Working notes for Inefficient Concessions

June 6, 2017

Want to develop result showing that mediator increases potential for peace

- i.e. want to show mediator makes (Trust, Trust) an equilibrium over a larger parameter space than without mediator

What do we know?

- Theorem 4: Under some parameters, optimal concessions aren't made when there isn't trust
  - This is still a separating equilibrium, but completely inefficient concessions are given. So welfare is lower because of inefficient concessions.
  - Where there is a possibility of mediation helping to achieve peace where it otherwise would not be attainable (not just improving welfare) is if this reduction in welfare due to the inefficient concessions means that for some parameters it's not worth separating so we don't get peace at all without the mediator.
    - i.e. when welfare for high type from separating falls below that for pooling, separating is no longer an equilibrium (convo with Jean-Guillaume).
- Theorem 5: Mediator eliminates inefficient concessions
  - Here, we're already in that parameter space where there *are* inefficient concessions

Need to start by assuming that $\delta_h <$ threshold where separating with no concessions works.

- **Check to see whether Theorem 4 proof has to change.**

Intermediate step for Theorem 4

- Low type IC constraint to solve for equilibrium high type gift
- Already having shown $g_l = 0$, can set $g_h = g$ for simplicity of notation
- Also, since all the discount factors are for the low type, I'll let $\delta_l = \delta$

- For the case where $e = 1$:

IC constraint:

$$X_{FF}^{l} \geq pX_{FT}^{l} + (1-p)X_{FF}^{l} - g$$

Expanded in basic terms (without efficiency issues)

$$\frac{W-D}{1-\delta} \geq p\left[T + W + \frac{\delta}{1-\delta}(W-D)\right] + (1-p)\frac{W-D}{1-\delta} - g$$

Multiply through by $(1-\delta)$

$$W - D \geq p(1-\delta)[T+W] + p\delta(W-D) + (1-p)[W-D] - (1-\delta)g$$

Now add complexity from Table 2

$$\begin{aligned} &W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1) \geq \\ &\quad p(1-\delta)[T(1+\alpha_2 g_1) + W(1+(1-\alpha_1)g_2)] + p\delta(W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)) \\ &\qquad + (1-p)[W(1+(1-\alpha_1)g_2) - D(1+(1-\alpha_2)g_1)] - (1-\delta)g \end{aligned}$$

Substitute in $\alpha_1 = 0$ everywhere since this is the IC for a low-type of player 1:

$$\begin{aligned} &W(1+g_2) - D(1+(1-\alpha_2)g_1) \geq \\ &\quad p(1-\delta)[T(1+\alpha_2 g_1) + W(1+g_2)] + p\delta(W(1+g_2) - D(1+(1-\alpha_2)g_1)) \\ &\qquad + (1-p)[W(1+g_2) - D(1+(1-\alpha_2)g_1)] - (1-\delta)g \end{aligned}$$

Set $g_1 = 0$ on the LHS and $g_1 = g$ on the RHS:

$$\begin{aligned} &W(1+g_2) - D \geq \\ &\quad p(1-\delta)[T(1+\alpha_2 g) + W(1+g_2)] + p\delta(W(1+g_2) - D(1+(1-\alpha_2)g)) \\ &\qquad + (1-p)[W(1+g_2) - D(1+(1-\alpha_2)g)] - (1-\delta)g \end{aligned}$$

Set $g_2 = 0$ and $\alpha_2 = 0$ wherever there is a $(1-p)$ and $g_2 = g$ and $\alpha_2 = 1$ wherever there is a p:

$$\begin{aligned} &pW(1+g) + (1-p)W - D \geq \\ &\quad p(1-\delta)[T(1+g) + W(1+g)] + p\delta(W(1+g) - D) \\ &\qquad + (1-p)[W - D(1+g)] - (1-\delta)g \end{aligned}$$

Expand

$$\begin{aligned} &pW + pWg + W - pW - D \geq \\ &\quad (p-p\delta)[T + Tg + W + Wg] + p\delta(W + Wg - D) \\ &\qquad + (1-p)[W - D - Dg] - (1-\delta)g \end{aligned}$$

Cancel some like terms and move $(1-\delta)\,g$ to LHS

$$
\begin{aligned}
(1-\delta)\,g + pWg + W - D \geq \\
& p\left[T + Tg + W + Wg\right] - p\delta\left[T + Tg\right] - p\delta D \\
&\qquad + (1-p)\left[W - D - Dg\right]
\end{aligned}
$$

