

Mass profiles of relaxed clusters of galaxies: an X-ray perspective

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Dark matter halos

Cold Dark Matter (CDM) N-body simulations of structure formation produce dark matter halos whose density profiles are well parametrized by a 2 parameter function regardless of their mass or cosmology used.

NFW density profile

(Navarro, Frenk & White 1997)

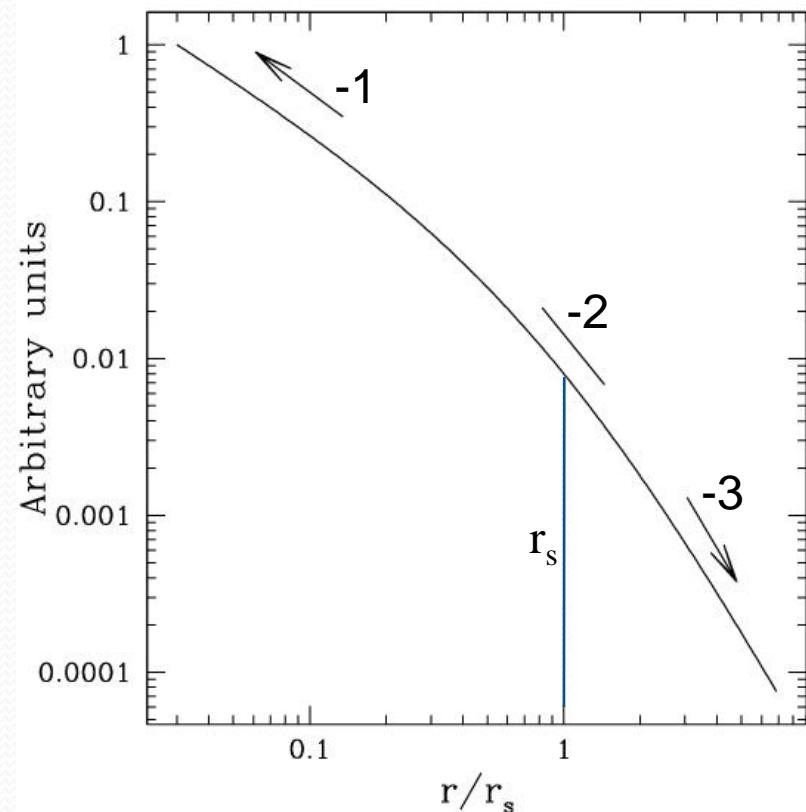
$$\rho(r) = \frac{\rho_c(z)\delta_c}{(r/r_s)(1 + r/r_s)^2}$$

The diagram shows the NFW density profile equation $\rho(r) = \frac{\rho_c(z)\delta_c}{(r/r_s)(1 + r/r_s)^2}$. Four annotations are shown as dashed boxes with arrows pointing to specific terms:

- A box on the left points to $\rho_c(z)$ and contains the text "critical density of the universe at redshift z".
- A box on the right points to δ_c and contains the text "characteristic dimensionless density".
- A box at the bottom points to r_s and contains the text "scale radius".
- A box at the top points to the term $(r/r_s)(1 + r/r_s)^2$.

NFW density profile

- Inner profile with logarithmic slope -1 giving rise to a cusp
- At large radius the slope reaches a value of -3
- The scale radius (r_s) defines the point in which the slope is -2



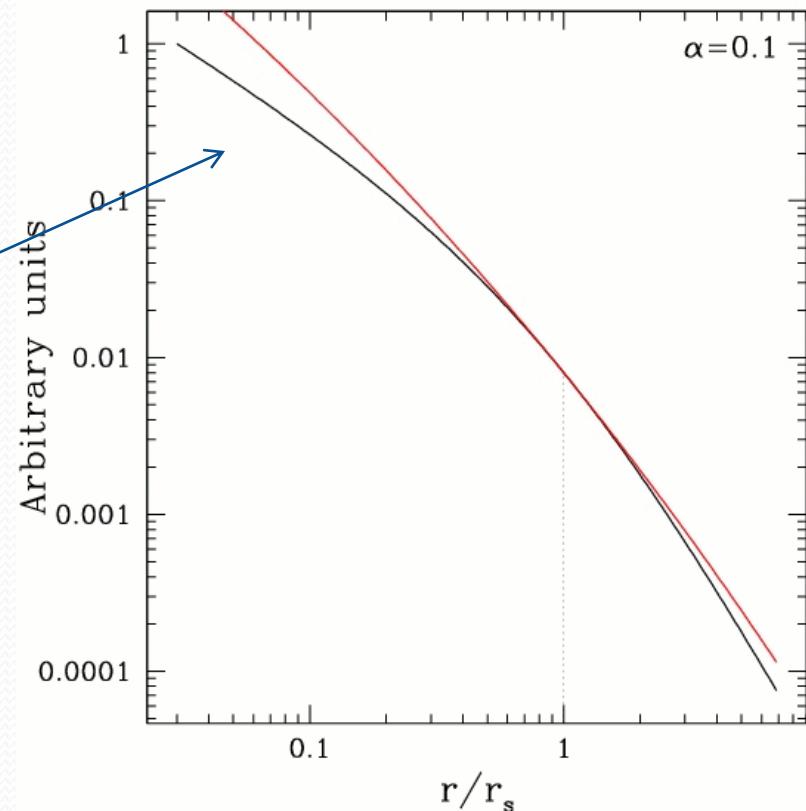
Navarro et al. 2004 profile

Navarro et al. 2004 (N04) with higher resolution simulations found halo inner slopes gradually flattening.

$$\rho(r) = \rho_{-2} \exp\left(\frac{2}{\alpha}\right) \exp\left(-\frac{2}{\alpha}x^\alpha\right)$$

$$x = r/r_s$$

- Same functional form of a Sersic profile
- Flattens gradually producing a constant density core
- $\alpha = 0.172 \pm 0.032$



Profiles modified by adiabatic contraction

From hydrodynamic simulations

with gas cooling:

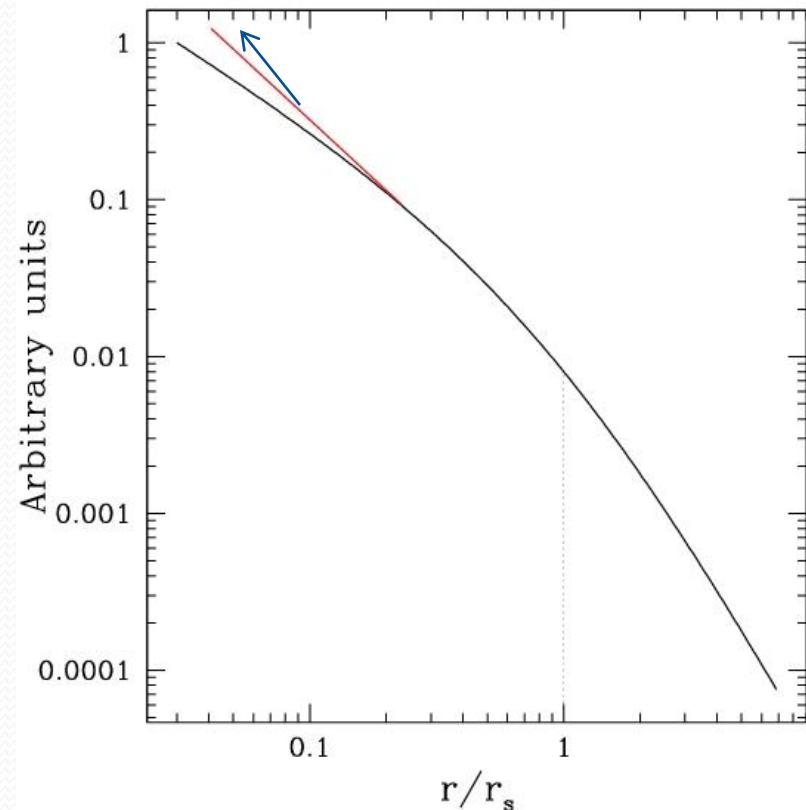
- the condensation of baryons in the cores of dark halos cause the dark matter to adiabatically contract further inward.

Process of Adiabatic Contraction (AC)

(Blumenthal et al. 1986; Gnedin et al. 2004)



The dark matter density further increases in the center.



Virial quantities and concentration parameter

Each profile can be related to the virial quantities of the halo.

Definitions:

- Virial radius r_{vir}  the radius within which the mean matter density is $\Delta_{\text{vir}} \rho_c(z)$
- Virial Mass $M_{\text{vir}} = 4/3 \pi \Delta_{\text{vir}} \rho_c(z) r_{\text{vir}}^3$  Mass enclosed in r_{vir}
- r_{-2} is the characteristic radial scale of the profile where the logarithmic slope of the density profile is -2 (*scale radius* for NFW)
- Concentration parameter $c = r_{\text{vir}}/r_{-2}$

Mass profiles of clusters of galaxies

Clusters of galaxies are excellent candidates for the study of mass and dark matter profiles



- dark matter can dominate deep down to $<0.01 r_{\text{vir}}$
(Lewis et al. 2003)
- Several powerful techniques can be used to recover their mass profiles

Mass profiles: different techniques

- dynamical analysis
 - redshift space caustics
 - gravitational lensing (strong, weak)
 - hydrostatic equilibrium (X-ray)
 - and many more...
-
- Each technique has advantages and disadvantages and are complementary:
 - E.g.
 - Caustics and lensing do not require dynamical equilibrium assumptions
 - Lensing probe the projected mass
 - Methods relying on galaxies → good at large radii
 - The X-ray method rely on the ICM → good in central regions

Mass profiles from X-rays

If the X-ray emitting IntraCluster Medium (ICM) is in hydrostatic equilibrium in the cluster potential



The mass enclosed within a radius r can be inferred by the hydrostatic equilibrium equation.

By assuming spherical symmetry and ideal gas law

$$M_{\text{grav}}(< r) = \frac{k_{\text{B}}}{G\mu m_p} r T \left(-\frac{d \ln \rho}{d \ln r} - \frac{d \ln T}{d \ln r} \right)$$

(Fabricant et al. 1980)

Mass profiles from X-rays

If the X-ray emitting IntraCluster Medium (ICM) is in hydrostatic equilibrium in the cluster potential

Reliable mass estimates

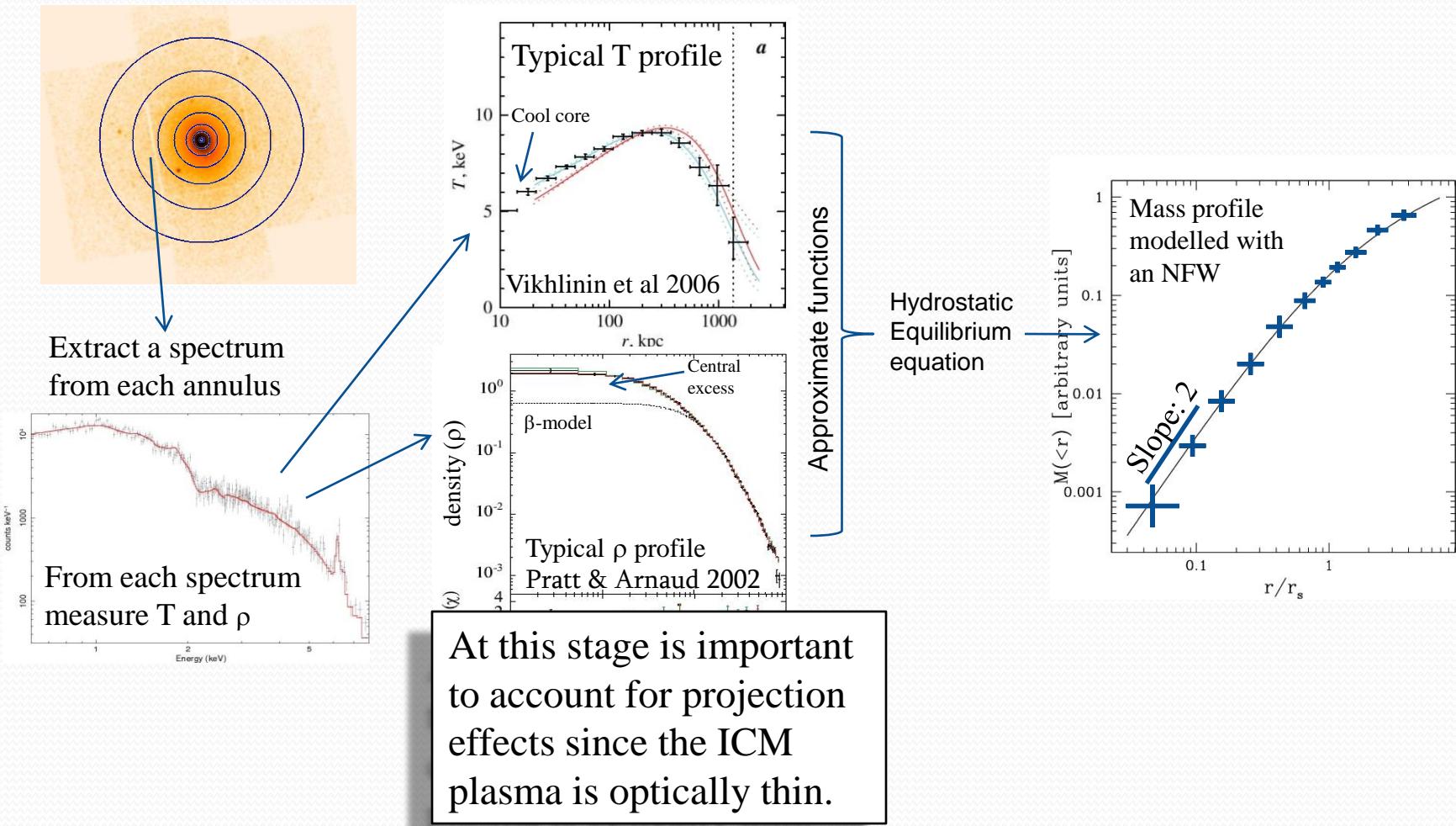
The hydrostatic equilibrium technique has been validated against other techniques

(Girardi et al. 1998, Allen et al. 2001, Diaferio et al. 2005, Sehgal et al. 2007)

$$M_{\text{grav}}(< r) = \frac{k_{\text{B}}}{G \mu m_p} r T \left(-\frac{d \ln \rho}{d \ln r} - \frac{d \ln T}{d \ln r} \right)$$

(Fabricant et al. 1980)

