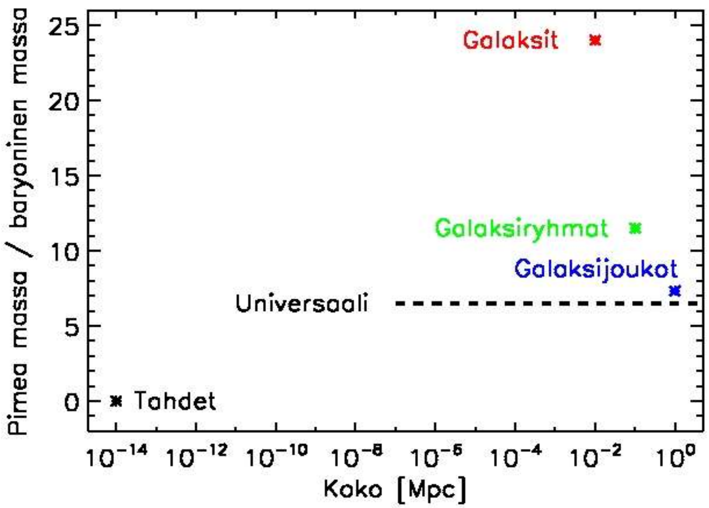
Cosmology with clusters of galaxies

- large scale structure, dark matter
- mass function
- baryonic fraction

Distribution of dark matter in the Universe



- Dark matter particles do not interact via electro-magnetism
- the mean free path due to gravity only is large →
- dark matter forms object only at very large (Mpc) scale
- There are no dark matter stars
- Dark matter dominates the mass budget of the Universe $\Omega_m \approx 0.25, \quad \Omega_{\mathrm{bar}} \approx 0.04$
- The fraction of dark matter decreases towards universal in clusters of galaxies

