

# A Survey on Distributed Randomness

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# 1 Building Blocks

## 1.1 Perfectly Synchronous Distributed Randomness

Suppose  $n$  participants attempt to source a common random number in a distributed yet agreeable manner. In an ideal world where the communication model is perfectly synchronous such that all participants are able to successfully send and receive their random shares  $s_i$  (which will make up the common random number) to and from all other participants at the same time, each participant can simply add all shares involved and call  $s = \sum s_i$  to be the randomness produced in that round. Such protocol would be not only simple, but also agreeable, as its nature in fact parallels that of rock-paper-scissors, which many play in real life as part of a quick decision-making process. While distributed randomness is hardly a hard problem in this setting, situations can change dramatically in practical settings where participants can go offline (perhaps temporarily) due to network failure, messages can be delayed either non-significantly or significantly, and Byzantine attackers that maliciously try to predict or bias the randomness to their benefit can exist.

## 1.2 Commit-Reveal

In the case of a strawman solution, the classical commit-reveal provides an intuitive way to source a random number from a group of nodes by first allowing each node to commit (in the cryptographic sense) to a secret random number and then adding all shares (revealed later) from all nodes to compute the final random number of a round. The problem with this is that the last person to reveal his share can in fact check the round's output faster than others and hence can decide not to reveal his number if he doesn't like the round's output, biasing the distributed randomness. This is called the *last revealer attack*.

## 1.3 Verifiable Secret Sharing

The issue with the classical Shamir's secret sharing scheme is that either the dealer or other participants could in fact be acting maliciously, e.g. Alice (a participant) needs to trust that she has received her share correctly from the dealer while she also needs to trust that other participants' revealed shares are correct.

This issue can be fixed by the notion of verifiable secret sharing (VSS). The idea is that we add an additional verification process to the usual Shamir's secret sharing scheme. Here, we take note of two commonly used VSS schemes: Feldman-VSS and Pedersen-VSS.

The mathematical setup is as follows. We choose primes  $p$  and  $q$  such that  $q \mid p - 1$  and let  $g$  be a generator of  $G_q$ , a cyclic subgroup of  $\mathbb{Z}_p^*$ . This setup has the effect of achieving the following:  $a \equiv b \pmod{q} \iff g^a \equiv g^b \pmod{p}$ . As a result, it should be understood that modular arithmetics are done in modulo  $q$

whenever the numbers involved concern the exponents while in modulo  $p$  otherwise. For convenience, we may omit  $\text{mod } p$  such that one can aptly assume arithmetics are done in modulo  $p$  given an equation unless stated otherwise.

### 1.3.1 Feldman-VSS

The following summarizes a simple VSS scheme (where  $t$  among  $n$  participants can reconstruct the group secret) proposed by Paul Feldman.

- $f(x) = \sum_{i=0}^{t-1} a_i x^i$  is randomly selected by the dealer, where  $a_i \in \mathbb{Z}_q$  and  $f(0) = a_0$  is the secret
- The shares are  $f(1), f(2), \dots, f(n)$  in mod  $q$  and are distributed to  $n$  participants, respectively
- Also distributed from the dealer are commitments to coefficients of  $f$ , i.e.  $c_j = g^{a_j}$  for  $j = 0, \dots, t-1$
- Given her share  $f(k)$  and the polynomial coefficient commitments, Alice (a participant) can verify her share by checking:

$$g^{f(k)} = g^{\sum_{i=0}^{t-1} a_i k^i} = \prod_{j=0}^{t-1} c_j^{k^j} = c_0 c_1^k c_2^{k^2} \cdots c_{t-1}^{k^{t-1}}$$

- Any  $t$  number of participants (say)  $i = 1, 2, \dots, t$  can recover the secret  $a_0$  by performing Lagrange interpolation involving Lagrange coefficients  $\lambda_i = \prod_{j \neq i} \frac{j}{j-i}$  in mod  $q$ :

$$a_0 = f(0) = \sum_{i=1}^t f(i) \lambda_i$$

What is new here (compared to Shamir's secret sharing scheme) is the inclusion of commitments to polynomial coefficients in the scheme. These commitments enable participants to verify the validity of their corresponding shares.

### 1.3.2 Pedersen-VSS

Reminiscent of Pedersen commitment, Pedersen-VSS is a variation that involves two random polynomials generated by the dealer as opposed to one. Namely, the scheme runs as follows.

