ASTRON

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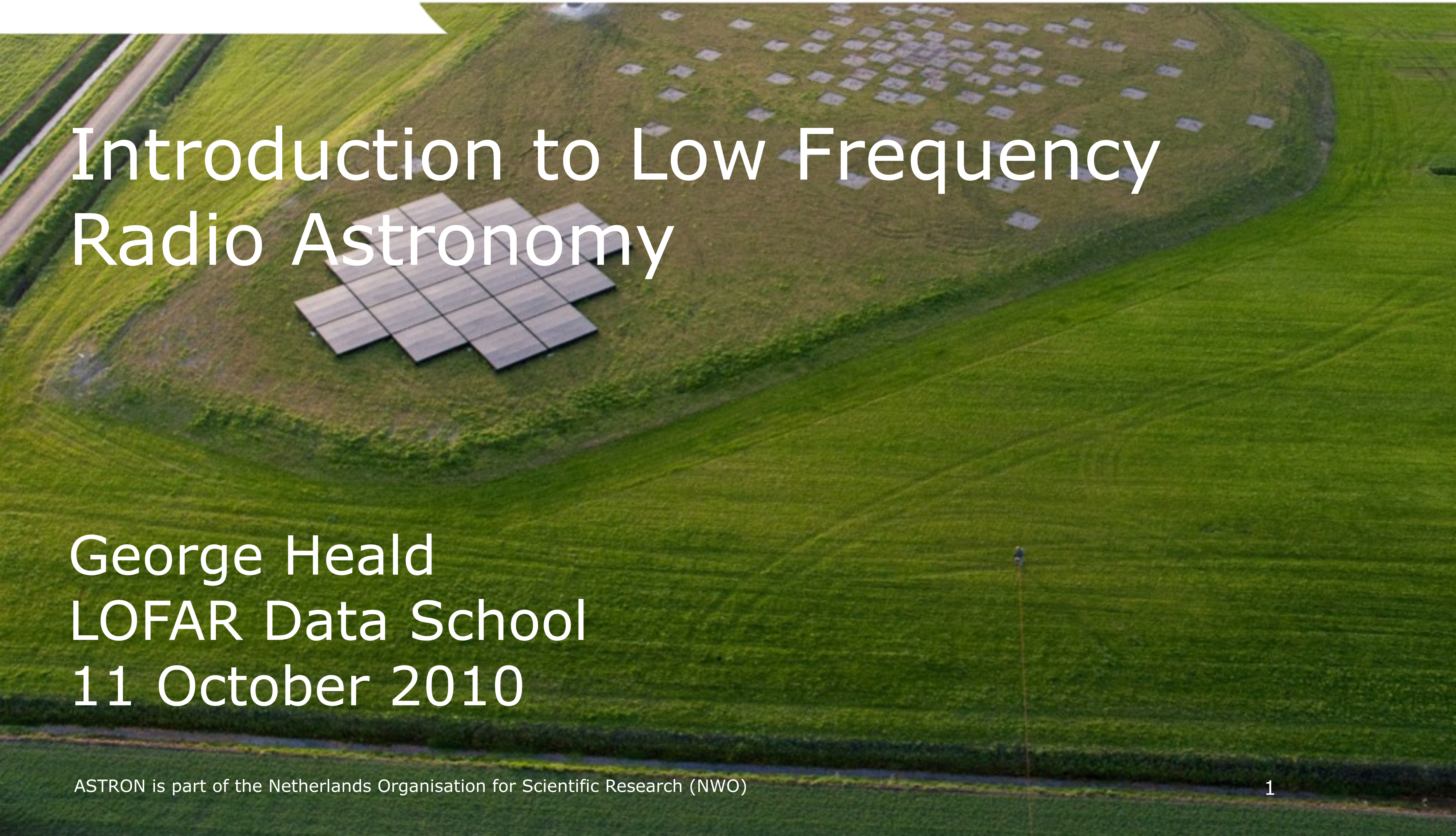

Introduction to Low Frequency Radio Astronomy

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LOFAR Data School

11 October 2010

Outline

- The low frequency sky
- Why aperture synthesis?
- What is it?
- How is it done?
- Issues specific to low frequencies
- How are images obtained from interferometers?

Acknowledgements

- Many of the general interferometry slides are adapted from lecture materials compiled by Tom Oosterloo

- Source material includes
  - Synthesis Imaging in Radio Astronomy II (VLA white book) editors: G. B. Taylor, C. L. Carilli, & R. A. Perley
  - Low frequency radio astronomy notes **http://www.gmrt.ncra.tifr.res.in/gmrt_hpage/Users/doc/WEBLF/LFRA/index.html**

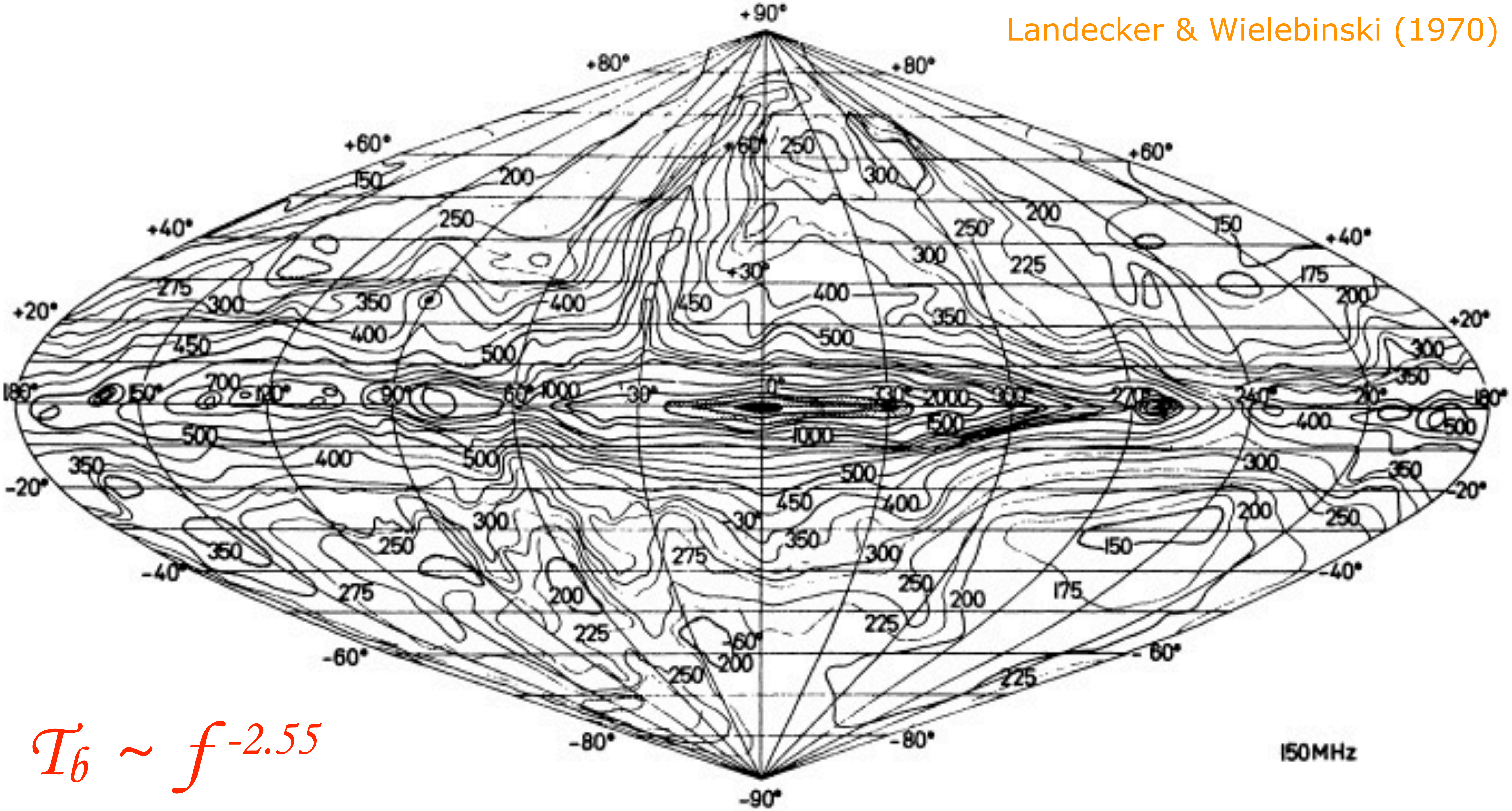
Key concepts

- Low frequency radio astronomy provides a unique view on the sky
- Aperture synthesis is used to increase angular resolution with small antennas
- Correlation takes place by multiplying and time-averaging antenna voltages
- Each baseline instantaneously measures the visibility function at a single location in the uv plane
- Earth rotation is exploited to fill the uv plane azimuthally, and bandwidth is exploited to fill the uv plane radially
- The visibility function is related to the intensity distribution on the sky via a Fourier transform relation
- The Measurement Equation is a useful tool for understanding the instrumental connection between visibilities and the sky brightness
- Aperture synthesis at low frequencies involves extra complications
- Sparse uv sampling causes image artifacts, which can be corrected (to a certain extent) with deconvolution routines

The low frequency sky

- Very high Galactic background! HII regions in absorption at low frequencies

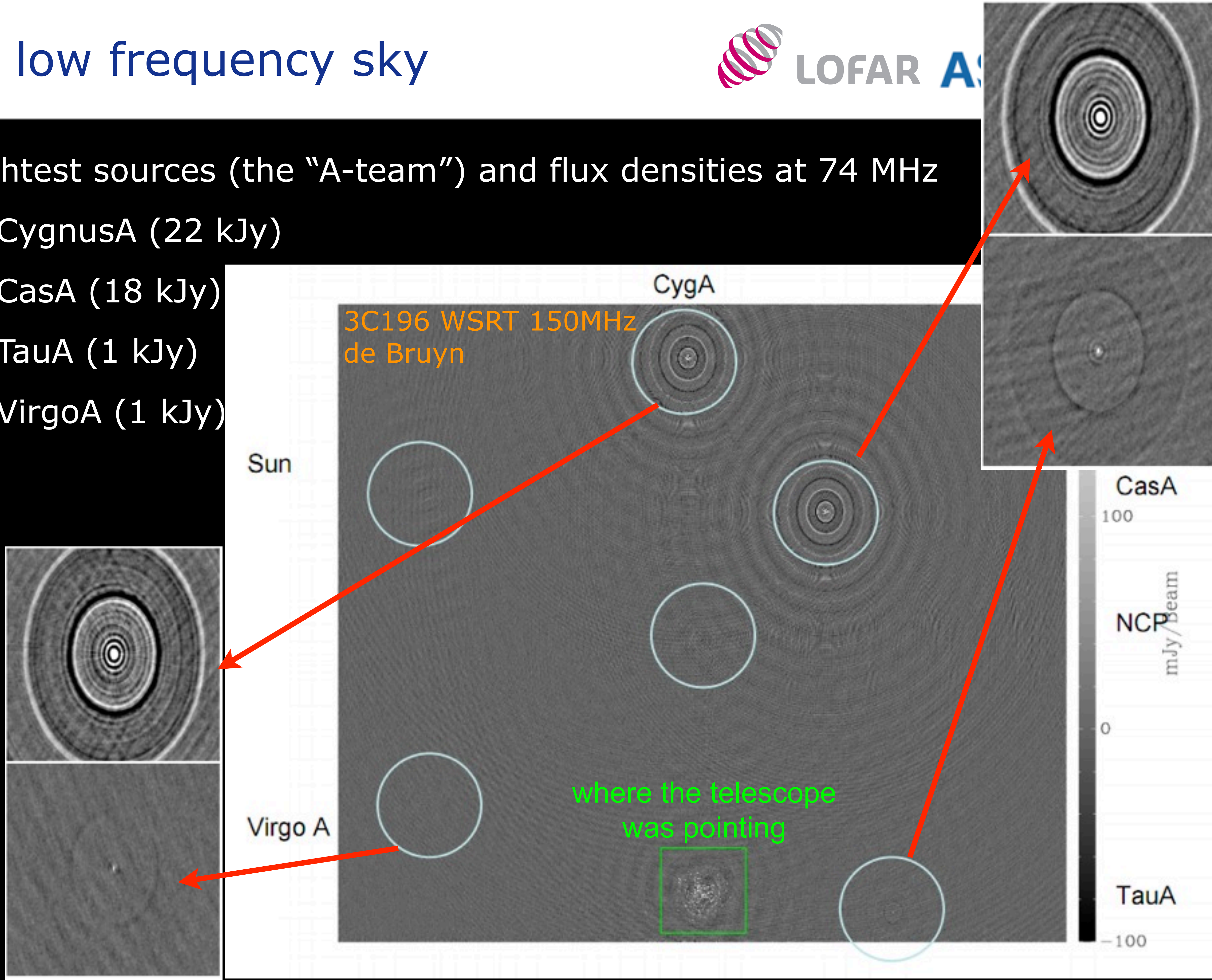
Landecker & Wielebinski (1970)



