7.1)

for simplification:

Let $A = \begin{bmatrix} L_{00} & 0 & 0 \\ \bar{z}_{10} e_L^T & 1 & v_{12}^T e_F \\ 0 & 0 & U_{22} \end{bmatrix}$

Let $B = \begin{bmatrix} D_{00} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_{22} \end{bmatrix}$

Let $X = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$

want to compute cost of $AB A^T X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Step 1, cost of AB:

$L_{00} D_{00}, \quad U_{22} E_{22}$.

L_{00} and U_{22} have bidiagonal structures, each costs $(2n-1)$

$Cost = (2n-1) + (2n-1) = 4n-2$

Step 2, cost of ABA^T

$L_{00} D_{00} L_{00}^T$, $U_{22} E_{22} U_{22}^T$

Still bidiagonal structures, each cost $2n-1$

cost $= (2n-1) + (2n-1) = 4n-2$

Step 3, $A^T X$:

$L_{00}^T X_0$, $U_{22}^T X_2$. each cost n

cost $= n + n = 2n$

total cost $= (4n-2) + (4n-2) + (2n) = 10n-4 \approx O(n)$