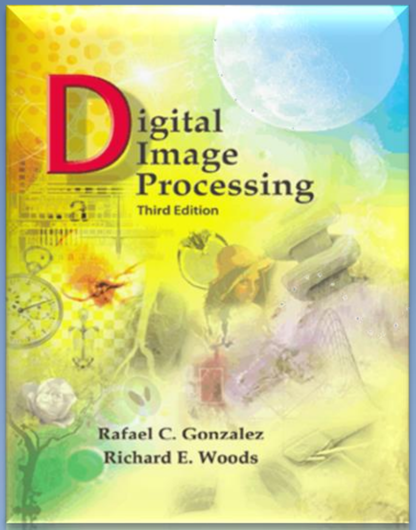
DIGITAL IMAGE PROCESSING



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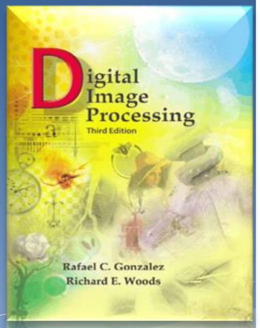
DIGITAL IMAGE PROCESSING



Chapter 3 - Intensity Transformations and Spatial Filtering

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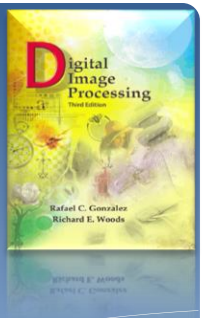
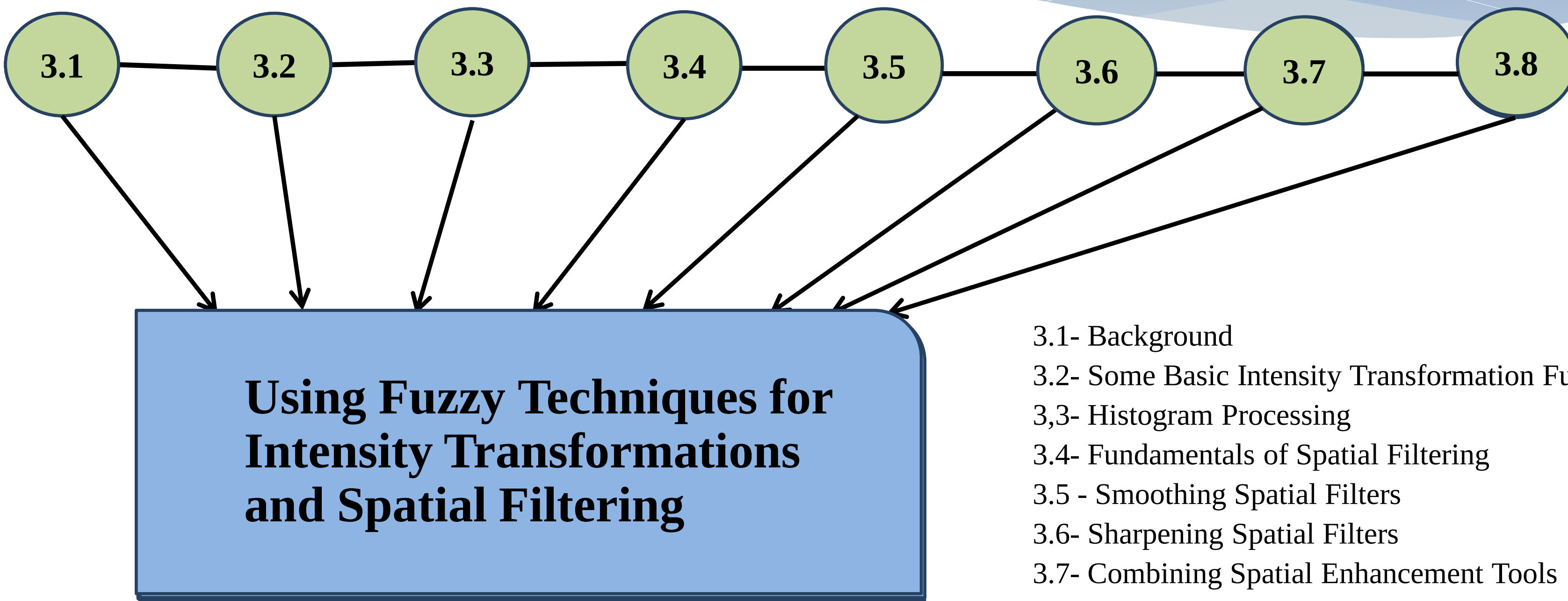
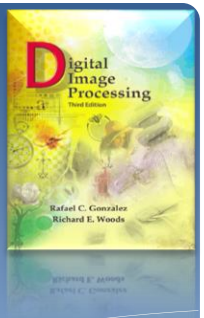
Road map of chapter 3

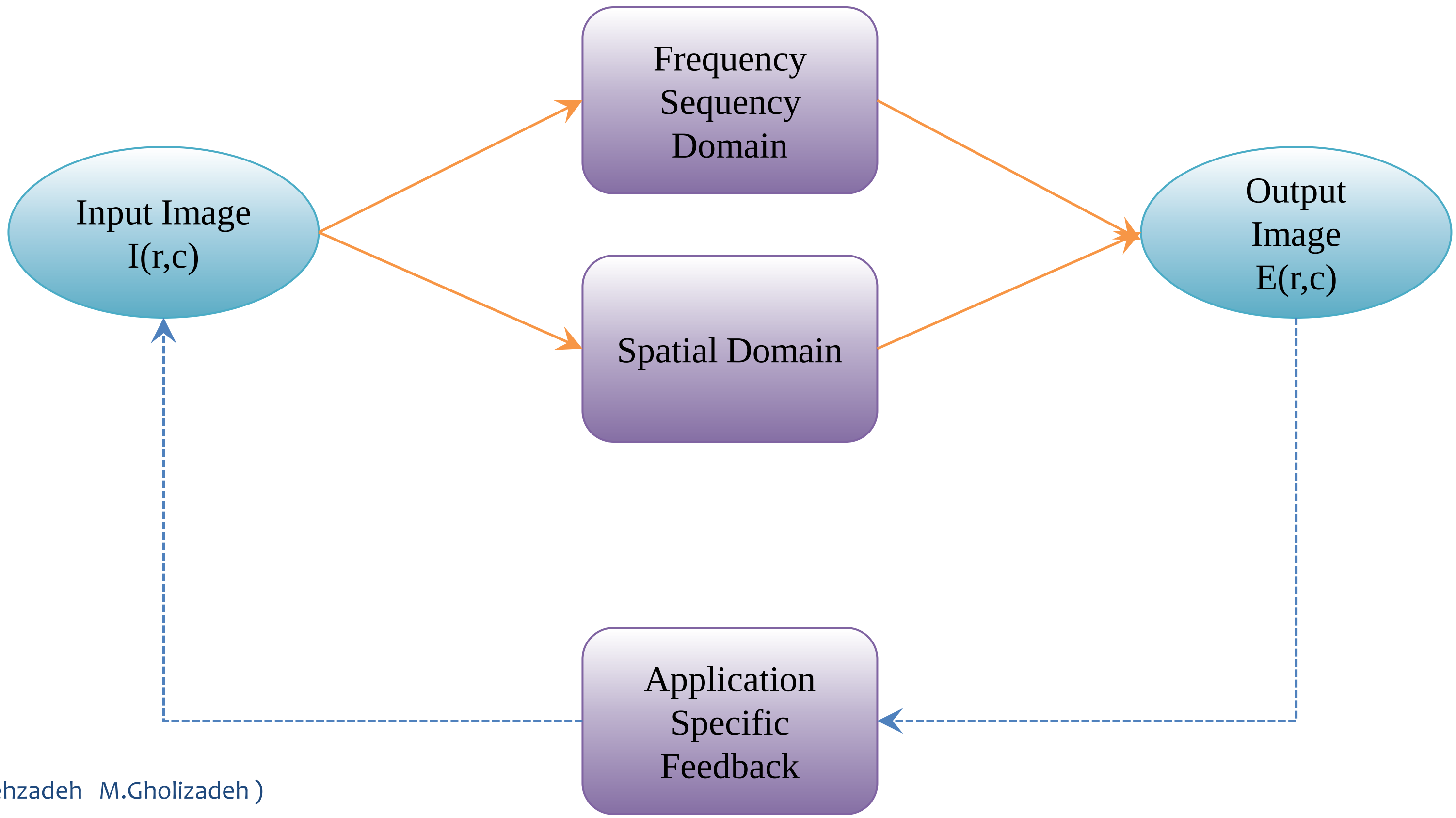




3.1- Background
3.2- Some Basic Intensity Transformation Functions
3,3- Histogram Processing
3.4- Fundamentals of Spatial Filtering
3.5 - Smoothing Spatial Filters
3.6- Sharpening Spatial Filters
3.7- Combining Spatial Enhancement Tools
3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

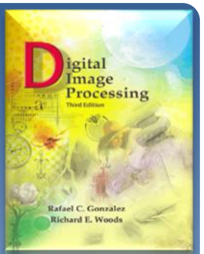
Image Enhancement





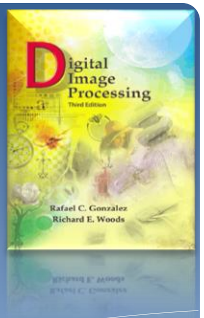
(J.Shanbehzadeh M.Gholizadeh)

Principle Objective of Enhancement



- Process an image so that the result will be more suitable than the original image for a specific application.
- The suitableness is up to each application.
- A method which is quite useful for enhancing an image may not necessarily be the best approach for enhancing another image

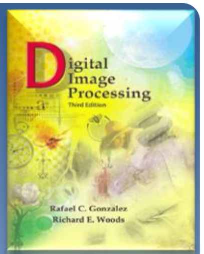
Image Enhancement Methods



- Spatial Domain : (image plane)
  - Techniques are based on direct manipulation of pixels in an image

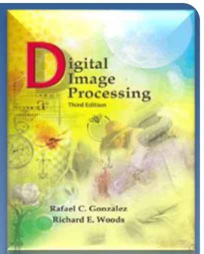
- Frequency Domain :
  - Techniques are based on modifying the Fourier transform of an image

- There are some enhancement techniques based on various combinations of methods from these two categories.



3.1Background

Elements of Visual Perception

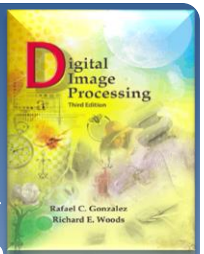


(J.Shanbehzadeh M.Gholizadeh)

The Basics of Intensity Transformations and Spatial Filtering

About the Examples in This Chapter

The Basics of Intensity Transformations and Spatial Filtering

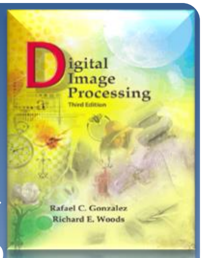


(J.Shanbehzadeh M.Gholizadeh)

- Neighborhood of a point (x,y) can be defined by using a square/rectangular (common used) or circular sub image area centered at (x,y)

- The center of the sub image is moved from pixel to pixel starting at the top of the corner

The Basics of Intensity Transformations and Spatial Filtering



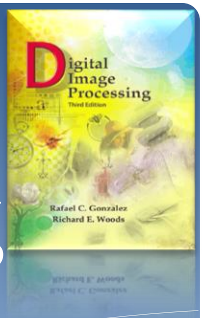
(J.Shanbehzadeh M.Gholizadeh)

- Spatial domain process

$$G(x,y)=T[f(x,y)]$$

Where f(x,y) is the input image, g(x,y) is the processed image, and T is an operator on f, defined over some neighborhood of (x,y)

The Basics of Intensity Transformations and Spatial Filtering



(J.Shanbehzadeh M.Gholizadeh)

- Neighborhood of a point (x,y) can be defined by using a square/rectangular (common used) or circular sub image area centered at (x,y)

- The center of the sub image is moved from pixel to pixel starting at the top of the corner

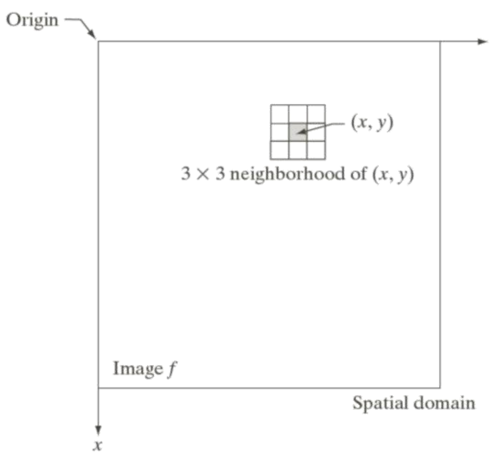


FIGURE 3.1 A 3×3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

The Basics of Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

- Gray-level transformation function

S=T(r)

where r is the gray level of f(x,y) and s is the gray level of g(x,y) at any point (x,y)

Elements of Visual Perception

(J.Shanbehzadeh M.Gholizadeh)

The Basics of Intensity Transformations and Spatial Filtering

About the Examples in This Chapter

3.2 Some Basic Inventory Transformation Functions

Some Basic Intensity Transformation Functions

(J.Shanbehzadeh   M.Gholizadeh)

Image Negatives

Log Transformations

Power-Law(Gamma) Transformations

Piecewise-Linear Transformation Functions

Image Negatives

(J.Shanbehzadeh M.Gholizadeh)

- An image with gray level in the range ***[0, L-1]*** where ***$L = 2^n$*** ; n = 1, 2…
- Negative transformation :

  $s = L - 1 - r$
- Reversing the intensity levels of an image.
- Suitable for enhancing white or gray detail embedded in dark regions of an image, especially when the black area dominant in size.

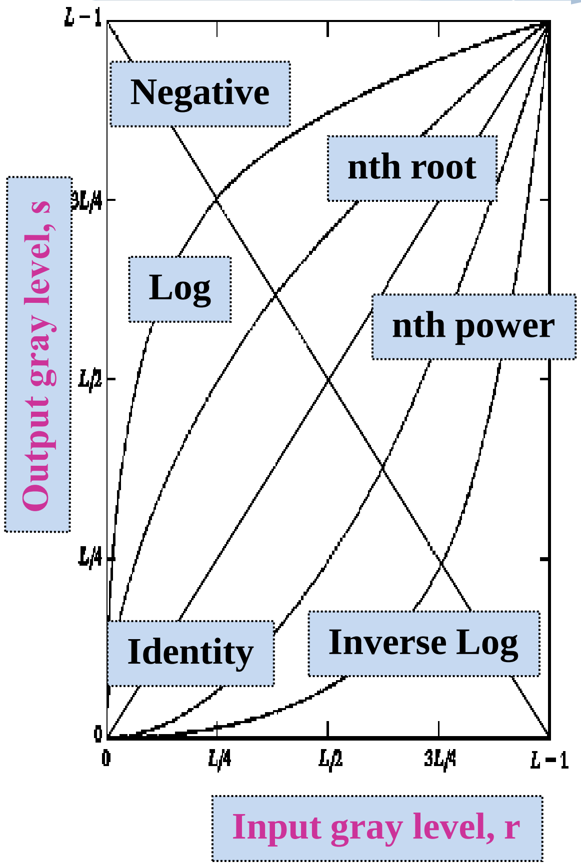
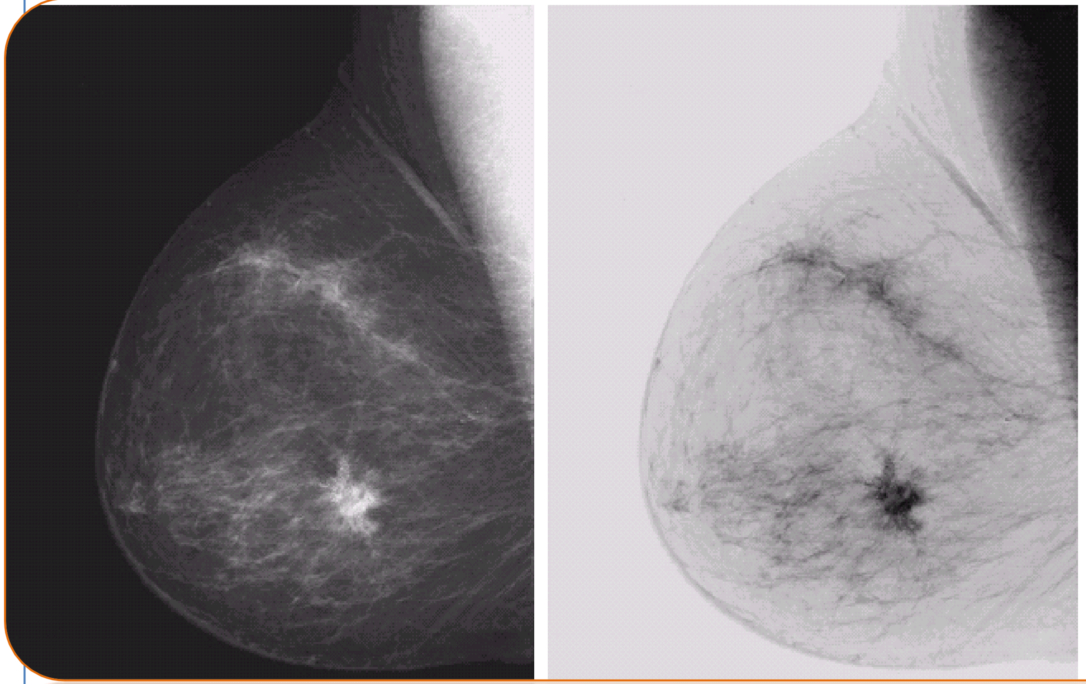


Image Negatives

- **Image negatives:**
  - Enhance white or gray details

(J.Shanbehzadeh M.Gholizadeh)

$$s=L-1-r$$



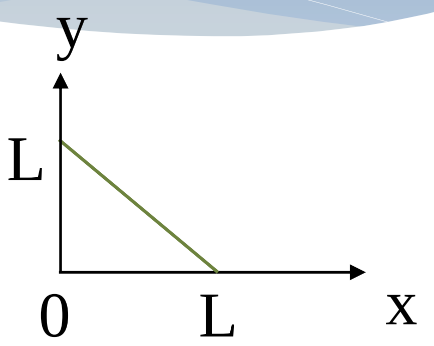
a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Image Negatives

(J.Shanbehzadeh M.Gholizadeh)

$$y=L-x$$



Some Basic Intensity Transformation Functions

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

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3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

Image Negatives

Log Transformations

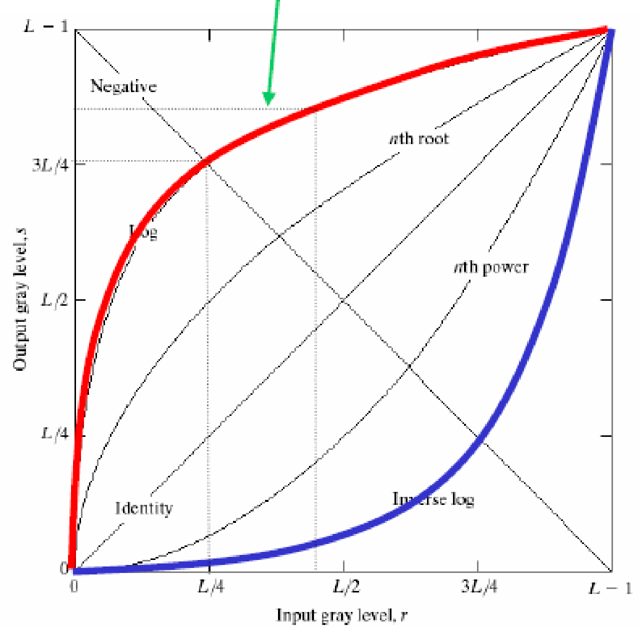
Power-Law(Gamma) Transformations

Piecewise-Linear Transformation Functions

Log Transformations

(J.Shanbehzadeh M.Gholizadeh)

$$s = c\log(1+r)$$

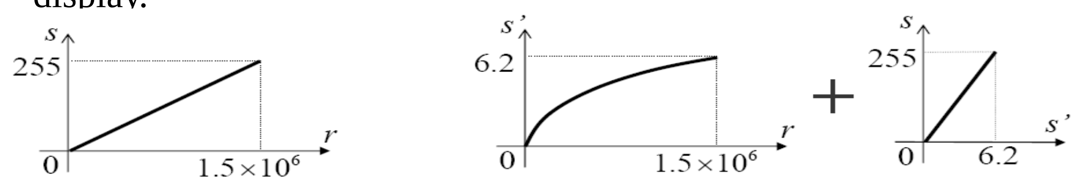


- c is a constant and r ≥ 0
- Log curve maps a narrow range of low gray-level values in the input image into a wider range of output levels.
- Used to expand the values of dark pixels in an image while compressing the higher-level values.

Log Transformations

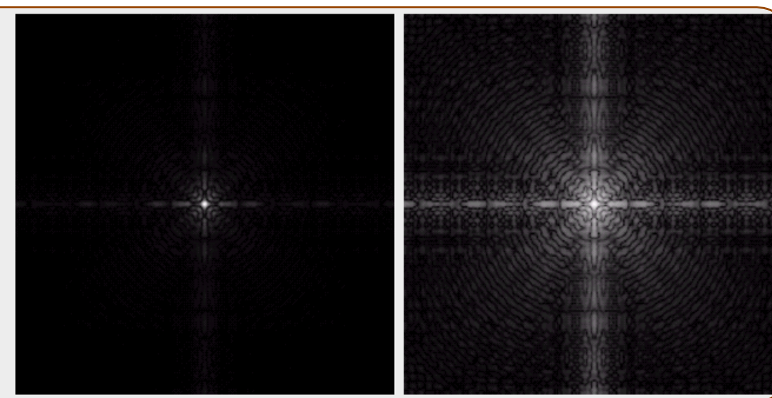
- Compress the dynamic range of images with large variations in pixel values
  - Stretch dark region, suppress bright region

- From the range 0 – 1.5×10^6 to the range 0 to 6.2
- We can't see the significant degree of detail as it will be lost in the display.



