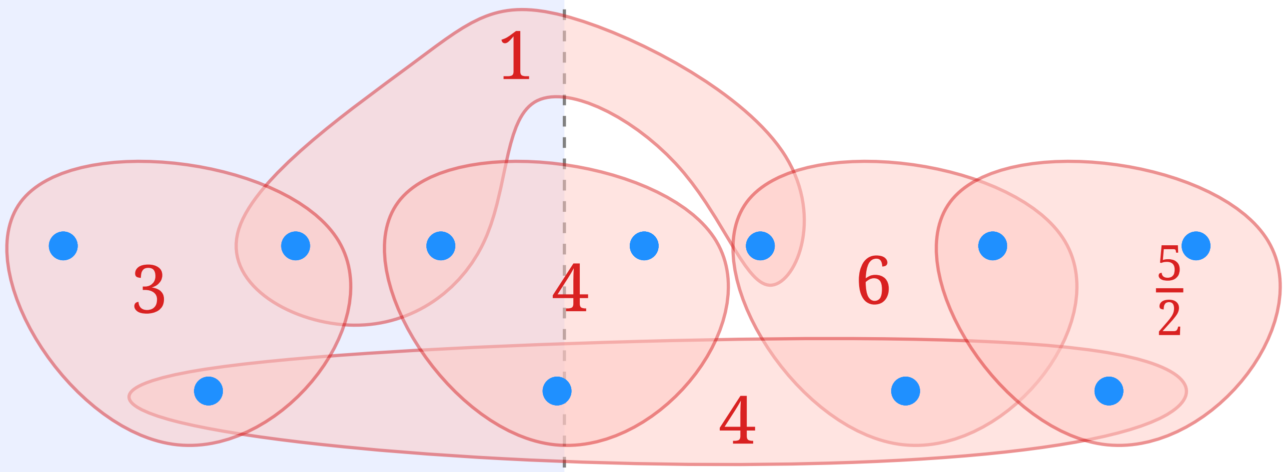
$C \leftarrow \emptyset$ `// covered elements`

$\mathcal{S}' \leftarrow \emptyset$ `// picked sets`

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

return $\mathcal{S}'$ `// Cover of U`



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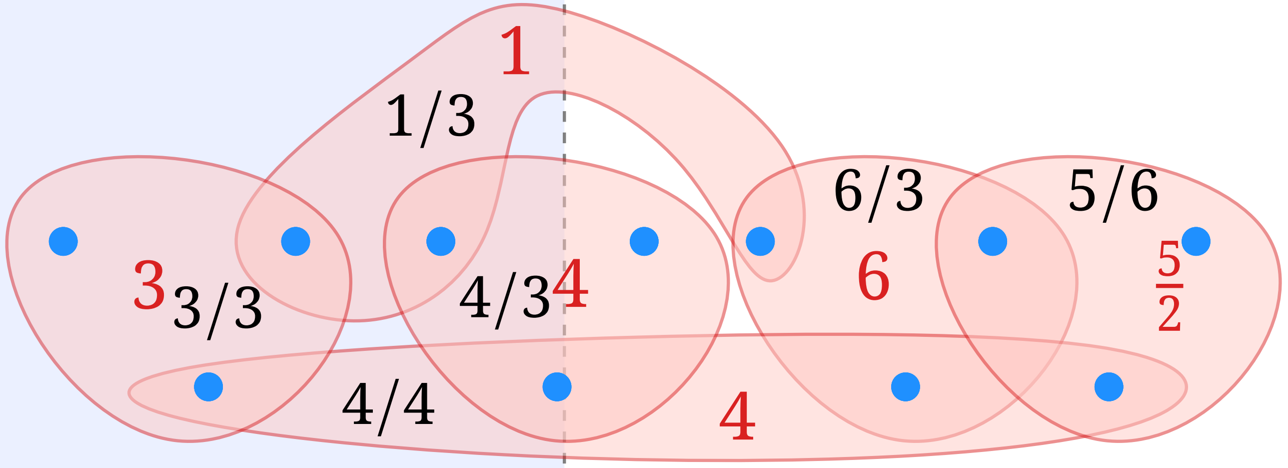

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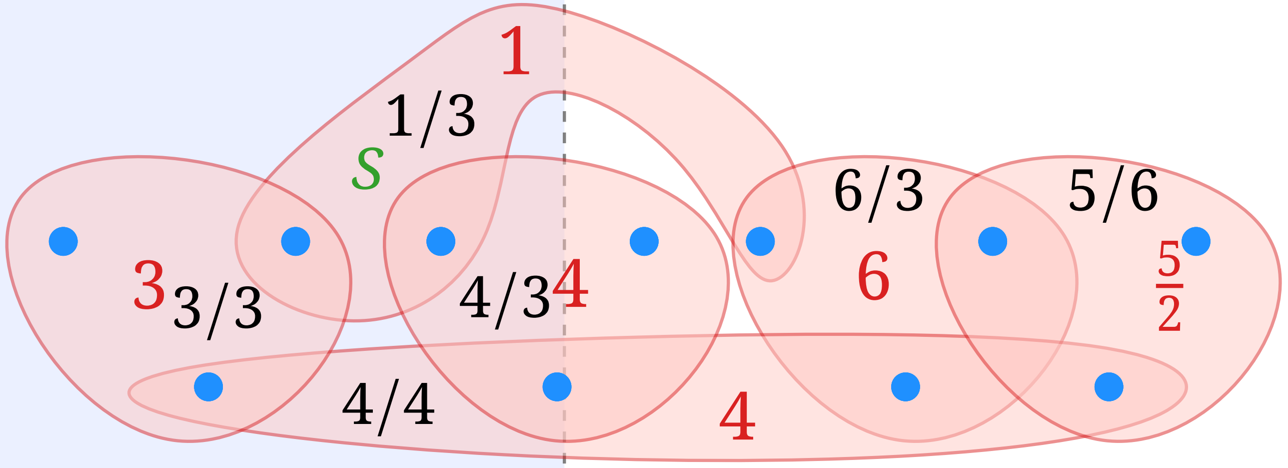

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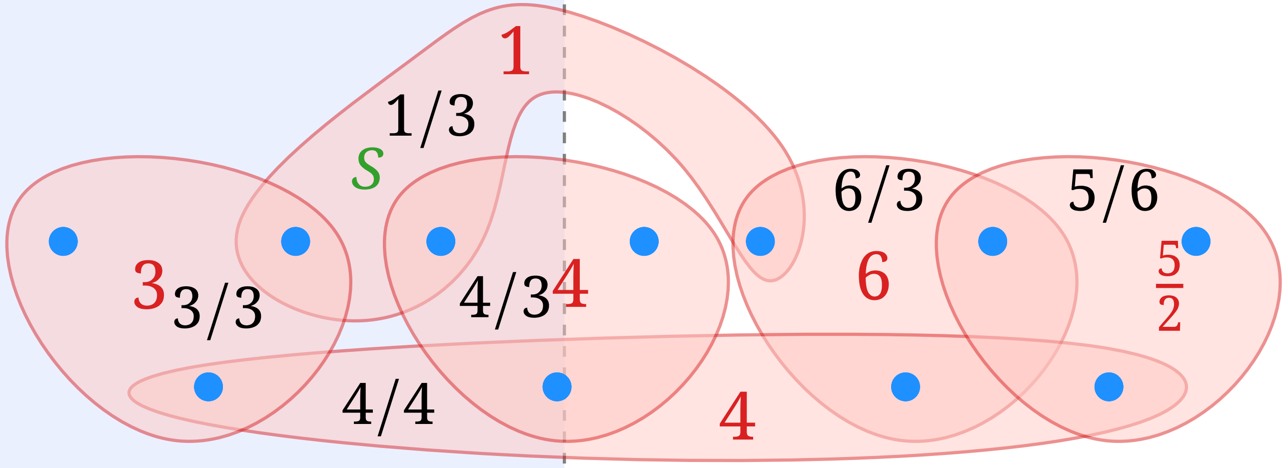

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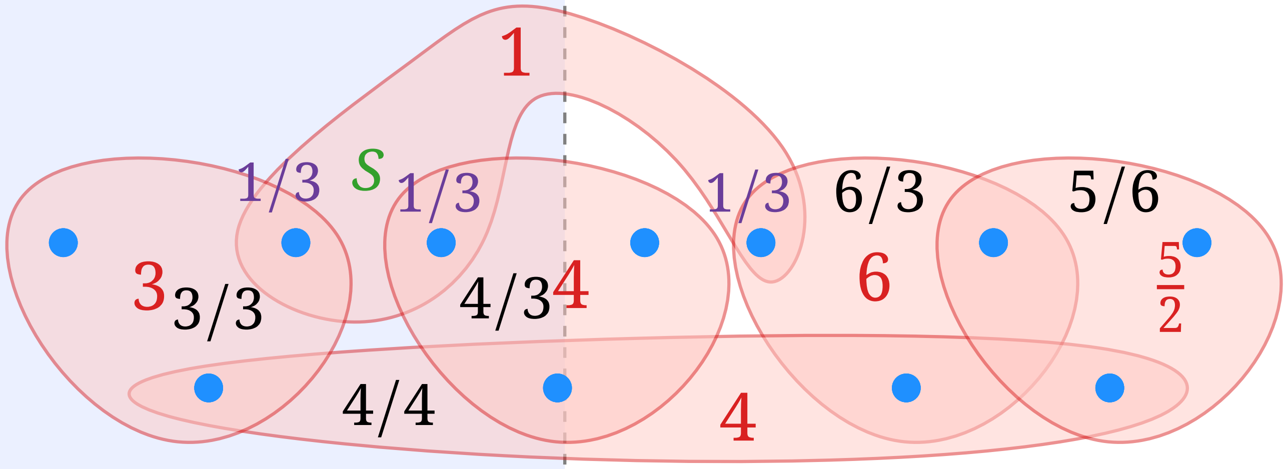

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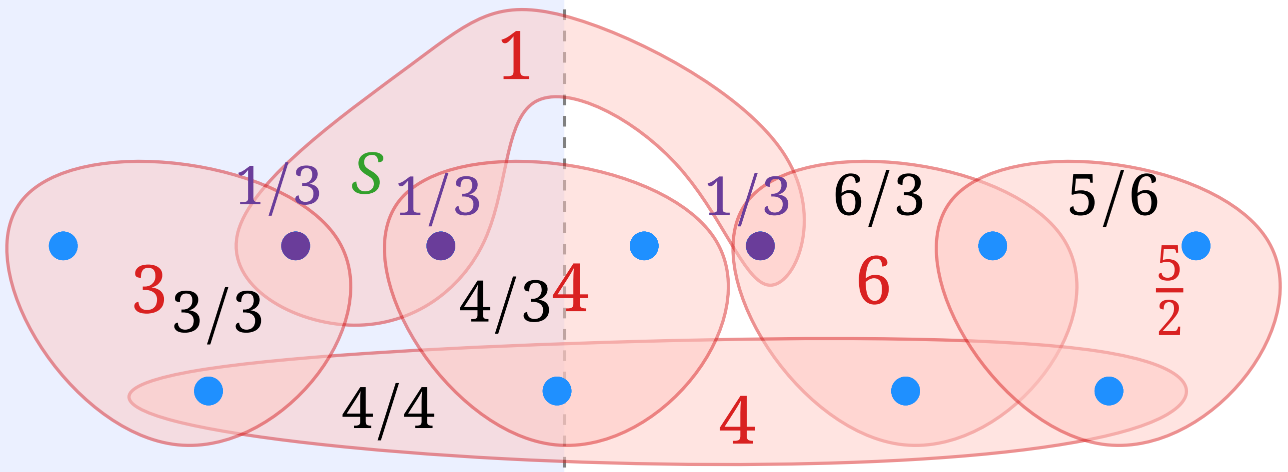

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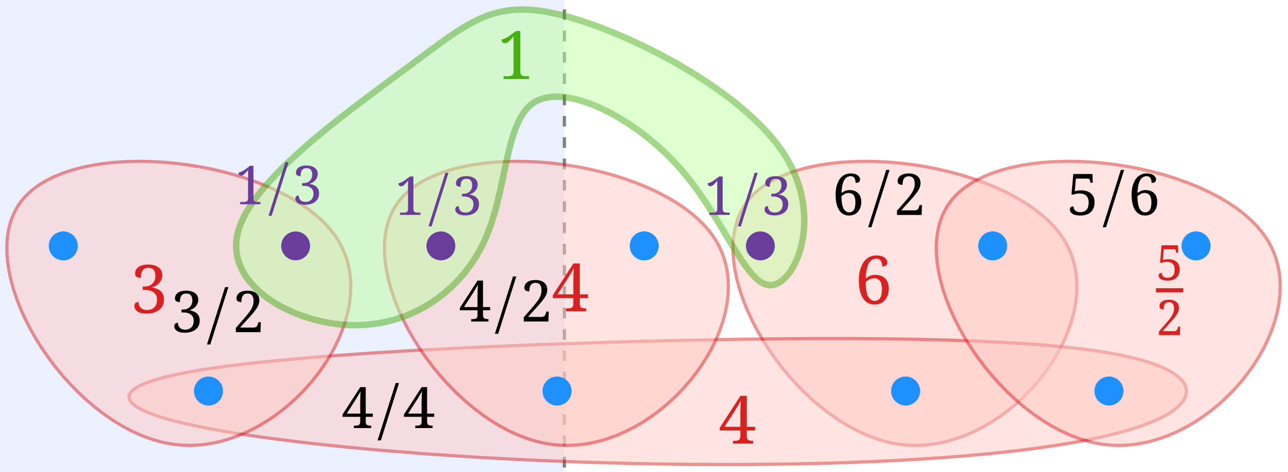

How does the algorithm continue?

Greedy SETCOVER: analysis

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

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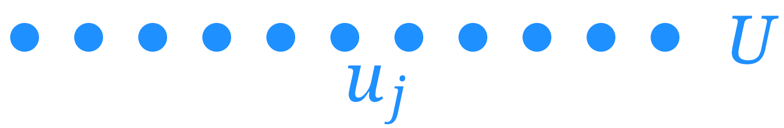

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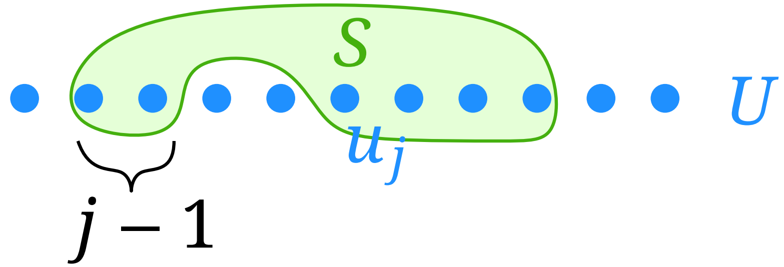

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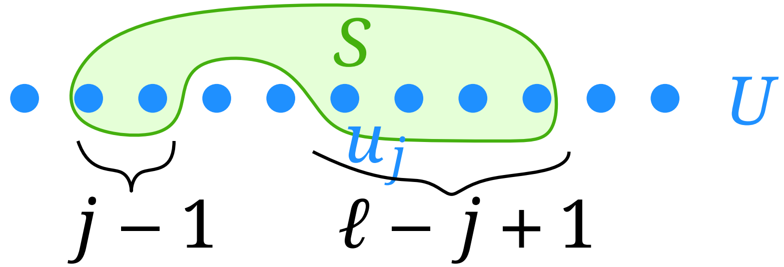

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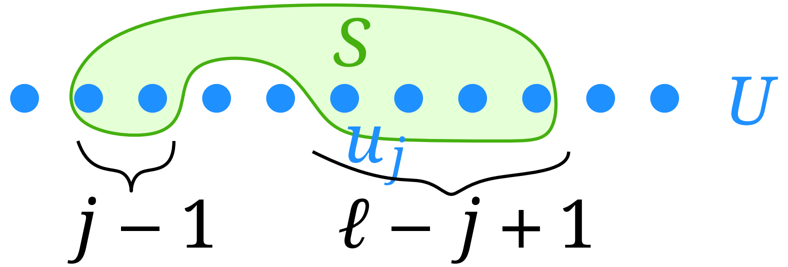

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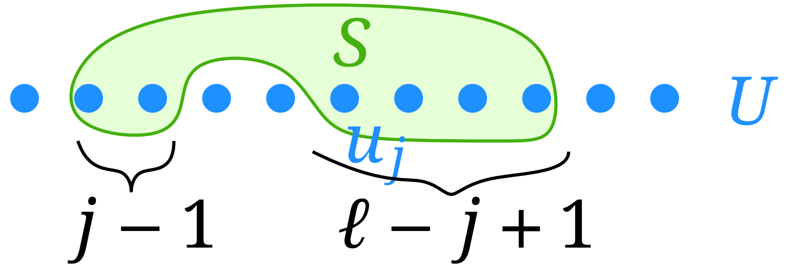

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- $\leq j - 1$ elements of S may already be bought
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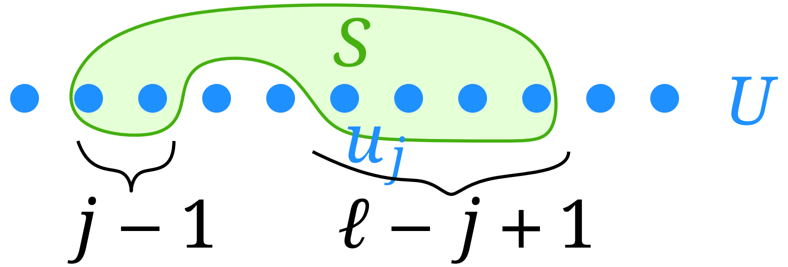
- price by alg. no larger due to greedy choice



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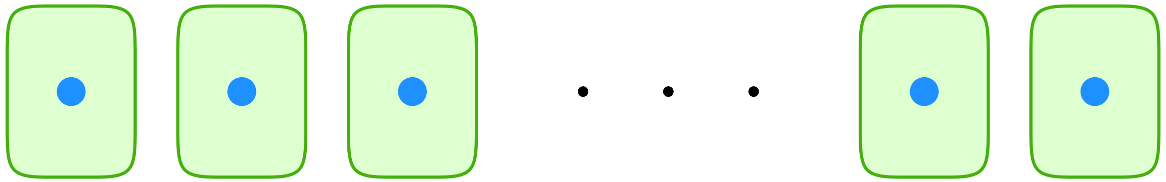

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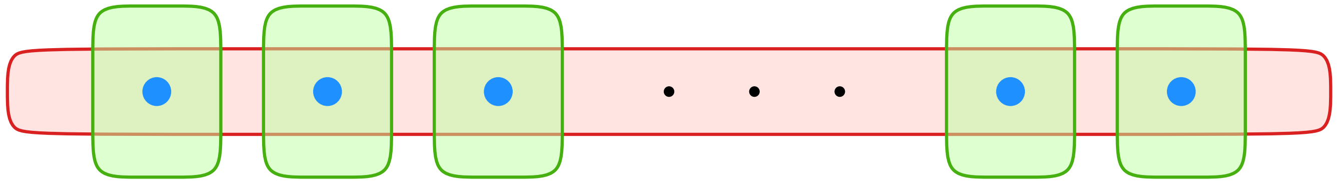

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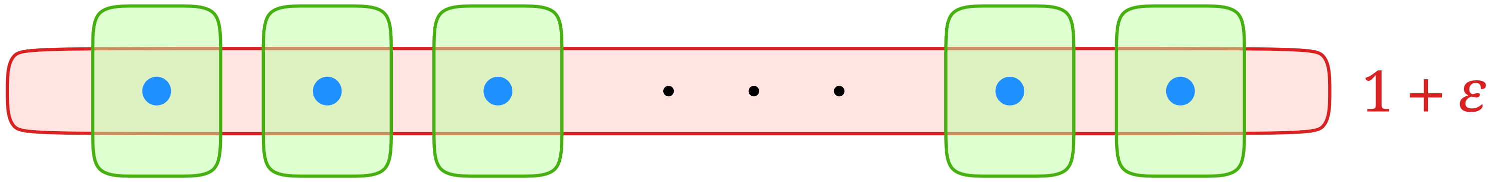

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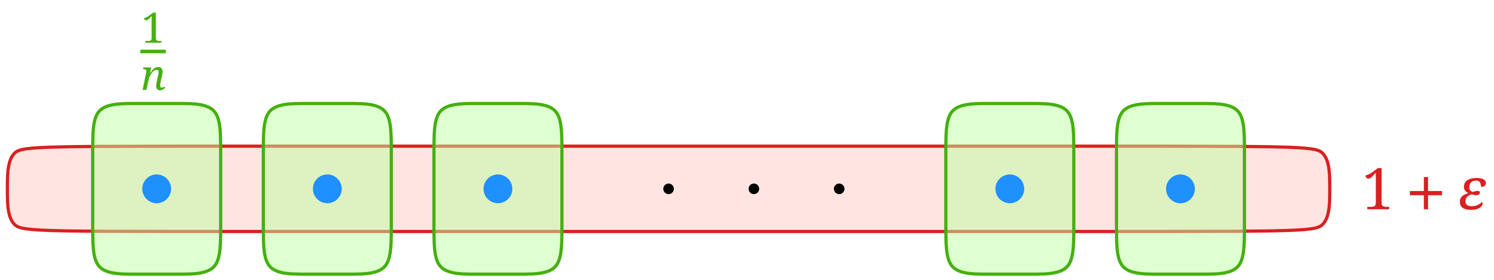

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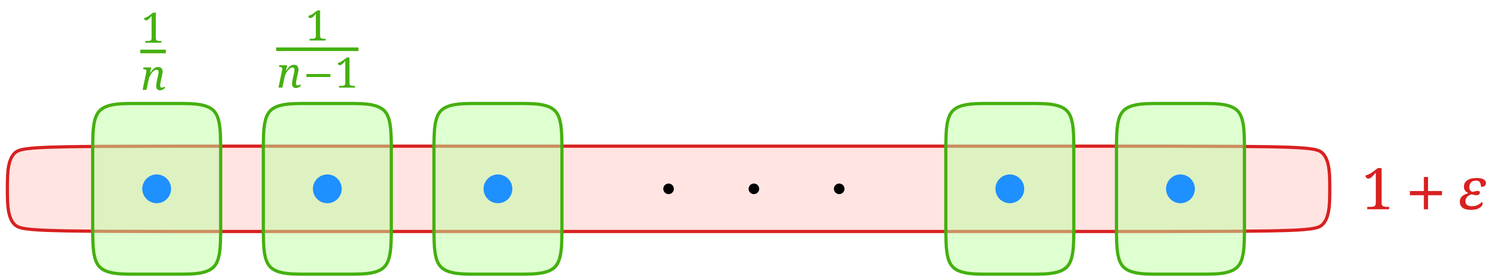

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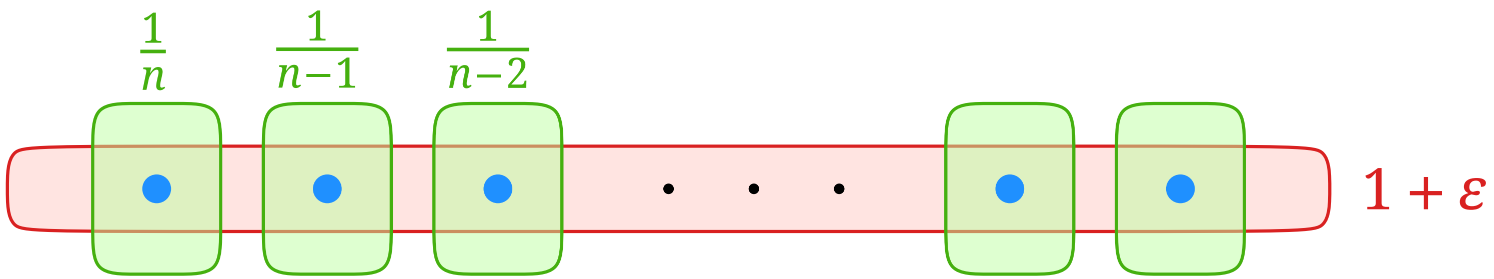

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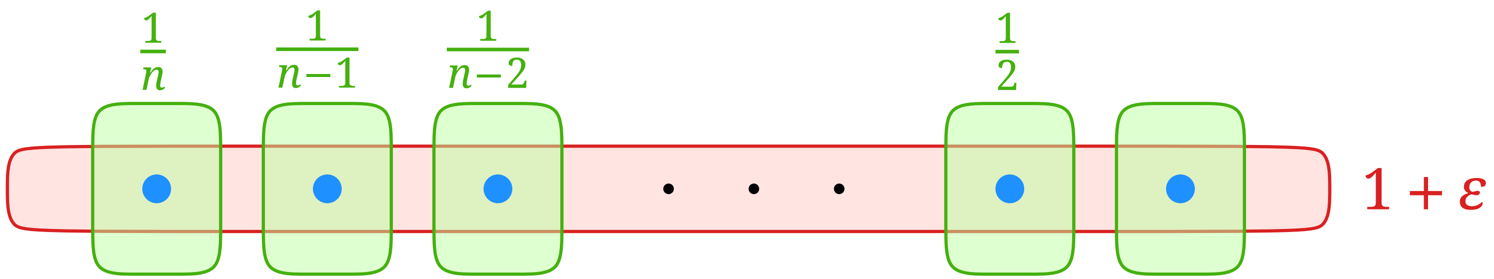

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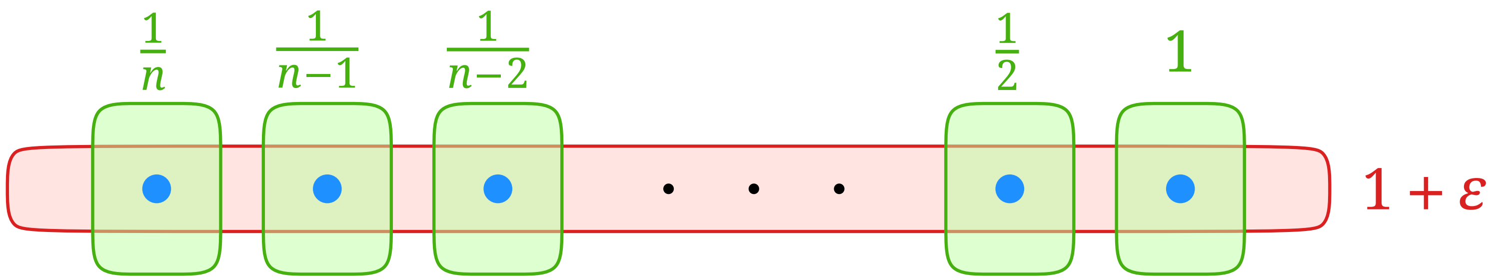

Q: which sets does the algorithm choose?
what is an optimal cover?