- $f(x) = \sum_{i=0}^{t-1} a_i x^i$  and  $f'(x) = \sum_{i=0}^{t-1} b_i x^i$  are randomly selected by the dealer, where  $a_i, b_i \in \mathbb{Z}_q$  and  $f(0) = a_0$  is the secret (as before)
- The shares are  $(f(1), f'(1)), \dots, (f(n), f'(n))$  in mod  $q$  and are distributed to  $n$  participants, respectively

- Also distributed from the dealer are commitments to coefficients of  $f$  and  $f'$ , i.e.  $c_j = g^{a_j} h^{b_j}$  for  $j = 0, \dots, t-1$
- Given her share  $(f(k), f'(k))$  and the polynomial coefficient commitments, Alice (a participant) can verify her share by checking:

$$g^{f(k)} h^{f'(k)} = \prod_{j=0}^{t-1} c_j^{k^j}$$

- Any  $t$  number of participants (say)  $i = 1, 2, \dots, t$  can recover the secret  $a_0$  by performing Lagrange interpolation involving Lagrange coefficients  $\lambda_i = \prod_{j \neq i} \frac{j}{j-i}$  in mod  $q$ :

$$a_0 = f(0) = \sum_{i=1}^t f(i) \lambda_i$$

Effectively, what Pedersen-VSS is able to achieve is the decoupling of  $g^{a_0}$  (as the public key corresponding to the secret key  $a_0$ ) and  $g^{a_0} h^{b_0}$  (as the published commitment for verification purposes). In other words, the verification process in which the participants verify their shares does not (even information-theoretically) leak any information regarding the initial secret  $a_0$ , a fact that is not true with Feldman-VSS.

#### 1.4 Distributed Key Generation

The motivation for distributed key generation (DKG), which basically comprises  $n$  parallel instances of a VSS (run by each participant), is to achieve and utilize a group secret in a leaderless manner such that any threshold  $t$  number of participants should be able to recover the same group secret. The basic idea is that each participant becomes a dealer (i.e. leader) for a VSS, in which case the protocol deals with  $n$  random polynomials in mod  $q$   $\{f_i\}_{i=1, \dots, n}$  as opposed to one. Though not computed explicitly, the implicit group polynomial  $f = \sum f_i$  then embeds the group secret in the form of  $f(0)$  (denoted by  $x$ ) as well as the corresponding public key  $g^{f(0)}$  (denoted by  $y$ ) via commitments to coefficients of  $f_i$  as per each VSS. Notably, this facet is the one that allows a leaderless configuration where there does not exist a leader for  $f$  even if each participant  $P_i$  remains a leader for  $f_i$  with the knowledge of  $f_i(0)$ . As a result, it is only collectively (i.e. with the collaboration of at least  $t$  number of nodes in a group) that the group can make use of  $x$  and  $y$  in the typical sense of asymmetric cryptography (e.g. signing a message) after performing a DKG.

Before delineating the process of a DKG, we include what it means for a DKG to be uniformly secure. Namely, a uniformly secure DKG should satisfy the following three correctness properties and one secrecy property.

**Definition 1.1.** In the setting where at most  $t - 1$  nodes can be controlled by an attacker without compromising the protocol, a DKG is *uniformly secure* if the following properties are satisfied.

**Correctness**

1. Any subset of shares of size  $t$  can be used to recover the same group secret key  $x$ .
2. All honest parties have access to the same public key  $y = g^x$ .
3.  $x$  is uniformly distributed in  $\mathbb{Z}_q$ , and thus  $y$  is uniformly distributed in  $G_q$  (subgroup of  $\mathbb{Z}_p^*$  generated by  $g$ ).

**Secrecy**

1. Other than from the fact that  $y = g^x$ , no information on  $x$  is leaked. Equivalently, this can be stated using a simulator *SIM*: for every probabilistic polynomial-time adversary  $\mathcal{A}$ , there exists a probabilistic polynomial-time simulator *SIM* that, given  $y \in G_q$  as input, produces an output distribution which is polynomially indistinguishable from  $\mathcal{A}$ 's view of a DKG protocol that ends with  $y$  as its public key output while  $\mathcal{A}$  is allowed to corrupt up to  $t - 1$  participants.

