Working notes for Inefficient Concessions

June 6, 2017

Want to develop result showing that mediator increases potential for peace

- i.e. want to show mediator makes (Trust, Trust) an equilibrium over a larger parameter space than without mediator

What do we know?

- Theorem 4: Under some parameters, optimal concessions aren't made when there isn't trust
  - This is still a separating equilibrium, but completely inefficient concessions are given. So welfare is lower because of inefficient concessions.
  - Where there is a possibility of mediation helping to achieve peace where it otherwise would not be attainable (not just improving welfare) is if this reduction in welfare due to the inefficient concessions means that for some parameters it's not worth separating so we don't get peace at all without the mediator.
    * i.e. when welfare for high type from separating falls below that for pooling, separating is no longer an equilibrium (convo with Jean-Guillaume).
- Theorem 5: Mediator eliminates inefficient concessions
  - Here, we're already in that parameter space where there *are* inefficient concessions

Need to start by assuming that $\delta_h <$ threshold where separating with no concessions works.

- **Check to see whether Theorem 4 proof has to change.**

When $e = 1$ and no material value, $g_h^* = p(T + D)$ is the smallest concession for CSE.
When $e = 1$ and concessions have material value, $g' = \frac{p(1-\delta_l)(D+T)}{(1-\delta_l)(1-pT)+(1-p)D}$ is the minimum separating concession [Theorem 3] I claim in Theorem 5 that the minimum separating concession under mediation is $g^* = \frac{g'}{1-p}$.

With $\alpha = 1$,

- SWOC (separating without concessions) when $\delta_h \geq \frac{W}{(1-p)W+p(T+D)}$

- CSE (concessions separating equilibrium) when $\delta_h \geq \frac{W}{T+D}$

With $\alpha \in [0, 1)$ and $e = 1$,

- SWOC doesn't change because there are no concessions
- WTS there are parameters where can't get STC outcome because of $\alpha < 1$
  - $e < 1$ means some of these *can* get to peace
  - but some can't under *any* value of e [assuming e chosen at same time as g]

Remember that decision between equilibria is made not by what *can* be achieved in terms of δ, but by best response, i.e. "Does high type do better by not giving concession?"

- Should check in my numerical examples

Intermediate step for Theorem 4

- Low type IC constraint to solve for equilibrium high type gift
- Already having shown $g_l = 0$, can set $g_h = g$ for simplicity of notation
- Also, since all the discount factors are for the low type, I'll let $\delta_l = \delta$
- For the case where $e = 1$:

IC constraint:

$$X^l_{FF} \geq pX^l_{FT} + (1-p)X^l_{FF} - g$$

Expanded in basic terms (without efficiency issues)

$$\frac{W-D}{1-\delta} \geq p\left[T + W + \frac{\delta}{1-\delta}(W-D)\right] + (1-p)\frac{W-D}{1-\delta} - g$$

Multiply through by $(1-\delta)$

$$W - D \geq p(1-\delta)[T+W] + p\delta(W-D) + (1-p)[W-D] - (1-\delta)g$$

Now add complexity from Table 2

$$\begin{aligned} &W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1) \geq \\ &\quad p(1-\delta)[T(1+\alpha_2 g_1) + W(1+(1-\alpha_1)g_2)] + p\delta(W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)) \\ &\qquad + (1-p)[W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)] - (1-\delta)g \end{aligned}$$

Substitute in $\alpha_1 = 0$ everywhere since this is the IC for a low-type of player 1:

$$\begin{aligned}&W(1+g_2) - D(1+(1-\alpha_2)g_1) \geq \\ &\qquad p\,(1-\delta)\,[T(1+\alpha_2 g_1) + W(1+g_2)] + p\delta\,(W(1+g_2) - D(1+(1-\alpha_2)g_1)) \\ &\qquad\qquad + (1-p)\,[W(1+g_2) - D(1+(1-\alpha_2)g_1)] - (1-\delta)\,g\end{aligned}$$

Set $g_1 = 0$ on the LHS and $g_1 = g$ on the RHS:

$$\begin{aligned}&W(1+g_2) - D \geq \\ &\qquad p\,(1-\delta)\,[T(1+\alpha_2 g) + W(1+g_2)] + p\delta\,(W(1+g_2) - D(1+(1-\alpha_2)g)) \\ &\qquad\qquad + (1-p)\,[W(1+g_2) - D(1+(1-\alpha_2)g)] - (1-\delta)\,g\end{aligned}$$

Set $g_2 = 0$ and $\alpha_2 = 0$ wherever there is a $(1-p)$ and $g_2 = g$ and $\alpha_2 = 1$ wherever there is a p:

$$\begin{aligned}&pW(1+g) + (1-p)W - D \geq \\ &\qquad p\,(1-\delta)\,[T(1+g) + W(1+g)] + p\delta\,(W(1+g) - D) \\ &\qquad\qquad + (1-p)\,[W - D(1+g)] - (1-\delta)\,g\end{aligned}$$

Expand

$$\begin{aligned}&pW + pWg + W - pW - D \geq \\ &\qquad (p - p\delta)\,[T + Tg + W + Wg] + p\delta\,(W + Wg - D) \\ &\qquad\qquad + (1-p)\,[W - D - Dg] - (1-\delta)\,g\end{aligned}$$

Cancel some like terms and move $(1-\delta)\,g$ to LHS

$$\begin{aligned}&(1-\delta)\,g + pWg + W - D \geq \\ &\qquad p\,[T + Tg + W + Wg] - p\delta\,[T + Tg] - p\delta D \\ &\qquad\qquad + (1-p)\,[W - D - Dg]\end{aligned}$$