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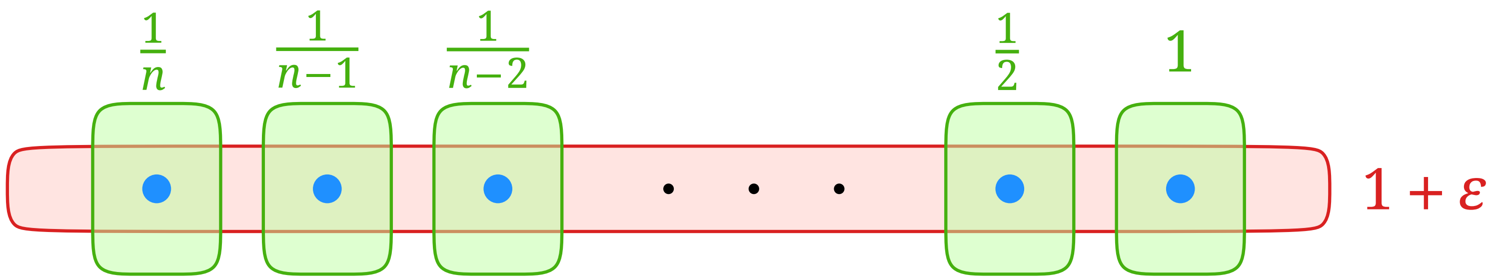

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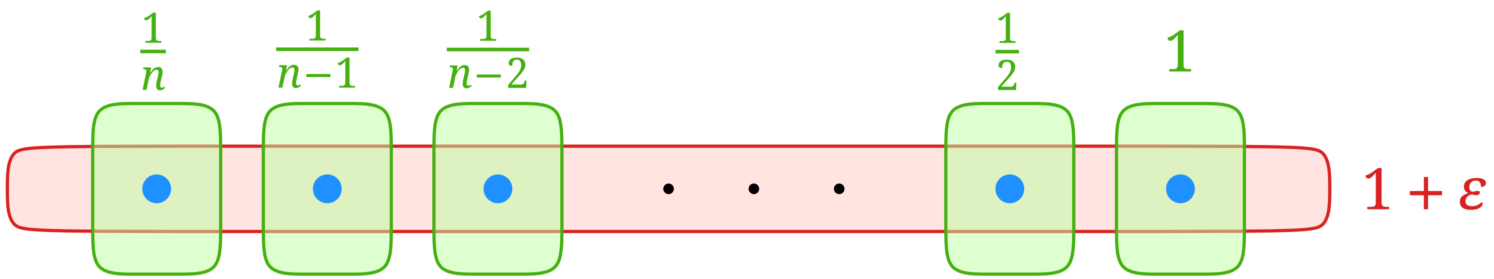

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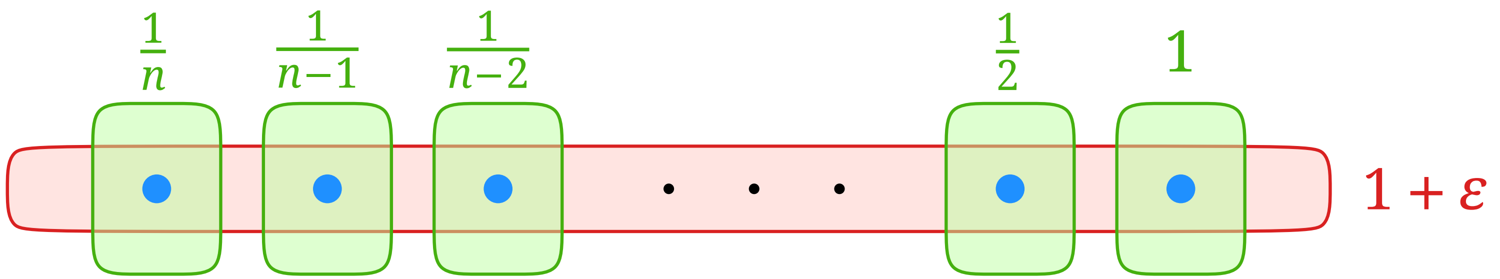

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Can we do better?

No – SetCover cannot be approximated within factor $(1 - o(1)) \cdot \ln n$ (unless P = NP). [Feige, JACM 1998]

VERTEXCOVER as SETCOVER

Vertex cover is a special case of set cover. Q: How?

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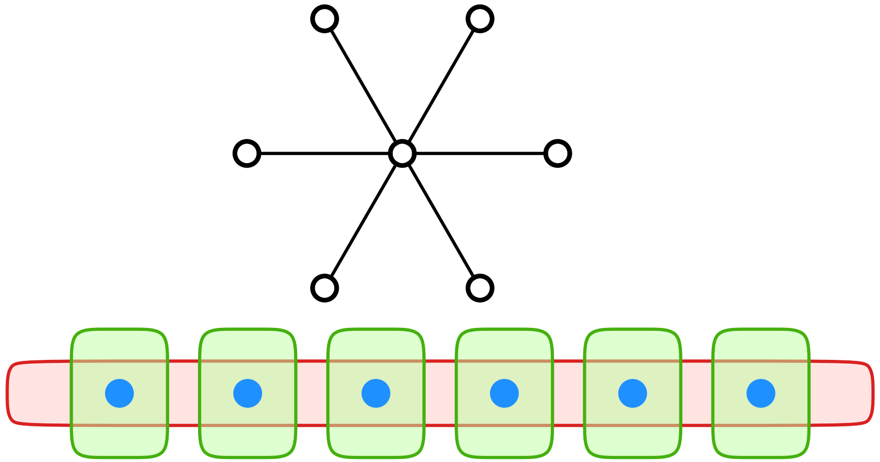

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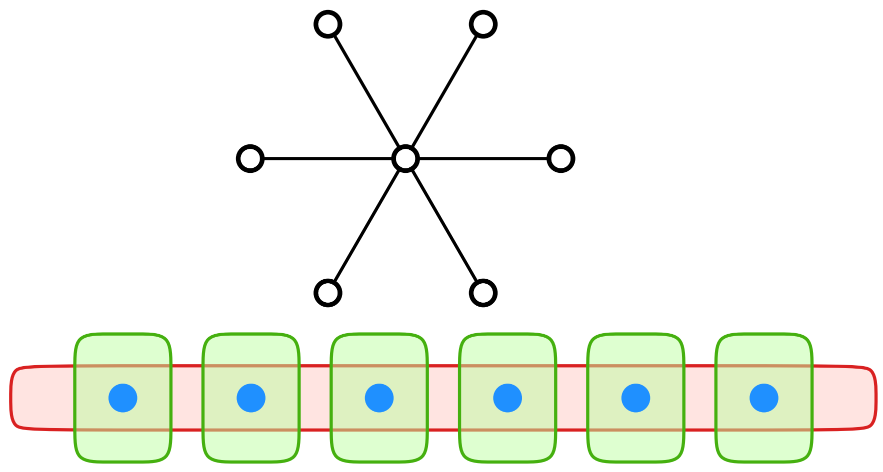

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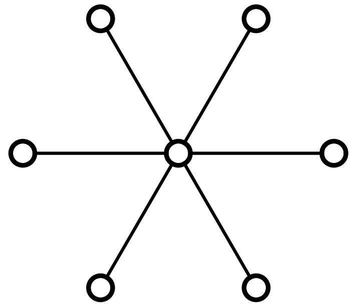

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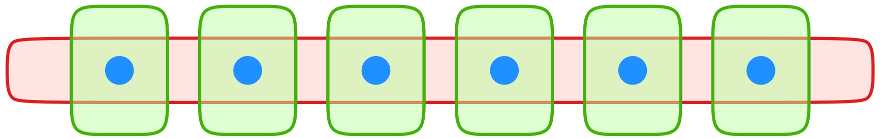

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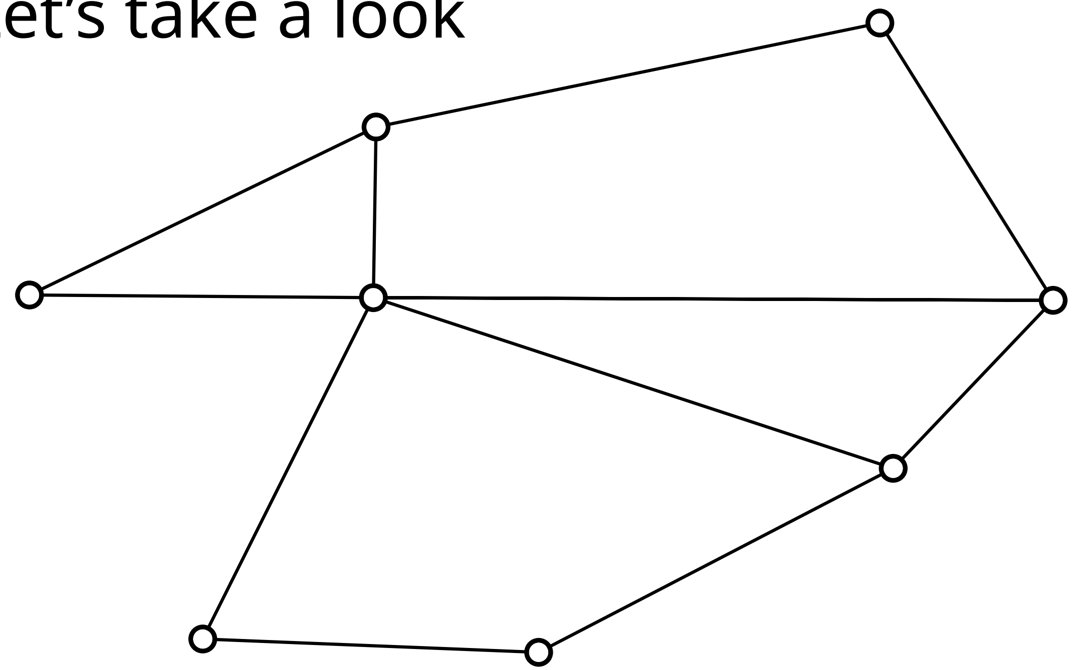

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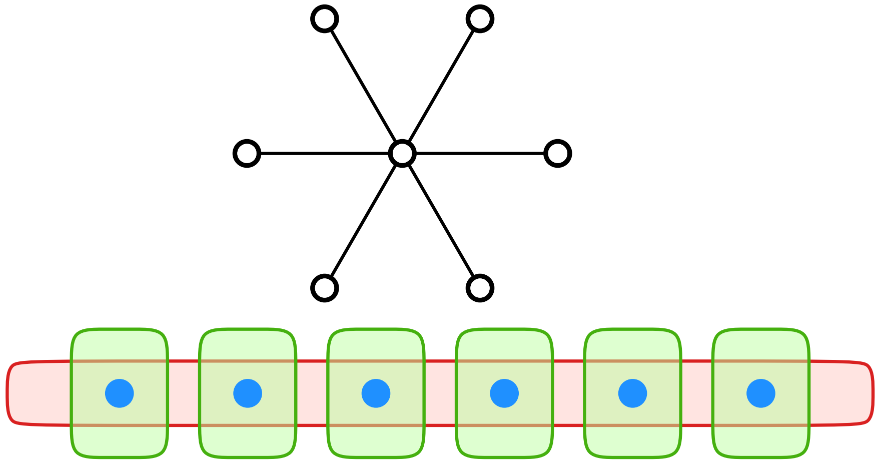

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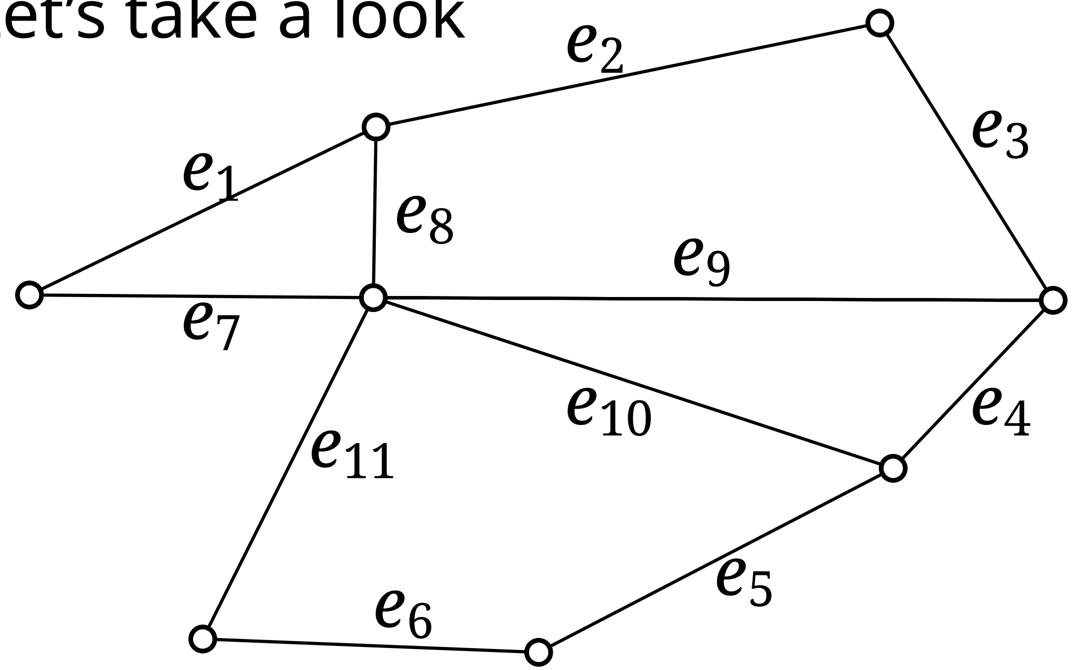

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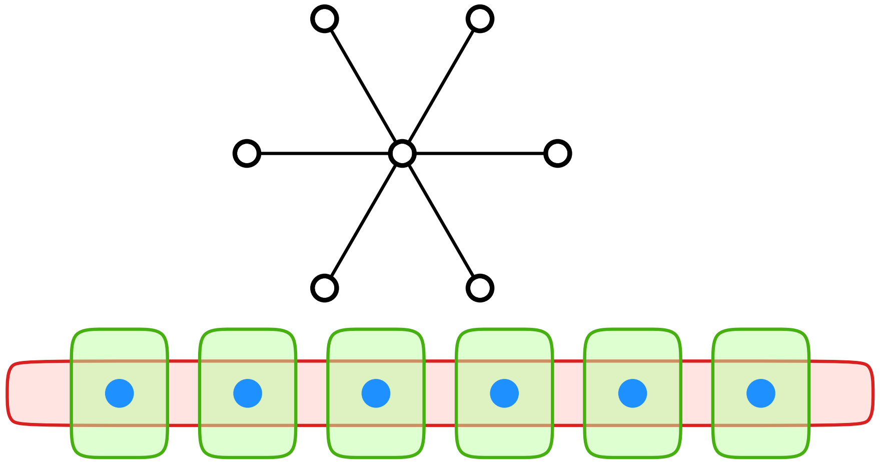

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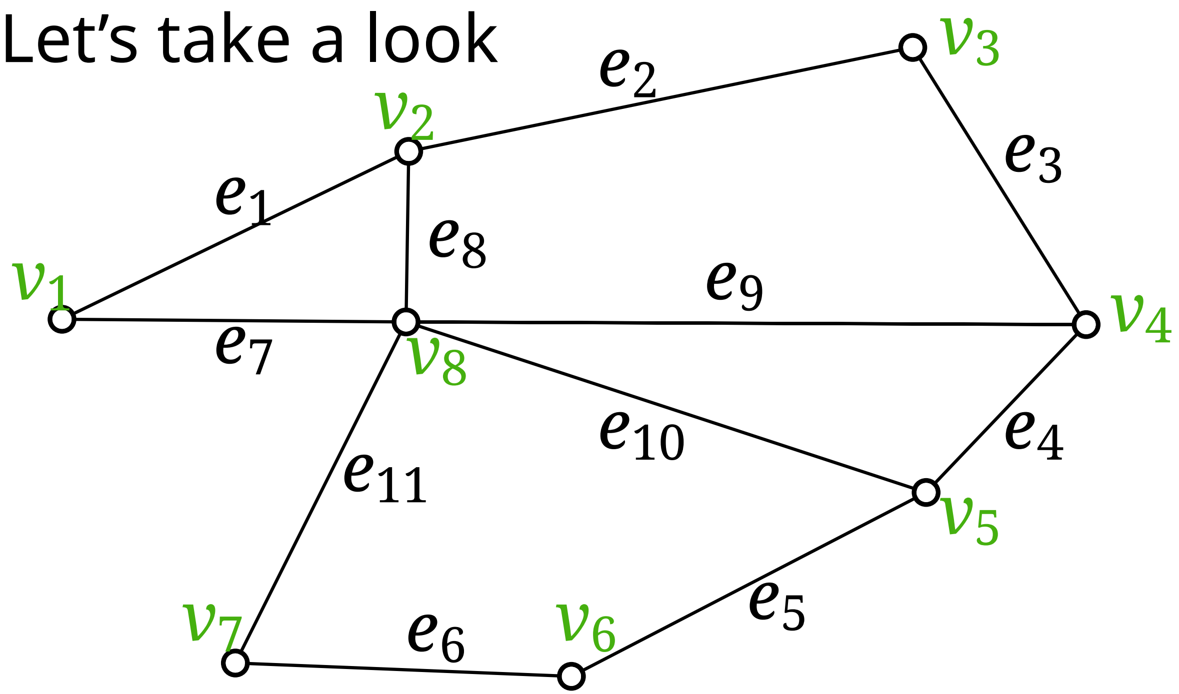

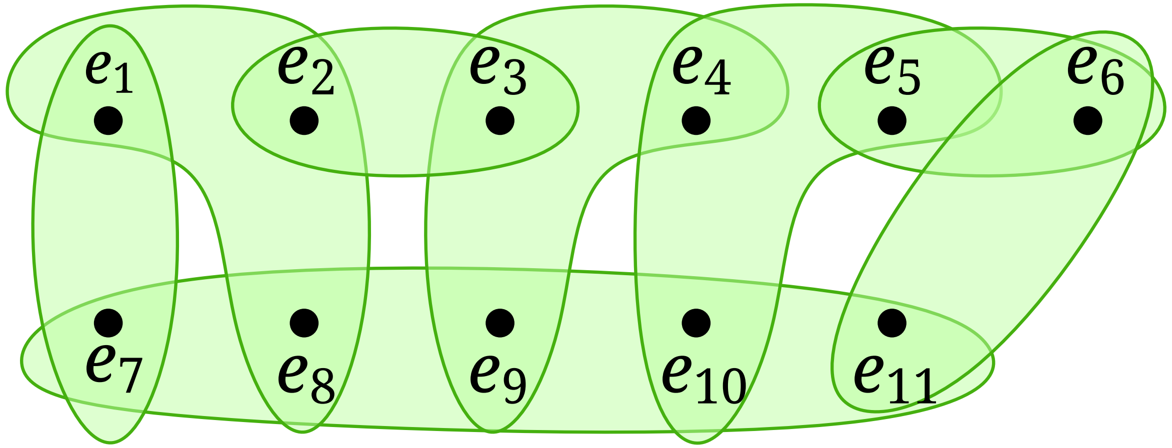

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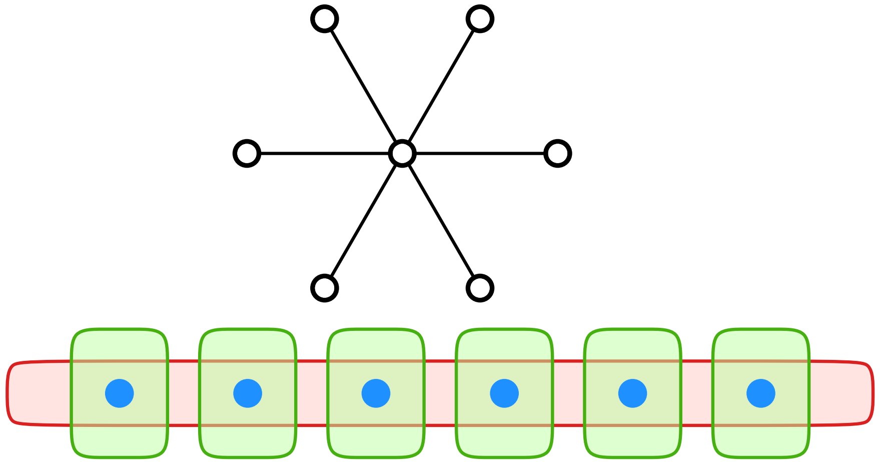