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

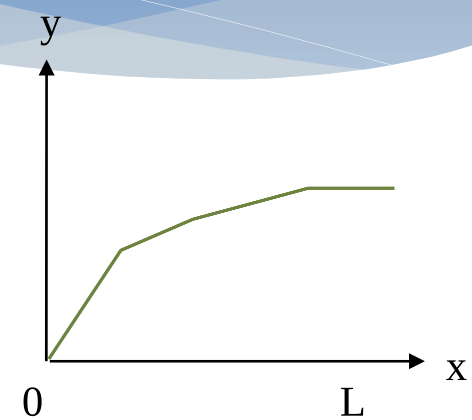


Range Compression

(J.Shanbehzadeh M.Gholizadeh)

$Y=c\log_{10dd}$

$$y = c\log_{10}(1+x)$$



c=100

Some Basic Intensity Transformation Functions

(J.Shanbehzadeh M.Gholizadeh)

- **Image Negatives**
- **Log Transformations**
- **Power-Law(Gamma) Transformations**
- **Piecewise-Linear Transformation Functions**

Power-Law(Gamma) Transformations

(J.Shanbehzadeh M.Gholizadeh)

- Power-law transformations:

  $s = cr^{\gamma}$ or $s = c(r+\varepsilon)^{\gamma}$

  - $\gamma < 1$ maps a narrow range of dark input values into a wider range of output values, while $\gamma > 1$ maps a narrow range of bright input values into a wider range of output values
  - γ : gamma, gamma correction

Power-Law(Gamma) Transformations

(J.Shanbehzadeh M.Gholizadeh)

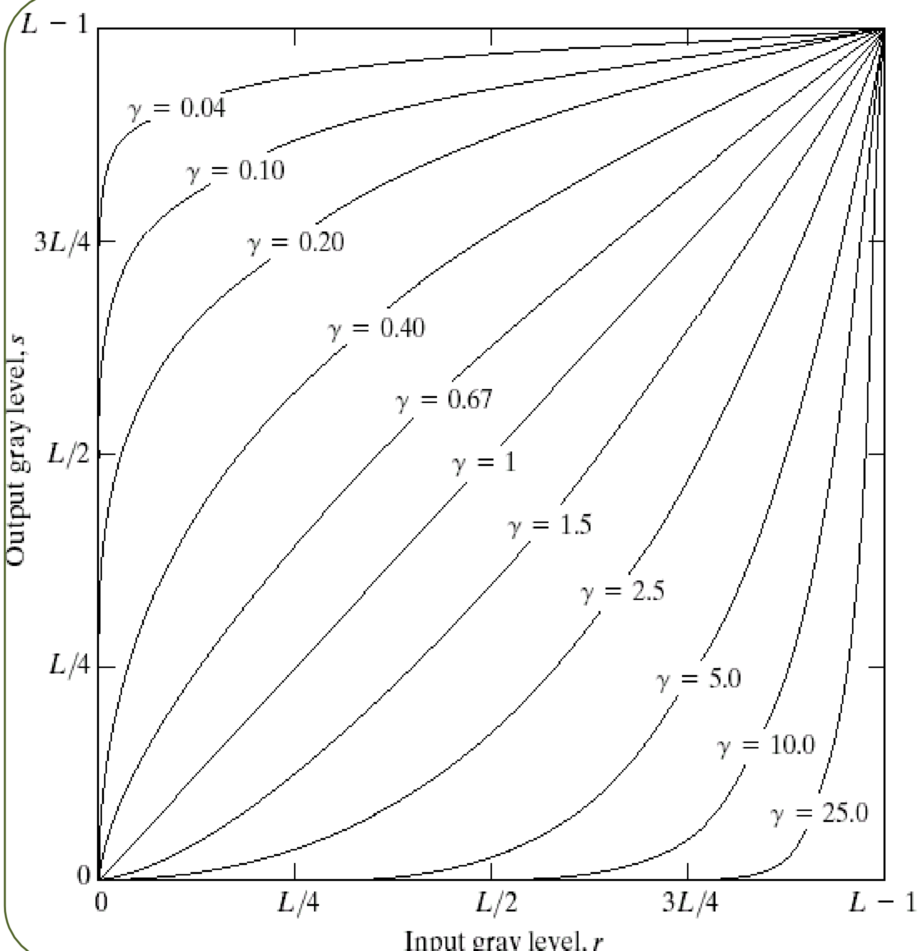
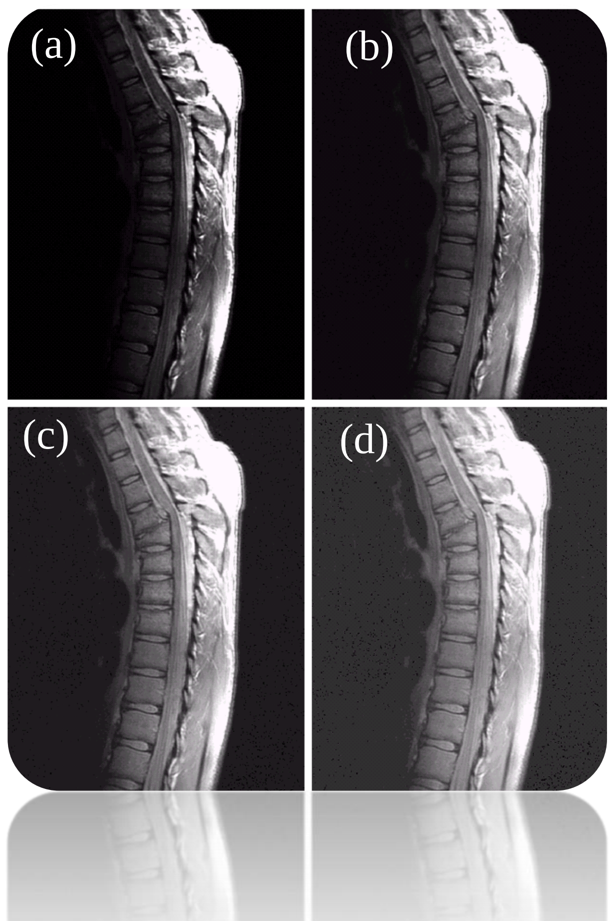


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

Power-Law(Gamma) Transformations

(J.Shanbehzadeh M.Gholizadeh)



(a) a magnetic resonance image(MRI) of an upper thoracic human spine with a fracture dislocation and spinal cord impingement

- The picture is predominately dark
- An expansion of gray levels are desirable ⇨ needs $\gamma < 1$

(b) result after power-law transformation with $\gamma = 0.6$, c=1

(c) transformation with $\gamma = 0.4$ (best result)

(d) transformation with $\gamma = 0.3$ (under acceptable level)

Power-Law(Gamma) Transformations

(Effect of decreasing gamma)

(J.Shanbehzadeh M.Gholizadeh)

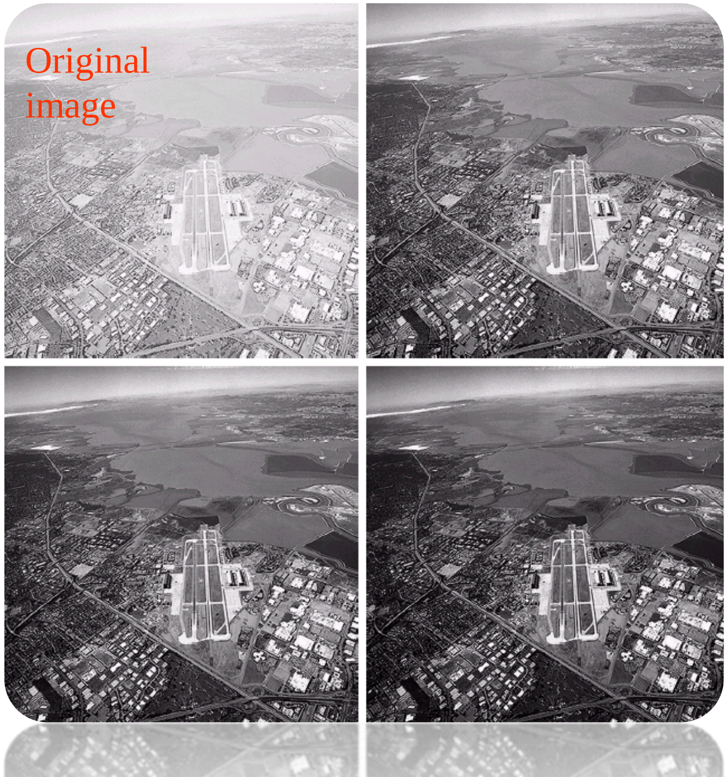
- When the γ is reduced too much, the image begins to reduce contrast to the point where the image started to have very slight "wash-out" look, specially in the background

Power-Law(Gamma) Transformations

(Effect of decreasing gamma)

(J.Shanbehzadeh   M.Gholizadeh)

a	b
c	d



(a) image has a washed-out appearance, it needs a compression of gray levels needs $\gamma > 1$

(b) result after power-law transformation with $\gamma = 3.0$ (suitable)

(c) transformation with $\gamma = 4.0$ (suitable)

(d) transformation with $\gamma = 5.0$ (high contrast, the image has areas that are too dark, some detail is lost)

Some Basic Intensity Transformation Functions

(J.Shanbehzadeh M.Gholizadeh)

Image Negatives

Log Transformations

Power-Law(Gamma) Transformations

Piecewise-Linear Transformation Functions

Piecewise-Linear Transformation Functions

(J.Shanbehzadeh M.Gholizadeh)

- **Advantage:**
  - The form of piecewise functions can be arbitrarily complex.
  - A Practical Implementation of some important transformations can be formulated only as piecewise functions.
- **Disadvantage:**
  - Their specification requires considerably more user input.

Gray-Scale Modification

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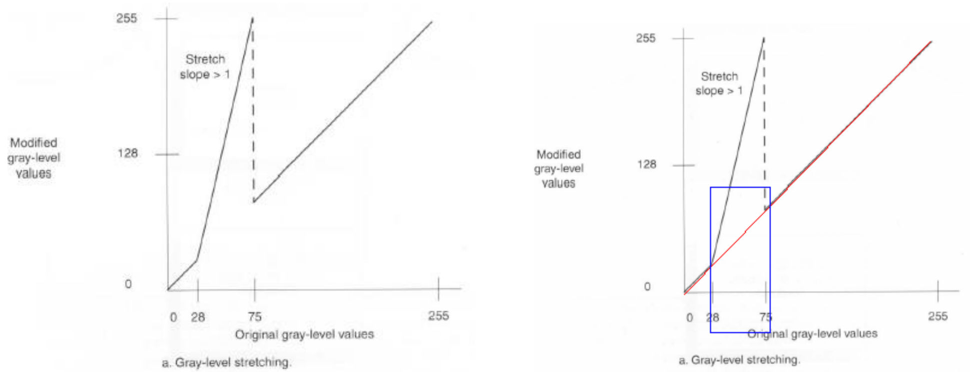
3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)



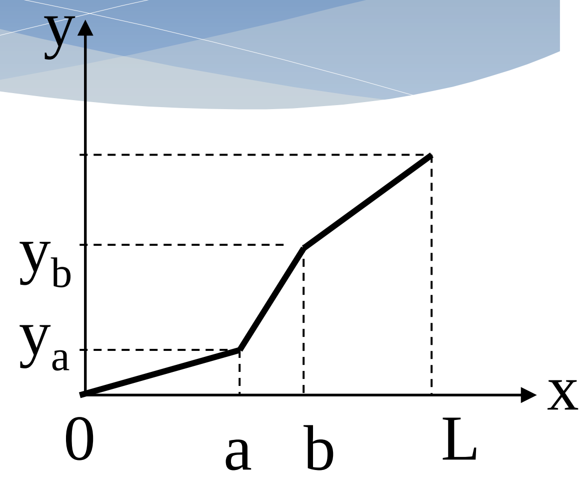
b. Original image.

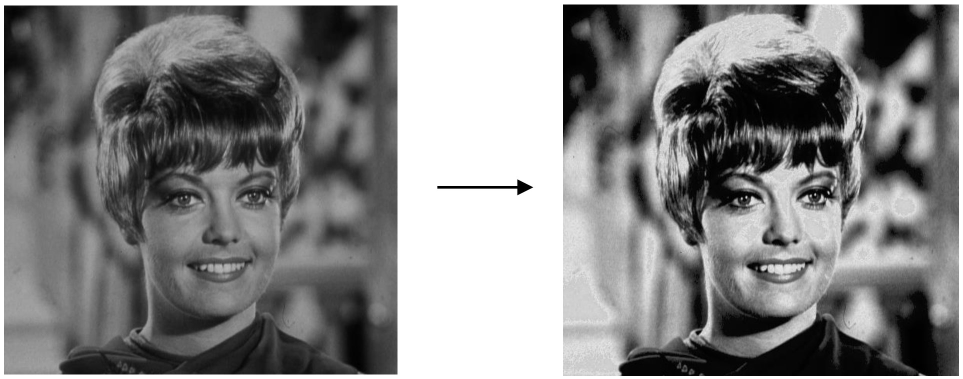
c. Image after modification.

Contrast Stretching

(J.Shanbehzadeh M.Gholizadeh)

$$y = \begin{cases} \alpha x & 0 \le x < a \\ \beta(x-a) + y_a & a \le x < b \\ \gamma(x-b) + y_b & b \le x < L \end{cases}$$

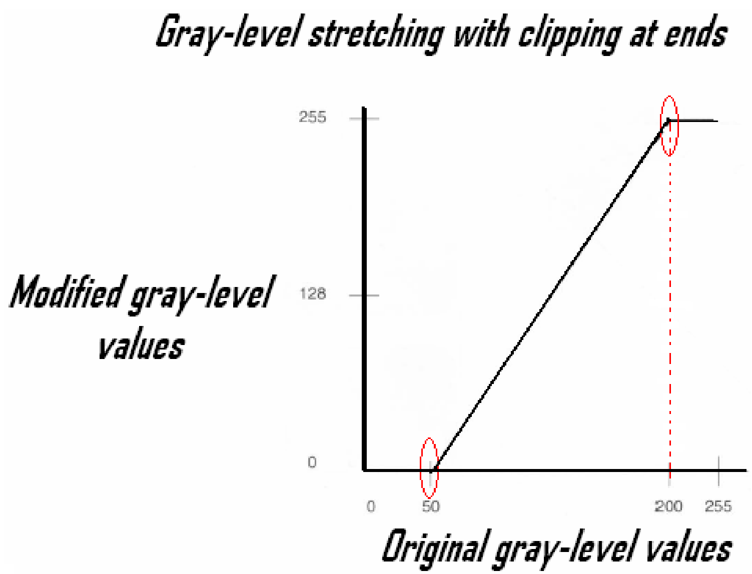




$$a = 50, b = 150, \alpha = 0.2, \beta = 2, \gamma = 1, y_a = 30, y_b = 200$$

Contrast Stretching

(J.Shanbehzadeh M.Gholizadeh)

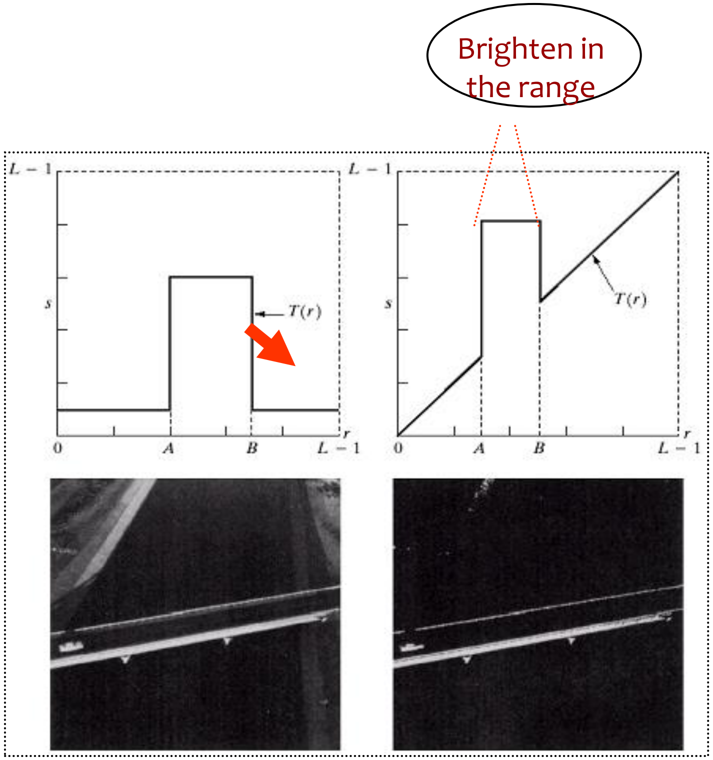


e. Original image.

f. Image after modification.

Gray-level Slicing

(J.Shanbehzadeh M.Gholizadeh)



Highlighting a specific range of gray levels in an image

Display a high value of all gray levels in the range of interest and a low value for all other gray levels

(a) transformation highlights range [A,B] of gray level and reduces all others to a constant level

(b) transformation highlights range [A,B] but preserves all other levels

Gray-level Slicing

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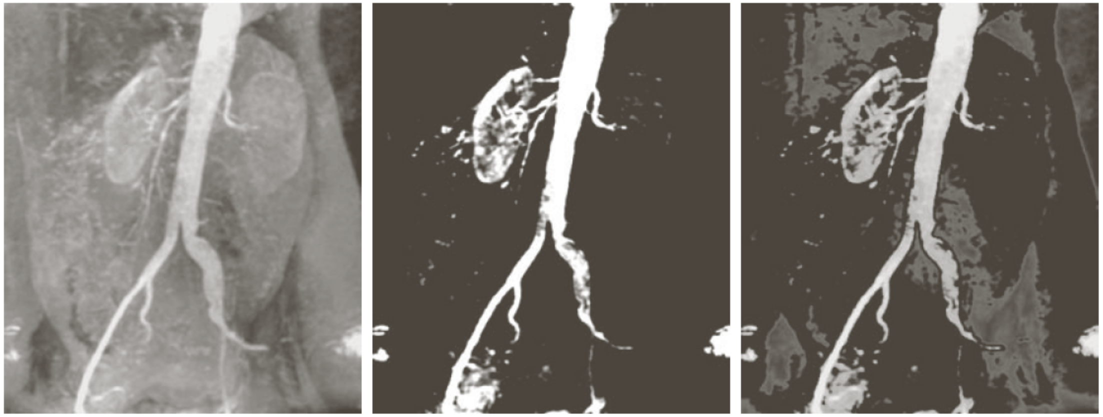
(J.Shanbehzadeh   M.Gholizadeh)

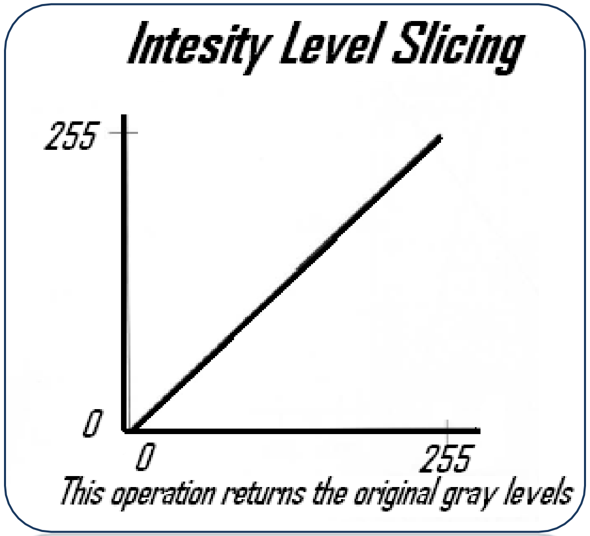


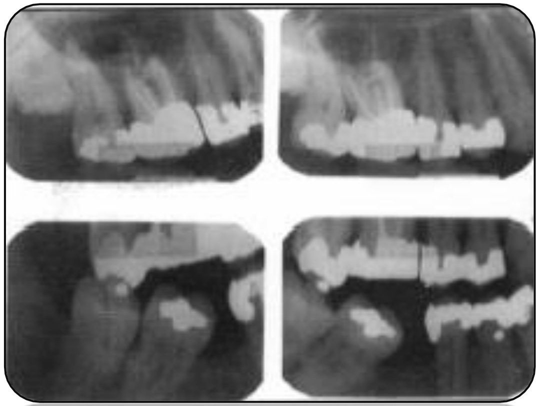
a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Gray-level Slicing

(J.Shanbehzadeh M.Gholizadeh)

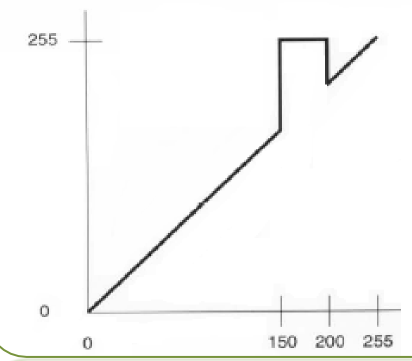


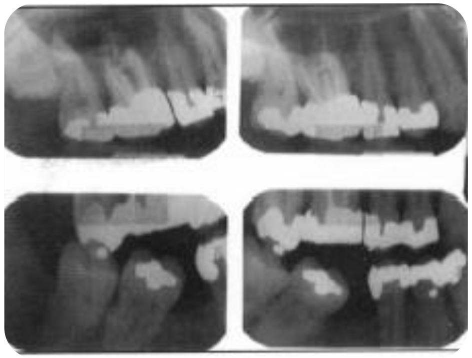


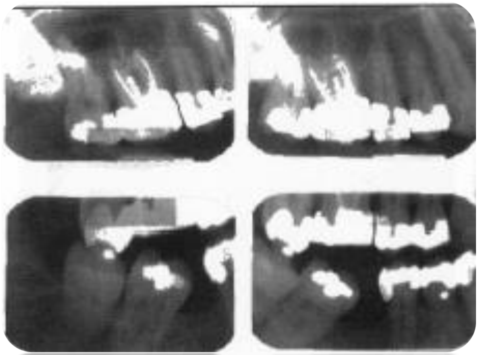
Gray-level Slicing

(J.Shanbehzadeh M.Gholizadeh)

This operation intensifies the desired gray-level range, while not changing the other values





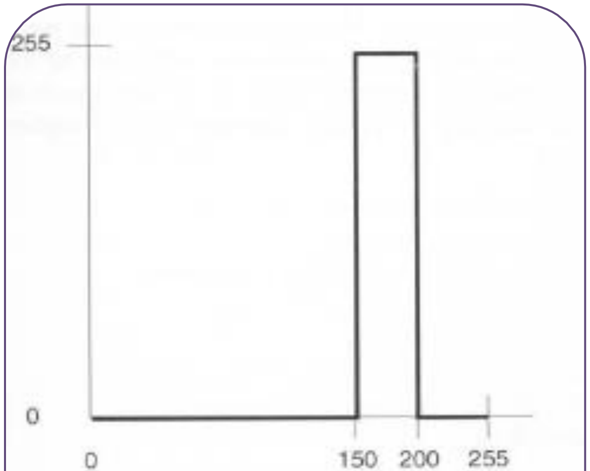


Digital Image Processing

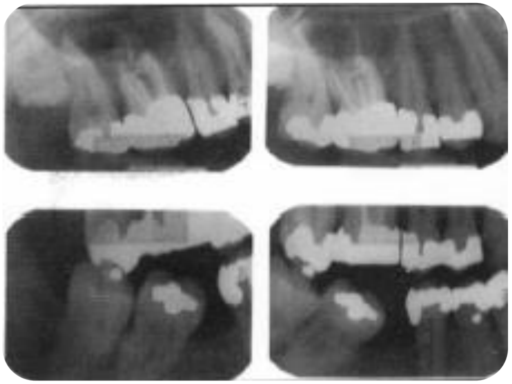
Piecewise-Linear Transformation Functions

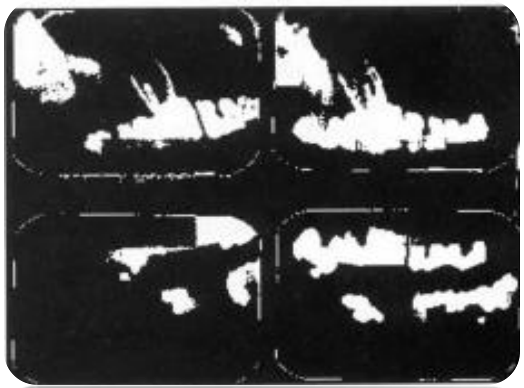
Gray-level Slicing

(J.Shanbehzadeh   M.Gholizadeh)



e. This operation intensifies the desired gray-level range, while changing the other values to black.