The third correctness property is the one that motivates the .....  
As the goal in practical terms is to use

**1.4.1 Joint-Feldman**

1. Each participant  $P_i$  runs a Feldman-VSS by choosing a random polynomial  $f_i(z) = \sum_{j=0}^{t-1} a_{ij}z^j$  and sending a “subshare”  $f_i(j)$  to player  $P_j$  for all  $j$
2. To satisfy the verifiability portion of the VSS,  $P_i$  broadcasts  $A_{ik} = g^{a_{ik}}$
3. Upon receiving the subshares and the corresponding commitments (e.g. in the form of a verification vector),  $P_j$  can use the verification mechanism per VSS to verify the subshares. If a verification fails,  $P_j$  can broadcast a complaint against  $P_i$ .
4. If  $P_i$  receives at least  $t$  complaints, then  $P_i$  is disqualified. Otherwise,  $P_i$  needs to reveal the subshare  $f_i(j)$  per  $P_j$  that has broadcasted a complaint. We call *QUAL* the set of non-disqualified players.
5. Once *QUAL* is set, we define  $f(z) = \sum_{i \in QUAL} f_i(z) = \sum_{i=0}^{t-1} a_i z^i$  such that each participant  $P_j$  in *QUAL* can compute the group public key  $y = g^{f(0)} = \prod_{i \in QUAL} A_{i0}$ , commitments to  $f$ 's coefficients  $A_k = g^{a_k} = \prod_{i \in QUAL} A_{ik}$ ,

and  $P_j$ 's share (of the group secret) from subshares  $f(j) = \sum_{i \in QUAL} f_i(j)$ . Though not computed explicitly, the group secret key  $x$  is then equal to both  $\sum_{i \in QUAL} a_{i0}$  and the Lagrange interpolation involving the shares  $\{f(j)\}_{j \in QUAL}$ .

#### 1.4.2 Joint-Pedersen

1. Each participant  $P_i$  runs a Pedersen-VSS by choosing two random polynomials  $f_i(z) = \sum_{j=0}^{t-1} a_{ij}z^j$  and  $f'_i(z) = \sum_{j=0}^{t-1} b_{ij}z^j$  and sending a "subshare"  $(f_i(j), f'_i(j))$  to player  $P_j$  for all  $j$
2. To satisfy the verifiability portion of the VSS,  $P_i$  broadcasts  $C_{ik} = g^{a_{ik}} h^{b_{ik}}$
3. Upon receiving the subshares and the corresponding commitments (e.g. in the form of a verification vector),  $P_j$  can use the verification mechanism per VSS to verify the subshares. If a verification fails,  $P_j$  can broadcast a complaint against  $P_i$ .
4. If  $P_i$  receives at least  $t$  complaints, then  $P_i$  is disqualified. Otherwise,  $P_i$  needs to reveal the subshare  $(f_i(j), f'_i(j))$  per  $P_j$  that has broadcasted a complaint. We call  $QUAL$  the set of non-disqualified players.
5. Once  $QUAL$  is set, we define  $f(z) = \sum_{i \in QUAL} f_i(z) = \sum_{i=0}^{t-1} a_i z^i$  such that each participant  $P_j$  in  $QUAL$  can compute the group public key  $y = g^{f(0)} = \prod_{i \in QUAL} A_{i0}$ , commitments to  $f$ 's coefficients  $A_k = g^{a_k} = \prod_{i \in QUAL} A_{ik}$ , and  $P_j$ 's share (of the group secret) from subshares  $f(j) = \sum_{i \in QUAL} f_i(j)$ . Though not computed explicitly, the group secret key  $x$  is then equal to both  $\sum_{i \in QUAL} a_{i0}$  and the Lagrange interpolation involving the shares  $\{f(j)\}_{j \in QUAL}$ .

### 1.5 Publicly Verifiable Secret Sharing

While VSS allows verification of the involved shares, the fact of the matter is that such verification can only be done by the involved participants in the secret sharing process. In other words, the verification process of a VSS is not public. Publicly verifiable secret sharing (PVSS), on the other hand, assumes a public setup where the verification process can be achieved publicly (e.g. by bystanders) such that the messages exchanged during the protocol are modified accordingly in order to reflect the fact that some information kept private in a typical VSS can be public in a PVSS. Naturally, this added facet of public verifiability is useful and desirable in models such as the public bulletin board model (e.g. for blockchains). We delineate Schoenmakers' PVSS scheme and Scrape's variations of it here.

