Notes on 2nd round of revisions for JIE of SOP_Repeated

1 Result 3

There is no interior solution in T under Referee 2's suggestion that gov't welfare in future periods be evaluated at the e that is realized at that time. I need to be able to explain Results 2 and 3 better as I defend the modeling choice.

- As T increases, both what the lobby has to pay (this also includes increase to τ^b) and the lobby's profit from the punishment phase increase.
- The increase in e as T increases comes from the interaction of a direct effect and an indirect effect.
  - When T increases, gov't feels punishment for longer so gov't is less willing to break the agreement. That means e has to be increased to make the gov't indifferent again (note that when e increases, the gov't becomes more willing to break the agreement because it places more weight on the lobby's profits).
- In my formulation, the direct effect decreases to 0 as $T \uparrow$; the indirect effect (denominator) increases to $\frac{\delta}{1-\delta}$. Thus $\frac{\partial \bar{e}}{\partial T}$ decreases to 0 as $T \to \infty$.
  - The lobby's benefit also decreases to 0 as $T \to \infty$, but at the same rate as the numerator of $\frac{\delta}{1-\delta}$ so $\frac{\delta}{1-\delta}$ goes to zero faster than the lobby's benefit.
- In Referee 2's formulation, the essential difference is that the denominator of $\frac{\delta}{1-\delta}$ is not a function of T because the current government's e does not show up in the expression for the punishment period (the current government evaluates the punishment period through the eyes of the government who will be in power at that time).
  - The terms involving T cancel out of both the cost and benefit side. Thus the constraint varies linearly in T in this formulation. The change is either a net positive (cost to lobby $\uparrow\uparrow$ while benefit only $\uparrow$) so want $T \to \infty$; or the change is a net negative (cost to lobby $\uparrow$ while benefit $\uparrow\uparrow$) so want $T \to 0$.

Useful equations

$$\frac{\delta_{\text{ML}} - \delta_{\text{ML}}^{T+1}}{1-\delta_{\text{ML}}} \left[W_{\text{ML}}(\gamma(e), \boldsymbol{\tau^a}) - W_{\text{ML}}(\gamma(e), \boldsymbol{\tau^{tw}}) \right] \geq$$
$$W_{\text{ML}}(\gamma(e), \tau^b(e), \tau^{*a}) - W_{\text{ML}}(\gamma(e), \boldsymbol{\tau^a})$$

$$\left(1 - \frac{\mathrm{d}\pi}{\mathrm{d}\bar{e}}\right) \frac{-\frac{\delta_{\mathrm{ML}}^{T+1} \ln \delta_{\mathrm{ML}}}{1-\delta_{\mathrm{ML}}} \left[W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{a}}) - W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{tw}})\right]}{\frac{\partial \gamma}{\partial e}\left[\pi(\tau^{b}(\bar{e})) - \pi(\tau^{a})\right] + \frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1-\delta_{\mathrm{ML}}} \frac{\partial \gamma}{\partial e}\left[\pi(\tau^{tw}) - \pi(\tau^{a})\right]} + \frac{\delta_{\mathrm{L}}^{T+1} \ln \delta_{\mathrm{L}}}{1-\delta_{\mathrm{L}}}\left[\pi(\tau^{tw}) - e_{tw} - \pi(\tau^{a}) + e_{a}\right] \quad (1)$$

$$\frac{\partial \bar{e}}{\partial T} = \frac{-\frac{\delta_{\mathrm{ML}}^{T+1} \ln \delta_{\mathrm{ML}}}{1-\delta_{\mathrm{ML}}} \left[W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{a}}) - W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{tw}})\right]}{\frac{\partial \gamma}{\partial e}\left[\pi(\tau^{b}(\bar{e})) - \pi(\tau^{a})\right] + \frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1-\delta_{\mathrm{ML}}} \frac{\partial \gamma}{\partial e}\left[\pi(\tau^{tw}) - \pi(\tau^{a})\right]}$$

$$\frac{\partial \Omega}{\partial e}\frac{\partial \bar{e}}{\partial T} + \frac{\partial \Omega}{\partial T} = 0$$

$$-\left[\frac{\partial \gamma}{\partial e}\left[\pi(\tau^{b}(\bar{e})) - \pi(\tau^{a})\right] + \frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1-\delta_{\mathrm{ML}}} \frac{\partial \gamma}{\partial e}\left[\pi(\tau^{tw}) - \pi(\tau^{a})\right]\right] \frac{\partial \bar{e}}{\partial T} + \left[-\frac{\delta_{\mathrm{ML}}^{T+1} \ln \delta_{\mathrm{ML}}}{1-\delta_{\mathrm{ML}}}\left[W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{a}}) - W_{\mathrm{ML}}(\gamma(\bar{e}), \boldsymbol{\tau^{tw}})\right]\right] = 0 \quad (2)$$

- First term: When e changes, it increases the weight on the lobby's profits relative to everything else. This loosens the constraint.
  - It gets loosened more the larger is T because the lobby's profits get bigger as T increases.
- Second term: the per-period punishment is felt for more periods, but the effect is decreasing b/c $\delta < 1$
- When T gets very large, the leg constraint is not being loosened much by an extra period of punishment [DIRECT effect]
  - BUT when e increases, you care about the benefit to the lobby in every period, not just the incremental one [INDIRECT effect].
  - So the tightening of the legislative constraint through e / the indirect effect is larger than the loosening through the direct effect as T grows large and so $\frac{\partial \bar{e}}{\partial T}$ must shrink.
  - At some T, the increase in e can no longer outweigh the increase in the benefit to the lobby