$T_6 \sim f^{-2.55}$

The low frequency sky

- Brightest sources (the “A-team”) and flux densities at 74 MHz
  - CygnusA (22 kJy)
  - CasA (18 kJy)
  - TauA (1 kJy)
  - VirgoA (1 kJy)

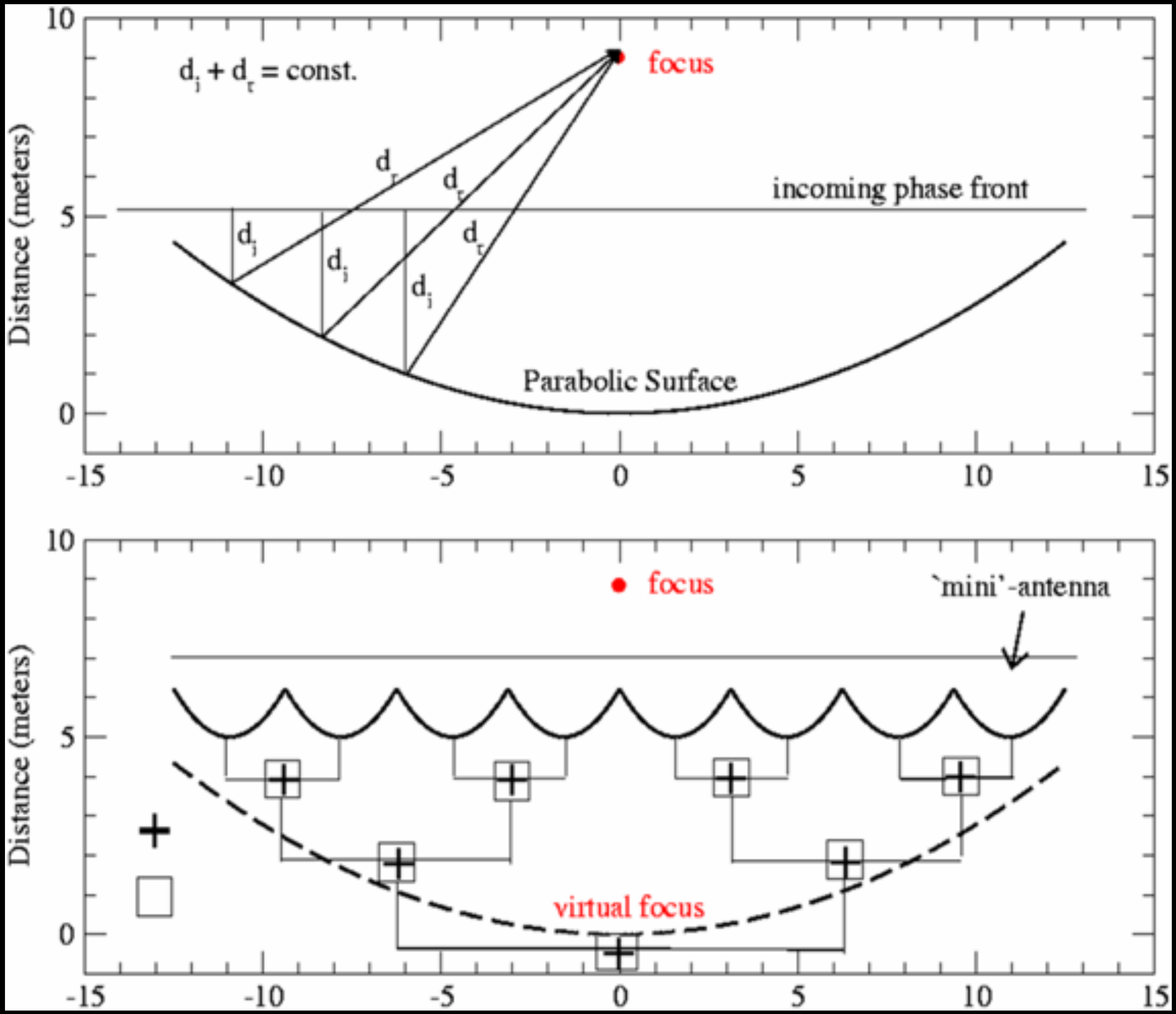


Aperture Synthesis

- The resolution of a single dish is poor: about 30 arcmin for 21cm radiation with a 25-m dish (like WSRT, VLA,...)
- How to get arcsecond, or even sub-arcsecond resolution?
  - For 21cm radiation, need a 43 km dish.... a bit impractical.
  - The largest steerable radio dish is 100m (still only ~7 arcmin at 21cm), while Arecibo is 300m in diameter.
  - The FAST telescope (under development in China) will be 500m, and will be difficult to build!
- So instead we synthesize an equivalent aperture, by combining smaller elements.
- The method was developed in the 1950s in England and Australia, and Martin Ryle (Cambridge) earned the Nobel Prize for his contributions.

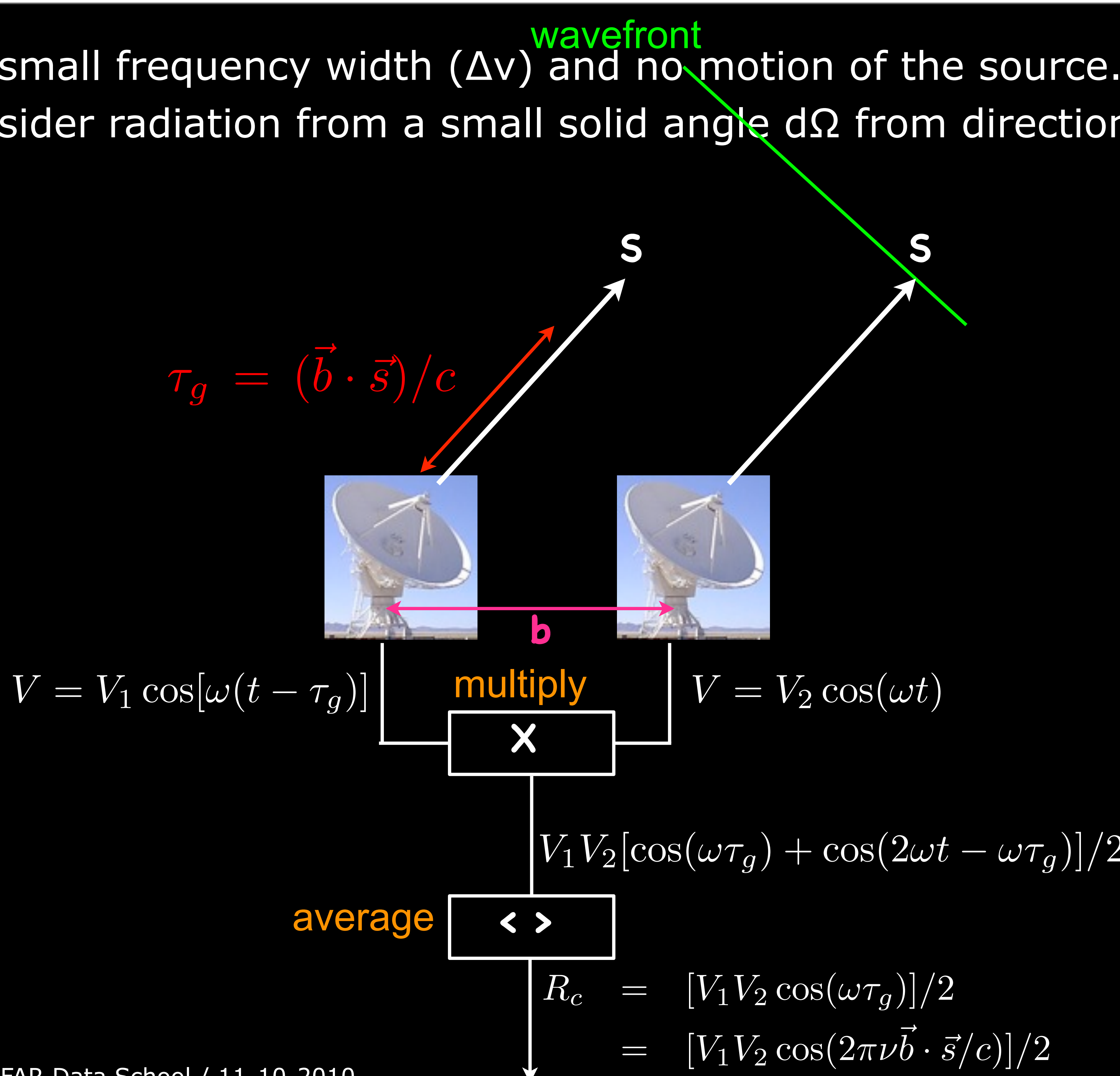
Aperture Synthesis

- If the source emission is non-variable, then we don't have to catch the entire wavefront at any given time.
- Instead we combine pairs of signals. If we have N elements, there are N(N-1)/2 pairs to combine.
- This is the basis of aperture synthesis.



Monochromatic interferometer

- Assume small frequency width (Δv) and no motion of the source. Now consider radiation from a small solid angle dΩ from direction S



Signal multiplication

- The averaged signal is independent of time, but it is dependent on the lag Since the lag is related to the direction, it is related to the source distribution on the sky --- so we can image the source distribution
- Here we have used V to indicate the voltage of the signal.
  How is that related to the source intensity?

  $$V \propto E \propto \sqrt{I}$$

  This means that the product $V_1 V_2$ is proportional to the source intensity I_ν which is measured in a unit called the Jansky

  $$10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}\,sr^{-1}} = 1\,\mathrm{Jansky(Jy)}$$

- The strength of the product also depends on the aperture areas of the antennas and the electronic gain factors, but these can be calibrated for.
- To determine the dependence of the response over an extended object, we integrate over solid angle.

Cosine correlator response

- The response from an extended source is just the integral of the response over the solid angle of the sky

$$R_c = \int\int I_\nu(\vec{s})\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

(neglecting any frequency dependence)

- NOTE: the vector s is a function of direction, so the phase in the cosine is dependent on the angle of arrival of the wavefront, and thus on the source structure.