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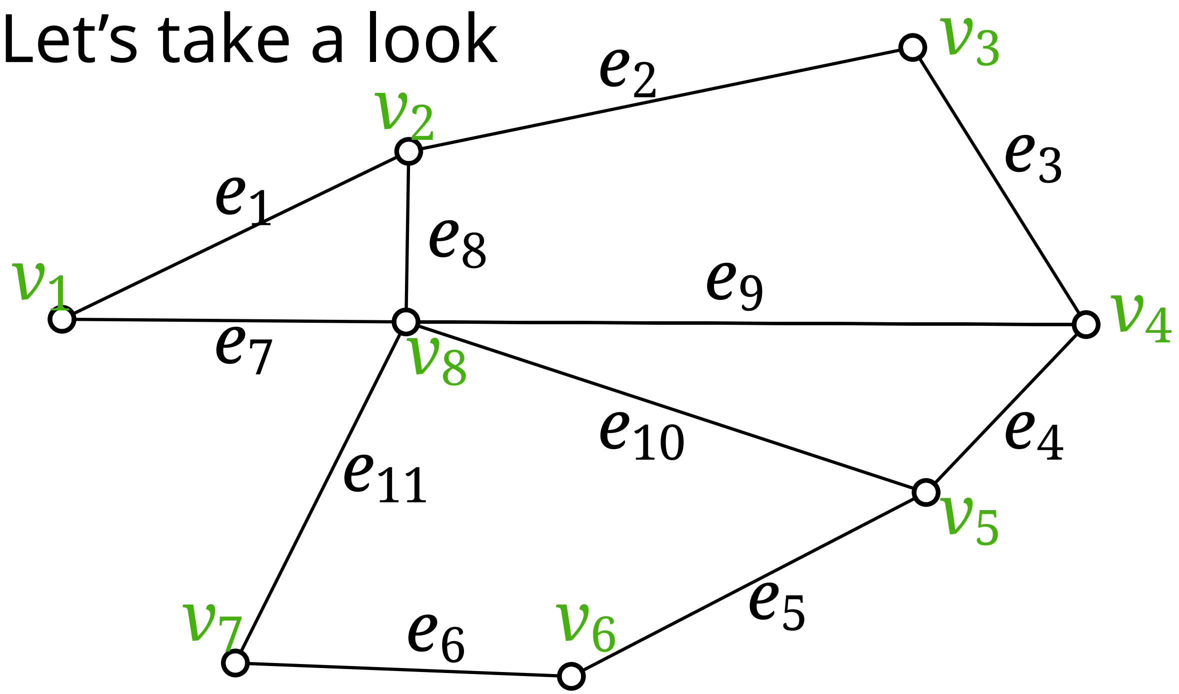
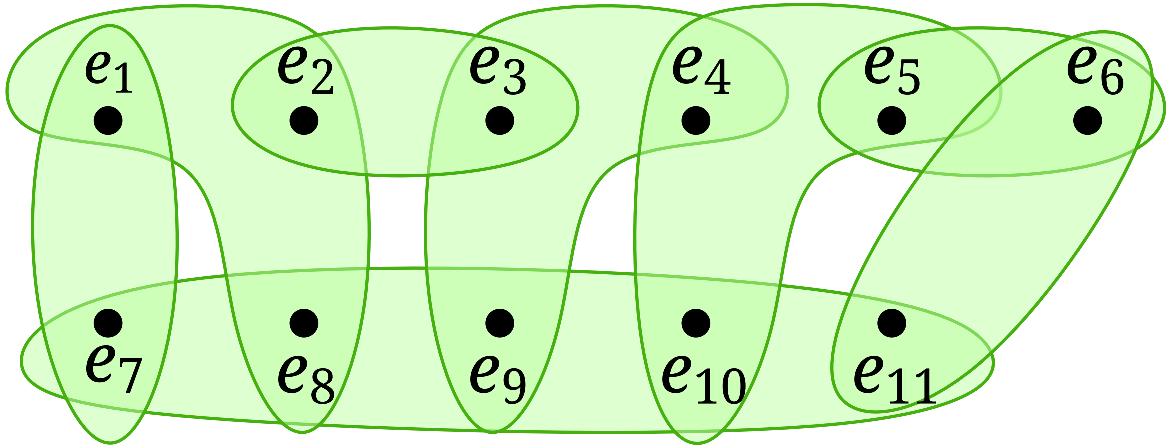


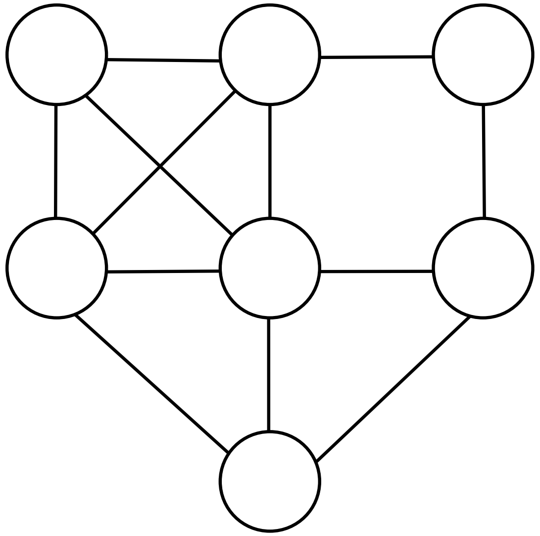


The greedy algorithm always chooses the vertex with largest remaining degree!

Layering Algorithm for Weighted VertexCover

Approximation Algorithm for Weighted VERTEXCOVER

Example:



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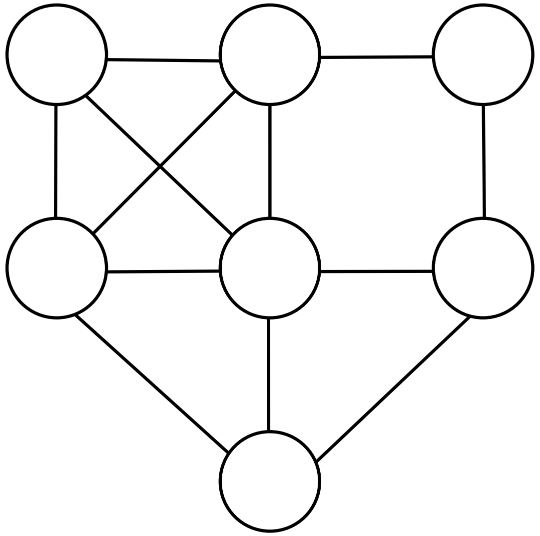

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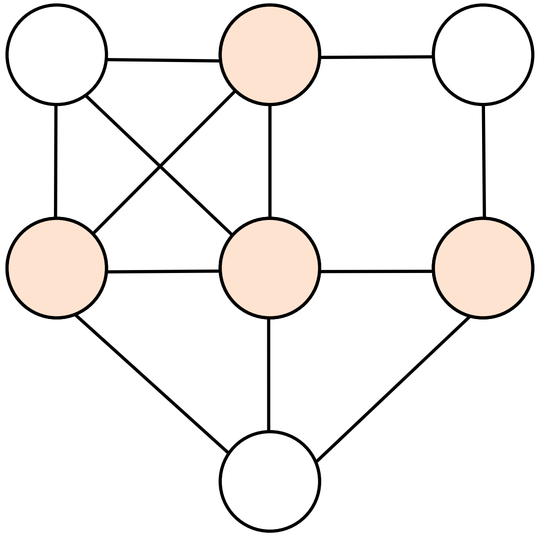

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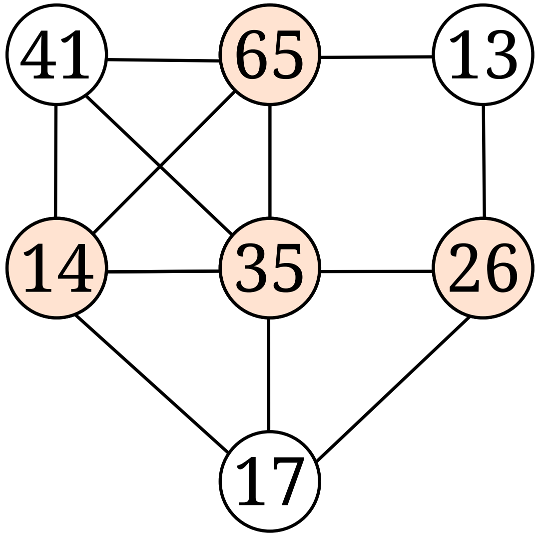

Q: What is a min cardinality vertex cover here?

Is this also a minimum weight vertex cover?

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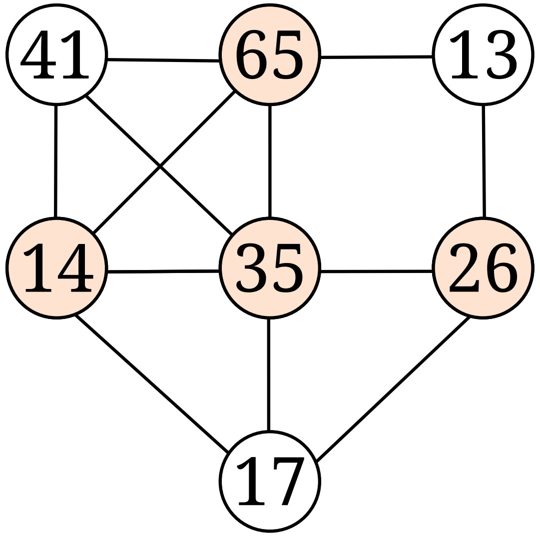

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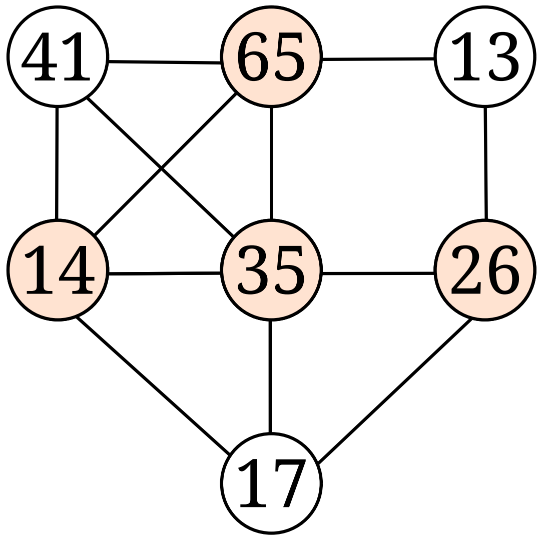

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Now instead the approach of local ratio / layering!

Approximation Algorithm for Weighted VertexCover

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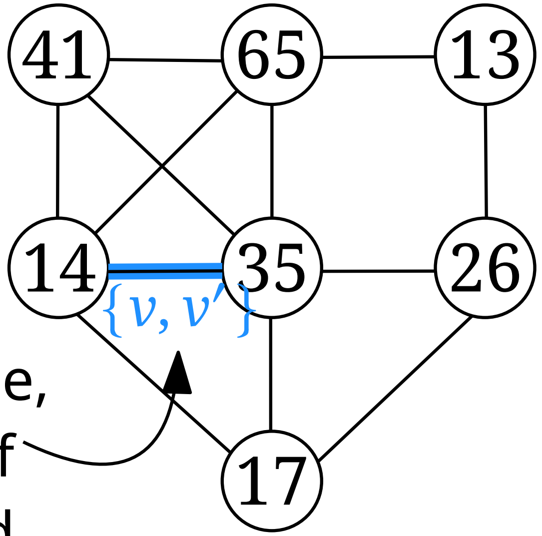

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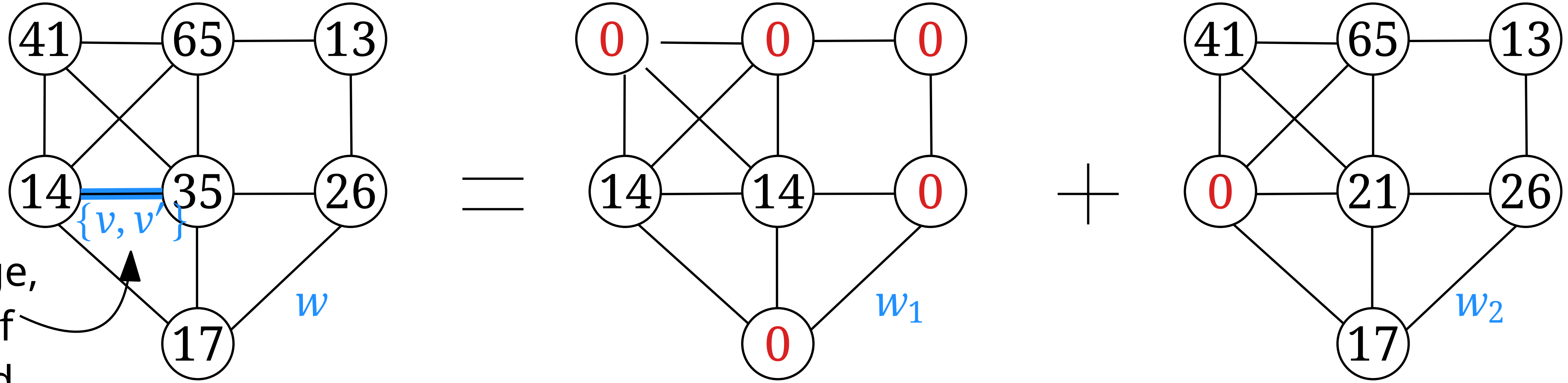

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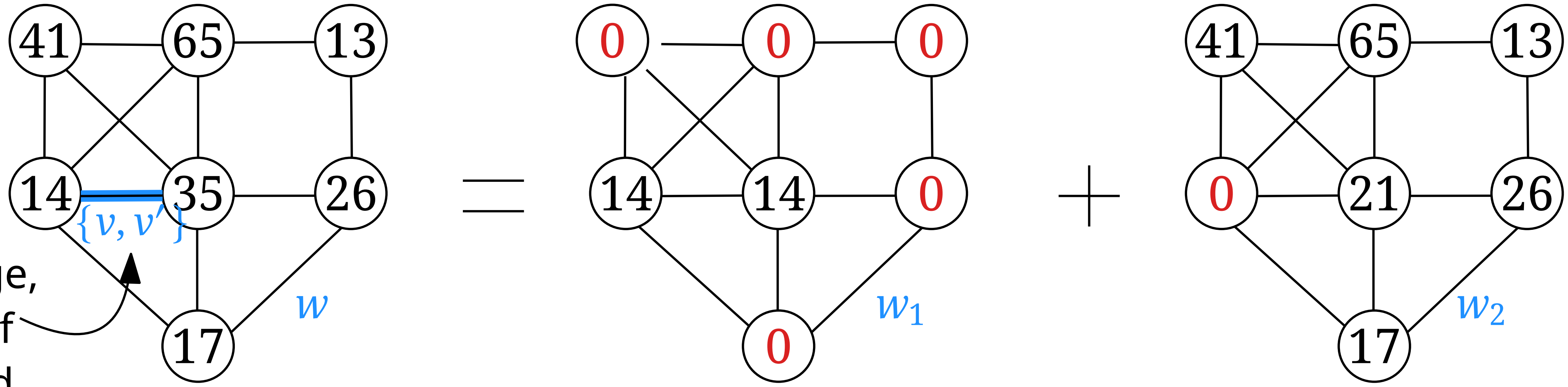

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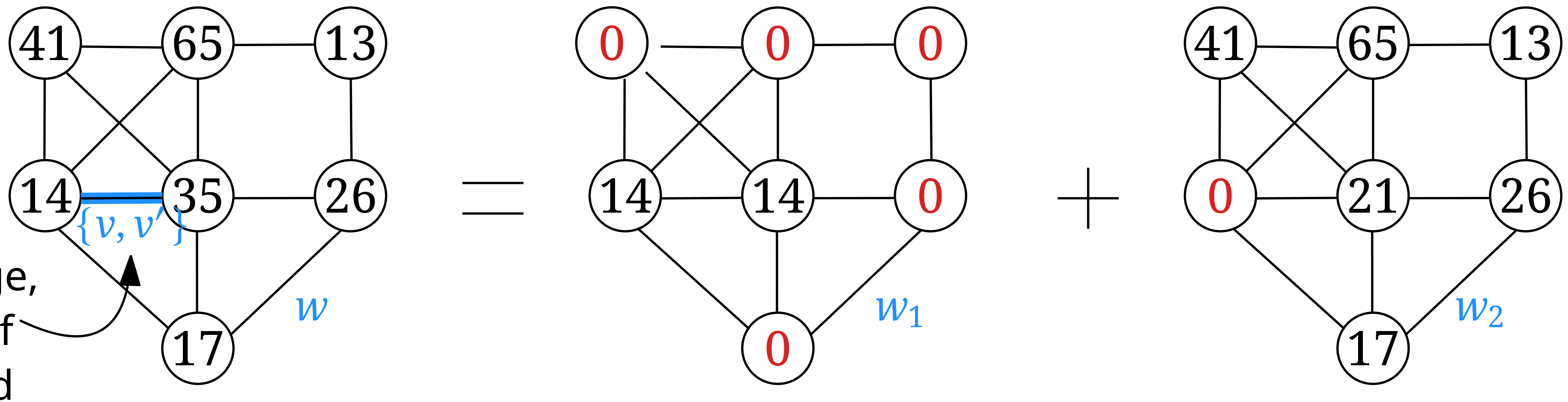

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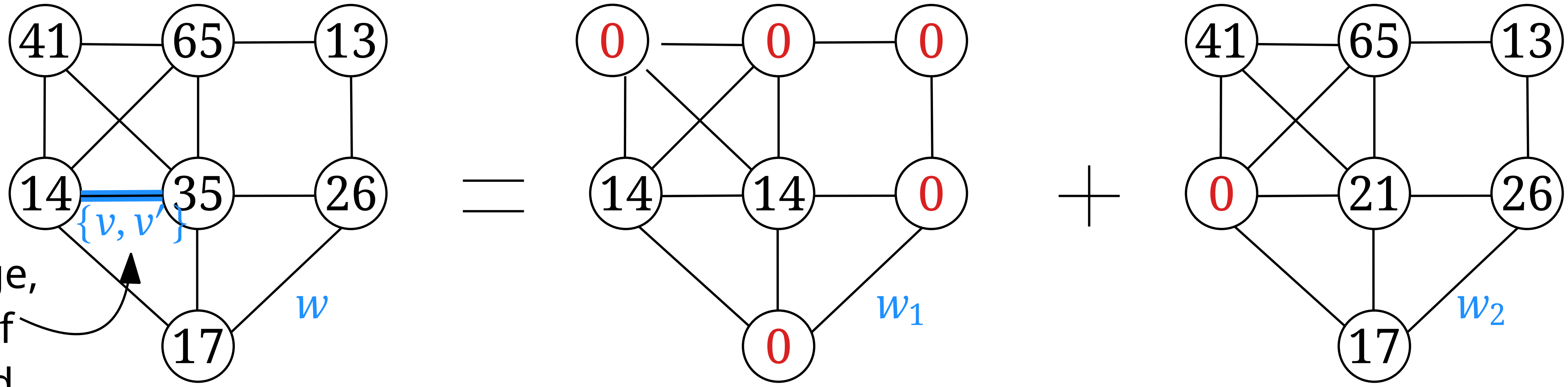

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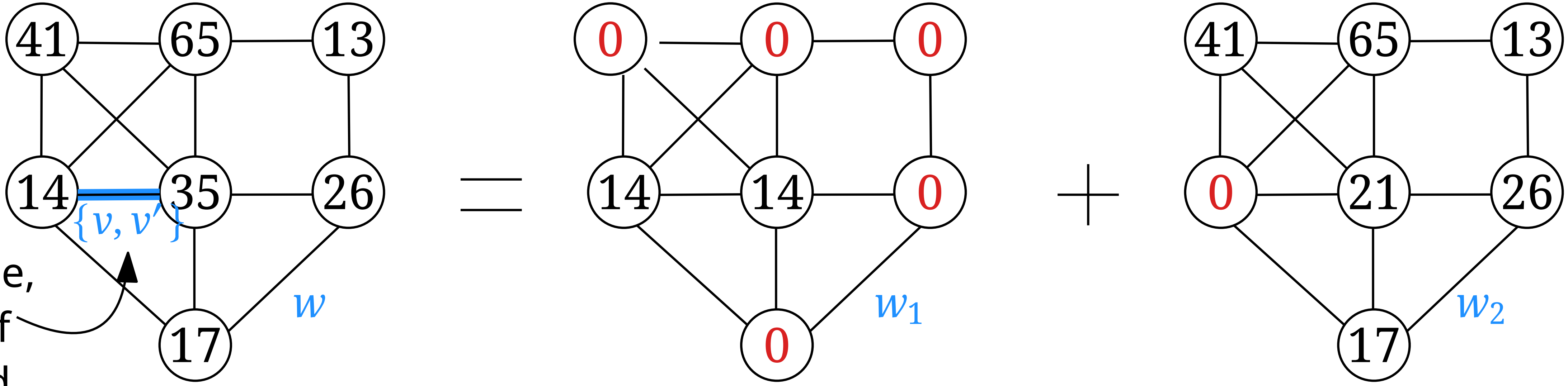

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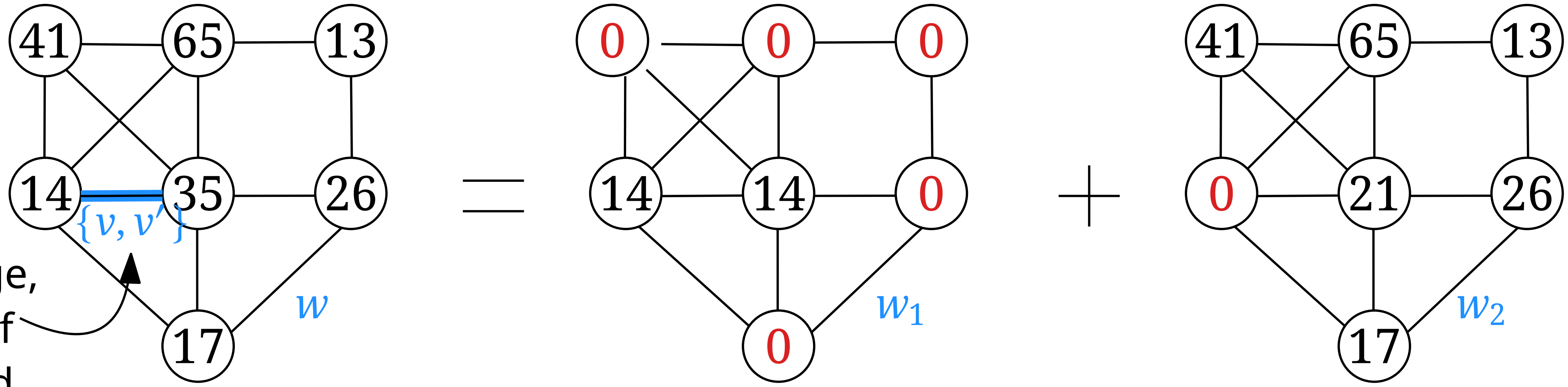

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Claim: A 2-approximation for w_2 is a 2-approximation for w

Local Ratio Theorem

Let $w = w_1 + w_2$. If x is an r-approximate solution for w_1 and w_2 then x is r-approximate with respect to w as well.