2 Dixit, Grossman and Helpman analogy

DGH97 paper:

- Has to be a truthful contribution schedule
  - Proposition 3: $G(a^0, e^T(a^0, u^0)) = \max_a G(a, 0)$
  - $e^T(a^0, u^0)$ is essentially $\phi(a^0, u^0)$, which is defined implicitly in eqn3 (p. 760) as

$$U\left[a, \phi(a^0, u^0)\right] = u^0$$

    * Careful: truthful contribution schedule is not a best response function. But the lobby will still have to be best responding in equilibrium.
    * Truthful contribution schedule is a device for solving equilibrium in their model. Doesn't mean I have to follow it (I think; I hope)
  - if $G = W + g(e)$ and $e = 0$ and $g(0) = 0$, then $a^* = \tau^{\text{opt}}$
  - Then rewrite as

$$G(\tau^0, e^T(\tau^0, u^0)) = \max_a G(\tau^{\text{opt}}, 0)$$

    * Right hand side provides a number
    * Ignoring arguments of e^T function, LHS traces out (τ, e) pairs that satisfy the equation given $g(e)$.
    * For lobby to be best responding, it MUST pick the pair that maximizes $\pi(\tau) - e$. *This* concern must be what sets u^0.
- Corollary to Prop 1 / Prop 3: Gov't gets utility equal to outside option. Is this true when there is just one lobby?
- Combining the two previous facts (if true in my case), then it must be that gov't getting outside option will set u^0 (eqm utility) and anchor contribution schedule (just have to be careful of zero contributions)

What editor proposes:

- Lobby offers contribution schedule (can be very simple: just one (e, τ) pair, $e = 0$ for everything else)
- Government maximizes $W + g(e)$
  - Note that $CS_X + \gamma(e) \cdot PS_X + CS_Y + PS_Y + TR = W + (\gamma(e) - 1)\, PS_X$

- Sectioning from my paper:

  3.1 Same (execs)

  3.2 Trade war

  - Government chooses unilateral τ as $\frac{\partial W}{\partial \tau} + \frac{\partial g(e)}{\partial \tau} = 0$
    - How to think about lobby's contribution schedule?
    - DGH Proposition 1: principal (lobby) has to provide at least agent's (gov'ts) outside option; as long as this constraint is satisfied, lobby can propose τ and payment that maximizes his own utility
    - DGH Proposition 3 (p. 760-61): simplifies so we don't need to look for contribution functions, only eqm values

Comparisons

- Note that in GH94, claim is that IN EQUILIBRIUM, government behaves as if it maximizes a weighted sum of the groups' utilities ($1 + a$ for those represented by lobbies; a for those not represented; pg. 10 of PDF). But this is in equilibrium: there's a lot going on out of equilibrium that leads us there!
  - I haven't proved this to myself, but that must also depend on lobbies having all the bargaining power
- In DGH, $G(\tau^0, e^0) = \max_\tau(\tau, 0)$
  - This is <u>not</u> generally true in my model
  - I think the slippage is in bargaining power. DGH paradigm assumes lobby essentially make TIOLI offer.
  - $\gamma(e)$ formulation in essence distributes bargaining power more generally
    - Is it okay to characterize it this way?
    - If so, does bargaining power vary with effort? Seems mixed up with diminishing returns to effort.
  - $W + \Phi(e)$ of Limao and Tovar gives decreasing returns; then explicitly models bargaining power through Nash bargain instead of menu auction (as far as I can tell–I can't find it clearly specified).
- In DGH, essentially lobby chooses its favorite (τ, e) pair from among all the possible ones that make the Gov't indifferent.
  - Is it possible that this is equivalent to $\frac{\partial \pi}{\partial e} = 1$?

- seems like the pair that satisfies that equation might not be available
- When I calculate the various (τ, e) pairs to 'compare' government welfare levels, this comes from the government welfare function: each e induces an optimal τ and then I get an optimal value function
  * Lobby doesn't care about these varying welfare levels. Chooses (τ, e) pair that maximizes $\pi - e$

Examples

- Use numeric example from BS2005 in R (DGH.r)
- Isomorphism I've already calculated: if $G = W + e$ then $\gamma(e) = 1 + \frac{e}{\pi_x}$
- Lobby's optimization:

$$W(\tau, e(\tau)) = W(0, 0)$$
$$W(\tau) + e = W(0)$$
$$e = W(0) - W(\tau)$$

  - This forms a contribution schedule
  - Lobby chooses the pair (τ, e) that maximizes $\pi(\tau) - e$
- Make sure these are unilateral changes in τ in the program
- Then compare to my old way: $\frac{\partial \pi}{\partial \tau} \frac{\partial \tau}{\partial \gamma} \frac{\partial \gamma}{\partial e} = 1$ (e changes with τ directly depending on shape of γ, then government maximizes w.r.t. τ
  - NEXT: go into R program and sort it out