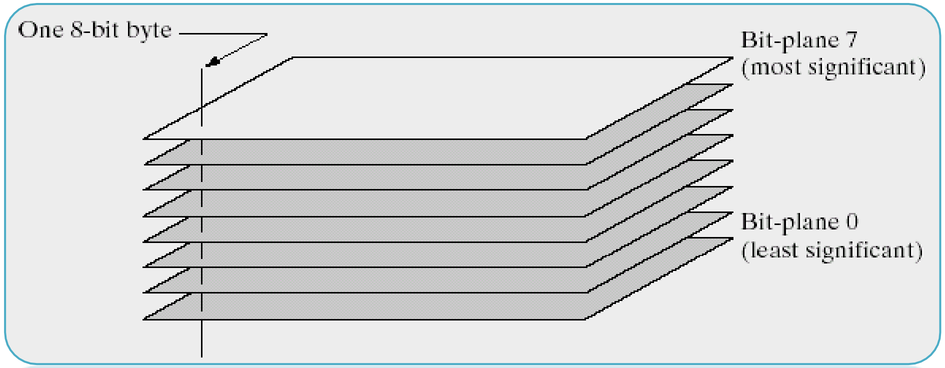




Bit-plane Slicing

(J.Shanbehzadeh M.Gholizadeh)

- Highlighting the contribution made to total image appearance by specific bits
- Suppose each pixel is represented by 8 bits
- Higher-order bits contain the majority of the visually significant data
- Useful for analyzing the relative importance played by each bit of the image



Bit-plane Slicing - Fractal Image

(J.Shanbehzadeh M.Gholizadeh)

- The (binary) image for bit-plane 7 can be obtained by processing the input image with a thresholding gray-level transformation.
  - Map all levels between 0 and 127 to 0
  - Map all levels between 129 and 255 to 255

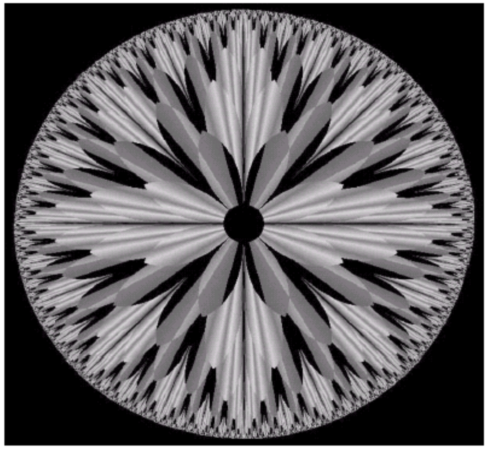


FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

Bit-plane Slicing - Fractal Image

(J.Shanbehzadeh M.Gholizadeh)

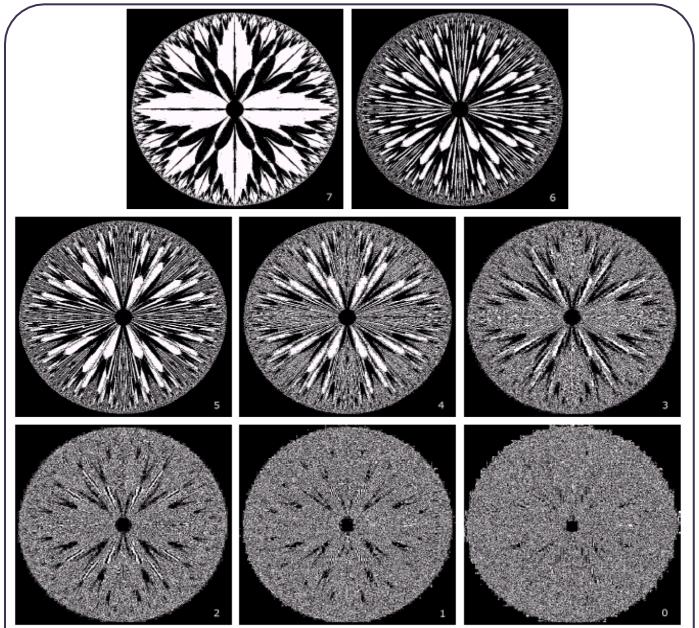


FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

Bit-plane 7	Bit-plane 6	
Bit-plane 5	Bit-plane 4	Bit-plane 3
Bit-plane 2	Bit-plane 1	Bit-plane 0

Bit-plane Slicing

(J.Shanbehzadeh M.Gholizadeh)

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit-plane Slicing

(J.Shanbehzadeh M.Gholizadeh)

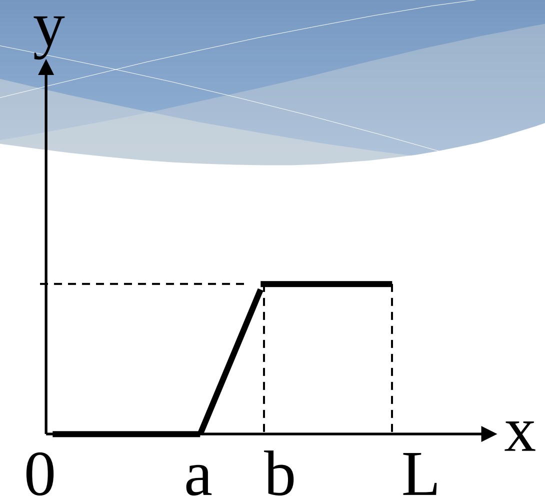
a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Clipping

(J.Shanbehzadeh M.Gholizadeh)

$$y = \begin{cases} 0 & 0 \le x < a \\ \beta(x-a) & a \le x < b \\ \beta(b-a) & b \le x < L \end{cases}$$



$a = 50, b = 150, \beta = 2$

Summary of Point Operation

(J.Shanbehzadeh M.Gholizadeh)

- So far, we have discussed various forms of f(x) that leads to different enhancement results
- The natural question is: How to select an appropriate f(x) for an arbitrary image?
- One systematic solution is based on the histogram information of an image.

3.3 Histogram Processing

Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

- **Histogram Modification**
- **Histogram Equalization**
- **Adaptive Contrast Enhancement (ACE)**
- **Histogram Matching (Specification)**
- **Local Histogram Processing**
- **Using Histogram Statistics for Image Enhancement**

Histogram Processing

(What is Histogram?)

(J.Shanbehzadeh M.Gholizadeh)

- Histogram:

$$\mathbf{h(r_k)=n_k}$$

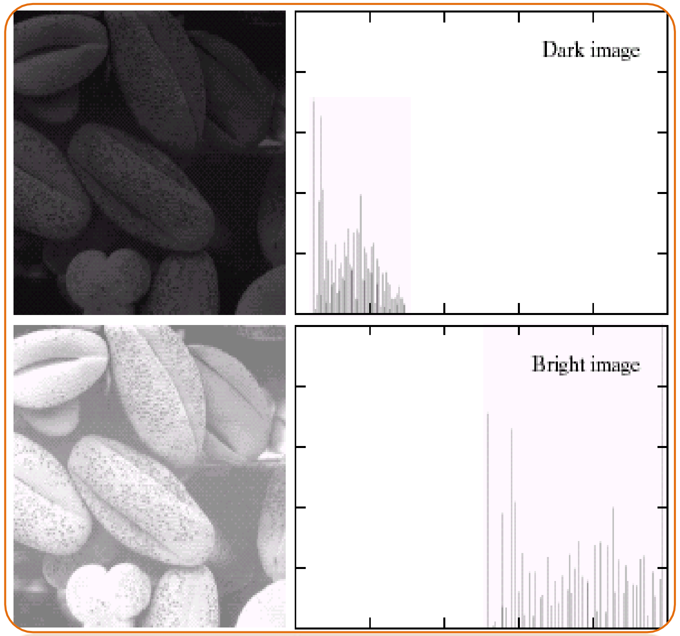
  - Where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k
  - Normalized histogram:

$$\mathbf{P(r_k)=n_k/n}$$

- Histogram of an image represents the relative frequency of occurrence of various gray levels in the image

Histogram Examples

(J.Shanbehzadeh M.Gholizadeh)



Dark image

Components of histogram are concentrated on the low side of the gray scale.

Bright image

Components of histogram are concentrated on the high side of the gray scale.

Histogram Examples

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

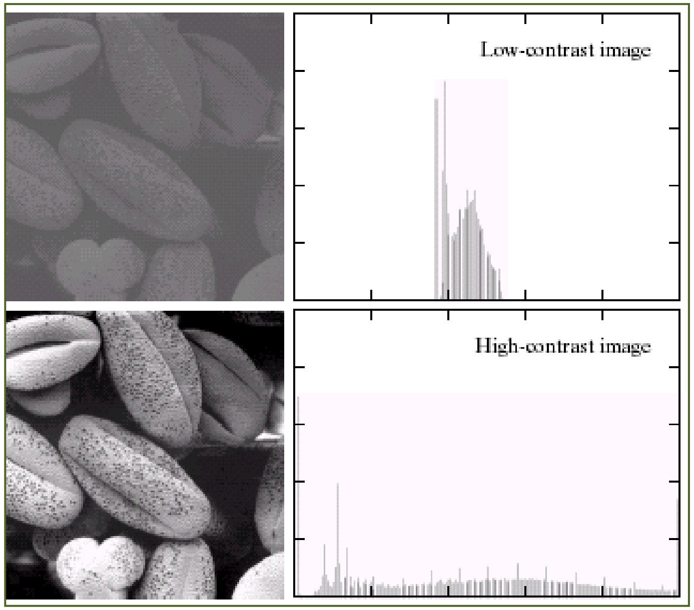
3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)



Low-contrast image

Histogram is narrow and centered toward the middle of the gray scale

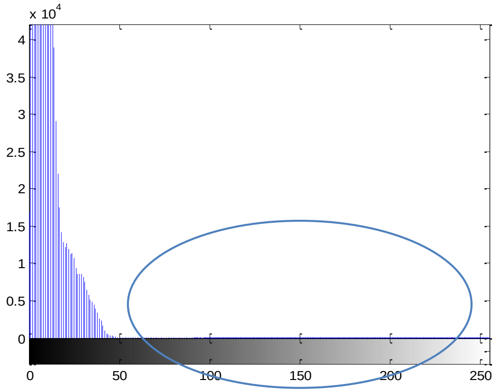
High-contrast image

Histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform, with very few vertical lines being much higher than the others

Why Histogram?

(J.Shanbehzadeh M.Gholizadeh)

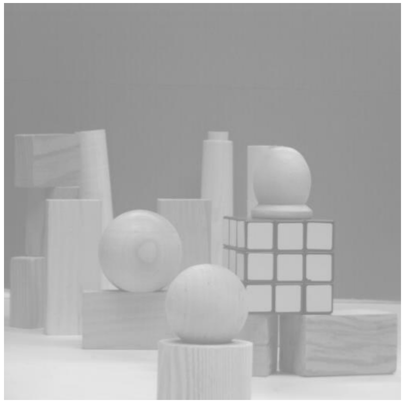
It is a baby in the cradle!



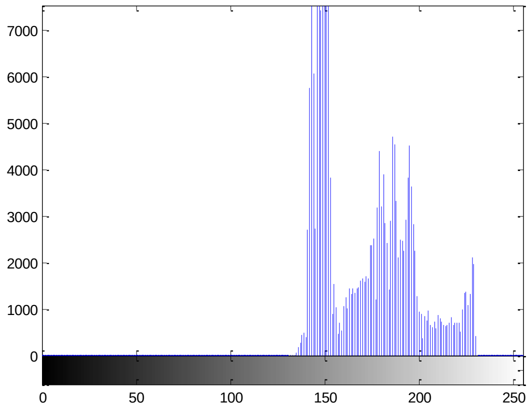
Histogram information reveals that image is under-exposed

Why Histogram?

(J.Shanbehzadeh M.Gholizadeh)



Over-exposed image



Histogram Modification

* Histogram Stretching
* Histogram Shrink
* Histogram Sliding

Histogram Modification

3.1- Background

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3,3- Histogram Processing

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3.5 - Smoothing Spatial Filters

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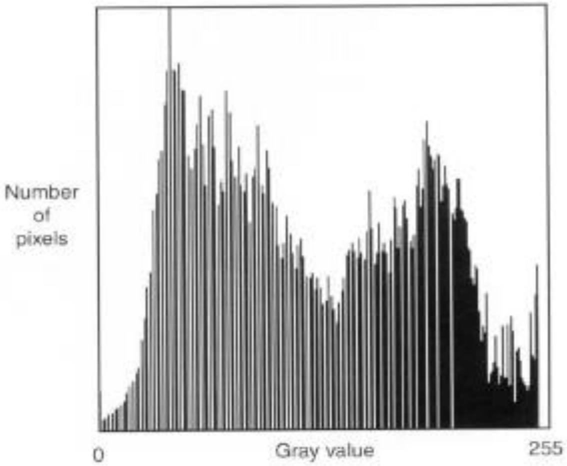
3.7- Combining Spatial Enhancement Tools

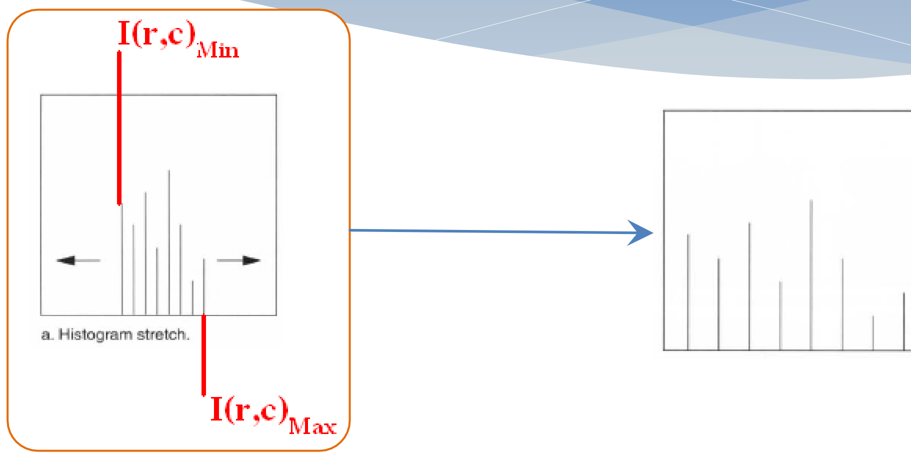
3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh   M.Gholizadeh)



Histogram Stretching

(J.Shanbehzadeh M.Gholizadeh)



$$\text{Stretch}(I(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\text{MIN}}}{I(r,c)_{\text{MAX}} - I(r,c)_{\text{MIN}}}\right][\text{MAX} - \text{MIN}] + \text{MIN}$$

$I(r,c)_{\text{MAX}}$ is the largest gray-level value in the image $I(r,c)$

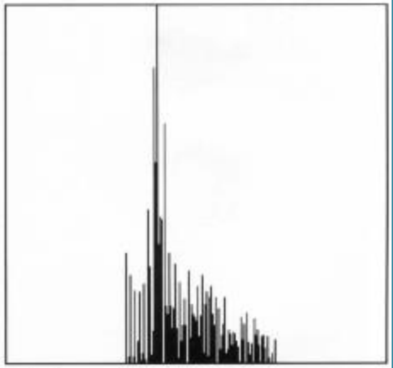
$I(r,c)_{\text{MIN}}$ is the smallest gray-level value in $I(r,c)$

MAX and MIN correspond to the maximum and minimum gray-level values possible (for an 8-bit image these are 0 and 255)

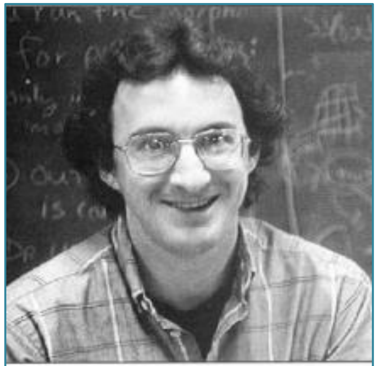
Histogram Stretching

(J.Shanbehzadeh M.Gholizadeh)

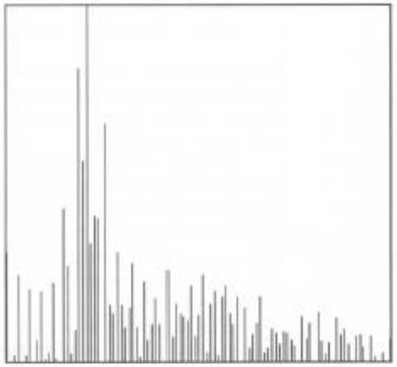
Low-contrast image.



b. Histogram of low-contrast image.



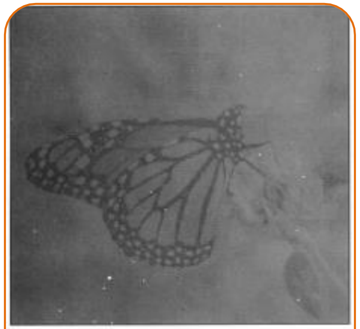
. Image after histogram stretching.



d. Histogram of image after stretching.

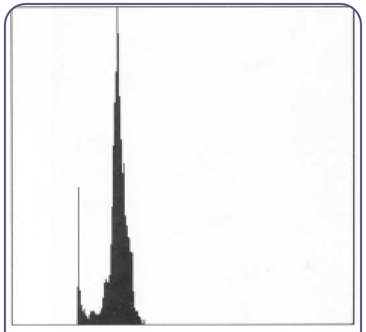
Histogram Stretching

(J.Shanbehzadeh M.Gholizadeh)

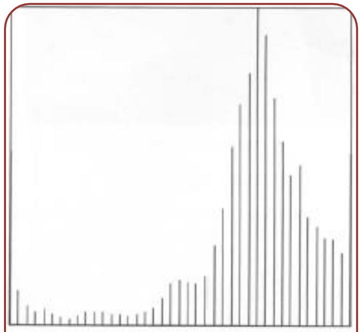


a. Original image.

e. image after histogram stretching with clipping 3% low and high values.



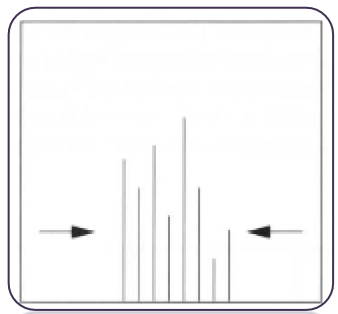
b. Histogram of original image.

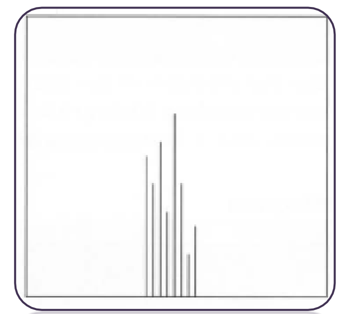


f. Histogram of image (e).

Histogram Shrinking

(J.Shanbehzadeh M.Gholizadeh)





$$\text{Shrink}(I(r,c)) = \left[\frac{\text{Shrink}_{\text{MAX}} - \text{Shrink}_{\text{MIN}}}{I(r,c)_{\text{MAX}} - I(r,c)_{\text{MIN}}}\right]\left[I(r,c) - I(r,c)_{\text{MIN}}\right] + \text{Shrink}_{\text{MIN}}$$

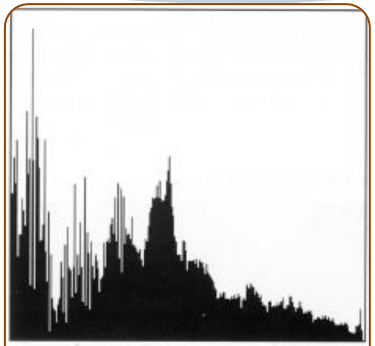
$I(r,c)_{\text{MAX}}$ is the largest gray-level value in the image $I(r,c)$

$I(r,c)_{\text{MIN}}$ is the smallest gray-level value in $I(r,c)$

Histogram Shrinking

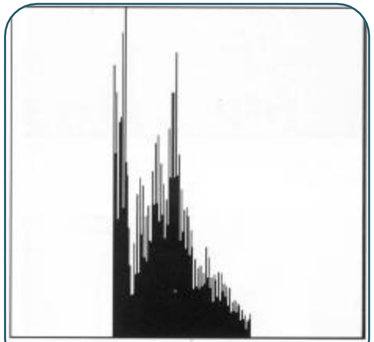
(J.Shanbehzadeh M.Gholizadeh)

a. Original image.



b. Histogram of image (a).

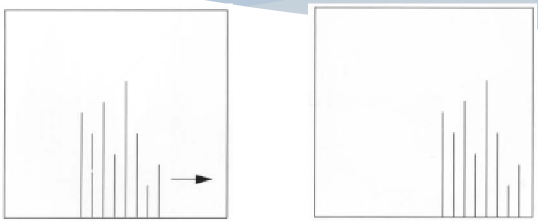
c. Image after shrinking the histogram to the range [75,175].



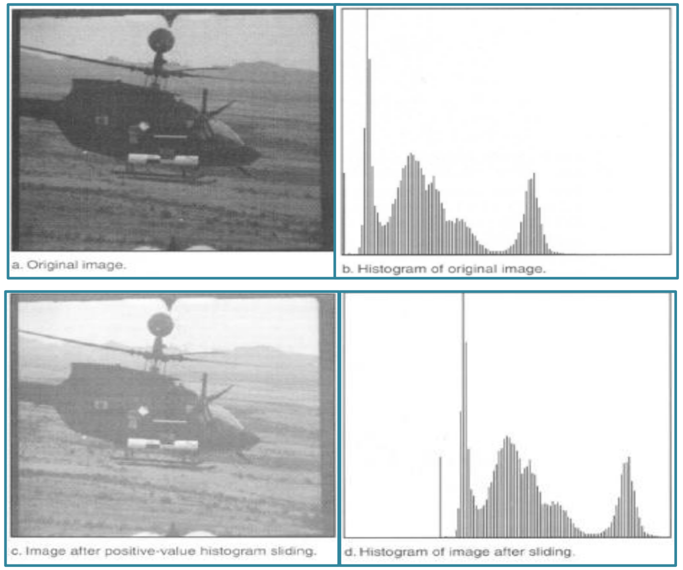
d. Histogram of image (c).

Histogram Sliding

(J.Shanbehzadeh M.Gholizadeh)



$$\text{Slide}(I(r,c)) = I(r,c) + \text{OFFSET}$$



Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

- **Histogram Modification**
- **Histogram Equalization**
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Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

- As the low-contrast image’s histogram is narrow and centered toward the middle of the gray scale, if we distribute the histogram to a wider range the quality of the image will be improved.

