New Equilibrium Construction

From "to_do_list.tex":

Take out renegotiation

- Add more basic tradeoff

- (??) Draw inverted U for lobby

- Now my short punishments don't rest on renegotiation

  - So now, for main analysis, must assume that we're constraining attention to a certain class of punishments: symmetric, and "Punish for T periods then go back to cooperation"

    * Go back to start if deviate should work for governments, but I think I need something else for lobbies since they would like that

  - Can I show that mine are optimal in this class?

January 17, 2015

- Must show players are best responding in every subgame, on and off the eqm path

- I'm going to try to use reversion to the static nash, but this is not necessarily subgame perfect (deviations can trigger changes in future periods)

  - Basic intuition: lobby wants punishment to go longer, leg wants it to go shorter

  - Ideally, want each to choose static BR in each period of punishment: in non-cooperative state, you can pick whatever you want, but the other guy is doing whatever he wants; τ^{tw} is independent of what he does

    * BUT it's not independent of lobby's effort

Equilibrium: Executives set trade agreement at $t = 0$. At $t \geq 1$, lobbies choose e, leg chooses applied τ

- $\forall\ t \geq 1$, leg applies τ^A if

  1. $\tau \leq \tau^A$ was applied last period

2. There have been T periods of punishment: I think $\tau \geq \tau^N$ and $e \leq e^N$

- Not sure how to specify lobby in these cooperation periods: $e = 0$ if $\tau \geq \tau^A$ (in any period? how are they involved in punishment? they're not really)

- if $\tau > \tau^A$ within the last T periods, leg applies $\tau^N(e^N)$

January 19, 2015

- Think of punishment scheme being designed either by execs or by supranational body like WTO

- Then want to know whether it's an eqm for leg and lobbies to follow the rules

Classes of subgames

1. $\tau \leq \tau^A$ and $e = 0$ last period; if there had ever been a violation, it was at least T periods previous.

   - Should I have "and $e < \bar{e}$" instead?

2. Conditions in (1) held in period $t - 2$, but there was a violation in period $t - 1$

   - Play static Nash this period and for $T - 1$ more periods before switching back to (1); more precisely, $\tau^D \geq \tau^N$ and $e^D \geq e^N$.

3. Static Nash punishment was initiated $i < T$ periods ago, and punishment has been followed since then

   - Punish this period and $T - i - 1$ more periods before switching back to (1)

4. In any punishment period, legislature does not follow punishment: i.e. $\tau^D < \tau^N$

   - Restart punishment at (2)

5. In any punishment period, lobby does not follow punishment: $e^D < e^N$

   - Legislature chooses (??) BR to e^D, then restart at (1); if anything else, restart punishment from (2)
     - Lobby *must* pay in final period of punishment, or else IC for leg will not hold. That is why the equilibrium is being re-worked.

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– But, if leg is going to BR to lobby's payment and then restart cooperation, lobby should want to continue with punishment. This seems like a realistic set-up (your protection ends if you don't hold up your end of the deal with the promised payments).

Conditions for equilibrium

- Checking that punishment is incentive compatible

  – Legislature:

$$W(\gamma(e^N), \tau^N) + \frac{\delta - \delta^{T+1}}{1 - \delta} W(\gamma(e^N), \tau^N) + \delta^{T+1} W(\gamma(0), \tau^a) \geq W(\gamma(e^N), \cdot) + \frac{\delta - \delta^{T+2}}{1 - \delta} W(\gamma(e^N), \tau^N)$$

  by definition, anything provides lower one-shot payoffs than τ^N, and Nash payoffs are lower than trade agreement payoffs (need to prove this–or is it just by assumption?)

    * Is this punishment IC for leg? Yes, condition is satisfied. Best responding in current period–would do the same in best deviation; future is trade agreement instead of restart of punishment for any other tariff.

  – Lobby:

$$\pi(\tau^N) - e^N + \frac{\delta - \delta^{T+1}}{1 - \delta} \left[\pi(\tau^N) - e^N \right] + \delta^{T+1} \pi(\tau^a) \geq \pi(\tau^D) - e^D + \frac{\delta - \delta^{T+2}}{1 - \delta} \left[\pi(\tau^a) \right]$$

  best deviation, given that leg will one-shot best respond is also e^N; given $\pi(\tau^N) - e^N \geq \tau^a$, which is necessary for any of this to be interesting, this condition holds.
  Since the best deviation is to the Nash tariff, it reduces to

$$\frac{\delta - \delta^{T+1}}{1 - \delta} \left[\pi(\tau^N) - e^N \right] \geq \frac{\delta - \delta^{T+1}}{1 - \delta} \left[\pi(\tau^a) \right]$$

$$\frac{\delta - \delta^{T+1}}{1 - \delta} \left[\pi(\tau^N) - e^N - \pi(\tau^a) \right] \geq 0 \tag{1}$$

  This now seems less of a conflict with the constraint in the main problem of

$$e^b \geq \pi(\tau^b) - \pi(\tau^a) + \frac{\delta - \delta^{T+1}}{1 - \delta} \left[\pi(\tau^N) - e^N - \pi(\tau^a) \right]$$

  there's still a push and pull, but it's easier to satisfy—in particular, we already assume that $\pi(\tau^N) - e^N - \pi(\tau^a) > 0$ or the problem is not interesting.
  Note that Expression 1 will hold for all T/i.

* In last period of punishment, will hold with equality
* Note that if a deviation were to occur (off eqm path), leg would apply a "punishment" tariff that is too low, which would normally cause the punishment to reset; but here, it's because the lobby deviated, so we re-start cooperation to punish the lobby further after giving the reduced tariff in the period in which the lobby deviates

From old construction, need to be rechecked:

- I've shown condition for lobby is constant through time except in last period, where they'll never pay (this was, obviously, for older model that doesn't enforce lobby paying in final period of punishment)

  - In general, need to check how conditions change through time in punishment
  - $\frac{\delta - \delta^{T+1}}{1 - \delta}$ is increasing in T, which means it gets smaller as you move toward the end of the punishment (there are fewer periods of punishment payoffs left)

- Need to pay special attention to leg's condition in the last period