Equilibrium Strategy Profiles (For JLEO Revision)

First, need to get timing clear for evolution of state variable. Take state variable to be q_{st}.

- q_{s1}: beginning of the world (or, $Q_{s0} - \mu = q_{s1}$)
- $Q_{s1} = q_{s1} + R_{p1} + R_{c1}$
  - $q_{s2} = Q_{s1} - \mu$

Strategies (this is for the most interesting state variable, but could write it out for the other five)

- For patron: function of q_{st}
- For c: function of $q_{st} + R_{pt}$
- For s: function of $Q_{st} = q_{st} + R_{pt} + R_{ct}$
- For g: not a function of q_{st} at all

According to Mailath and Samuelson (2006) p. 177.

- The strategy profile σ is a stationary Markov strategy if for any two ex post histories $\tilde{h}^t$ and $\tilde{h}^\tau$ (of equal or different lengths) terminating in the same state, $\sigma\left(\tilde{h}^t\right) = \sigma\left(\tilde{h}^\tau\right)$.
- The strategy profile σ is a stationary Markov equilibrium if σ is a stationary Markov strategy profile and a subgame-perfect equilibrium.

We have six state variables, so $s = (q_s, q_g, l_s, l_g, w_s, w_g)$.

- Strategies are a function of all six state variables
  - We can restrict attention to relevant ranges of the state variables. CAN I CHANGE PROP 1 PARTS 1 AND 2 TO BE INITIAL VALUES OF STATE VARIABLES? If so, then these conditions and what is necessary in each period to make the game continue.

- Strategies for the patron and c are also how much to invest in each of the six state variables. Some can be ruled out by preference assumptions:
  - c dislikes war, so will never invest in w_s or w_g. It would also not want to make the government lose, so won't invest in l_g either.
  - Because the patron's preferences are aligned with the secessionists and against the government, it never invests in w_g or l_s.

This leaves four state varibles in which the patron may invest: q_s, q_g, l_g and w_s. Three in which c may invest: q_s, q_g, l_s.

- Also, Gov't / Secessionists: Choose unilateral, simultaneous best responses depending on magnitudes of Q_{i1}, L_{i1} and ω_{i1}
  - Game only continues if (SQ,SQ) or (Cede, Cede) was played

Assumptions

- $Q_{s0} \geq L_{s0} \Rightarrow q_{s1} + \mu \geq l_{s1}$
- $Q_{g0} \geq L_{g0} \Rightarrow q_{g1} \geq l_{g1}$

Period 1

1. Patron would like $Q_{s1} \geq L_{s1}$ to prevent c from incentivizing secessionists to Cede
   - Equivalent to $q_{s1} + R_{p1} \geq l_{s1} + R_{c1}$ / $Q_{s0} + \mu + R_{p1} \geq L_{s0} + R_{c1}$ / $R_{p1} \geq R_{c1} - (Q_{s0} - \mu - L_{s0})$
     - $R_{c1} \leq \frac{\beta}{1-\delta} \Rightarrow R_{p1} \geq \frac{\beta}{1-\delta} - (Q_{s0} - \mu - L_{s0})$ allows the patron to ensure the original inequality
     - Initial assumption means $R_{p1} \geq \frac{\beta}{1-\delta} + \mu$ is a tighter condition
     - Since patron is willing to pay up to $R_{p1} = \frac{\alpha}{1-\delta}$, when $\frac{\alpha}{1-\delta} \geq \frac{\beta}{1-\delta} + \mu$, it will invest $R_{p1} = \frac{\beta}{1-\delta} - (Q_{s0} - \mu - L_{s0})$ to augment q_{s1} if this is greater than 0. Else, $R_{p1} = 0$.
       (a) If $R_{p1} + (Q_{s0} - \mu - L_{s0}) \geq \frac{\beta}{1-\delta}$, $R_{c1} = 0$. (SQ,SQ) is played and game continues.
       (b) If instead $R_{p1} + (Q_{s0} - \mu - L_{s0}) < \frac{\beta}{1-\delta}$ (i.e. assumption doesn't hold), $R_{c1} = l_{s1} - (q_{s1} + R_{p1}) + \varepsilon$ to augment l_{s1}. Note optimal R_{p1} in this case is 0.