$$M_{bar}/M_{tot} = \Omega_{bar}/\Omega_m$$

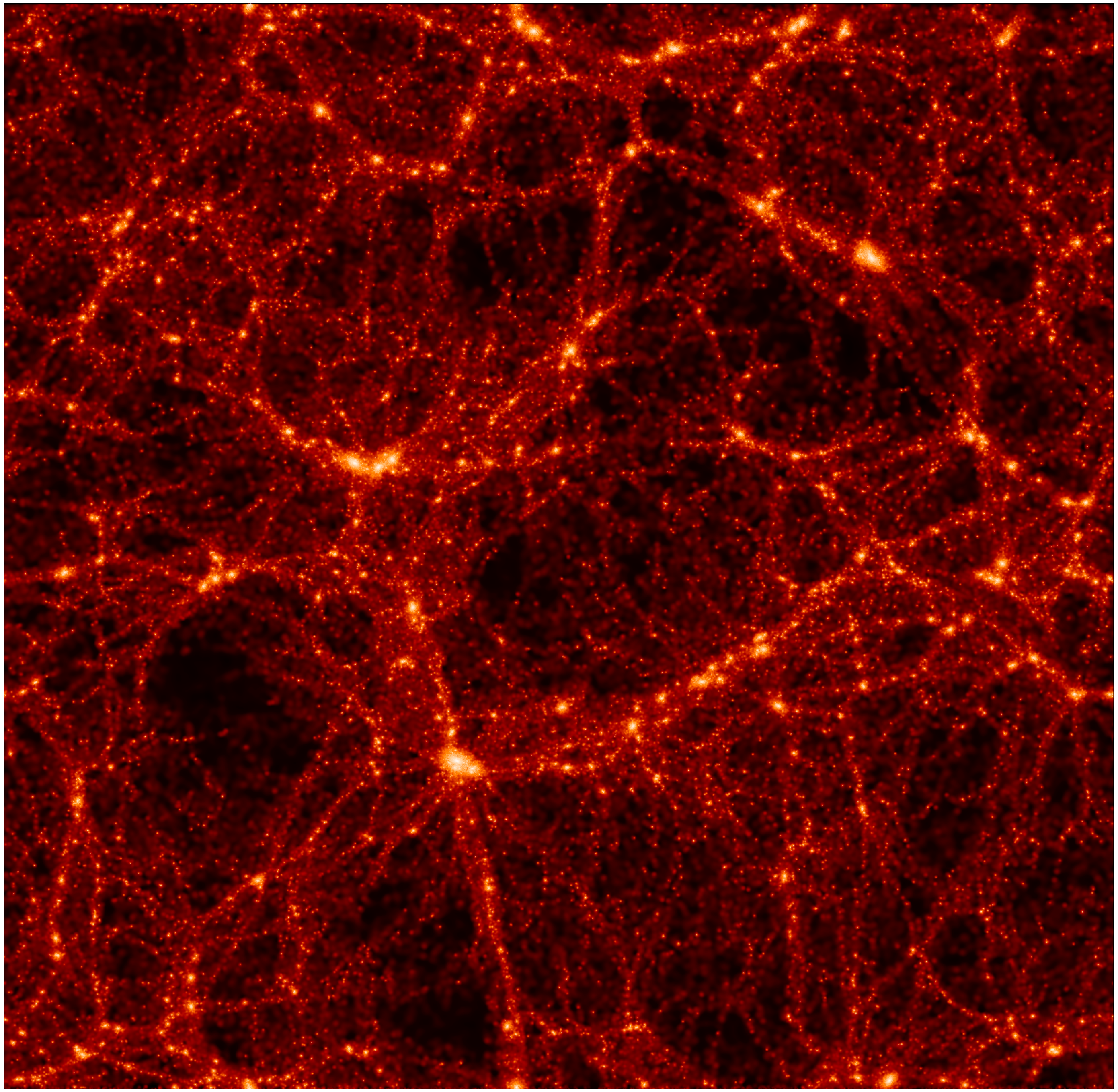
Large scale structure

simulations of Jenkins et al., 1998, ApJ, 499, 20

- homogenous primordial matter + dark matter density fluctuations

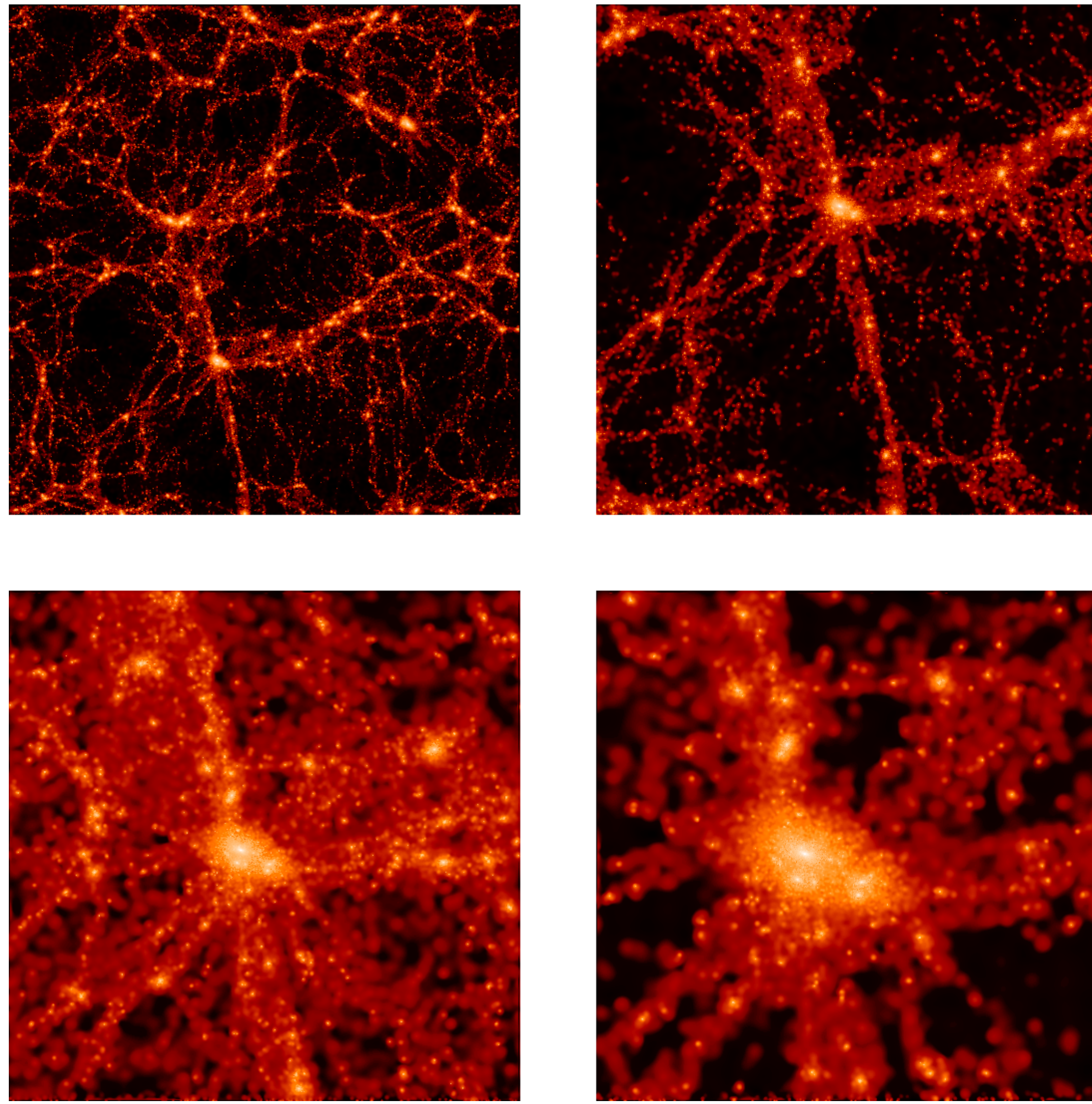
→

- scale here is 200 Mpc
- filaments
- voids
- concentrations



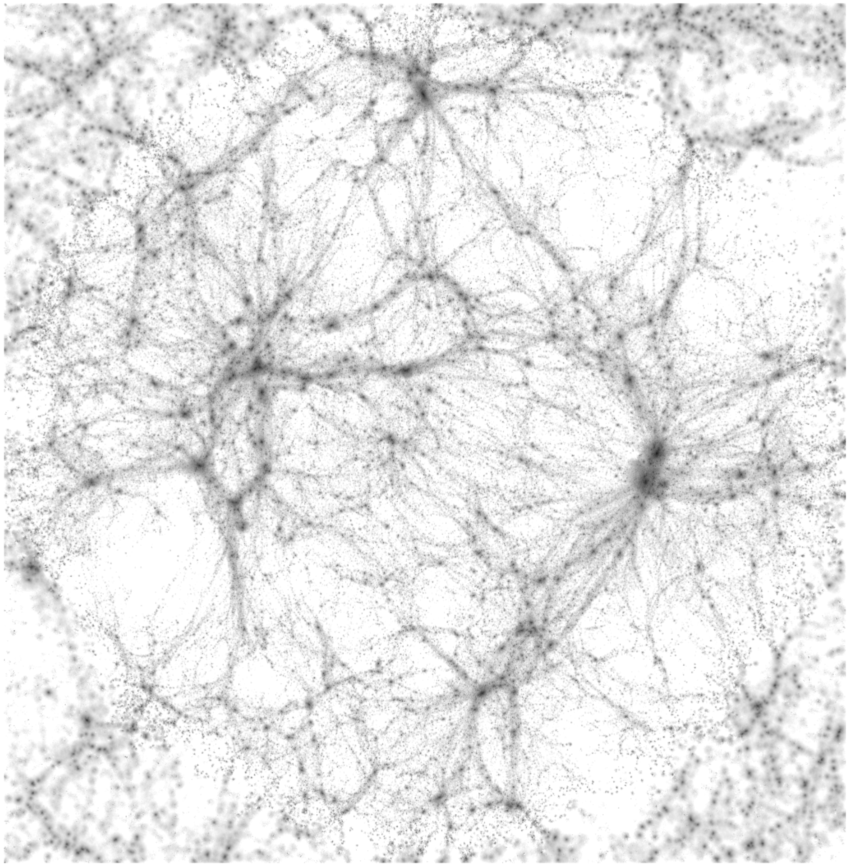
Large scale structure

- Zooming into a concentration at filament crossing = galaxy cluster

