1 Make mediator work

November 29, 2020: In making revisions in April 2020 to respond to the EJPE referee reports, I discovered the IC constraint does not hold for the low type in Section 5: Mediation. Here I will see what adjustments might make the mediator work.

- November 30: I showed formally and in great detail in Section 1.4 that the IC constraint does not bind for $e = 1$ with our standard assumptions.

- December 1: I began working on the $e < 1$ case in Section 1.5

1.1 Mediation: Basic Details and Overview

- If the mediator gets two 'high' reports, both countries are instructed to exchange gifts and play 'Trust'

- If the mediator gets one or zero 'high' reports, there is no gift and they both play 'Fight'

I show the constraints for both types in Section 1.2.
I detail the choices of all types in regards to investments in civil society in Section 1.3.
I show in Section 1.4 that the low type's IC constraint cannot hold if $e = 1$.

1.2 IC constraints for Mediation

- High type IC:
$$pX_{TT}^h + (1-p)X_{FF}^h - pg_h + peg_h \geq X_{FF}^h$$

  - Relative to Section 4.2, $-g_h$ there becomes $-pg_h$ here (again, only give gift if matched with high type). Also $+peg_h$ in Section 4.2 disappears on RHS.

- Low type IC:
$$X_{FF}^l \geq pX_{FT}^l + (1-p)X_{FF}^l - pg_h + peg_h$$

  - If a low type reveals truthfully: you get X_{FF} and no gift with probability p; you get the same with probability $(1-p)$ (you need both countries to be high types in order to do anything)

- If a low type lies: get X_{FT} and give gift (g_h) and receive gift (eg_h) with probability p; X_{FF} again with probability $(1-p)$

- Relative to Section 4.2, there is no peg_h on the LHS (in Section 4.2, it cancels out with the peg_h on the RHS): no benefit of getting the gift from the high type. Also, $-g_h$ on the RHS becomes $-pg_h$ since you only send the gift if matched with a high type

1.3 Civil society investment choices

In a concessions separating equilibrium or in the mechanism, the high type always has the incentive to invest all of the gift in civil socity

- If there is an exchange of gifts (whether in the mechanism or a truthfully revealed separating equilibrium), you know you're both receiving from and giving to a high type

- **I should check this in all the scenarios to make sure we don't invoke it when it's not right**

  - **I also want to know whether it depends on the linear form**
  - **Make more explicit in the text**

α_i is the proportion of the gift that country i receives that it invests in civil society.

- $\alpha_i = 1$ means the whole value of the gift is invested in civil society. High types make this choice

  - $\alpha_i = 0$ means the whole value of the gift is invested in military buildup

- High type of Country 1 chooses $\alpha_1 = 1$ (all into civil society)

  - $(1 - \alpha_1) = 0$ (nothing into military)

- Low type of Country 1 chooses $\alpha_1 = 0$ (nothing into civil society)

  - $(1 - \alpha_1) = 1$ (all into military)

- High type of Country 2 chooses $\alpha_2 = 1$ (all into civil society)

  - $(1 - \alpha_2) = 0$ (nothing into military)

- Low type of Country 2 chooses $\alpha_2 = 0$ (nothing into civil society)

  - $(1 - \alpha_2) = 1$ (all into military)

1.4 Why does low type IC not hold with efficient gifts?

$$X_{FF}^l \geq pX_{FT}^l + (1-p)X_{FF}^l - pg_h + peg_h \tag{1}$$

With our restrictions (mostly the linear assumption, IIRC), High type gives gift with efficiency $e = 1$, so gift giving and receiving cancels out. This leaves

$$X_{FF}^l \geq pX_{FT}^l + X_{FF}^l - pX_{FF}^l$$

which simplifies to

$$X_{FF}^l \geq X_{FT}^l.$$

Expanding, we have

$$\frac{W() - D()}{1 - \delta^l} \geq T() + W() + \frac{\delta^l}{1 - \delta^l}(W() - D())$$

Now we have to fill in all the complicated bits in parentheses from Table 3 in Section 4.2

$$W(m_1 + (1-\alpha_1)eg_2) - D(m_2 + (1-\alpha_2)eg_1) \geq$$
$$(1-\delta^l)[T(s_2 + \alpha_2 eg_1) + W(m_1 + (1-\alpha_1)eg_2)]$$
$$+ \delta^l (W(m_1 + (1-\alpha_1)eg_2) - D(m_2 + (1-\alpha_2)eg_1)) \quad (2)$$

We've already assumed $e = 1$ here as given in Lemma 2 for a concessions separating equilibrium

- We *should* assume s_1, s_2, m_1, m_2 to all be 1 to be consistent with the theory though. If we implement all that AND $e = 1$, we get

$$W(1 + (1-\alpha_1)g_2) - D(1 + (1-\alpha_2)g_1) \geq (1-\delta^l)[T(1 + \alpha_2 g_1) + W(1 + (1-\alpha_1)g_2)]$$
$$+ \delta^l (W(1 + (1-\alpha_1)g_2) - D(1 + (1-\alpha_2)g_1))$$

- We also know that there are no gifts exchanged for X_{FF}^l, so $g_1 = g_2 = 0$ on the LHS. On the RHS, gifts ARE exchanged because player 2 is a high type and the low type of player 1 lied. So we can set $g_1 = g_2 = g_M$

$$W - D \geq (1-\delta^l)[T(1 + \alpha_2 g_M) + W(1 + (1-\alpha_1)g_M)]$$
$$+ \delta^l (W(1 + (1-\alpha_1)g_M) - D(1 + (1-\alpha_2)g_M))$$

- The α_i's. Recall α_i represents the portion of the received gift that Player i invests in civil society.