- $*$ (SQ,Cede) is played and game ends

2. Patron would like $L_{g1} > Q_{g1}$ to achieve recognition directly. Must invest to augment l_{g1}

   - Equivalent to $l_{g1} + R_{p1} > q_{g1} + R_{c1}$ / $L_{g0} + R_{p1} > Q_{g0} + R_{c1}$ / $R_{p1} > R_{c1} + (Q_{g0} - L_{g0})$
     - $R_{c1} \leq \frac{\nu}{1-\delta} \Rightarrow R_{p1} > \frac{\nu}{1-\delta} + (Q_{g0} - L_{g0})$ allows the patron to ensure the original inequality
     - Since patron is willing to pay up to $R_{p1} = \frac{\lambda+\mu+\beta}{1-\delta}$, when $\frac{\lambda+\mu+\beta}{1-\delta} \leq \frac{\nu}{1-\delta}$, patron will invest 0 in l_{g1}. $R_{c1} = 0$ as well.
     - If instead assumption 4 did not hold and $R_{p1} > \frac{\nu}{1-\delta} + (Q_{g0} - L_{g0})$, it will invest this amount to augment l_{g1}. Again, $R_{c1} = 0$. (Cede, SQ) is played and game ends.

3. Patron might want to instigate fighting to have a p_{1s} probability of achieving recognition indirectly. If $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta}$ is positive, invest in w_{s1} to change ω_{s1}. Will not invest when the quantity is non-positive. Inequality is $\omega_{s1} > Q_{s1}$

   - $\omega_{s1} = -\zeta_s(1-\delta) + W_{s1}p_{1s} + L_{s1}(1-p_{1s}) > Q_{s1}$
   - $-\zeta_s(1-\delta) + (W_{s0} + R_{p1})p_{1s} + L_{s0}(1-p_{1s}) > Q_{s0} - \mu + R_{c1}$
   - $R_{p1}p_{1s} > R_{c1} - \mu + Q_{s0} - (-\zeta_s(1-\delta) + W_{s0}p_{1s} + L_{s0}(1-p_{1s}))$
   - $R_{c1} \leq \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta}$, so if $R_{p1}p_{1s} > \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta} - \mu + Q_{s0} - (-\zeta_s(1-\delta) + W_{s0}p_{1s} + L_{s0}(1-p_{1s}))$, $R_{c1} = 0$ and (SQ,Fight) is played in third stage. Outcome depends on war lottery.
   - Patron will spend at most $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu$
     - So need

$$p_{1s}\left[\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu\right] < \frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu \leq \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta} - \mu$$
$$\leq \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta} - \mu + Q_{s0} - (-\zeta_s(1-\delta) + W_{s0}p_{1s} + L_{s0}(1-p_{1s})) \quad (1)$$

4. Patron could also invest in q_g to counter investment by c in government's payoffs. But this would only been needed if c had incentive to invest in g playing Fight, and we've ruled that out.

Now generalize profiles for all t

1. Patron would like $Q_{st} \geq L_{st}$ to prevent c from incentivizing secessionists to Cede

   - Equivalent to $q_{st} + R_{pt} \geq l_{st} + R_{ct}$ / $Q_{s,t-1} + \mu + R_{pt} \geq L_{s,t-1} + R_{ct}$ / $R_{pt} \geq R_{ct} - (Q_{s,t-1} - \mu - L_{s,t-1})$

