

## Working notes for Inefficient Concessions

June 6, 2017

Want to develop result showing that mediator increases potential for peace

- i.e. want to show mediator makes (Trust, Trust) an equilibrium over a larger parameter space than without mediator

What do we know?

- Theorem 4: Under some parameters, optimal concessions aren't made when there isn't trust
  - This is still a separating equilibrium, but completely inefficient concessions are given. So welfare is lower because of inefficient concessions.
  - Where there is a possibility of mediation helping to achieve peace where it otherwise would not be attainable (not just improving welfare) is if this reduction in welfare due to the inefficient concessions means that for some parameters it's not worth separating so we don't get peace at all without the mediator.
    - \* i.e. when welfare for high type from separating falls below that for pooling, separating is no longer an equilibrium (convo with Jean-Guillaume).
- Theorem 5: Mediator eliminates inefficient concessions
  - Here, we're already in that parameter space where there *are* inefficient concessions

Need to start by assuming that  $\delta_h < \text{threshold}$  where separating with no concessions works.

- **Check to see whether Theorem 4 proof has to change.**

With  $\alpha = 1$ ,

- SWOC (separating without concessions) when  $\delta_h \geq \frac{W}{(1-p)W+p(T+D)}$
- CSE (concessions separating equilibrium) when  $\delta_h \geq \frac{W}{T+D}$

With  $\alpha \in [0, 1)$  and  $e = 1$ ,

- SWOC doesn't change because there are no concessions
- WTS there are parameters where can't get STC outcome because of  $\alpha < 1$ 
  - $e < 1$  means some of these *can* get to peace
  - but some can't under *any* value of  $e$  [assuming  $e$  chosen at same time as  $g$ ]

Remember that decision between equilibria is made not by what *can* be achieved in terms of  $\delta$ , but by best response, i.e. “Does high type do better by not giving concession?”

- Should check in my numerical examples

Intermediate step for Theorem 4

- Low type IC constraint to solve for equilibrium high type gift
- Already having shown  $g_l = 0$ , can set  $g_h = g$  for simplicity of notation
- Also, since all the discount factors are for the low type, I'll let  $\delta_l = \delta$
- For the case where  $e = 1$ :

IC constraint:

$$X_{FF}^l \geq pX_{FT}^l + (1-p)X_{FF}^l - g$$

Expanded in basic terms (without efficiency issues)

$$\frac{W-D}{1-\delta} \geq p \left[ T + W + \frac{\delta}{1-\delta} (W-D) \right] + (1-p) \frac{W-D}{1-\delta} - g$$

Multiply through by  $(1-\delta)$

$$W-D \geq p(1-\delta)[T+W] + p\delta(W-D) + (1-p)[W-D] - (1-\delta)g$$

Now add complexity from Table 2

$$\begin{aligned} W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1) \geq \\ p(1-\delta)[T(1+\alpha_2g_1) + W(1+(1-\alpha_1)g_2)] + p\delta(W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)) \\ + (1-p)[W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)] - (1-\delta)g \end{aligned}$$

Substitute in  $\alpha_1 = 0$  everywhere since this is the IC for a low-type of player 1:

$$\begin{aligned} W(1 + g_2) - D(1 + (1 - \alpha_2)g_1) &\geq \\ p(1 - \delta) [T(1 + \alpha_2 g_1) + W(1 + g_2)] + p\delta (W(1 + g_2) - D(1 + (1 - \alpha_2)g_1)) \\ &\quad + (1 - p) [W(1 + g_2) - D(1 + (1 - \alpha_2)g_1)] - (1 - \delta) g \end{aligned}$$

Set  $g_1 = 0$  on the LHS and  $g_1 = g$  on the RHS:

$$\begin{aligned} W(1 + g_2) - D &\geq \\ p(1 - \delta) [T(1 + \alpha_2 g) + W(1 + g_2)] + p\delta (W(1 + g_2) - D(1 + (1 - \alpha_2)g)) \\ &\quad + (1 - p) [W(1 + g_2) - D(1 + (1 - \alpha_2)g)] - (1 - \delta) g \end{aligned}$$

