

## Miscellaneous notes on JIE R&R of SOP\_Repeated

Legislative constraint as a function of  $e$

- I thought it would be positive at  $e = 0$  and turn negative as  $e$  increases
- What does it mean that for some values it's negative at 0, becomes positive, and then goes negative again?
  - For sure I have to be careful in numerical examples

# Numerical examples

$\delta_L = \delta_{ML} = .95$					
	E=.35	E=.4	E=.41	E=.42	E=.45
$\tau^{tw}$				.074	.0654
$e^{tw}$					.00123
T = 2		.07500			.057407
T = 3		.074716	.070243	.066284	<b>.0570802</b>
T = 4		<b>.074708</b>	<b>.070233</b>	<b>.066275</b>	.0570806
T = 5		.074795	.07033	.06638	.057185
T = 6	.1080	.07492			
T = 7	.1081				.057
T = 8	.10814				

I have another sheet of notes that conflicts with the first column. It just says “ $\delta = .95$ ”:

	E=.35
$\tau^{tw}$	.1213
$e^{tw}$	.006003
T = 3	.1023044
T = 4	<b>.1022411</b>
T = 5	.1022427
T = 6	.10227
T = 7	.102305

This one just says “ $\delta = .99$ ”:

	E=.4	.39
$\tau^{tw}$		
$e^{tw}$		
T = 2	.068510	.06391
T = 3	<b>.068261</b>	<b>.06374</b>
T = 4	.068289	.06378
T = 5	.06841	.06391

This has the note, “This at least works in the direction I thought it would” with “ $\delta_L = .94$ ,  $\delta_{ML} = .95$ ”:

	E=.4
$\tau^{tw}$	
$e^{tw}$	
T = 4	.07464
T = 5	<b>.07421</b>
T = 6	.07481
T = 7	.07492

(“Really want to know if reducing  $\delta_L$  — making future term less important — will give me the  $\sigma$  result I’ve been after; really, no result at all; depends on other parameters.)

Some summaries

- $E = .4$ ,  $\delta_L = .99$ ,  $\delta_{ML} = .95$ ,  $e_{tw} = .00232$ ,  $\tau^{tw} = .08185$ . Optimal  $\tau^a = .07494$  at  $T = 3$ .
- $E = .5$ , assume I kept  $\delta_L = .99$ ,  $\delta_{ML} = .95$ . Optimal  $\tau^a = .04864$  at  $T = 3$ .
- $E = .4$ ,  $\delta_L = \delta_{ML} = .99$ ,  $e_{tw} = .00232$ ,  $\tau^{tw} = .08185$ . Optimal  $\tau^a = .07470$  at  $T = 3$ .
- $E = .4$ ,  $\delta_L = .99$ ,  $\delta_{ML} = .5$ ,  $e_{tw} = .00232$ ,  $\tau^{tw} = .08185$ . Optimal  $\tau^a = .07802$  at  $T = 2$ .
- $E = .4$ ,  $\delta_L = .99$ ,  $\delta_{ML} = .75$ ,  $e_{tw} = .00232$ ,  $\tau^{tw} = .08185$ . Optimal  $\tau^a = .07629$  at  $T = 3$ .

Trying to understand what is really going on with constraint in terms of  $T$

- Want to get good intuition for why  $T$  can go up as  $\sigma \downarrow$ .
  - Not obvious that it always does; I think it's possible that the direction of  $T$  in response to  $\sigma$  is indeterminant
- I think I need to show effect of  $\sigma$  on  $\bar{e}$  first (I have informally that  $\sigma \uparrow \Rightarrow \bar{e} \downarrow$ )
  - Then, impact of  $\sigma$  on  $\tau^a$ . Next look to see if net profits at  $\tau^a$  increase more than those at  $\tau^{tw}$ , then lobby's future incentives are muted

**Result 1.**  $\frac{d\bar{e}}{d\sigma} > 0$

Proof: Corollary 4 shows that  $\frac{d\bar{e}}{d\gamma} < 0$ . All that is left is to show that  $\frac{d\gamma}{d\sigma} < 0$ .