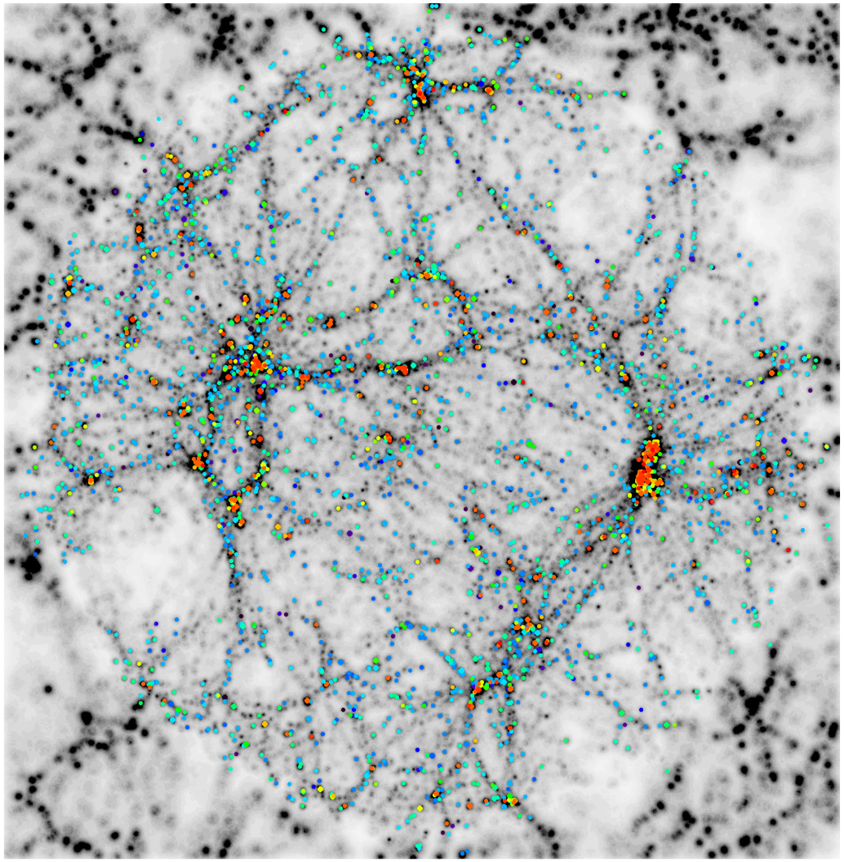


Large scale structure

dark matter

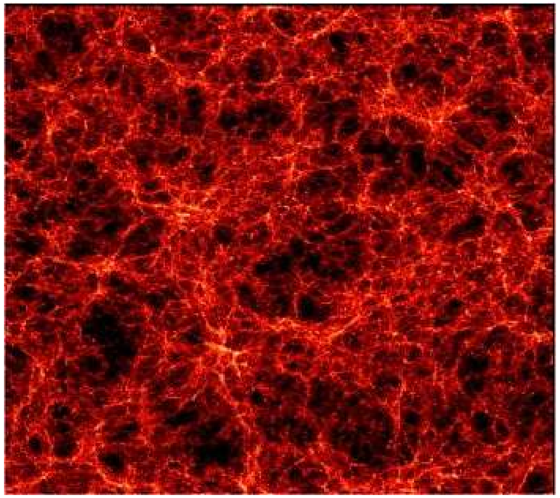


galaxies and other baryons



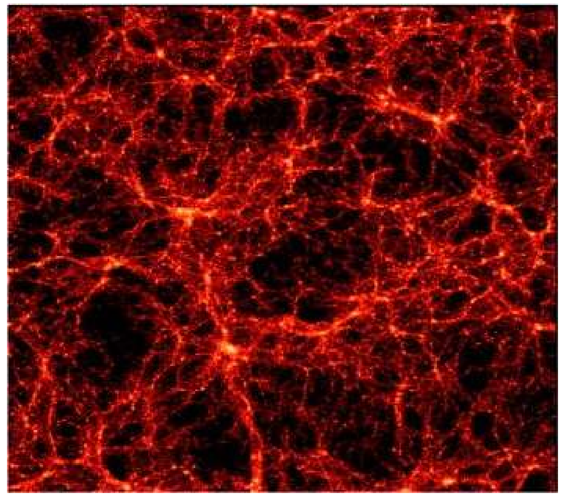
Large scale structure

z = 3



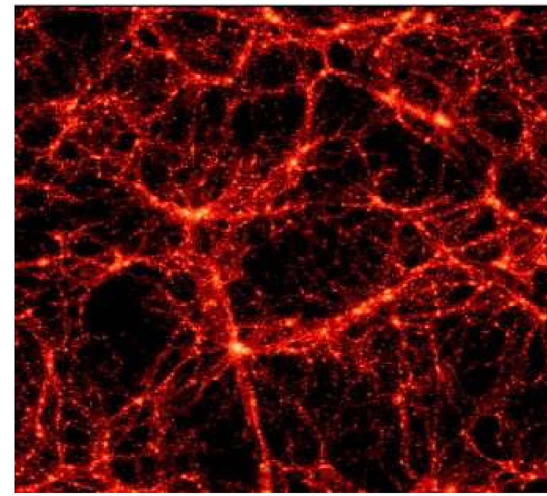
age = 2 Gyr

z = 1



age = 6 Gyr

z = 0



age = 14 Gyr

- Virgo consortium simulations: http://www.mpa-garching.mpg.de/Virgo/

Concordance cosmology≡ΛCDM

- Cosmic Microwave background measurements with WMAP (Spergel et al., 2007, ApJS, 170, 377)