#### 1.5.1 Schoenmakers

**Initialization.** The group  $G_q$  and the generators  $g, G$  are selected using an appropriate public procedure. Participant  $P_i$  generates a private key  $x_i \leftarrow \mathbb{Z}_q^*$  and

registers  $y_i = G^{x_i}$  as its public key.

**Distribution.** The protocol consists of two steps:

1. *Distribution of the shares.* Suppose without loss of generality that the dealer wishes to distribute a secret among participants  $P_1, \dots, P_n$ . The dealer picks a random polynomial  $p$  of degree at most  $t - 1$  with coefficients in  $\mathbb{Z}_q$

$$p(x) = \sum_{i=0}^{t-1} a_i x^i$$

and sets  $s = a_0$ . The dealer keeps this polynomial secret but publishes the related commitments  $C_j = g^{a_j}$  for  $0 \leq j < t$ . The dealer also publishes the encrypted shares  $Y_i = y_i^{p(i)}$  for  $1 \leq i \leq n$  using the public keys of the participants. Finally, let  $X_i = \prod_{j=0}^{t-1} C_j^{i^j}$ . The dealer shows that the encrypted shares are consistent by producing a proof of knowledge of the unique  $p(i)$  for  $1 \leq i \leq n$  satisfying:

$$X_i = g^{p(i)}, \quad Y_i = y_i^{p(i)}.$$

The non-interactive proof is the  $n$ -fold parallel composition of the protocols for  $DLEQ(g, X_i, y_i, Y_i)$ . Applying Fiat-Shamir's technique, the challenge  $c$  for the protocol is computed as a cryptographic hash of  $X_i, Y_i, a_{1i}, a_{2i}, 1 \leq i \leq n$ . The proof consists of the common challenge  $c$  and the  $n$  responses  $r_i$ .

2. *Verification of the shares.* The verifier computes  $X_i = \prod_{j=0}^{t-1} C_j^{i^j}$  from the  $C_j$  values. Using  $y_i, X_i, Y_i, r_i, 1 \leq i \leq n$  and  $c$  as input, the verifier computes  $a_{1i}, a_{2i}$  as

$$a_{1i} = g^{r_i} X_i^c, \quad a_{2i} = y_i^{r_i} Y_i^c$$

and checks that the hash of  $X_i, Y_i, a_{1i}, a_{2i}, 1 \leq i \leq n$  matches  $c$ .

**Reconstruction.** The protocol consists of two steps:

1. *Decryption of the shares.* Using its private key  $x_i$ , each participant finds the share  $S_i = G^{p(i)}$  from  $Y_i$  by computing  $S_i = Y_i^{1/x_i}$ . They publish  $S_i$  plus a proof that the value  $S_i$  is a correct decryption of  $Y_i$ . To this end, it suffices to prove knowledge of an  $\alpha$  such that  $y_i = G^\alpha$  and  $Y_i = S_i^\alpha$ , which is accomplished by the non-interactive version of the protocol  $DLEQ(G, y_i, S_i, Y_i)$ .
2. *Pooling the shares.* Suppose without loss of generality that participants  $P_i$  produce correct values for  $S_i$ , for  $i = 1, \dots, t$ . The secret  $G^s$  is obtained by Lagrange interpolation:

$$\prod_{i=1}^t S_i^{\lambda_i} = \prod_{i=1}^t \left( G^{p(i)} \right)^{\lambda_i} = G^{\sum_{i=1}^t p(i)\lambda_i} = G^{p(0)} = G^s$$

where  $\lambda_i = \prod_{j \neq i} \frac{j}{j-i}$  is a Lagrange coefficient.



Note that the participants do not need nor learn the values of the exponents  $p(i)$ . Only the related values  $S_i = G^{p(i)}$  are required to complete the reconstruction of the secret value  $S = G^s$ . Also, note that participant  $P_i$  does not expose its private key  $x_i$ ; consequently participant  $P_i$  can use its key pair in several runs of the PVSS scheme. The type of encryption used for the shares has been optimized for performance; however, if desired, it is also possible to use standard ElGamal encryption instead.

Clearly, the scheme is homomorphic. For example, given the dealer's output for secrets  $G^{s_1}$  and  $G^{s_2}$ , the combined secret  $G^{s_1+s_2}$  can be obtained by applying the reconstruction protocol to the combined encrypted shares  $Y_{i1}Y_{i2}$ .