Measuring a mass profile from X-rays: a schematic view



Mass profiles from X-rays: the observer point of view

Thanks to XMM and Chandra

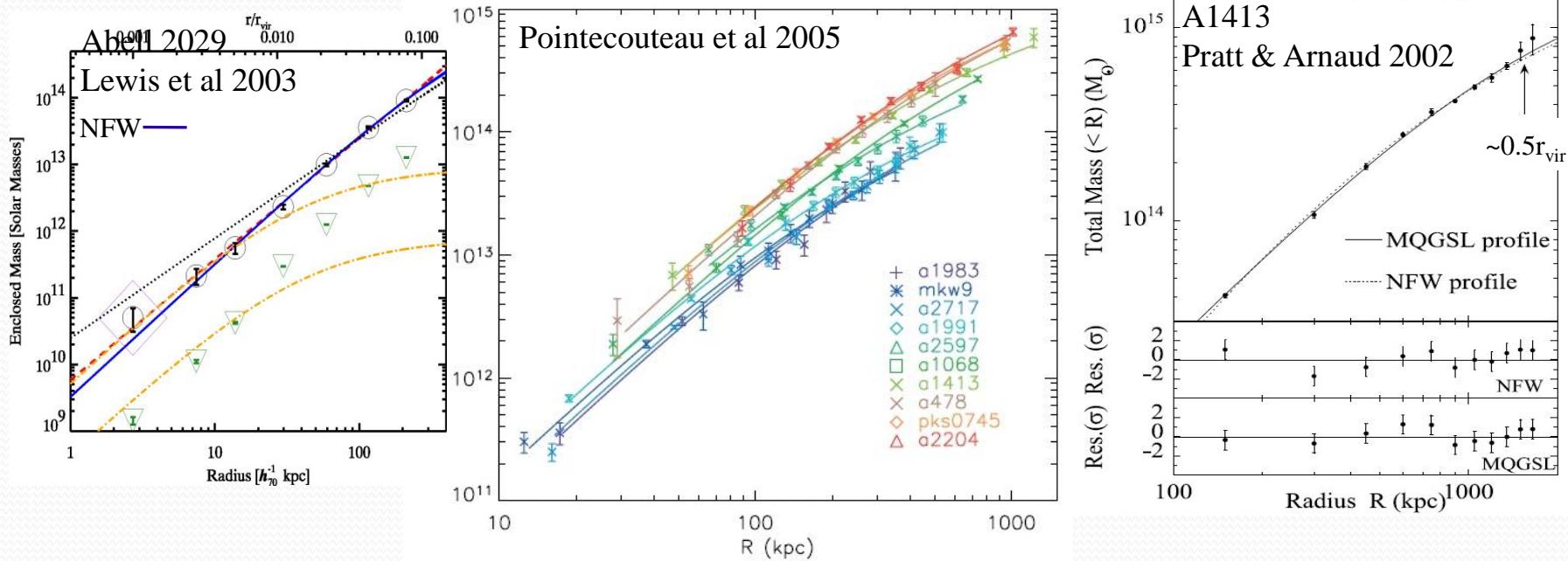


High sensitivity, high
spatial and spectral
resolution



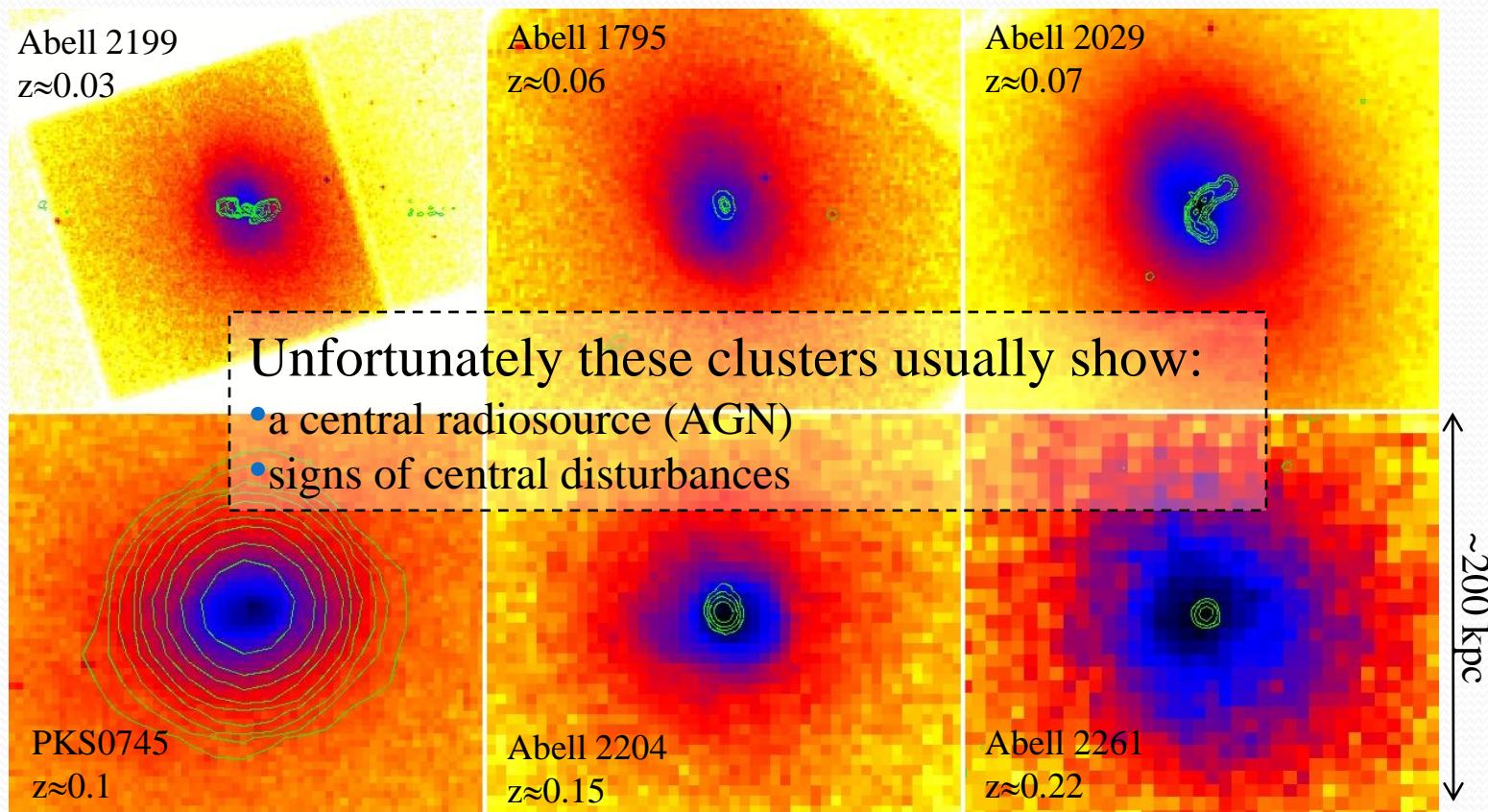
- Mass profiles of relaxed X-ray clusters of galaxies can be measured in great detail from $< 0.01r_{\text{vir}}$ to $\geq 0.5r_{\text{vir}}$
(Lewis et al 2003, Pratt & Arnaud 2002)
- Generally an NFW provide a good description of the total mass profiles for clusters of galaxies
(Pointecouteau et al. 2005)

Mass profiles from X-rays: the observer point of view II



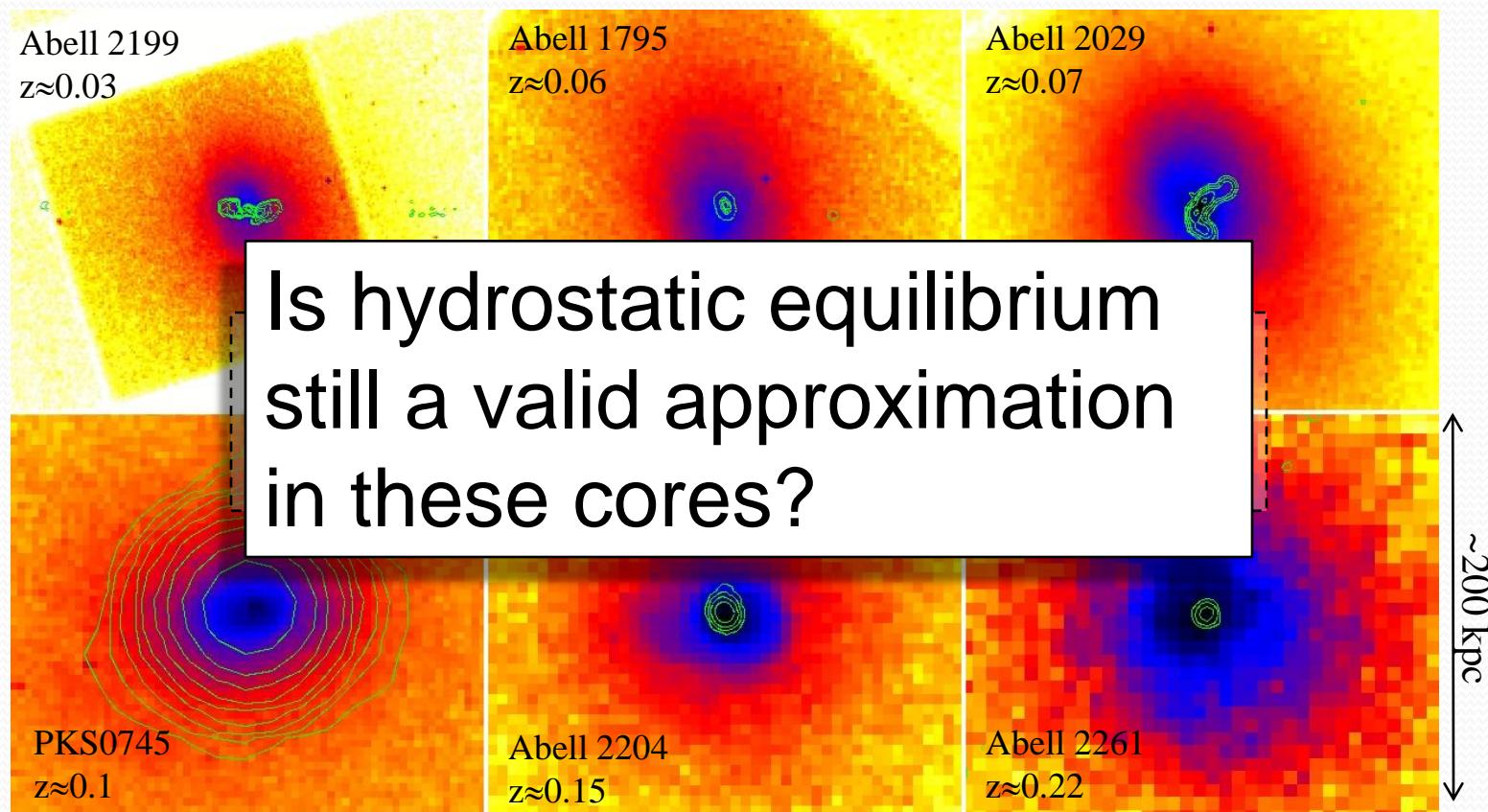
Mass profiles from X-rays: the observer point of view III

Only clusters with regular morphologies indicative of a relaxed dynamical state. To validate the hydrostatic equilibrium assumption.



Mass profiles from X-rays: the observer point of view III

Only clusters with regular morphologies indicative of a relaxed dynamical state. To validate the hydrostatic equilibrium assumption.

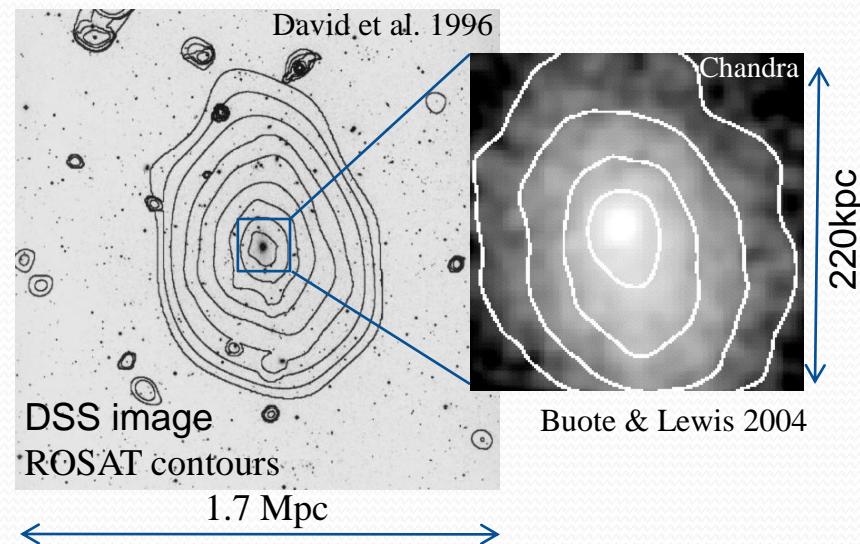


The relaxed radio-quiet cluster Abell 2589

One way to examine the mass profiles in cores of clusters is
to select and study relaxed radio-quiet clusters

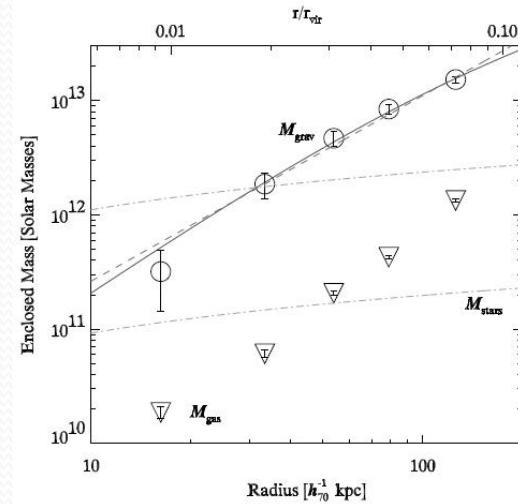
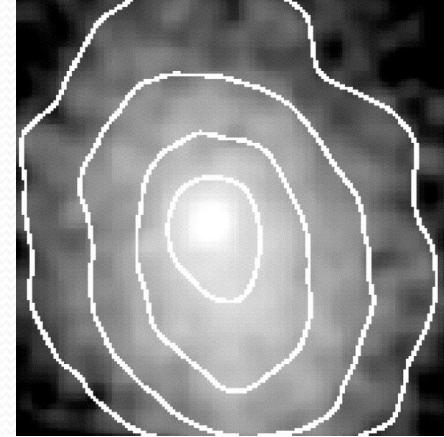
One of the most promising
candidates is: **Abell 2589**

- $z=0.0414$
- No central radio emission
at 1.4 GHz (NVSS maps)
- Relaxed from \sim Mpc scales
down to kpc scales



Chandra analysis of A2589

- Buote & Lewis 2004 analysed a 14ks mildly flared Chandra observation
- Cluster relaxed in the core apart from a $\sim 12\text{kpc}$ center shift (between the core and the outermost observed region).
- Isothermal temperature profile at $\sim 3.2 \text{ keV}$
- Mass profile: NFW
- $r_{\text{vir}} \sim 1 \text{ Mpc}$ (for both total mass and dark matter)
- Inner mass and dark matter profile slope: $\sim 1.6 \pm 0.2$ (measured at $\geq 0.02 r_{\text{vir}}$)



XMM analysis of Abell 2589

(Zappacosta et al. 2006)

- Abell 2589 was observed with XMM for \sim 46ks
- only 1/3 of the original exposure per detector could be used for the analysis (because of background flarings)
- Data quality better than the Chandra observation analysed in Buote & Lewis 2004

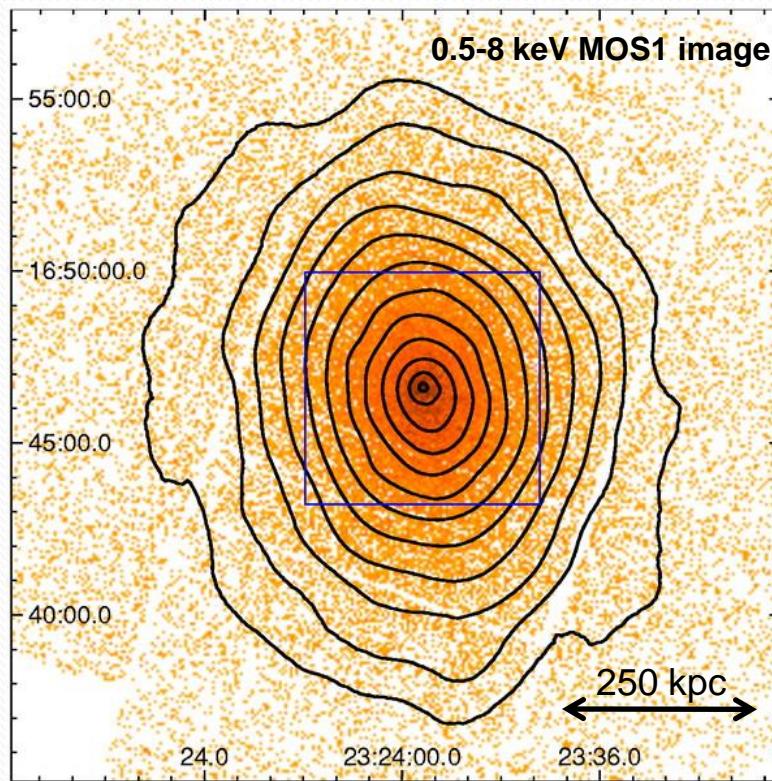


- Improve the Chandra constraints
- Test the NFW vs N04 profile
- Test the occurrence of AC in the cluster core