  - Player 1 is low type (it is the low type IC constraint that I'm analyzing) and so $\alpha_1 = 0$.

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- Player 2 is a high type in what's left of the RHS, so $\alpha_2 = 1$.
    * The $(1-p)$ part of the RHS was combined with the LHS; it was for the low type of Player 2, so it matched with the LHS where there are no gifts since the mediator instructs no gifts if one low-type player tells the truth, which Player 2 does when we're looking at Player 1's IC constraint.
- With $a_1 = 0$ and $a_2 = 1$, we have

$$W - D \geq \left(1 - \delta^l\right) \left[T \left(1 + g_M\right) + W \left(1 + g_M\right)\right] + \delta^l \left(W \left(1 + g_M\right) - D\right) \qquad (3)$$

- Distributing, we have

$$W - D \geq T + Tg_M + W + Wg_M - \delta^l T - \delta^l Tg_M - \delta^l W - \delta^l Wg_M + \delta^l W + \delta^l Wg_M - \delta^l D$$

- Canceling terms:
$$-D \geq T + Tg_M + Wg_M - \delta^l T - \delta^l Tg_M - \delta^l D$$

- Rearranging:
$$0 \geq \left(1 - \delta^l\right) T + \left(1 - \delta^l\right) Tg_M + Wg_M + \left(1 - \delta^l\right) D$$

Assuming $g_M > 0$, this inequality can never hold since each of $T, W, D \geq 0$ and $0 \leq \left(1 - \delta^l\right) \leq 1$

- What would it mean to have negative gifts?

1.5   What if gifts are inefficient?

Here I attempt to use inefficient gifts to get the low type to truthfully reveal.

Equation 1 is the general form of the IC constraint. In the next step in Section 1.4, I let the gifts cancel out because we assume in that section that $e = 1$. I need to go back to Equation 1 here since e is not assumed to be 1.

$$X_{FF}^l \geq pX_{FT}^l + (1 - p)X_{FF}^l - pg_h + peg_h$$

Leaving the two gift terms on the end intact, I simplify in the same way as in Section 1.4 to arrive at the correct analog to Equation 2.

$$W \left(m_1 + (1 - \alpha_1) eg_2\right) - D \left(m_2 + (1 - \alpha_2) eg_1\right) \geq$$
$$\left(1 - \delta^l\right) \left[T \left(s_2 + \alpha_2 eg_1\right) + W \left(m_1 + (1 - \alpha_1) eg_2\right)\right]$$
$$+ \delta^l \left(W \left(m_1 + (1 - \alpha_1) eg_2\right) - D \left(m_2 + (1 - \alpha_2) eg_1\right)\right) - pg_h + peg_h$$

- We assume s_1, s_2, m_1, m_2 to all be 1 to be consistent with Section 4.2

$$W \left(1 + (1 - \alpha_1) eg_2\right) - D \left(1 + (1 - \alpha_2) eg_1\right) \geq$$
$$\left(1 - \delta^l\right) \left[T \left(1 + \alpha_2 eg_1\right) + W \left(1 + (1 - \alpha_1) eg_2\right)\right]$$
$$+ \delta^l \left(W \left(1 + (1 - \alpha_1) eg_2\right) - D \left(1 + (1 - \alpha_2) eg_1\right)\right) - pg_h + peg_h$$

- We know that there are no gifts exchanged for X_{FF}^l, so $g_1 = g_2 = 0$ on the LHS. On the RHS, gifts ARE exchanged because player 2 is a high type and the low type of player 1 lied. So we can set $g_1 = g_2 = g_M$:

$$W (1) - D (1) \geq \left(1 - \delta^l\right) \left[T \left(1 + \alpha_2 eg_M\right) + W \left(1 + (1 - \alpha_1) eg_M\right)\right]$$
$$+ \delta^l \left(W \left(1 + (1 - \alpha_1) eg_M\right) - D \left(1 + (1 - \alpha_2) eg_M\right)\right) - pg_M + peg_M$$

- With $a_1 = 0$ and $a_2 = 1$ (Recall the discussion above Equation 3), we have

$$W - D \geq \left(1 - \delta^l\right) \left[T \left(1 + eg_M\right) + W \left(1 + eg_M\right)\right] + \delta^l \left(W \left(1 + eg_M\right) - D\right) - pg_M + peg_M$$

- It's time to start rearranging:

$$pg_M \left(1 - e\right) \geq \left(1 - \delta^l\right) \left[T \left(1 + eg_M\right) + W \left(1 + eg_M\right)\right] + \delta^l W \left(1 + eg_M\right) - \delta^l D - W + D$$

$$pg_M \left(1 - e\right) \geq \left(1 - \delta^l\right) \left[T \left(1 + eg_M\right) + W \left(1 + eg_M\right)\right] + \delta^l W \left(1 + eg_M\right) + \left(1 - \delta^l\right) D - W$$

$$pg_M \left(1 - e\right) \geq T \left(1 + eg_M\right) + W \left(1 + eg_M\right) - \delta^l T \left(1 + eg_M\right) - \delta^l W \left(1 + eg_M\right)$$
$$+ \delta^l W \left(1 + eg_M\right) + \left(1 - \delta^l\right) D - W$$

$$pg_M \left(1 - e\right) \geq T \left(1 + eg_M\right) + W \left(1 + eg_M\right) - \delta^l T \left(1 + eg_M\right) + \left(1 - \delta^l\right) D - W$$

$$pg_M \left(1 - e\right) \geq T \left(1 + eg_M\right) + W eg_M - \delta^l T \left(1 + eg_M\right) + \left(1 - \delta^l\right) D$$

$$pg_M \left(1 - e\right) \geq \left(1 - \delta^l\right) T \left(1 + eg_M\right) + W eg_M + \left(1 - \delta^l\right) D \qquad (4)$$