- $R_{ct} \leq \frac{\beta}{1-\delta} \Rightarrow R_{pt} \geq \frac{\beta}{1-\delta} - (Q_{s,t-1} - \mu - L_{s,t-1})$ allows the patron to ensure the original inequality
- Assumption 1 combined with equilibrium play (equivalently, if $L_{s,t-1} > Q_{s,t-1}$, the game would have ended before the start of period t) means $R_{pt} \geq \frac{\beta}{1-\delta} + \mu$ is a tighter condition
- Since patron is willing to pay up to $R_{pt} = \frac{\alpha}{1-\delta}$, when $\frac{\alpha}{1-\delta} \geq \frac{\beta}{1-\delta} + \mu$, it will invest $R_{pt} = \frac{\beta}{1-\delta} - (Q_{s,t-1} - \mu - L_{s,t-1})$ to augment q_{st} if this is greater than 0. Else, $R_{pt} = 0$.
  (a) If $R_{pt} + (Q_{s,t-1} - \mu - L_{s,t-1}) \geq \frac{\beta}{1-\delta}$, $R_{ct} = 0$. (SQ,SQ) is played and game continues.
  (b) If instead $R_{pt} + (Q_{s,t-1} - \mu - L_{s,t-1}) < \frac{\beta}{1-\delta}$ (i.e. assumption 3 doesn't hold), $R_{ct} = l_{st} - (q_{st} + R_{pt}) + \varepsilon$ where ε is small to augment l_{st}. Note optimal R_{pt} in this case is 0.
    * (SQ,Cede) is played and game ends

2. Patron would like $L_{gt} > Q_{gt}$ to achieve recognition directly. Must invest to augment l_{gt}

- Equivalent to $l_{gt} + R_{pt} > q_{gt} + R_{ct}$ / $L_{g,t-1} + R_{pt} > Q_{g,t-1} + R_{ct}$ / $R_{pt} > R_{ct} + (Q_{g,t-1} - L_{g,t-1})$
  - $R_{ct} \leq \frac{\nu}{1-\delta} \Rightarrow R_{pt} > \frac{\nu}{1-\delta} + (Q_{g,t-1} - L_{g,t-1})$ allows the patron to ensure the original inequality
  - Since patron is willing to pay up to $R_{pt} = \frac{\lambda+\mu+\beta}{1-\delta} - \mu$, when $\frac{\lambda+\mu+\beta}{1-\delta} - \mu \leq \frac{\nu}{1-\delta}$, patron will invest 0 in l_{gt}. $R_{ct} = 0$ as well.
  - If instead assumption 4 did not hold and $R_{pt} > \frac{\nu}{1-\delta} + (Q_{g,t-1} - L_{g,t-1})$, patron will invest this amount to augment l_{gt}. Again, $R_{ct} = 0$. (Cede, SQ) is played and game ends.

3. Patron might want to instigate fighting to have a p_{1s} probability of achieving recognition indirectly. If $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu$ is positive, invest in w_{st} to change ω_{st}. Will not invest when the quantity is non-positive. Inequality is $\omega_{st} > Q_{st}$

- $\omega_{st} = -\zeta_s(1-\delta) + W_{st}p_{1s} + L_{st}(1-p_{1s}) > Q_{st}$
- $-\zeta_s(1-\delta) + (W_{s,t-1} + R_{pt})p_{1s} + L_{s,t-1}(1-p_{1s}) > Q_{s,t-1} - \mu + R_{ct}$
- $R_{pt}p_{1s} > R_{ct} - \mu + Q_{s,t-1} - (-\zeta_s(1-\delta) + W_{s,t-1}p_{1s} + L_{s,t-1}(1-p_{1s}))$
- $R_{ct} \leq \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta}$, so if $R_{pt}p_{1s} > \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta} - \mu + Q_{s,t-1} - (-\zeta_s(1-\delta) + W_{s,t-1}p_{1s} + L_{s,t-1}(1-p_{1s}$ $R_{ct} = 0$ and (SQ,Fight) is played in third stage. Outcome depends on war lottery.
- Patron will spend at most $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu$

- So need

$$p_{1s}\left[\tfrac{\lambda p_{1s}-\alpha(1-p_{1s})+\mu+\beta}{1-\delta}-\mu\right] < \tfrac{\lambda p_{1s}-\alpha(1-p_{1s})+\mu+\beta}{1-\delta}-\mu \leq \tfrac{\nu p_{1s}-\beta(1-p_{1s})}{1-\delta}-\mu$$
$$\leq \frac{\nu p_{1s}-\beta(1-p_{1s})}{1-\delta}-\mu+Q_{s,t-1}-(-\zeta_s(1-\delta)+W_{s,t-1}p_{1s}+L_{s,t-1}(1-p_{1s})) \tag{2}$$

4. Patron could also invest in q_g to counter investment by c in government's payoffs. But this would only been needed if c had incentive to invest in g playing Fight, and we've ruled that out.