Morphological analysis

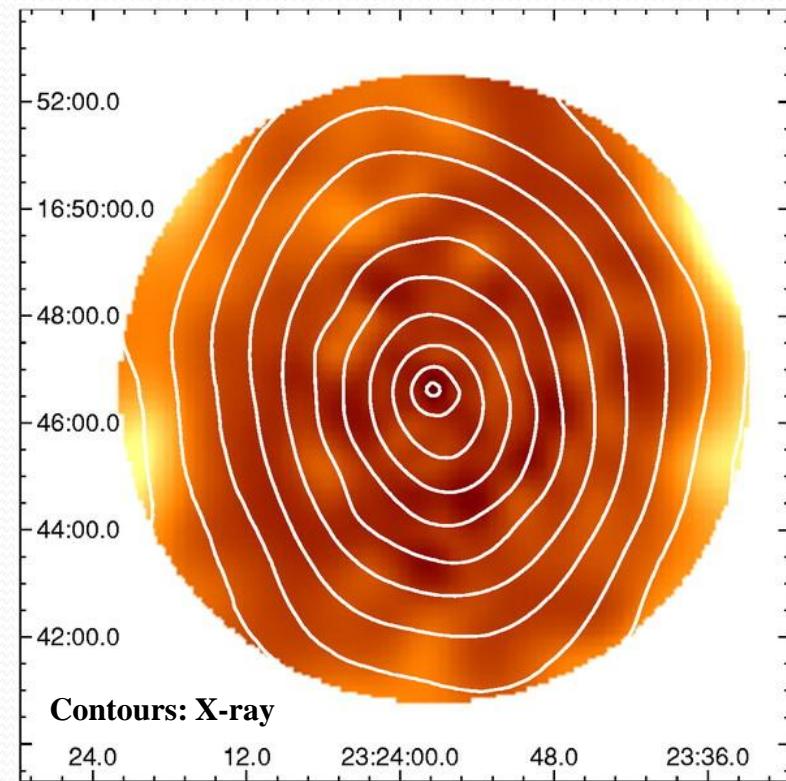
X-ray image

Very relaxed apart from a center shift of 16kpc between the 0-15kpc and 45-60kpc regions



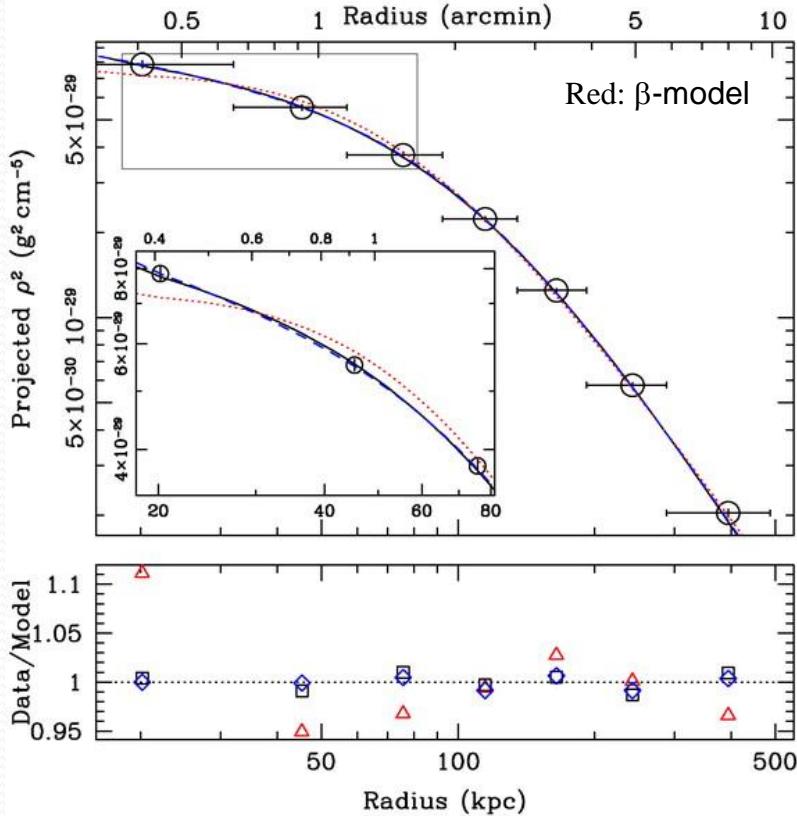
Hardness ratio map

No evidence of significant deviation in the temperature structure

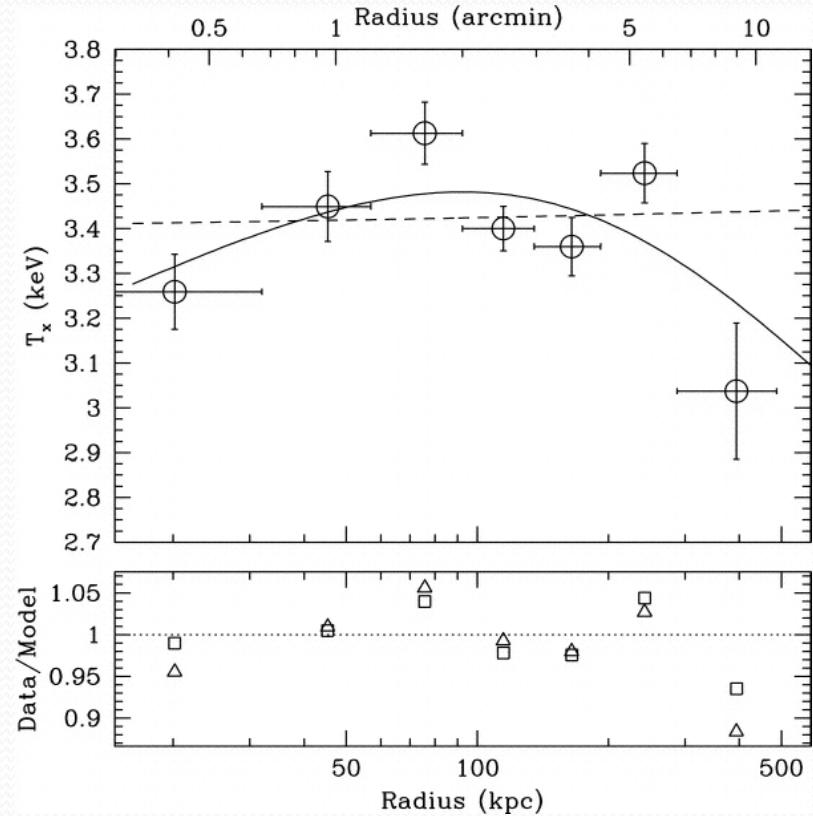


Density and Temperature profiles

Density profile well parametrized by a β -model with central cusp

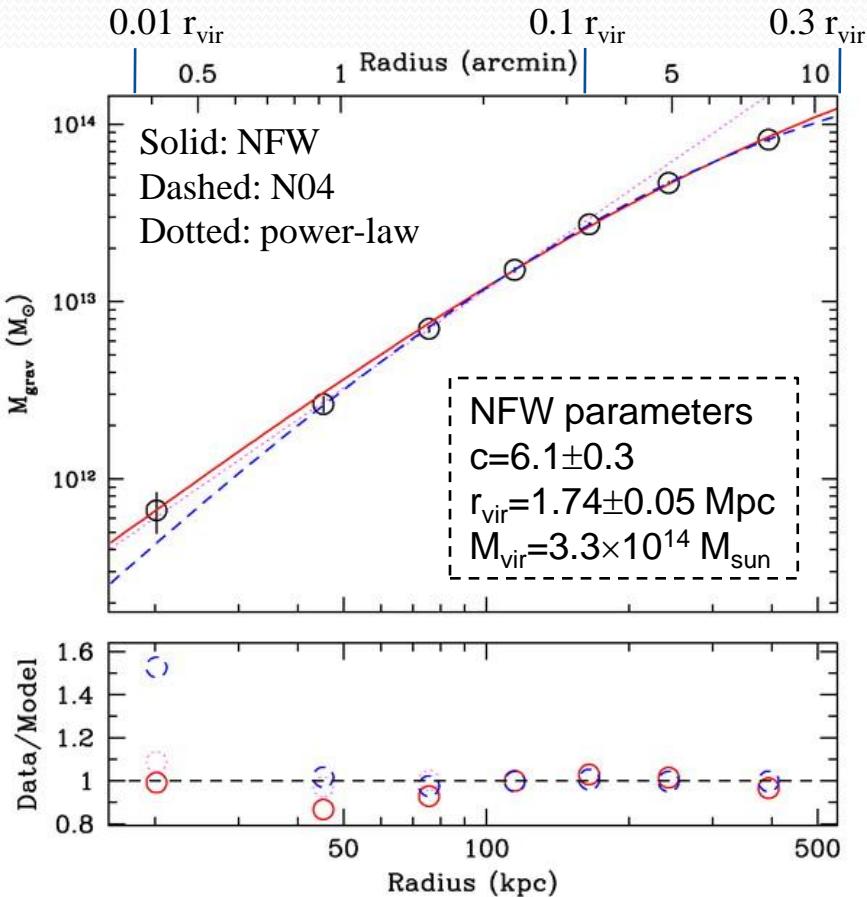


The temperature profiles is nearly isothermal with a hint of cool core



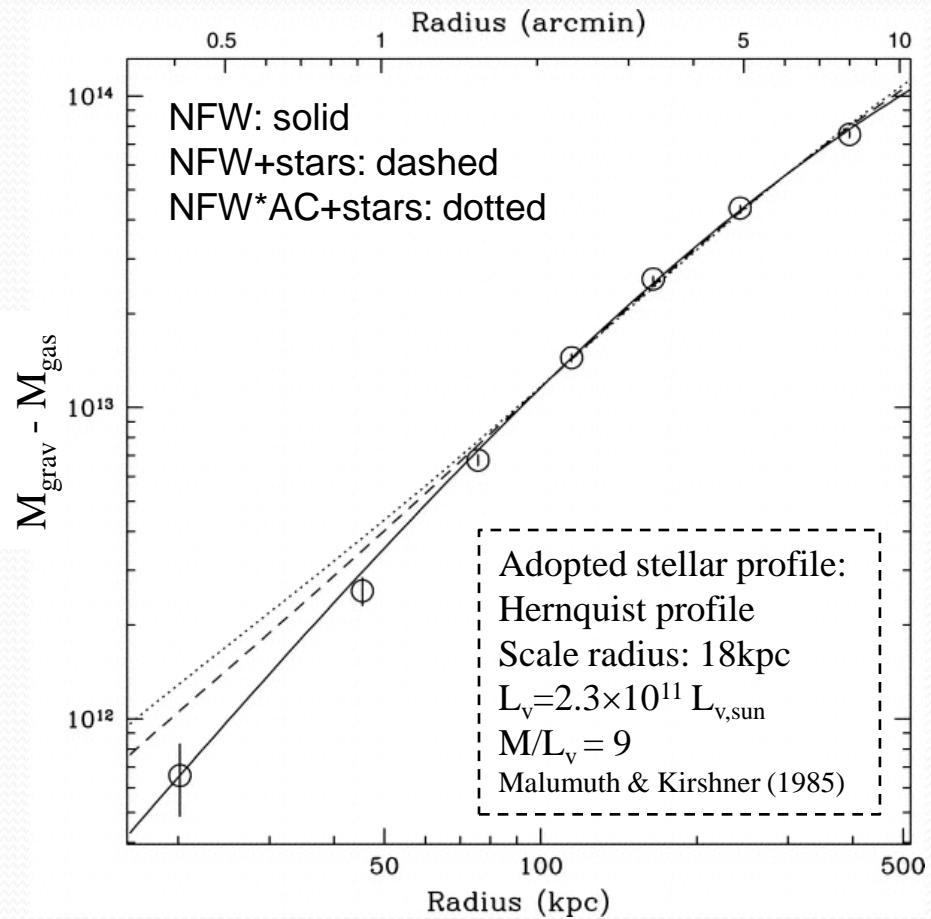
Gravitating mass profile

- The total mass profile is well parametrized by an NFW
- The sersic-like N04 profile underestimate the inner data point
- The N04 index $\alpha=0.4\pm0.05$ is inconsistent with the predicted 0.172 ± 0.032
- The slope computed in the range 0.01 - 0.05 r_{vir} is 1.84 ± 0.18 (agrees at 1σ with NFW)



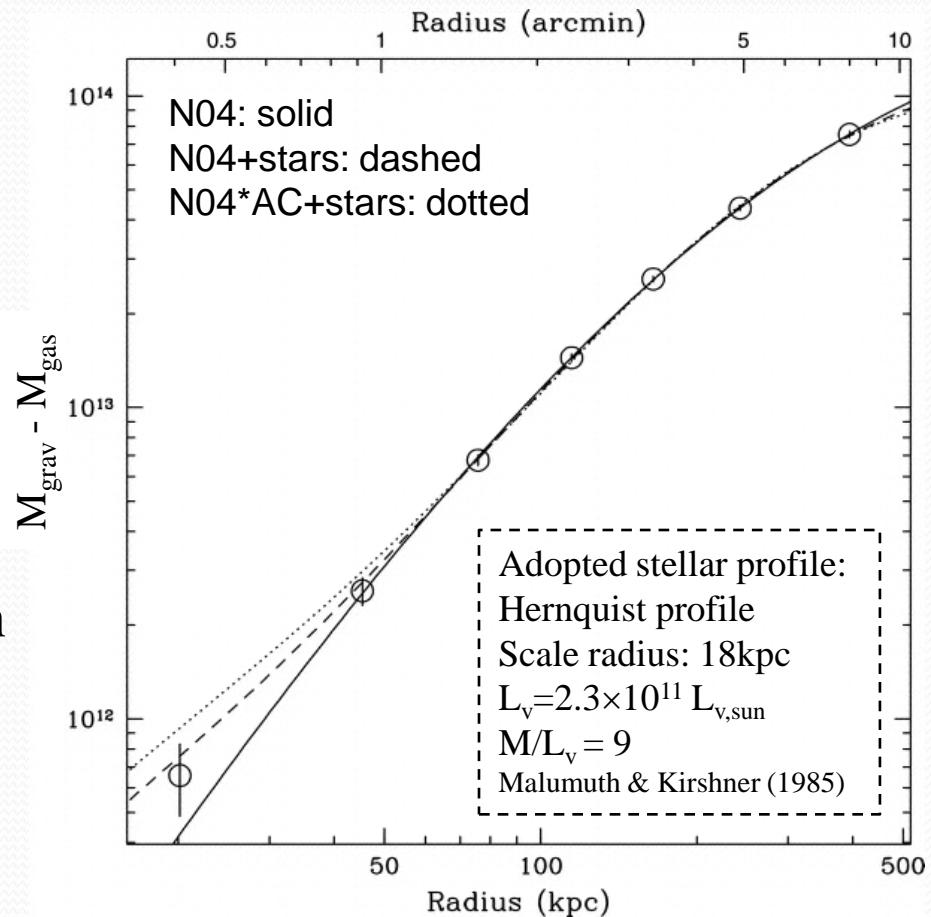
Dark matter profile I

- We fitted dark matter+stellar profiles to $M_{\text{grav}} - M_{\text{gas}}$
- A simple NFW still fits very well
- Adding stellar mass does not improve the fit
- Accounting for AC further worsen the fit
- If we let M/L_v to be a free parameter, it assumes unphysically low values (i.e. <0.15)



Dark matter profile II

- A simple N04 still underestimate the inner data point
- Adding stellar mass improves the fit
- Accounting for AC worsen the fit
- An M/L_v free produce values in the range 3-5
- These results have to be taken with caution because the N04 index α reaches very high values (>0.6) to accomodate more central stellar mass



Adiabatic contraction in Abell 2589

Apparently AC does not take place in the core of A2589

- Can departures from hydrostatic equilibrium like additional pressure support from non-thermal processes reconcile the mass profile with the AC scenario?