1.5.1 Is it possible to pick e and g to satisfy low-type IC?

NOW, it's time to explore this equation and see if it's even possible to pick a gift high enough and an e low enough to satisfy the low type's incentive constraint. Continuing on from Equation 4 and simplifying so that $g_M = g$ and $\delta^l = \delta$,

$$pg \left(1 - e\right) \geq \left(T - \delta T\right) \left(1 + eg\right) + W eg + \left(1 - \delta\right) D$$

- Distributing AGAIN

$$pg\,(1-e) \geq T + Teg - \delta T - \delta Teg + Weg + (1-\delta)\,D$$

$$pg\,(1-e) \geq (1-\delta)\,T + (1-\delta)\,D + Teg - \delta Teg + Weg$$

$$pg - pge \geq (1-\delta)\,(T+D) + eg\,(T - \delta T + W) \tag{5}$$

- From here we can solve either for the constraint in terms of g or in terms of e

1.5.2 Solve for g

First solve for g starting from Expression 5:

$$pg - pge \geq (1-\delta)\,(T+D) + eg\,(T - \delta T + W)$$

$$pg - pge - eg\,(T - \delta T + W) \geq (1-\delta)\,(T+D)$$

$$g\,(p - e\,(p + T - \delta T + W)) \geq (1-\delta)\,(T+D) \tag{6}$$

$$g \geq \frac{(1-\delta)\,(T+D)}{p - e\,(p + T - \delta T + W)} \tag{7}$$

- THIS ASSUMES $p - e\,(p + T - \delta T + W)$ IS POSITIVE

  - Note that if we set $e = 0$, this is a very simple constraint on g

$$g \geq \frac{(1-\delta)\,(T+D)}{p} \tag{8}$$

- $T + D$, $1 - \delta$ non-negative by assumption $\Rightarrow$ RHS of Expression 6 non-negative

  - If $(p - pe - e\,(T - \delta T + W))$ is negative, the only way the IC constraint in Expression 6 can hold is if $g < 0$. AGAIN WITH THE NEGATIVE GIFTS.

- To compare to the optimal gift size in the other sections:

  - In Section 3 (Concessions Separating Equilibrium): $p\,(T+D)$
  - Material value with $e = 0$ same as in Section 3: $p\,(T+D)$
  - Material value with $e = 1$:

$$g \geq \frac{p(1-\delta)(T+D)}{(1-\delta) - p(1-\delta)T + (1-p)D} \tag{9}$$

- Under the assumption that payoffs are multiplied by something linear like $1 + \alpha_2 g_1$, there's no reason to choose any intermediate value of e. The mediator, of course, can specify an intermediate value.
- We can simplify to see that the minimum gift when $e = 1$ is larger when

$$(1 - \delta)pT > (1 - p)D$$

BIG NEW NOTE: This condition only true if denominator above is positive

1.5.3 Solve for e

Now solve for e starting from Expression 5:

$$pg - (1 - \delta)(T + D) \geq peg + eg(T - \delta T + W)$$

$$pg - (1 - \delta)(T + D) \geq eg(p + T - \delta T + W) \tag{10}$$

- $g(p + T - \delta T + W)$ is non-negative as long as g is positive: $p, 1 - \delta, T$ and W are all assumed non-negative and $T + D > W$ so that T is strictly positive. It can only possibly be zero if $p = 0$ and $\delta = 1$.

  - For the IC constraint to hold then (with non-negative gifts), we must at least have $pg - (1 - \delta)(T + D)$ non-negative, which requires g to be set according to Expression 8. This only works if e is set to zero; otherwise we need to look back at Expression 7. This is just a weak condition that, if violated, means the IC constraint does not hold.

- If $g(p + T - \delta T + W)$ is positive, we can divide through to get

$$\frac{pg - (1 - \delta)(T + D)}{g(p + T - \delta T + W)} \geq e$$

To get a sense of the magnitudes, I will plug $g = p(T + D)$ from the $e = 0$ CSE case (just because it's easier to work with than the optimal gift when $e = 1$ into Expression 10. First I divide through by g (this is okay if g is assumed to be positive)

$$p - \frac{(1 - \delta)(T + D)}{g} \geq e(p + T - \delta T + W) \tag{11}$$

$$p - \frac{(1 - \delta)(T + D)}{p(T + D)} \geq e(p + T - \delta T + W)$$

$$p - \frac{1 - \delta}{p} \geq e(p + T - \delta T + W)$$

- For the LHS to be positive, we need $p^2 \geq 1 - \delta$, e.g. if $p = 0.5$, we need $\delta \geq .75$

1.5.4 Check with some parameterizations

Parameterization that is in the paper now:

- Let $T = 10$, $W = 8$ and $D = 5$

- Let $\delta_h = .6$ and $\delta_l = .3$. Assume $p = .2$.

