Exercise 4: 3×3 Gridworld

(1) compute the value functions of states A B D E F H at k=1

	$V_k(s)$	$V_{k+1}(s)$	$V_{k+2}(s)$
A	0	-1	-2
B	0	-1	-1.75
D	0	-1	-1.75
E	0	-1	-2
F	0	-1	-1.5
H	0	-1	-1.5

step 1 A at k=1

(1) $V_{k+1}(A)$ ← → ↓ ↑

↳ $= \frac{1}{4}[(-1+V(A)) + (-1+V(B)) + (-1+V(D)) + (-1+V(A))]$

$-1+0 \quad + -1+0 \quad + -1+0 \quad + -1+0$

$V_{k+1}(A) = -1$

B at k=1

(2) $V_{k+1}(B)$ ← ↑ → ↓

↳ $= \frac{1}{4}[(-1+V(A)) + (-1+V(B)) + (-1+V(C)) + (-1+V(E))]$

$-1+0 \quad + -1+0 \quad + -1+0 \quad + -1+0$

(3) $V_{k+1}(D)$ ↓ → ← ↑

↳ $\frac{1}{4}[(-1+V(G)) + (-1+V(E)) + (-1+V(D)) + (-1+V(A))]$

$-1+0 \quad + \quad -1+0 \quad -1+0 \quad + -1+0 \quad = \frac{-4}{4} = -1$

(4) $V_{k+1}(E)$

↳ $\frac{1}{4}[(-1+V(F)) + (-1+V(H)) + (-1+V(D)) + (-1+V(B))]$

$= -1$

(5) $V_{k+1}(F)$ ↑ ↓ ←

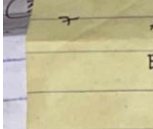
↳ $\frac{1}{4}(-1+V(F)) + (-1+V(C)) + (-1+V(I)) + (-1+V(E))$

-1

(6) $V_{k+1}(H) = \frac{1}{4}[(-1+V(G)) + (-1+V(H)) + (-1+V(I)) + (-1+V(E))]$

(-1)

(7)



at k=1

-1	-1	0
-1	-1	-1
0	-1	0

Step 2

(8) $q_{k+1}(A, \text{LEFT}) = -2$

$\frac{1}{4}(-1 + V(A)) = -1$

(9) $q_{k+1}(A, \text{right}) = -2$

$\frac{1}{4}((-1+V(B)))$

(10) $q_{k+1}(A, \text{up}) = -2$

(11) $q_{k+1}(A, \text{Down}) = -2$

(12) $\pi_{k+1}(A) = \leftarrow$ up, down, R

(13) $q_{k+1}(B, \text{LEFT}) = -2$

(14) $q_{k+1}(B, \text{RIGHT}) = -1$

(15) $q_{k+1}(B, \text{up}) = -2$

(16) $q_{k+1}(B, \text{Down}) = -2$

(17) $\pi_{k+1}(B) \rightarrow$ right

(18) $q_{k+1}(D, \text{LEFT}) = -2$

(20) $q_{k+1}(D, \text{up}) = -2$

(21) $q_{k+1}(D, \text{Down}) = -1$

(19) $q_{k+2}(D, \text{right}) = -2$

$\pi_{k+1}(D) =$ Down

(22) 1

(23) $q_{k+1}(E, \text{LEFT}) = -2$

(24) $q_{k+1}(E, \text{RIGHT}) = -2$

(25) $q_{k+1}(E, \text{up}) = -2$

(26) $q_{k+1}(E, \text{DOWN}) = -2$

(27) $\pi_{k+1}(E) = \leftrightarrow$

(28) $Q_{k+1}(F, \text{LEFT}) = -2$

(29) $Q_{k+1}(F, \text{RIGHT}) = -2$

(30) $Q_{k+1}(F, \text{UP}) = -1$

(31) $Q_{k+1}(F, \text{DOWN}) = -1$

(32) $\pi_{k+1}(F) =$ ↕ up, down

(33) ~~$Q_{k+1}(H, \text{LEFT}) = -1$~~

(34) ~~$Q_{k+1}(H, \text{RIGHT}) = -1$~~

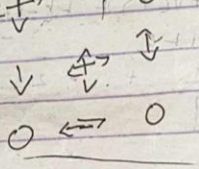
(35) ~~$Q_{k+1}(H, \text{up}) = -2$~~

(36) $Q_{k+1}(\text{any}, \text{DOWN}) = -2$

(37) $\pi_{k+1}(H) =$ ↔ L, R

(38)

Step 2.