- We can do it by adjusting the probability density function of the original histogram of the image so that the probability spread equally

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

- Histogram equalization:

$$s = T(r), \; 0 \le r \le 1$$

$$r = T^{-1}(s), \; 0 \le s \le 1$$

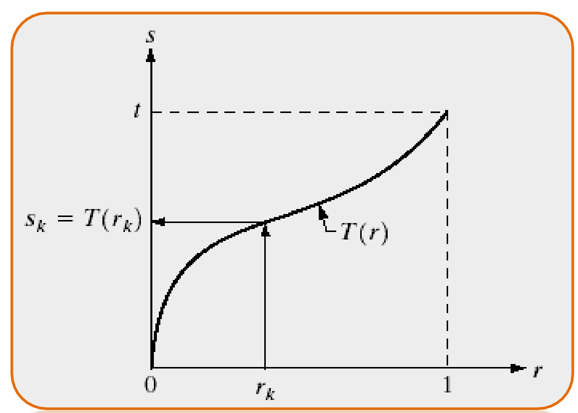
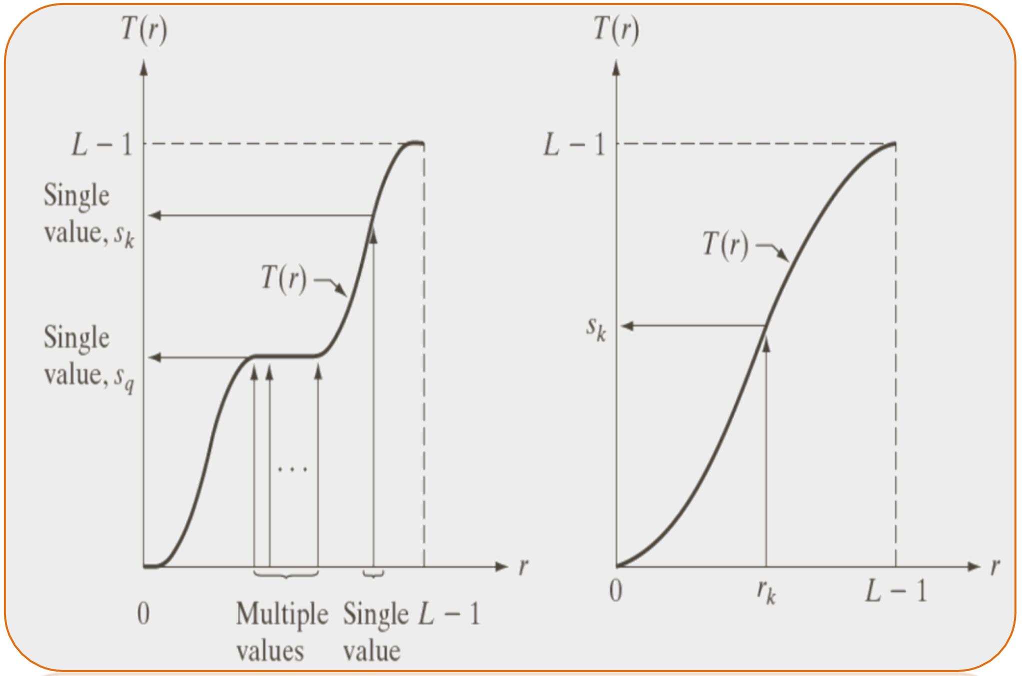


FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)



a b

FIGURE 3.17
(a) Monotonically increasing function, showing how multiple values can map to a single value.
(b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

- Probability density functions (PDF):

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right] = (L-1)p_r(r)$$

$$\Longrightarrow \quad p_s(s) = \frac{1}{L-1}$$

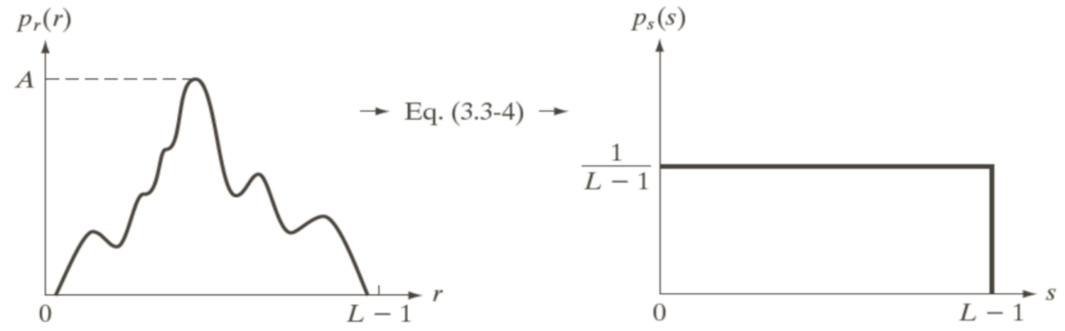
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{n}, \quad k = 0,1,2,\dots,L-1$$

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.



Histogram Equalization

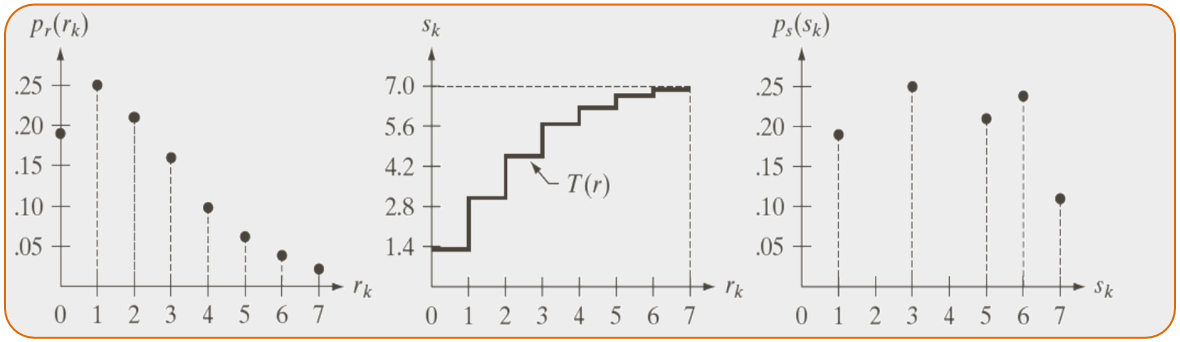
(J.Shanbehzadeh M.Gholizadeh)

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)



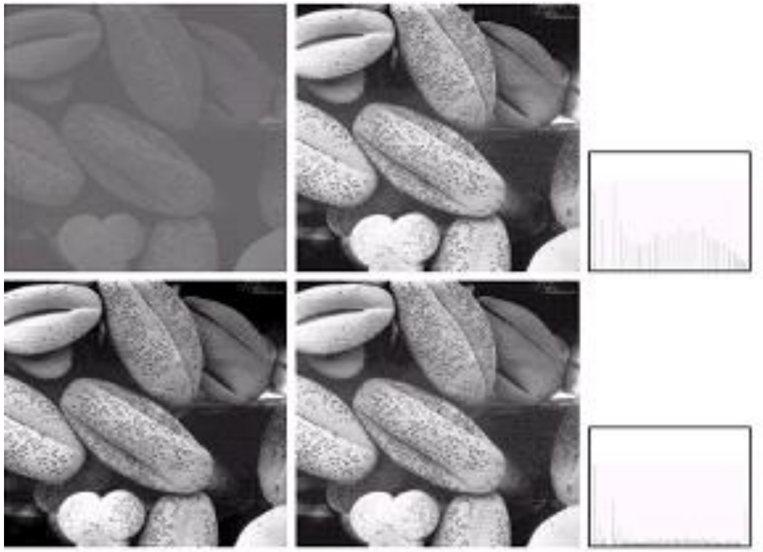
a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

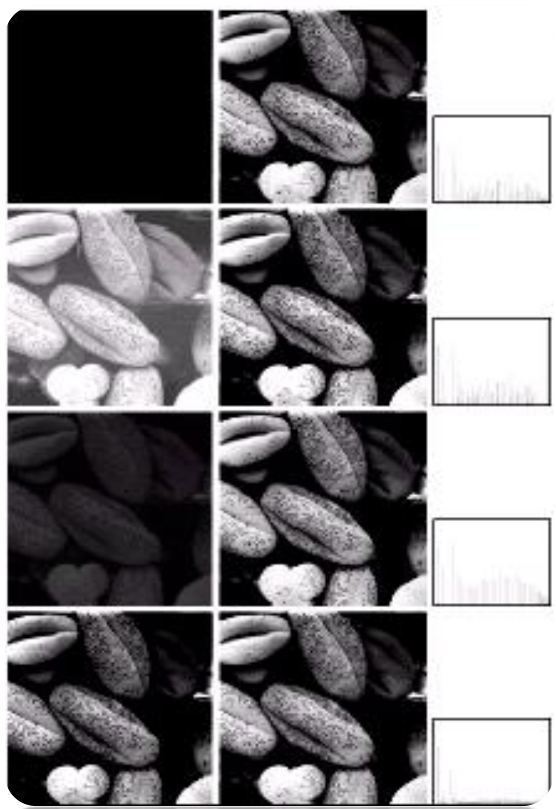
Before After



The quality is not improved much because the original image already has a broaden gray-level scale

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)



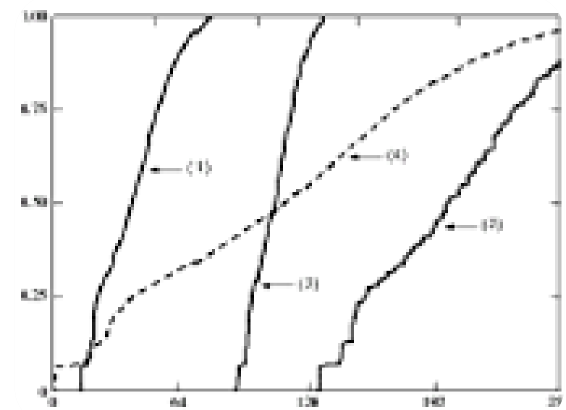
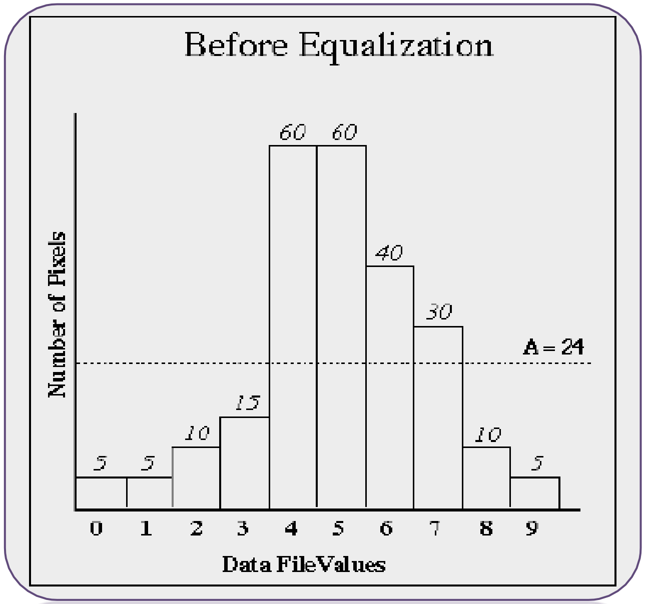


FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)



Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

Total Number of Pixels:
5+5+10+15+60+60+40+30+10+5 = 240.

Histogram

Step 1: H(0) = 5 ; H(1) = 5 ; H(2) = 10 ;
H(3) = 15 ; H(4) = 60 ; H(5) = 60 ;
H(6) = 40 ; H(7) = 30 ; H(8) = 10 ;
H(9) = 5 ;

Cumulative Normalized Histogram

Step 2: T(0) = 5/240 ; T(1) = 10/240 ;
T(2) = 20/240 ; T(3) = 35/240 ;
T(4) = 95/240 ; T(5) = 155/240 ;
T(6) = 195/240 ; T(7) = 225/240 ;
T(8) = 235/240 ; T(9) = 240/240 ;

Histogram Equalization

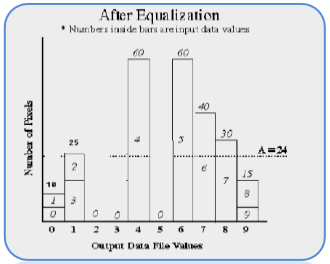
(J.Shanbehzadeh M.Gholizadeh)

transformed intensity $g_k = (L-1) * T(k)$. To encompass the whole dynamic range.

- Step 3: G0 = 0.1875 →0; G1 = 0.3750→0;
  G2 = 0.7500→1 ; G3 = 1.3125→1;
  G4 = 3.5625→4 ; G5 = 5.8125→6;
  G6 = 7.3125→7 ; G7 = 8.4375→8 ;
  G8 = 8.8125→9 ; G9 = 9.0000→9 ;
- Step 4:

Original Intensity →New Intensity → Number

0→0→5; 1→0→5 ; 2→1→10 ;
3→1→15; 4→4→60; 5→6→60 ;
6→7→40; 7→8→30; 8→9→10;
9→9→5;



Histogram Equalization

- 3.1- Background
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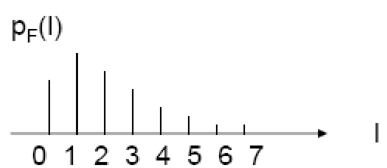
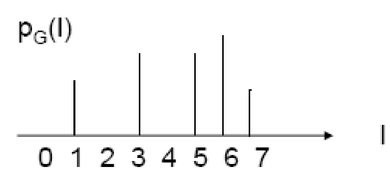
- 3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

f_k	$p_F(l)$	$\tilde{g}_l = \sum_{k=0}^{l} p_F(k)$	$g_l = [\tilde{g}_l * 7]$	$p_G(l)$	g_k
0	0.19	0.19	[1.33]=1	0	0
1	0.25	0.44	[3.08]=3	0.19	1
2	0.21	0.65	[4.55]=5	0	2
3	0.16	0.81	[5.67]=6	0.25	3
4	0.08	0.89	[6.03]=6	0	4
5	0.06	0.95	[6.65]=7	0.21	5
6	0.03	0.98	[6.86]=7	0.16+0.08=0.24	6
7	0.02	1.00	[7]=7	0.06+0.03+0.02=0.11	7

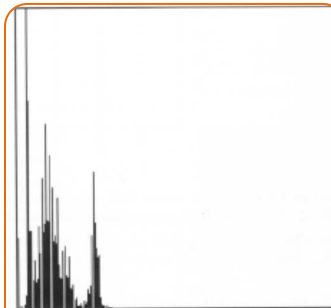




Histogram Equalization

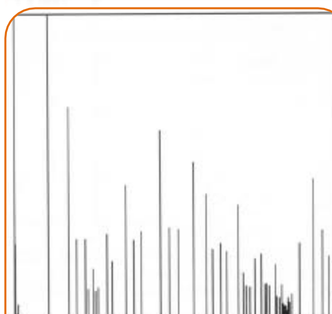
(J.Shanbehzadeh M.Gholizadeh)

a. Original dark image.



b. Histogram of image (a).

c. Dark image after histogram equalization.

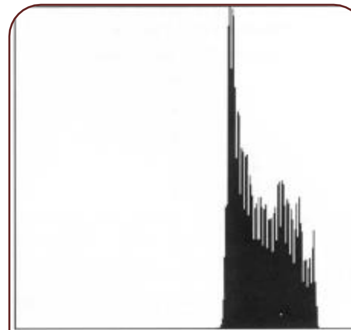


d. Histogram of image (c).

Histogram Equalization

(J.Shanbehzadeh M.Gholizadeh)

e. Original light image.



f. Histogram of image (e).

g. Light image after histogram equalization.

h. Histogram of image (g).

Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

Histogram Modification

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Histogram Matching (Specification)

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Using Histogram Statistics for Image Enhancement

Adaptive Contrast Enhancement (ACE)

(J.Shanbehzadeh M.Gholizadeh)

$$\mathrm{ACE} = k_1 \left[\frac{m_{I(r,c)}}{\sigma_l(r,c)} \right] \left[I(r,c) - m_l(r,c) \right] + k_2\, m_l(r,c)$$

$m_{I(r,c)}$ = is the mean for the entire image $I(r,c)$

σ_l = local standard deviation (in the window under consideration)

m_l = local mean (average in the window under consideration)

k_1, k_2 = constants, vary between 0 and 1

Adaptive Contrast Enhancement (ACE)

(J.Shanbehzadeh M.Gholizadeh)

a. Original image.

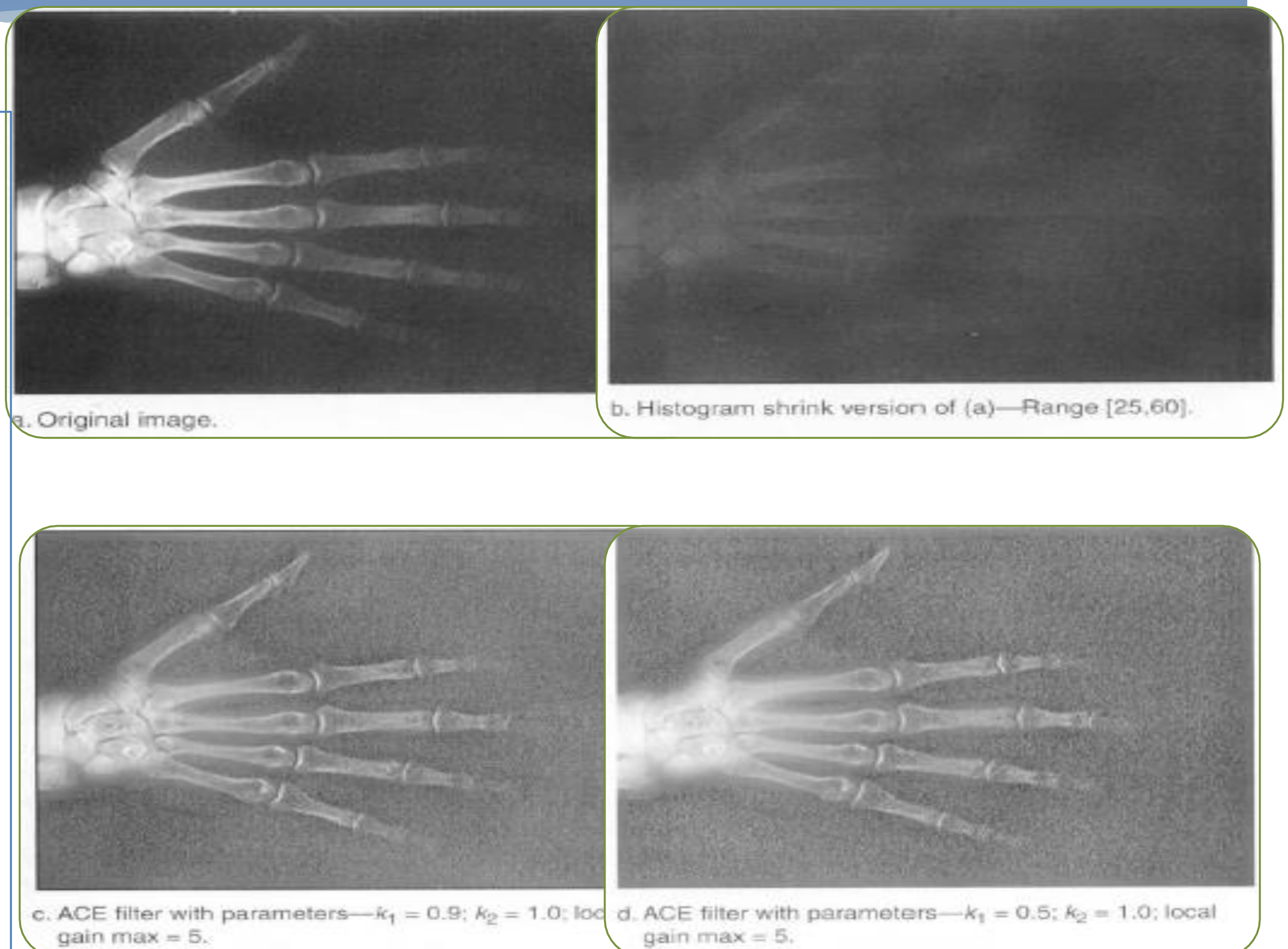
b. Image after global histogram equalization.

c. Image after local histogram equalization, window = 7 × 7.

d. Image after local histogram equalization, window = 15 × 15.

Adaptive Contrast Enhancement (ACE)

(J.Shanbehzadeh M.Gholizadeh)



a. Original image.

b. Histogram shrink version of (a)—Range [25,60].

c. ACE filter with parameters—$k_1 = 0.9$; $k_2 = 1.0$; loc gain max = 5.

d. ACE filter with parameters—$k_1 = 0.5$; $k_2 = 1.0$; local gain max = 5.

Adaptive Contrast Enhancement (ACE)

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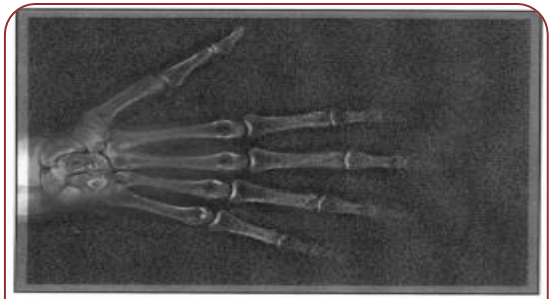
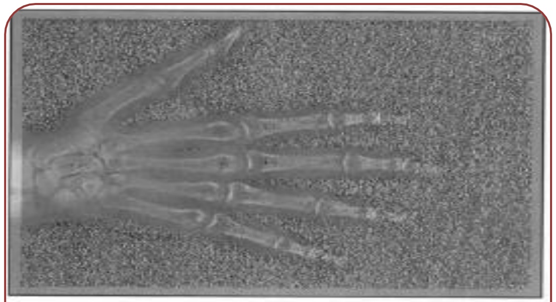
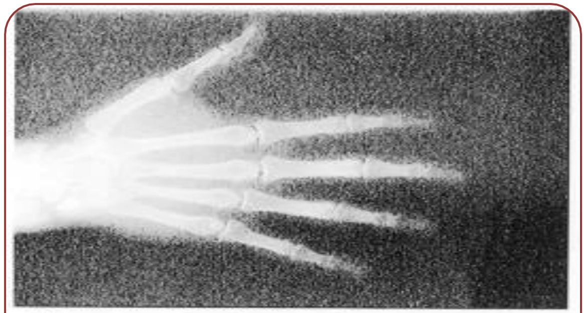
(J.Shanbehzadeh M.Gholizadeh)



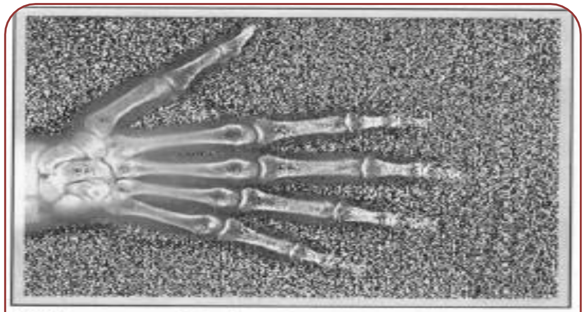
e. ACE filter with parameters—$k_1 = 0.9$; $k_2 = 0.5$; local gain max = 5.



f. ACE filter with parameters—$k_1 = 0.9$; $k_2 = 0.5$; local gain max = 25.



g. Histogram equalized version of original image (a).



h. Histogram equalized version of image (f).

Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

Histogram Modification

Histogram Equalization

Adaptive Contrast Enhancement (ACE)

Histogram Matching (Specification)

Local Histogram Processing

Using Histogram Statistics for Image Enhancement

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

- Histogram equalization has a disadvantage which is that it can generate only one type of output image.
- With Histogram Specification, we can specify the shape of the histogram that we wish the output image to have.
- It doesn't have to be a uniform histogram

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

- Given a desired histogram, the goal is to transform the image intensity such that the **transformed image has a histogram matches the desired histogram**.
- Let the initial image histogram be p_r, the desired image histogram be p_z.
- Let ***T*** be the function that equalizes the original image and ***G*** be the function that equalizes the desired image.
  - – r: The initial image intensity.
  - – s: The image intensity after equalization
  - – z: The image intensity of the desired image.
  - – v: The image intensity after equalization of the desired image.

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

- Histogram matching (specification)

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}(s) = G^{-1}[T(r)]$$

$\Rightarrow$ $p_z(z)$ is the desired PDF

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k} \frac{n_j}{n}, \quad k = 0,1,2,\ldots, L-1$$

$$v_k = G(z_k) = (L-1)\sum_{i=0}^{k} p_z(z_i) = s_k, \quad k = 0,1,2,\ldots, L-1$$

$$\Rightarrow \quad z_k = G^{-1}[T(r_k)], \quad k = 0,1,2,\ldots, L-1$$

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

- Histogram matching:
  - Obtain the histogram of the given image, T(r)
  - Precompute a mapped level S_k for each level r_k
  - Obtain the transformation function G from the given $p_z(z)$
  - Precompute Z_k for each value of r_k
  - Map r_k to its corresponding level S_k; then map level S_k into the final level Z_k

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

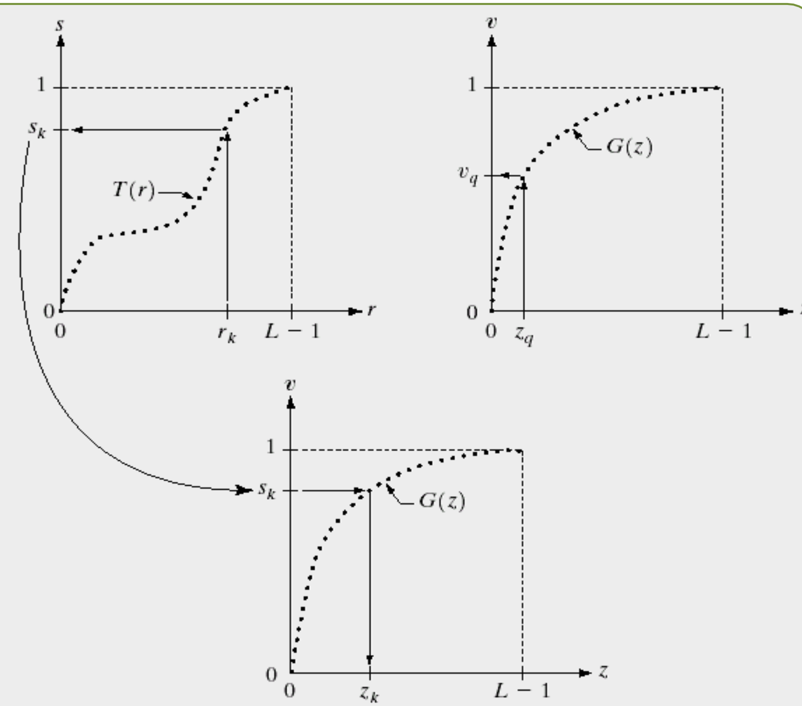


FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via $T(r)$. (b) Mapping of z_q to its corresponding value v_q via $G(z)$. (c) Inverse mapping from s_k to its corresponding value of z_k.

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

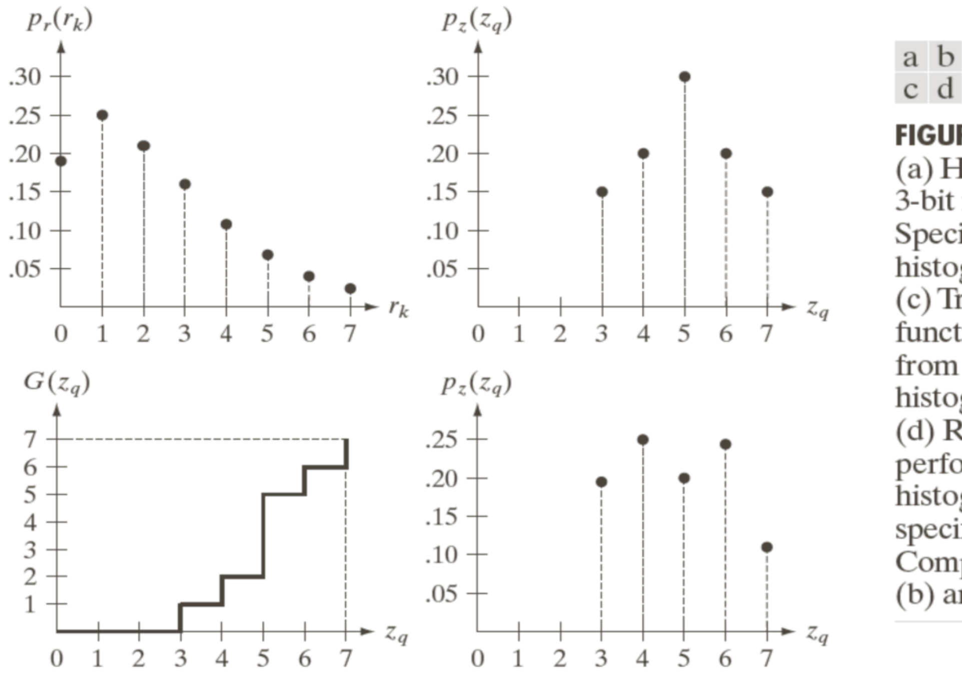


FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2
Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

TABLE 3.3
All possible values of the transformation function G scaled, rounded, and ordered with respect to z.

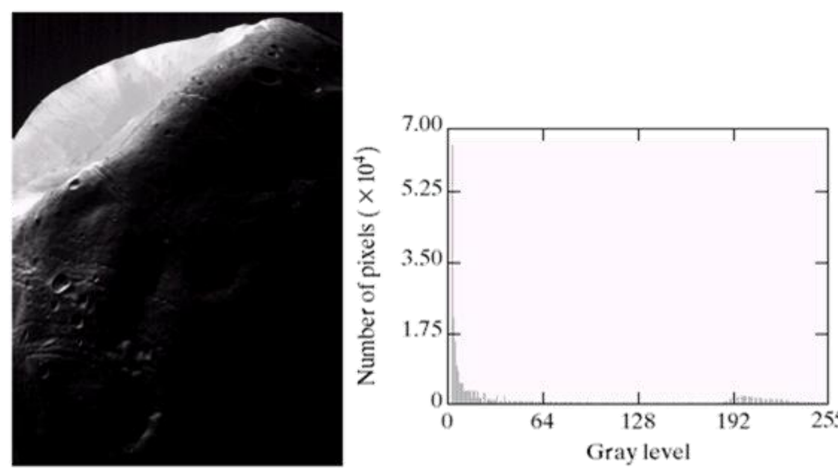
s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

TABLE 3.4
Mappings of all the values of s_k into corresponding values of z_q.

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

Image is dominated by large, dark areas, resulting in a histogram characterized by a large concentration of pixels in pixels in the dark end of the gray scale

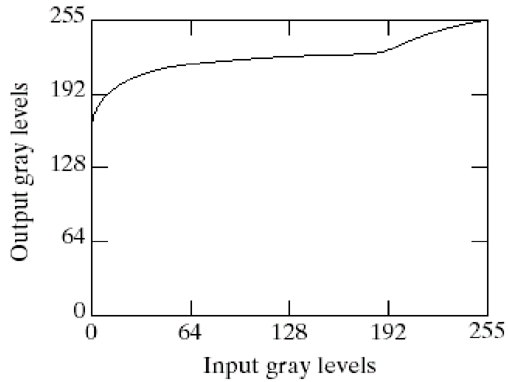


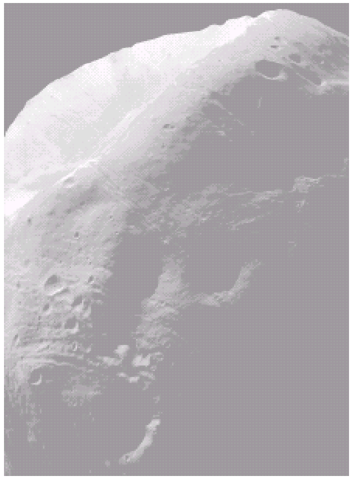
a b

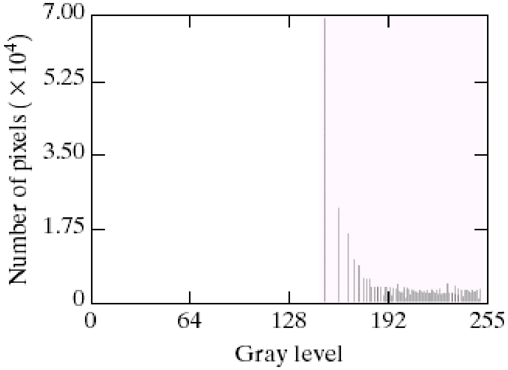
FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)







a b
c

FIGURE 3.21 (a) Transformation function for histogram equalization. (b) Histogram-equalized image (note the washed-out appearance). (c) Histogram of (b).

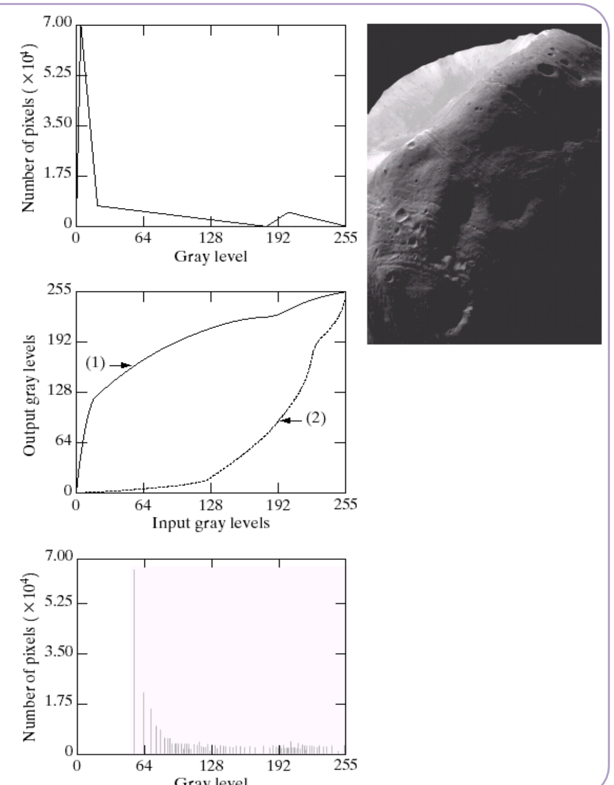
Notice that the output histogram's low end has shifted right toward the lighter region of the gray scale as desired.

Histogram Matching (Specification)

(J.Shanbehzadeh M.Gholizadeh)

a c
b
d

FIGURE 3.22
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).



Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

Histogram Modification

Histogram Equalization

Adaptive Contrast Enhancement (ACE)

Histogram Matching (Specification)

Local Histogram Processing

Using Histogram Statistics for Image Enhancement

Local Histogram Processing

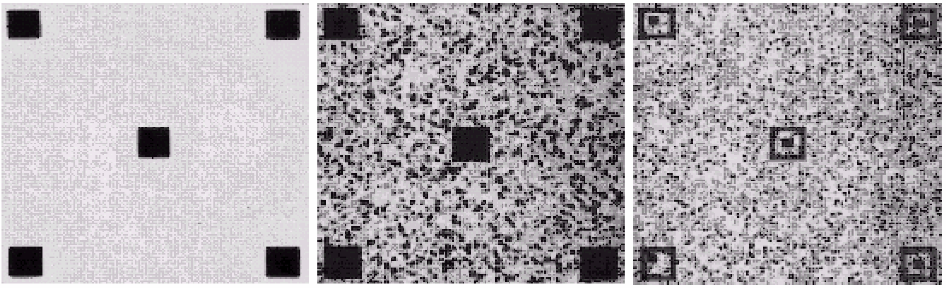
(J.Shanbehzadeh M.Gholizadeh)

- Histogram specification is a trial-and-error process
- There are no rules for specifying histograms, and one must resort to analysis on a case-by-case basis for any given enhancement task.
- Histogram processing methods are global processing, in the sense that pixels are modified by a transformation function based on the gray-level content of an entire image.
- Sometimes, we may need to enhance details over small areas in an image, which is called a local enhancement.

Local Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

- Local enhancement:
  - Histogram using a local neighborhood, for example 7*7 neighborhood



a) Original image (slightly blurred to reduce noise)

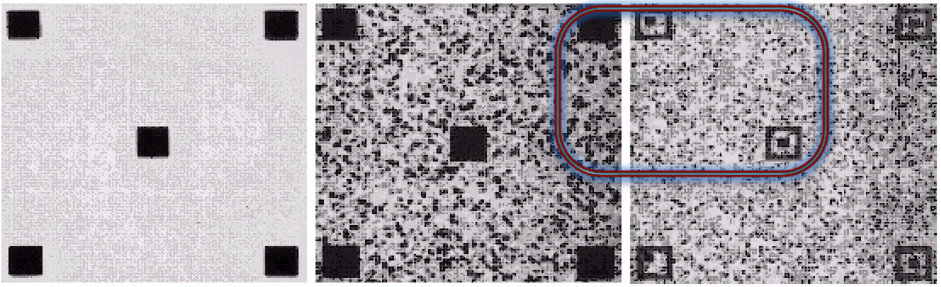
b) global histogram equalization (enhance noise & slightly increase contrast but the construction is not changed)

c) local histogram equalization using 7x7 neighborhood (reveals the small squares inside larger ones of the original image.

- define a square or rectangular neighborhood and move the center of this area from pixel to pixel.
- at each location, the histogram of the points in the neighborhood is computed and either histogram equalization or histogram specification transformation function is obtained.
- another approach used to reduce computation is to utilize no overlapping regions, but it usually produces an undesirable checkerboard effect.

Local Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

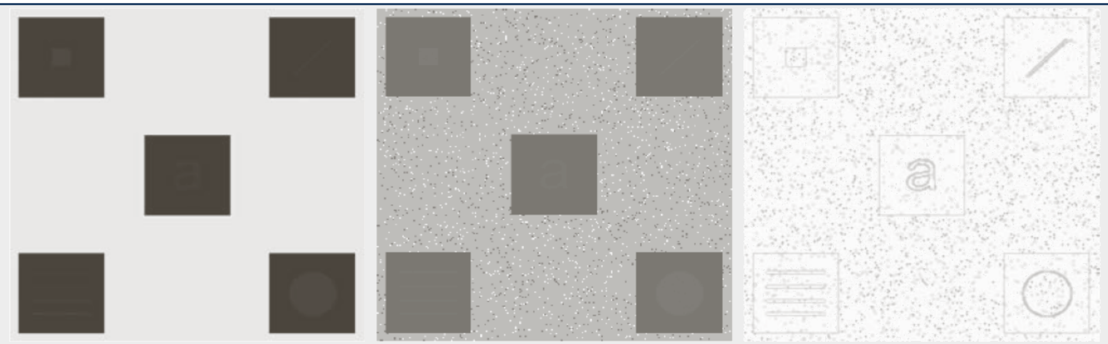


- Basically, the original image consists of many small squares inside the larger dark ones.
- However, the small squares were too close in gray level to the larger ones, and their sizes were too small to influence global histogram equalization significantly.
- So, when we use the local enhancement technique, it reveals the small areas.
- Note also the finer noise texture is resulted by the local processing using relatively small neighborhoods.

Local Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

- Histogram using a local 3*3 neighborhood



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

Histogram Processing

(J.Shanbehzadeh M.Gholizadeh)

Histogram Modification

Histogram Equalization

Adaptive Contrast Enhancement (ACE)

Histogram Matching (Specification)

Local Histogram Processing

Using Histogram Statistics for Image Enhancement

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)

- Use of histogram statistics for image enhenancement:
  - r denotes a discrete random variable
  - $P(r_i)$ denotes the normalized histogram component corresponding to the i^{th} value of r
  - Mean: $$m = \sum_{i=0}^{L-1} r_i p(r_i)$$
  - The n^{th} moment: $$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$
  - The second moment: $$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)

- **Global enhancement**: The global mean and variance are measured over an entire image
- **Local enhancement**: The local mean and variance are used as the basis for making changes

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)

- $r_{s,t}$ is the gray level at coordinates (s,t) in the neighborhood
- $P(r_{s,t})$ is the neighborhood normalized histogram component
- mean: $$m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t}\, p(r_{s,t})$$
- local variance: $$\sigma^2_{S_{xy}} = \sum_{(s,t)\in S_{xy}} [r_{s,t} - m_{S_{xy}}]^2\, p(r_{s,t})$$

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)

- E, K_0, K_1, K_2 are specified parameters
- M_G is the global mean
- D_G is the global standard deviation

- Mapping:

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{if } m_{S_{xy}} \le k_0 M_G \\ & \text{and } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

Using Histogram Statistics

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

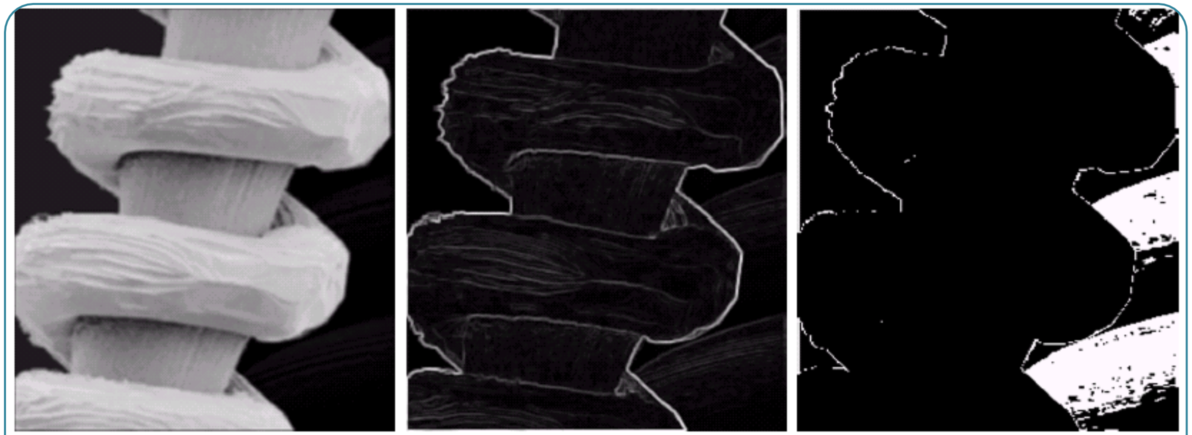
3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

Using Histogram Statistics

(J.Shanbehzadeh M.Gholizadeh)

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

3.4 Fundamentals of Spatial Filtering

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

The Mechanics of Spatial Filtering

Spatial Correlation and Convolution

Vector Representation of Linear Filtering

Generating Spatial Filter Masks

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

- The Mechanics of Spatial Filtering:

$$
\begin{aligned}
R = {} & w(-1,-1)\,f(x-1,y-1) + \\
& w(-1,0)\,f(x-1,y) + \cdots + \\
& w(0,0)\,f(x,y) + \cdots + \\
& w(1,0)\,f(x+1,y) + \\
& w(1,1)\,f(x+1,y+1)
\end{aligned}
$$

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

- Image size: M×N
- Mask size: m×n

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)\,f(x+s,y+t)$$

- a=(m-1)/2 and b=(n-1)/2
- x= 0,1,2,…,M-1 and y= 0,1,2,…,N-1

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

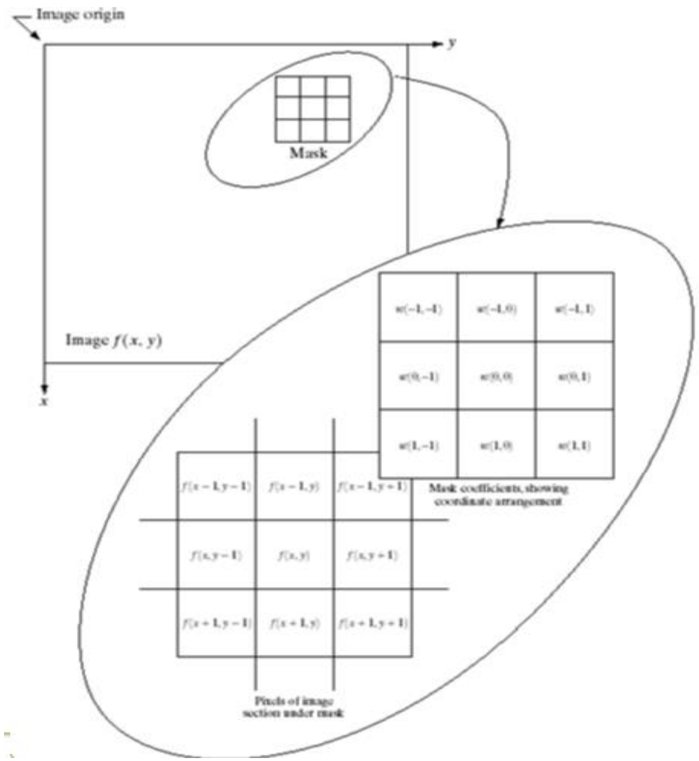


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

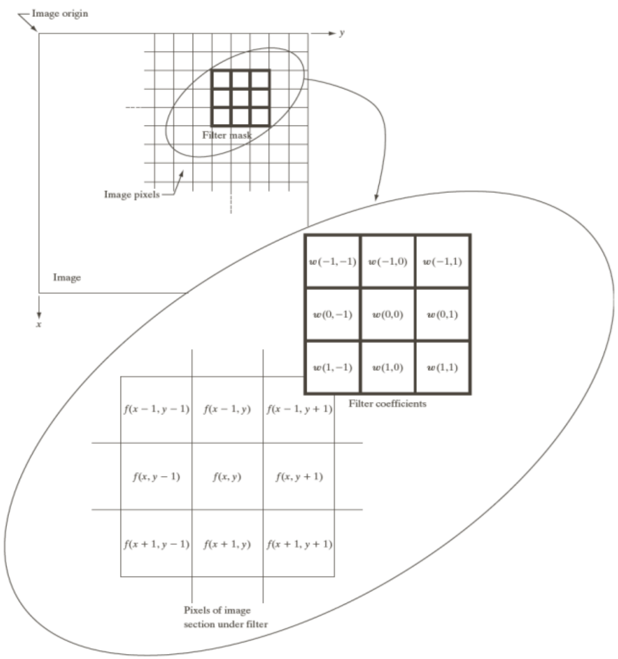


FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

Fundamentals of Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

The Mechanics of Spatial Filtering

Spatial Correlation and Convolution

Vector Representation of Linear Filtering

Generating Spatial Filter Masks

Spatial Correlation and Convolution

(J.Shanbehzadeh M.Gholizadeh)

