Equilibrium Strategy Profiles (For JLEO Revision)

First, need to get timing clear for evolution of state variable. Take state variable to be q_{st}.

- q_{s1}: beginning of the world (or, $Q_{s0} - \mu = q_{s1}$)
- $Q_{s1} = q_{s1} + R_{p1} + R_{c1}$
  - $q_{s2} = Q_{s1} - \mu$

Strategies (this is for the most interesting state variable, but could write it out for the other five)

- For patron: function of q_{st}
- For c: function of $q_{st} + R_{pt}$
- For s: function of $Q_{st} = q_{st} + R_{pt} + R_{ct}$
- For g: not a function of q_{st} at all

According to Mailath and Samuelson (2006) p. 177.

- The strategy profile σ is a stationary Markov strategy if for any two ex post histories $\tilde{h}^t$ and $\tilde{h}^\tau$ (of equal or different lengths) terminating in the same state, $\sigma\left(\tilde{h}^t\right) = \sigma\left(\tilde{h}^\tau\right)$.
- The strategy profile σ is a stationary Markov equilibrium if σ is a stationary Markov strategy profile and a subgame-perfect equilibrium.

We have six state variables, so $s = (q_s, q_g, l_s, l_g, w_s, w_g)$.

- Strategies are a function of all six state variables
  - We can restrict attention to relevant ranges of the state variables. CAN I CHANGE PROP 1 PARTS 1 AND 2 TO BE INITIAL VALUES OF STATE VARIABLES? If so, then these conditions and what is necessary in each period to make the game continue.

- Strategies for the patron and c are also how much to invest in each of the six state variables. Some can be ruled out by preference assumptions:
  - c dislikes war, so will never invest in w_s or w_g. It would also not want to make the government lose, so won't invest in l_g either.
  - Because the patron's preferences are aligned with the secessionists and against the government, it never invests in w_g or l_s.

This leaves four state varibles in which the patron may invest: q_s, q_g, l_g and w_s. Three in which c may invest: q_s, q_g, l_s.

- Also, Gov't / Secessionists: Choose unilateral, simultaneous best responses depending on magnitudes of Q_{i1}, L_{i1} and ω_{i1}
  - Game only continues if (SQ,SQ) or (Cede, Cede) was played

Assumptions

- $Q_{s0} \geq L_{s0} \Rightarrow q_{s1} + \mu \geq l_{s1}$
- $Q_{g0} \geq L_{g0} \Rightarrow q_{g1} \geq l_{g1}$

Period 1

1. Patron would like $Q_{s1} \geq L_{s1}$ to prevent c from incentivizing secessionists to Cede
   - Equivalent to $q_{s1} + R_{p1} \geq l_{s1} + R_{c1}$ / $Q_{s0} + \mu + R_{p1} \geq L_{s0} + R_{c1}$ / $R_{p1} \geq R_{c1} - (Q_{s0} - \mu - L_{s0})$
     - Equivalent to $R_{c1} \leq \frac{\beta}{1-\delta} \Rightarrow R_{p1} \geq \frac{\beta}{1-\delta} - (Q_{s0} - \mu - L_{s0})$ allows the patron to ensure the original inequality
     - Initial assumption means $R_{p1} \geq \frac{\beta}{1-\delta} + \mu$ is a tighter condition
     - Since patron is willing to pay up to $R_{p1} = \frac{\alpha}{1-\delta}$, when $\frac{\alpha}{1-\delta} \geq \frac{\beta}{1-\delta} + \mu$, it will invest $R_{p1} = \frac{\beta}{1-\delta} - (Q_{s0} - \mu - L_{s0})$ to augment q_{s1} if this is greater than 0. Else, $R_{p1} = 0$.
       (a) If $R_{p1} + (Q_{s0} - \mu - L_{s0}) \geq \frac{\beta}{1-\delta}$, $R_{c1} = 0$. (SQ,SQ) is played and game continues.
       (b) If instead $R_{p1} + (Q_{s0} - \mu - L_{s0}) < \frac{\beta}{1-\delta}$ (i.e. assumption doesn't hold), $R_{c1} = l_{s1} - (q_{s1} + R_{p1}) + \varepsilon$ to augment l_{s1}. Note optimal R_{p1} in this case is 0.

* (SQ,Cede) is played and game ends

2. Patron might also want to invest to encourage recognition directly. How much to augment l_{g1}?

3. Patron might also want to instigate fighting. If $\frac{\lambda p_{1s} - \alpha(1-p_{1s}) + \mu + \beta}{1-\delta}$ is positive, invest in w_{s1} to change ω_{s1}. Will not invest in w_{s1} when the quantity is non-positive.

   - c will counter up to $\frac{\nu p_{1s} - \beta(1-p_{1s})}{1-\delta}$

4. Patron could also invest in q_g to counter investment by c in government's payoffs. But this would only been needed if c had incentive to invest in g playing Fight, and we've ruled that out.