### 1.5.2 Scrape's Variations

Scrape offers two variations of Schoenmakers' PVSS: one that relies on the DDH (decisional Diffie-Hellman) assumption in the random oracle model and one that relies on the DBS (decisional bilinear square) assumption in the plain model. Accordingly, the former utilizes the same NIZK (i.e. *DLEQ* by Chaum and Pedersen) from Schoenmakers' whereas the latter utilizes a bilinear pairing, which serves as a valid size-efficient substitute to the NIZK at the cost of pairing-related computations. Both variations make novel use of Reed-Solomon codes from coding theory in a way that the total number of exponentiations needed to verify the shares distributed by the dealer in a PVSS is decreased from  $O(nt)$  (which becomes quadratic if  $t = \frac{n}{2}$  for instance) to  $O(n)$ . In other words, the approaches optimize the verifier's computational complexity before the Reconstruction phase.

The idea is that the verifier does not have to compute  $t$  exponentiations via  $X_i = \prod_{j=0}^{t-1} C_j^{i^j}$  per share, before which the dealer publishes the commitments  $C_j$  to the polynomial coefficients. Instead, the dealer publishes  $v_i = g^{p(i)}$  directly alongside the dual code  $\mathcal{C}^\perp$  of the  $[n, k, n-k+1]$  Reed-Solomon code  $\mathcal{C}$  corresponding to the shares calculated by the dealer in the form  $\mathcal{C} = \{(p(1), \dots, p(n)) \mid p(x) \in \mathbb{Z}_q[x], \deg(p(x)) \leq t-1\}$ , in which case the verifier can randomly sample a codeword  $c^\perp = (c_1^\perp, \dots, c_n^\perp)$  from  $\mathcal{C}^\perp$ , indirectly compute the inner product of  $c^\perp$  and the share vector  $(s_1, \dots, s_n)$ , and verify that  $\prod v_i^{c_i^\perp} = g^{\sum s_i c_i^\perp} = g^0 = 1$  with  $O(n)$  exponentiations.

## 1.6 Verifiable Delay Function

### 1.6.1 Wesolowski's

### 1.6.2 Pietrzak's

### 1.6.3 Group Instantiation

RSA group vs class group of  $\mathbb{Q}(\sqrt{p})$  (non-trusted setup)

## 2 Protocols

### 2.1 PVSS-based

#### 2.1.1 RandHound

One-off protocol

#### 2.1.2 Scrape

Protocol  $\pi_{DDH}$  is run between  $n$  parties  $P_1, \dots, P_n$  who have access to a public ledger where they can post information for later verification. A PVSS protocol is used as a subprotocol and it is assumed that the Setup phase is already done and the public keys  $pk_i$  of each party  $P_i$  are already registered in the ledger. The protocol proceeds as follows:

1. **Commit:** For  $1 \leq j \leq n$ , party  $P_j$  executes the Distribution phase of the PVSS subprotocol as the Dealer with threshold  $t = \frac{n}{2}$ , publishing the encrypted shares and the verification information  $PROOF_D^j$  on the public ledger, and also learning the random secret  $h^{s^j}$  and  $s^j$ .  $P_j$  also publishes a commitment to the secret exponent  $\text{Com}(s^j, r_j)$  (with fresh randomness  $r_j \leftarrow \mathbb{Z}_q$ ), for  $1 \leq j \leq n$ .
2. **Reveal:** For every set of encrypted shares and the verification information  $PROOF_D^j$  published in the public ledger, all parties run the Verification phase of the PVSS subprotocol. Let  $\mathcal{C}$  be the set of all parties who published commitments and valid shares. Once  $\frac{n}{2}$  parties have posted their commitments and valid shares on the ledger, party  $P_j$  opens its commitment, posting  $\text{Open}(s^j, r_j)$  on the ledger, for  $j \in \mathcal{C}$ .
3. **Recovery:** For every party  $P_a \in \mathcal{C}$  that does not publish  $\text{Open}(s^a, r_a)$  in the Reveal phase, party  $P_j$  runs the Reconstruction phase of the PVSS protocol posting  $\tilde{s}_j^a$  and  $PROOF_j^a$  to the public ledger, for  $1 \leq j \leq n$ . Once  $\frac{n}{2}$  valid decrypted shares are published, every party reconstructs  $h^{s^a}$ .

The final randomness is computed as  $\rho = \prod_{j \in \mathcal{C}} h^{s^j}$ .

