2024 DIP Final

Order Space-Based Morphology for Color Image Processing

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Motivation

Morphological processing (e.g. erosion) on……

Binary image

$$(F \ominus B)(x) = \bigwedge_{b \in B} F(x+b)$$

Grayscale

$$(F \ominus B)(x) = \min_{b \in B} F(x+b)$$

RGB Color

?

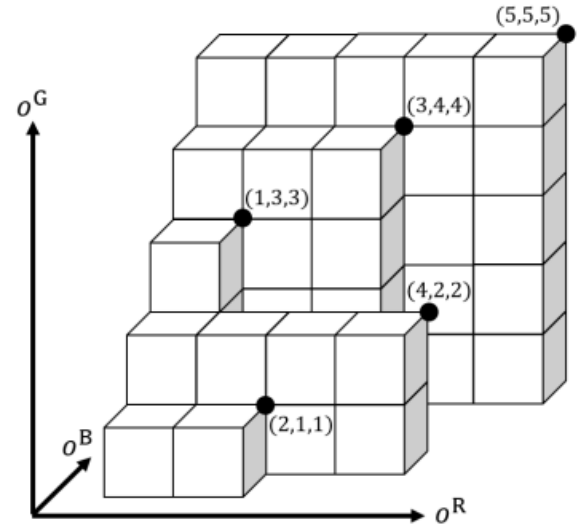
No ordering for 3 channels!

I.e. the image is not an ordered set

Problem Definition

Given n pixels $f_1, f_2, \dots, f_n$, each with R,G,B values.

Find the order $o_1, o_2, \dots, o_n$ of the pixels,
where o_i is the order of f_i among all pixels.



"Order space"

Proposed Algorithm

Algorithm 1 Mapping from RGB color space to order space

Require: a set of pixels $\{f_{i+k,\,j+l} = [f^{\mathrm{R}}_{i+k,\,j+l},\ f^{\mathrm{G}}_{i+k,\,j+l},\ f^{\mathrm{B}}_{i+k,\,j+l}] \mid (k,l) \in S\}$ // S is the structuring element used in operation

Ensure: a set of coordinates in order space $\{(o^{\mathrm{R}}_{\xi}, o^{\mathrm{G}}_{\xi}, o^{\mathrm{B}}_{\xi}) \mid \xi \in \{1,2,\ldots,|S|\}\}$

1: Assign serial numbers $\xi \in \{1,2,\ldots,|S|\}$ to all pixels in the required set to have a set of re-indexed pixels $\{f_{\xi} = [f^{\mathrm{R}}_{\xi},\ f^{\mathrm{G}}_{\xi},\ f^{\mathrm{B}}_{\xi}] \mid \xi \in \{1,2,\ldots,|S|\}\}$. // Flatten the image

2: **for** $\mathrm{X} \in \{\mathrm{R},\mathrm{G},\mathrm{B}\}$ **do**

3: $\quad [a^{\mathrm{X}}_1, a^{\mathrm{X}}_2, \ldots, a^{\mathrm{X}}_{|S|}] = \mathrm{argsort}(f^{\mathrm{X}}_1, f^{\mathrm{X}}_2, \ldots, f^{\mathrm{X}}_{|S|})$ // Find the order in each channel

4: $\quad$ **for** $\xi \in \{1,2,\ldots,|S|\}$ **do**

5: $\quad\quad o^{\mathrm{X}}_{a^{\mathrm{X}}_{\xi}} = \xi$

6: $\quad$ **end for**

7: **end for**

8: Return $\{(o^{\mathrm{R}}_{\xi}, o^{\mathrm{G}}_{\xi}, o^{\mathrm{B}}_{\xi}) \mid \xi \in \{1,2,\ldots,|S|\}\}$ // Combine the order in each channel

Reduced Order Space

Choose from functions that map triplet to singlet

$$o_\xi^S = o_\xi^{\mathrm{R}} + o_\xi^{\mathrm{G}} + o_\xi^{\mathrm{B}},$$
$$o_\xi^P = o_\xi^{\mathrm{R}} o_\xi^{\mathrm{G}} o_\xi^{\mathrm{B}},$$
$$o_\xi^M = \mathrm{med}\left\{ o_\xi^{\mathrm{R}}, o_\xi^{\mathrm{G}}, o_\xi^{\mathrm{B}} \right\},$$

Order-preserving: $o_\xi^X > o_{\xi'}^X$ for $X \in \{R, G, B\}$ guarantees $o_\xi > o_{\xi'}$.