We consider turbulent motions and magnetic field
(how much is needed to bring AC in agreement with the data?)

- Turbulent motions: turbulent velocity of $\sim 1.3c_s$ much larger than the predicted 0.1-0.3 c_s (Nagai et al 2003; Faltenbacher et al. 2005)
- Magnetic field: 56 μ G (current evidences favor magnetic field values of 1-10 μ G; Govoni & Feretti 2004)

What did we learn from Abell 2589

- A simple NFW profile is all we need to model A2589 mass and dark matter profile in the range $0.015\text{-}0.3r_{\text{vir}}$
- N04 profile underestimate the central region and gives an index α too large (i.e. core too shallow)
- Adiabatic contraction does not seem have taken place during this cluster formation
- Other processes may counteract the effect of AC in cores
- El-Zant et al. 2004 propose that during halo formation dynamical friction experienced by member galaxies may heat-up the dark matter flattening the central cusp and leading to a total mass profile consistent with NFW

Future prospects

- We need better data for A2589 to better constrain the profile in the very inner core (i.e. $< 0.015r_{\text{vir}}$) and place more stringent limits on the degree of adiabatic contraction that could be present in this object



80ks of Chandra time awarded and already observed in AO7

- We need to carry this study on a sample of objects to draw more general conclusions



Chandra and XMM archives contain ~ 15 radio quiet clusters

Predicted concentration-mass relation

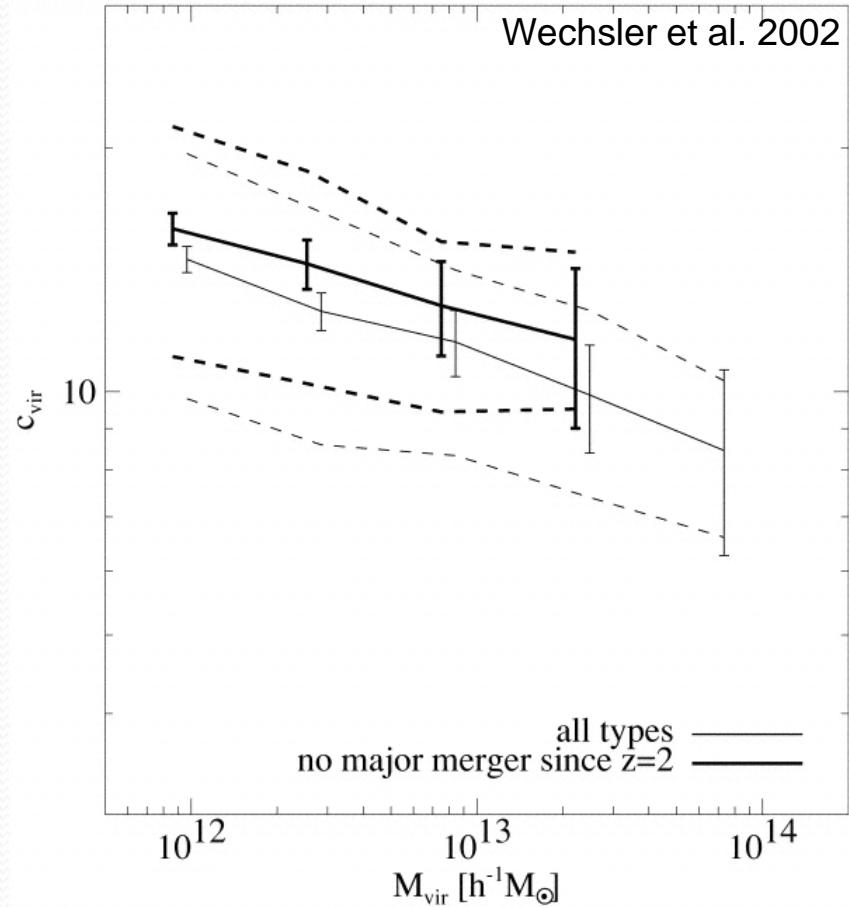
CDM N-body simulations find:

- A relation between concentration parameter (c) and the virial mass (M_{vir}) of a halo *at any given fixed epoch*.

c - M_{vir} relation

(Navarro et al. 1997, Bullock et al. 2001, Dolag et al. 2004, Kuhlen et al. 2005, Shaw et al. 2006, Neto et al. 2007)

- An intrinsic scatter of the c - M_{vir} relation of $0.14\Delta\log(c)$
(Bullock et al. 2001)
- relaxed halos: c - M_{vir} relation with $\sim 10\%$ higher normalization and $\sim 0.1\Delta\log(c)$ of intrinsic scatter.
(Wechsler et al. 2002)



Qualitative explanation of the c - M_{vir} relation

The hierarchical scenario of structure formation



Less massive halos form at earlier times



They will reflect the high density of the universe exhibiting higher concentration than late forming halos

The intrinsic scatter can be explained with a scatter in formation times for halos of fixed mass

Modeling the c-M_{vir} relation

- Several semianalytic models have been proposed to explain quantitatively the relation
(Navarro et al. 1997, Bullock et al. 2001, Eke et al. 2001)
- They predict qualitatively the same c-M_{vir} but differ for details on low/high masses
- Dolag et al. 2004 introduced a power-law parametrization to empirically describe the relation on small ranges of mass:

$$c \propto c_0 / (1+z) M_{\text{vir}}^a$$

Typical values:
 $a \approx -0.1 - -0.14$
 $c_0 \approx 6 - 9$

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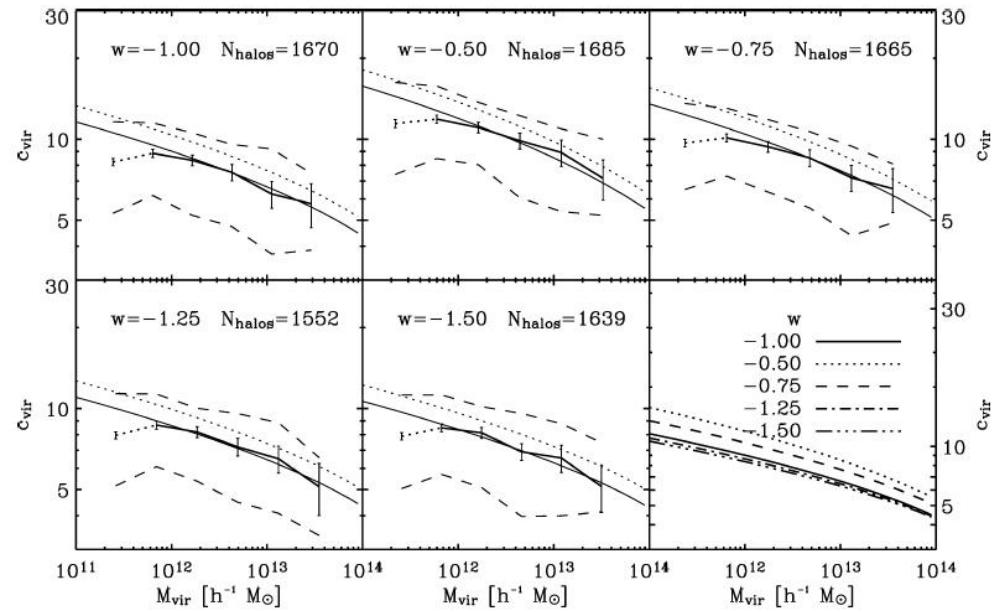
- They details In the following we will compare our results with the Bullock et al. 2001 model and use the Dolag et al. 2004 power-law parametrization
- Dolag to empirically describe the relation on small ranges of mass.

$$c \propto c_0 / (1+z) M_{\text{vir}}^a$$

Typical values:
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Cosmology with the c - M_{vir} relation

- The normalization of the c - M_{vir} relation is sensitive to the cosmology used.
- In particular it depends on Ω_m , w and σ_8

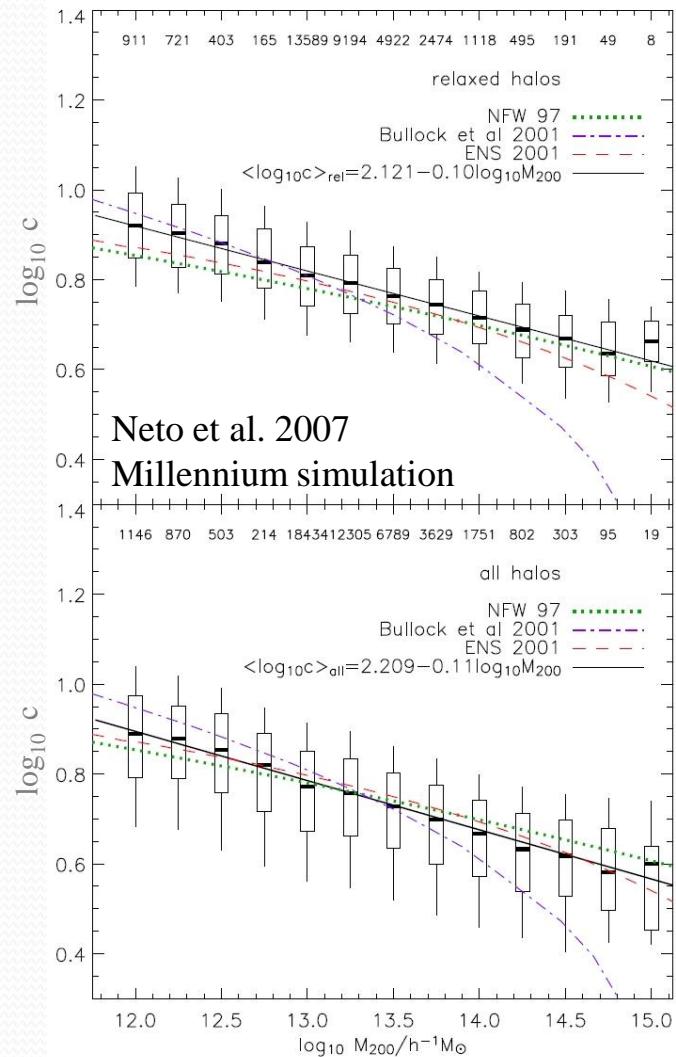


Kuhlen et al. 2005

c-M_{vir} on cluster scales

- c-M_{vir} not well tested on cluster scales
- Difficult to create enough clusters for statistical studies if simulated boxes are not large enough
- Only few studies exploring the cluster scale (i.e. 10^{14} - $10^{15} M_{\odot}$)

(Dolag et al. 2004, Shaw et al. 2006, Neto et al. 2007)



Predicted concentration redshift relation

From N-body simulations

- Predicted dependence between c and redshift *at fixed mass*

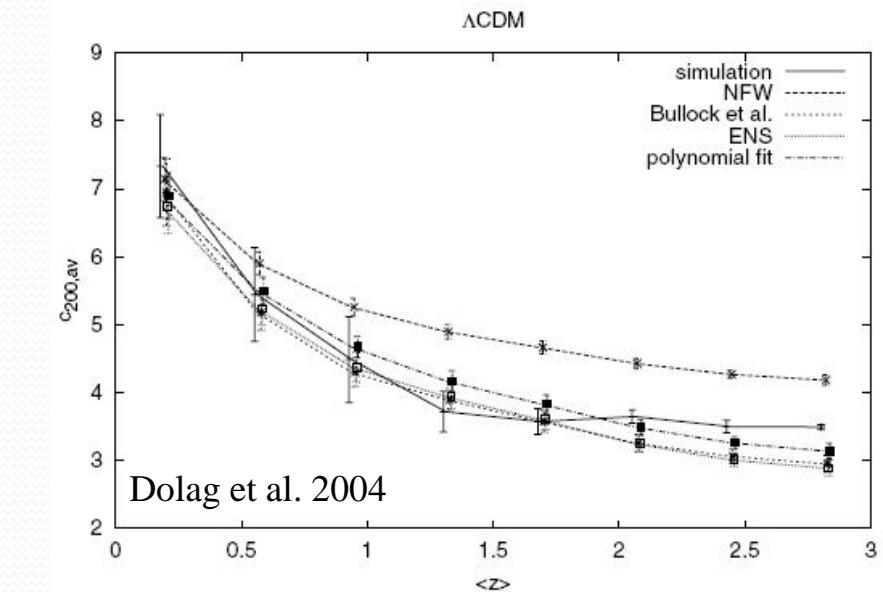
c-z relation

(Bullock et al. 2001)

- $c \propto 1/(1+z)$



c is roughly related to the halo central density over the varying (with time) background density of the universe

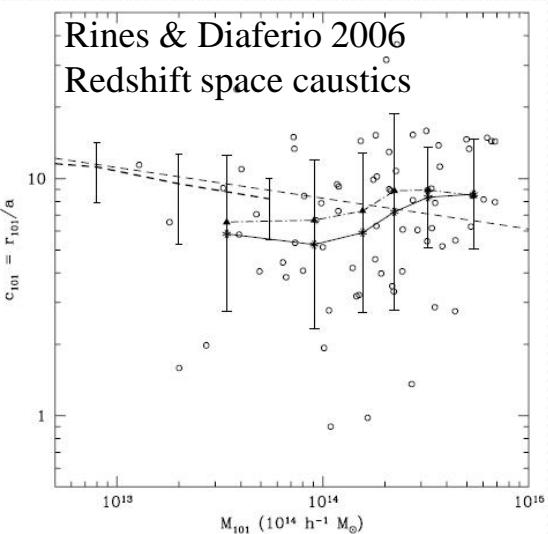
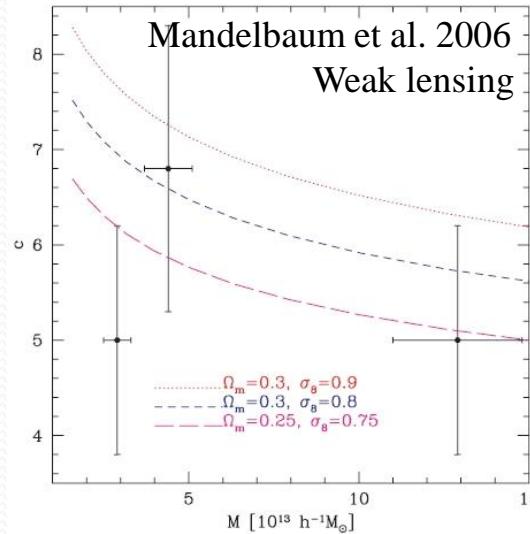
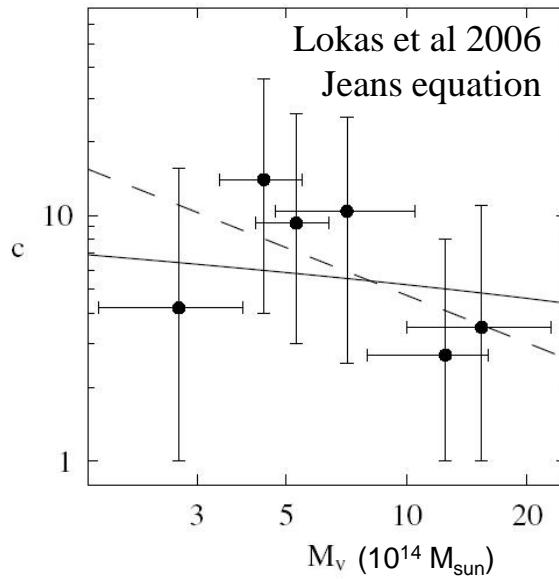


$$c \propto (\rho_{\text{halo}}/\rho_{\text{univ}}(z))^{1/3} = (1+z_{\text{form}})/(1+z)$$

