Miscellaneous notes on JIE R&R of SOP_Repeated

Legislative constraint as a function of e

- I thought it would be positive at $e = 0$ and turn negative as e increases

- What does it mean that for some values it's negative at 0, becomes positive, and then goes negative again?

  - For sure I have to be careful in numerical examples

Numerical examples

$\delta_L = \delta_{ML} = .95$

	E=.35	E=.4	E=.41	E=.42	E=.45
τ^{tw}				.074	.0654
e^{tw}					.00123
T = 2		.07500			.057407
T = 3		.074716	.070243	.066284	**.0570802**
T = 4		**.074708**	**.070233**	**.066275**	.0570806
T = 5		.074795	.07033	.06638	.057185
T = 6	.1080	.07492			
T = 7	.1081				.057
T = 8	.10814				

I have another sheet of notes that conflicts with the first column. It just says "$\delta = .95$":

	E=.35
τ^{tw}	.1213
e^{tw}	.006003
T = 3	.1023044
T = 4	**.1022411**
T = 5	.1022427
T = 6	.10227
T = 7	.102305

This one just says "$\delta = .99$":

	E=.4	.39
τ^{tw}		
e^{tw}		
T = 2	.068510	.06391
T = 3	**.068261**	**.06374**
T = 4	.068289	.06378
T = 5	.06841	.06391

This has the note, "This at least works in the direction I thought it would" with "$\delta_L = .94$, $\delta_{ML} = .95$":

$$\tau^{tw}$$

$$\begin{array}{ccc}
 & & E=.4 \\
 & & \tau^{tw} \\
 & & e^{tw} \\
\end{array}$$

	e^{tw}
T = 4	.07464
T = 5	**.07421**
T = 6	.07481
T = 7	.07492

("Really want to know if reducing δ_L — making future term less important — will give me the σ result I've been after; really, no result at all; depends on other parameters.)

Some summaries

- $E = .4$, $\delta_L = .99$, $\delta_{ML} = .95$, $e_{tw} = .00232$, $\tau^{tw} = .08185$. Optimal $\tau^a = .07494$ at $T = 3$.

- $E = .5$, assume I kept $\delta_L = .99$, $\delta_{ML} = .95$. Optimal $\tau^a = .04864$ at $T = 3$.

- $E = .4$, $\delta_L = \delta_{ML} = .99$, $e_{tw} = .00232$, $\tau^{tw} = .08185$. Optimal $\tau^a = .07470$ at $T = 3$.

- $E = .4$, $\delta_L = .99$, $\delta_{ML} = .5$, $e_{tw} = .00232$, $\tau^{tw} = .08185$. Optimal $\tau^a = .07802$ at $T = 2$.

- $E = .4$, $\delta_L = .99$, $\delta_{ML} = .75$, $e_{tw} = .00232$, $\tau^{tw} = .08185$. Optimal $\tau^a = .07629$ at $T = 3$.

Trying to understand what is really going on with constraint in terms of T

Write constraint:

$$\bar{e}(\tau^a) - \pi(\tau^b(\bar{e}(\tau^a))) + \pi(\tau^a) - e_a - \frac{\delta_{\mathrm{L}} + \delta_{\mathrm{L}}^{T+1}}{1 - \delta_{\mathrm{L}}}\left[\pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) + e_a\right] = 0$$

For now, assume this has an interior solution so calculus works.

First, what does T do to $\bar{e}(\tau^a)$?
By the Implicit Function Theorem:

$$\frac{\mathrm{d}\bar{e}}{\mathrm{d}\tau^a} = -\frac{\frac{\partial\Omega}{\partial T}}{\frac{\partial\Omega}{\partial\bar{e}}} = \frac{-\frac{\delta_{\mathrm{ML}}^{T+1}\ln\delta_{\mathrm{ML}}}{1-\delta_{\mathrm{ML}}}\left[W_{\mathrm{ML}}(\gamma(\bar{e}),\boldsymbol{\tau^a}) - W_{\mathrm{ML}}(\gamma(\bar{e}),\boldsymbol{\tau^{tw}})\right]}{\frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1-\delta_{\mathrm{ML}}}\frac{\partial\gamma}{\partial\bar{e}}[\pi(\tau^a) - \pi(\tau^{tw})] - \frac{\partial\gamma}{\partial\bar{e}}\left[\pi(\tau^b(\bar{e})) - \pi(\tau^a)\right]} > 0 \tag{1}$$

So if $T\uparrow$ then $\bar{e}\uparrow$.

Let $\gamma(e) = 1 + \frac{1}{1-\theta}e^{1-\theta}$. Or $\gamma(e) = 1 + \frac{1}{\sigma}e^{\sigma}$.

- $\frac{\partial\gamma}{\partial e} = e^{\sigma-1} > 0 \ \forall\sigma$

- $\frac{\partial^2\gamma}{\partial\sigma\partial e} = \ln e \cdot e^{\sigma-1} < 0$ for $e < 1$

i.e. as $\sigma\downarrow$, $\frac{\partial\gamma}{\partial e}\uparrow$.

Can I show that the optimal T can go either way when σ changes? That is, a counterexample to my quasi-result?

- My result says that as lobby gets stronger ($\frac{\partial\gamma}{\partial e}\uparrow$, so $\sigma\downarrow$), T should have to decrease.

- If optimal T increases when σ decreases (i.e. if T is decreasing in σ), this is a counterexample.

    - I think it's possible that both cases can happen depending on other parameters, like δ.

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Look at legislature's constraint:

$$\frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1 - \delta_{\mathrm{ML}}} \left[W_{\mathrm{ML}}(\gamma(e_b), \boldsymbol{\tau^a}) - W_{\mathrm{ML}}(\gamma(e_b), \boldsymbol{\tau^{tw}}) \right] \geq$$

$$W_{\mathrm{ML}}(\gamma(e_b), \tau^b(e_b), \tau^{*a}) - W_{\mathrm{ML}}(\gamma(e_b), \boldsymbol{\tau^a})$$

If T is too small, future gap can be smaller than current-period gap. So if T gets too short, can't enforce on legislature.