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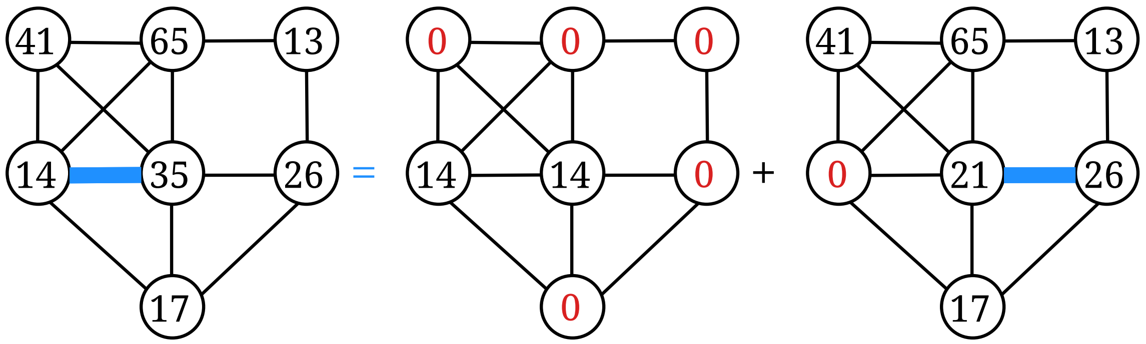
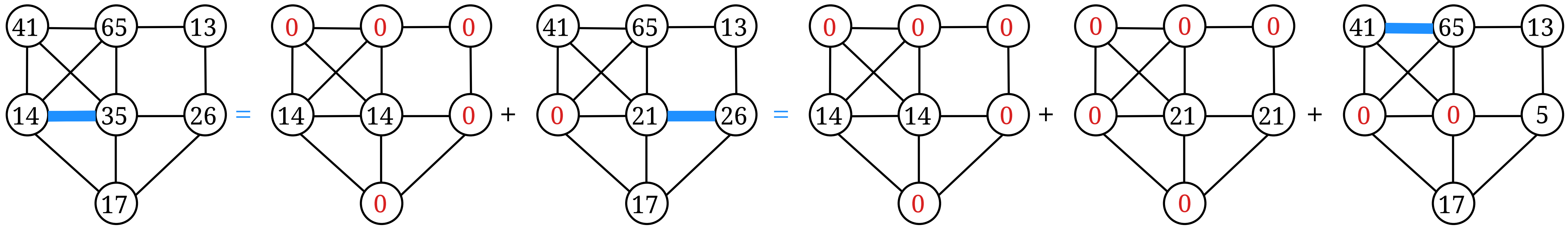
$$w(x) = w_1(x) + w_2(x) \leq r \cdot (w_1(x^*) + w_2(x^*)) = r \cdot w(x^*)$$

Example – continued



Example – continued



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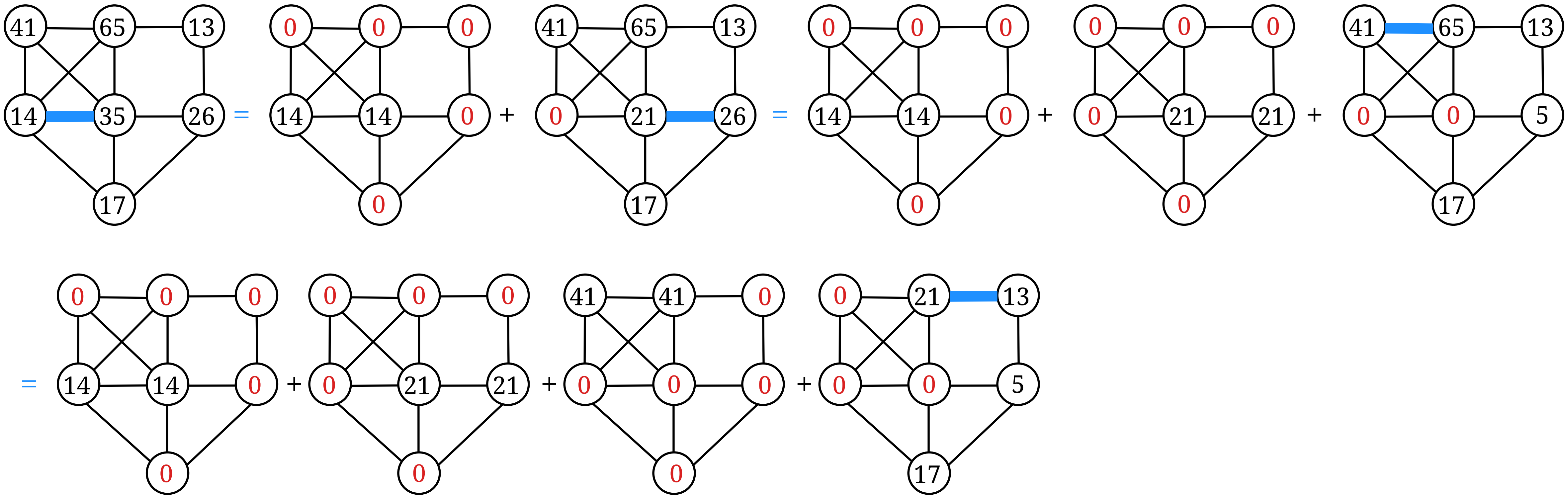

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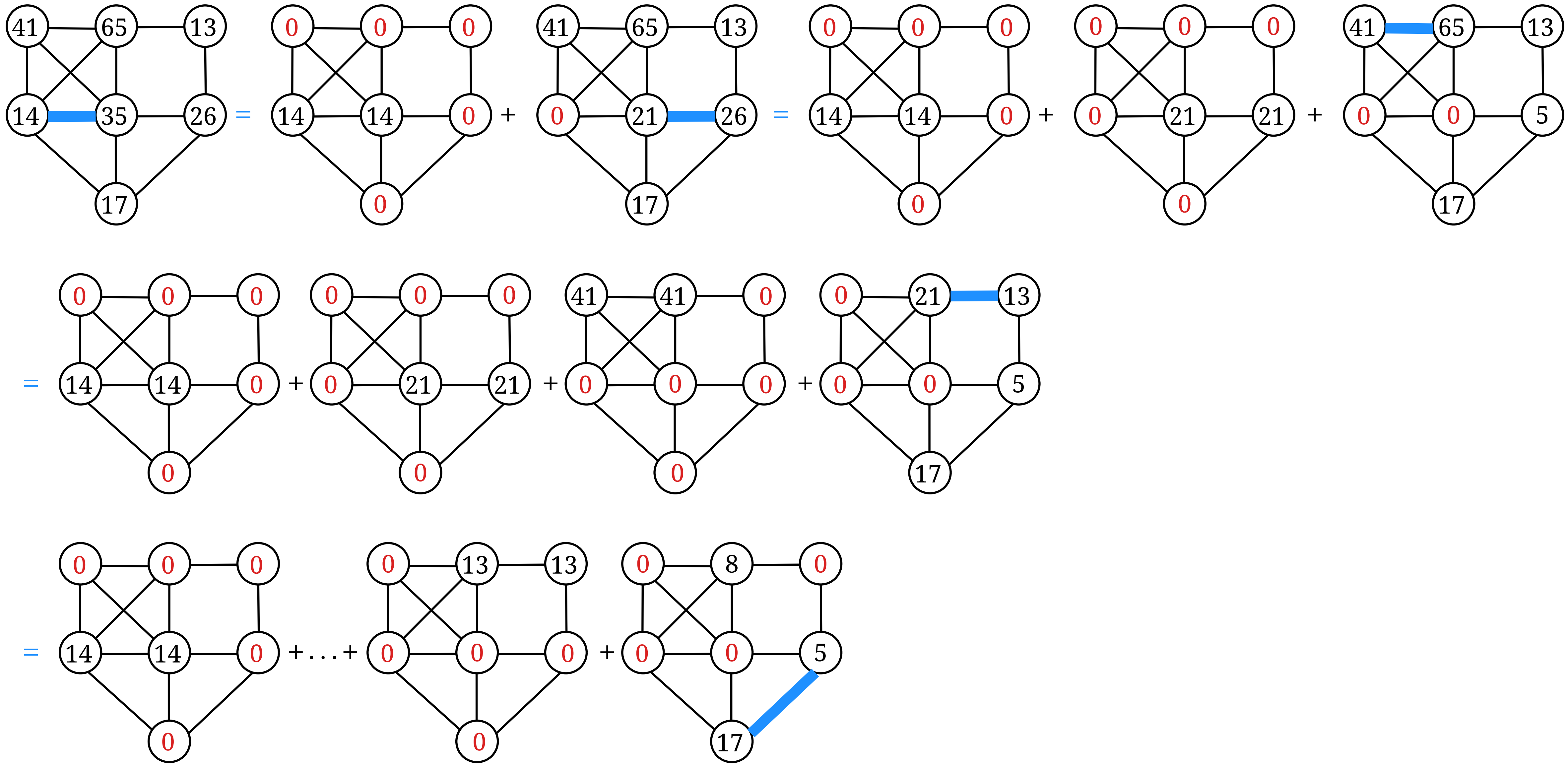

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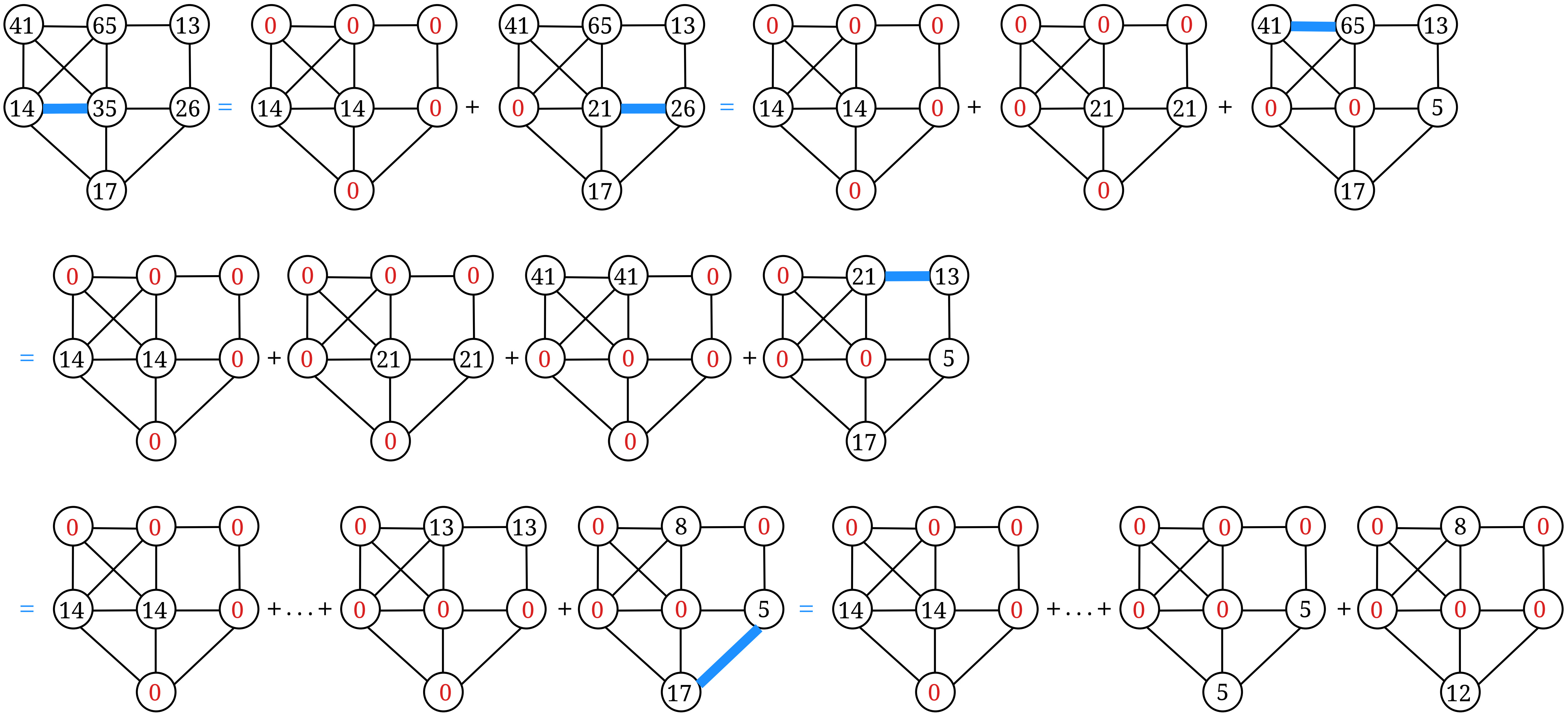

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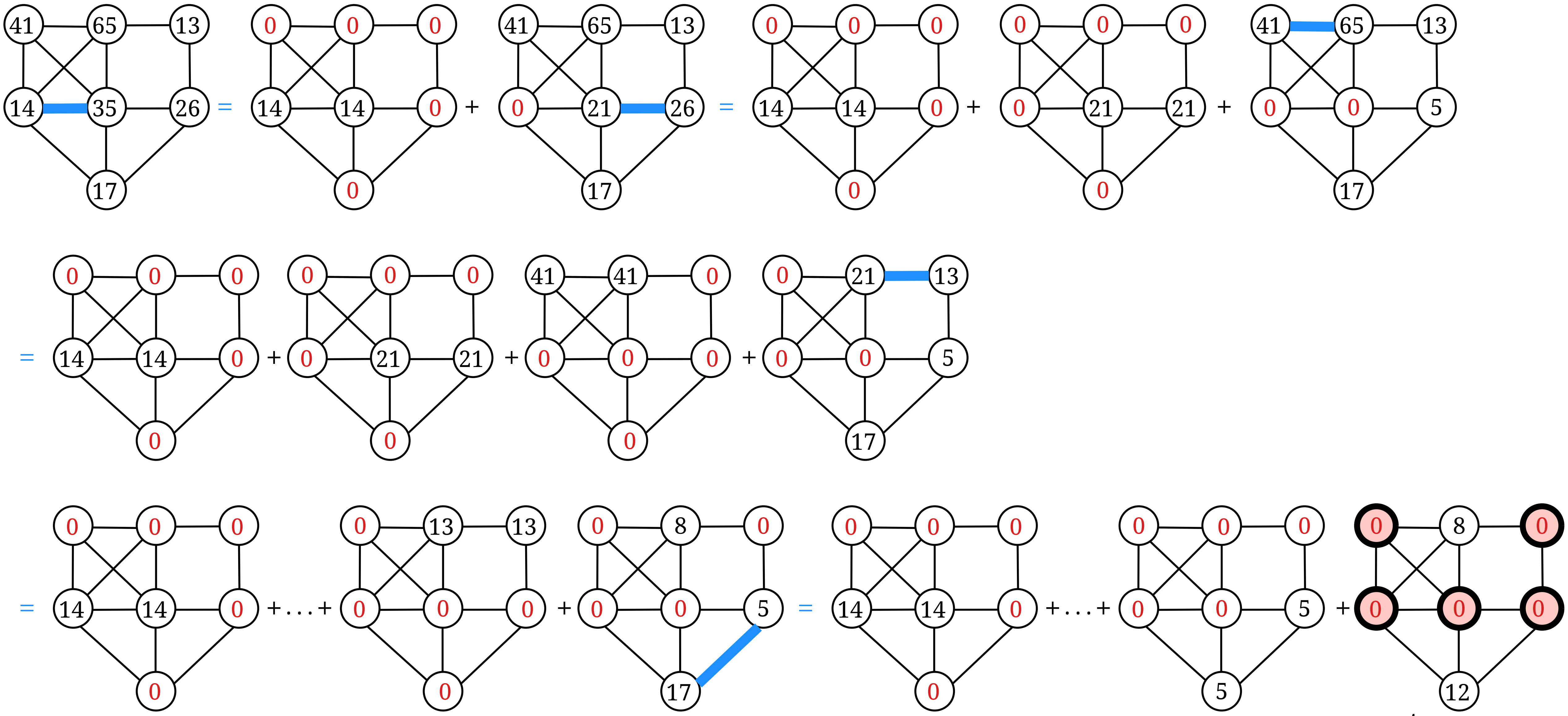
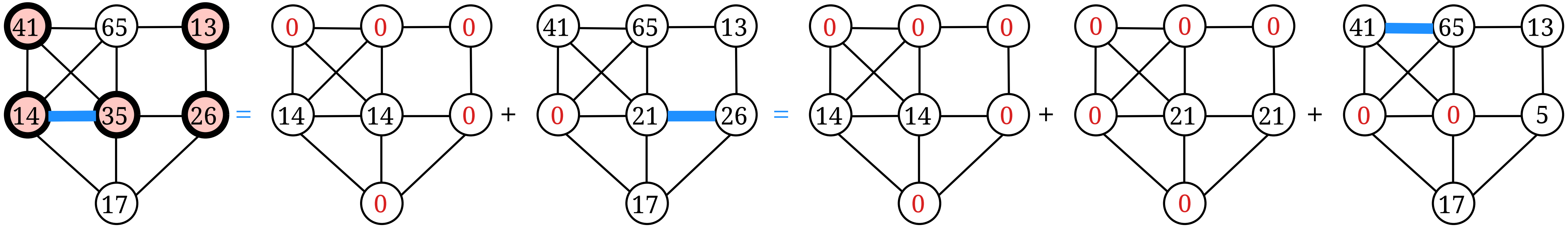
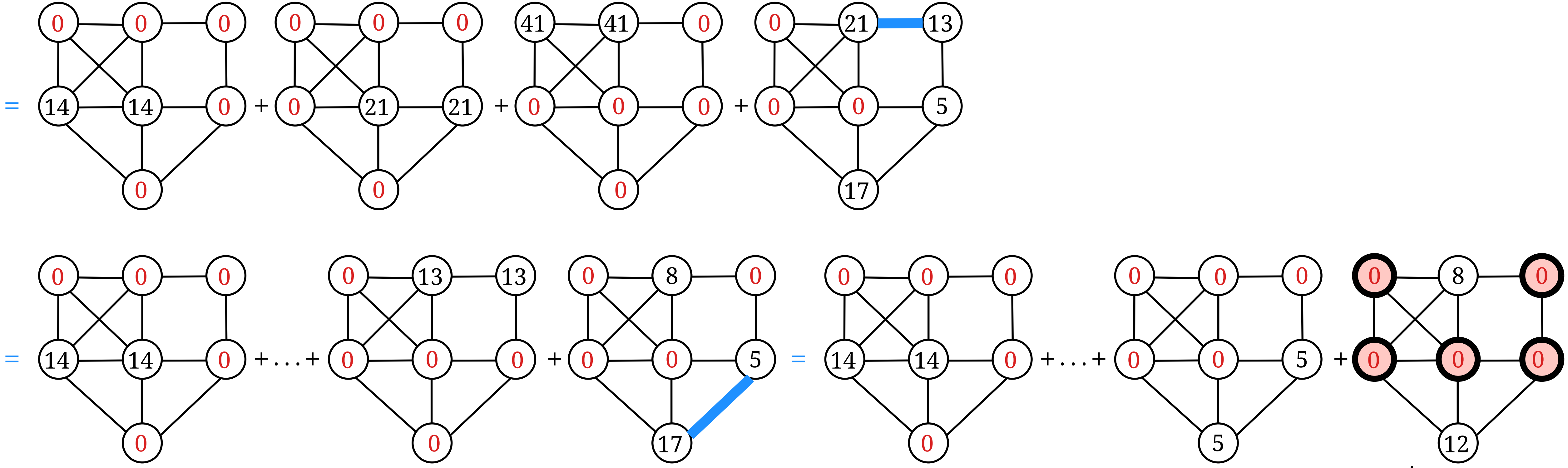


vertex cover of
weight 0

Example – continued

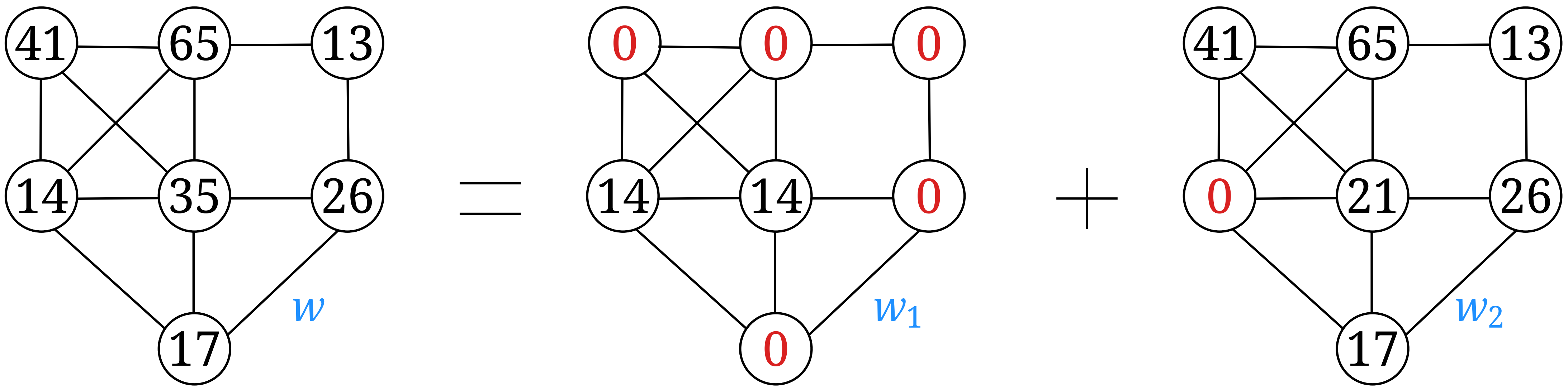


2-approximation



vertex cover of weight 0

Technique: Layering / Local Ratio



decompose weights: $w = w_1 + w_2$, and
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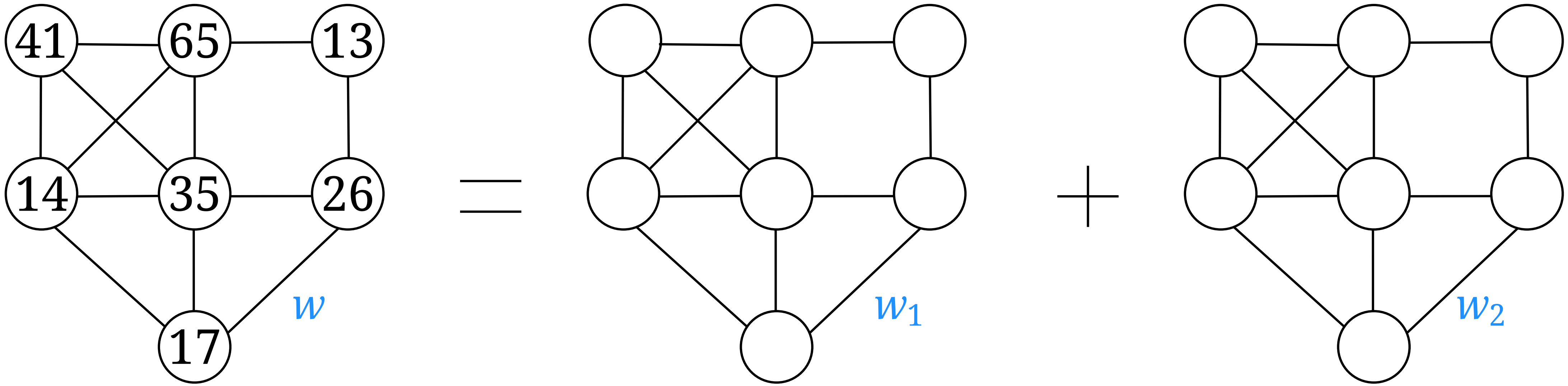

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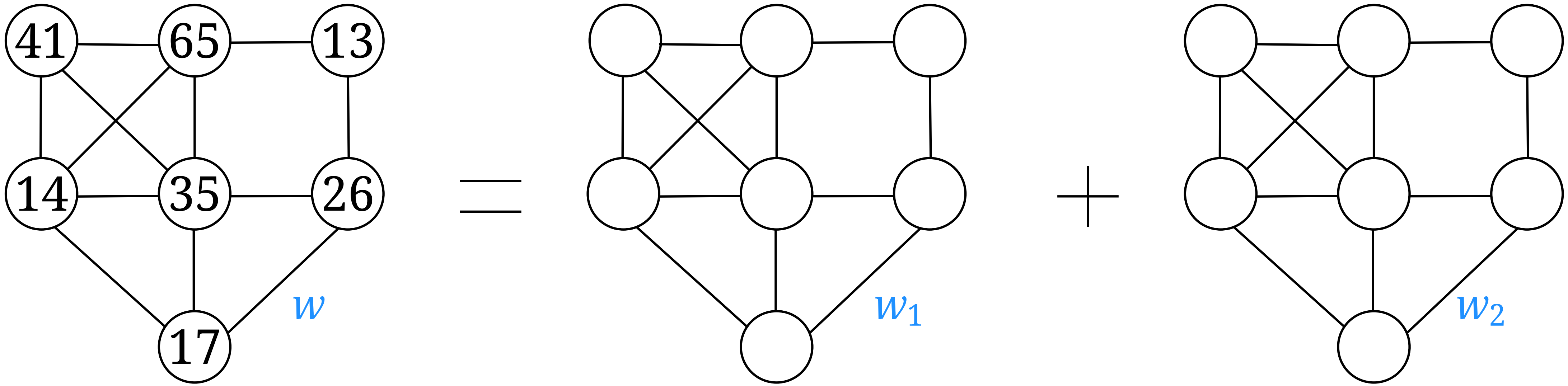

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$c := \min(w(u)/\text{degree}(u), u \in V)$