- Now we have a relationship between the quantity of interest (I_ν, the source brightness on the sky) and an observable quantity (Rc, the interferometer response)

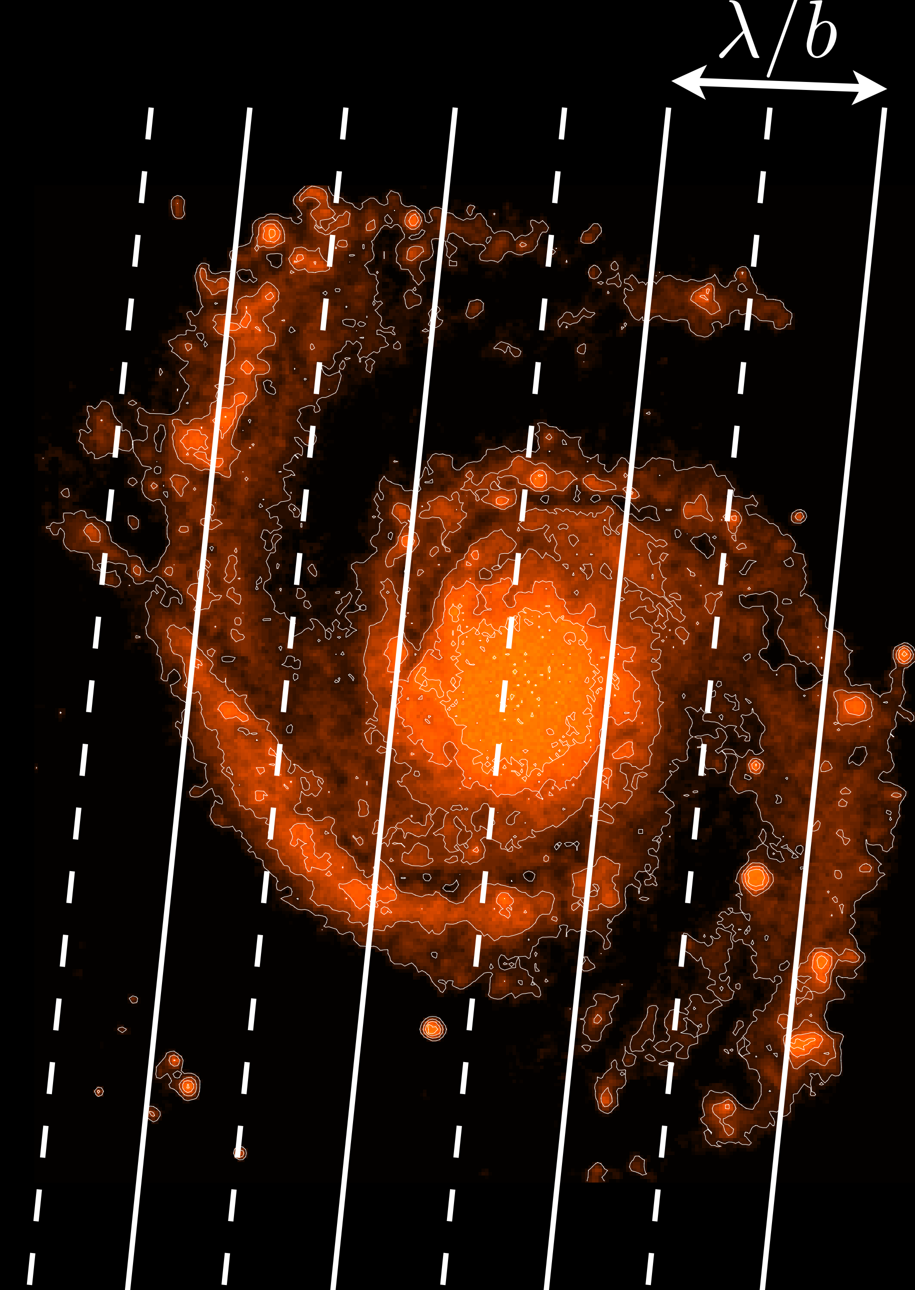
Schematic

- The cosine correlator can be thought of as casting a sinusoidal fringe pattern on the sky (of angular scale λ/b). The correlator multiplies the source intensity distribution by this fringe, and integrates the product over the sky.

$$R_c = \int \int I_\nu(\vec{s}) \cos(2\pi\nu\vec{b}\cdot\vec{s}/c)\, d\Omega$$

source brightness

fringe pattern

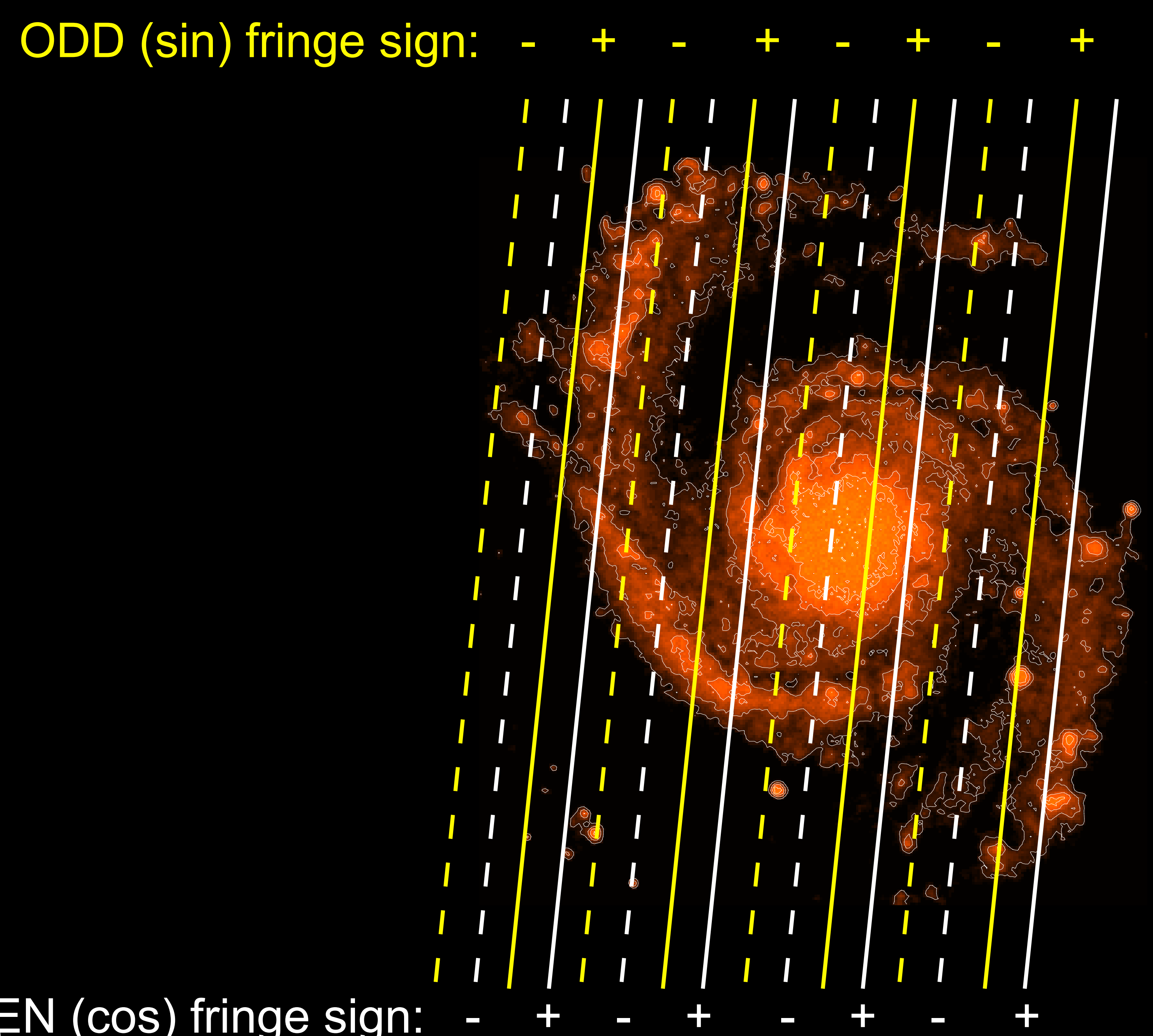


- The orientation of the fringe is set by the baseline geometry

  The fringe separation is set by baseline length, and the observing wavelength

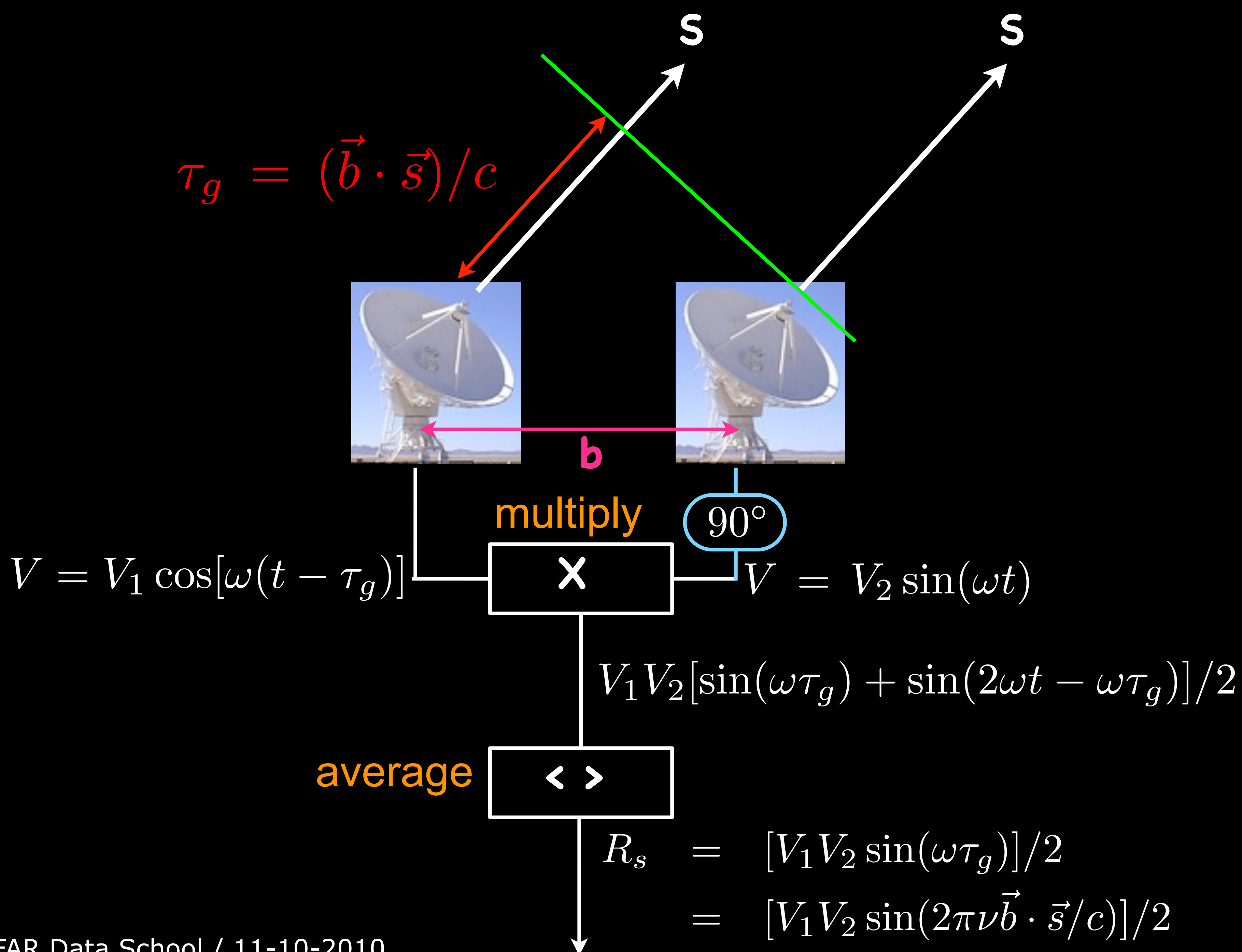
Even and odd coherence patterns

- To obtain the odd part, shift the cos pattern by 1/4 period to get a sin pattern



Sin correlator

- Just add a pi/2 phase shift to one of the signal paths...



