## Modified Legislative Constraint

Here I change the legislative constraint so that future political economy weights are evaluated according to anticipated lobbying effort, instead of the current period, or "break" lobbying effort as in the draft submitted to the JIE

• This is an attempt to keep the legislative constraint from collapsing when acknowledging the endogenous impact of lobbying effort on the break tariff, which I had not fully incorporated in the previous drafts

We can write the executives' joint problem as

$$\max_{\boldsymbol{\tau}^a} \frac{\boldsymbol{W}_{\mathrm{E}}(\boldsymbol{\tau}^a)}{1 - \delta_{\mathrm{E}}} \quad \text{subject to} \tag{1}$$

$$\frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1 - \delta_{\mathrm{ML}}} \left[ W_{\mathrm{ML}}(\gamma(0), \boldsymbol{\tau}^{\boldsymbol{a}}) - W_{\mathrm{ML}}(\gamma(e_{tw}), \boldsymbol{\tau}^{\boldsymbol{tw}}) \right] \ge W_{\mathrm{ML}}(\gamma(e_b), \tau^b(e_b), \tau^{*a}) - W_{\mathrm{ML}}(\gamma(e_b), \boldsymbol{\tau}^{\boldsymbol{a}})$$
(2)

and

$$e_b \ge \pi(\tau^b(e_b)) - \pi(\tau^a) + \frac{\delta_{L} - \delta_{L}^{T+1}}{1 - \delta_{L}} \left[ \pi(\tau^{tw}) - e_{tw} - \pi(\tau^a) \right]$$
 (3)

- Lobby's condition can't hold unless  $\overline{e} \geq e_{tw}$ 
  - Intuition: lobby's net profit is maximized at  $e_{tw}$ : if you're using an effort level below this to reduce the net profit and make the lobbying constraint hold, the lobby will just increase effort up to  $e_{tw}$ . So have to force lobbying effort above lobby's optimal level
  - Only holds at  $\overline{e} = e_{tw}$  if net profit is exactly  $\pi(\tau^a)$ , in which case lobby is indifferent between trade war and trade agreement.
- So, the question is: when does there exist a  $\tau^a$  such that  $\overline{e} \geq e_{tw}$  and the pair  $(\tau^a, \overline{e})$  satisfies the lobby's constraint?
  - Intuitively, how do you need to set  $\tau^a$  so that  $\overline{e}$ , which is derived from Expression 2 at equality, implies that  $\tau^b$  is enough larger than  $\tau^{tw}$  so that Expression 3 holds?
  - Or, am I even thinking about  $\bar{e}$  anymore?

- \* Yes, but it has to be driven by setting  $\tau^a$  high enough (otherwise, lobby would choose lower effort level, and Expression 3 would fail
- Take  $\tau^b = \tau^{tw}$  (note that lobby's constraint is still probably a problem, but this is a benchmark). Then it must be that  $e_b = e_{tw}$ .

$$\frac{\delta_{\mathrm{ML}} - \delta_{\mathrm{ML}}^{T+1}}{1 - \delta_{\mathrm{ML}}} \left[ W_{\mathrm{ML}}(\gamma(0), \boldsymbol{\tau}^{\boldsymbol{a}}) - W_{\mathrm{ML}}(\gamma(e_{tw}), \boldsymbol{\tau}^{\boldsymbol{tw}}) \right] \ge W_{\mathrm{ML}}(\gamma(e_{tw}), \boldsymbol{\tau}^{tw}, \tau^{*a}) - W_{\mathrm{ML}}(\gamma(e_{tw}), \boldsymbol{\tau}^{\boldsymbol{a}})$$

$$(4)$$

- \* Can we say anything about what  $\tau^a$  must be?
- \* Not sure if there is some  $\tau^a$  that makes this equal. The left hand side is positive and largest when  $\tau^a = 0$  and becomes negative and smallest when  $\tau^a = \tau^{tw}$ . The right hand side varies from positive to zero. I haven't tried very hard, but don't see a way to sign the total expression at  $\tau^a = 0$