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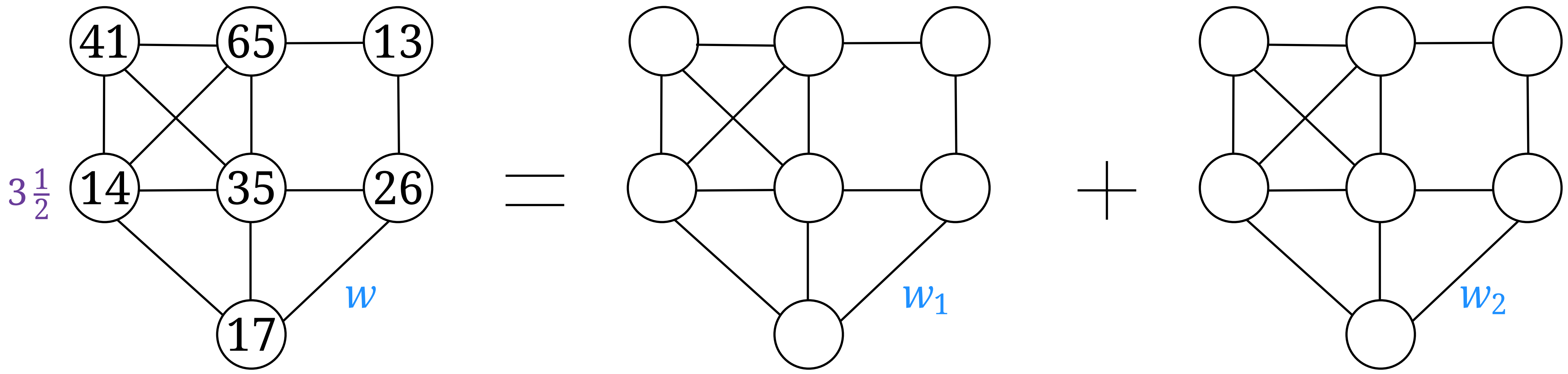

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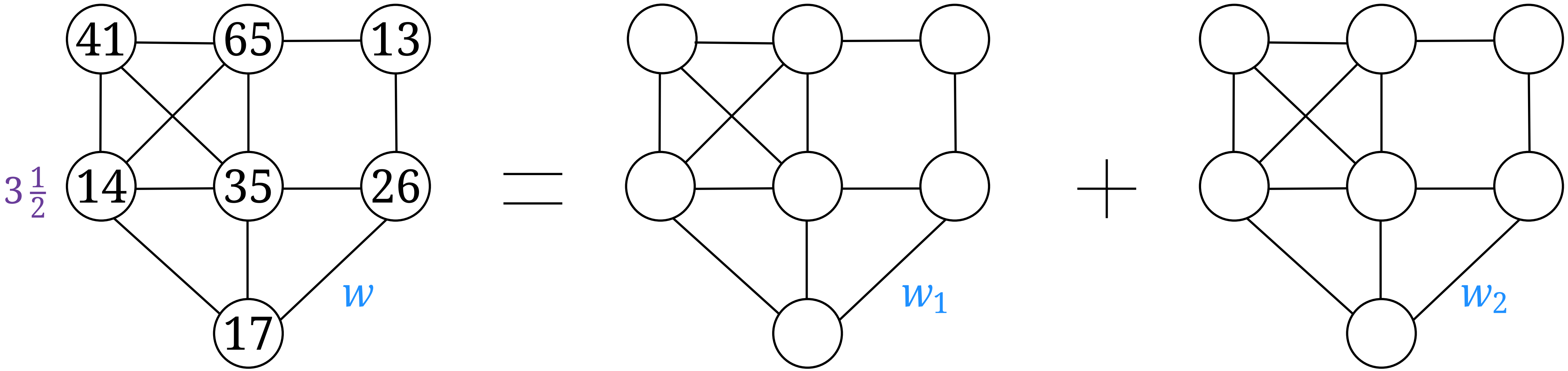

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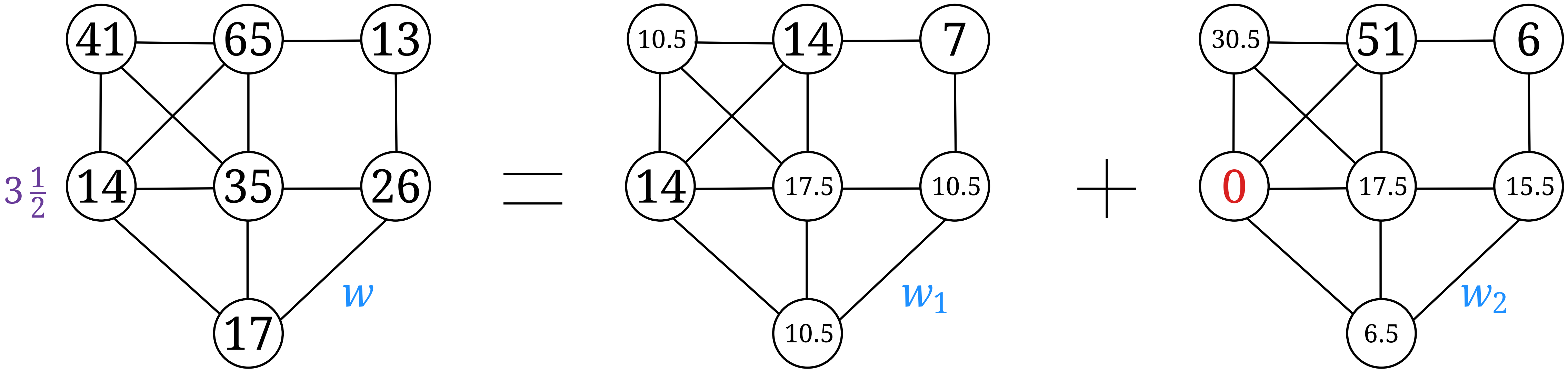

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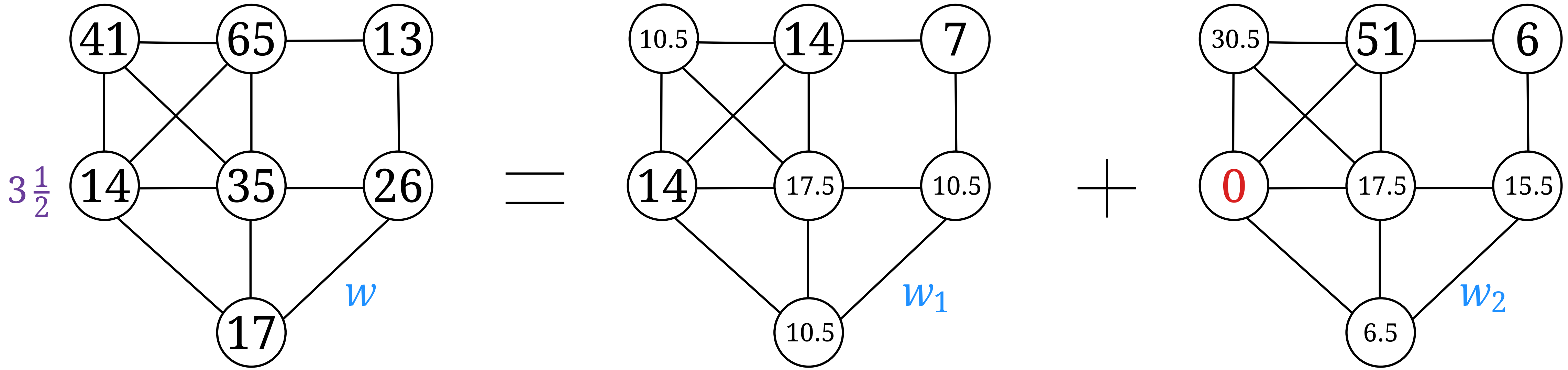

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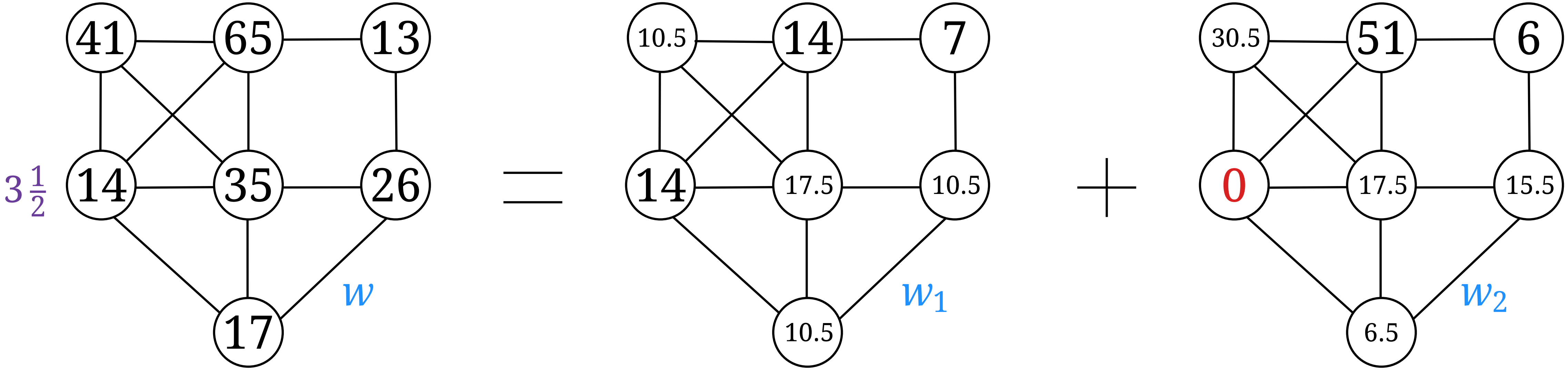

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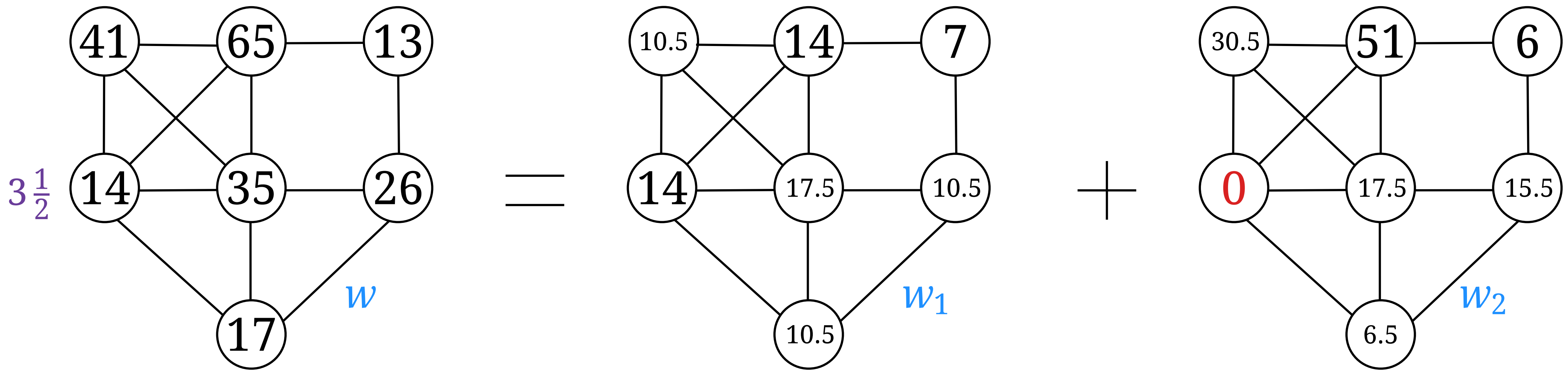

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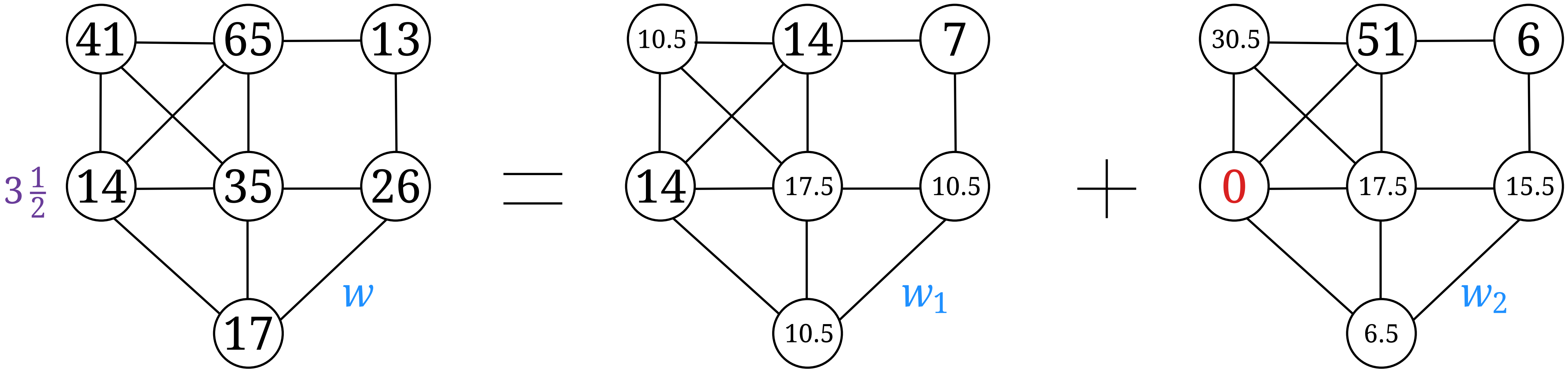

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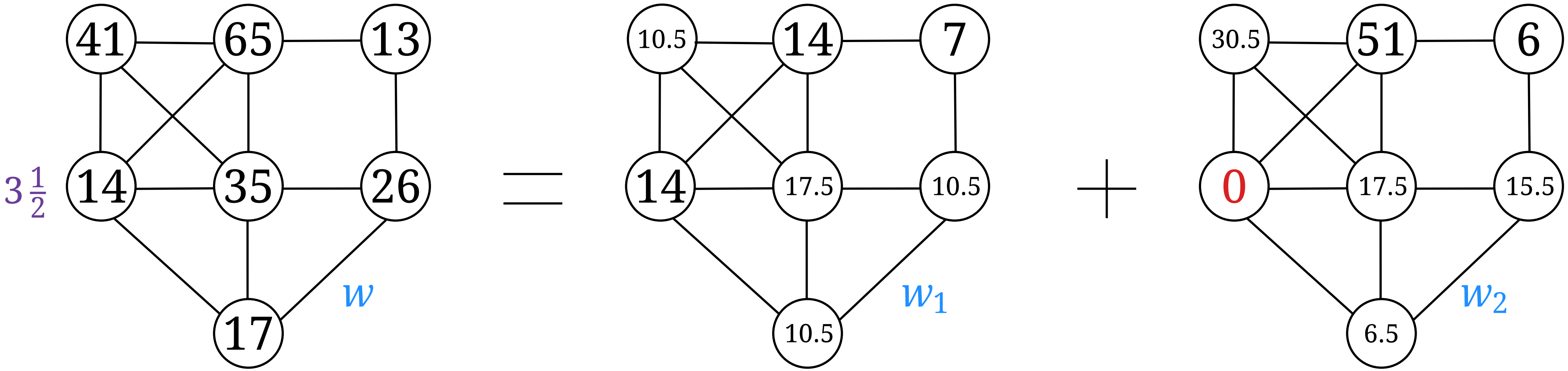

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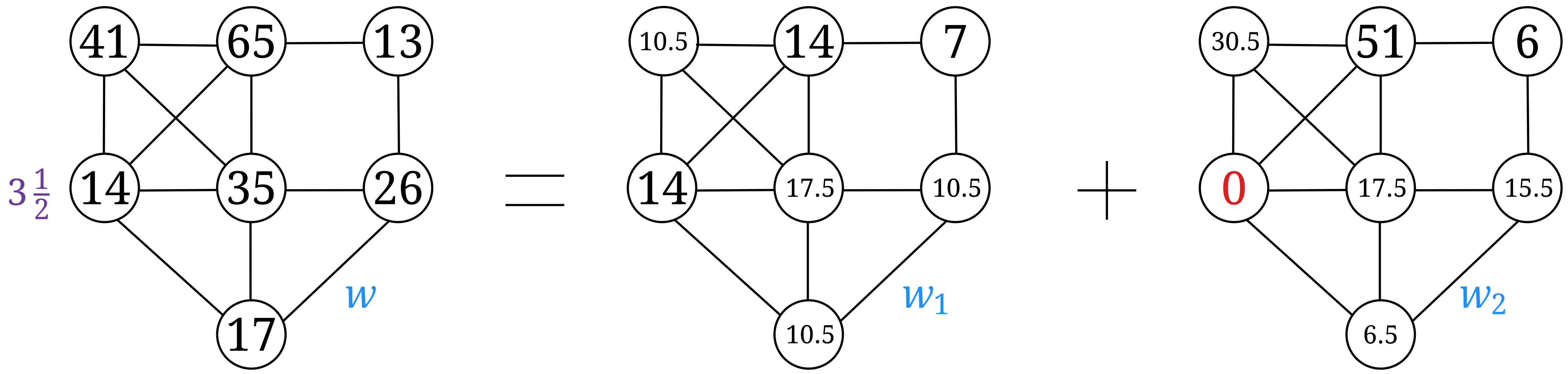

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$\Rightarrow$ 2-approximation, generalizes to k-approximation for set cover with every element occuring in at most k sets.

Greedy SetCover applied to ShortestSuperString

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

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$abcbaa$ “covers” all strings in U

abc

bcb

$cbaa$

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$$\begin{array}{l} abc \\ bcb \\ cbaa \end{array}$$

W.l.o.g.: No string s_i is a substring of any other string s_j.

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Greedy approach: iteratively combine two strings with maximum overlap

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$U' := \{ab^k, b^k c, b^{k+1}\} \longrightarrow \{ab^k c, b^{k+1}\} \longrightarrow \{ab^k cb^{k+1}\}$

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SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

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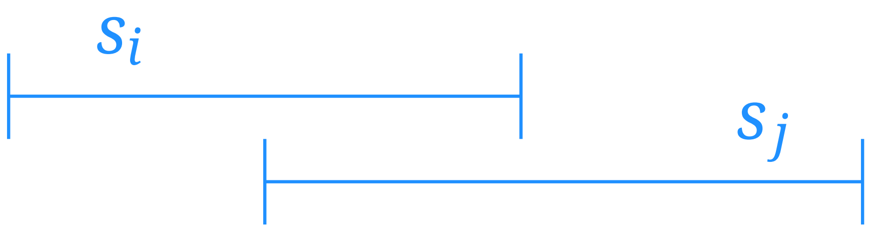

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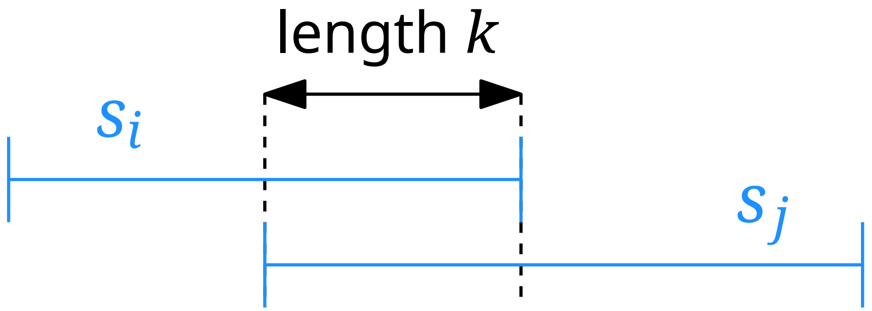

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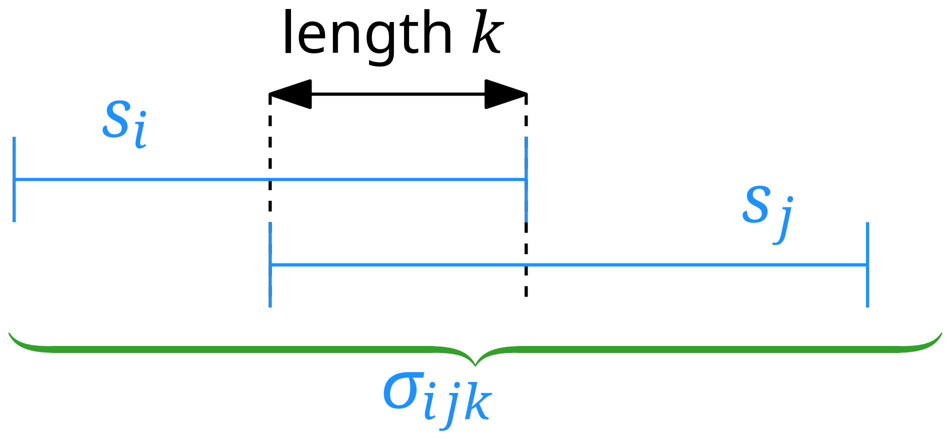

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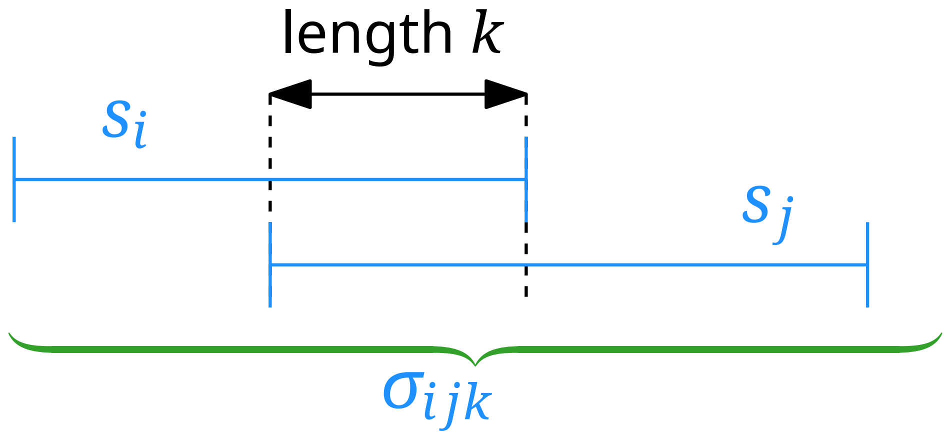

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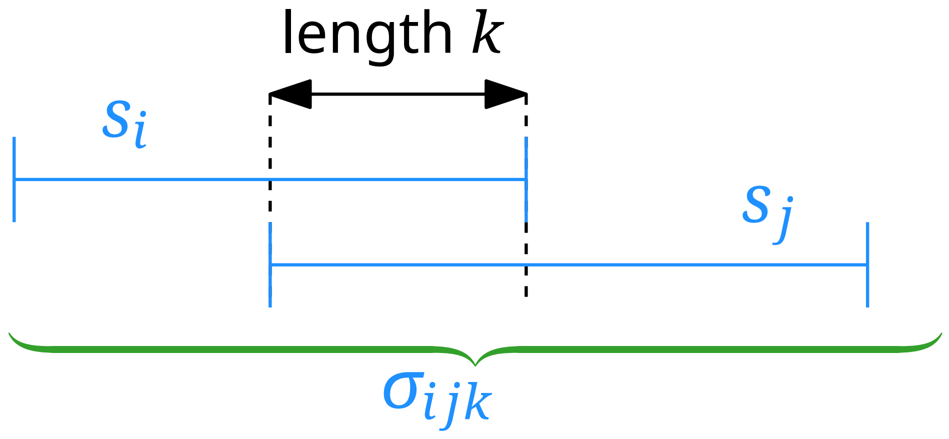

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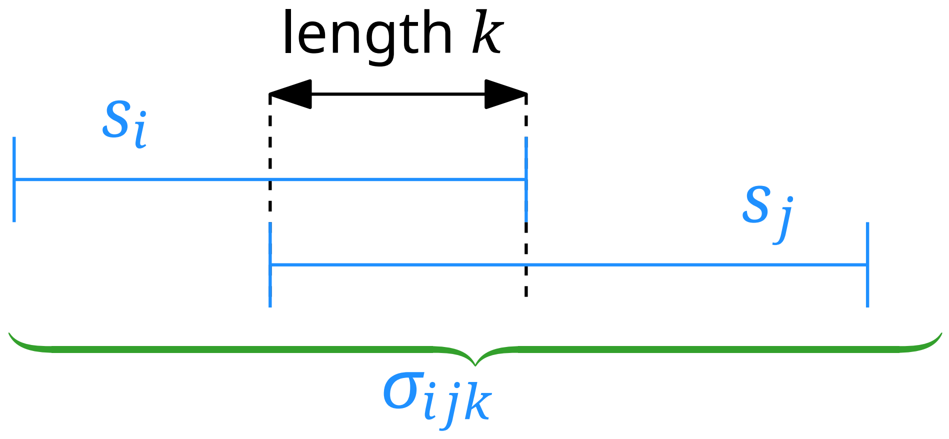

