Introduction on
finding DOA-MUSIC algorimth

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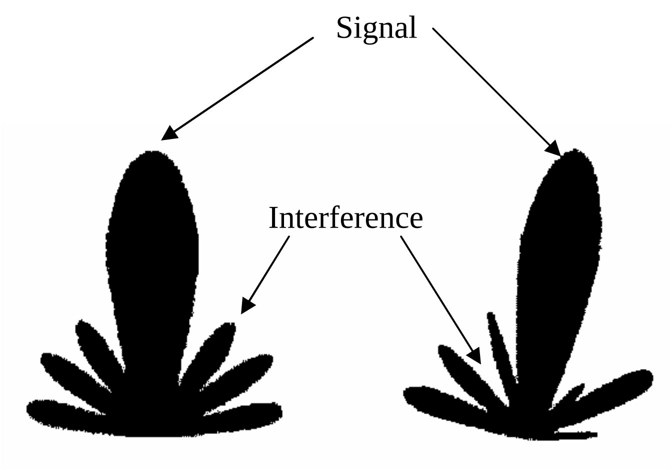
Brief Words

- DOA: **D**irection **o**f **A**rrival
- MUSIC: **MU**ltiple **SI**gnal **C**lassification
- ULA: **U**niform **L**inear **A**rray

Purpose of DOA Estimation

- Position location:
  - target finding
  - frequency management
  - rescue mission
  - improve SIR (a part of adaptive beamformer)



Data Model

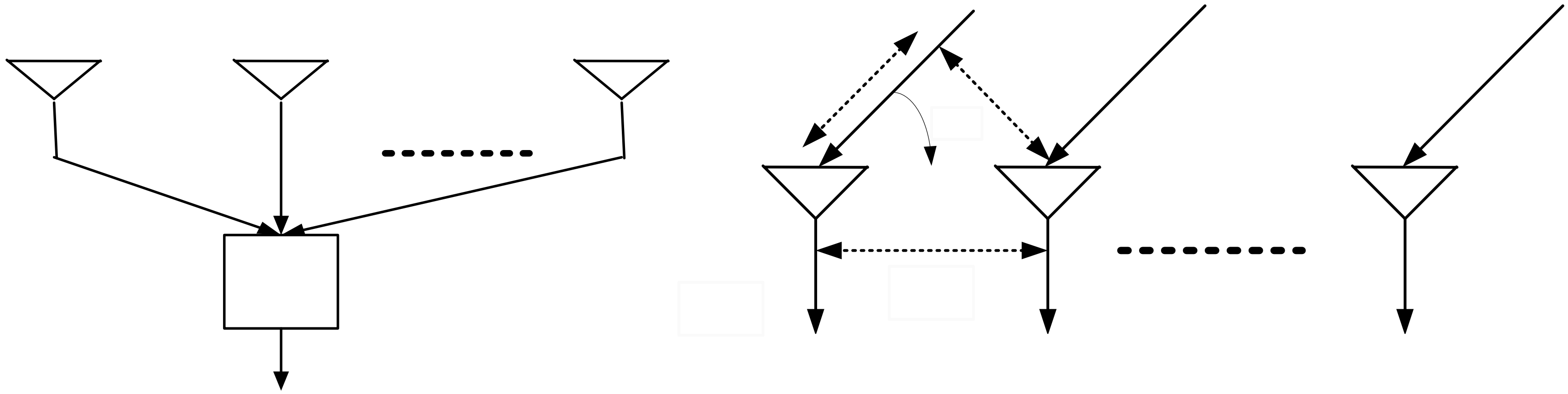


Fig.1 Data Model

- M antenna elements
- D uncorrelated sources

$$U(t)=\begin{bmatrix} u_0(t) \\ u_1(t) \\ \dots \\ u_{M-1}(t) \end{bmatrix}=[A(\theta_1)A(\theta_2)\dots A(\theta_D)]\begin{bmatrix} s_1(t) \\ s_2(t) \\ \dots \\ s_D(t) \end{bmatrix}+\begin{bmatrix} n_0(t) \\ n_1(t) \\ \dots \\ n_{M-1}(t) \end{bmatrix}=AS(t)+N(t) \qquad A(\theta)=\begin{bmatrix} 1 \\ e^{-j\beta d\cos\theta} \\ \dots \\ e^{-j\beta d(M-1)\cos\theta} \end{bmatrix}$$

MUSIC Algorithm

- decomposes the **autocorrelation matrix** into **signal subspace** and **noise subspace** to estimate DOA.