Do some more canceling and expanding

$$
\begin{aligned}
(1-\delta)\,g + W - D \geq \\
& p\left[T + Tg + W\right] - p\delta T - p\delta Tg - p\delta D \\
&\qquad + W - D - Dg - pW + pD + pDg
\end{aligned}
$$

$$(1-\delta)\,g \geq pT + pTg + pW - p\delta T - p\delta Tg - p\delta D - Dg - pW + pD + pDg$$

$$(1-\delta)\,g \geq pT + pTg - p\delta T - p\delta Tg - p\delta D - Dg + pD + pDg$$

Now just rearrange to get all the g terms on the left

$$(1-\delta)\,g - pTg + p\delta Tg + Dg - pDg \geq pT - p\delta T - p\delta D + pD$$

$$\left[(1-\delta) - p(1-\delta)T + (1-p)D\right]g \geq p(1-\delta)(T+D)$$

$$g \geq \frac{p(1-\delta)(T+D)}{(1-\delta) - p(1-\delta)T + (1-p)D}$$

In comparing to the minimum separating gift when $e = 0$, which is

$$g^* \geq \frac{p(1-\delta)(T+D)}{(1-\delta)}$$

We can simplify to see that the minimum gift when $e = 1$ is larger when

$$(1-\delta)pT > (1-p)D$$

Next we get the threshold for δ_h that is necessary for a concessions separating eqm to exist:

- Use the high-type IC constraint:

$$pX^h_{TT} + (1-p)X^h_{FF} - g_h \geq X^h_{FF}$$

- We'll need to expand for material effects but then solve for both $e = 0$ and $e = 1$

- For $e = 1$:

$$\frac{p}{1-\delta_h}T(1+g) + \frac{1-p}{1-\delta_h}(W - D(1+g)) - g_h \geq \frac{1}{1-\delta_h}(W - D)$$

$$pT + pTg + (1-p)(W - D - Dg) - g_h(1-\delta_h) \geq W - D$$

$$pT + pTg + W - D - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq W - D$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

To-do list after Venice CesIfo 06/15/17:

- From Eli four years ago: why can mediator do what the two sides cannot?
- Add discussion of intuitive criterion to text (if I end up needing it): out of eqm beliefs put zero weight on types that can never gain from deviating from a fixed eqm outcome
- Incorporate mediator result into text; re-organize text so it matches June presentation (re-worded Theorem 4, new Theorem 5)
  - Careful of SWOC being chosen over STC (δ_h threshold restriction?)
- Note for mediator result:
  - For two high types to cooperate in the mechanism, need large enough gift to screen out low types. They're still just reporting, so don't give a gift if not matched with another high type. PLUS when they're exchanging gifts, the gifts will actually be used for good:

$$T(1+g) \geq (1-\delta)[T(1+g)+W] + \delta(W-D)$$

$$\delta \geq \frac{W}{T+Tg+D}$$

    Note that this threshold is smaller than $\frac{W}{T+D}$, the one we get without concessions having material benefit / harm
- Jim Fearon: maybe get rid of repeated game and just parameterize a one-shot game (doens't work because concessions required for separation are too large; require future to recoup cost
  - he has a paper about concessions being used against you; he also gave me the reference for another one but I've forgotten it
- Someone who *can't* cooperate under either $e=0$ or $e=1$ CAN cooperate under mediator
  - In the absence of mediation, being able to burn money can help, but you still lose something
  - I still have a question about the overall game: if $e=0$ provides higher welfare if it were enforceable but it's not, do we just go with $e=1$?
  - Some parameters. For all of these, I take $W=8$, $D=5$, $T=10$
    * $p=.4, \delta_l=.45, \delta_h=\delta_{STC}=.53$. Choose $e=1$, but $\delta_1=.65$
    * $p=.3, \delta_l=.45, \delta_h=\delta_{STC}=.53$. Choose $e=1$, but $\delta_1>1$; even worse at $p=.2$
    * $p=.4, \delta_l=.5, \delta_h=\delta_{STC}=.53$. Choose $e=1$, $\delta_1 \geq .4$ (works fine)
    * $p=.1, \delta_l=.1, \delta_h=\delta_{STC}=.53$. Choose $e=0$, but $\delta_0 \geq 6.7$

- $p = .05, \delta_l = .5, \delta_h = .7$. Choose $e = 0$, but $\delta_0 = 7.6$
- $p = .1, \delta_l = .5, \delta_h = .75$. Choose $e = 0$, but $\delta_0 = 6.7$ and $\delta_1 \geq 3$