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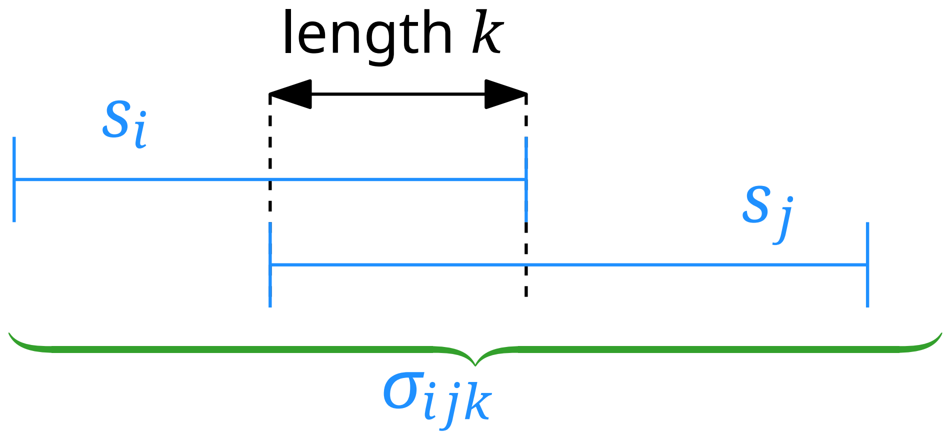

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```
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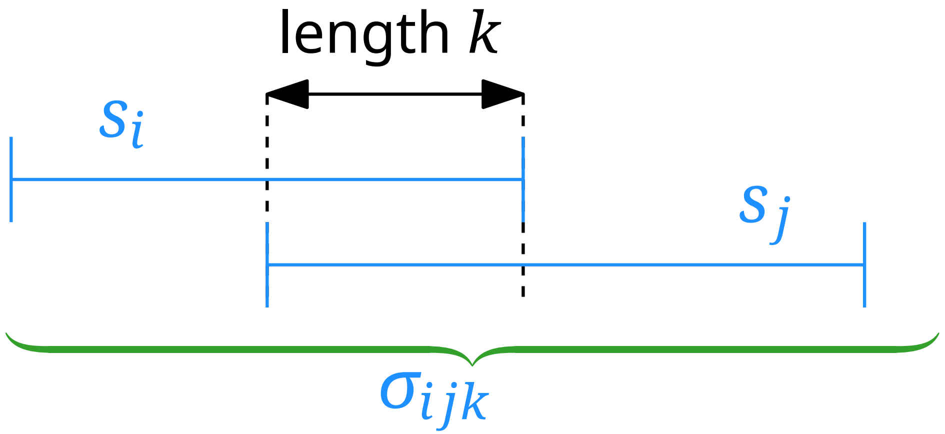

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cabab
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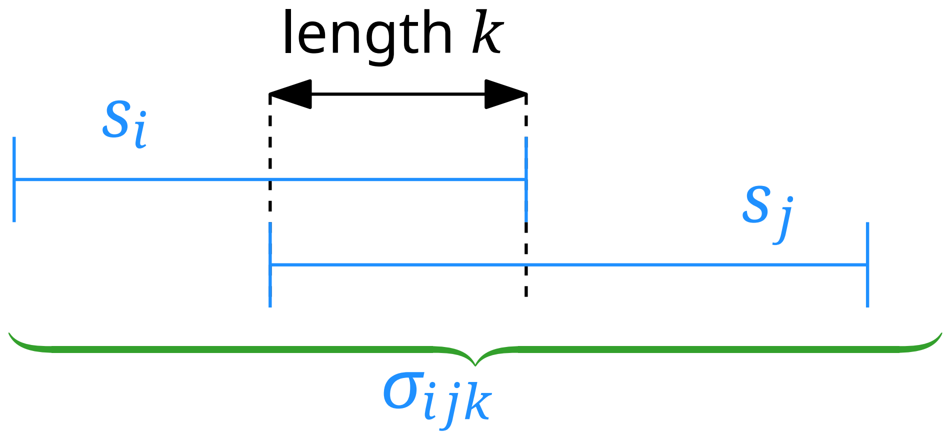

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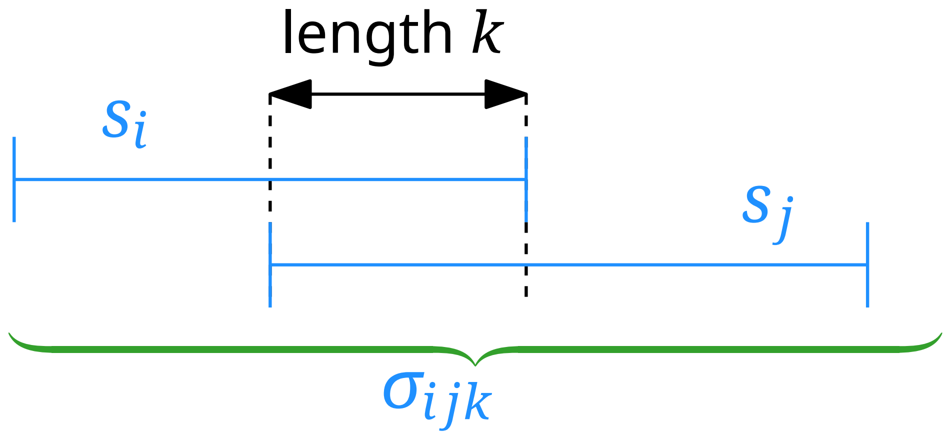

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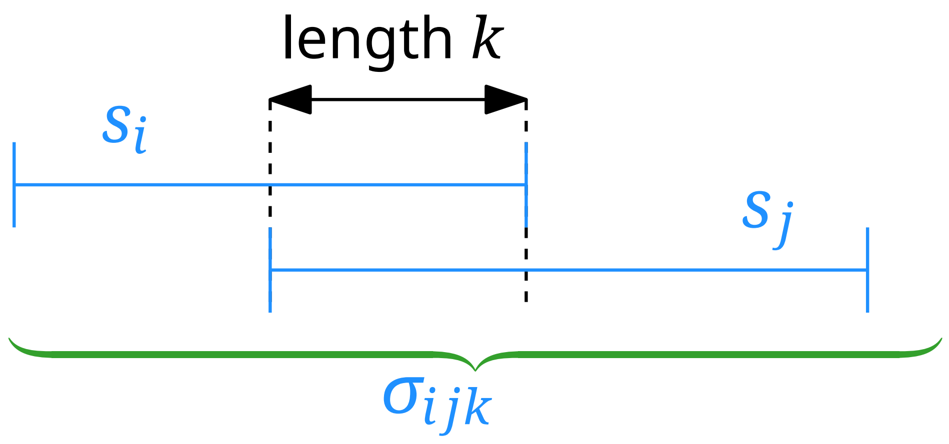

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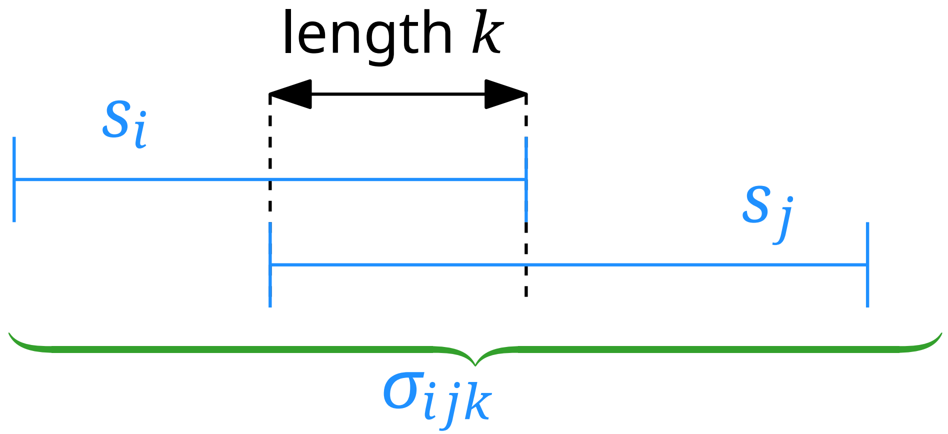

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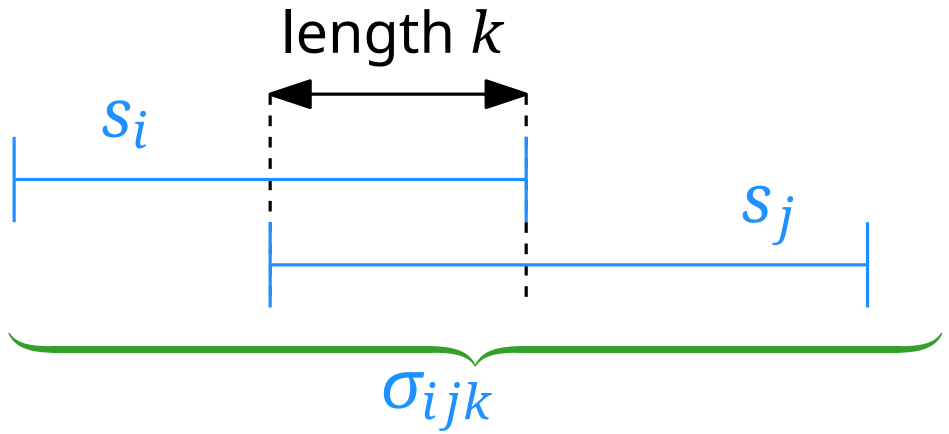

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Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U, and let $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

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Proof.

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$$\mathrm{OPT}_{\mathrm{SSS}} \leq \mathrm{OPT}_{\mathrm{SC}}.$$

Proof.

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Then $s := \pi_1 \circ \dots \circ \pi_k$ is a superstring of U of length $\sum_{i=1}^{k} |\pi_i| = \sum_{i=1}^{k} c(S(\pi_i)) = \mathrm{OPT}_{\mathrm{SC}}$.

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Thus, $\mathrm{OPT}_{\mathrm{SSS}} \leq |s| = \mathrm{OPT}_{\mathrm{SC}}$.

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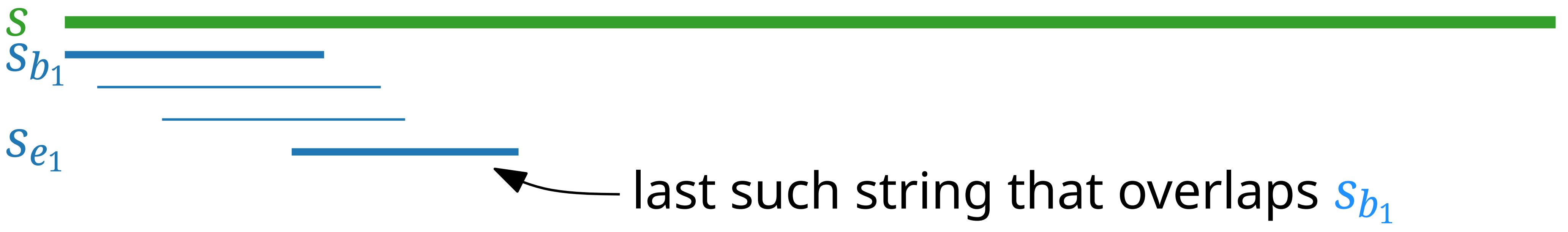

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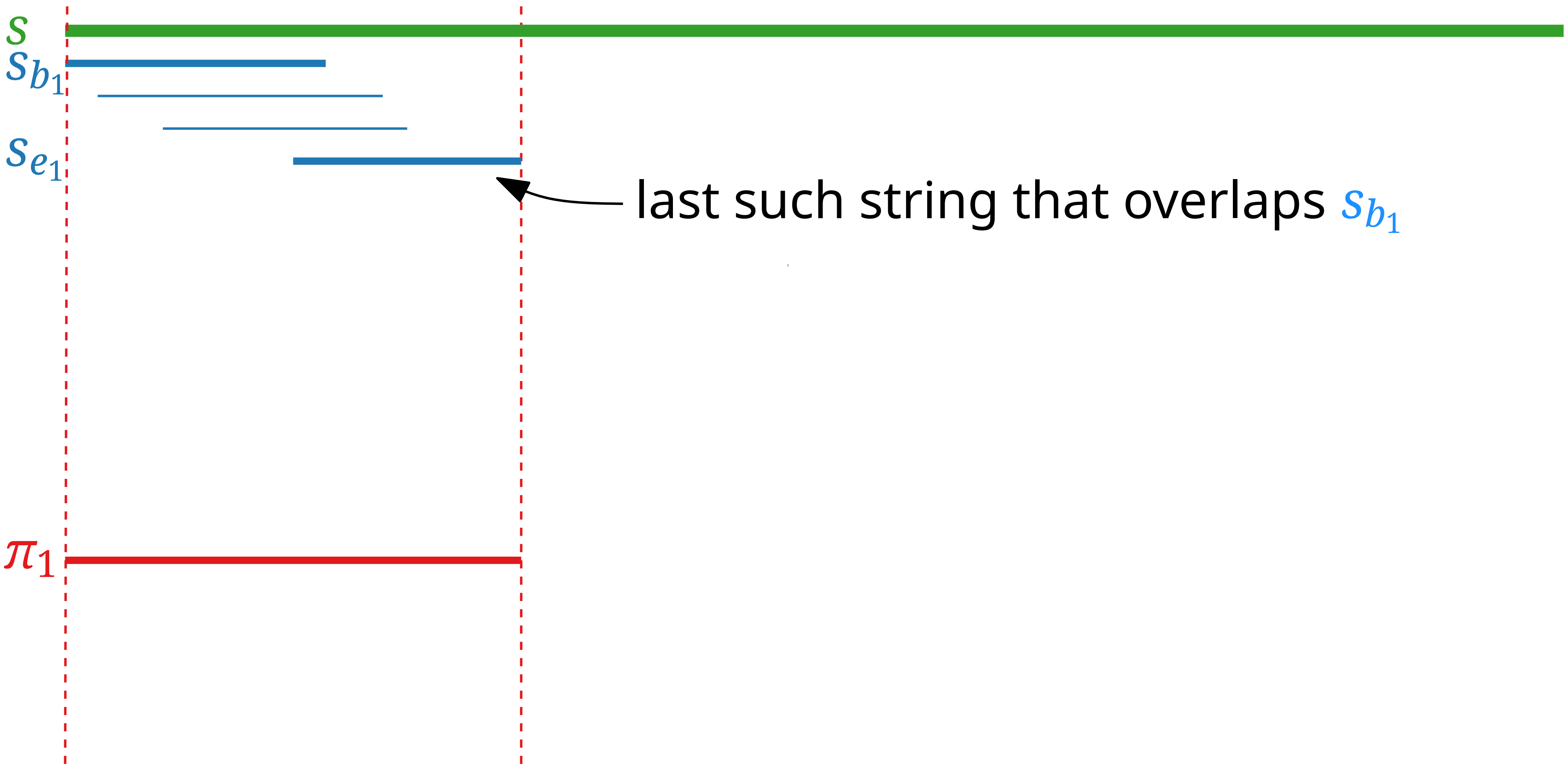

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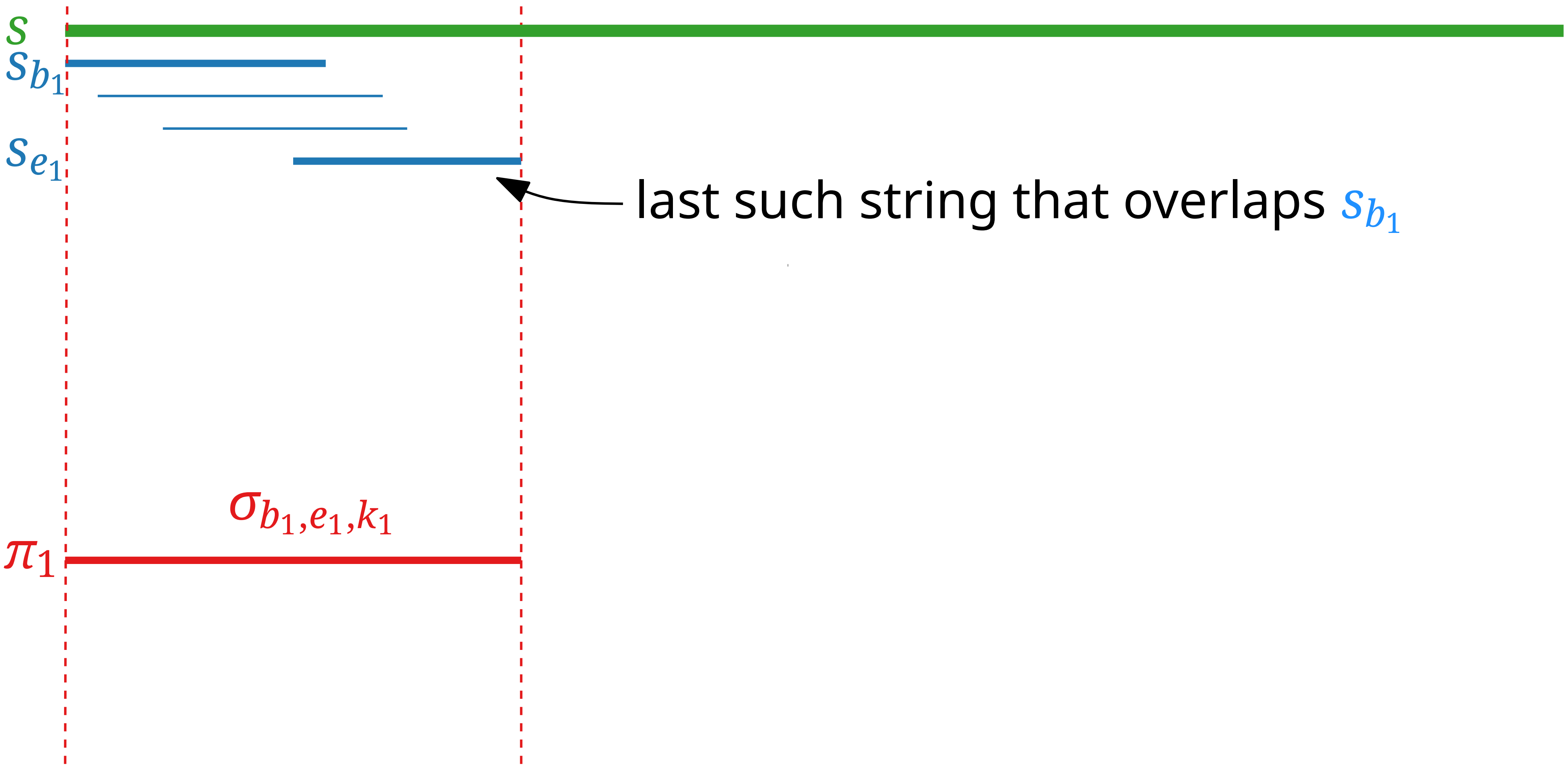

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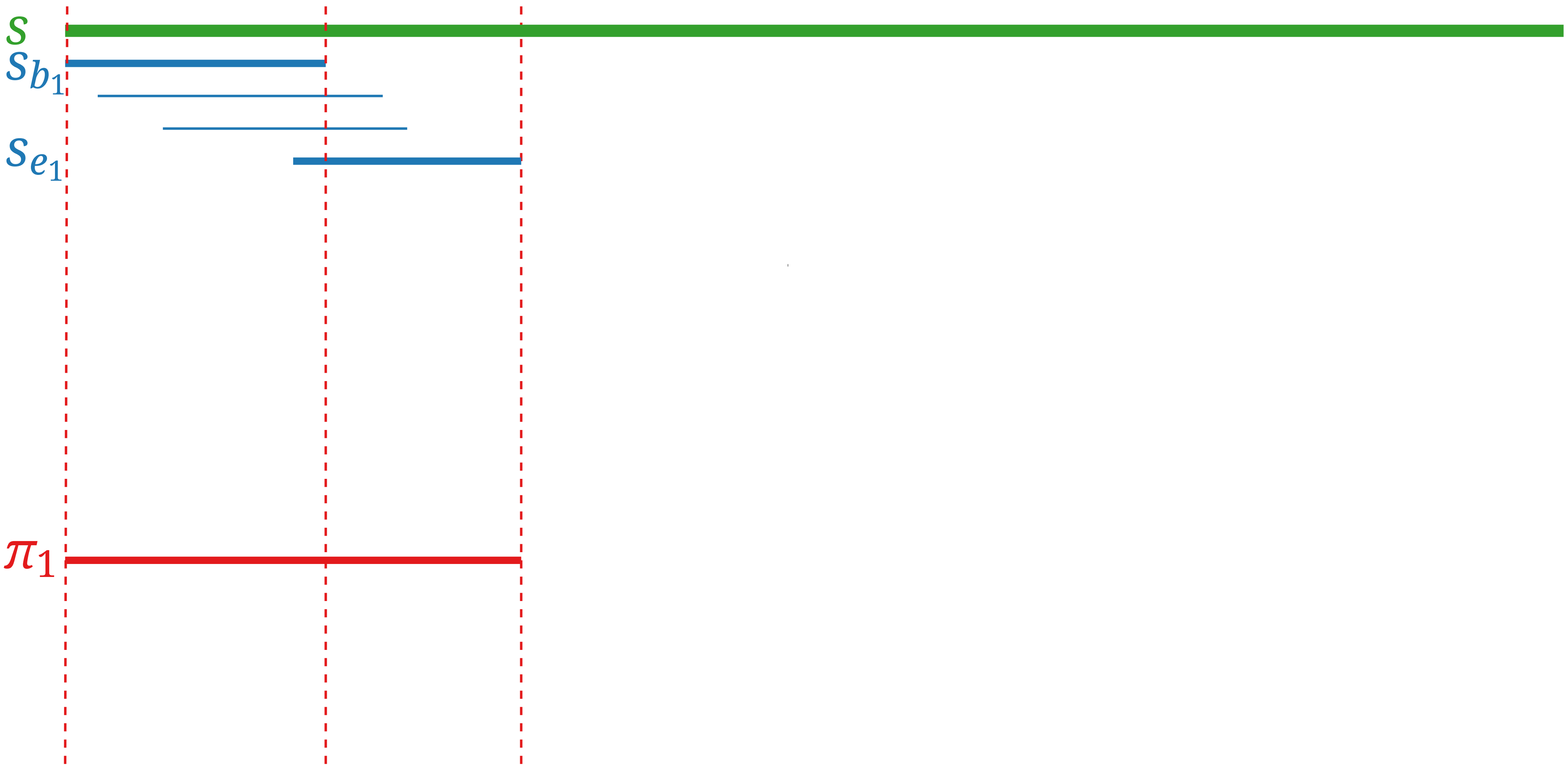