Complex visibilities

- We define a complex visibility to be

$$V \;=\; R_c - iR_s \;=\; Ae^{-i\phi}$$

where

$$A \;=\; (R_c^2 + R_s^2)^{1/2}$$

$$\phi \;=\; \tan^{-1}\left(\frac{R_s}{R_c}\right).$$

and now we have

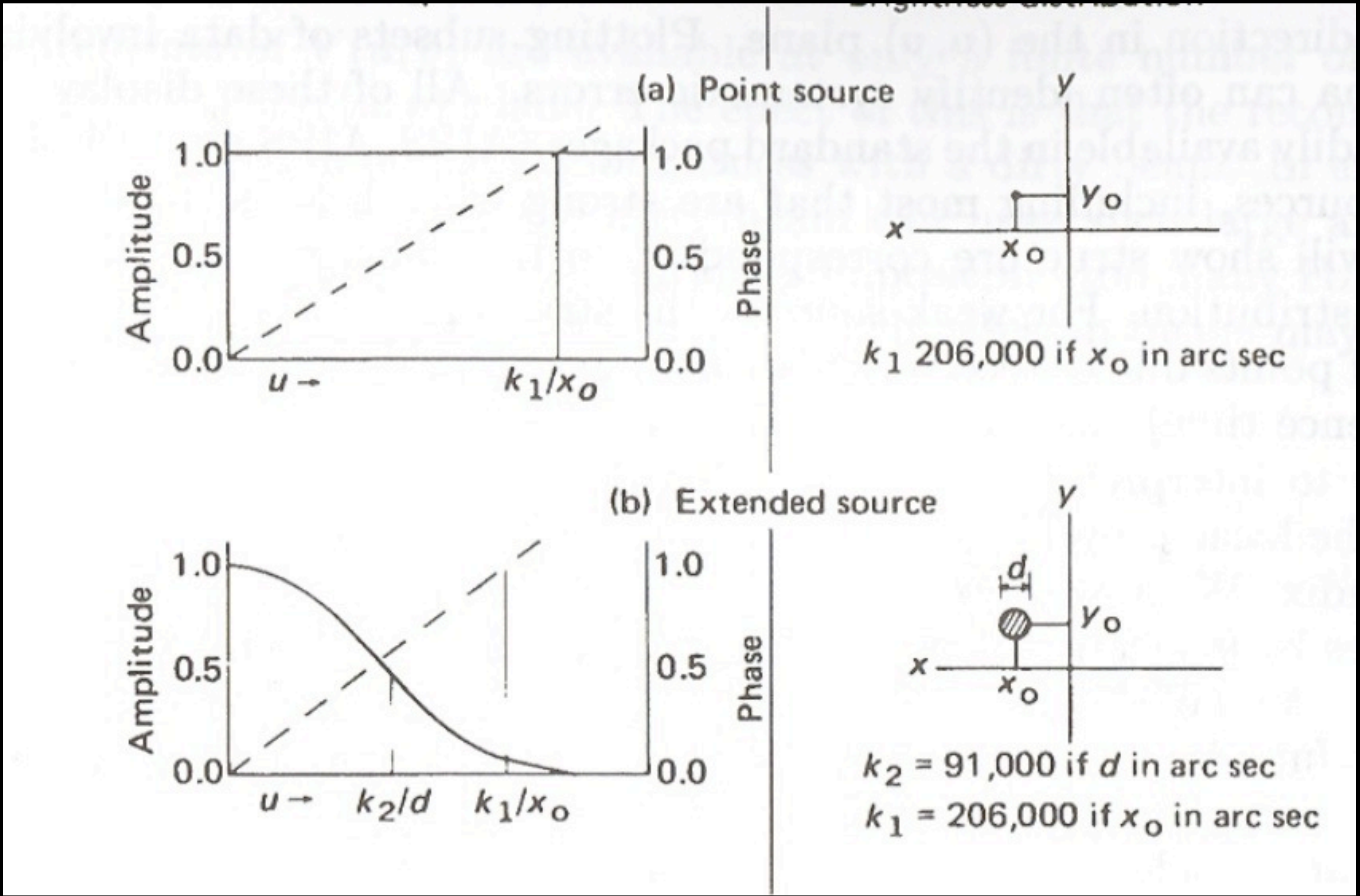
$$V(\vec{b}) = R_c - iR_s = \int\int I_\nu(\vec{s}) e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c} d\Omega$$

- So now we have a beautiful and useful relationship between the source brightness and the response of the interferometer!
- This can be inverted to get I(s) from V(b).

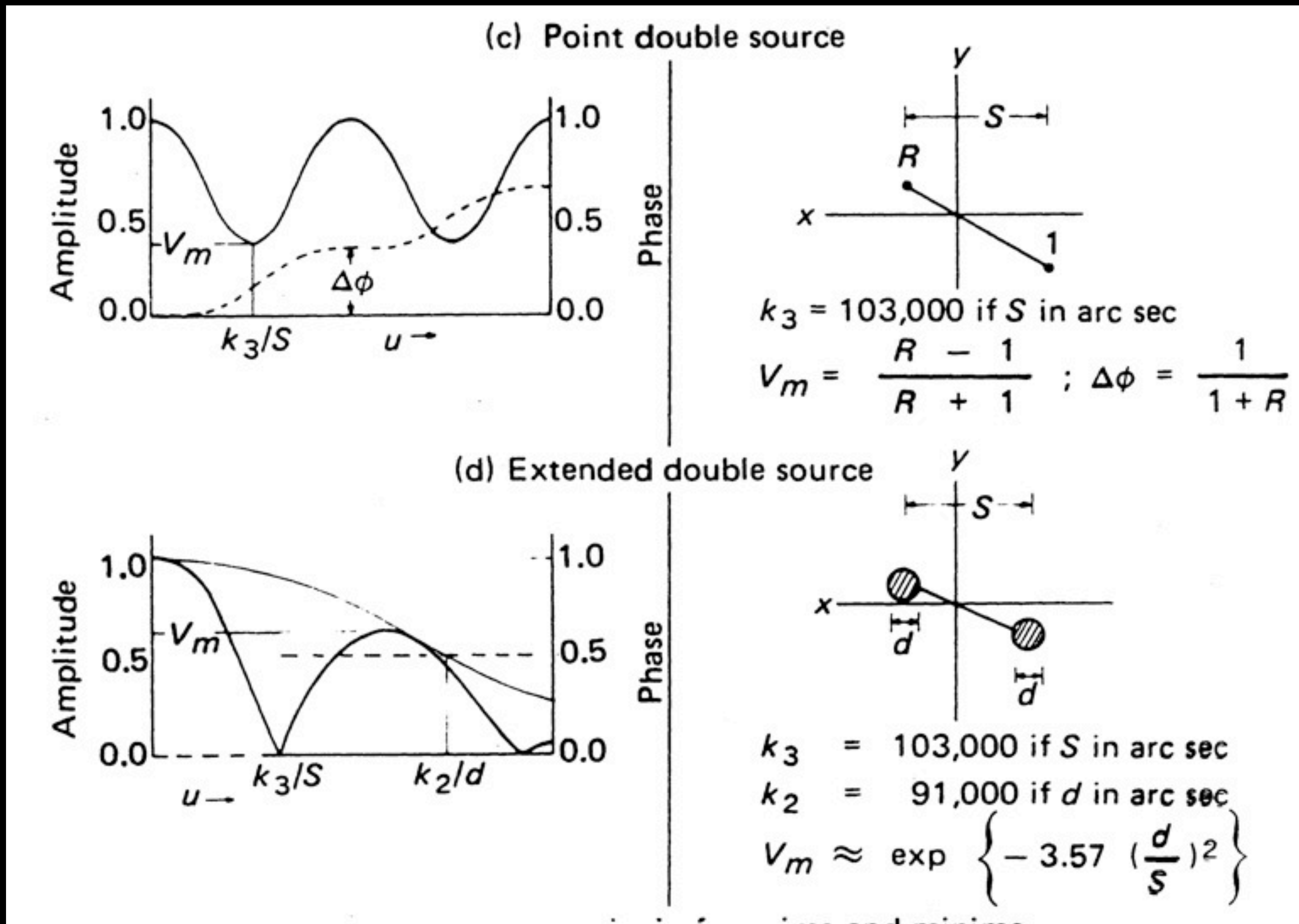
About visibilities

- The visibility is a function of the baseline and the source (intensity) structure
- Note that it is NOT dependent on the absolute positions of the individual antennas: we only care about the distances between elements.
- Visibilities are Hermitian, in other words $V(u,v) = V^*(-u,-v)$.
  This is a consequence of the fact that the source brightness distribution is real.
  (More about this later in the lecture...)
- There is a unique relationship between a given source brightness distribution, and the visibility function.
- Every measurement of the source with a given baseline length and orientation gives one measure of the visibility.
- If we sample enough of the visibility function, we can make a reasonable estimate of the source brightness distribution.

Example: single source



Example: double source



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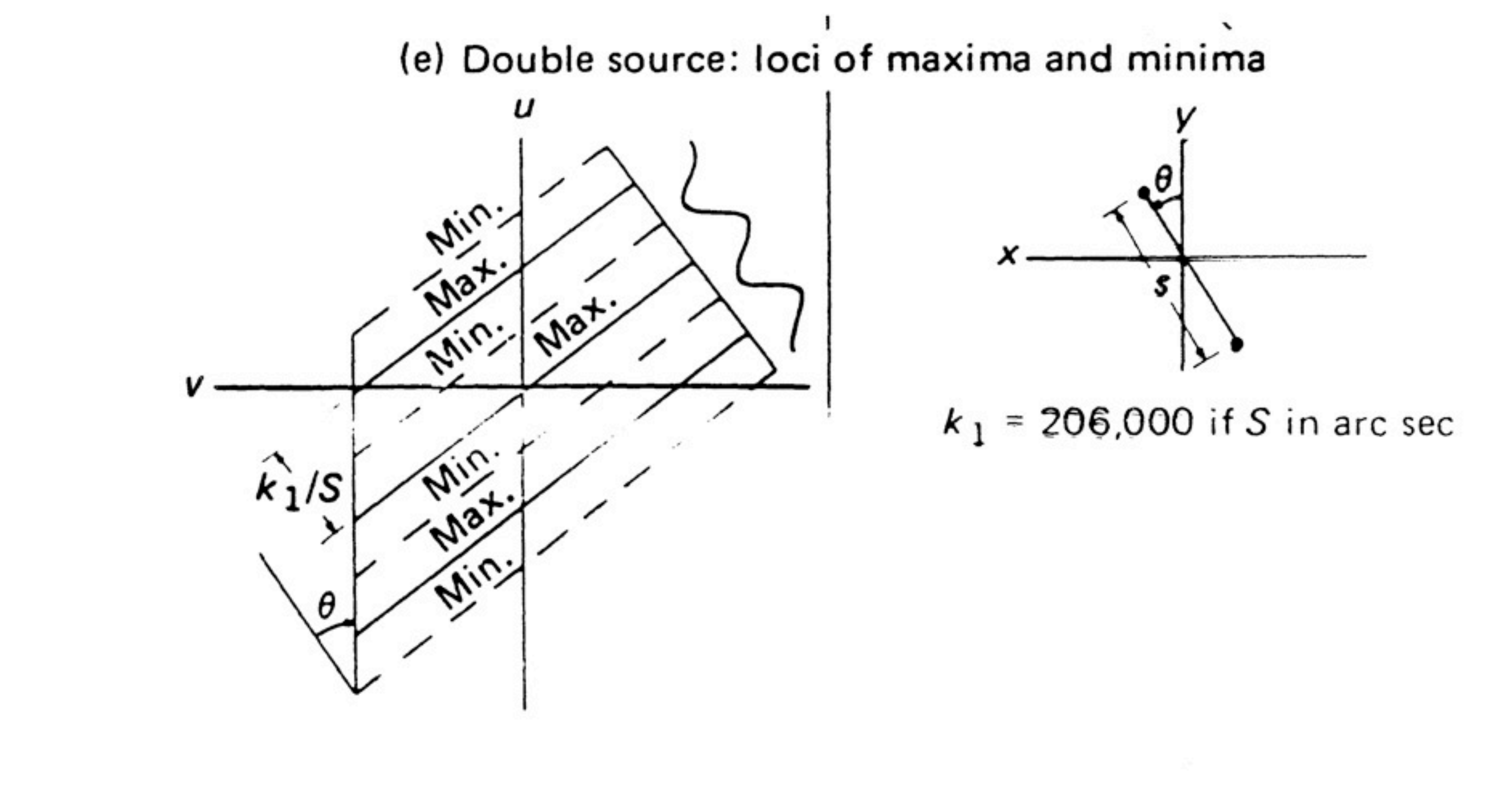