- Hubble Space Telescope observations of distant supernovae (e.g Riess et al., 2004, ApJ, 607, 665)
- Baryonic acoustic oscillations
- Big Band Nucleosynthesis

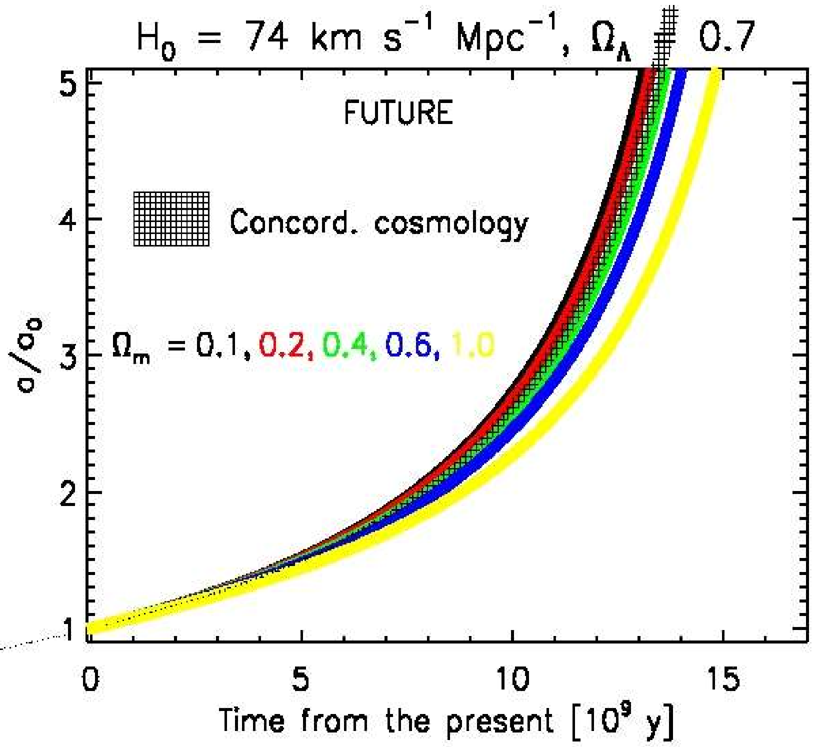
→

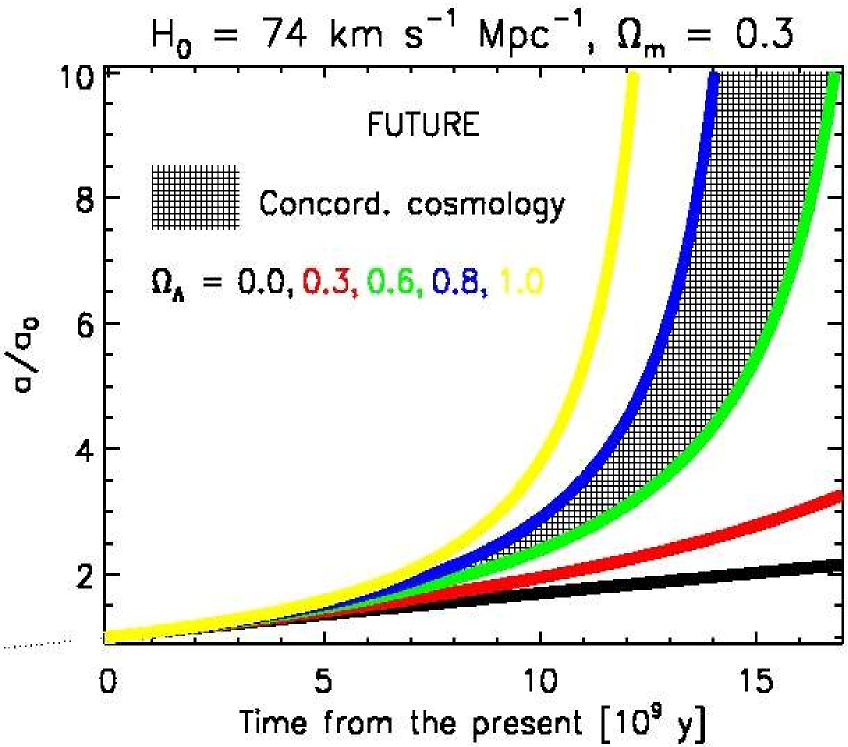
- Universe is flat
- The expansion is accelerating (cosmological constant **Λ)**
- Matter density is dominated by non-baryonic (cold) dark matter (CDM)
- Age of the Universe ~ 14 Gyr