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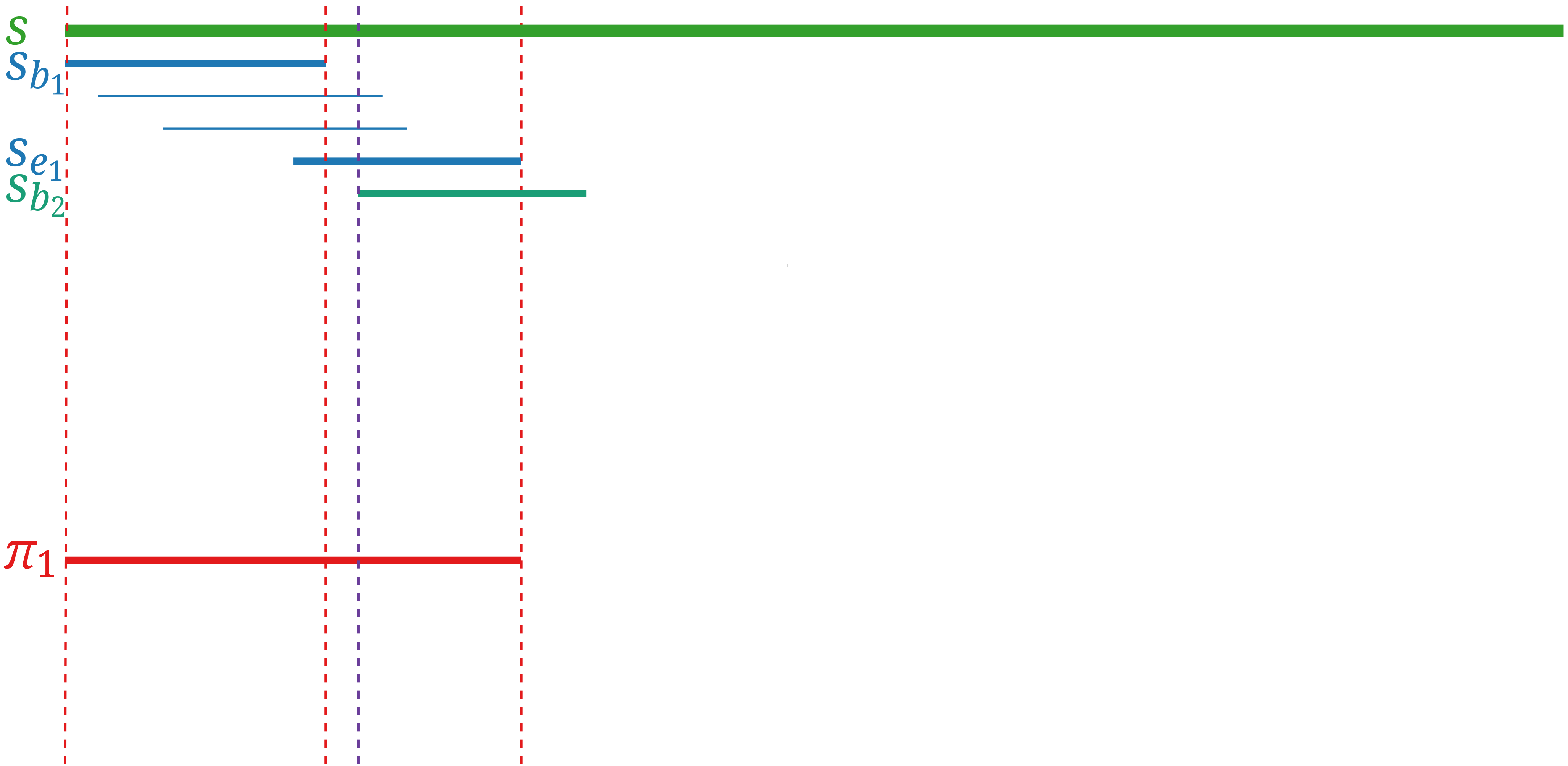

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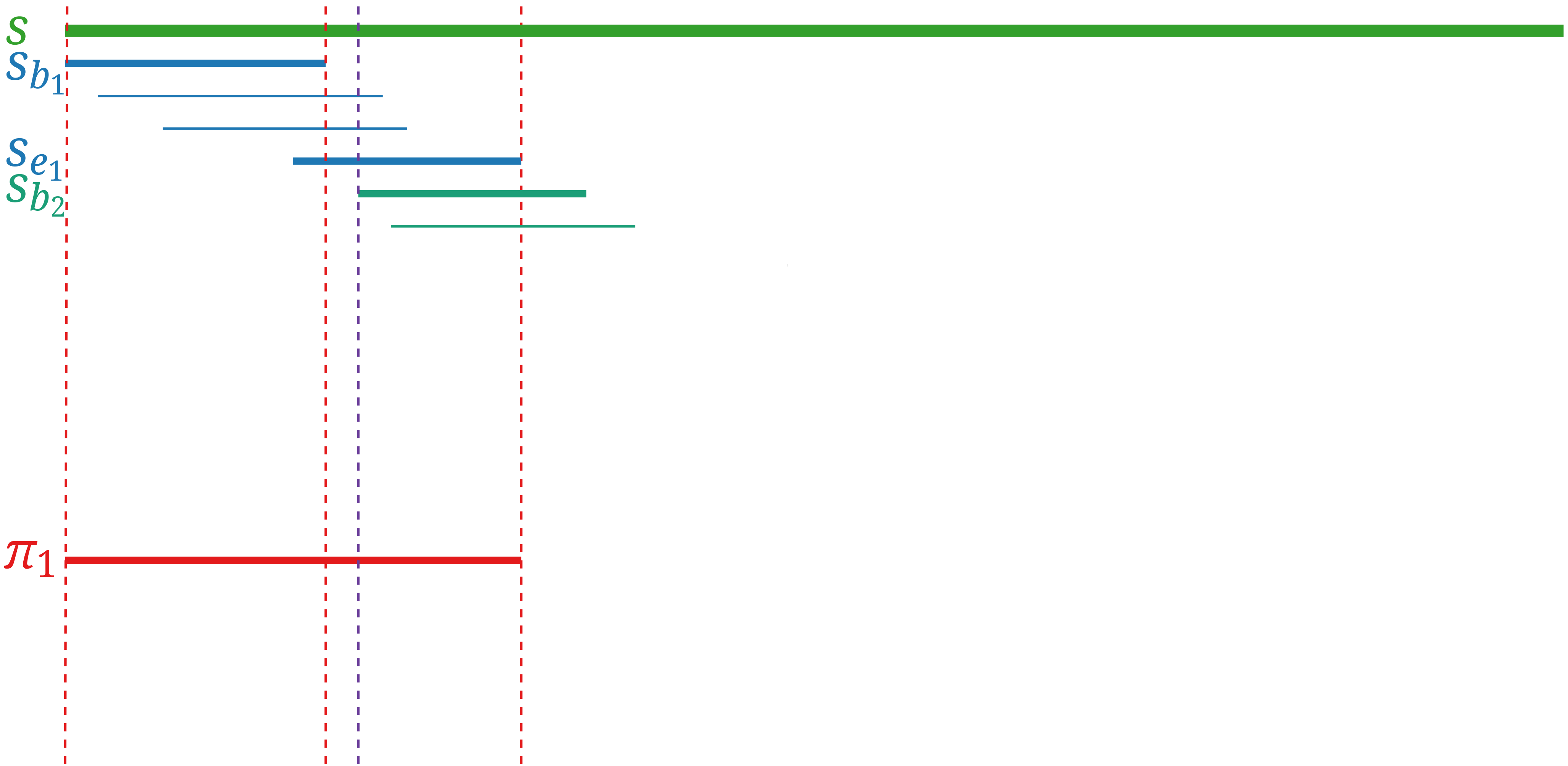

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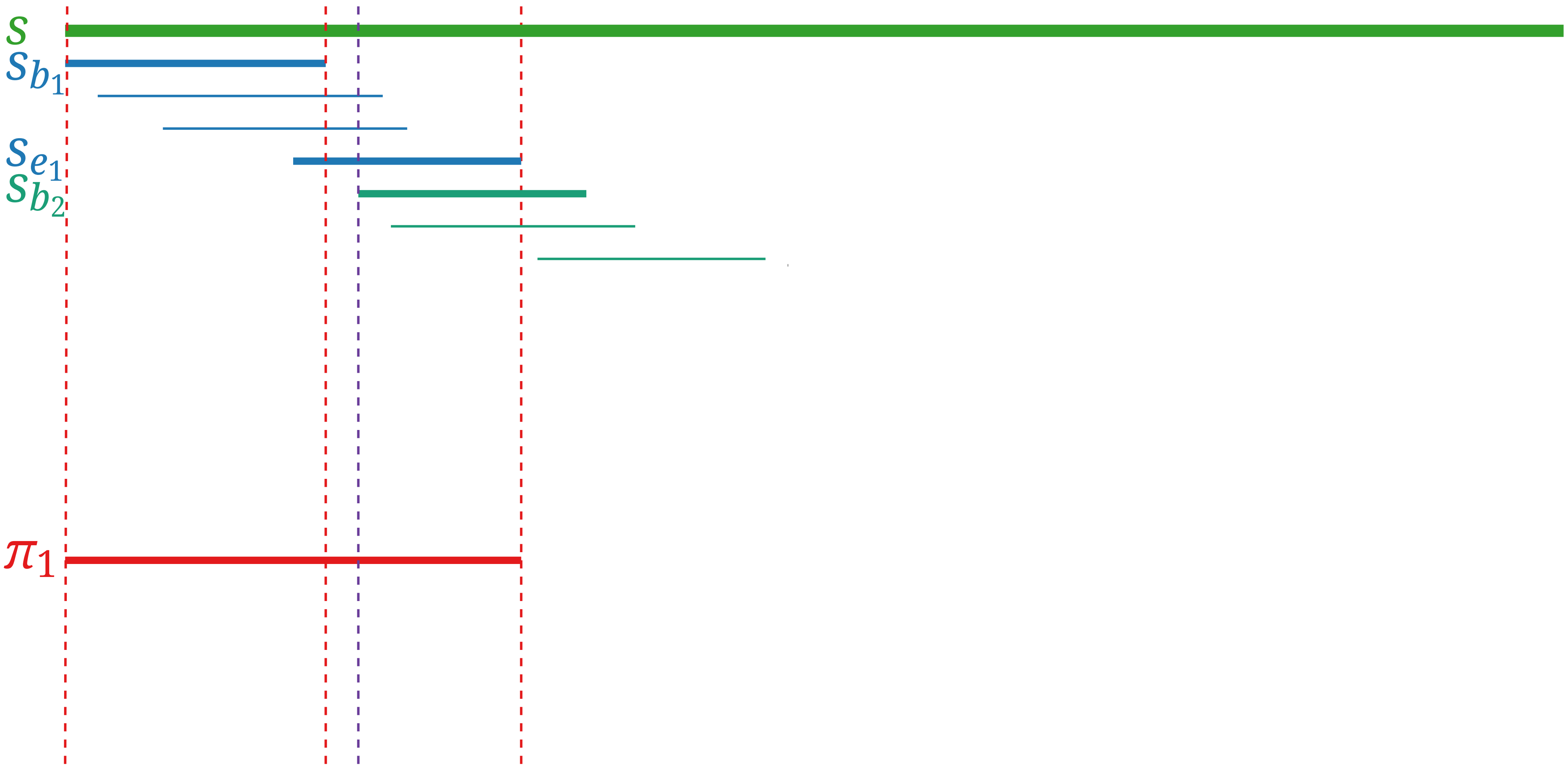

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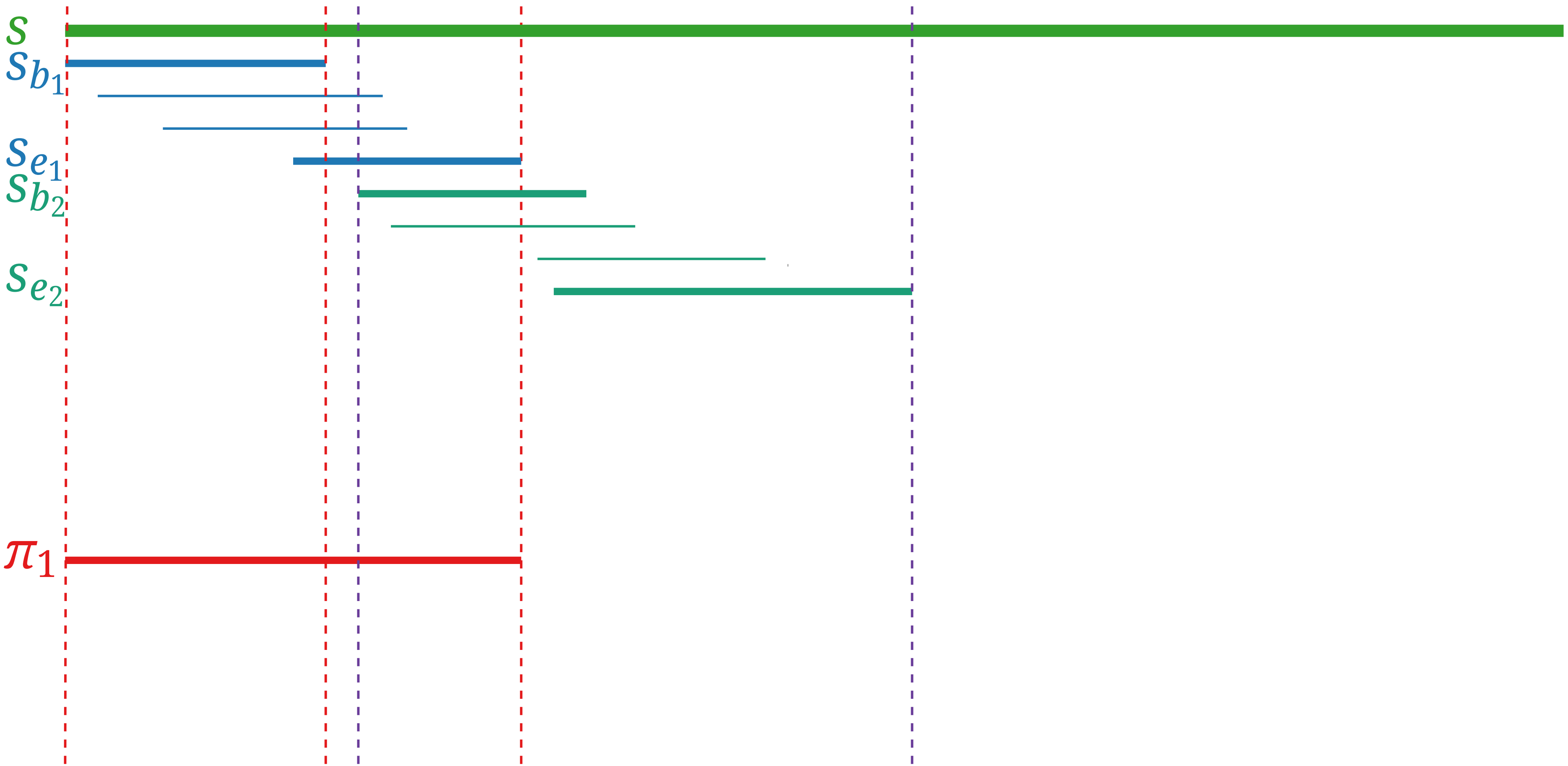

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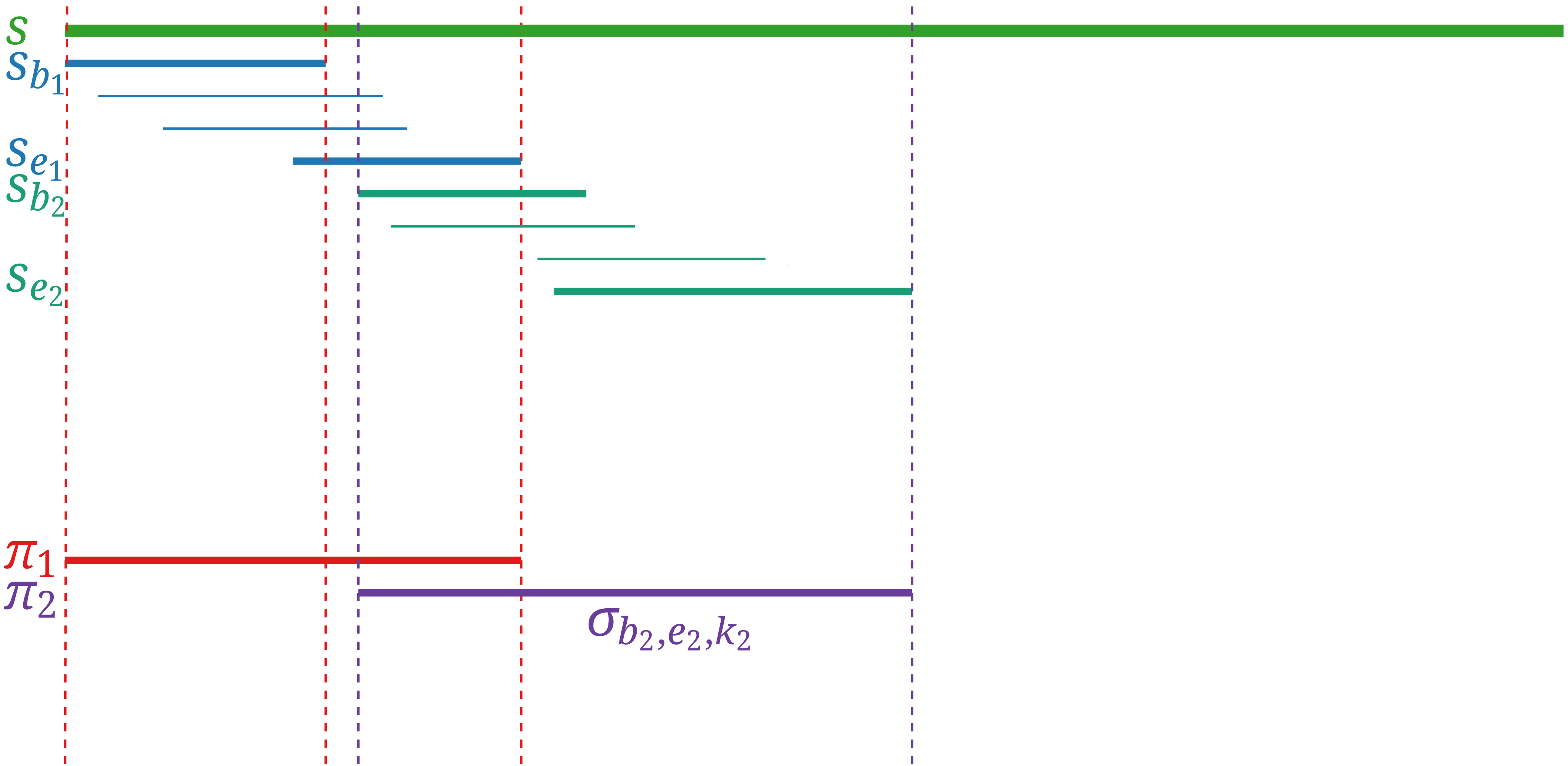

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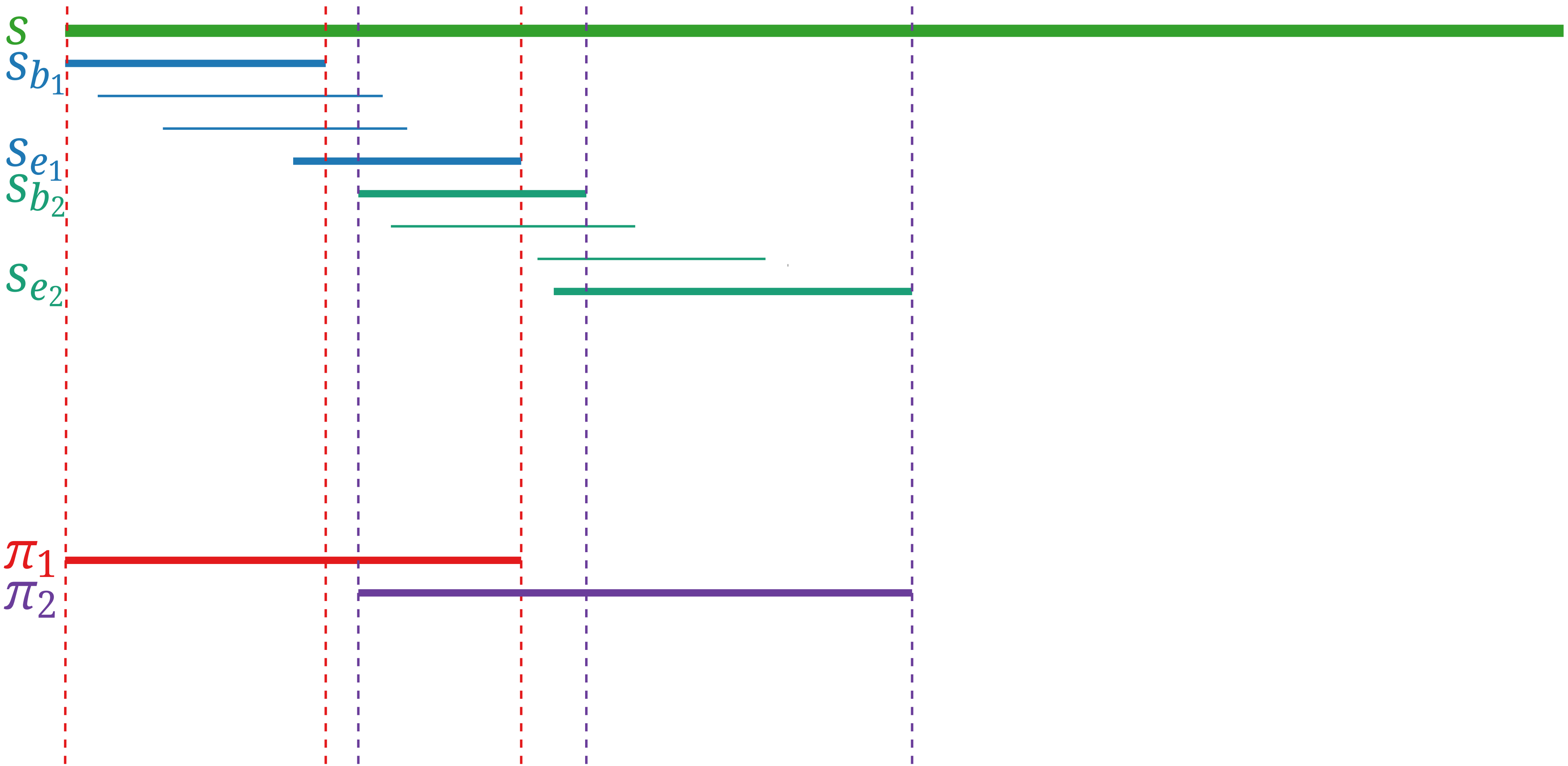

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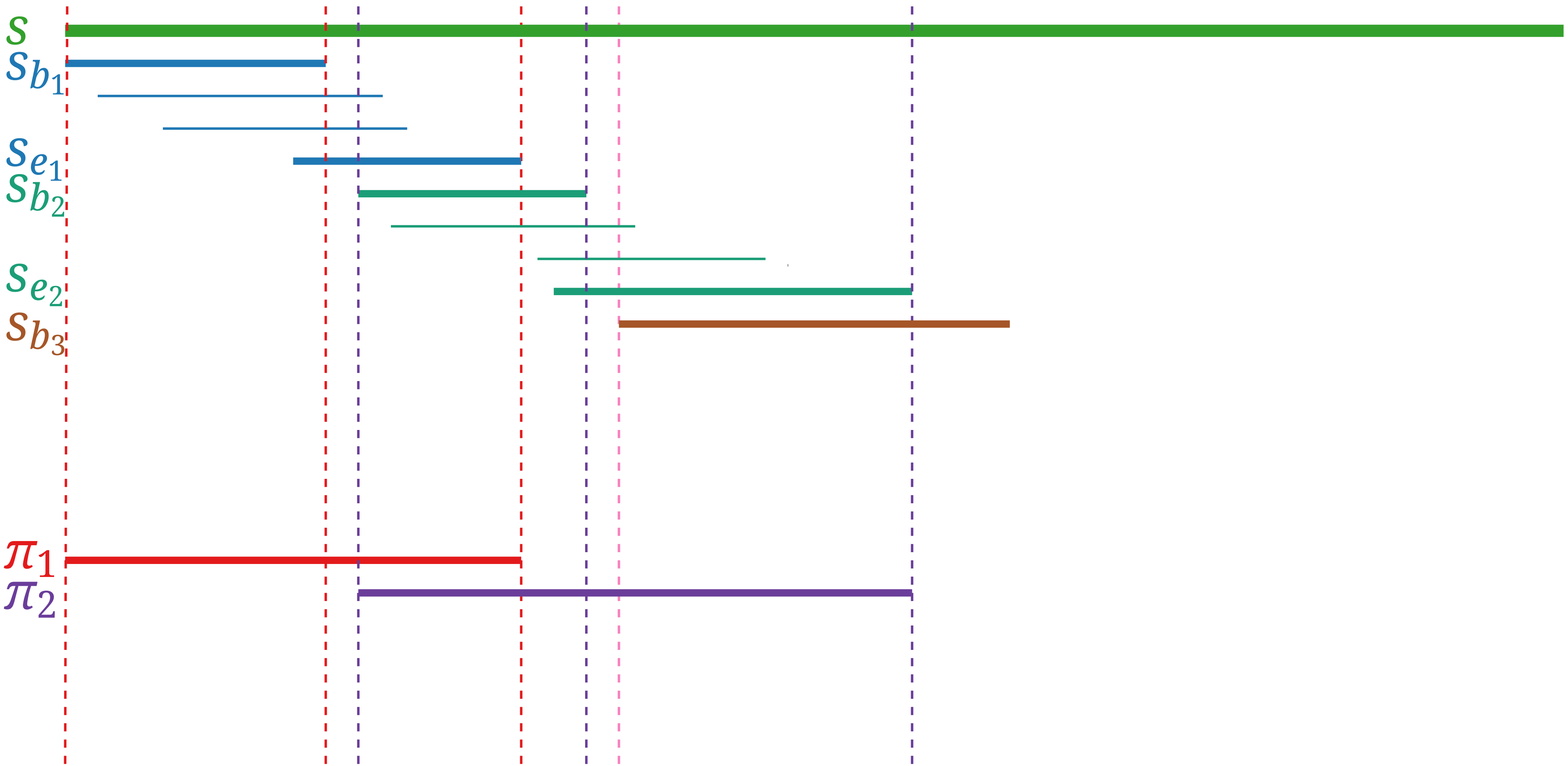