Morphological Operations

Similar to grayscale, use o_ξ as "key" instead

$$D(F,S) = [d_{ij}], \quad d_{ij} = f_{\xi^{\max}}, \quad \xi^{\max} = \arg \max_{\xi \in \{1,2,\ldots,|S|\}} o_\xi,$$

$$E(F,S) = [e_{ij}], \quad e_{ij} = f_{\xi^{\min}}, \quad \xi^{\min} = \arg \min_{\xi \in \{1,2,\ldots,|S|\}} o_\xi.$$

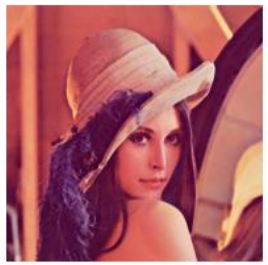
Fuzzy Morphological Operations

Average with $\exp(\pm\alpha o_\xi)$ as weighted

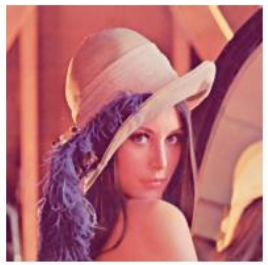
$$D^{\mathrm{FUZ}}(F,S,\alpha)=\left[d_{ij}^{\mathrm{FUZ}}\right],\quad d_{ij}^{\mathrm{FUZ}}=\frac{\sum_{\xi=1}^{|S|}\exp\left(\alpha o_\xi\right)f_\xi}{\sum_{\xi=1}^{|S|}\exp\left(\alpha o_\xi\right)},\qquad E^{\mathrm{FUZ}}(F,S,\alpha)=\left[e_{ij}^{\mathrm{FUZ}}\right],\quad e_{ij}^{\mathrm{FUZ}}=\frac{\sum_{\xi=1}^{|S|}\exp\left(-\alpha o_\xi\right)f_\xi}{\sum_{\xi=1}^{|S|}\exp\left(-\alpha o_\xi\right)}.$$

Results (1)


(a) dilation with sum


(b) erosion with sum


(c) opening with sum


(d) closing with sum


(e) dilation with prod


(f) erosion with prod


(g) opening with prod


(h) closing with prod


(i) dilation with median


(j) erosion with median


(k) opening with median


(l) closing with median

No significant difference between sum, product & median.

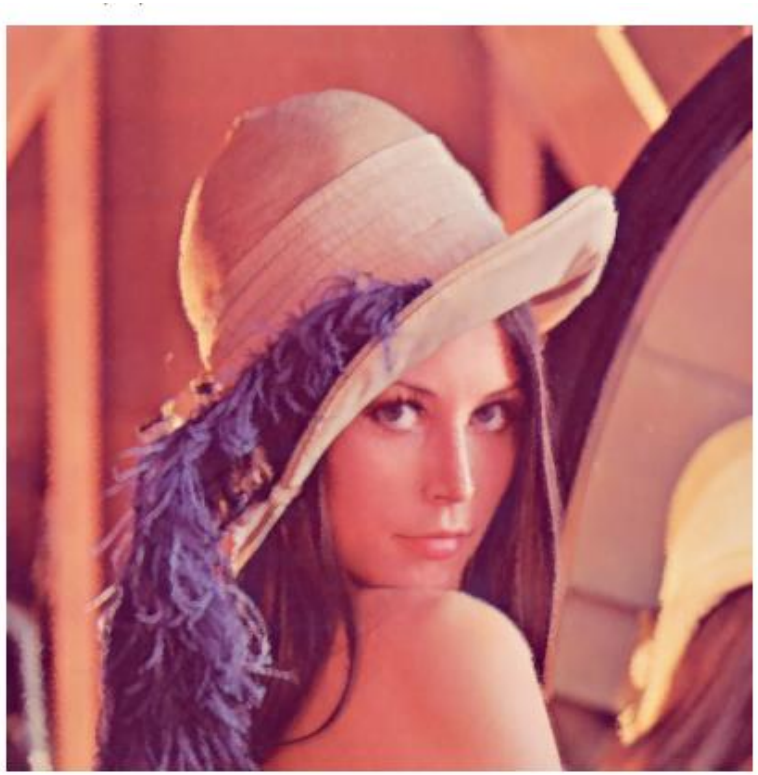
Compared with grayscale

(a) dilation, color (b) erosion, color (c) opening, color (d) closing, color

(e) dilation, gray (f) erosion, gray (g) opening, gray (h) closing, gray

Fuzzy operation

Erosion



Fuzzy erosion

Denoising (1)

Pepper noise

Pepper noise after opening

Denoising (2)

Salt noise

Salt noise after opening

Reference

Sun, S.; Huang, Y.; Inoue, K.; Hara, K. Order Space-Based Morphology for Color Image Processing. J. Imaging 2023, 9, 139. https://doi.org/10.3390/jimaging9070139