Concordance cosmology $\equiv \Lambda$CDM

- $H_0 = 74$ km s^{-1} Mpc^{-1} (Hubble constant)
- $\Omega_m = 0.3$ (matter density parameter = ρ_m / ρ_{crit})
- $\Omega_\Lambda = 0.7$ (cosmological constant = $\rho_\Lambda / \rho_{crit}$)
- $\Omega_b = 0.04$ (baryon density parameter = ρ_b / ρ_{crit})

Scale of the Universe in the future





- higher Ω_m, slower expansion in the future
- higher Ω_Λ, faster and accelerating expansion in the future

Mass function

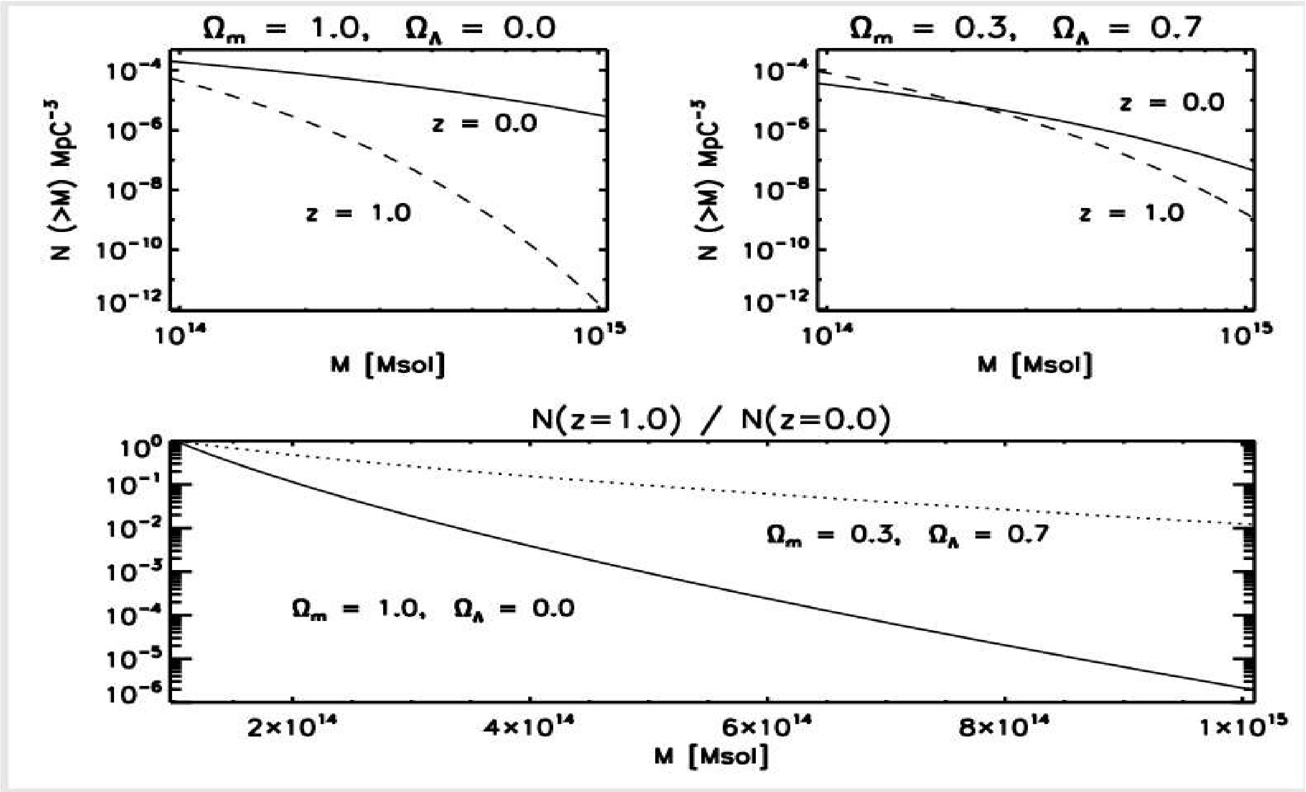
- N(M) = mass function of collapsed objects
- P(k) = spectrum of primordial density fluctuations, k is a scale parameter

$$P(k) \propto k^n \rightarrow \sigma(M) = \sigma_8 \left(\frac{M}{M_8} \right)^{-(n+3)/6}$$

- σ(M) = mass fluctuation spectrum
- masses exceeding the critical threshold δ_c collapse
- evolution of the Universe $(\Omega_{m,} \Omega_{\Lambda}) \rightarrow$ N(M)

$$\frac{dN}{dM} = \sqrt{2/\pi} \frac{\rho_m}{M^2} \frac{\delta_c}{\sigma} \left| \frac{n+3}{6} \right| \exp\left(-0.5 \left(\frac{\delta_c}{\sigma}\right)^2\right)$$