#### 2.1.3 Hydrand

Hydrand can be interpreted as an improvement over Scrape, where at most one operation of PVSS distribution or reconstruction is conducted every round such that we are able to reduce the protocol's communication complexity by a factor of  $n$ , i.e. from  $O(n^3)$  to  $O(n^2)$ . Basically, this is made possible by introducing the concept of a round leader (which is nonexistent in the case of Scrape) such that one of the following two scenarios is to happen every round: the round leader reveals his

secret committed in his previous round (i.e. the last time he was the round leader as per some predefined, deterministic leader selection mechanism) and commits to a new secret to be utilized in a future round by distributing the corresponding PVSS shares via a message of size  $O(n)$  (hence at most one PVSS distribution), or the leader fails to reveal his secret committed in his previous round in a timely manner, in which case the non-leader nodes are able to reconstruct the secret by aggregating the shares that are received previously (hence at most one PVSS reconstruction). This indeed contrasts with Scrape, where there are  $O(n)$  fresh operations of PVSS distribution and reconstruction every round due to the fact that each node has to perform one each round.

Again, the difference is that PVSS shares distributed by a leader in HydRand are used not in the current round (which is the case for Scrape), but the next time the same leader is selected in some future round. Combined with the fact that HydRand’s randomness each round is defined as the hash of the previous round’s randomness as well as the round leader’s secret corresponding to the current round (but committed previously), this can potentially cause the beacon’s unpredictability property to fail without any remedy, as the leader selection mechanism can theoretically select only the adversarial nodes as the round leaders such that it becomes possible for the adversarial nodes to predict the entire randomness chain in advance. To remedy this, HydRand takes the approach that a round leader is not allowed to be selected again for the next  $f$  rounds (where  $f$  denotes the number of Byzantine nodes). As a result, the protocol guarantees that there exists at least one honest node in  $f + 1$  consecutive rounds such that it is impossible to achieve predictability for up to  $f + 1$  rounds. Nonetheless, it remains possible to achieve predictability for up to  $f$  rounds with a low probability whenever  $f$  Byzantine nodes are consecutively selected as leaders, and this remains a tradeoff incurred by HydRand.

Employing Scrape’s PVSS as a subprotocol, HydRand’s rounds are implemented in three distinct phases: propose, acknowledge, and vote.

**I. Propose.** During this phase, the round leader  $\ell$  reveals his previously committed value  $s_\ell$  and provides a new commitment  $Com(s_\ell^*)$ . For this purpose, it is the leader’s task to propose a new dataset  $D_r$  for the current round  $r$ . As a performance optimization, we split a dataset into two parts: a header and a body. For certain operations, we only require sending the header of the dataset. The header  $header(D_r)$  of dataset  $D_r$  contains:

- the hash of the dataset’s body  $H(body(D_r))$
- the current round index  $r$
- the round’s random beacon value  $R_r$
- the revealed secret value  $s_\ell$
- the round index  $\tilde{r}$  of the previous dataset  $D_{\tilde{r}}$

- the hash  $H(D_{\tilde{r}})$  of the previous dataset  $D_{\tilde{r}}$  if  $\tilde{r} > 0$
- a list of random beacon values  $\{R_k, R_{k+1}, \dots\}$  for all recovered rounds between  $\tilde{r}$  and  $r$  (if any)
- the Merkle tree root hash  $M_r$  over all encrypted shares in the new commitment  $Com(s_\ell^*)$

Then the body  $body(D_r)$  of dataset  $D_r$  contains:

- a confirmation certificate  $CC(D_{\tilde{r}})$  which confirms that  $D_{\tilde{r}}$  was previously accepted as a valid dataset
- a recovery certificate  $RC(k)$  for all rounds  $k \in \{\tilde{r} + 1, \tilde{r} + 2, \dots, r - 1\}$  which confirms that there exists a recovery for all rounds between  $\tilde{r}$  and  $r$
- the commitment  $Com(s_\ell^*)$  to a new randomly chosen secret  $s_\ell^*$

Note that the leader selects  $\tilde{r} < r$  as the most recent regular round, for which the leader is not aware of any successful recovery.