Observational results. I

cM_{vir} the optical view

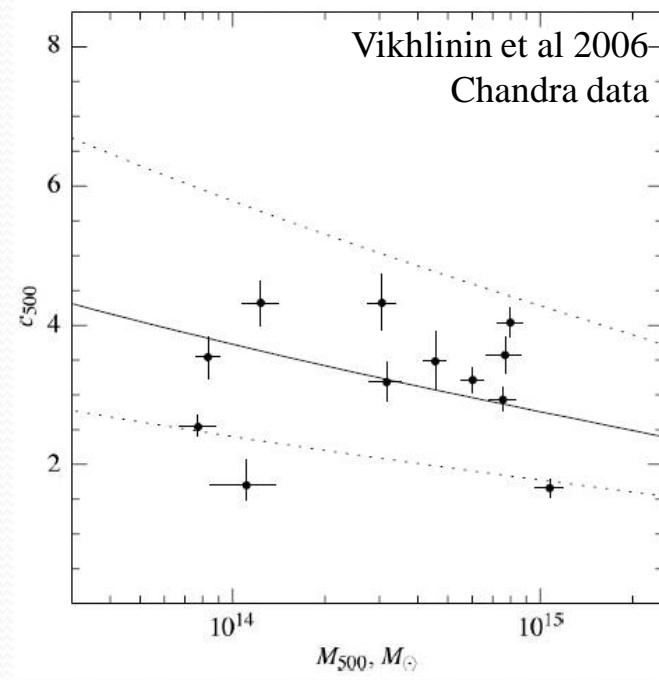
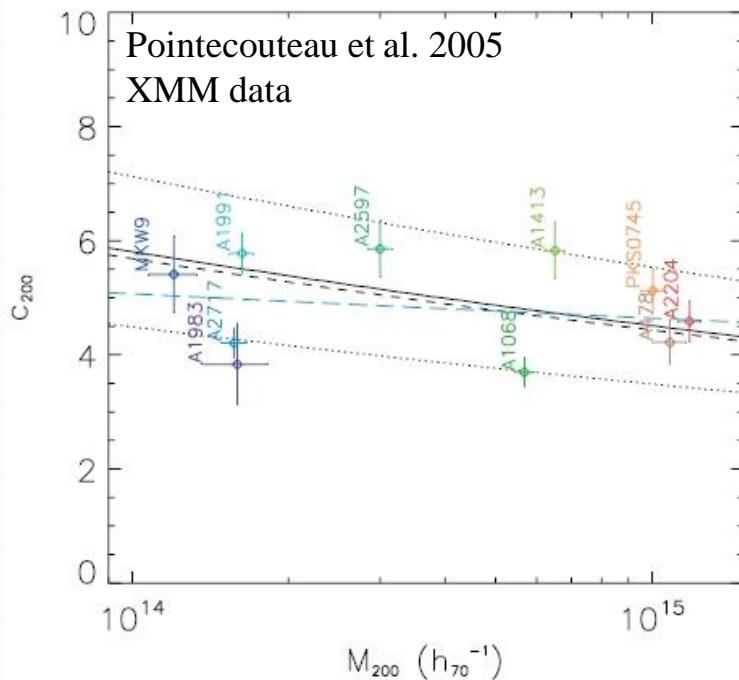
- Optical observers using a variety of techniques obtain results consistent with Λ CDM
(Biviano & Salucci 2006, Łokas et al. 2006, Rines & Diaferio 2006, Mandelbaum et al. 2006)



Observational results. II

c-M_{vir} the X-ray view

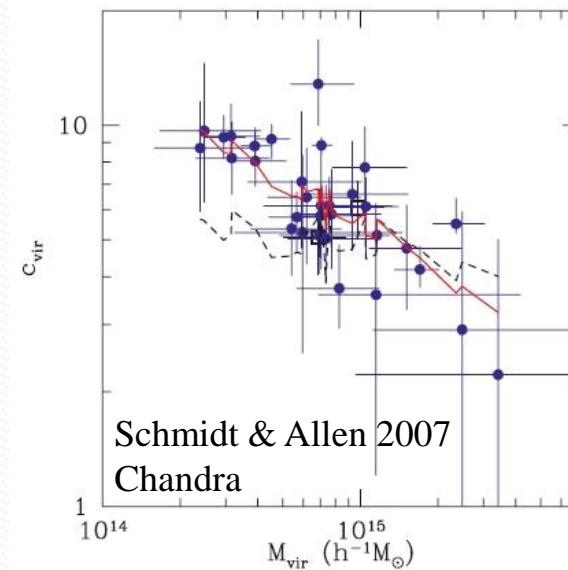
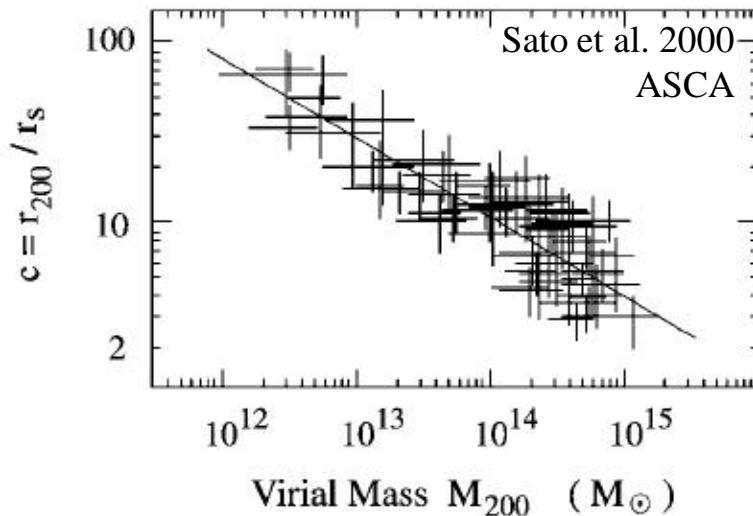
- Also X-ray observers find consistency with predictions
(Pointecouteau et al. 2005 , Vikhlinin et al. 2006)



Observational results. III

c - M_{vir} the X-ray view

- ... not always!
- Some author find c - M_{vir} slopes too steep than predicted
(Sato et al. 2000, Schmidt & Allen 2007)



Observational results. IV

c-z relation

So far only few studies

Observational results. IV

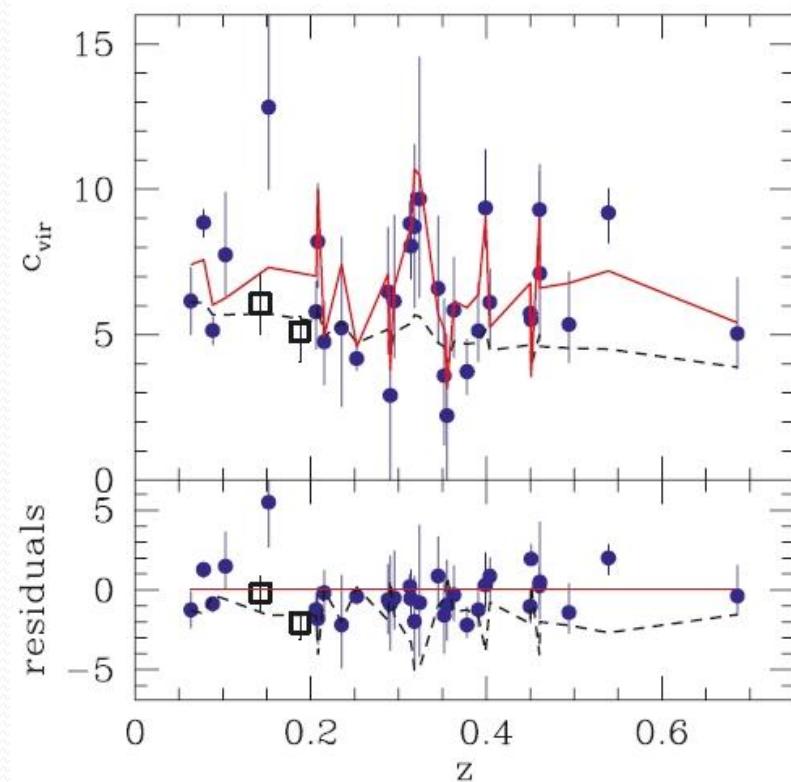
c-z relation

X-ray

- Schmidt & Allen (2007; SA07 hereafter) analyzing Chandra observations for 34 clusters of galaxies observed in the redshift range 0.06-0.7.

$$c \propto (1 + z)^{-b} M_{\text{vir}}^a$$

- $b=0.71 \pm 0.52$ (but $a=-0.45 \pm 0.12$)
- no variation of c-z ($b=0.3 \pm 0.49$) assuming a predicted c-M slope



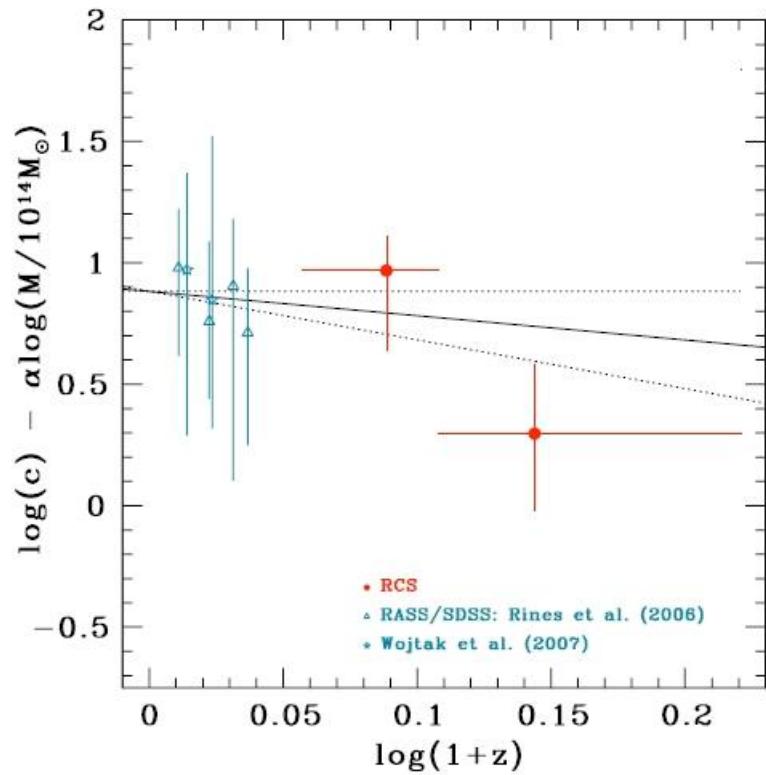
Observational results. IV

c-z relation

Optical

- Blindert (2007) using RCS clusters at $\langle z \rangle = 0.33$ find a somewhat steeper c-z

$$c \propto (1 + z)^{-b} M_{\text{vir}}^a$$



Constraining the c- M_{vir} relation

(Buote et al. 2007)

- Aim: significantly improve the constraints by enlarging the mass range and employing many more systems
- Sample of 39 relaxed galaxy systems spanning a mass range of $(0.06\text{-}20) \times 10^{14} M_{\text{sun}}$ (i.e. from ellipticals to clusters)



- Accurate mass profiles have been obtained by us for 24 low mass systems in the mass range $6 \times 10^{12} \leq M_{\text{vir}} \leq 3 \times 10^{14} M_{\text{sun}}$
[\(Humphrey et al. 2006, Zappacosta et al. 2006, Gastaldello et al. 2007\)](#)
- Data for more massive systems have been taken from the literature
[\(Pointecouteau et al 2005, Vikhlinin et al 2006\)](#)

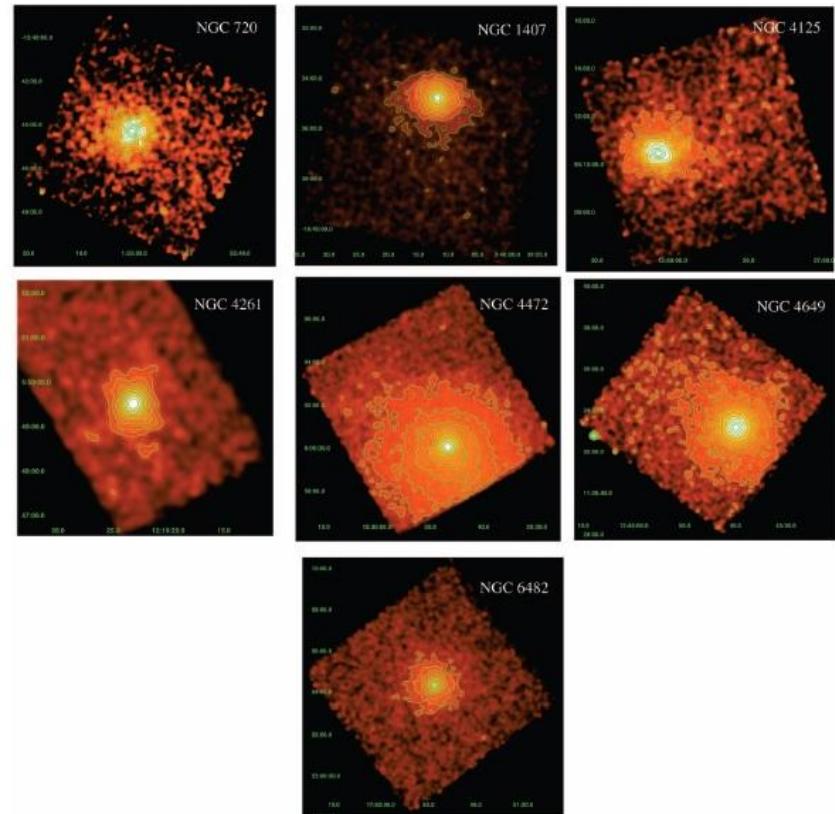
Brief highlights on the galaxy scale

(Humphrey et al. 2006)