Mass function



- Einstein-deSitter → flat ΛCDM: more power at high M at high z, less evolution
- need data at high z to break degeneracy btw. Ω_m and Ω_Λ
- at z=0 one can determine n and σ_8

Gas mass fraction

$$f_{gas} \equiv \frac{M_{gas}}{M_{tot}} \approx \frac{M_{gas} + M_{gal}}{M_{tot}} \equiv f_b = \Upsilon \frac{\Omega_b}{\Omega_m} \longrightarrow$$

$$\Omega_m \approx \Upsilon \, \Omega_b / f_{gas} \qquad \text{Eq. 1}$$

$f_{gas} =$ gas mass fraction

M_{gas}, $M_{tot} =$ gas mass and total mass (from X-rays)

$\Omega_b, \Omega_m =$ density parameter of baryonic and total mass

Υ = local baryon density enhangement factor

Ω_b and Υ

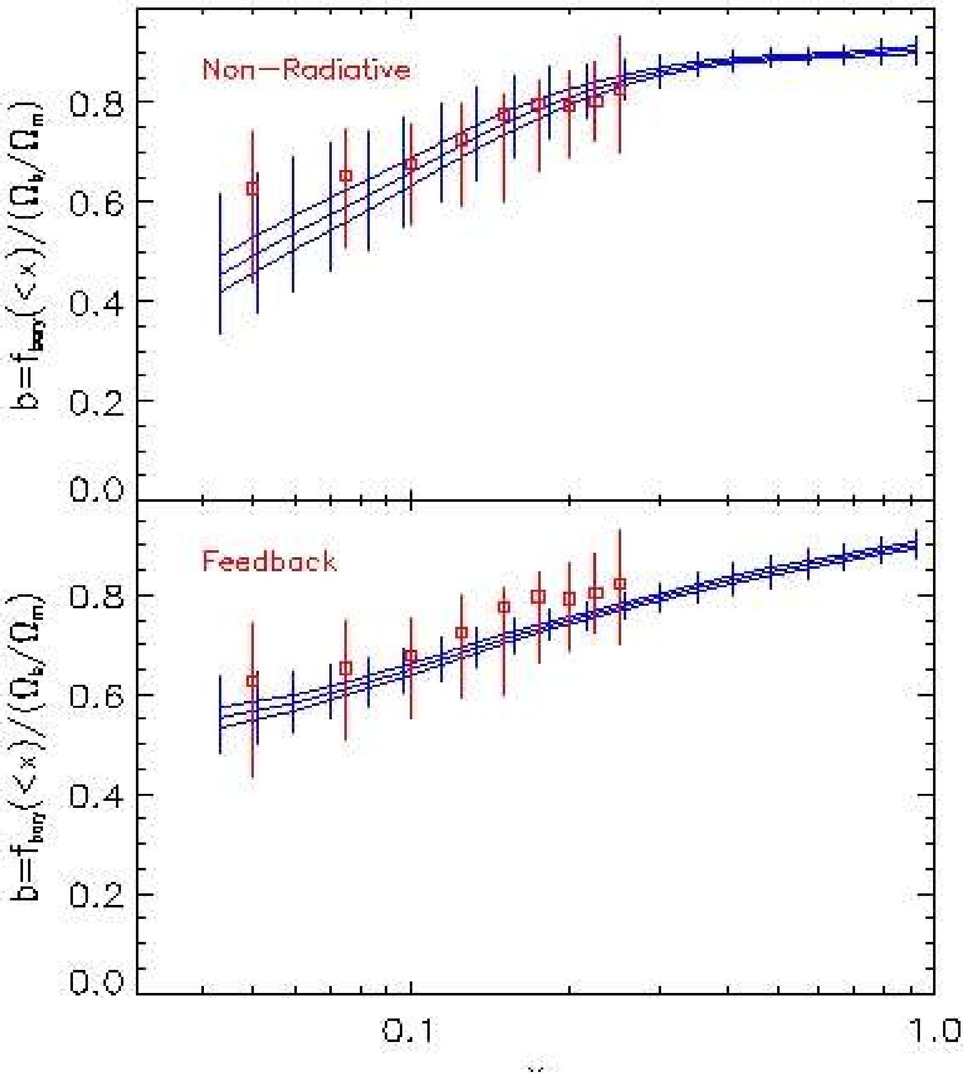
- CMB peak positions in WMAP 3yr (Spergel et al., 2007):

  a) $\Omega_b h^2 = 0.0223 \pm 0.0008$ (when $h = H_0$ / 100 km s^{-1} Mpc^{-1})

  b) $\Omega_b h^2 = 0.0407 \pm 0.0015$ (when $h = H_0$ / 74 km s^{-1} Mpc^{-1}

- Υ increases with radius:

  $\Upsilon(r_{vir}) \approx 0.9$ (e.g. Kay et al., 2004, MNRAS, 355, 1091) → need to evaluate f_{gas} at large r



Virial radius r_{vir} and overdensity Δ

- primordial density fluctuations grow by accumulating material
- when density exceeds a critical threshold, the system collapses
- released potential energy propagates as a shock wave through the matter
- shock pushes matter around → bulk motions and irregular e velocity distribution
- relaxation proceeds from the center towards the edges t_{eq} after the shock
- within virial radius, all the released E_{pot} is converted into thermal energy of e
- virial radius r_{vir} is the largest radius inside which hydrostatic equilibrium holds