- The derivative of  $\gamma = 1 + \frac{1}{\sigma}e^\sigma$  w.r.t.  $\sigma$  is

$$\frac{1}{\sigma} \ln \sigma e^\sigma + e^\sigma \left( -\frac{1}{\sigma^2} \right)$$

Both terms are negative given  $\sigma \in (0, 1)$  and  $e \geq 0$ . QED.

Now for the result on  $\tau^a$ . Differentiating the lobby's condition with respect to  $\sigma$ , we have

$$\frac{\partial \Pi}{\partial \tau^a} \frac{d\tau^a}{d\gamma} \frac{d\gamma}{d\sigma} + \frac{\partial \Pi}{\partial \bar{e}} \frac{d\bar{e}}{d\gamma} \frac{d\gamma}{d\sigma} + \frac{\partial \Pi}{\partial \gamma} \frac{d\gamma}{d\sigma} = 0$$

$$\frac{d\tau^a}{d\gamma} \frac{d\gamma}{d\sigma} = \left[ -\frac{\frac{\partial \Pi}{\partial \bar{e}} \frac{d\bar{e}}{d\gamma} + \frac{\partial \Pi}{\partial \gamma}}{\frac{\partial \Pi}{\partial \tau^a}} \right] \frac{d\gamma}{d\sigma} \quad (1)$$

We know from Corollary 5 that  $\frac{d\tau^a}{d\gamma}$  is positive, and we've just shown that  $\frac{d\gamma}{d\sigma}$  is negative. Thus  $\frac{d\tau^a}{d\sigma} < 0$ .

Write constraint:

$$\bar{e}(\tau^a) - \pi(\tau^b(\bar{e}(\tau^a))) + \pi(\tau^a) - e_a - \frac{\delta_L + \delta_L^{T+1}}{1 - \delta_L} [\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) + e_a] = 0$$

For now, assume this has an interior solution so calculus works.

First, what does  $T$  do to  $\bar{e}(\tau^a)$ ?

By the Implicit Function Theorem:

$$\frac{d\bar{e}}{dT} = -\frac{\frac{\partial \Omega}{\partial T}}{\frac{\partial \Omega}{\partial \bar{e}}} = \frac{-\frac{\delta_{ML}^{T+1} \ln \delta_{ML}}{1-\delta_{ML}} [W_{ML}(\gamma(\bar{e}), \tau^a) - W_{ML}(\gamma(\bar{e}), \tau^{tw})]}{\frac{\delta_{ML} - \delta_{ML}^{T+1}}{1-\delta_{ML}} \frac{\partial \gamma}{\partial \bar{e}} [\pi(\tau^a) - \pi(\tau^{tw})] - \frac{\partial \gamma}{\partial \bar{e}} [\pi(\tau^b(\bar{e})) - \pi(\tau^a)]} > 0 \quad (2)$$

So if  $T \uparrow$  then  $\bar{e} \uparrow$ .

Now, want to know about effect of  $T$  on  $\tau^a$ .

$$\frac{\partial \Pi}{\partial \tau^a} \frac{d\tau^a}{dT} + \frac{\partial \Pi}{\partial \bar{e}} \frac{d\bar{e}}{dT} + \frac{\partial \Pi}{\partial T} = 0$$

Because  $\frac{\partial \Pi}{\partial T} = \frac{\ln \delta_L \delta_L^{T+1}}{1-\delta_L} [\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) + e_a]$ , we are looking for

$$\frac{d\tau^a}{dT} = -\frac{\frac{\partial \Pi}{\partial \bar{e}} \frac{d\bar{e}}{dT} + \frac{\partial \Pi}{\partial T}}{\frac{\partial \Pi}{\partial \tau^a}} = \frac{-\left(1 - \frac{d\pi}{d\bar{e}}\right) \cdot \frac{d\bar{e}}{dT} - \frac{\ln \delta_L \delta_L^{T+1}}{1-\delta_L} [\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) + e_a]}{\left(1 + \frac{\delta_L - \delta_L^{T+1}}{1-\delta_L}\right) \left[\frac{\partial \pi(\tau^a)}{\partial \tau^a} - \frac{\partial e_a}{\partial \tau^a}\right]} \quad (3)$$

The proof of Corollary ?? shows that  $(1 - \frac{d\pi}{d\bar{e}})$  is positive, and the above result shows that  $\frac{d\bar{e}}{dT}$  is positive. The second term is positive since net profits are maximized at  $e_{tw}$  and  $\delta_L < 1$  so that its log is negative. With the leading negative signs, the numerator has both a negative and a positive part. This is not changed by the denominator, as the arguments given in the proof of Corollary ?? show that the denominator is positive.