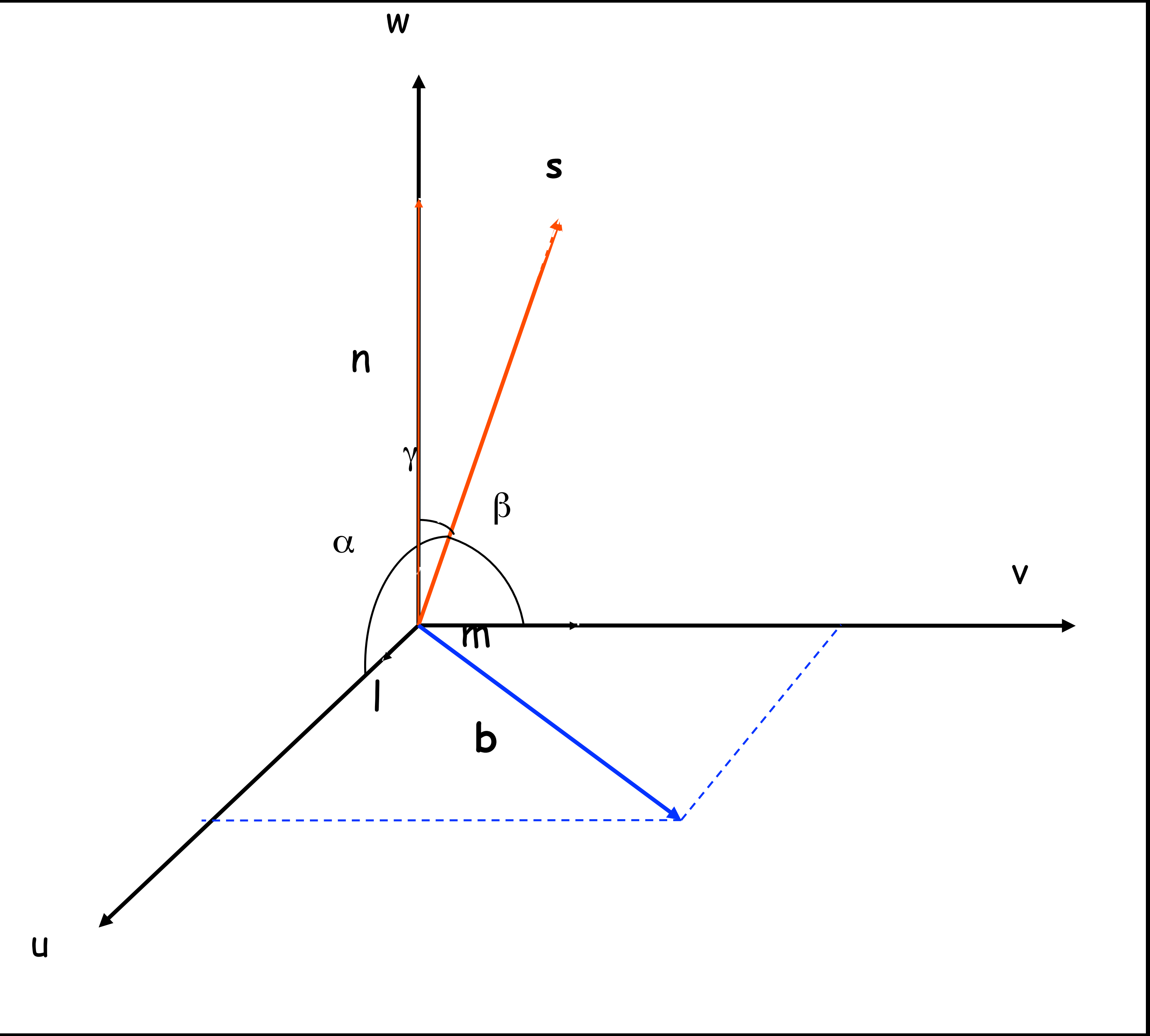
Imaging geometry

- The unit direction vector s is defined by its projections on the (u,v,w) axes. These components are called the direction cosines.

$$l = \cos(\alpha)$$
$$m = \cos(\beta)$$
$$n = \cos(\gamma) = \sqrt{1 - l^2 - m^2}$$

The baseline vector b is specified by its coordinates (u,v,w) which are measured in wavelengths

$$\vec{b} = (\lambda u, \lambda v, \lambda w)$$



2D Fourier transform relation

- If the array is 2-D (like WSRT) then we can write

  $$\nu\vec{b}\cdot\vec{s}/c \;=\; ul + vm + wn \;=\; ul + vm$$

  from which we find that:

  $$V_\nu(u,v) \;=\; \int\int \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}}\, e^{-2i\pi(ul+vm)}\, dl\, dm$$

  which is a 2-dimensional Fourier transform between:

  - the projected brightness
  - and the visibility function

- We know how to invert the Fourier transform relation:

  $$I_\nu(l,m) = \cos(\gamma) \int\int V_\nu(u,v)\, e^{+2i\pi(ul+vm)}\, du\, dv$$

  So, with enough measures of V, we can derive I.

Fourier transform properties

- Fourier transforms are linear:

$$\mathcal{F}[ax(t) + by(t)] = a\mathcal{F}[x(t)] + b\mathcal{F}[y(t)]$$

- Coordinate shifts lead to phase changes:

$$\mathcal{F}[x(t \pm t_0)] = X(j\omega)e^{\pm j\omega t_0}$$

- Symmetry is preserved:

$$\text{if } \; x(t) = x(-t) \quad \text{then} \quad X(j\omega) = X(-j\omega)$$

$$\text{if } \; x(t) = -x(-t) \quad \text{then} \quad X(j\omega) = -X(-j\omega)$$

- Scaling of coordinates:

$$\mathcal{F}[x(at)] = \frac{1}{a}X(\frac{\omega}{a}) \quad \text{or} \quad \mathcal{F}[ax(at)] = X(\frac{\omega}{a})$$

- Convolution relation:

$$\mathcal{F}[x(t)\,y(t)] = X(j\omega) * Y(j\omega)$$

- For (much) more detail:

  e.g. **`<fourier.eng.hmc.edu/e101/lectures/handout3/node2.html>`**

3D interferometer arrays

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- More complicated: for a 3-D measurement volume
- What if the interferometer does not measure the visibilities within a plane, but instead in a volume? We need a slightly different coordinate system.
- Remember the full expression,

$$V_\nu(u,v,w) = \int\int \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm+wn)}\, dl\, dm$$

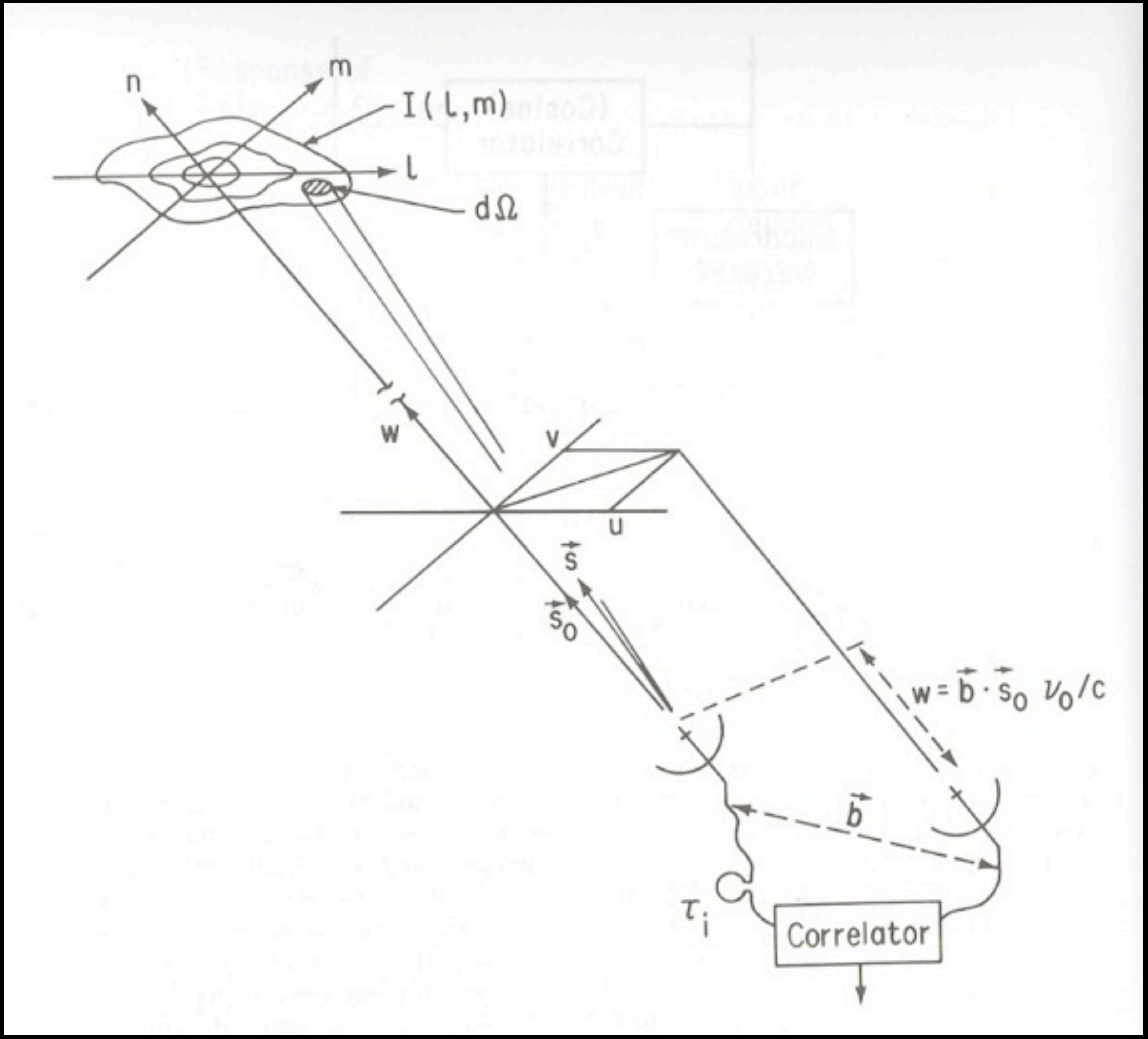
  NOTE, this is not a 3-D Fourier transform.
- Now we
  - orient the coordinate system so that the w-axis points to the center of the region of interest (so that u points east and v points north)
  - make use of the small angle approximation,

$$n = \cos(\gamma) = \sqrt{1-\sin^2\gamma} \simeq \sqrt{1-\theta^2} \simeq 1-\theta^2/2$$

    where θ is the polar angle from the center of the image.
    The w-coordinate is the "delay distance" of the baseline.