Virial radius r_{vir} and overdensity Δ

- Overdensity within radius r : $\Delta(r) = \langle\rho\rangle_r/\rho_{crit}$

where the mean density is $$\langle \rho \rangle_r = \frac{M(r)}{4/3\,\pi\, r^3}$$

and the critical density is $$\rho_{crit} = \frac{3}{8\pi}\frac{H^2}{G}$$

- Based on simulations and theory, at the maximum radius of the virialised region $\Delta(r) = 200–500$ → often used approximations $r_{vir} = r_{500}$ or $r_{vir} = r_{200}$
- for a cluster with temperature T = 5 keV, $r_{200} \sim 1.7$ Mpc (H=70 , $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, Arnaud et al. 2005, A&A, 441, 893)

Cosmological formulae

- M_{gas} and M_{tot} (needed for f_{gas}) values depend on cosmology
- Observational scale is 2D (arcmin) , physics is 3D (Mpc) ➔

  angular diameter distance D_A
- Photon observations are in Flux [erg s^{-1} cm^{-2}] , physics is in Luminosity [erg s^{-1}] ➔ luminosity distance D_L

$$D_L = (1+z)^1 D_c \text{(luminosity distance)} \quad \text{Eq. 2}$$

$$D_A = (1+z)^{-1} D_c \text{(angular diameter distance)} \quad \text{Eq. 3}$$

$$D_C = (c/H_0) \int_0^z \frac{dz}{E} \qquad \text{(co-moving distance)} \quad \text{Eq. 4}$$

$$E = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda + \Omega_k (1+z)^2}$$

$$\Omega_k = 1.0 - \Omega_m - \Omega_\Lambda$$

M_{tot} dependence on D_A and H_0

- Hydrostatic total mass (from X-ray analysis):

$$M_{tot}(<r) = -\frac{k}{\mu m_p G} T_g(r)\, r \left(\frac{d \ln \rho_g(r)}{d \ln r} + \frac{d \ln T_g(r)}{d \ln r} \right) \rightarrow \qquad M_{tot} \propto r_{\mathrm{metric}}$$

- The X-ray measurements obtained in regions measured by the angular distance from the center θ_{arcmin}

$$\theta_{radians} = \frac{r_{metric}}{D_A} \quad \text{(trigonometry)}$$

$$\theta_{radians} = \frac{\theta_{arcmin}}{60} \frac{\pi}{180} \text{(arcmin to radians)}$$

$$r_{metric} = \theta_{radians} D_A \rightarrow$$

$$M_{tot} \propto D_A$$

Since $D_A \propto H_0^{-1}$ (see Eq. 3) ➜ $M_{tot} \propto H_0^{-1}$ Eq. 5

M_{gas} dependence on D_A and H_0

- Observations F [erg s^{-1} cm^{-2}] -- physics L [erg s^{-1}] (normalisation of the gas density profile)

$$norm_{MEKAL} = \frac{\int \rho^2 dV}{10^{-14} 4\pi [D_L/(1+z)]^2} = \frac{\int \rho^2 dV}{10^{-14} 4\pi [D_A(1+z)]^2} \rightarrow$$

$$\rho \propto \frac{D_L}{\sqrt{V}} \propto \frac{D_A (1+z)^2}{D_A^{3/2}} \propto D_A^{-1/2} (1+z)^2$$

$$M_{gas} \propto \rho V \propto D_A^{-1/2} (1+z)^2 D_A^3 = D_A^{5/2} (1+z)^2 \rightarrow$$

$$M_{gas} \propto D_A^{5/2} (1+z)^2$$

$$\rightarrow M_{gas} \propto H_0^{-5/2}$$

Eq. 6

Ω_M dependence on H_0

- Repeat Eq. 1: $\Omega_m \propto \Omega_b / f_{gas}$
- Repeat Eq. 5: $M_{tot} \propto H_0^{-1}$
- Repeat Eq. 6: $M_{gas} \propto H_0^{-5/2}$

➔ $$f_{gas} = \frac{M_{gas}}{M_{tot}} \propto \frac{H_0^{-5/2}}{H_0^{-1}} = H_0^{-3/2}$$

Since $\Omega_b \propto H_0^{-2}$ (see page 13)

➔ $$\Omega_m \propto H_0^{-2} \times H_0^{3/2} = H_0^{-1/2}$$

Example calculation

- Using $h = H_0 / 74$ km s^{-1} Mpc^{-1}
- $M_{tot} = (1 \pm 0.2)\ 10^{15}\ h^{-1}\ M_\odot$
- $M_{gas} = (1.5 \pm 0.1)\ 10^{14}\ h^{-5/2}\ M_\odot$
- $f_{gas} = M_{gas} / M_{tot} = 0.15\ h^{-3/2}$
- $\Omega_b = 0.0407 \pm 0.0015\ h^{-2}$
- $\Upsilon = 0.9$
- Repeat (Eq. 1): $\Omega_m = \Upsilon\, \Omega_b / f_{gas}$

$$\Omega_m(h=0.74) = 0.9 \times \frac{0.0407}{0.15} h^{-1/2} = 0.24\, h^{-1/2}$$

Error propagation

$f(x,y) \qquad\qquad \sigma_x, \sigma_y,$

$$\sigma_f^{\,2} = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^{\,2} + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^{\,2}$$