- Threshold for high types to separate when concessions do not have future material value is $\delta = \frac{8}{10+5} = .5\overline{3}$. By Theorem 2, separation through concessions is possible.

- Minimum separating concession is $g^1 = \frac{p(1-d_l)(D+T)}{(1-d_l)(1-pT)+(1-p)D} = .\overline{63}$

- Minimum patience level required for separation is $\delta_h^1 = D(1-p)+1-pT+\frac{p(W-(T+D))}{g^1} = .7\overline{9}$. Because the specified δ_h is not involved in any of these calculations, we can see that for any $\delta_h \in [.5\overline{3}, .7\overline{9})$, separating through concessions is possible when concessions have no material value but is not possible when they do have future material value.

- Minimum e from Expression 11:

$$0.2 - \frac{(1 - 0.3)(10 + 5)}{g} \geq e\,(0.2 + 10 - 0.3 \cdot 10 + 8)$$

$$0.2 - \frac{0.7\,(15)}{g} \geq e\,(11.2)$$

$$\frac{0.2}{11.2} - \frac{0.7\,(15)}{11.2g} \geq e$$

$$0.01786 - \frac{9.375}{g} \geq e$$

To keep $e \geq 0$, we need $g \geq 525$ (this is REALLY big compared to W for instance). It's also huge compared to the optimal gifts when $e = 0$ of 3 and when $e = 1$ of $0.\overline{63}$.

Now let's try the same parameterization with $p = 0.9$.

- Minimum e from Expression 11:

$$0.9 - \frac{(1 - 0.3)(10 + 5)}{g} \geq e\,(0.9 + 10 - 0.3 \cdot 10 + 8)$$

$$0.9 - \frac{0.7\,(15)}{g} \geq e\,(11.9)$$

$$\frac{0.9}{11.9} - \frac{0.7\,(15)}{11.9g} \geq e$$

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$$0.07563 - \frac{8.8235}{g} \geq e$$

To keep $e \geq 0$, we need $g \geq 116.66$. This is still nearly an order of magnitude larger than W, and both the optimal gifts when $e = 0$ of 13.5 and when $e = 1$ of $\frac{p(1-d_l)(D+T)}{(1-d_l)(1-pT)+(1-p)D} = \frac{0.9(0.7)(15)}{0.7\cdot(1-0.9)+(.1)5} = \frac{0.63\cdot15}{0.7\cdot0.1+.5} = 16.58$.

"Parameterization 2": goal was to make welfare higher under mediator; $We > W0$ and $e = 1$ is not incentive compatible

$$0.9 - \frac{(1 - 0.3)\,(7 + 5)}{g} \geq e\,(0.9 + 7 - 0.3\cdot7 + 8)$$

$$0.9 - \frac{8.4}{g} \geq e\,(18)$$

$$0.05 - \frac{0.466}{g} \geq e$$

Parameterization 3: $W1 > W0$ and $g1 > g0$. Welfare under mediator is never positive.

- $T = 7$, $W = 8$, $D = 5$, $p = .5$, $\delta_l = .25$, $\delta_h = .6$

Parameterization 4: $W0 > W1 > We$ (mediator). Mediator optimally chooses e=0 anyway

- $T = 7$, $W = 8$, $D = 5$, $p = .3$, $\delta_l = .4$, $\delta_h = .9$

Parameterization 5: $W1 > W0 > We$ (mediator). Mediator optimally chooses $e = 0.03$

- $T = 6$, $W = 8$, $D = 5$, $p = .5$, $\delta_l = .2$, $\delta_h = .9$

Parameterization 6: $We > W1 > W0$, $54 > 7.33 > 7$. Mediator optimally chooses $e = 0.24$. Optimal gift is 50, effective gift is 12, vs. $g1 = .667$ and $g0 = 1$

- $T = 1$, $W = 1$, $D = 1$, $p = .5$, $\delta_l = .5$, $\delta_h = 15/16 = 0.9375$

Welfare comparisons (always for high type)

- When $e = 1$: $\frac{1}{1-\delta_h} \left\{ pT(1 + g^*) + (1 - p)(W - D(1 + g^*)) \right\} - \frac{p(1-d_l)(D+T)}{(1-d_l)(1-pT)+(1-p)D}$

    - Parameterization in the paper: welfare is 7.18; gift is 0.63
    - Parameterization 2: g^1 is negative, creates negative upper bound for δ_h. Separating with concessions equilibrium can't be sustained.

- When $e = 0$: $\frac{1}{1-\delta_h} \left\{ pT + (1 - p)(W - D) \right\} - p(T + D)$

    - Parameterization in the paper: welfare is 8; gift is 3
    - Parameterization 2: welfare is 5.7; gift is 10.8

- When $0 < e < 1$ under a mediator: $\frac{1}{1-\delta_h} \left\{ pT(1 + eg_M) + (1 - p)(W - D) \right\} - g_M$

    - Optimal g_M: Expression 7 creates a menu of (e, g) pairs that satisfy the constraint:

    $$g_M \geq \frac{(1 - \delta)(T + D)}{p - e(p + T - \delta T + W)}$$

    - We're assuming $p - e(p + T - \delta T + W) > 0$ (assuming gifts must be non-negative)
    - Also $pg - (1 - \delta)(T + D) \geq 0$ (e version; this is WEAKER than the condition on e)
    - For the parameterization in the paper, welfare is negative for $e = 0$ and $e = 0.01$, the only values of e that lead to non-negative gifts; gifts range from 52.5 to 218.75
    - Parameterization 2: welfare is 10.083 at optimal gift-efficiency $e = 0.06$, $g = 116.67$, $eg = 7$

