New Equilibrium Construction

From "to_do_list.tex":

Take out renegotiation

- Add more basic tradeoff
- (??) Draw inverted U for lobby
- Now my short punishments don't rest on renegotiation
  - So now, for main analysis, must assume that we're constraining attention to a certain class of punishments: symmetric, and "Punish for T periods then go back to cooperation"
    - Go back to start if deviate should work for governments, but I think I need something else for lobbies since they would like that
  - Can I show that mine are optimal in this class?

January 17, 2015

- Must show players are best responding in every subgame, on and off the eqm path
- I'm going to try to use reversion to the static nash, but this is not necessarily subgame perfect (deviations can trigger changes in future periods)
  - Basic intuition: lobby wants punishment to go longer, leg wants it to go shorter
  - Ideally, want each to choose static BR in each period of punishment: in non-cooperative state, you can pick whatever you want, but the other guy is doing whatever he wants; τ^{tw} is independent of what he does
    - BUT it's not independent of lobby's effort

Equilibrium: Executives set trade agreement at $t = 0$. At $t \geq 1$, lobbies choose e, leg chooses applied τ

- $\forall\ t \geq 1$, leg applies τ^A if
  1. $\tau \leq \tau^A$ was applied last period

2. There have been T periods of punishment: I think $\tau \geq \tau^N$ and $e \leq e^N$

- Not sure how to specify lobby in these cooperation periods: $e = 0$ if $\tau \geq \tau^A$ (in any period? how are they involved in punishment? they're not really)
- if $\tau > \tau^A$ within the last T periods, leg applies $\tau^N(e^N)$

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- Think of punishment scheme being designed either by execs or by supranational body like WTO
- Then want to know whether it's an eqm for leg and lobbies to follow the rules

Classes of subgames

1. $\tau \leq \tau^A$ and $e = 0$ last period; if there had ever been a violation, it was at least T periods previous.
   - Should I have "and $e < \bar{e}$" instead?
2. Conditions in (1) held in period $t-2$, but there was a violation in period $t-1$
   - Play static Nash this period and for $T-1$ more periods before switching back to (1); more precisely, $\tau^D \geq \tau^N$ and $e^D \geq e^N$.
3. Static Nash punishment was initiated $i < T$ periods ago, and punishment has been followed since then
   - Punish this period and $T - i - 1$ more periods before switching back to (1)
4. In any punishment period, legislature does not follow punishment: i.e. $\tau^D < \tau^N$
   - Restart punishment at (2); lobby happy to do this
5. In any punishment period, lobby does not follow punishment: $e^D < e^N$
   - Legislature chooses (??) BR to e^D, then restart at (1); if anything else, restart punishment from (2). BR + eqm, so okay
     - Lobby *must* pay in final period of punishment, or else IC for leg will not hold. That is why the equilibrium is being re-worked.

- But, if leg is going to BR to lobby's payment and then restart cooperation, lobby should want to continue with punishment. This seems like a realistic set-up (your protection ends if you don't hold up your end of the deal with the promised payments).

Conditions for equilibrium

- Checking that punishment (2) is incentive compatible given (4) and (5)
  - Legislature:

    $$W(\gamma(e^N), \tau^N) + \frac{\delta - \delta^{T+1}}{1-\delta} W(\gamma(e^N), \tau^N) + \delta^{T+1} W(\gamma(0), \tau^a) \geq W(\gamma(e^N), \cdot) + \frac{\delta - \delta^{T+2}}{1-\delta} W(\gamma(e^N), \tau^N)$$

    by definition, anything provides lower one-shot payoffs than τ^N, and Nash payoffs are lower than trade agreement payoffs (need to prove this–or is it just by assumption?)

    * Is this punishment IC for leg? Yes, condition is satisfied. Best responding in current period–would do the same in best deviation; future is trade agreement instead of restart of punishment for any other tariff.
    * This conditions is always satisfied; the above equation is for the first period of the punishment. In later periods, there are more periods of the punishment that get repeated, so the constraint becomes looser
  - Lobby [(2) is incentive compatible given (4) and (5)]:

    $$\pi(\tau^N) - e^N + \frac{\delta - \delta^{T+1}}{1-\delta} \left[\pi(\tau^N) - e^N\right] + \delta^{T+1} \pi(\tau^a) \geq \pi(\tau^D) - e^D + \frac{\delta - \delta^{T+2}}{1-\delta} \left[\pi(\tau^a)\right]$$

    best deviation, given that leg will one-shot best respond is also e^N; given $\pi(\tau^N) - e^N \geq \tau^a$, which is necessary for any of this to be interesting, this condition holds.
    Since the best deviation is to the Nash tariff, it reduces to

    $$\frac{\delta - \delta^{T+1}}{1-\delta} \left[\pi(\tau^N) - e^N\right] \geq \frac{\delta - \delta^{T+1}}{1-\delta} \left[\pi(\tau^a)\right]$$

    $$\frac{\delta - \delta^{T+1}}{1-\delta} \left[\pi(\tau^N) - e^N - \pi(\tau^a)\right] \geq 0 \tag{1}$$

    This now seems less of a conflict with the constraint in the main problem of

    $$e^b \geq \pi(\tau^b) - \pi(\tau^a) + \frac{\delta - \delta^{T+1}}{1-\delta} \left[\pi(\tau^N) - e^N - \pi(\tau^a)\right]$$

    there's still a push and pull, but it's easier to satisfy—in particular, we already assume that $\pi(\tau^N) - e^N - \pi(\tau^a) > 0$ or the problem is not interesting.
    Note that Expression 1 will hold for all T/i.

    * In last period of punishment, will hold with equality. Is tighest at the end of the punishment, looser at beginning. Holds for all T, i.e. always holds.
    * Note that if a deviation were to occur (off eqm path), leg would apply a "punishment" tariff that is too low, which would normally cause the punishment to reset; but here, it's because the lobby deviated, so we re-start cooperation to punish the lobby further after giving the reduced tariff in the period in which the lobby deviates
- Legislature's constraint holds $\forall\ T$ (but is tighter for large T)
- Lobby's constraint also holds $\forall\ T$ (but is tighter for small T)
    * $\frac{\delta-\delta^{T+1}}{1-\delta}$ is increasing in T, which means it gets smaller as you move toward the end of the punishment (there are fewer periods of punishment payoffs left)