Do some more canceling and expanding

$$\begin{aligned}&(1-\delta)\,g + W - D \geq \\ &\qquad p\,[T + Tg + W] - p\delta T - p\delta Tg - p\delta D \\ &\qquad\qquad + W - D - Dg - pW + pD + pDg\end{aligned}$$

$$(1-\delta)\,g \geq pT + pTg + pW - p\delta T - p\delta Tg - p\delta D - Dg - pW + pD + pDg$$

$$(1-\delta)\,g \geq pT + pTg - p\delta T - p\delta Tg - p\delta D - Dg + pD + pDg$$

Now just rearrange to get all the g terms on the left

$$(1-\delta)\,g - pTg + p\delta Tg + Dg - pDg \geq pT - p\delta T - p\delta D + pD$$

$$[(1-\delta) - p(1-\delta)T + (1-p)D]\,g \geq p(1-\delta)(T+D)$$

$$g \geq \frac{p(1-\delta)(T+D)}{(1-\delta) - p(1-\delta)T + (1-p)D}$$

In comparing to the minimum separating gift when $e = 0$, which is

$$g^* \geq \frac{p(1-\delta)(T+D)}{(1-\delta)}$$

We can simplify to see that the minimum gift when $e = 1$ is larger when

$$(1-\delta)pT > (1-p)D$$

Next we get the threshold for δ_h that is necessary for a concessions separating eqm to exist:

- Use the high-type IC constraint:

$$pX^h_{TT} + (1-p)X^h_{FF} - g_h \geq X^h_{FF}$$

- We'll need to expand for material effects but then solve for both $e = 0$ and $e = 1$
- For $e = 1$:

$$\frac{p}{1-\delta_h}T(1+g) + \frac{1-p}{1-\delta_h}(W - D(1+g)) - g_h \geq \frac{1}{1-\delta_h}(W-D)$$

$$pT + pTg + (1-p)(W - D - Dg) - g_h(1-\delta_h) \geq W - D$$

$$pT + pTg + W - D - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq W - D$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

Note that all the g's should be g_h's in the five lines above:

$$g_h\delta_h \geq -pT - pTg_h + Dg_h + pW - pD - pDg_h + g_h$$

$$\delta_h \geq \frac{p(W-D-T)}{g_h} + (1-p)D + 1 - pT$$

- For $e = 0$, you're back in the original case with no material value, i.e. enforceable for same δ_h as no material value case

$$\frac{p}{1-\delta_h}T + \frac{1-p}{1-\delta_h}(W-D) - g_0 \geq \frac{1}{1-\delta_h}(W-D)$$
$$pT + (1-p)(W-D) - (1-\delta_h)g_0 \geq (W-D)$$
$$pT - p(W-D) + \delta_h g_0 \geq g_0$$
$$\delta_h g_0 \geq g_0 + p(W-D-T)$$
$$\delta_h \geq 1 + \frac{p(W-D-T)}{g_0}$$

  Optimal separating gift in this setting is $g_0 = p(D+T)$. Substituting this in, we have:

$$\delta_h \geq 1 + \frac{p(W-D-T)}{p(D+T)} = 1 + \frac{pW - p(D-T)}{p(D+T)} = 1 + \frac{pW}{p(D+T)} - 1 = \frac{W}{D+T}$$

  - But check: when $e = 0$, also wipe out immediate monetary value? YES, in terms of benefit, but NOT cost, which is the only part that doesn't cancel out of IC constraints (same for high type and low type; benefit is on both sides; cost is on only one side)
    * Not from utility standpoint, but from IC standpoint because direct benefit of gift cancels out

- But high type utility goes down
  - Interestingly, if $e = 0$, if high types would have separated without material value, they will here with material value that is destroyed (they don't do better in pooling). We see this by calculating the high type utility for the case where $e = 0$, which is

$$\frac{1}{1-\delta_h}\left[pT + (1-p)(W-D)\right] - g_0$$

    Substituting in the value of the g_0 (the optimal separating gift with $e = 0$) and the threshold value to separate when $e = 0$, we have

$$\frac{T+D}{T+D-W}\left[pT + (1-p)(W-D)\right] - p(T+D)$$
$$\frac{T+D}{T+D-W}\left[p(T+D-W) + (W-D)\right] - p(T+D)$$
$$p(T+D) + \frac{T+D}{T+D-W}\left[W-D\right] - p(T+D)$$
$$\frac{T+D}{T+D-W}\left[W-D\right]$$

This is exactly the payoff to the pooling equilibrium when $\delta = \frac{W}{T+D}$. So $U_{g_0} = U_{pooling}$ at the cutoff for separating, and U_{g_0} is larger for any discount factor higher than the cutoff. The value of the change of peace for ever is outweighed by having to pay for the gift even though the value of the gift will never be received.

To-do list after Venice CesIfo 06/15/17:

- Jim Fearon: maybe get rid of repeated game and just parameterize a one-shot game (doens't work because concessions required for separation are too large; require future to recoup cost)
  - he has a paper about concessions being used against you; he also gave me the reference for another one but I've forgotten it

Notes from CPEG, Oct. 25, 2019 presentation:

  - Arnaud
    * δ are not exogenous (look at Brexit)
    * Proposes a one-shot game to replace the repeated game. Simple bayesian game with difference preferences depening on type.
  - Raphael Godefroy
    * Can δ_i be a function of g? That is, likelihood of defection decreases as concession gets better