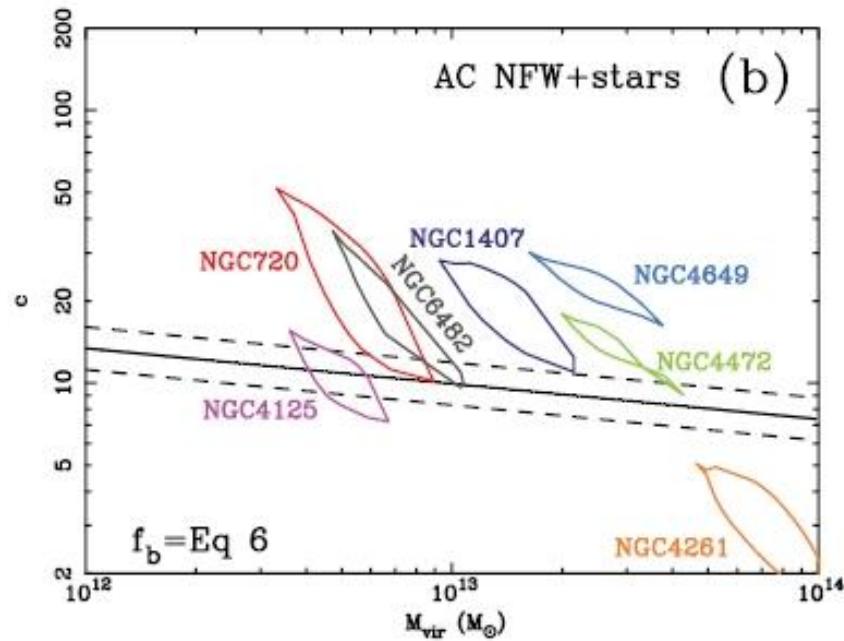
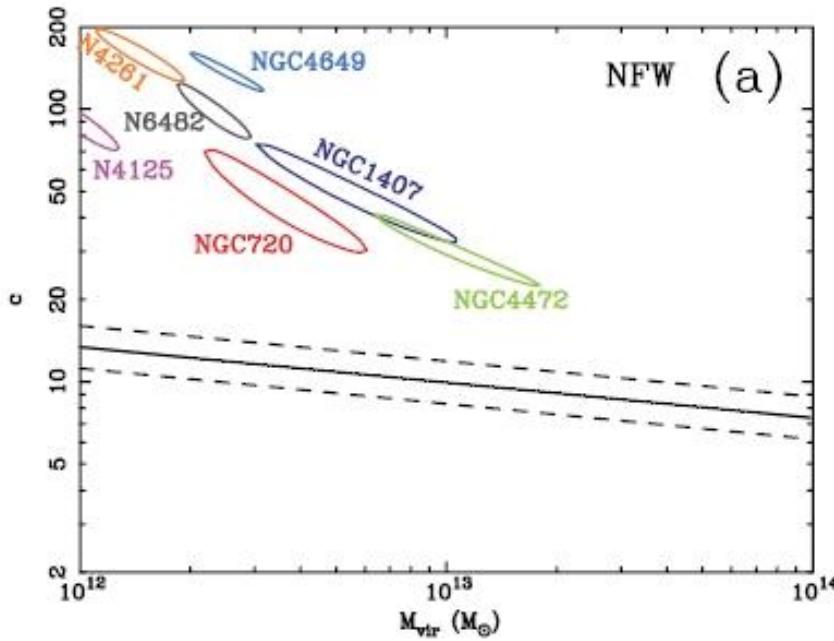
- Sample of 7 elliptical galaxies observed with Chandra
- Regular X-ray morphologies

MAIN RESULTS

- c - M_{vir} relation agrees with the predictions
- M_{tot} - M_{gas} profiles are parametrized by NFW+stellar component
- AC make M/L discrepant



Brief highlights on the galaxy scale (Humphrey et al. 2006)

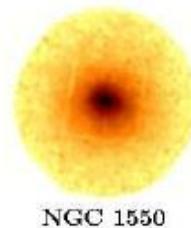


Brief highlights on the group scales (Gastaldello et al. 2007)

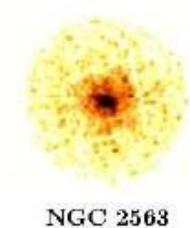
Joint analysis of Chandra and XMM observations of 16 groups/poor clusters

MAIN RESULTS

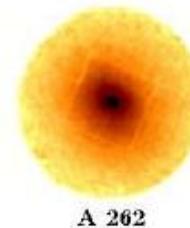
- $M_{\text{tot}}-M_{\text{gas}}$ profiles are modeled by an NFW+stellar component for 8 systems the others do not require any stellar mass
- $c-M_{\text{vir}}$ in agreement with the predictions and inconsistent at 3σ with no variation
- AC is not required (does not improve the fits)



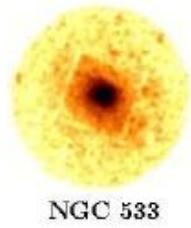
NGC 1550



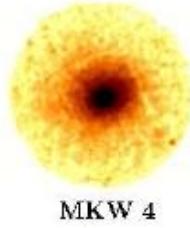
NGC 2563



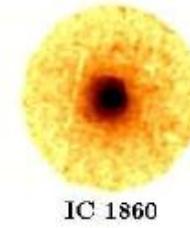
A 262



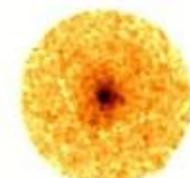
NGC 533



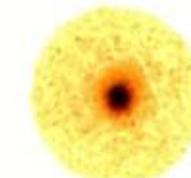
MKW 4



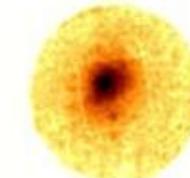
IC 1860



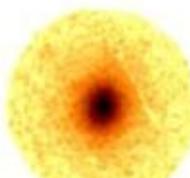
NGC 5129



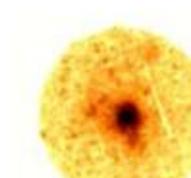
NGC 4325



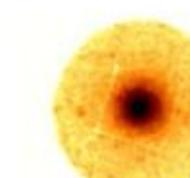
ESO 5520200



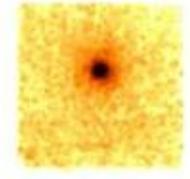
AWM 4



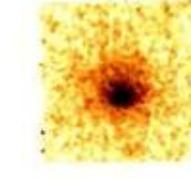
RGH 80



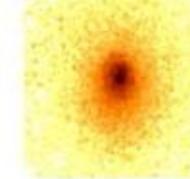
A 2717



RXJ 1159.8+5531

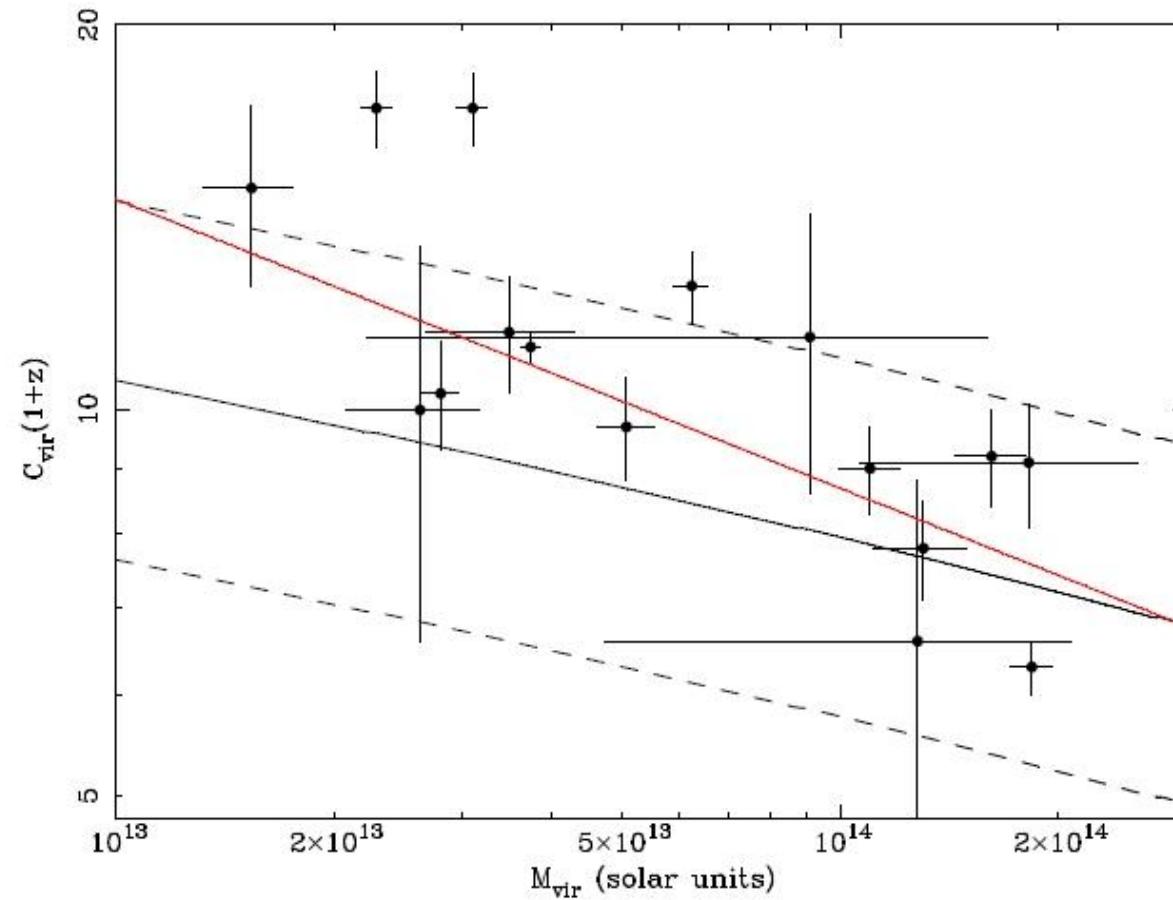


MS 0116.3-0115



ESO 3060170

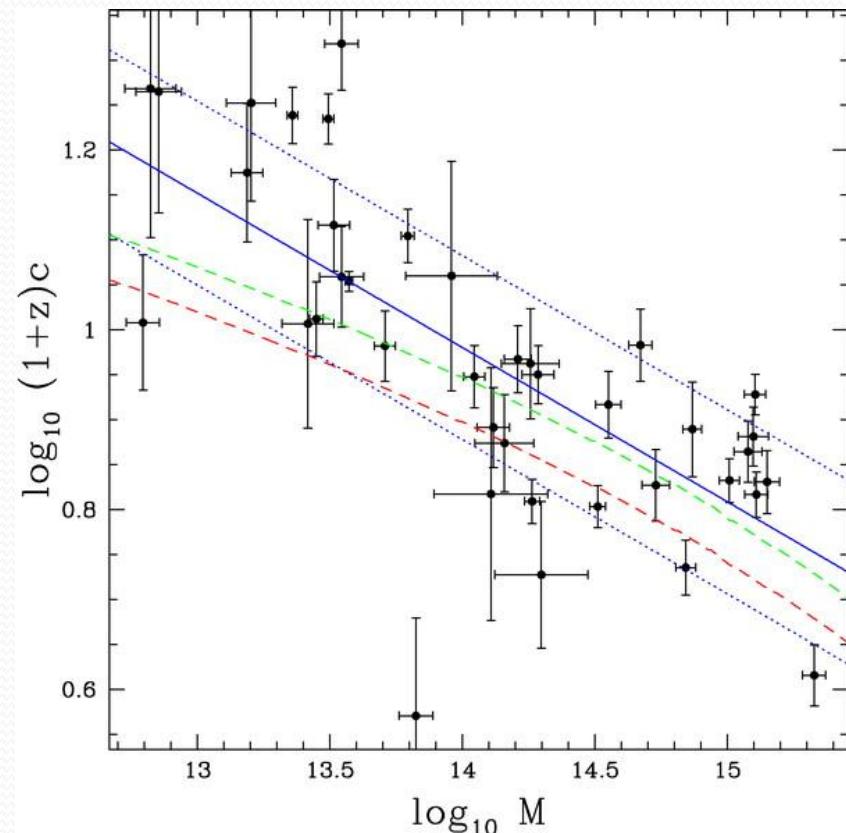
Brief highlights on the group scales (Gastaldello et al. 2007)



Constraining the c - M_{vir} relation continued... (Buote et al. 2007)

c - M_{vir} relation

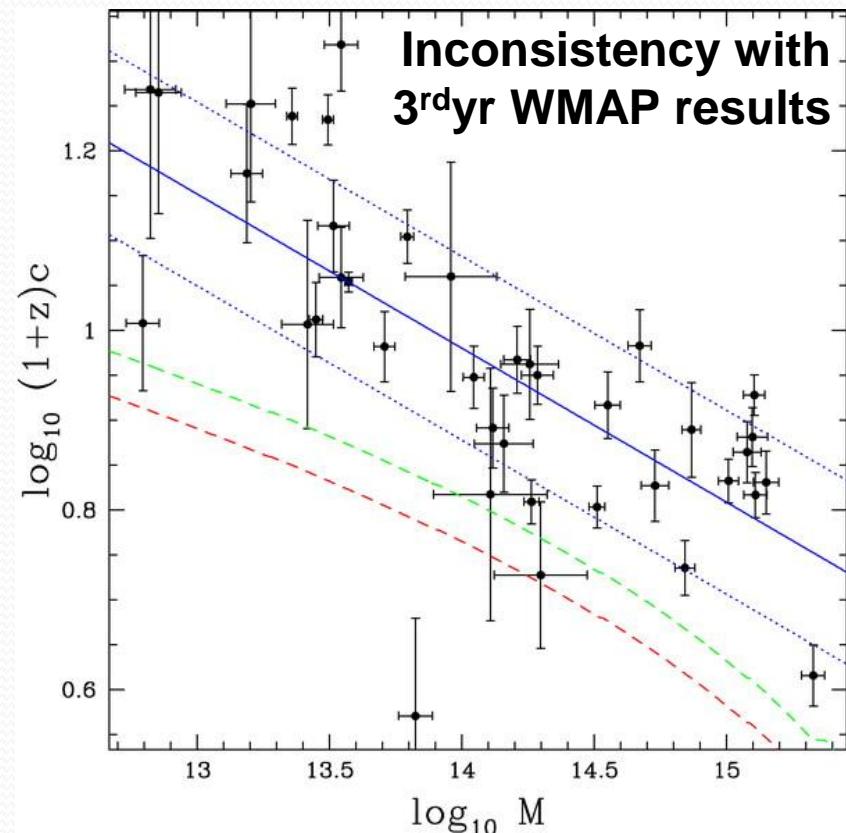
- Power-law good approximate description
- c - M_{vir} relation significant at 6.6σ
- Slope $a = -0.172 \pm 0.026$
- Consistency with Bullock et al. 2001 models assuming cosmological parameters from 1styr WMAP results (assuming 10% higher normalization for early forming halos)
- Intrinsic scatter and normalization consistent with early forming halos



Constraining the c - M_{vir} relation continued... (Buote et al. 2007)