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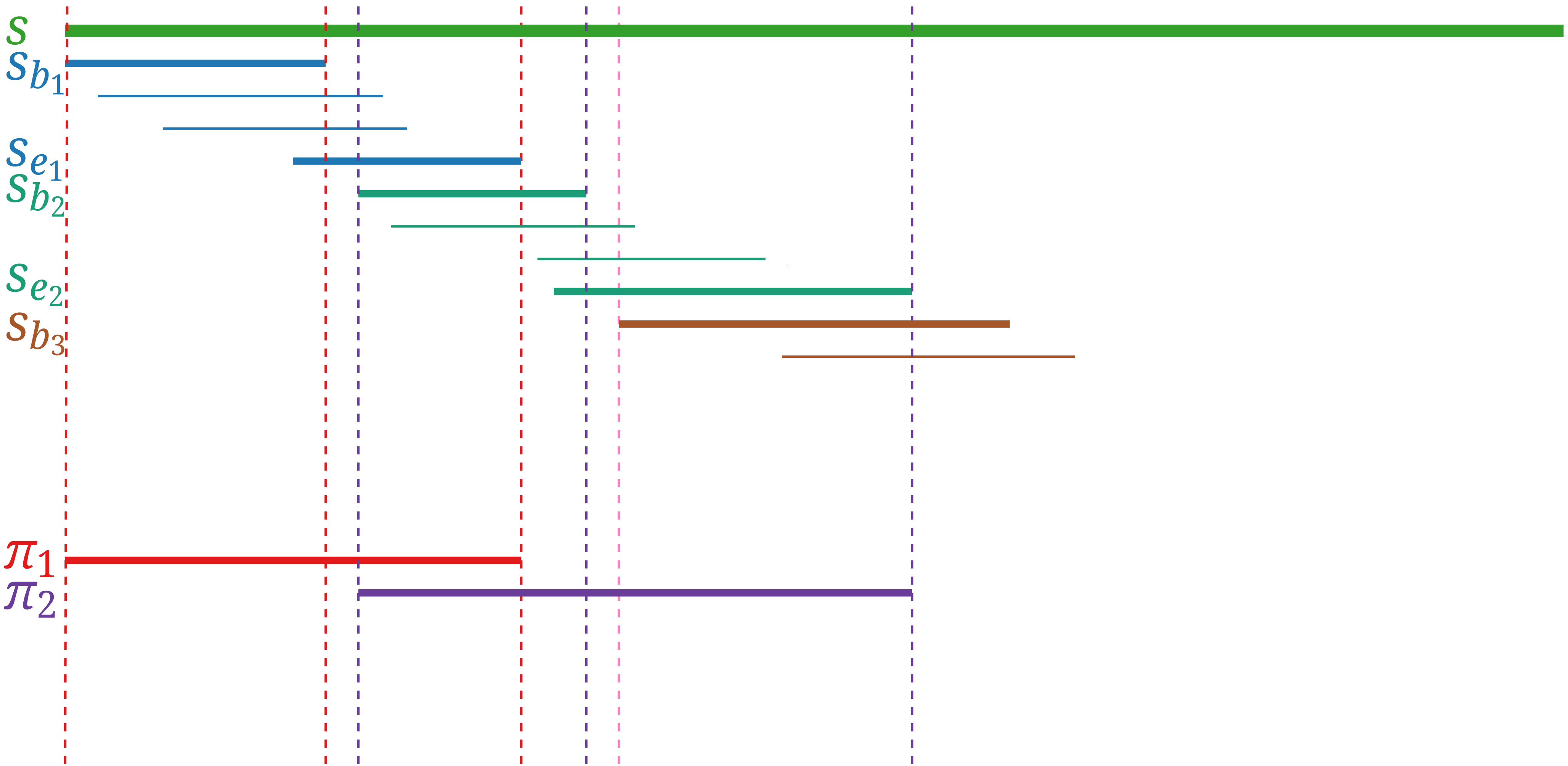

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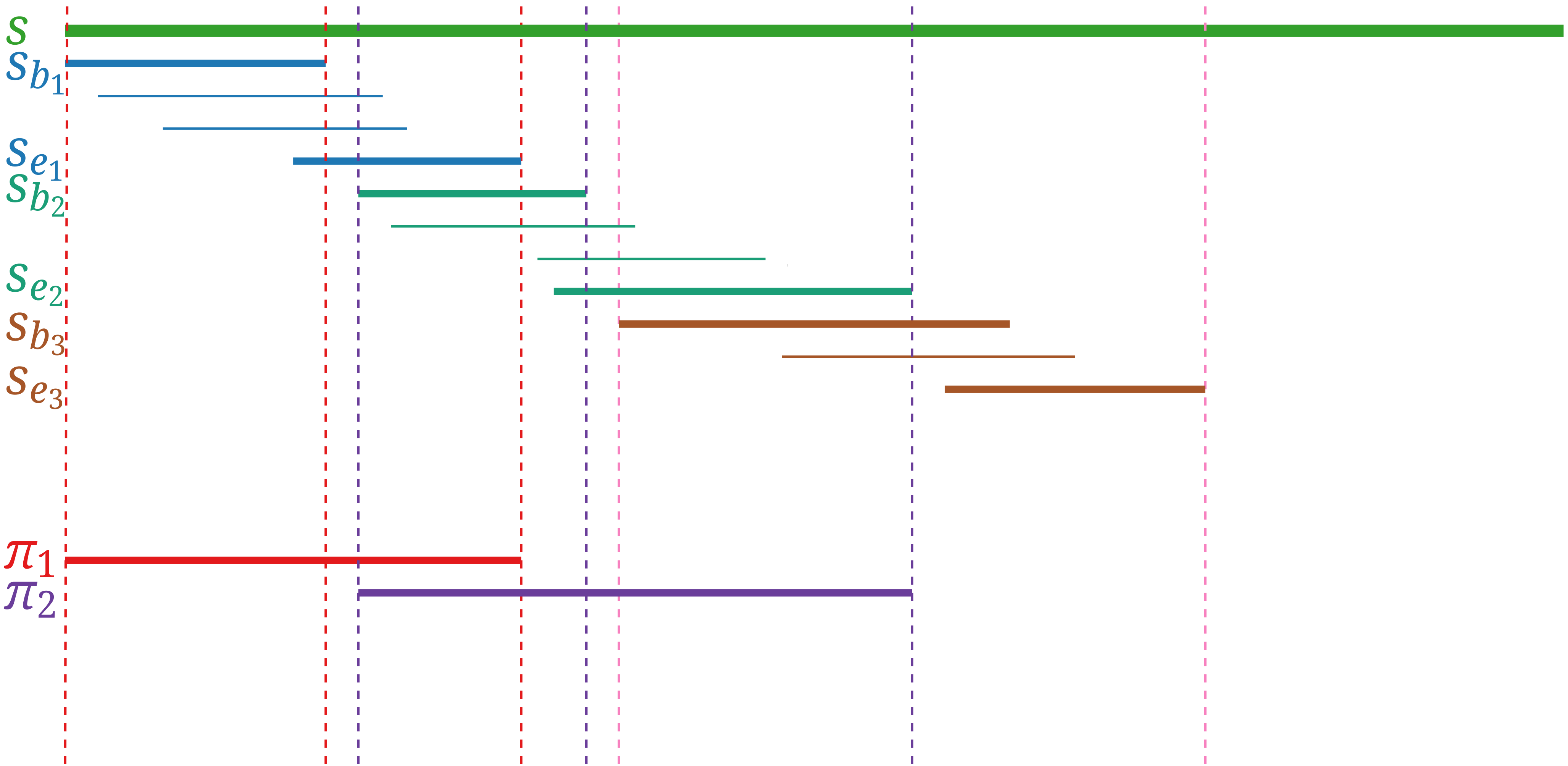

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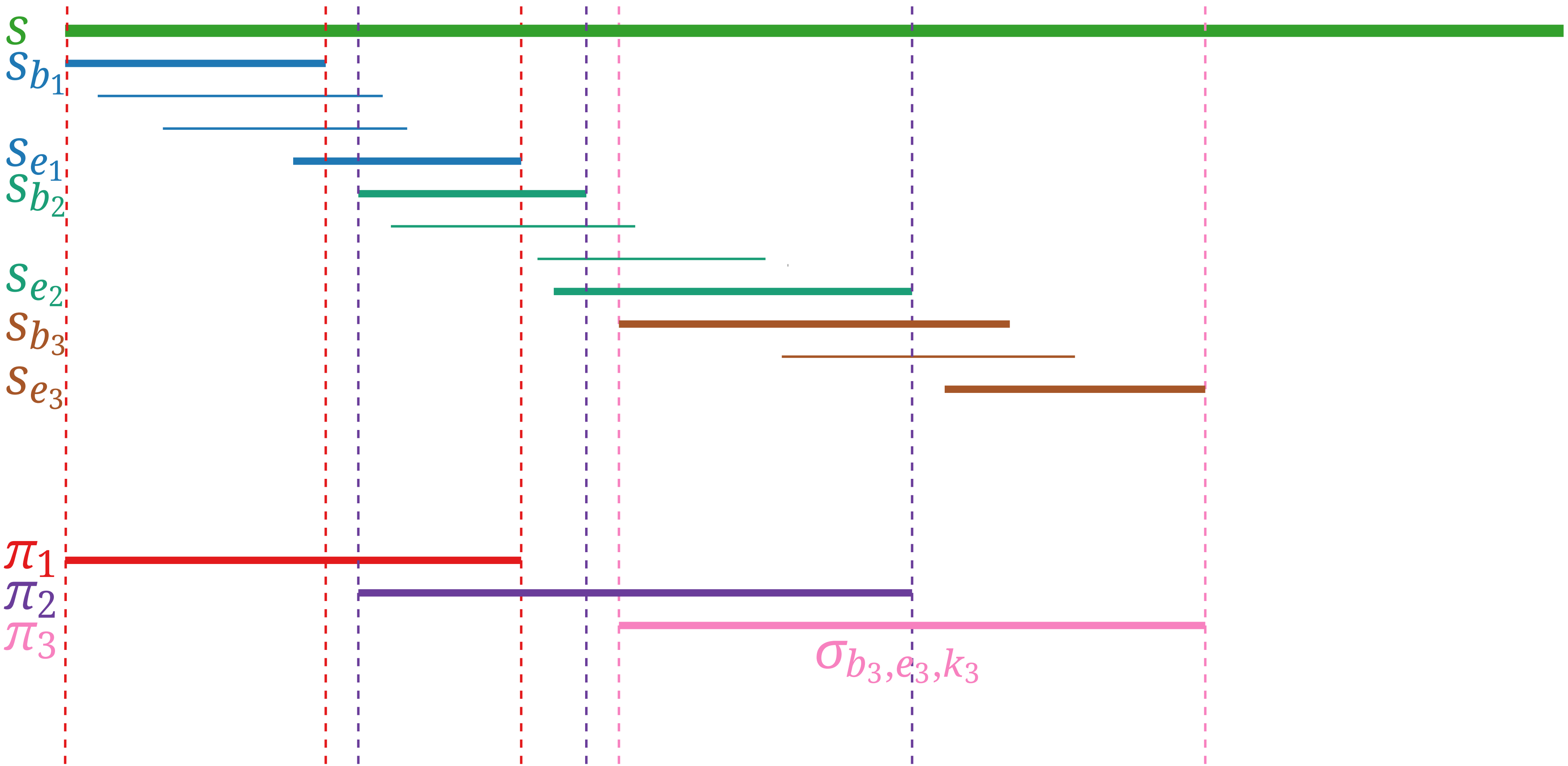

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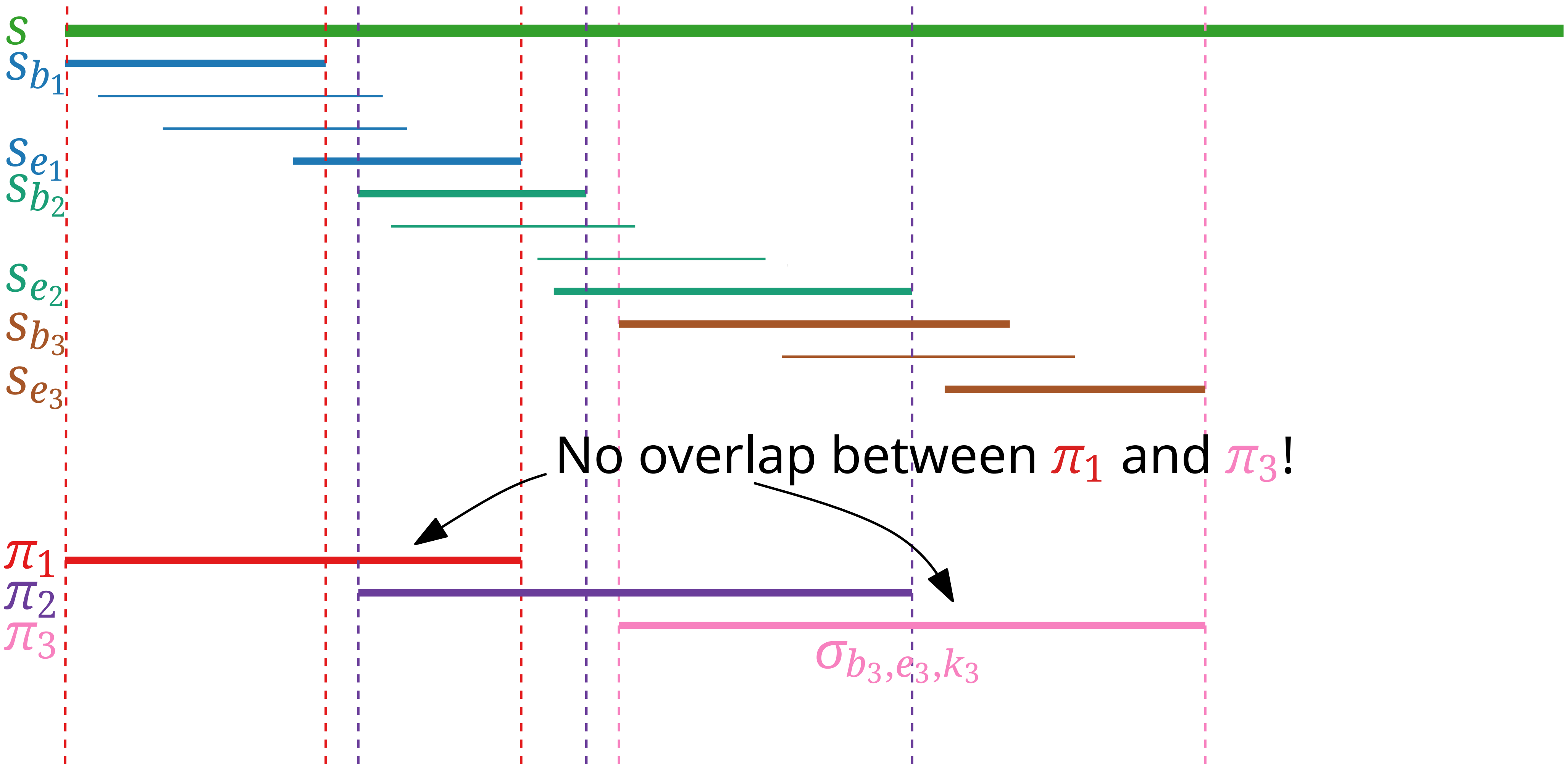

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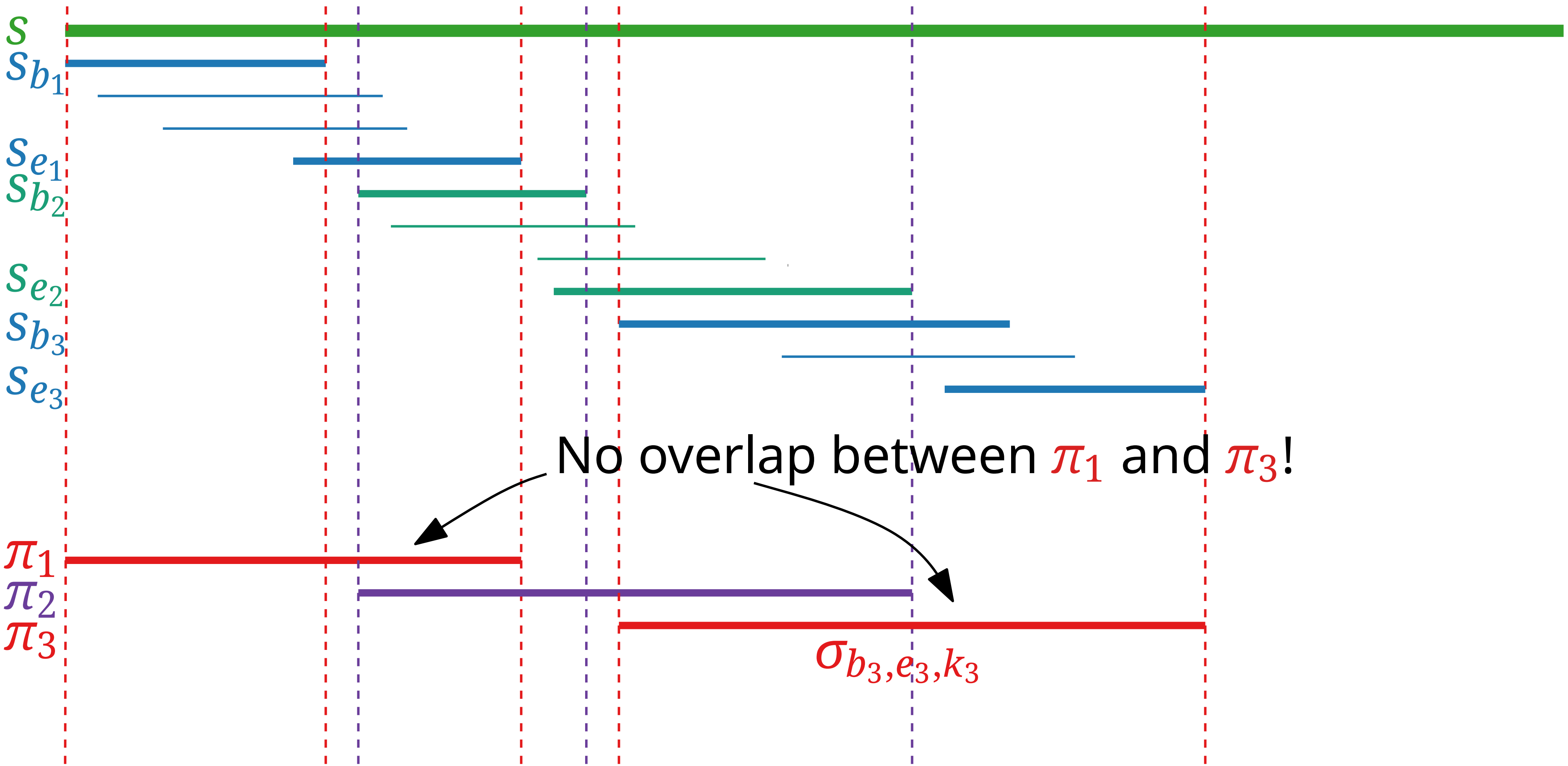

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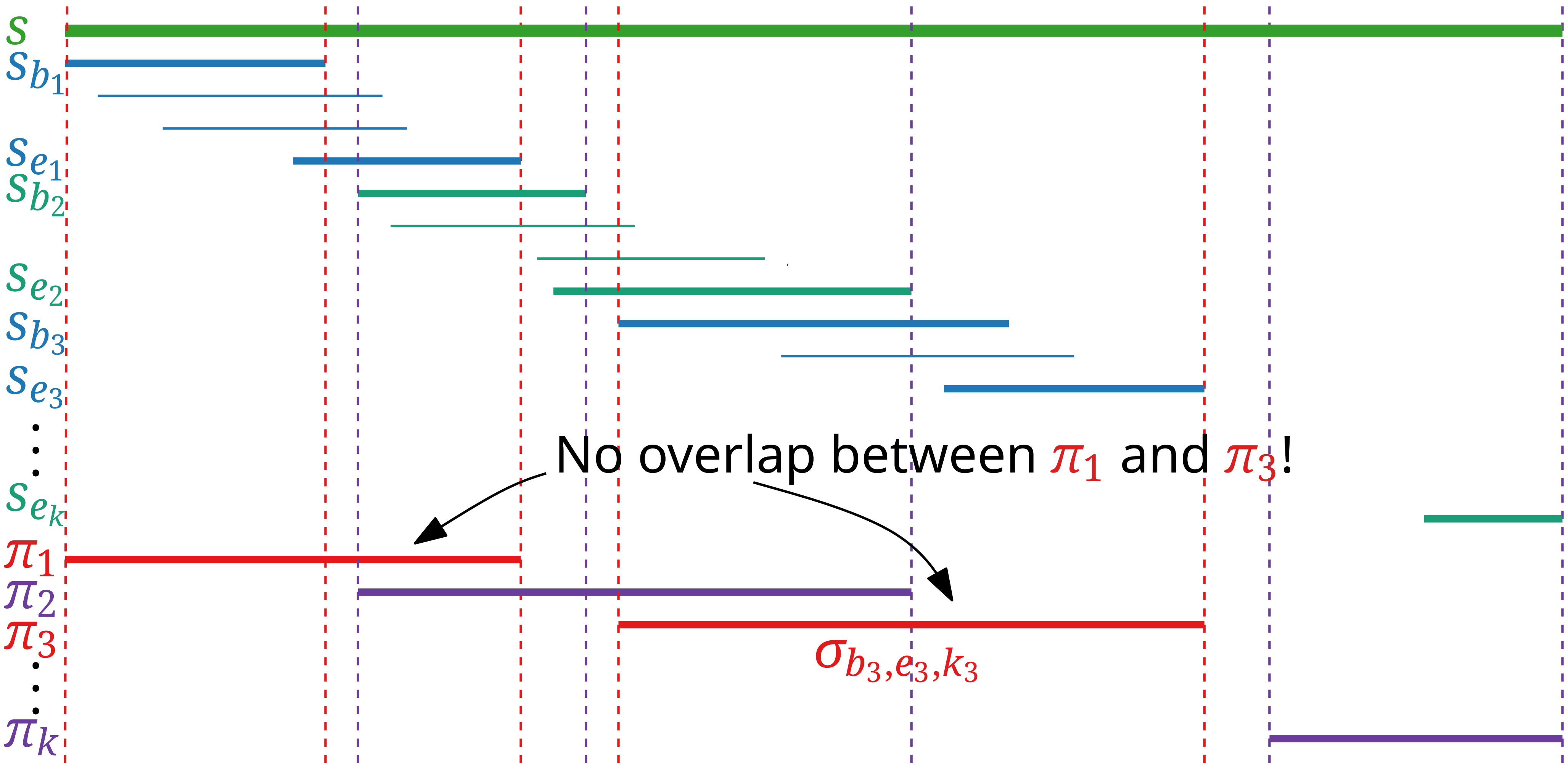

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Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and
$\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

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- The best known approximation factor for ShortestSuperString is $\frac{71}{30} \approx 2.367$.
- ShortestSuperString cannot be approximated within factor $\frac{333}{332} \approx 1.003$ (unless $\mathrm{P} = \mathrm{NP}$).

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