	Correlation	Convolution	
	Origin f w	Origin f w rotated 180°	
(a)	0 0 0 1 0 0 0 0 1 2 3 2 8	0 0 0 1 0 0 0 0 8 2 3 2 1	(i)
(b)	0 0 0 1 0 0 0 0	0 0 0 1 0 0 0 0	(j)
	1 2 3 2 8	8 2 3 2 1	
	Starting position alignment		
	Zero padding		
(c)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	(k)
	1 2 3 2 8	8 2 3 2 1	
(d)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	(l)
	1 2 3 2 8	8 2 3 2 1	
	Position after one shift		
(e)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	(m)
	1 2 3 2 8	8 2 3 2 1	
	Position after four shifts		
(f)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	(n)
	1 2 3 2 8	8 2 3 2 1	
	Final position		
	Full correlation result	Full convolution result	
(g)	0 0 0 8 2 3 2 1 0 0 0 0	0 0 0 1 2 3 2 8 0 0 0 0	(o)
	Cropped correlation result	Cropped convolution result	
(h)	0 8 2 3 2 1 0 0	0 1 2 3 2 8 0 0	(p)

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

Spatial Correlation and Convolution

(J.Shanbehzadeh M.Gholizadeh)

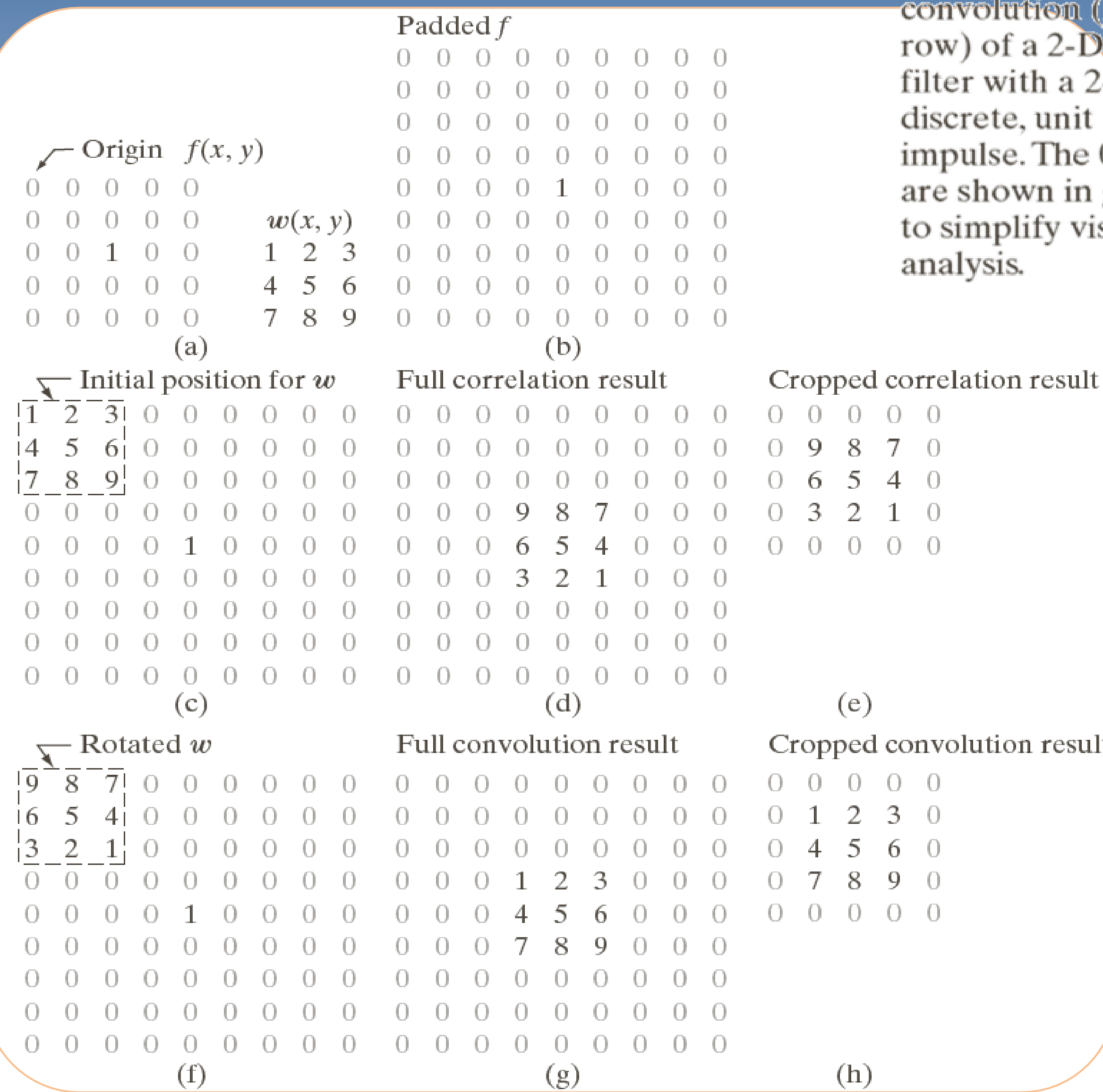


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

Fundamentals of Spatial Filtering

(J.Shanbehzadeh   M.Gholizadeh)

The Mechanics of Spatial Filtering

Spatial Correlation and Convolution

Vector Representation of Linear Filtering

Generating Spatial Filter Masks

Vector Representation of Linear Filtering

(J.Shanbehzadeh M.Gholizadeh)

- Vector Representation of Linear Filtering:

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9$$
$$= \sum_{i=1}^{9} w_i z_i$$

FIGURE 3.33 Another representation of a general 3×3 spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3.5 Smoothing Spatial Filters

Smoothing Spatial Filters

(J.Shanbehzadeh   M.Gholizadeh)

Smoothing Linear Filters

Order-Static (Nonlinear) Filters

Smoothing Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

- Smoothing Linear Filters :
  - Noise reduction
  - Smoothing of false contours
  - Reduction of irrelevant detail

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

Smoothing Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

- Image size: M×N
- Mask size: m×n

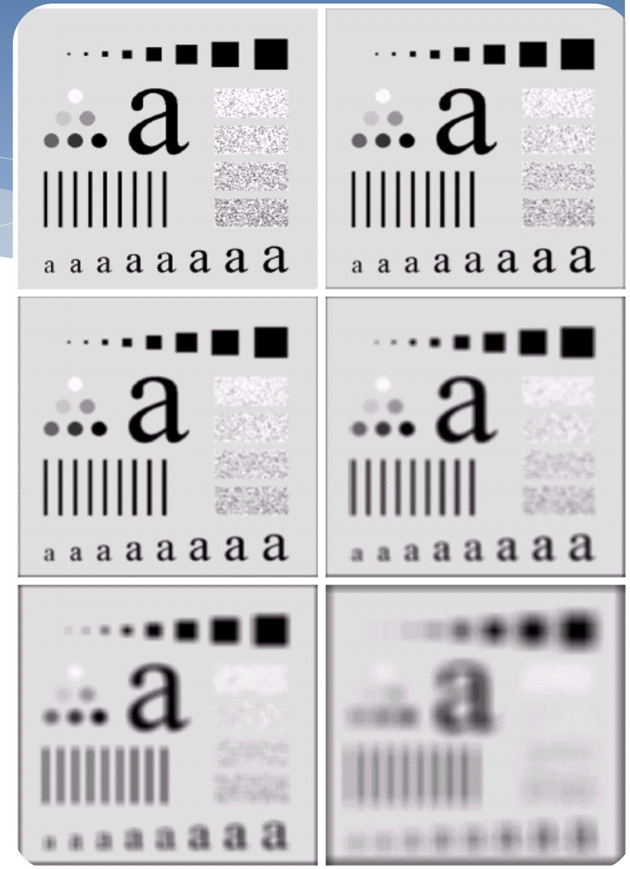
$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

- a=(m-1)/2 and b=(n-1)/2
- x= 0,1,2,…,M-1 and y= 0,1,2,…,N-1

Smoothing Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

- Image smoothing
- with masks of various sizes.

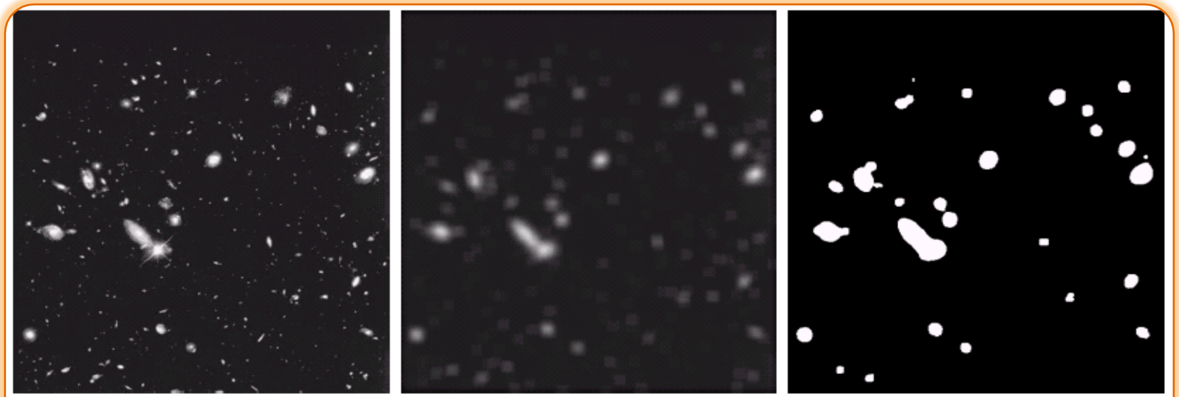


a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes n = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Smoothing Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Smoothing Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

Smoothing Linear Filters

Order-Static (Nonlinear) Filters

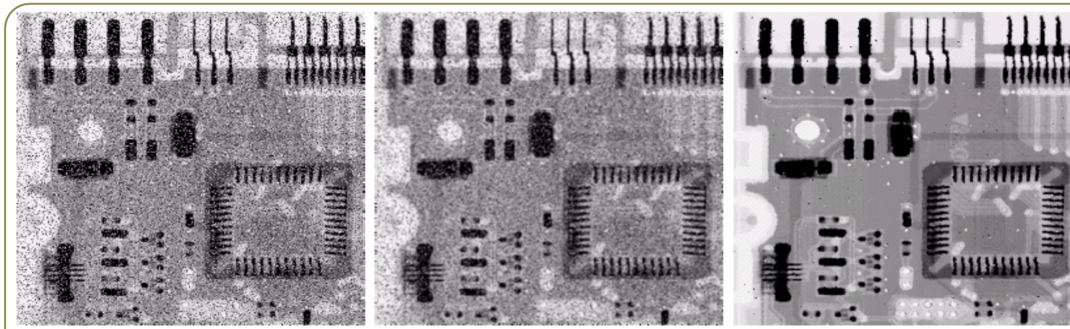
Order-Static (Nonlinear) Filters

(J.Shanbehzadeh M.Gholizadeh)

- Order-statistic filters:
  - **Median filter**: Replace the value of a pixel by the median of the gray levels in the neighborhood of that pixel
  - **Noise-reduction**

Order-Static (Nonlinear) Filters

(J.Shanbehzadeh M.Gholizadeh)



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

3.6 Sharpening Spatial Filters

Sharpening Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

Foundation

Using the Second Derivative for Image Sharpening - The Laplacian

Unsharp Masking and Highboost Filtering

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

Foundation

(J.Shanbehzadeh M.Gholizadeh)

- The first-order derivative:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Foundation

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

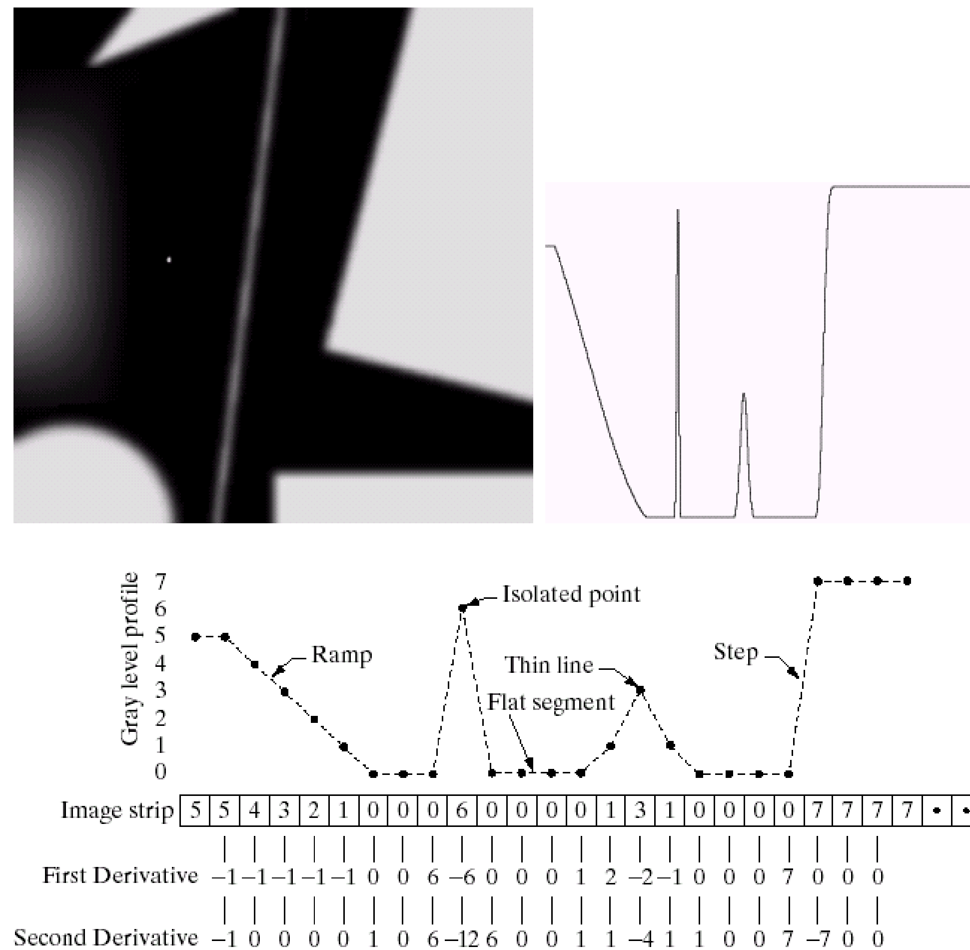
3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

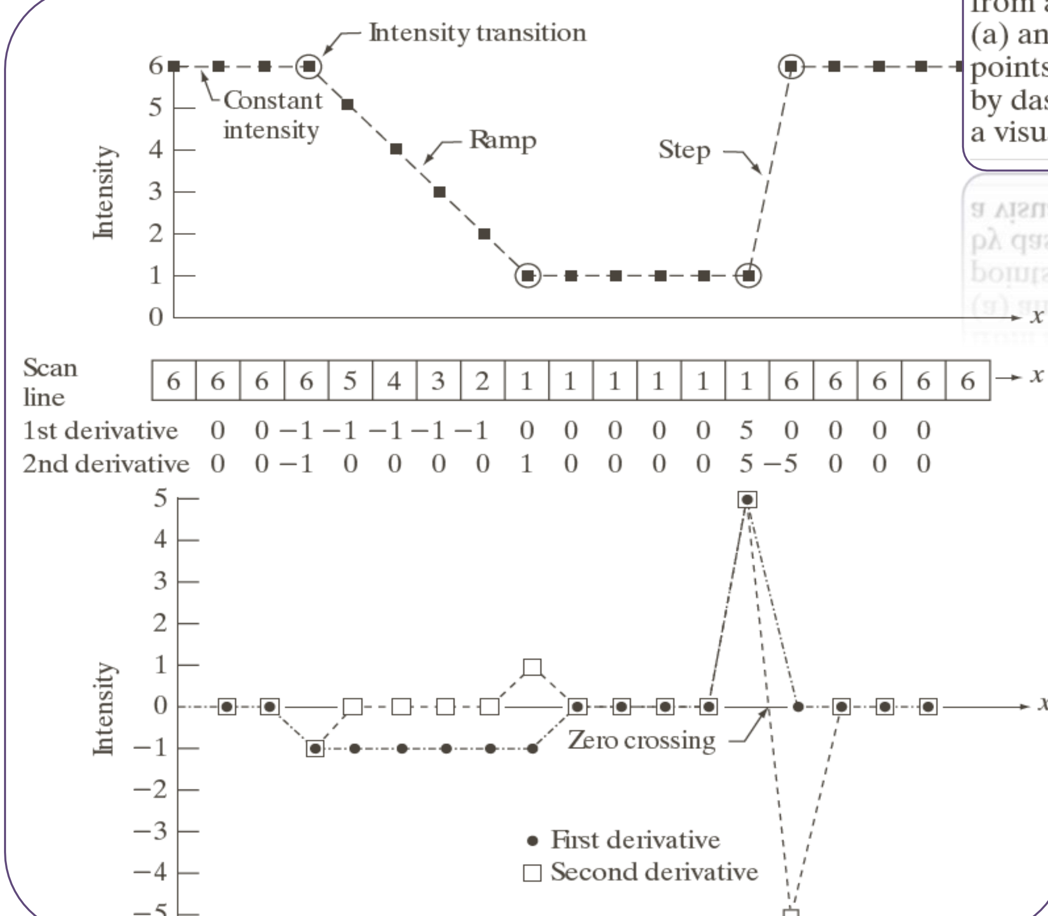
a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).



Foundation

(J.Shanbehzadeh M.Gholizadeh)



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

Sharpening Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

Foundation

Using the Second Derivative for Image Sharpening - The Laplacian

Unsharp Masking and Highboost Filtering

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

Laplacian

(J.Shanbehzadeh M.Gholizadeh)

- Use of second derivatives for enhancement-The Laplacian:
  - Development of the method

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian

(J.Shanbehzadeh M.Gholizadeh)

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \begin{array}{l} \text{if the center coefficient} \\ \text{of the Laplacian mask} \\ \text{is negative} \end{array} \\ f(x,y) + \nabla^2 f(x,y) & \begin{array}{l} \text{if the center coefficient} \\ \text{of the Laplacian mask} \\ \text{is positive} \end{array} \end{cases}$$

Laplacian

(J.Shanbehzadeh M.Gholizadeh)

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

(d)

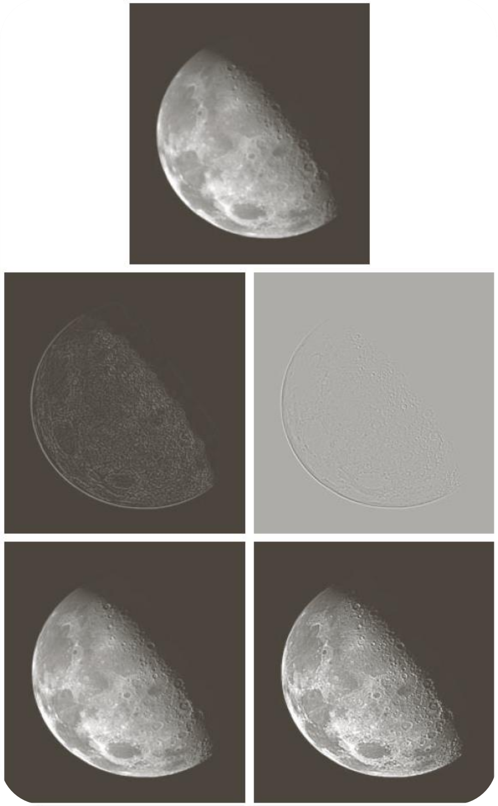
−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Laplacian

(J.Shanbehzadeh M.Gholizadeh)



a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Laplacian

(J.Shanbehzadeh M.Gholizadeh)

- Simplifications:`

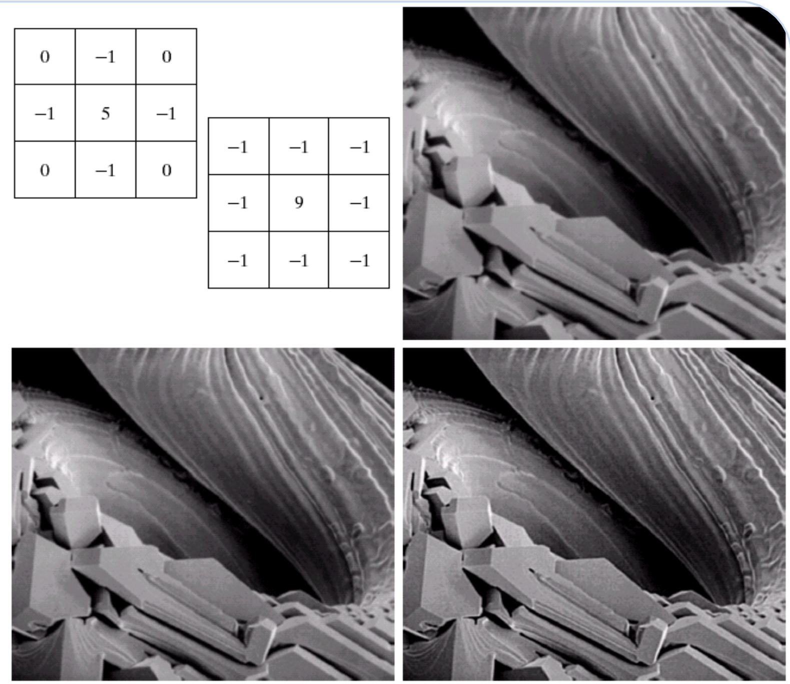
$$
\begin{aligned}
g(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] + 4f(x,y) \\
&= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]
\end{aligned}
$$

Laplacian

(J.Shanbehzadeh M.Gholizadeh)

0	–1	0
–1	5	–1
0	–1	0

–1	–1	–1
–1	9	–1
–1	–1	–1



a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Sharpening Spatial Filters

(J.Shanbehzadeh M.Gholizadeh)

Foundation

Using the Second Derivative for Image Sharpening - The Laplacian

Unsharp Masking and Highboost Filtering

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

Unsharp Masking and Highboost Filtering

(J.Shanbehzadeh M.Gholizadeh)

- Unsharp masking
  - Subtract a blurred version of an image from the image itself