        * Compared to $e = 0$, welfare (gross of gift) goes up from 16.5 to 126.75. The entire difference is due to the positive effect of the material value of the gift (the full term is $\frac{1}{1-\delta_h} pTeg$). The effective gift is 7, which is much larger than the 1 in $\frac{1}{1-\delta_h} pT \cdot 1$ that is there in both terms.
        * The increase in the size (cost) of the gift from 10.8 to 116.67 is smaller than the increase in gross-of-gift welfare

1.5.5 Welfare under Mediator vs. e=0

Here I'll try to derive a general result for when the welfare under a mediator would be higher than under $e = 0$ separating with concessions equilibrium.

When $0 < e < 1$ under a mediator, the optimal g_M is the one that chooses the highest welfare from a menu of (e, g) pairs:

$$\frac{1}{1 - \delta_h} \{pT(1 + eg_M) + (1 - p)(W - D)\} - \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)}$$

When $e = 0$, welfare is

$$\frac{1}{1 - \delta_h} \{pT + (1 - p)(W - D)\} - p(T + D)$$

We compare them to find when the welfare under the mediator is larger:

$$\frac{1}{1 - \delta_h} \{pT(1 + eg_M) + (1 - p)(W - D)\} - \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)} \geq$$
$$\frac{1}{1 - \delta_h} \{pT + (1 - p)(W - D)\} - p(T + D)$$

Eliminating common terms:

$$\frac{1}{1 - \delta_h} \{pTeg_M\} - \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)} \geq -p(T + D)$$

Rearranging:

$$\frac{1}{1 - \delta_h} \{pTe - 1\} \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)} \geq -p(T + D)$$

Rearranging:

$$\frac{pTeg_M}{1 - \delta_h} \geq \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)} - p(T + D)$$

This is what I implement in Mathematica Notebook "We greater than W0.nb"

- So far, I've been able to find an instance that satisfies this equation: e = 11/64, δ_h = 15/16, δ_l = 0.5, p = 0.5, T = 1, W = 1, D = 1

- This is "Parameterization 6" above and in R file "e-g calculations.R"

1.5.6 Welfare under Mediator vs. e=1

Here I'll try to derive a general result for when the welfare under a mediator would be higher than under $e = 1$ separating with concessions equilibrium.

When $0 < e < 1$ under a mediator, the optimal g_M is the one that chooses the highest welfare from a menu of (e, g) pairs:

$$\frac{1}{1 - \delta_h} \{pT(1 + eg_M) + (1 - p)(W - D)\} - \frac{(1 - \delta_l)(T + D)}{p - e(p + T - \delta_l T + W)}$$

When $e = 1$, welfare is $\frac{1}{1-\delta_h}\{pT(1+g^1) + (1-p)(W - D(1+g^1))\} - \frac{p(1-d_l)(D+T)}{(1-d_l)(1-pT)+(1-p)D}$

We compare them to find when the welfare under the mediator is larger:

$$\frac{1}{1-\delta_h}\{pT(1+eg_M) + (1-p)(W-D)\} - \frac{(1-\delta_l)(T+D)}{p - e(p+T-\delta_l T + W)} \geq$$
$$\frac{1}{1-\delta_h}\{pT(1+g^1) + (1-p)(W - D(1+g^1))\} - \frac{p(1-d_l)(T+D)}{(1-d_l)(1-pT)+(1-p)D}$$

Eliminating common terms:

$$\frac{1}{1-\delta_h}\{pTeg_M\} - \frac{(1-\delta_l)(T+D)}{p-e(p+T-\delta_l T + W)} \geq$$
$$\frac{1}{1-\delta_h}\{pTg^1 + (1-p)(-Dg^1))\} - \frac{p(1-d_l)(T+D)}{(1-d_l)(1-pT)+(1-p)D}$$

Rearranging:

$$\frac{1}{1-\delta_h}\{pTeg_M\} - g_M \geq \frac{1}{1-\delta_h}\{pTg^1 + (1-p)(-Dg^1))\} - g^1$$

I think it's possible that this condition can never be satisfied (i.e. if e=1 is IC, you'd never want to call in the mediator). But I haven't tried to prove it yet

1.5.7   To do and Notes

- NEXT STEP: check condition for sustainable $e = 1$ but prefer mediator with $e < 1$

    - I've found parameterizations where $e = 1$ is preferred (P5)

    - Try to find a parameterization that shows the reverse (counterexample to my hypothesis that if $e = 1$ is sustainable, you never invoke mediator) or see if I can prove that it never happens

- Can I derive an analytical expression for when the mediator enhances welfare? See Section 1.5.5

    - This is likely (surely?) to be when efficient gifts can't be sustained as a CSE.

    - If you can sustain $e = 1$ with no mediator, would you ever want a mediator?

- When g^1 turns negative, is that exactly when $e = 1$ can't be sustained? If we can get positive g^1, not sure we'd need the mediator? It's not about g^1, but the δ_h^1 condition.

    - When g^1 is negative, the constraint on δ_h^1 (see proof of Thm 3) becomes an upper bound. If it's negative, it means that no δ_h exists to make separating possible.