3D coordinate system

- Now w points to the source, u to the east, and v toward the NCP. The direction cosines l and m increase to the east and the north respectively



3D to 2D

- After making the small angle approximation, we have

$$V_\nu(u,v,w) \; = \; e^{-2i\pi w} \int\int \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} \, e^{-2i\pi(ul+vm-w\theta^2/2)} \, dl\, dm$$

The quadratic term in the phase can be neglected if it is much less than 1

$$w\theta^2 << 1$$

Or in other words, if the maximum angle from the center is

$$\theta_{\mathrm{max}} < \sqrt{\frac{1}{w}} \leq \sqrt{\frac{\lambda}{B}} \sim \sqrt{\theta_{\mathrm{syn}}}$$

then the relation between intensity and visibility again becomes a 2-D FT:

$$V'_\nu(u,v) \; = \; \int\int \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} \, e^{-2i\pi(ul+vm)} \, dl\, dm$$

where the modified visibility is defined as

$$V'_\nu = V_\nu e^{2i\pi w}$$

and is the visibility that we would have measured if we had been able to put the baseline on the w=0 plane.

3D to 2D: approximation validity

- This coordinate system, coupled with the small-angle approximation, allows us to use two-dimensional transforms for many interferometer arrays.
- Breaks down for instance for the current Dutch LOFAR
  - Maximum baselines ~40 km
  - At 150 MHz, the 2-D approximation isn't even valid out to half a degree! (But the field of view is ~5 degrees FWHM...)
- How do we make images when the small angle approximation breaks down?
  - W-Projection, Faceting. See LOFAR Imaging Cookbook.
  - In short: We know how to do it, and it takes a lot of compute power.
  - Ronald Nijboer's lecture on Wednesday will present the details.

Making images

- In general the Fourier transform relation can be solved to give

$$I_\nu(l,m) = \cos(\gamma) \int \int V_\nu(u,v)\, e^{+2i\pi(ul+vm)}\, du\, dv$$

This relationship presumes knowledge of V(u,v) for all values of u and v.
But in fact, we have a finite number, N, measures of the visibility.
So to obtain an image, the integrals are replaced by a sum:

$$I_\nu(l,m) \;=\; \frac{1}{N} \sum_{n=1}^{N} V_n(u_n, v_n) e^{2i\pi(u_n l + v_n m)} \Delta u\, \Delta v$$

If we have N_v visibilities, and N_m cells (pixels) in the image, we have of order $\sim N_v N_m$ calculations to perform - a number that can exceed 10^{12} !

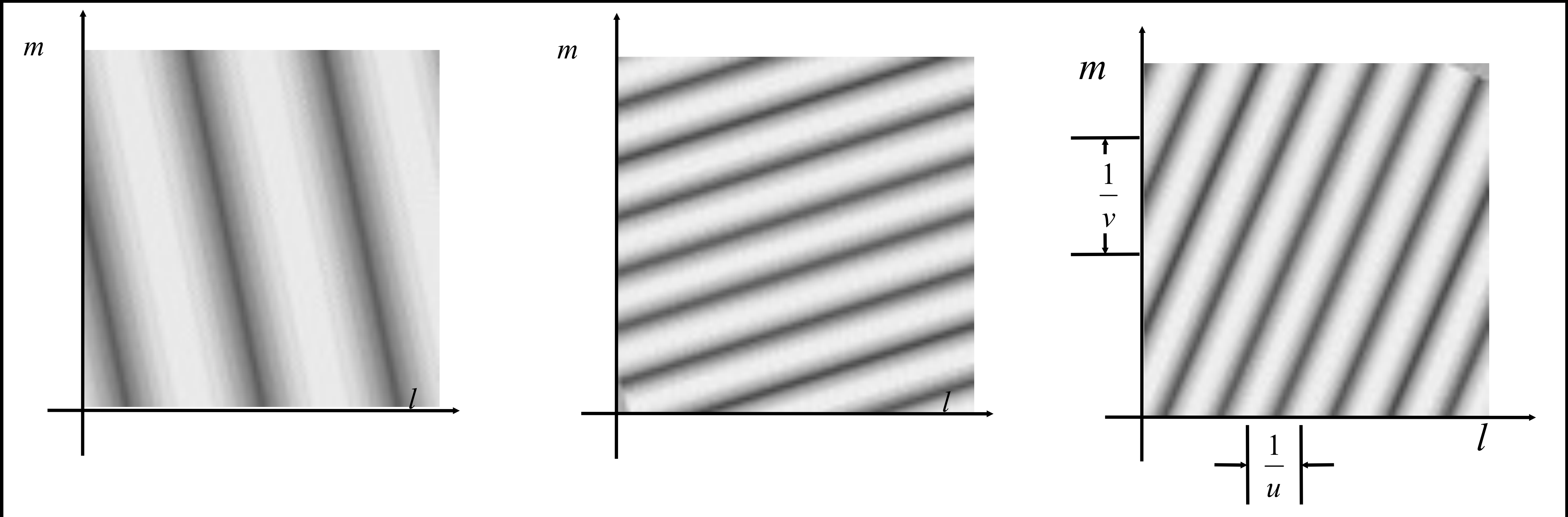
Real sky ⇒ Hermitian visibilities

- The sum on the last page is in general complex, while the sky brightness is real. What's wrong here?
- In fact, each measured visibility represents two visibilities, since V(-u,-v) = V*(u,v)
- This is because interchanging two antennas leaves R_c unchanged, but changes the sign of R_s (which is the imaginary part).
- Mathematically, since the sky is real, the visibility must be Hermitian.
- So we can modify the sum to read:

$$I_\nu(l,m) = \frac{1}{N}\sum_{n=1}^{N} A_n \cos[2\pi(u_n l + v_n m) + \phi_n]\,\Delta u\,\Delta v$$

Image synthesis

- Intensity distribution built up out of fringes like these:



- Spatial frequency / angle determined by location of visibility in u,v plane.
- Brightness of the fringe determined by the visibility amplitude, and its shift relative to phase center is determined by the visibility phase.

Earth rotation synthesis

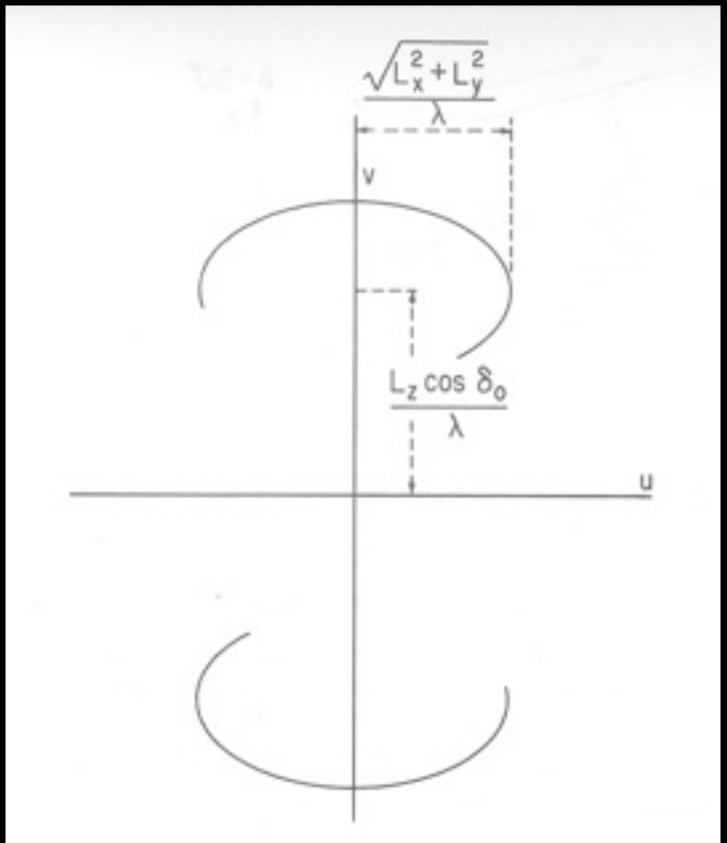
- u,v,w coordinates:
  X pointing to ha=0h, dec=0º
  Y pointing to ha=-6h, dec=0º
  Z pointing to dec=90º
- Lx, Ly, and Lz represent a single baseline, h is hour angle, δ is dec

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \times \begin{pmatrix} \sin h & \cos h & 0 \\ -\sin\delta\cos h & \sin\delta\sin h & \cos\delta \\ \cos\delta\cos h & -\cos\delta\sin h & \sin\delta \end{pmatrix} \begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix}$$

so that

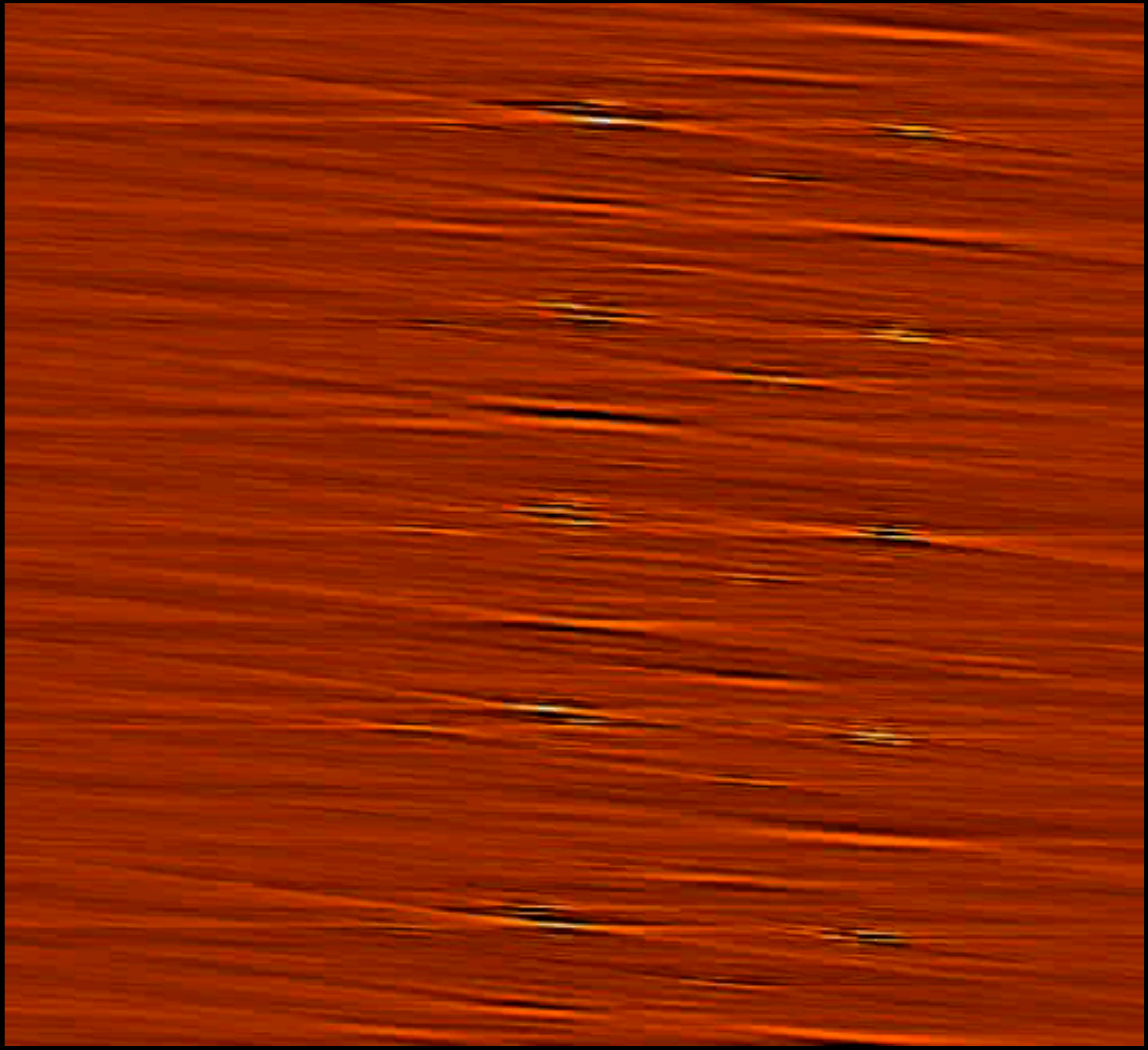
$$u^2 + \left(\frac{v - (L_z/\lambda)\cos\delta}{\sin\delta}\right)^2 = \frac{L_x^2 + L_y^2}{\lambda^2}$$