As  $\sigma \downarrow$ ,  $\bar{e} \downarrow$  for a given  $\tau^a$ .

- Remember that change in  $\bar{e}$  is really a change in the  $(\tau^a, \bar{e}(\tau^a))$  schedule that derives from legislature's constraint
- So  $\tau^a$  has to be raised to satisfy lobby's constraint
  - Net profits are greatest at  $\tau^{tw}$ , so relative gap between net profits at  $\tau^a$  and  $\tau^{tw}$  (future) closes faster than that between break profits and trade agreement profits (present)
- How much  $\tau^a$  adjusts depends on magnitude of  $\frac{\delta_L + \delta_L^{T+1}}{1-\delta_L}$

- $\pi(\tau^b(\bar{e}(\tau^a))) - \bar{e}(\tau^a)$  is negative, gets less negative when  $\bar{e}$  is reduced.
- $\pi(\tau^a) - e_a$  is positive, becomes larger as  $\tau^a$  rises

$$0 \geq - [\bar{e}(\tau^a) - \pi(\tau^b(\bar{e}(\tau^a))) + \pi(\tau^a) - e_a] + \frac{\delta_L + \delta_L^{T+1}}{1 - \delta_L} [\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) + e_a]$$

Where present part of constraint is on left and future part is on right. Remember present part must be negative.

Let  $\gamma(e) = 1 + \frac{1}{1-\theta}e^{1-\theta}$ . Or  $\gamma(e) = 1 + \frac{1}{\sigma}e^\sigma$ .

- $\frac{\partial \gamma}{\partial e} = e^{\sigma-1} > 0 \forall \sigma$
- $\frac{\partial^2 \gamma}{\partial \sigma \partial e} = \ln e \cdot e^{\sigma-1} < 0$  for  $e < 1$

i.e. as  $\sigma \downarrow$ ,  $\frac{\partial \gamma}{\partial e} \uparrow$ .

Can I show that the optimal  $T$  can go either way when  $\sigma$  changes? That is, a counterexample to my quasi-result?

- My result says that as lobby gets stronger ( $\frac{\partial \gamma}{\partial e} \uparrow$ , so  $\sigma \downarrow$ ),  $T$  should have to decrease.
- If optimal  $T$  increases when  $\sigma$  decreases (i.e. if  $T$  is decreasing in  $\sigma$ ), this is a counterexample.
  - I think it's possible that both cases can happen depending on other parameters, like  $\delta$ .

Look at legislature's constraint:

$$\frac{\delta_{ML} - \delta_{ML}^{T+1}}{1 - \delta_{ML}} [W_{ML}(\gamma(e_b), \tau^a) - W_{ML}(\gamma(e_b), \tau^{tw})] \geq W_{ML}(\gamma(e_b), \tau^b(e_b), \tau^{*a}) - W_{ML}(\gamma(e_b), \tau^a)$$

If  $T$  is too small, future gap can be smaller than current-period gap. So if  $T$  gets too short, can't enforce on legislature.

Note that changing  $\sigma$  changes  $\tau^{tw}$

- $e_{tw}$  is solution to  $\frac{\partial \pi}{\partial \tau} \frac{\partial \tau}{\partial \gamma} \frac{\partial \gamma}{\partial e} = 1$ ; or  $\frac{\partial \pi}{\partial \tau} \frac{\partial \tau}{\partial \gamma} = \frac{1}{\frac{\partial \gamma}{\partial e}}$
- When  $\frac{\partial \gamma}{\partial e} \uparrow$ , RHS  $\downarrow$ , so LHS must go down.