Set  $g_2 = 0$  and  $\alpha_2 = 0$  wherever there is a  $(1 - p)$  and  $g_2 = g$  and  $\alpha_2 = 1$  wherever there is a  $p$ :

$$\begin{aligned} pW(1 + g) + (1 - p)W - D &\geq \\ p(1 - \delta) [T(1 + g) + W(1 + g)] + p\delta (W(1 + g) - D) \\ &\quad + (1 - p) [W - D(1 + g)] - (1 - \delta) g \end{aligned}$$

Expand

$$\begin{aligned} pW + pWg + W - pW - D &\geq \\ (p - p\delta) [T + Tg + W + Wg] + p\delta (W + Wg - D) \\ &\quad + (1 - p) [W - D - Dg] - (1 - \delta) g \end{aligned}$$

Cancel some like terms and move  $(1 - \delta) g$  to LHS

$$\begin{aligned} (1 - \delta) g + pWg + W - D &\geq \\ p[T + Tg + W + Wg] - p\delta [T + Tg] - p\delta D \\ &\quad + (1 - p) [W - D - Dg] \end{aligned}$$

Do some more canceling and expanding

$$\begin{aligned} (1 - \delta) g + W - D &\geq \\ p[T + Tg + W] - p\delta T - p\delta Tg - p\delta D \\ &\quad + W - D - Dg - pW + pD + pDg \end{aligned}$$

$$(1 - \delta) g \geq pT + pTg + pW - p\delta T - p\delta Tg - p\delta D - Dg - pW + pD + pDg$$

$$(1 - \delta) g \geq pT + pTg - p\delta T - p\delta Tg - p\delta D - Dg + pD + pDg$$

Now just rearrange to get all the  $g$  terms on the left

$$(1 - \delta)g - pTg + p\delta Tg + Dg - pDg \geq pT - p\delta T - p\delta D + pD$$

$$[(1 - \delta) - p(1 - \delta)T + (1 - p)D]g \geq p(1 - \delta)(T + D)$$

$$g \geq \frac{p(1 - \delta)(T + D)}{(1 - \delta) - p(1 - \delta)T + (1 - p)D}$$

In comparing to the minimum separating gift when  $e = 0$ , which is

$$g^* \geq \frac{p(1 - \delta)(T + D)}{(1 - \delta)}$$

We can simplify to see that the minimum gift when  $e = 1$  is larger when

$$(1 - \delta)pT > (1 - p)D$$

Next we get the threshold for  $\delta_h$  that is necessary for a concessions separating eqm to exist:

- Use the high-type IC constraint:

$$pX_{TT}^h + (1 - p)X_{FF}^h - g_h \geq X_{FF}^h$$

- We'll need to expand for material effects but then solve for both  $e = 0$  and  $e = 1$
- For  $e = 1$ :

$$\frac{p}{1 - \delta_h}T(1 + g) + \frac{1 - p}{1 - \delta_h}(W - D(1 + g)) - g_h \geq \frac{1}{1 - \delta_h}(W - D)$$

$$pT + pTg + (1 - p)(W - D - Dg) - g_h(1 - \delta_h) \geq W - D$$

$$pT + pTg + W - D - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq W - D$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

- For  $e = 0$ , you're back in the original case with no material value, i.e. enforceable for same  $\delta_h$  has no material value case

– But check: when  $e = 0$ , also wipe out immediate monetary value?

\* Not from utility standpoint, but from IC standpoint because direct benefit of gift cancels out

- But high type utility goes down

To-do list after Venice CesIfo 06/15/17:

- Jim Fearon: maybe get rid of repeated game and just parameterize a one-shot game (doesn't work because concessions required for separation are too large; require future to recoup cost)
  - he has a paper about concessions being used against you; he also gave me the reference for another one but I've forgotten it