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

  - f(x,y) : The image, $\bar{f}$(x,y): The blurred image

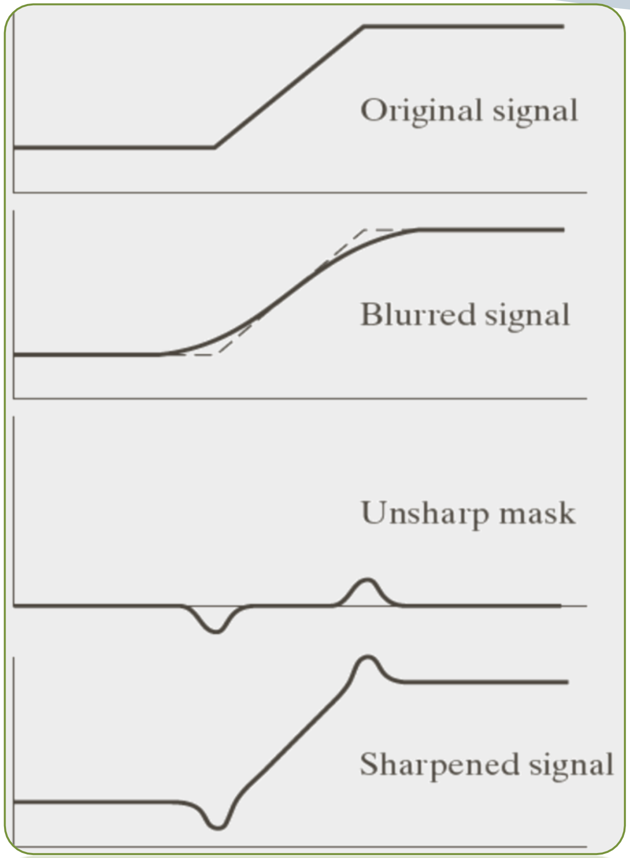
$$g(x,y) = f(x,y) + k * g_{mask}(x,y) \qquad ,k=1$$

- High boost Filtering:

$$g(x,y) = f(x,y) + k * g_{mask}(x,y) \qquad ,k>1$$

Unsharp Masking and Highboost Filtering

(J.Shanbehzadeh M.Gholizadeh)



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Unsharp Masking and Highboost Filtering

(J.Shanbehzadeh M.Gholizadeh)

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

(J.Shanbehzadeh M.Gholizadeh)

Foundation

Using the Second Derivative for Image Sharpening - The Laplacian

Unsharp Masking and Highboost Filtering

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

(J.Shanbehzadeh M.Gholizadeh)

- Using first-order derivatives for (nonlinear) image sharpening, The gradient:
  - The gradient:

$$\nabla \mathbf{f} = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

  - The magnitude is rotation invariant (**isotropic**)

$$M(x, y) = \text{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2 \right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$$M(x, y) \approx |g_x| + |g_y|$$

Using First-Order Derivative for (Nonlinear) Image Sharpening - The Gradient

(J.Shanbehzadeh M.Gholizadeh)

- Computing using cross differences, Roberts cross-gradient operators

$$g_x = (z_9 - z_5) \quad \text{and} \quad g_y = (z_8 - z_6)$$

$$M(x,y) = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2}$$

$$M(x,y) \approx |z_9 - z_5| + |z_8 - z_6|$$

Using the Gradient for Image Sharpening

(J.Shanbehzadeh M.Gholizadeh)

- Sobel operators:
  - A weight value of 2 is to achieve some smoothing by giving more importance to the center point

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

Using the Gradient for Image Sharpening

(J.Shanbehzadeh M.Gholizadeh)

a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

(a)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(b)

−1	0
0	1

(c)

0	−1
1	0

(d)

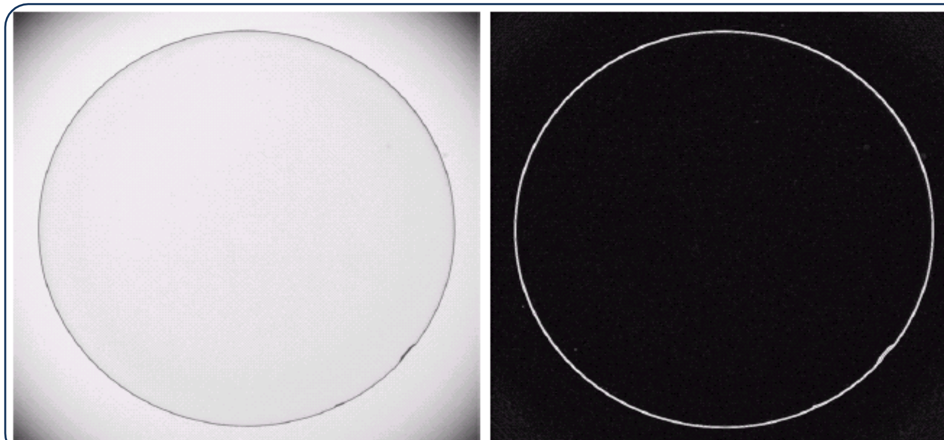
−1	−2	−1
0	0	0
1	2	1

(e)

−1	0	1
−2	0	2
−1	0	1

Using the Gradient for Image Sharpening

(J.Shanbehzadeh M.Gholizadeh)



a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

3.7 Combining Spatial Enhancement Tools

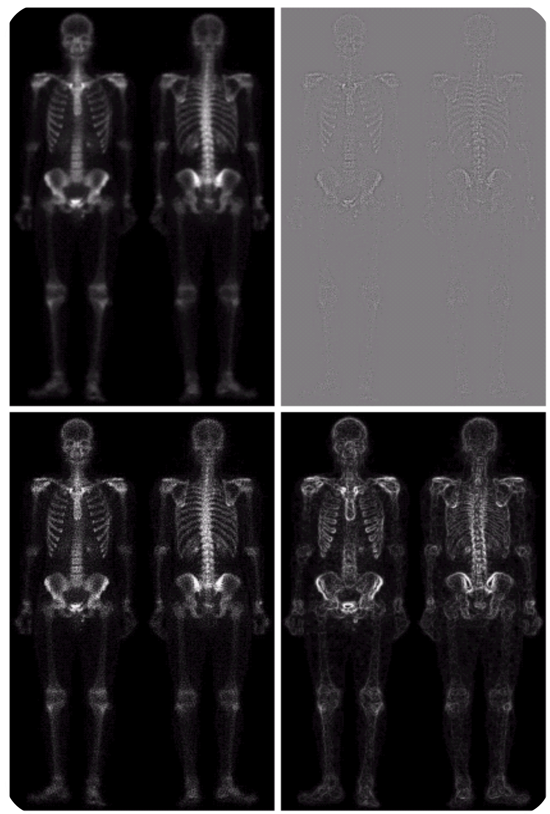
Combining Spatial Enhancement Tools

(J.Shanbehzadeh M.Gholizadeh)

* **An example:**
  * Laplacian to highlight fine detail
  * Gradient to enhance prominent edges
  * Smoothed version of the gradient image used to mask the Laplacian image
  * Increase the dynamic range of the gray levels by using a gray-level transformation

Combining Spatial Enhancement Tools

(J.Shanbehzadeh M.Gholizadeh)

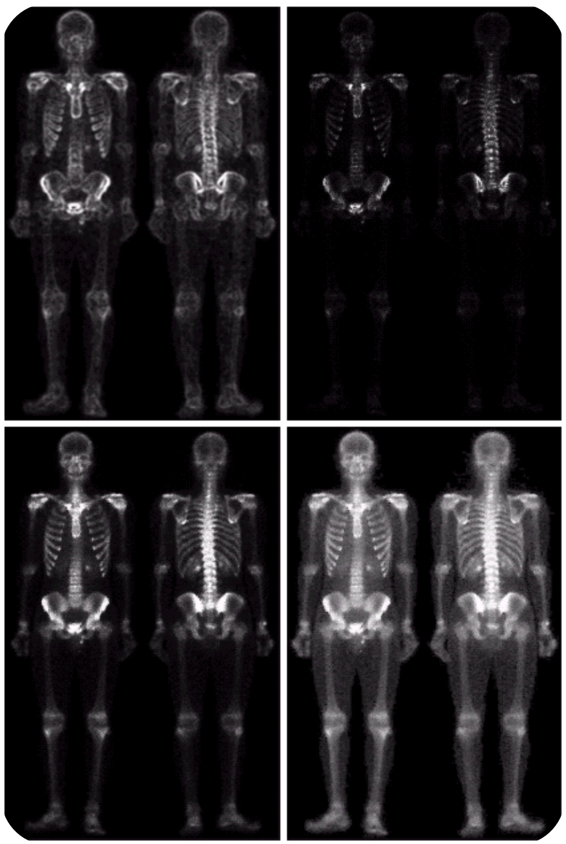


a b
c d

FIGURE 3.46
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).

Combining Spatial Enhancement Tools

(J.Shanbehzadeh M.Gholizadeh)



e f
g h

FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

3.8 Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

(J.Shanbehzadeh M.Gholizadeh)

Introduction

Principles of Fuzzy Set Theory

Using Fuzzy Sets

Using Fuzzy Sets for Intensity Transformations

Using Fuzzy Sets for spatial Filtering

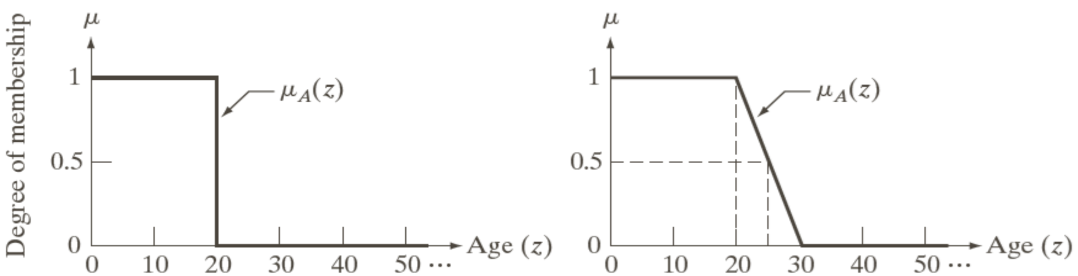
Introduction

(J.Shanbehzadeh M.Gholizadeh)

- Fuzzy set theory is the extension of conventional (crisp) set theory
- It handles the concept of partial truth (truth values between 1 (completely true) and 0 (completely false))
- It was introduced by Prof. Lotfi A. Zadeh of UC/Berkeley in 1965 as a mean to model the vagueness and ambiguity in complex systems
- The idea of fuzzy sets is simple and natural

Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

(J.Shanbehzadeh M.Gholizadeh)



a b

FIGURE 3.44 Membership functions used to generate (a) a crisp set, and (b) a fuzzy set.

Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

(J.Shanbehzadeh M.Gholizadeh)

- **Introduction**
- **Principles of Fuzzy Set Theory**
- **Using Fuzzy Sets**
- **Using Fuzzy Sets for Intensity Transformations**
- **Using Fuzzy Sets for spatial Filtering**

Principles of Fuzzy Set Theory

(J.Shanbehzadeh M.Gholizadeh)

- A *fuzzy set* A in Z (the universe of discourse) is characterized by a *membership function*
- for all :
  - **Empty set**
    - $\mu_A(z) = 0$
  - **Equality**
    - A = B, if and only if $\mu_A(z) = \mu_B(z)$
  - **Complement**
    - $\mu_{\bar{A}}(z) = 1 - \mu_A(z)$
  - **Subset**
    - , if and only if $\mu_A(z) \leq \mu_B(z)$
  - **Union**
    - $\mu_U(z) = \max[\mu_A(z), \mu_B(z)]$
  - **Intersection**
    - $\mu_I(z) = \min[\mu_A(z), \mu_B(z)]$

Principles of Fuzzy Set Theory

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

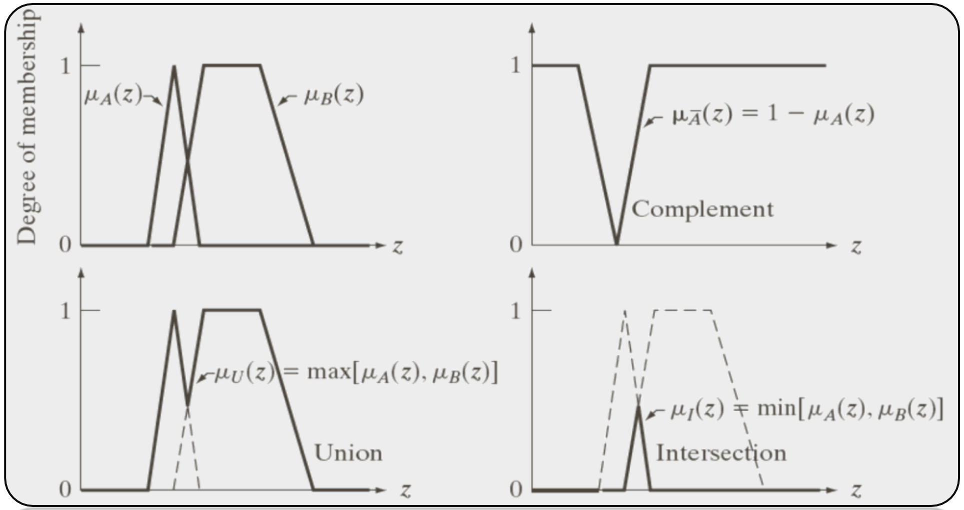
3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)



a b
c d

FIGURE 3.45 (a) Membership functions of two sets, A and B. (b) Membership function of the complement of A. (c) and (d) Membership functions of the union and intersection of the two sets.

Some Common Membership Functions

(J.Shanbehzadeh M.Gholizadeh)

- Triangular:

$$\mu(z)=\begin{cases}1-(a-z)/b; a-b\le z<a\\ 1-(z-a)/c; a\le z\le a+c\\ 0; otherwise\end{cases}$$

- Sigma

$$\mu(z)=\begin{cases}1-(a-z)/b; a-b\le z\le a\\ 1; z>a\\ 0; otherwise\end{cases}$$

- Trapezoidal:

$$\mu(z)=\begin{cases}1-(a-z)/c; a-c\le z<a\\ 1; a\le z<b\\ 1-(z-b)/d; b\le z\le b+d\\ 0; otherwise\end{cases}$$

- S-shape

$$S(z;a,b,c)=\begin{cases}0; z<a\\ 2\left(\dfrac{z-a}{c-a}\right)^2; a\le z\le b\\ 1-2\left(\dfrac{z-c}{c-a}\right)^2; b<z\le c\\ 1; z>c\end{cases}$$

Some Common Membership Functions

(J.Shanbehzadeh M.Gholizadeh)

- Bell-shape

$$\mu(z) = \begin{cases} S(z; c-b, c-b/2, c); z \le c \\ 1 - S(z; c, c+b/2, c+b); z > c \end{cases}$$

- Truncated Gaussian

$$\mu(z) = \begin{cases} e^{-\frac{(z-a)^2}{2b^2}}; a-c \le z \le a+c \\ 0; otherwise \end{cases}$$

Some Common Membership Functions

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

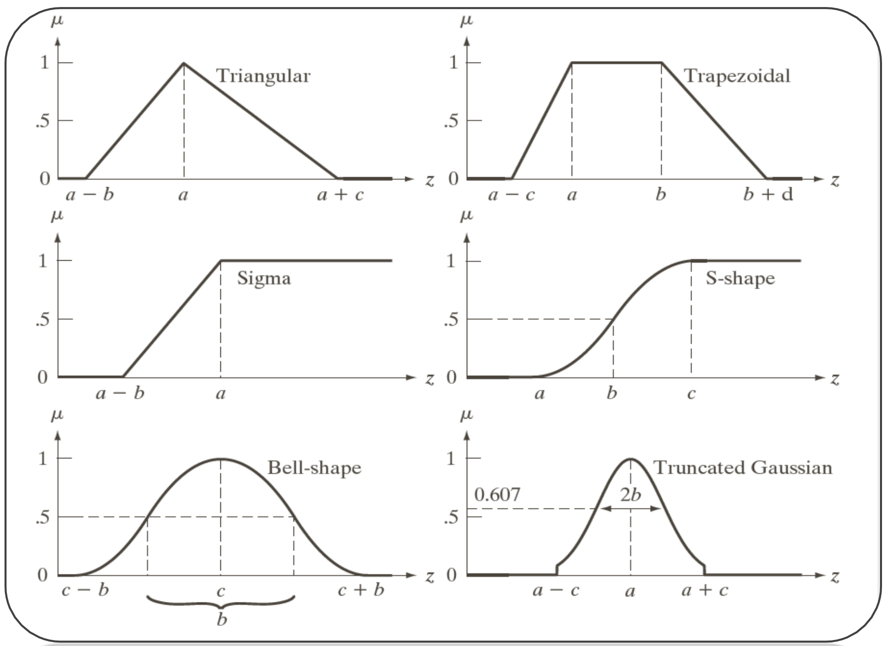
3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)



a	b
c	d
e	f

FIGURE 3.46 Membership functions corresponding to Eqs. (3.8-6)–(3.8-11).

Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

(J.Shanbehzadeh M.Gholizadeh)

- **Introduction**
- **Principles of Fuzzy Set Theory**
- **Using Fuzzy Sets**
- **Using Fuzzy Sets for Intensity Transformations**
- **Using Fuzzy Sets for spatial Filtering**

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)

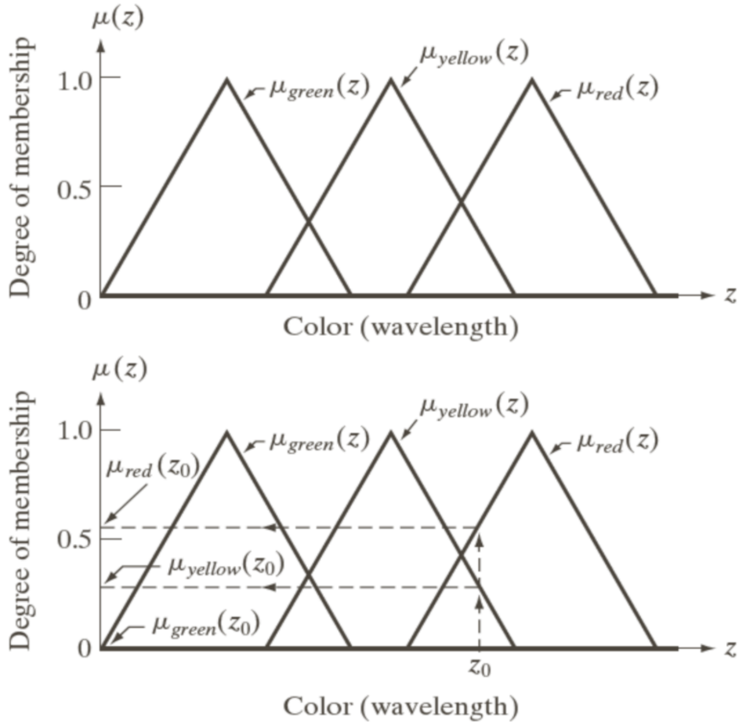
- R1: IF the color is *green,*
  THEN the fruit is *verdant*

- R2: IF the color is *yellow,*
  THEN the fruit is *half-mature*

- R3: IF the color is *red,*
  THEN the fruit is *mature*

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)



a
b

FIGURE 3.47
(a) Membership functions used to fuzzify color.
(b) Fuzzifying a specific color z_0. (Curves describing color sensation are bell shaped; see Section 6.1 for an example. However, using triangular shapes as an approximation is common practice when working with fuzzy sets.)

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)

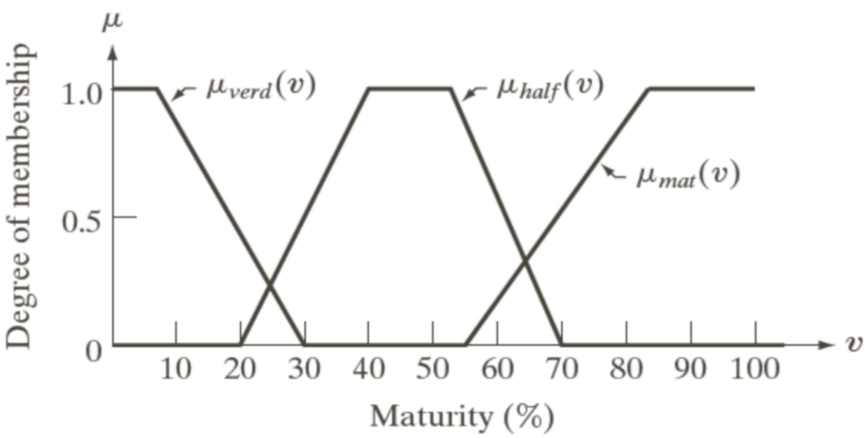
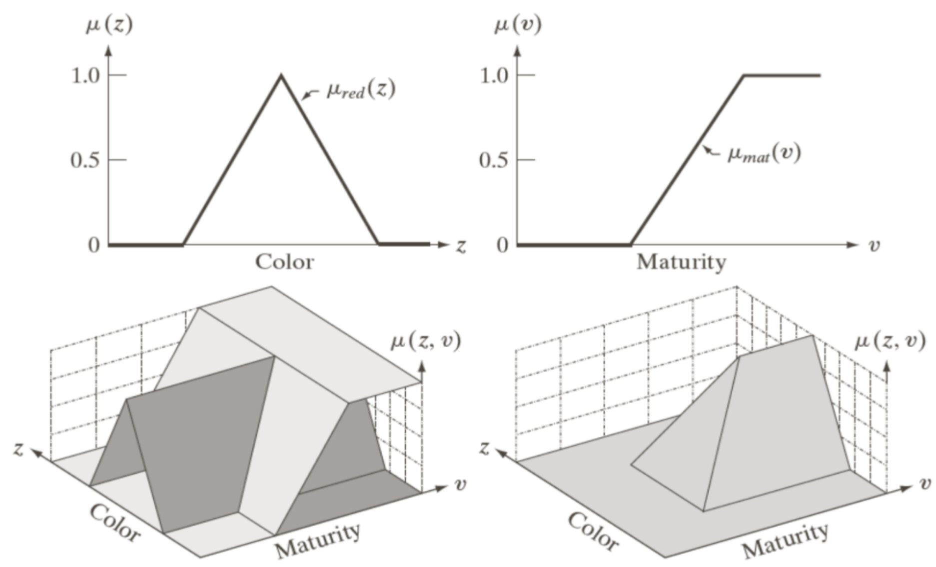


FIGURE 3.48 Membership functions characterizing the outputs *verdant*, *half-mature*, and *mature*.

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)



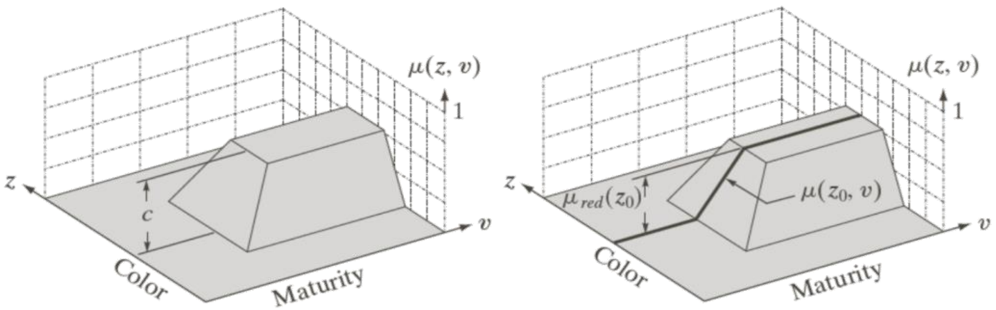
a b
c d

FIGURE 3.49 (a) Shape of the membership function associated with the color red, and (b) corresponding output membership function. These two functions are associated by rule R_3. (c) Combined representation of the two functions. The representation is 2-D because the independent variables in (a) and (b) are different. (d) The AND of (a) and (b), as defined in Eq. (3.8-5).

$$\mu_3(z,v) = \min\{\mu_{red}(z), \mu_{mat}(v)\}$$

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)



$$Q_3(v) = \min\{\mu_{red}(z_0), \mu_3(z_0,v)\}$$

$$Q_2(v) = \min\{\mu_{yellow}(z_0), \mu_2(z_0,v)\}$$

$$Q_1(v) = \min\{\mu_{green}(z_0), \mu_1(z_0,v)\}$$

a b

FIGURE 3.50 (a) Result of computing the minimum of an arbitrary constant, c, and function $\mu_3(z, v)$ from Eq. (3.8-12). The minimum is equivalent to an AND operation. (b) Cross section (dark line) at a specific color, z_0.