Step 3. Use DP to find the optimal val $f(V_k)$ of states A, B, D, E, F, H

(39) $V_{k+1}(A) = \frac{1}{4}[(-1+V(A)) + (-1+V(A)) + (-1+V(D)) + (-1+V(B))]$

$= -1 + -1 + -1 + -1$

$= -2$

$V_{k+1}(A) = -2$

(40) $V_{k+1}(B) = \frac{1}{4}[(-1+V(A)) + (-1+V(B)) + (-1+V(C)) + (-1+V(E))]$

(41) $= -1 + -1 + -1 + -1 + -1 + 0 + -1 + -1$

$= -2 + -2 + -1 + -2$

$= -7/4 = -1.75$

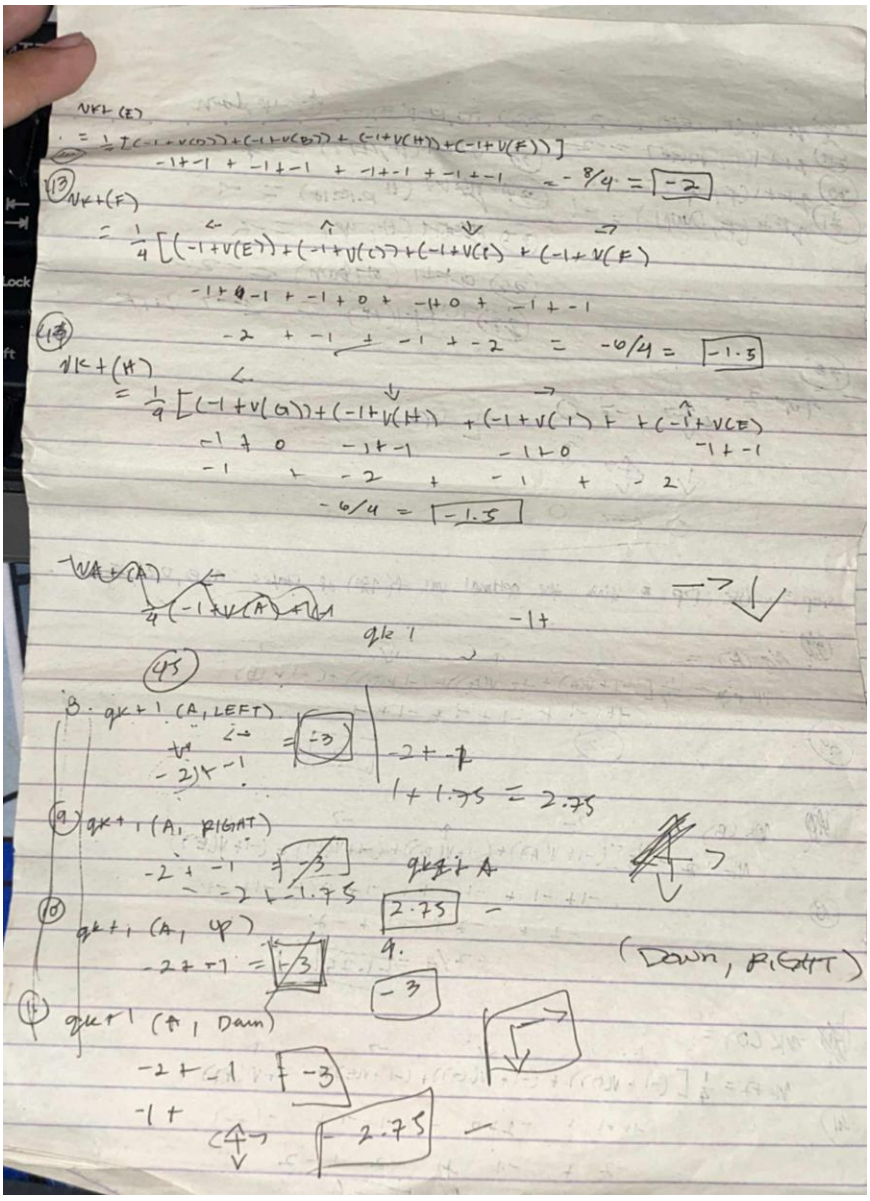
$V_{k+1}(D) =$

$V_{k+1} = \frac{1}{4}[(-1+V(D)) + (-1+V(G)) + (-1+V(E)) + (-1+V(A))]$

$= -1+1 + -1+0 + -1+-1 + -1+-1$

$= -2 + -1 + -2 + -2$

$= -7/4 = -1.75$



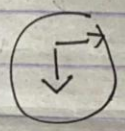
(10)

$E_{k+1}(E, \text{LEFT}) =$

$-1 + 1.75 = 2.75$

(E, RIGHT)

$-1 + 1.5 = 2.5$

$\Rightarrow E =$ 

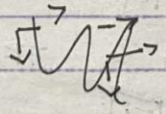
E, UP

$-1 + 1.75 = 2.75$

(E, Down)

$-1 + + 1.5 = 2.5$

Final



↓→	→	0
↓	↓→	↕
0	←	0

(46)

q_{k+1} (B, LEFT)
$-1 + -2 = -3$

q_{k+1} (B, UP)
$-1 + -1.75 = -2.75$

q_{k+1} (B, RIGHT)
$-1 + 0 = -1$

q_{k+1} (B, DOWN)
$-1 + -2 = -3$

π_k = (RIGHT)

(47)

q_{k+1} (D, LEFT)
$-1 + 1.75 = -2.75$

q_{k+1} (D, UP)
$-1 + -2 = -3$

q_{k+1} (D, RIGHT)
~~$-1 + -1.5 = -2.5$~~
$-1 + -2 = -3$

q_{k+1} (D, DOWN)
$-1 + 0 = -1$

π_k = (DOWN)

(49)

q_{k+1} (~~F LEFT~~)
$-1 + -2 = -3$

q_{k+1} (F, UP)
$-1 + 0 = -1$

q_{k+1} (F, RIGHT)
$-1 + -1.5 = -2.5$

q_{k+1} (F, DOWN)
$-1 + 0 = -1$

π_k = (UP, DOWN)

(50)

q_{k+1} (H, LEFT)
$-1 + 0 = -1$

q_{k+1} (H, UP)
$-1 + -2 = -3$

q_{k+1} (H, RIGHT)
$-1 + 0 = -1$

q_{k+1} (H, DOWN)
$-1 + -1.5 = -2.5$

π_k (LEFT, RIGHT)

(51) $\pi_k(A)$ = (DOWN, RIGHT)

(52) $\pi_k(B)$ = (RIGHT)

(53) $\pi_k(D)$ = (DOWN)

(54) $\pi_k(E)$ = (DOWN, RIGHT)

(55) $\pi_k(F)$ = (UP, DOWN)

(56) $\pi_k(H)$ = (LEFT, RIGHT)

(57) (58)

A B C
D E F
G H I