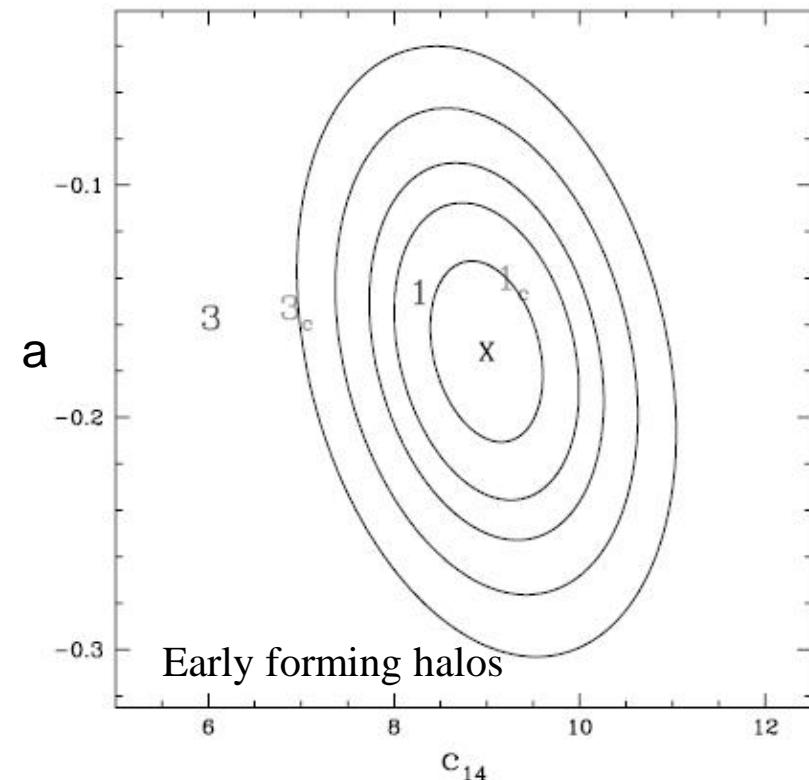
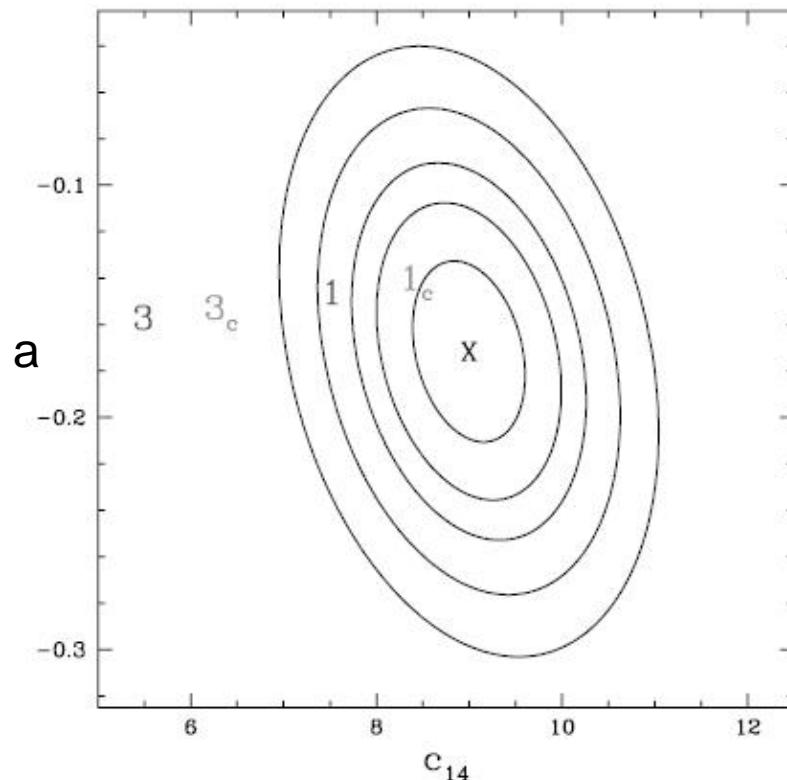
c - M_{vir} relation

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Constraining the c - M_{vir} relation (Buote et al. 2007)

Disagreement mainly due to the lower value of $\sigma_8=0.76$, but also the tilt of the power spectrum and the low value of Ω_m may play a role. Increasing w may also help.



Constraining the c- M_{vir} relation

(Buote et al. 2007)

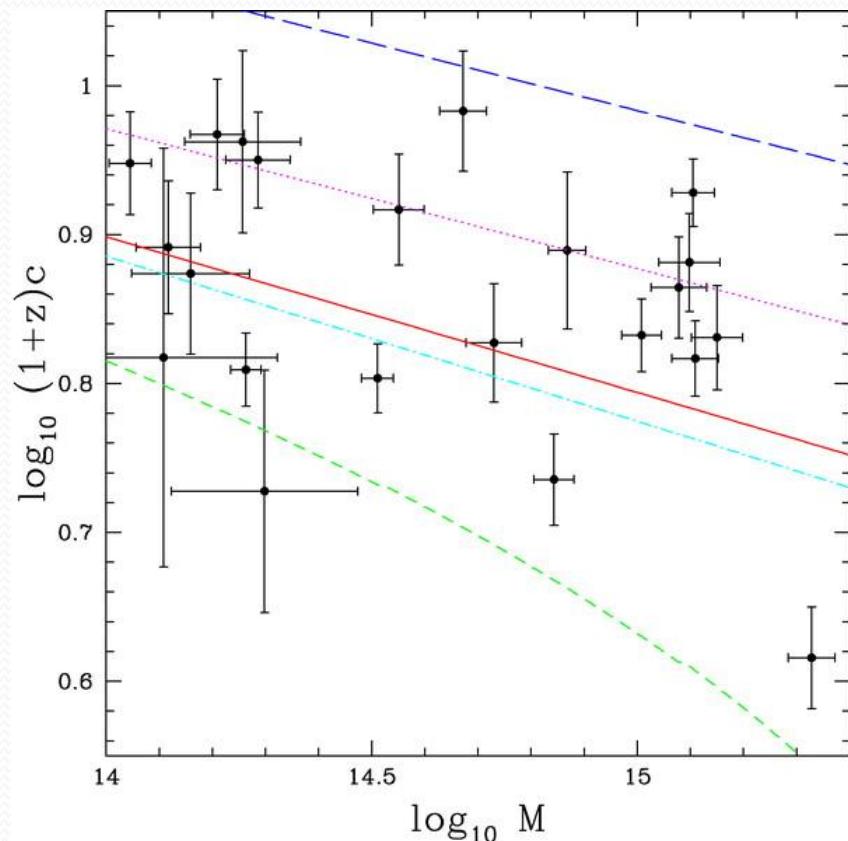
Only clusters

- Slope not constrained and consistent with 0
- Normalization may help in discriminating several different cosmological models

TABLE 2
COSMOLOGICAL MODEL PARAMETERS

Name	Ω_m	Ω_Λ	$\Omega_B h^2$	h	σ_8	n_s	$-w$
Λ CDM1.....	0.30	0.70	0.022	0.7	0.90	1.00	1.0
Λ CDM3.....	0.24	0.76	0.022	0.73	0.76	0.96	1.0
DECDM	0.30	0.70	0.022	0.7	0.90	1.00	0.6
QCDEM.....	0.30	0.70	0.022	0.7	0.82	1.00	≈ 0.8
OCDM.....	0.30	0.00	0.022	0.7	0.90	1.00	...

NOTES.— Ω_m is the energy density parameter for matter in the universe; Ω_Λ is the energy density parameter associated with a cosmological constant or, more generally, dark energy; Ω_B is the energy density parameter of baryons; h is $H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$; σ_8 is the rms mass fluctuation within spheres of comoving radius $8 \text{ h}^{-1} \text{ Mpc}$. See § 4.



Constraining the c-M_{vir} relation (Buote et al. 2007)

From Cluster abundance studies:

$$\Omega_m \approx 1 \rightarrow \sigma_8 \approx 0.45$$

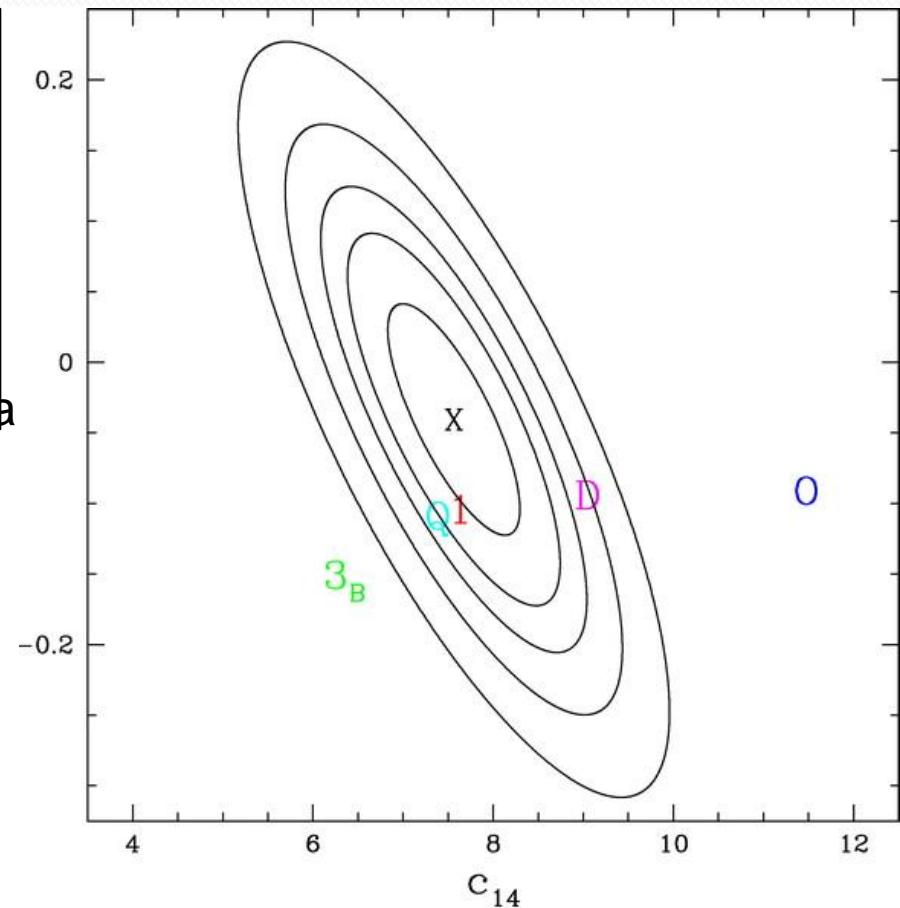
(e.g. Hoekstra et al. 2002, Van Waerbeke et al. 2005, Rines et al. 2007)

Cosmological models

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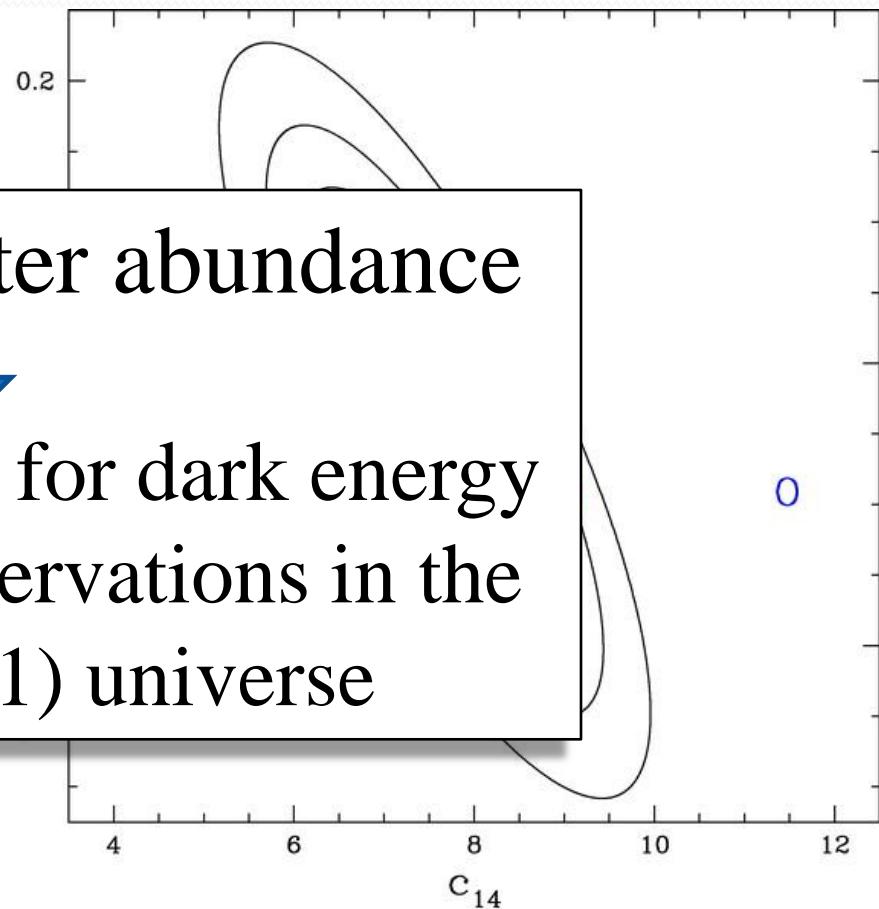
Constraining the c - M_{vir} relation (Buote et al. 2007)

From Cluster abundance studies:

$$\Omega_m \approx 1$$

(e.g. Hoekstra et al. 2005, Rines et al. 2006)

Name	Ω_m
Λ CDM 1	0.30
Λ CDM 3	0.24
DECDM	0.30
QCDM	0.30
OCDM	0.30



NOTES.— Ω_m is the energy density parameter for matter in the universe; Ω_Λ is the energy density parameter associated with a cosmological constant or, more generally, dark energy; Ω_B is the energy density parameter of baryons; h is $H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$; σ_8 is the rms mass fluctuation within spheres of comoving radius $8 h^{-1} \text{ Mpc}$. See § 4.

Constraining the c-z relation

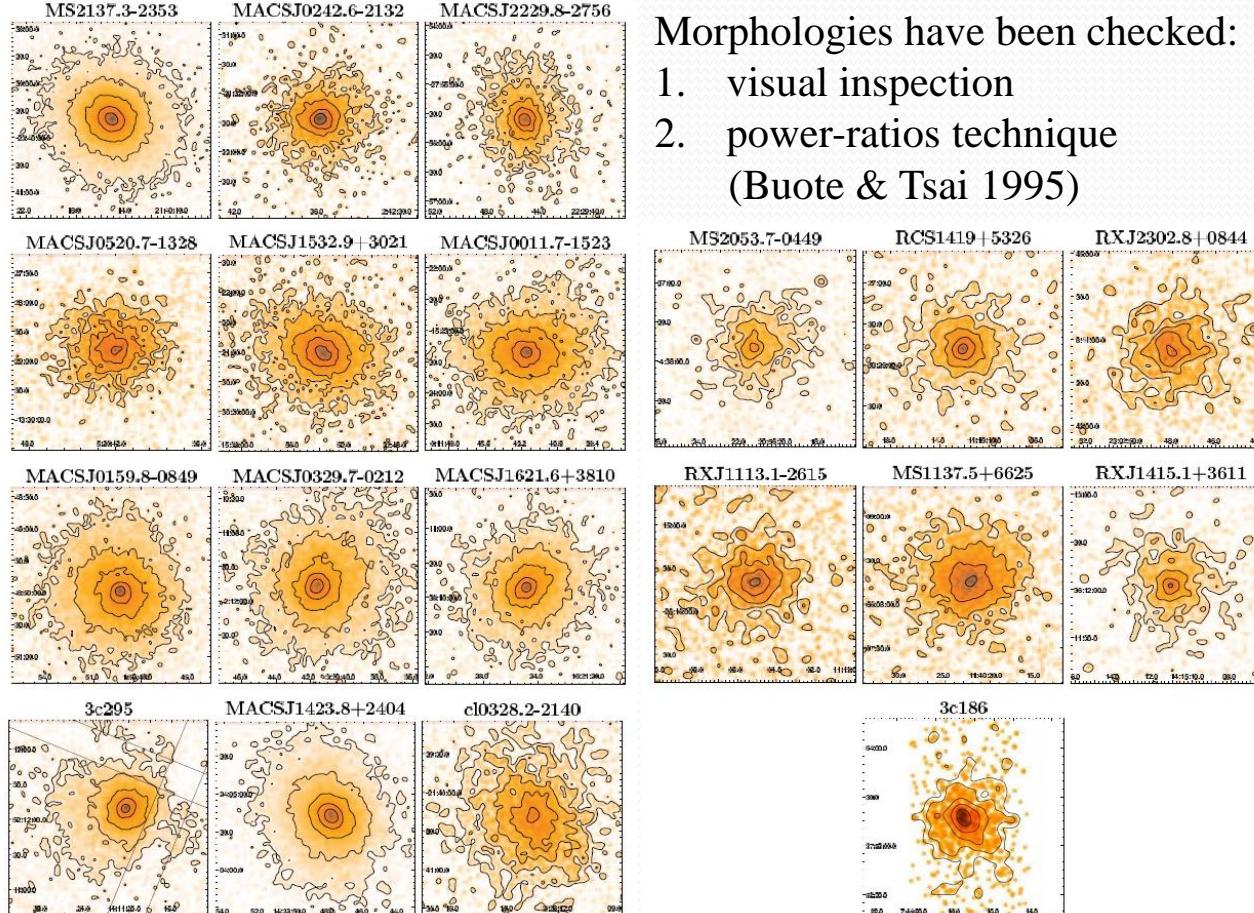
(Zappacosta et al. in prep.)