Notes:

- I think Mediator could instruct them to give $e < 1$ gifts without having to disturb the rest of the theory.

  - **If I go with $e < 1$, I need to come back to verify this.**

2 Develop result: mediator increases potential for peace

June 6, 2017

- i.e. want to show mediator makes (Trust, Trust) an equilibrium over a larger parameter space than without mediator

What do we know?

- Theorem 4: Under some parameters, optimal concessions aren't made when there isn't trust

  - This is still a separating equilibrium, but completely inefficient concessions are given. So welfare is lower because of inefficient concessions.
  - Where there is a possibility of mediation helping to achieve peace where it otherwise would not be attainable (not just improving welfare) is if this reduction in welfare due to the inefficient concessions means that for some parameters it's not worth separating so we don't get peace at all without the mediator.
    * i.e. when welfare for high type from separating falls below that for pooling, separating is no longer an equilibrium (convo with Jean-Guillaume).

- Theorem 5: Mediator eliminates inefficient concessions

  - Here, we're already in that parameter space where there *are* inefficient concessions

Need to start by assuming that $\delta_h <$ threshold where separating with no concessions works.

- **Check to see whether Theorem 4 proof has to change.**

When $e = 1$ and no material value, $g_h^* = p(T + D)$ is the smallest concession for CSE.
When $e = 1$ and concessions have material value, $g' = \frac{p(1-\delta_l)(D+T)}{(1-\delta_l)(1-pT)+(1-p)D}$ is the minimum separating concession [Theorem 3] I claim in Theorem 5 that the minimum separating concession under mediation is $g^* = \frac{g'}{1-p}$.

With $\alpha = 1$,

- SWOC (separating without concessions) when $\delta_h \geq \frac{W}{(1-p)W+p(T+D)}$

- CSE (concessions separating equilibrium) when $\delta_h \geq \frac{W}{T+D}$

With $\alpha \in [0, 1)$ and $e = 1$,

- SWOC doesn't change because there are no concessions

- WTS there are parameters where can't get STC outcome because of $\alpha < 1$

  - $e < 1$ means some of these *can* get to peace
  - but some can't under *any* value of e [assuming e chosen at same time as g]

Remember that decision between equilibria is made not by what *can* be achieved in terms of δ, but by best response, i.e. "Does high type do better by not giving concession?"

- Should check in my numerical examples

Intermediate step for Theorem 4

- Low type IC constraint to solve for equilibrium high type gift

- Already having shown $g_l = 0$, can set $g_h = g$ for simplicity of notation

- Also, since all the discount factors are for the low type, I'll let $\delta_l = \delta$

- For the case where $e = 1$:

IC constraint:
$$X_{FF}^l \geq pX_{FT}^l + (1 - p)X_{FF}^l - g$$

Expanded in basic terms (without efficiency issues)
$$\frac{W - D}{1 - \delta} \geq p \left[T + W + \frac{\delta}{1 - \delta} (W - D) \right] + (1 - p)\frac{W - D}{1 - \delta} - g$$

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Multiply through by $(1 - \delta)$

$$W - D \geq p(1 - \delta)[T + W] + p\delta(W - D) + (1 - p)[W - D] - (1 - \delta)g$$

Now add complexity from Table 2

$$W(1 + (1 - \alpha_1)g_2) - D(1 + (1 - \alpha_2)g_1) \geq$$
$$p(1 - \delta)[T(1 + \alpha_2 g_1) + W(1 + (1 - \alpha_1)g_2)] + p\delta(W(1 + (1 - \alpha_1)g_2) - D(1 + (1 - \alpha_2)g_1))$$
$$+ (1 - p)[W(1 + (1 - \alpha_1)g_2) - D(1 + (1 - \alpha_2)g_1)] - (1 - \delta)g$$

Substitute in $\alpha_1 = 0$ everywhere since this is the IC for a low-type of player 1:

$$W(1 + g_2) - D(1 + (1 - \alpha_2)g_1) \geq$$
$$p(1 - \delta)[T(1 + \alpha_2 g_1) + W(1 + g_2)] + p\delta(W(1 + g_2) - D(1 + (1 - \alpha_2)g_1))$$
$$+ (1 - p)[W(1 + g_2) - D(1 + (1 - \alpha_2)g_1)] - (1 - \delta)g$$

Set $g_1 = 0$ on the LHS and $g_1 = g$ on the RHS:

$$W(1 + g_2) - D \geq$$
$$p(1 - \delta)[T(1 + \alpha_2 g) + W(1 + g_2)] + p\delta(W(1 + g_2) - D(1 + (1 - \alpha_2)g))$$
$$+ (1 - p)[W(1 + g_2) - D(1 + (1 - \alpha_2)g)] - (1 - \delta)g$$

Set $g_2 = 0$ and $\alpha_2 = 0$ wherever there is a $(1 - p)$ and $g_2 = g$ and $\alpha_2 = 1$ wherever there is a p:

$$pW(1 + g) + (1 - p)W - D \geq$$
$$p(1 - \delta)[T(1 + g) + W(1 + g)] + p\delta(W(1 + g) - D)$$
$$+ (1 - p)[W - D(1 + g)] - (1 - \delta)g$$

Expand

$$pW + pWg + W - pW - D \geq$$
$$(p - p\delta)[T + Tg + W + Wg] + p\delta(W + Wg - D)$$
$$+ (1 - p)[W - D - Dg] - (1 - \delta)g$$