Prose version of equilibrium strategy profiles:

On the equilibrium path, the patron invests to avoid the reunification outcome. That is, it invests in the secessionists' status quo payoffs to deter the international community from investing in the secessionists' payoffs from ceding. In period t, the patron invests $R_{pt} = \max\left\{\frac{\beta}{1-\delta} - (Q_{s,t-1} - \mu - L_{s,t-1}), 0\right\}$ to augment the status quo payoffs of the secessionists, q_{st}. Player c then chooses $R_{ct} = 0$ so that playing status quo is a best response for both inside players. (SQ,SQ) is played in the third stage and the game continues to period $t+1$.
If instead $R_{pt} < \frac{\beta}{1-\delta} - (Q_{s,t-1} - \mu - L_{s,t-1})$ (i.e. assumption 3 doesn't hold), player c will invest $R_{ct} = l_{st} - (q_{st} + R_{pt}) + \varepsilon$, for ε small, to augment l_{st}. In this case, status quo remains a best response for the government but cede becomes the best response for the secessionists. The game ends with the secessionist territory being reunited. This path is ruled out by Assumption 3 (see Lemma 2).
The patron would prefer to invest to provoke the recognition outcome, which it can attempt to do directly by investing in the government's payoffs from ceding, l_{gt}. If the patron invests $R_{pt} > \frac{\nu}{1-\delta} + (Q_{g,t-1} - L_{g,t-1})$ to augment l_{gt}, player c will then choose $R_{ct} = 0$ and cede will be the best response for the government. Status quo remains the best response for the secessionists, so the game ends with the secessionists gaining recognition. This path is ruled out by Assumption 4 (see Lemma 3).
As long as $R_{pt} \leq \frac{\nu}{1-\delta} + (Q_{g,t-1} - L_{g,t-1})$, player c will counter with $R_{ct} = R_{pt} - (Q_{g,t-1} - L_{g,t-1})$ to augment l_{gt}. This makes status quo a best response for the government as well as for the secessionists so that the status quo equilibrium is played and the game continues to period $t+1$. Note that the optimal investments in the government's payoffs zero.

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   - $\omega_{st} = -\zeta_s(1-\delta) + W_{st}p_{1s} + L_{st}(1-p_{1s}) > Q_{st}$
   - $-\zeta_s(1-\delta) + (W_{s,t-1} + R_{pt})p_{1s} + L_{s,t-1}(1-p_{1s}) > Q_{s,t-1} - \mu + R_{ct}$
   - $R_{pt}p_{1s} > R_{ct} - \mu + Q_{s,t-1} - (-\zeta_s(1-\delta) + W_{s,t-1}p_{1s} + L_{s,t-1}(1-p_{1s}))$
   - $R_{ct} \leq \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta}$, so if $R_{pt}p_{1s} > \frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta} - \mu + Q_{s,t-1} - (-\zeta_s(1-\delta) + W_{s,t-1}p_{1s} + L_{s,t-1}(1-p_{1s}$ $R_{ct} = 0$ and (SQ,Fight) is played in third stage. Outcome depends on war lottery.
   - Patron will spend at most $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta} - \mu$

– So need

$$p_{1s}\left[\tfrac{\lambda p_{1s}-\alpha(1-p_{1s})+\mu+\beta}{1-\delta}-\mu\right] < \tfrac{\lambda p_{1s}-\alpha(1-p_{1s})+\mu+\beta}{1-\delta}-\mu \leq \tfrac{\nu p_{1s}-\beta(1-p_{1s})}{1-\delta}-\mu$$
$$\leq \frac{\nu p_{1s}-\beta(1-p_{1s})}{1-\delta}-\mu+Q_{s,t-1}-(-\zeta_s(1-\delta)+W_{s,t-1}p_{1s}+L_{s,t-1}(1-p_{1s})) \tag{3}$$

2. Patron could also invest in q_g to counter investment by c in government's payoffs. But this would only been needed if c had incentive to invest in g playing Fight, and we've ruled that out.