if $f(x,y) = \dfrac{x}{y} \rightarrow$

$\dfrac{\partial f}{\partial x} = \dfrac{1}{y} = \dfrac{f}{x}$ and

$\dfrac{\partial f}{\partial y} = \dfrac{-x}{y^2} = \dfrac{-f}{y} \rightarrow$

$$\sigma_f^{\,2} = \left(\frac{f}{x}\sigma_x\right)^2 + \left(\frac{f}{y}\sigma_y\right)^2 = f^2\left[\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2\right] \rightarrow$$

$$\sigma_f = f\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

Error propagation

$$f_{gas} = \frac{M_{gas}}{M_{tot}} \rightarrow \sigma_{fgas} = f_{gas}\sqrt{\left(\frac{\sigma_{Mgas}}{M_{gas}}\right)^2 + \left(\frac{\sigma_{Mtot}}{M_{tot}}\right)^2} \rightarrow$$

$$\sigma_{fgas} = 0.15\sqrt{\left(\frac{0.1}{1.5}\right)^2 + \left(\frac{0.2}{1.0}\right)^2} = 0.03 \rightarrow$$

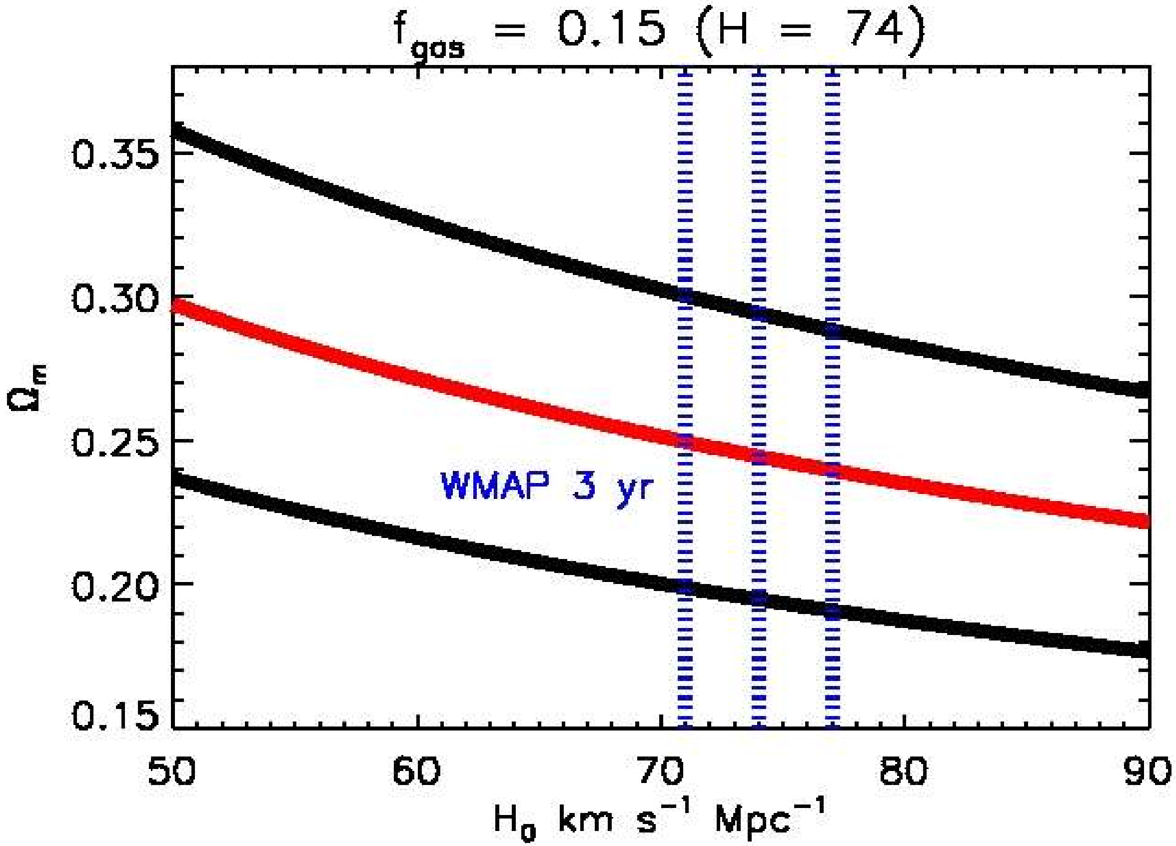
$$f_{gas} = 0.15 \pm 0.03$$

$$\Omega_m \propto \frac{\Omega_b}{f_{gas}} \rightarrow \sigma_{\Omega m} = \Omega_m\sqrt{\left(\frac{\sigma_{\Omega b}}{\Omega_b}\right)^2 + \left(\frac{\sigma_{fgas}}{f_{gas}}\right)^2} \rightarrow$$

$$\sigma_{\Omega m} = 0.24 \times \sqrt{\left(\frac{0.0015}{0.0407}\right)^2 + \left(\frac{0.03}{0.15}\right)^2} = 0.05 \rightarrow$$

$$\Omega_m = 0.24 \pm 0.05$$

$\Omega_m\ (H_0)$



- $\Omega_m = 0.24 \pm 0.05$ at $H_0 = 74$ km s^{-1} Mpc^{-1}
- considering H_0 uncertainties (3 km s^{-1} Mpc^{-1}) yields $\Omega_m = 0.24 \pm 0.06$