Cancel some like terms and move $(1 - \delta)g$ to LHS

$$(1 - \delta)g + pWg + W - D \geq$$
$$p[T + Tg + W + Wg] - p\delta[T + Tg] - p\delta D$$
$$+ (1 - p)[W - D - Dg]$$

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Do some more canceling and expanding

$$(1 - \delta) g + W - D \geq$$

$$p\,[T + Tg + W] - p\delta T - p\delta Tg - p\delta D$$
$$+ W - D - Dg - pW + pD + pDg$$

$$(1 - \delta) g \geq pT + pTg + pW - p\delta T - p\delta Tg - p\delta D - Dg - pW + pD + pDg$$

$$(1 - \delta) g \geq pT + pTg - p\delta T - p\delta Tg - p\delta D - Dg + pD + pDg$$

Now just rearrange to get all the g terms on the left

$$(1 - \delta) g - pTg + p\delta Tg + Dg - pDg \geq pT - p\delta T - p\delta D + pD$$

$$[(1 - \delta) - p(1 - \delta)T + (1 - p)D]\, g \geq p(1 - \delta)(T + D)$$

$$g \geq \frac{p(1 - \delta)(T + D)}{(1 - \delta) - p(1 - \delta)T + (1 - p)D}$$

In comparing to the minimum separating gift when $e = 0$, which is

$$g^* \geq \frac{p(1 - \delta)(T + D)}{(1 - \delta)} = p(T + D)$$

We can simplify to see that the minimum gift when $e = 1$ is larger when

$$(1 - \delta)pT > (1 - p)D$$

Next we get the threshold for δ_h that is necessary for a concessions separating eqm to exist:

- Use the high-type IC constraint:

$$pX^h_{TT} + (1 - p)X^h_{FF} - g_h \geq X^h_{FF}$$

- We'll need to expand for material effects but then solve for both $e = 0$ and $e = 1$

- For $e = 1$:

$$\frac{p}{1 - \delta_h}T(1 + g) + \frac{1 - p}{1 - \delta_h}(W - D(1 + g)) - g_h \geq \frac{1}{1 - \delta_h}(W - D)$$

$$pT + pTg + (1 - p)(W - D - Dg) - g_h(1 - \delta_h) \geq W - D$$

$$pT + pTg + W - D - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq W - D$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

$$pT + pTg - Dg - pW + pD + pDg - g_h + g_h\delta_h \geq 0$$

Note that all the g's should be g_h's in the five lines above:

$$g_h\delta_h \geq -pT - pTg_h + Dg_h + pW - pD - pDg_h + g_h$$

$$\delta_h \geq \frac{p(W - D - T)}{g_h} + (1 - p)D + 1 - pT$$

- For $e = 0$, you're back in the original case with no material value, i.e. enforceable for same δ_h as no material value case

$$\frac{p}{1 - \delta_h}T + \frac{1 - p}{1 - \delta_h}(W - D) - g_0 \geq \frac{1}{1 - \delta_h}(W - D)$$

$$pT + (1 - p)(W - D) - (1 - \delta_h)g_0 \geq (W - D)$$

$$pT - p(W - D) + \delta_h g_0 \geq g_0$$

$$\delta_h g_0 \geq g_0 + p(W - D - T)$$

$$\delta_h \geq 1 + \frac{p(W - D - T)}{g_0}$$

Optimal separating gift in this setting is $g_0 = p(D + T)$. Substituting this in, we have:

$$\delta_h \geq 1 + \frac{p(W - D - T)}{p(D + T)} = 1 + \frac{pW - p(D - T)}{p(D + T)} = 1 + \frac{pW}{p(D + T)} - 1 = \frac{W}{D + T}$$

  - But check: when $e = 0$, also wipe out immediate monetary value? YES, in terms of benefit, but NOT cost, which is the only part that doesn't cancel out of IC constraints (same for high type and low type; benefit is on both sides; cost is on only one side)

    * Not from utility standpoint, but from IC standpoint because direct benefit of gift cancels out

- But high type utility goes down

  - Interestingly, if $e = 0$, if high types would have separated without material value, they will here with material value that is destroyed (they don't do better in pooling). We see this by calculating the high type utility for the case where $e = 0$, which is

$$\frac{1}{1 - \delta_h}\left[pT + (1 - p)(W - D)\right] - g_0$$

Substituting in the value of the g_0 (the optimal separating gift with $e = 0$, which I have as $p(T + D)$ but I should check again to make sure it's correct for this situation) and the threshold value to separate when $e = 0$ (i.e. $\delta = \frac{W}{T+D}$), we have

$$\frac{T+D}{T+D-W}\left[pT + (1-p)(W-D)\right] - p(T+D)$$

$$\frac{T+D}{T+D-W}\left[p(T+D-W) + (W-D)\right] - p(T+D)$$

$$p(T+D) + \frac{T+D}{T+D-W}\left[W-D\right] - p(T+D)$$

$$\frac{T+D}{T+D-W}\left[W-D\right]$$

This is exactly the payoff to the pooling equilibrium (that is, both types pool on fight with no gift giving) when $\delta = \frac{W}{T+D}$, since $U_{pooling} = \frac{W-D}{1-\delta}$.

So $U_{g_0} = U_{pooling}$ at the cutoff for separating, and U_{g_0} is larger for any discount factor higher than the cutoff.