As Earth rotates, baselines describe an ellipse
For EW baselines, Lz=Lx=0
⇒ concentric, coplanar ellipses



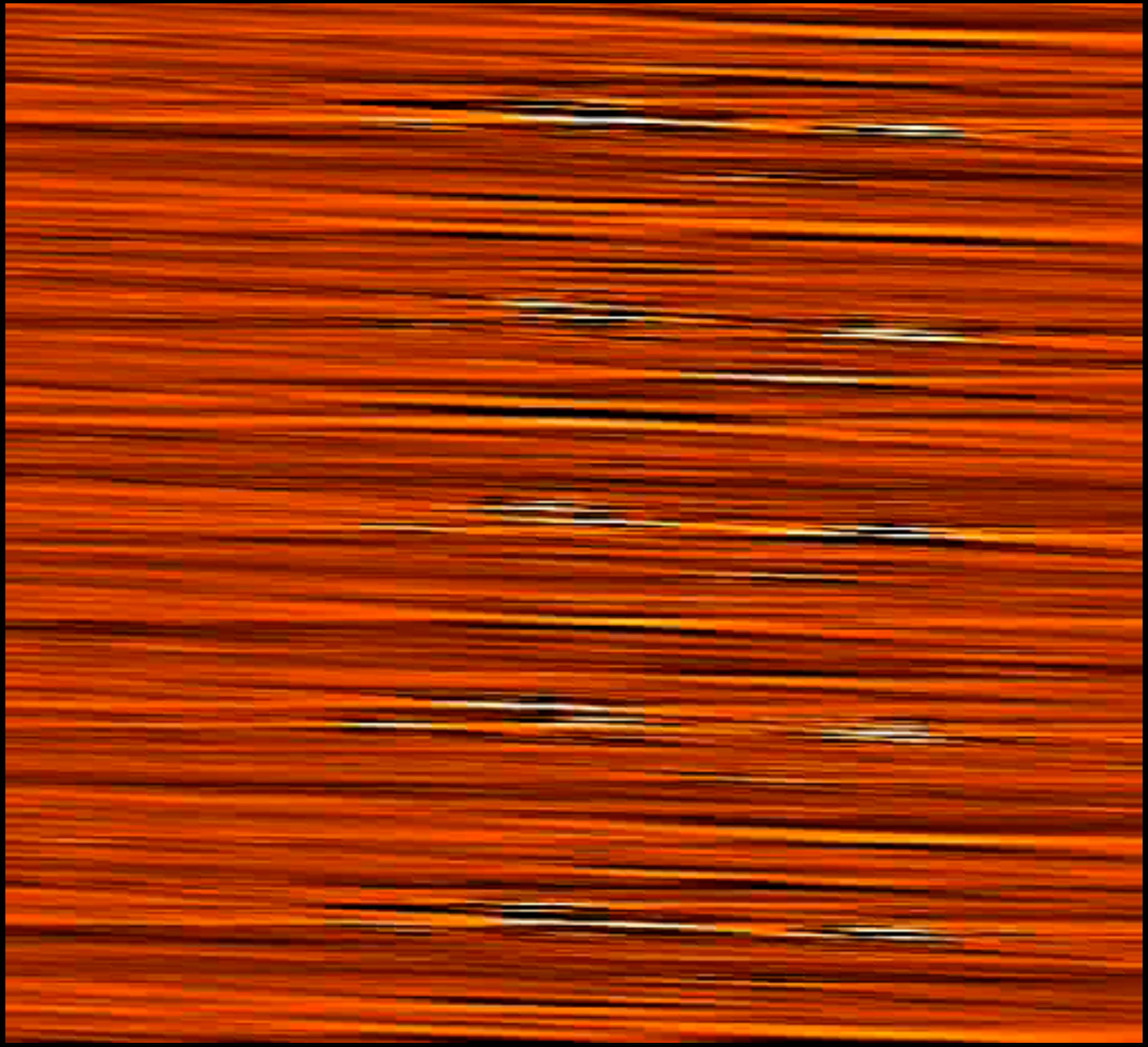
How an EW array sees the sky

- 12 hr sequence of instantaneous snapshots (WSRT)



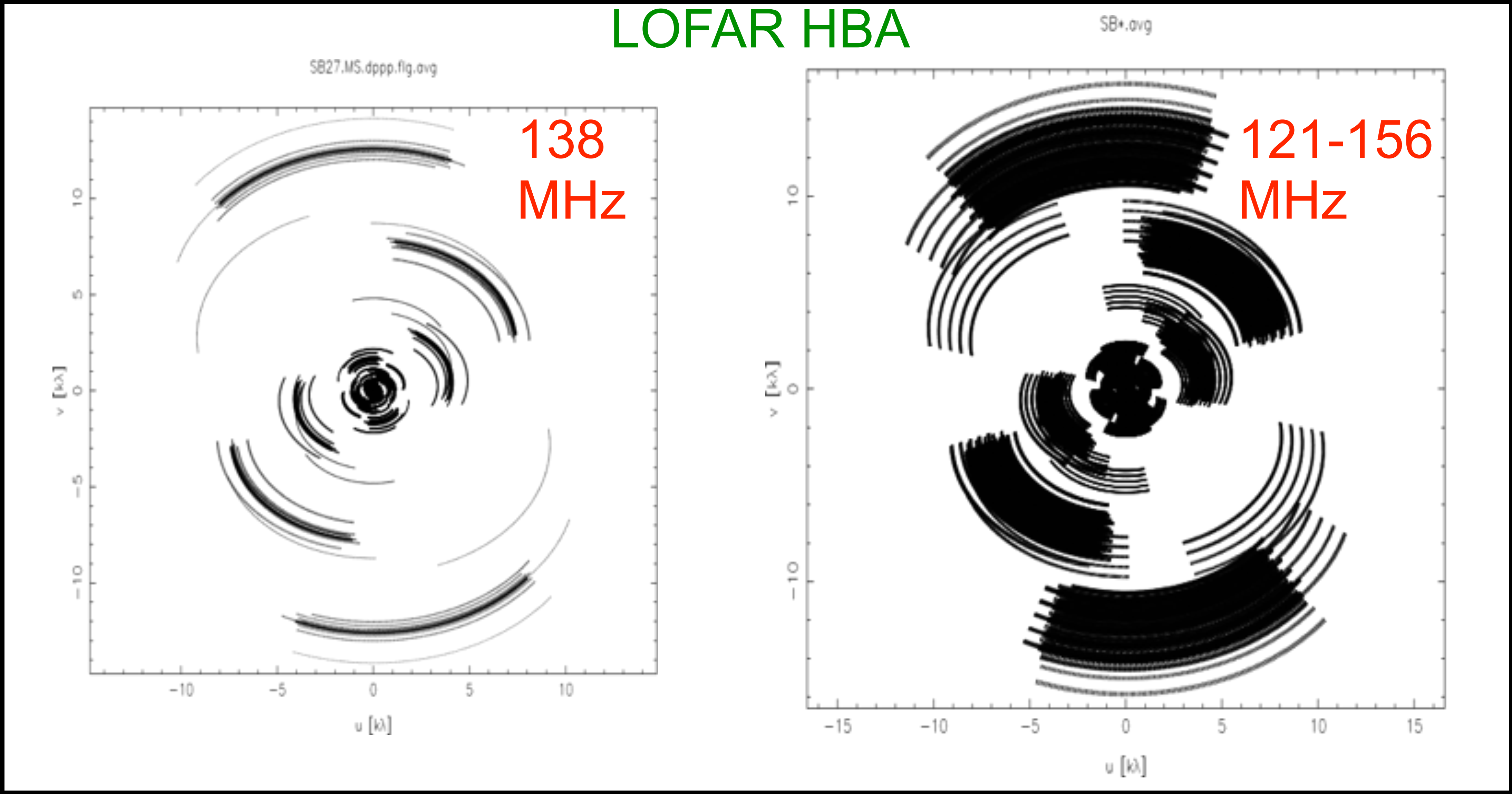
How an EW array sees the sky

- 12 hr cumulative buildup of an image of the sky



Bandwidth synthesis

- Another way to fill in the uv plane is with bandwidth. Since uv coordinates are expressed in units of the wavelength, correlating at multiple frequencies fills in the uv plane in the radial direction.
- An excellent example of this is LOFAR: 48 MHz is a huge fractional bandwidth! (Especially in the low band, where Δν/ν ≈ 100%!)