MUSIC Algorithm

- **Input Autocorrelation matrix:**

$$R_{UU} = E\left[UU^H\right] = AE\left[SS^H\right]A^H + E\left[NN^H\right]$$
$$= AR_{SS}A^H + \sigma_n^2 I$$

- **If $\{\lambda_0, \lambda_1, \ldots, \lambda_{M-1}\}$ is eigenvalues of $\mathbf{R}_{UU}$:**

$$\left|\mathrm{R}_{\mathrm{UU}} - \lambda_i I\right| = 0$$

$$\left|AR_{SS}A^H + \sigma_n^2 I - \lambda_i I\right| = \left|AR_{SS}A^H + \left(\sigma_n^2 - \lambda_i\right)I\right| = 0$$

$\Rightarrow$ $\left(\sigma_n^2 - \lambda_i\right)$ is eigenvalues of $A\mathrm{R}_{\mathrm{SS}}A^H$ includes noise subspace (zero eigenvalues) and signal subspace

MUSIC Algorithm

- **With M-D noise vectors then:**

$$\lambda_{D \to M-1} = \sigma_n^2$$

- **So:**

$$\Rightarrow \left(R_{UU} - \sigma_n^2 I\right) q_i = A\mathrm{R}_{SS} A^H q_i + \sigma_n^2 I - \sigma_n^2 I = 0$$

$$A\mathrm{R}_{SS} A^H q_i = 0$$

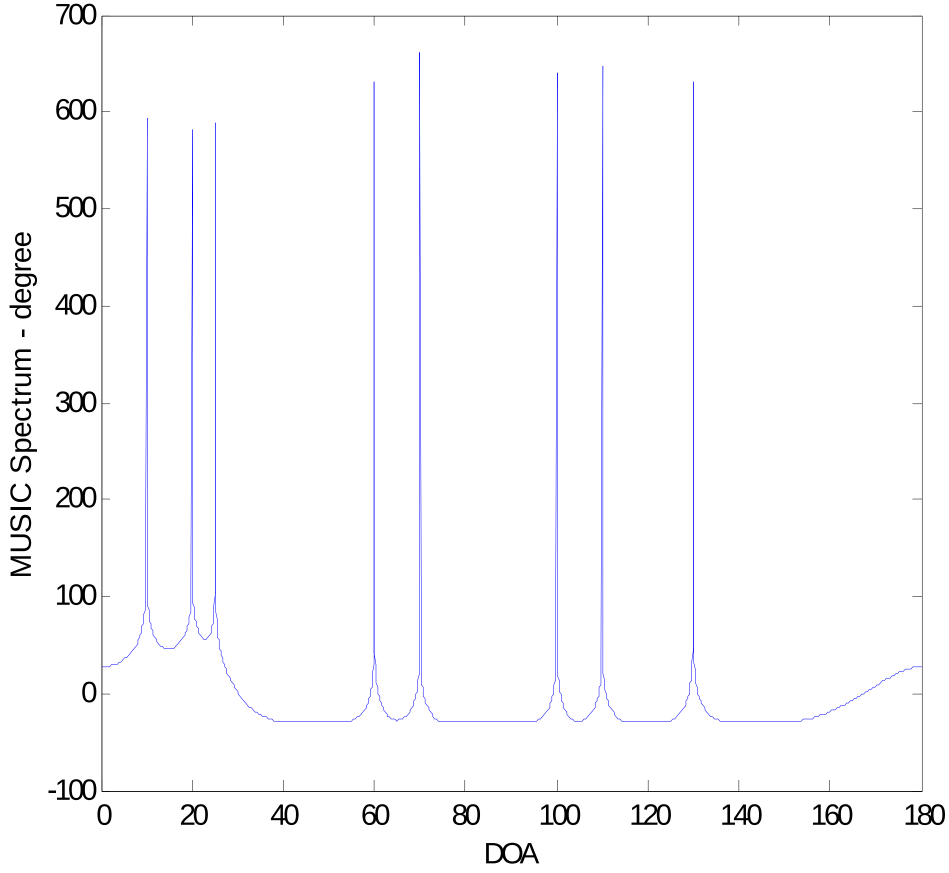
$$AR_{SS} \neq 0$$

$$\Rightarrow \boxed{\mathrm{A}^H q_i = 0}$$

$\Rightarrow$ D signal vectors orthogonal to M-D noise vectors

$$\Rightarrow \boxed{P(\theta) = \frac{a^H(\theta) a(\theta)}{a^H(\theta) V_n V_n^H a(\theta)}} \qquad V_n = \{q_D, \ldots, q_{M-1}\}$$

MATLAB Simulation



References

- [1]Joseph C. Liberti, Jr. & Theodore S.Rappaport, *Smart Antenna for Wireless Communications IS-95 and Third Generation CDMA Applications*, Prentice Hall PTR.