    * This can be seen, for instance, by decomposing $U_{pooling}$ along the same lines as U_{g_0} above into components for the cooperation vs. punishment and cost of the gift (even though that's not what's going on in pooling). The whole thing increases, including the cost of the gift portion for the pooling when δ increases. But the cost of the gift does not scale up in U_{g_0}.

We can also do this more generally. Take

$$U_{g_0} = \frac{1}{1-\delta}(W-D)$$

$$U_{pooling} = \frac{1}{1-\delta_h}\left[pT + (1-p)(W-D)\right] - p(T+D)$$

$$= pT + pD + \frac{\delta}{1-\delta}(pT+pD) + \frac{1}{1-\delta}(W-D-pW) - pT - pD$$

$$= \frac{\delta}{1-\delta}(pT+pD) + \frac{1}{1-\delta}(W-pW-D)$$

Now compare the two, $U_{g_0} \lessgtr U_{pooling}$:

$$\frac{\delta}{1-\delta}(pT+pD) + \frac{1}{1-\delta}(W-pW-D) \lessgtr \frac{1}{1-\delta}(W-D)$$

$$\frac{\delta}{1-\delta}(pT+pD) - \frac{1}{1-\delta}(pW) \lessgtr 0$$

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$$\delta\left(pT + pD\right) \lessgtr pW$$

$$\delta \lessgtr \frac{pW}{pT + pD} = \frac{W}{T + D}$$

So that the utility from U_{g_0} is greater exactly when the discount factor is above the threshold for concessions separating equilibrium to be possible without material value.

How does utility compare under $e = 1$ and $e = 0$? Compare the two, $U_{g_1} \lessgtr U_{g_0}$:

$$\frac{p}{1 - \delta_h} T(1 + g) + \frac{1 - p}{1 - \delta_h}(W - D(1 + g)) - g_h \lessgtr \frac{1}{1 - \delta_h}\left[pT + (1 - p)(W - D)\right] - g_0$$

$$pT(1 + g_1) + (1 - p)\left(W - D(1 + g_1)\right) - (1 - \delta_h)g_1 \lessgtr pT + (1 - p)(W - D) - (1 - \delta_h)g_0$$

$$pTg_1 + (1 - p)\left(W - D - Dg_1\right) - (1 - \delta_h)g_1 \lessgtr (1 - p)(W - D) - (1 - \delta_h)g_0$$

$$pTg_1 - (1 - p)Dg_1 - (1 - \delta_h)g_1 \lessgtr - (1 - \delta_h)g_0$$

$$pTg_1 - (1 - p)Dg_1 \lessgtr (1 - \delta_h)g_1 - (1 - \delta_h)g_0$$

$$pT - (1 - p)D \lessgtr \frac{(1 - \delta_h)(g_1 - g_0)}{g_1} \tag{12}$$

We know (from above) that the minimum gift when $e = 1$ is larger than the minimum gift when $e = 0$ when

$$(1 - \delta)pT > (1 - p)D$$

Let's case this out.

1. Assume $g_0 = g_1$; then $(1 - \delta)pT = (1 - p)D$. The right hand side of 12 is 0 and the left hand side is

$$pT - (1 - \delta)pT = pT - pT + \delta pT = \delta pT$$

And utility under $e = 1$ is higher.

2. When $g_0 > g_1$; then $(1 - \delta)pT < (1 - p)D$. The right hand side of 12 is negative and the left hand side is larger (more positive) than in case 1, so utility under $e = 1$ is also higher in this case.

3. When $g_1 > g_0$; then $(1 - \delta)pT > (1 - p)D$. The right hand side of 12 becomes positive and the left hand side becomes smaller (less positive, and possibly negative) than in case 1, so utility under $e = 1$ is higher when g_1 is only a little larger than g_0, but eventually as g_1 gets much larger than g_0, utility under $e = 0$ becomes larger than utility under $e = 1$.

3 To-do list after Venice CesIfo 06/15/17

- Jim Fearon: maybe get rid of repeated game and just parameterize a one-shot game (doens't work because concessions required for separation are too large; require future to recoup cost)

  - he has a paper about concessions being used against you; he also gave me the reference for another one but I've forgotten it

  Notes from CPEG, Oct. 25, 2019 presentation:

  - Arnaud
    * δ are not exogenous (look at Brexit)
    * Proposes a one-shot game to replace the repeated game. Simple bayesian game with difference preferences depening on type.
  - Raphael Godefroy
    * Can δ_i be a function of g? That is, likelihood of defection decreases as concession gets better

4 Review 1 from EJPE (Email from Toke Aidt, August 19, 2018)

While clearly these are interesting issues, I have difficulties reading this paper. Everything is presented in a very unclear manner. Game-theoretical concepts are invoked, but typically not correctly. For instance, an equilibrium is a strategy profile (one strategy for each type of each player). Hence it does not make sense to say, as the authors do in Theorem 1 on page 10, that something is an equilibrium strategy for one type without also specifying what others do. As another example, on page 18, the authors say that "The revelation principle is used to attain truthful self-identification." But in reality, the revelation principle is just an argument for why it is without loss of generality to restrict attention to direct mechanisms. The truthfulness of messages is guaranteed by requiring the mechanism to be incentive compatible, something the authors do not address.

Another thing. It is not considered a good idea to introduce new assumptions, which have not been mentioned before, in the proof of a proposition, as happens in the alleged proof of Theorem 4 on page 27.