- Aim: Constrain the c-z relation out to the highest redshift possible
- Sample: 19 clusters of galaxies at redshifts $0.3 < z < 1.1$
- Selection: the most relaxed and with best available Chandra data

Sample heavily biased toward high redshifts:

- Highest redshift probed: ~ 1.1
- 9 clusters at $z > 0.5$
- SA07 only probed up to 0.7, with 2 clusters at $z > 0.5$ (one of them is manifestly disturbed)

Morphologies



Morphologies have been checked:

1. visual inspection
2. power-ratios technique
(Buote & Tsai 1995)

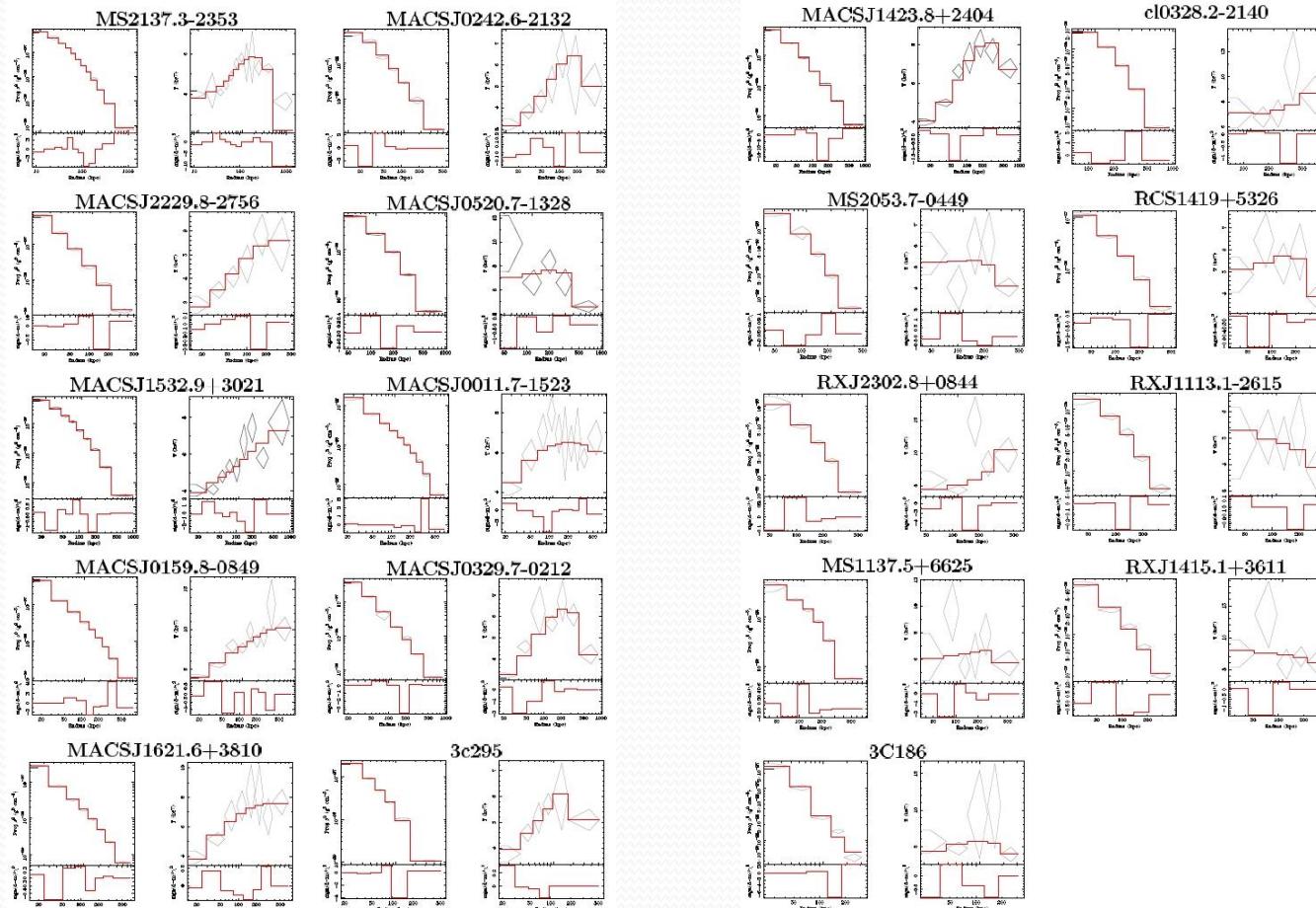
Mass profiles

- Computed inverting the equation of hydrostatic equilibrium and solving for the temperature.

$$T_g(r) = T_0 \frac{\rho_{g0}}{\rho_g(r)} - \frac{\mu m_p G}{k_B \rho_g(r)} \int_{r_0}^r \frac{\rho_g M dr}{r^2}$$

- (cusp/simple) β -model and NFW are assumed for gas density and mass profiles
- The temperature profile is determined by the hydrostatic equilibrium assumption according to the data

Gas density and temperature profiles



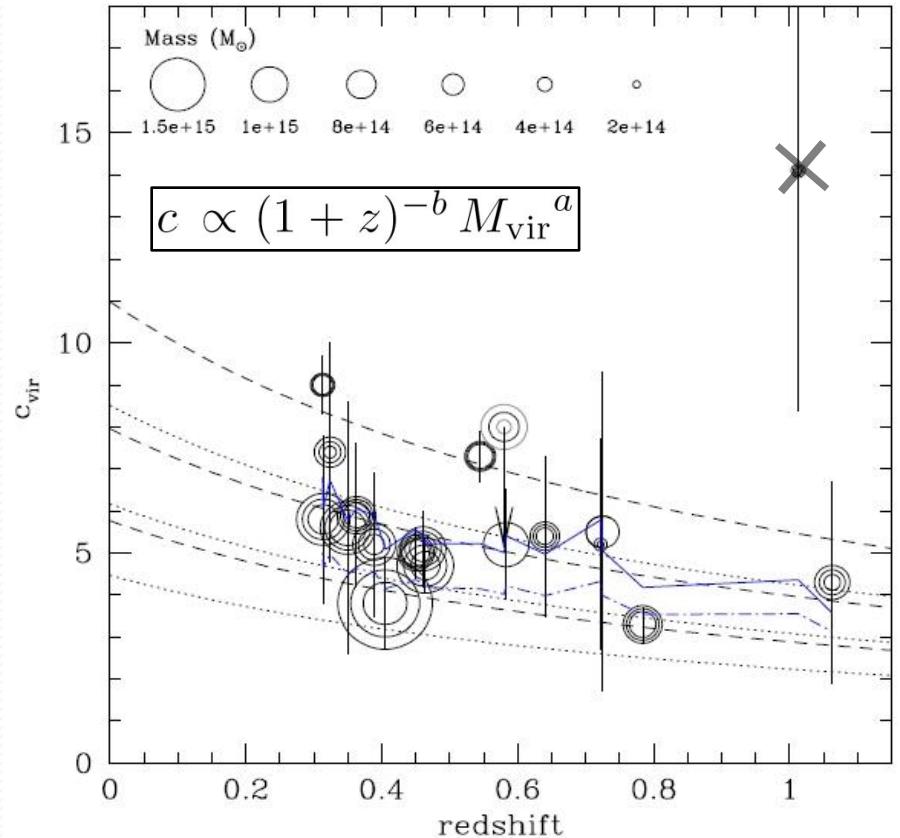
c-z with only high-z clusters

Assumptions:

- c - M_{vir} slope $a=-0.172$
(Buote et al. 2007)
- Intrinsic scatter $0.1\Delta\log(c)$

Results:

- $b \approx 1.3 \pm 0.4$
 - consistent with predictions
 - Inconsistent at 3σ with no variation
- Slightly high normalization

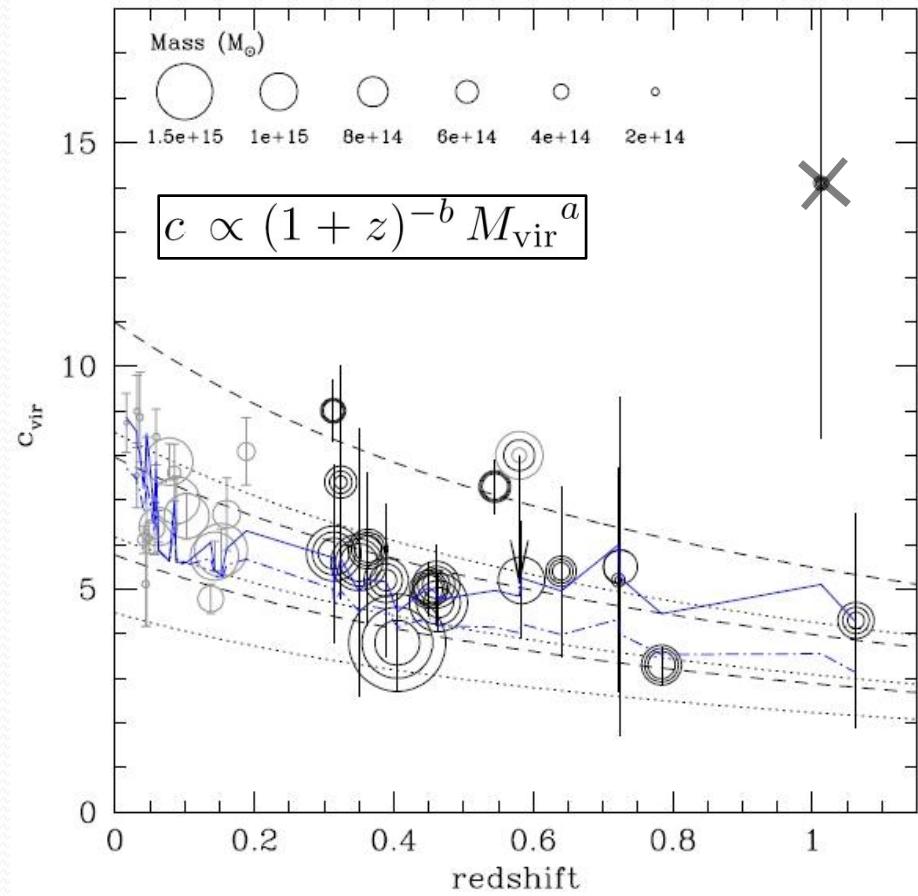


c-z including local clusters

- Inclusion of low-z systems with $M_{\text{vir}} > 10^{14} M_{\text{sun}}$
(from Buote et al. 2007)

RESULTS:

- $b = 0.52 \pm 0.19$
- Inconsistent with no variation at 2.7σ
- Inconsistent at 2.5σ with the predictions
- Normalization: consistency at 1.5σ
- Larger a values predicted by CDM improve the agreement at 2σ level



c-z including local clusters

- Inclusion of low-z systems with $M_{\text{vir}} > 10^{14} M_{\text{sun}}$
(from Buote et al. 2007)

RESULTS

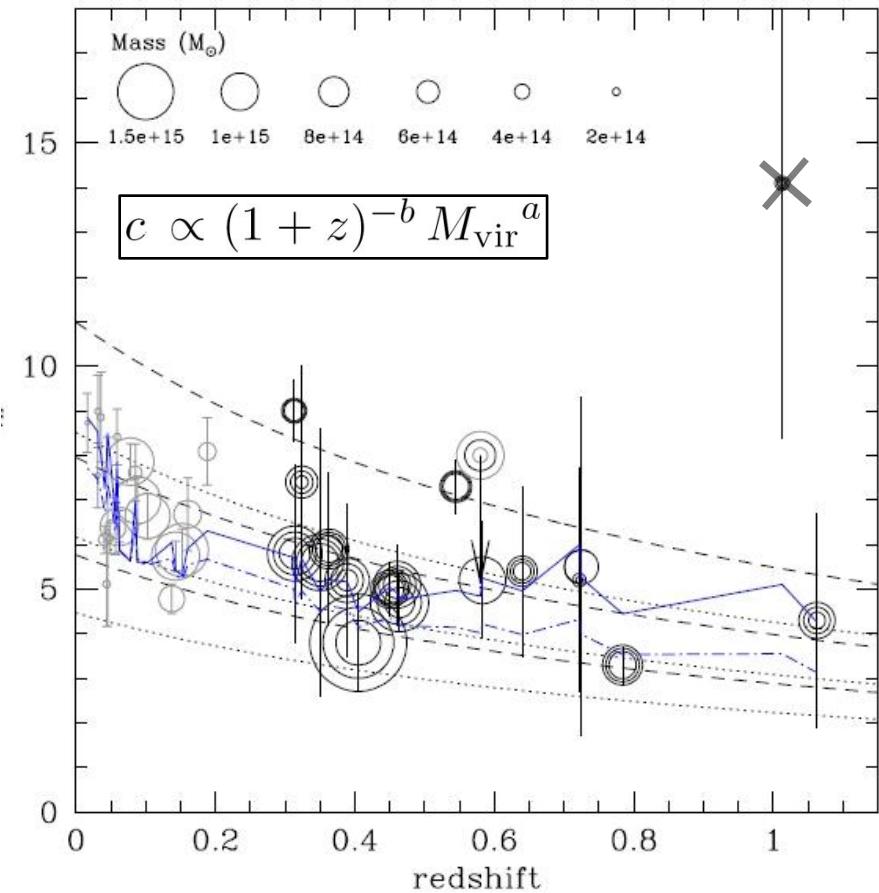
RESULTS:

Assuming $a = -0.45$ as found
by SA07



$b = 0.0 \pm 0.3$
(no c-z relation)

at 20 level



Future prospects on the c-z relation

- Enlarge the sample (intrinsic scatter)
- Improve our redshift coverage especially at $z > 0.5$
- Doubling the sample at $z > 0.3$ $\rightarrow b \sim 0.7 \pm 0.2$
- Halving also the errors in c and M_{vir} $\rightarrow b \sim 0.8 \pm 0.16$
(assuming a c-z relation as predicted)

Conclusions on the c- M_{vir} and c-z relations

- Large sample of local ($z < 0.2$) relaxed galaxy systems from ellipticals to massive clusters
 - significant variation of c with M_{vir}
 - consistency with CDM predictions (slope, scatter and normalization)
 - Clusters only: exclude an open cosmology and provide new evidences for the existence of dark energy *at low redshift* (including cluster abundance studies results)
- Including distant relaxed galaxy clusters ($z = 0.3 - 1.1$)
 - We detect variation of c with redshift at $\sim 3\sigma$ level
 - Marginal consistency with CDM predictions