After the construction of the above dataset, a correct leader  $\ell$  broadcasts a signed *propose* message to all nodes. Each node  $i$ , which receives such a message from the leader before the end of the propose phase, checks the validity of the dataset  $D_r$ . For this purpose  $i$  verifies that  $D_r$  is constructed as previously defined and properly signed. This includes a check that the revealed secret  $s_\ell$  corresponds to the previous commitment of the current leader. Additionally, the validity of the confirmation and recovery certificates is checked. A *confirmation certificate* for dataset  $D_{\tilde{r}}$  is valid iff it consists of  $f + 1$  signed messages from  $f + 1$  different senders. Similarly, a *recovery certificate* for some round  $k$  is a collection of  $f + 1$  signed messages from  $f + 1$  different senders.

**II. Acknowledge.** If a node  $i$  receives a valid dataset  $D_r$  from the round's leader  $\ell$  during the propose phase, it constructs and broadcasts a signed acknowledge message thereby also forwarding the revealed secret value  $s_\ell$  as part of the header. Further, each node  $i$  collects and validates acknowledge messages from other nodes.

**III. Vote.** Each node  $i$  checks the following conditions:

- During the current propose phase a valid dataset  $D_r$  was received.
- During the current acknowledge phase at least  $2f + 1$  valid acknowledge messages from different senders have been received.
- All acknowledge messages received refer to the dataset's hash  $H(D_r)$ . Valid acknowledge messages for more than one value of  $H(D_r)$  form a cryptographic proof of leader equivocation.

If all conditions are met, node  $i$  broadcasts a signed confirmation message. Otherwise, node  $i$  broadcasts a recover message. Note that a Merkle tree root hash is required to enable nodes which are not in possession of  $Com(s_\ell)$  to verify the share decryption proof.  $R_{r-1}$  is included for efficient external verification.

At the end of this phase, each node  $i$  can obtain the round's random beacon value  $R_r$ . We distinguish between the following two cases: (i) node  $i$  already knows the secret value  $s_\ell$ , because it received the dataset  $D_r$  or an acknowledge message for  $D_r$ , and (ii) node  $i$  has received at least  $f + 1$  valid recover messages which include at least  $f + 1$  decrypted secret shares for  $s_\ell$ . In the latter case, the reconstruction procedure of Scrape's PVSS can be executed to produce the value  $h^{s_\ell}$ . In both cases,  $R_r$  is then obtained by computing:

$$R_r \leftarrow H(R_{r-1} || h^{s_\ell})$$

#### 2.1.4 RandPiper

RandPiper offers two protocols: GRandPiper (Good Pipelined Random beacon) and BRandPiper (Better Pipelined Random beacon). While GRandPiper roughly follows HydRand's protocol, it improves upon it via its idiosyncratic use of BFT SMR (state machine replication) protocol, which uses erasure coding and cryptographic accumulators (either of the bilinear type incurring a trusted setup tradeoff or of the Merkle tree type) to implement the public bulletin board model in a Byzantine broadcast fashion such that HydRand's worst-case communication complexity of  $O(n^3)$  is improved to  $O(n^2)$ . The main idea is that RandPiper's SMR protocol allows a round leader to Byzantine broadcast a message of size  $O(n)$  within a total communication complexity of  $O(n^2)$  rather than  $O(n^3)$ . While this improvement benefits the side of communication complexity indeed, the same issue of predictability present in HydRand still remains in GRandPiper, where an adversary can predict up to  $f$  number of rounds (albeit with low probability).

### 2.2 DVRF-based

Distributed verifiable random function (DVRF) is quite naturally a distributed version of VRF, where the VRF's secret key is distributed among a group of participants.

Hash of VUF  $\rightarrow$  VRF

Threshold signature can be interpreted under this bucket

#### 2.2.1 RandHerd

Involves threshold Schnorr + CoSi (collective signing = multisig aggregation + communication tree) in a fairly complicated way

### **2.2.2 Dfinity**

League of Entropy's drand

Stalling the network? 100% eventual liveness assumption?

### **2.2.3 DDH-DVRF**

Does not use pairings

## **2.3 VDF-based**

### **2.3.1 Extending Commit-Reveal**

Unicorn: pre-VDF

RANDAO + VDF

### **2.3.2 Extending Public Randomness**

Taking some closing stock price or a block hash + VDF

### **2.3.3 RandRunner**

A VDF chain involving a group of trapdoor VDFs

## References

- [1] Feldman, Paul. “A practical scheme for non-interactive verifiable secret sharing.” 28th Annual Symposium on Foundations of Computer Science (sfcs 1987). IEEE, 1987.
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