Using Fuzzy Sets

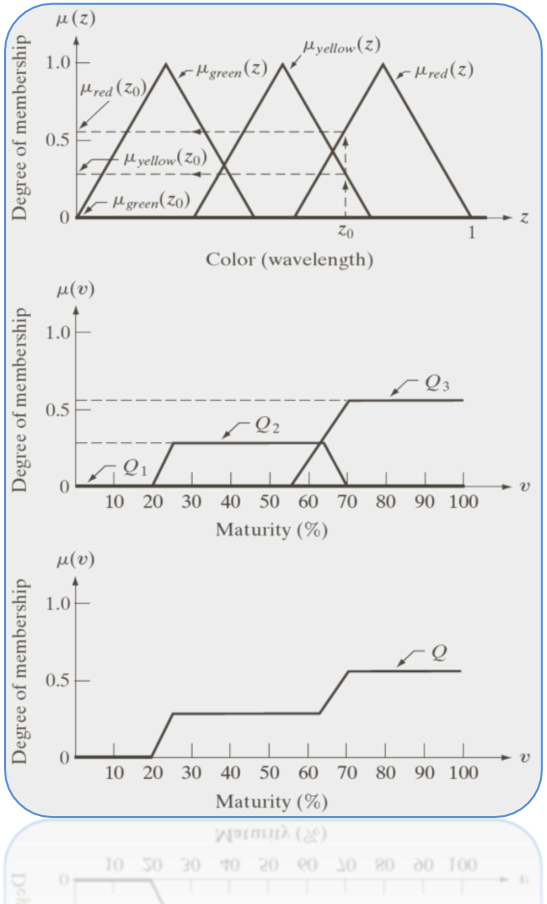
(J.Shanbehzadeh M.Gholizadeh)

- $Q = Q_1 \text{ OR } Q_2 \text{ OR } Q_3$
- $Q(v) = \max_r\{\min_s\{\mu_s(z_0), \mu_r(z_0,v)\}\}$

$$v_0 = \frac{\sum_{v=1}^{K} vQ(v)}{\sum_{v=1}^{K} Q(v)}$$

Using Fuzzy Sets

(J.Shanbehzadeh M.Gholizadeh)



a
b
c

FIGURE 3.51
(a) Membership functions with a specific color, z_0, selected.
(b) Individual fuzzy sets obtained from Eqs. (3.8-13)–(3.8-15). (c) Final fuzzy set obtained by using Eq. (3.8-16) or (3.8-17).

A Fuzzy Rule-Based System

(J.Shanbehzadeh M.Gholizadeh)

- Fuzzify the inputs
- Perform any required fuzzy logical operation
- Apply an implication method
- Apply an aggregation method
- Defuzzify the final output

A Fuzzy Rule-Based System

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

(J.Shanbehzadeh M.Gholizadeh)

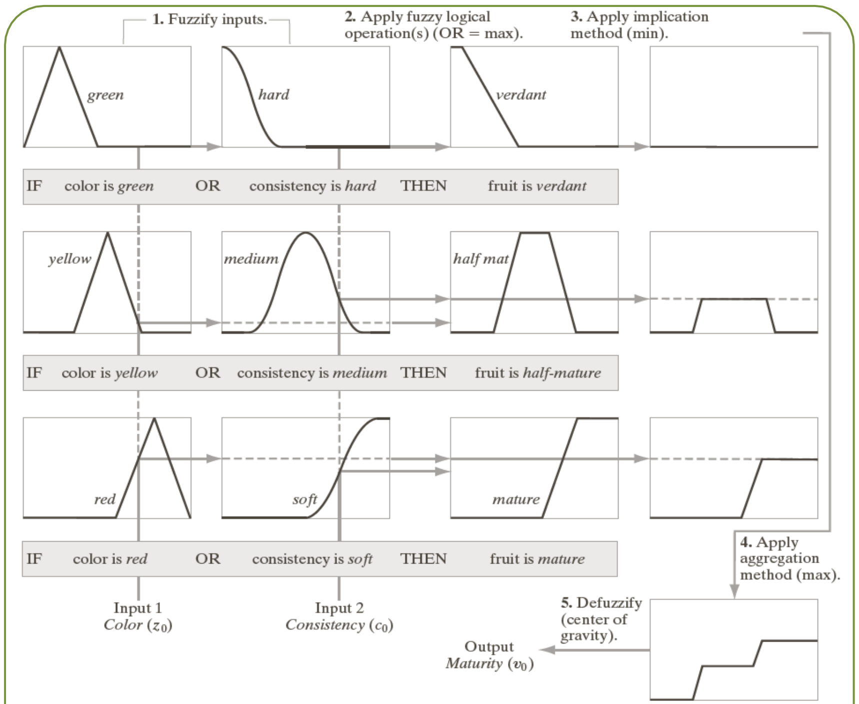


FIGURE 3.52 Example illustrating the five basic steps used typically to implement a fuzzy, rule-based system: (1) fuzzification, (2) logical operations (only OR was used in this example), (3) implication, (4) aggregation, and (5) defuzzification.

A Fuzzy Rule-Based System

(J.Shanbehzadeh M.Gholizadeh)

- Dealing with
  - M IF-THEN rules
  - N input variables : $z_1,\ldots,z_N$
  - One output variable : v
  - A_{ij} : the fuzzy set associated with the *i*th rule and *j*th input variable
  - B_i : the fuzzy set associated with the *i*th rule

IF (z_1,A_{11}) AND (z_2,A_{12}) AND … AND (z_N,A_{1N}) THEN (v,B_1)

IF (z_1,A_{21}) AND (z_2,A_{22}) AND … AND (z_N,A_{2N}) THEN (v,B_2)

……………..

IF (z_1,A_{M1}) AND (z_2,A_{M2}) AND … AND (z_N,A_{MN}) THEN (v,B_M)

ELSE (v,B_E)

- $\lambda_i = \min\{\mu_{Aij}(z_j);\ j = 1,2,\ldots,M\}$
- $\lambda_E = \min\{1- \lambda_i\ ;\ i = 1,\ldots,M\}$

What does Fuzzy Image Processing Mean?

(J.Shanbehzadeh M.Gholizadeh)

- For instance, we want to define a set of gray levels that share the property dark
  - In classical set theory, we have to determine a threshold, say the gray level 100
  - All gray levels between 0 and 100 are element of this set, the others do not belong to the set
  - But the darkness is a matter of degree
  - So, a fuzzy set can model this property much better
  - To define this set, we also need two thresholds, say gray levels 50 and 150

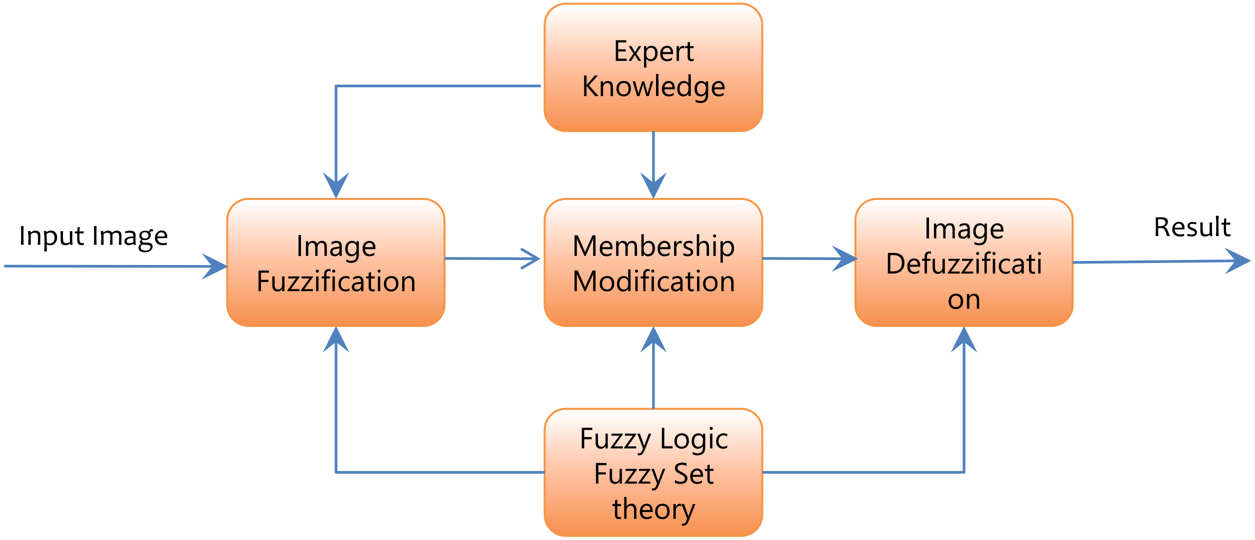
- Fuzzy image processing is not a unique theory. It is a collection of different fuzzy approaches to image processing, Nevertheless, the following definition can be regarded as an attempt to determine the boundaries:

- **Fuzzy image processing** is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved.

What does Fuzzy Image Processing Mean?

(J.Shanbehzadeh M.Gholizadeh)

- Fuzzy image processing has three main stages:
  - Image fuzzification
  - Modification of membership values
  - if necessary, image defuzzification



Fuzzy Image Processing

(J.Shanbehzadeh M.Gholizadeh)

- The fuzzification and defuzzification steps are due to the fact that we do not possess fuzzy hardware. Therefore, the coding of image data (fuzzification) and decoding of the results (defuzzification) are steps that make possible to process images with fuzzy techniques.

- The main power of fuzzy image processing is in the middle step (modification of membership values). After the image data are transformed from gray-level plane to the membership plane (fuzzification), appropriate fuzzy techniques modify the membership values. This can be a fuzzy clustering, a fuzzy rule-based approach, a fuzzy integration approach and so on.

Using Fuzzy Techniques for Intensity Transformations and Spatial filtering

(J.Shanbehzadeh M.Gholizadeh)

- **Introduction**
- **Principles of Fuzzy Set Theory**
- **Using Fuzzy Sets**
- **Using Fuzzy Sets for Intensity Transformations**
- **Using Fuzzy Sets for spatial Filtering**

Using Fuzzy Sets for Intensity Transformations

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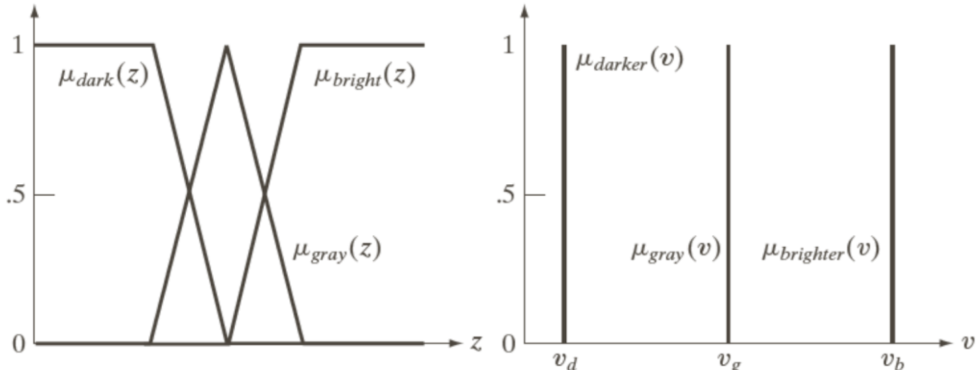
- Rules :
  - IF a pixel is *dark*, THEN make it *darker*.
  - IF a pixel is *gray*, THEN make it *gray*.
  - IF a pixel is *bright*, THEN make it *brighter*.



a b

FIGURE 3.53 (a) Input and (b) output membership functions for fuzzy, rule-based contrast enhancement.

Using Fuzzy Sets for Intensity Transformations

3.1- Background

3.2- Some Basic Intensity Transformation Functions

3,3- Histogram Processing

3.4- Fundamentals of Spatial Filtering

3.5 - Smoothing Spatial Filters

3.6- Sharpening Spatial Filters

3.7- Combining Spatial Enhancement Tools

3.8- Using Fuzzy Techniques for Intensity Transformations and Spatial Filtering

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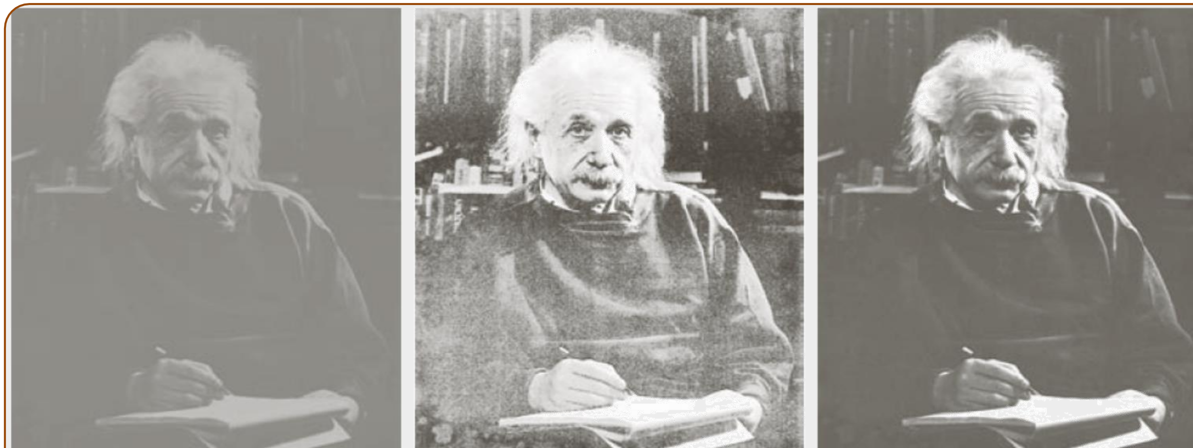

a b c

FIGURE 3.54 (a) Low-contrast image. (b) Result of histogram equalization. (c) Result of using fuzzy, rule-based contrast enhancement.

Using Fuzzy Sets for Intensity Transformations

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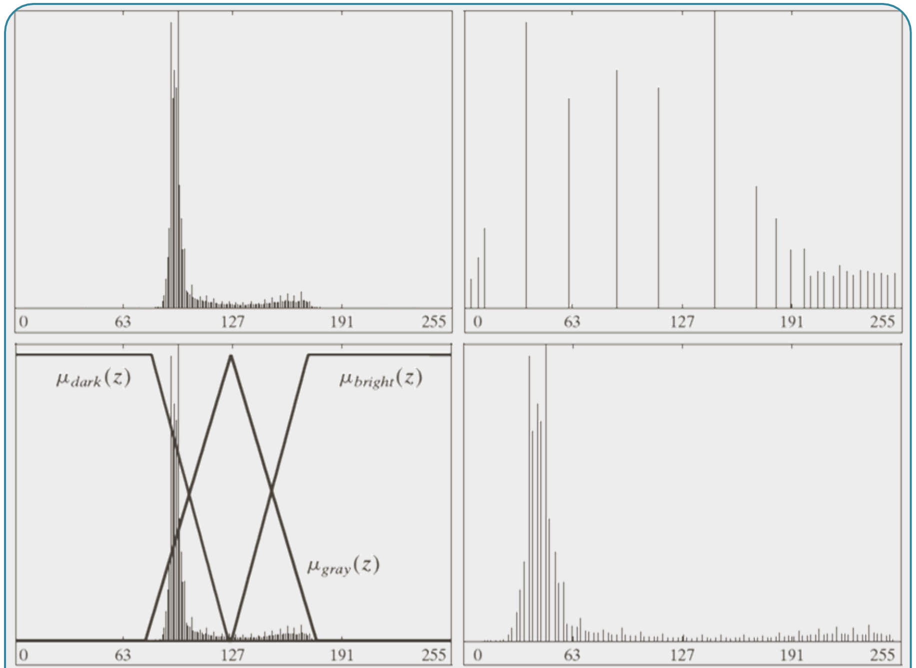

a b
c d

FIGURE 3.55 (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).

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Using Fuzzy Sets for spatial Filtering

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z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Pixel neighborhood

d_1	d_2	d_3
d_4	0	d_6
d_7	d_8	d_9

Intensity differences

a b

FIGURE 3.56 (a) A 3×3 pixel neighborhood, and (b) corresponding intensity differences between the center pixels and its neighbors. Only d_2, d_4, d_6, and d_8 were used in the present application to simplify the discussion.

Using Fuzzy Sets for spatial Filtering

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- $d_i = z_i - z_5$
- A simple set of rules :
  - IF d_2 is *zero* AND d_6 is *zero* THEN z_5 is *white*
  - IF d_6 is *zero* AND d_8 is *zero* THEN z_5 is *white*
  - IF d_8 is *zero* AND d_4 is *zero* THEN z_5 is *white*
  - IF d_4 is *zero* AND d_2 is *zero* THEN z_5 is *white*
  - ELSE z_5 is *black*

Using Fuzzy Sets for spatial Filtering

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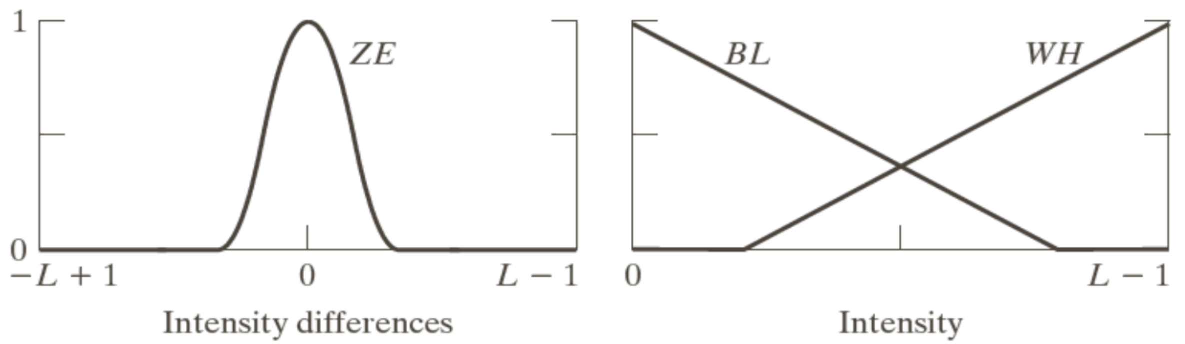

a b

FIGURE 3.57 (a) Membership function of the fuzzy set *zero*. (b) Membership functions of the fuzzy sets *black* and *white*.

Using Fuzzy Sets for spatial Filtering

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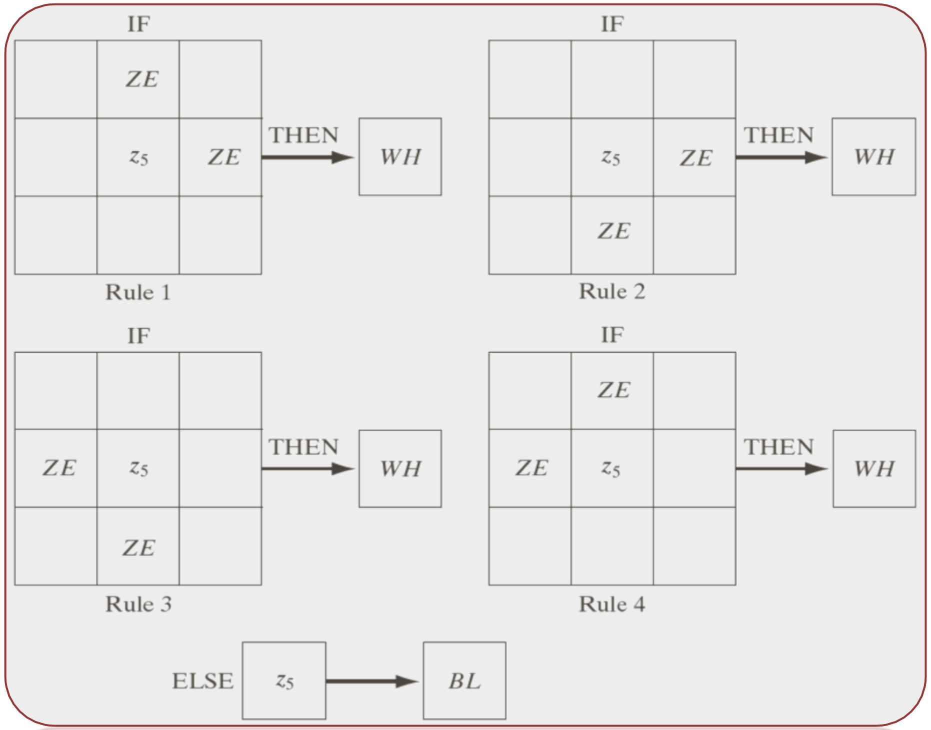

FIGURE 3.58 Fuzzy rules for boundary detection.

Using Fuzzy Sets for spatial Filtering

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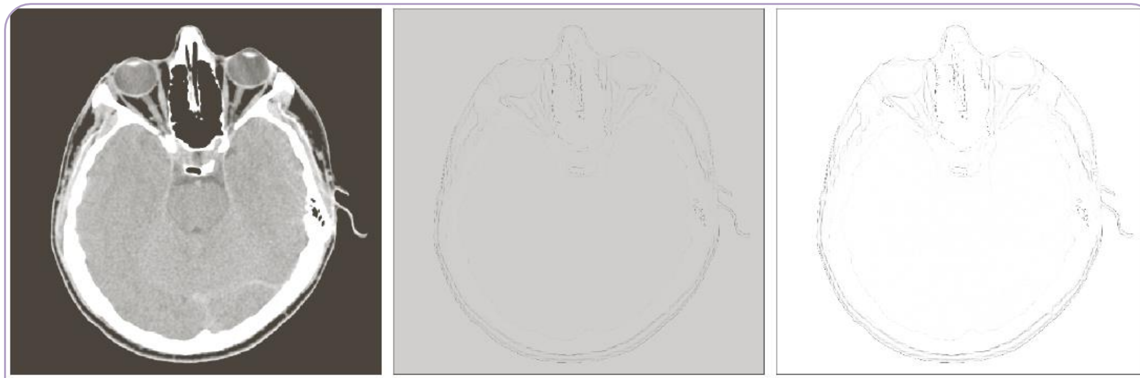

a b c

FIGURE 3.59 (a) CT scan of a human head. (b) Result of fuzzy spatial filtering using the membership functions in Fig. 3.57 and the rules in Fig. 3.58. (c) Result after intensity scaling. The thin black picture borders in (b) and (c) were added for clarity; they are not part of the data. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)