Lobby's condition will hold if  $\pi(\tau^{tw}) - e_{tw} = \pi(\tau^a)$ 

- But we'd like to be able to reduce  $\tau^a$  below this level so that lobby actually cares to create a trade war
- This is possible only if  $\overline{e} \geq e_{tw}$ . So  $\tau^b$  is at minimum  $\tau^{tw}$
- Looking back at Expression 2, if  $\tau^a$  is above optimal tariff at e = 0, raising  $\tau^a$  reduces the LHS
  - At the same time, raising  $\tau^a$  reduces the RHS
- Lemma 1 is going to need some modifications:  $\frac{\partial \bar{e}}{\partial \tau^a} > 0$  IF increase in  $\gamma(e_b)$  term outweighs potential decrease in  $\gamma(0)$  term

To be clear about how legislature's decision is made:

- In break stage, if  $e^b$  is such that Expression 2
  - holds, choose  $\tau^a$  (that is, there are lots of  $e^b$  that make the leg unilaterally prefer a tariff higher than  $\tau^a$ , but dynamic incentives lead the leg to stick with the trade agreement)
  - does not hold, choose  $\tau^b$  (if breaking the agreement, want to choose static unilateral optimal tariff)

For legislature, there is some  $\tau^a$  (call it  $\tau^A$ ) such that  $W_{\rm ML}(\gamma(0), \tau^A, \tau^{*A}) = W_{\rm ML}(\gamma(e_{tw}), \boldsymbol{\tau^{tw}})$ 

- Trade war will only look like a punishment if  $\tau^a < \tau^A$ .
- So must have  $\tau^a \leq \tau^A$  or else legislature will cheat. Take  $\tau^A$  as rightmost boundary?
  - Can I prove something about the comparison of  $W_{\rm ML}(\gamma(e_b), \tau^b(e_b), \tau^{*A})$  and  $W_{\rm ML}(\gamma(e_b), \tau^A, \tau^{*A})$ ?
- Must have  $\overline{e}$  rise above  $e_{tw}$  before  $\tau^a$  exceeds  $\tau^A$ 
  - Remember:  $\frac{\partial \bar{e}}{\partial \tau^a} > 0$  IF increase in  $\gamma(e_b)$  term outweighs decrease in  $\gamma(0)$  term (loss from  $\gamma(0)$  term is zero at optimal unilateral tariff by envelope theorem)
    - \* I don't know where  $\frac{\partial \overline{e}}{\partial \tau^a} = 0$  (that is, where  $\overline{e}$  starts to decrease in  $\tau^a$ )
    - \* If the curve starts to decrease before lobby's constraint is satisfied, whether there's a solution or not depends on whether  $\pi(\tau^b(e_b)) e_b$  decreases faster than  $\pi(\tau^a)$

## What do I need to show?

- That, taking into account the dependency of  $\tau^b$  on  $e_b$ , there exists a cutoff  $\overline{e}$
- And, given this  $\overline{e}$ , the lobby's constraint binds
  - It's really the same procedure as before: use Expression 2 to define pairs of trade agreement tariffs and cutoff expenditure levels; then choose the pair with the lowest  $\tau^a$  that satisfies Expression 3
- In terms of Lemma 4 (old Lemma 3): when  $\tau_{NL}^a < \tau^{tw}$  (recall, at  $\tau_{NL}^a$  there is no lobby, so  $\overline{e} = 0$ ), I say "Lobby will increase profits." This is not necessarily true. If if the gain is negative, the constraint becomes tighter up to  $\overline{e} = e_{tw}$ .
  - If A.2 holds at  $\tau^{tw}$  (only when  $\tau^a$  is such that lobby doesn't care about triggering trade war), then  $\tau^a_{NL}$  is solution
  - If A.2 fails, need to raise  $\tau^a$ 
    - \* Is there a condition that would guarantee that eventually raising  $\tau^a$  makes the constraint hold?