$$P_m = x_1 \cdot \underbrace{x_2^2 \cdot \ldots \cdot x_m^m}$$

$$S_n = \sum_{i=1}^{m} x_i = m$$

$\frac{\partial P_m}{\partial x} = \text{const}$

Lagrange multiplier

$$g = \sum_1^m x_i - m$$

$$\Lambda(x_1, \ldots x_m, \lambda) = \prod_1^m x_i^i + \lambda\left(\sum_1^m x_i - m\right)$$

$$\frac{\partial \Lambda}{\partial x_1} = \prod_2^m x_i^i + \lambda$$

$$\frac{\partial \Lambda}{\partial x_2} = 2x_1 x_2 x_3^3 \ldots + \lambda$$

$$\frac{\partial \Lambda}{\partial x_k} = \prod_{i \neq k}^{m} x_i^i \cdot x^{k-1} \cdot k + \lambda$$

$$\frac{\partial \Lambda}{\partial \lambda} = \sum_1^m x_i - m = 0$$

$$\cancel{x_1 = -\frac{1}{\lambda} \prod_2^m x_i^i}$$

$$\cancel{x_2\left[2x_1 \cdot x_3^3 \ldots x_m^m + \lambda\right] = 0}$$

$$\cancel{-2\frac{1}{\lambda} x_2^2 x_3^{2\cdot 3} \ldots x_m^{2m} + \lambda = 0}$$

$$\cancel{x_2^2 = \frac{\lambda^3}{2\prod_{i=3}^{m} x_i^{2\cdot i}}}$$

$$\frac{\prod_{2}^{n} x_i^i}{2 x_1 x_2 x_3^3 \ldots} = \frac{x_2^2 \cancel{x_3^3 \ldots x_n^n}}{2 x_1 x_2 \cancel{x_3^3} \ldots} = \frac{x_2}{2x_1} = 1$$

$\vdots$

$$\frac{x_2^2 x_3^3 \ldots x_k^k \cancel{\ldots x_n^n}}{k\, x_1 x_2^2 \ldots x_k^{k-1} \cancel{\ldots x_n^n}} = \frac{x_k}{k x_1} = 1$$

$$\sum_i x_i = n$$

$$\frac{x_2}{2x_1} = 1$$

$1+2+3=6$

$$\frac{x_3}{3x_1} = 1$$

$\frac{3 \cdot 4}{2} =$

$$\frac{x_2}{2} + \frac{x_3}{3} + \ldots \frac{x_n}{n} = (n-1)x_1$$

$$\frac{x_4}{4x_1} = 1$$

$\vdots$

$$\prod_1^n x_i^i (1+2+\ldots+n) = -\lambda x_1 - \lambda x_2 - \ldots = -\lambda \sum x_i = -\lambda n$$

$$\frac{\cancel{n}(n+1)}{2} \prod x_i^i = -\cancel{n}\lambda$$

$$\boxed{\lambda = -\frac{(n+1)}{2} \prod_1^n x_i}$$

$$\frac{2}{x_2} \prod_1 x_i^i = \frac{n+1}{2} \prod$$

$$x_2 = \frac{4}{n+1}$$

$$\frac{1}{x_1} \prod_1^n x_i^i = \frac{(n+1)}{2} \prod_1^n x_i$$

$$\frac{1}{x_1} = \frac{n+1}{2}$$

$$x_1 = \frac{2}{n+1}$$

$$\frac{k}{x_k} \prod_1^n x_i^i = \frac{n+1}{2} \prod$$

$$x_k = \frac{2 \cdot k}{n+1}$$

$$\sum_1^n x_k = \frac{2}{n+1} \sum_1^{n+1} k = \frac{2}{n+1} \cdot \frac{n(n+1)}{2} = n$$

$$x_k = \frac{2\cdot k}{m+1}$$

$\frac{2\cdot 1}{m+1}$ $\quad 2\cdot 2 \quad 2\cdot 3 \dots$

$x_1 + x_2 = 2$

3

$x \cdot x_2^2 = max$

$$\cancel{\prod^m x_k = \frac{2^m \cdot m!}{(m+1)^m}}$$

$m=2$

$x_1 = \frac{2}{3}$

$x_2 = \frac{4}{3}$

$$x_1 \cdot x_2^2 = \frac{2}{3}\cdot\frac{16}{9} = \frac{32}{27}$$

$$\cancel{\frac{2\cdot 1}{m+1}\cdot\frac{2\cdot 2}{m+1}\cdot\frac{2\cdot 3}{m+1}\cdots}$$

$$\cancel{\frac{2^m\cdot m!}{(m+1)^m}} \qquad \cancel{\frac{2}{3}}$$

$$= \prod_1^m \left[\frac{2\cdot k}{m+1}\right]^k =$$

$2^k \cdot 2^k \cdot 2^k \dots$

$$x_k^k = \left[\frac{2\cdot k}{m+1}\right]^k = \frac{2^k \cdot k^k}{(m+1)^k}$$

$$\prod_1^m x_k^k = \prod_1^m \frac{2^k \cdot k^k}{(m+1)^k} = \frac{2 \prod k^k}{} =$$

$$2^1\cdot 2^2\cdot 2^3 \cdots 2^m = 2^{(1+\dots m)} = 2^{\frac{m(m+1)}{2}}$$

$$(m+1)(m+1)^2 - \quad = (m+1)^{\frac{m(m+1)}{2}}$$

$$\boxed{= \left(\frac{2}{m+1}\right)^{\frac{m(m+1)}{2}} \prod_1^m k^k}$$

for $m=2$: $\left(\frac{2}{3}\right)^3 \cdot 2^2 = \frac{8\cdot 4}{3\cdot 3\cdot 3}$

match!