Primary beam pattern

- For a perfect interferometer,

$$V(\vec{b}) = \int\int I_\nu(\vec{s}) e^{-2\pi i \nu \vec{b}\cdot\vec{s}/c} d\Omega$$

but each antenna sees only a small region of the sky :

$$A(\theta, \phi, \nu)$$

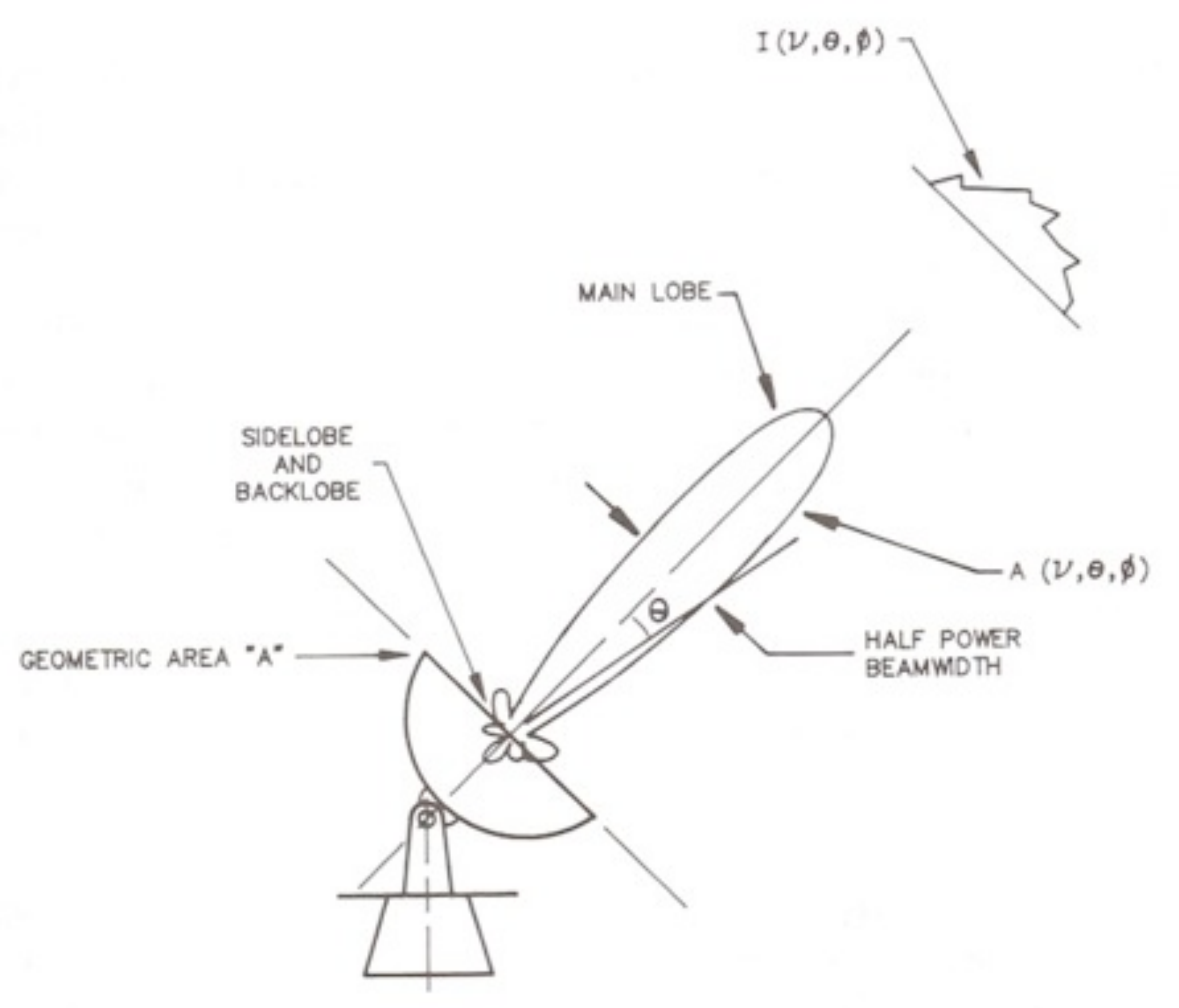


Figure 3–2. The reception pattern of an antenna.

- The sky is multiplied by the reception pattern of the antennae which results in

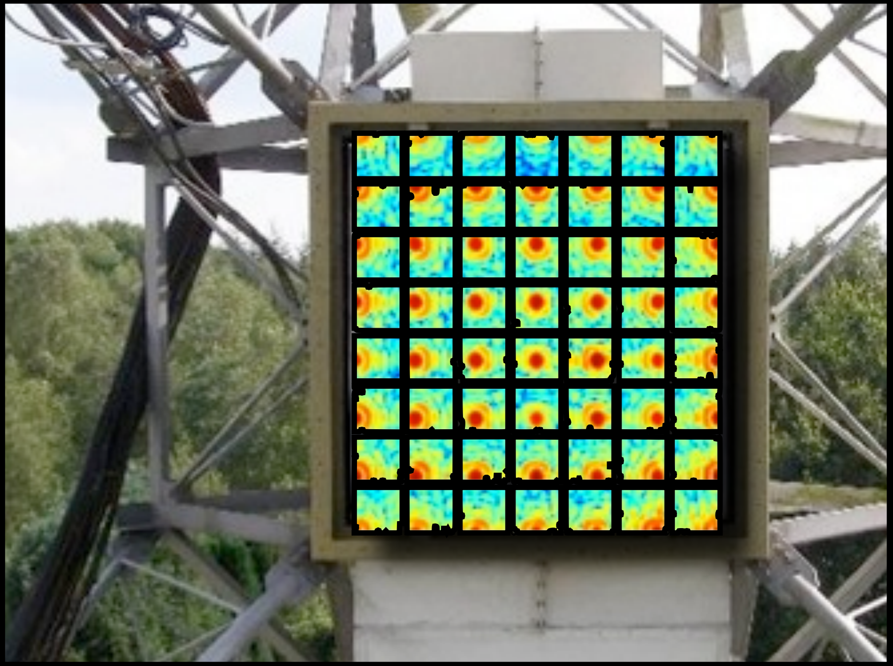
$$V(\vec{b}) = \int\int A_\nu(\vec{s}) I_\nu(\vec{s}) e^{-2\pi i \nu \vec{b}\cdot\vec{s}/c} d\Omega$$

or in general for two elements

$$V(\vec{b}) = \int\int \sqrt{A'_\nu(\vec{s})}\sqrt{A''_\nu(\vec{s})} I_\nu(\vec{s}) e^{-2\pi i \nu \vec{b}\cdot\vec{s}/c} d\Omega$$

Example: DIGESTIF

- DIGESTIF is a demonstrator for APERTIF, and has one of the WSRT antennas equipped with a Phased Array Feed (PAF)



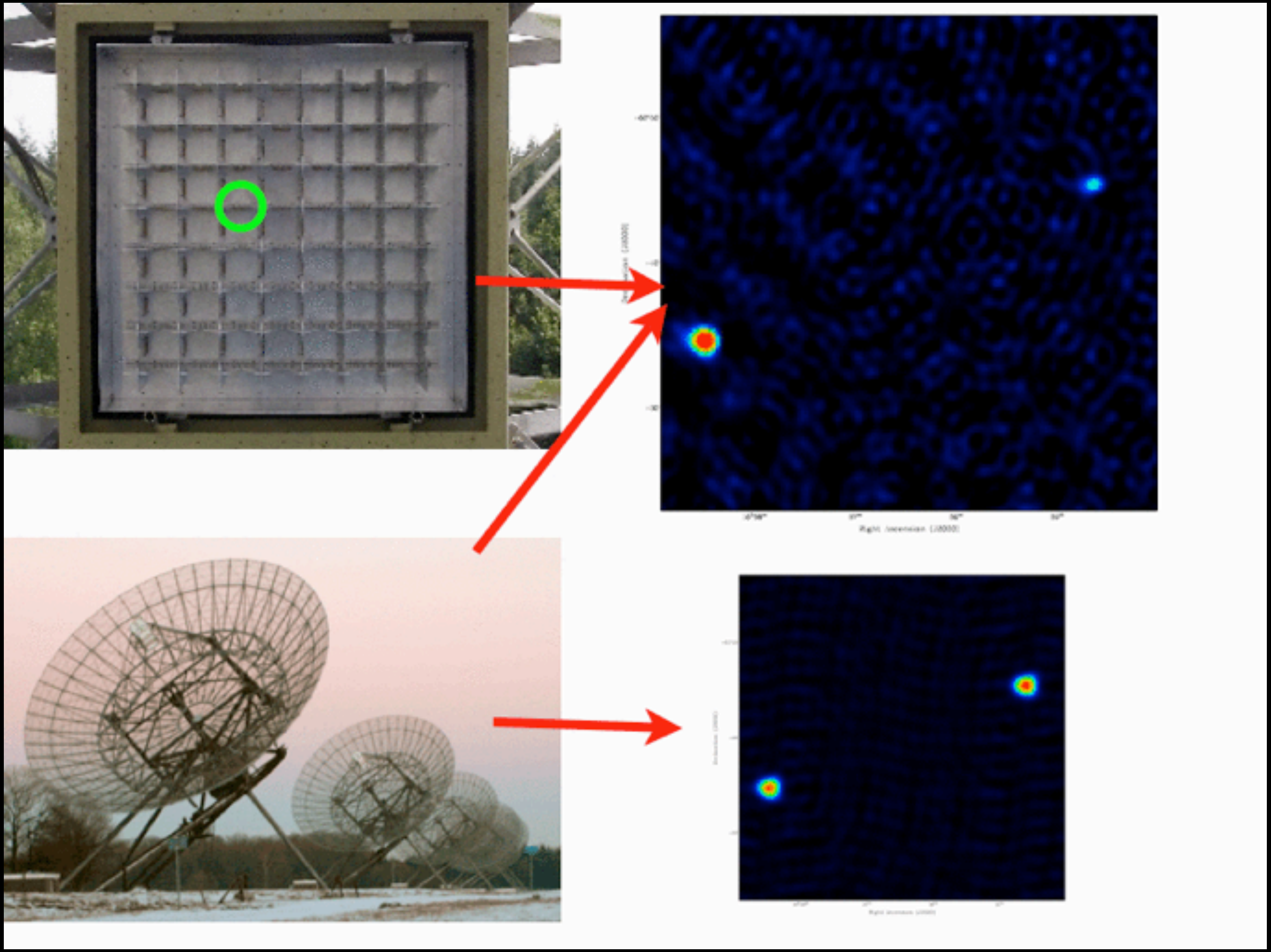
- Each FPA element has a unique sensitivity pattern on the sky
- Experiment has been done correlating these FPA elements with other (normal) WSRT antennas, resulting in multiple versions of the same baseline, but with very different antenna response patterns.

$$V(\vec{b}) = \int\int \sqrt{A'_\nu(\vec{s})}\sqrt{A''_\nu(\vec{s})}I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

- What do the visibilities of a double source (3C343) look like?

Example: DIGESTIF

- DIGESTIF interferometric observation of 3C343



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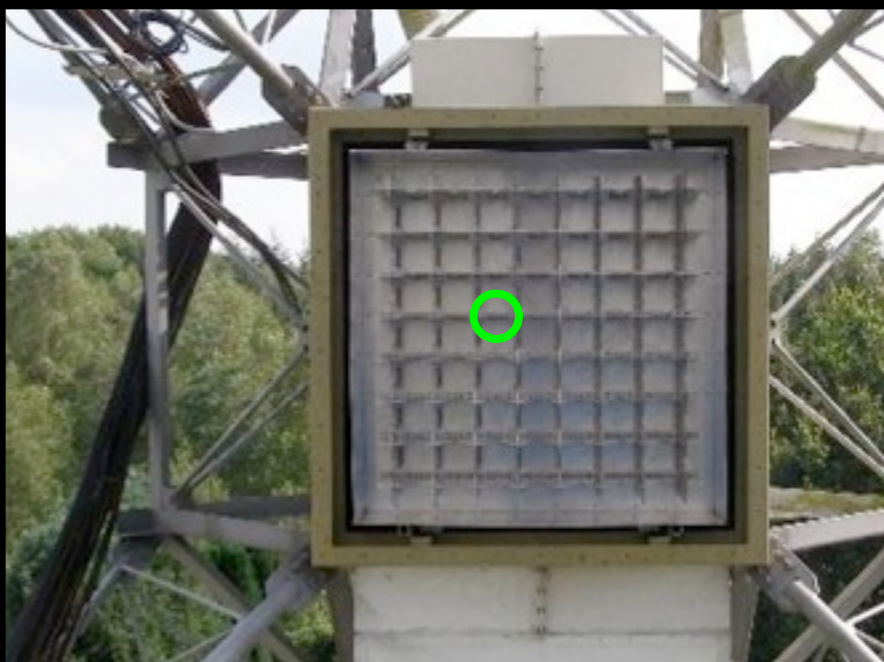

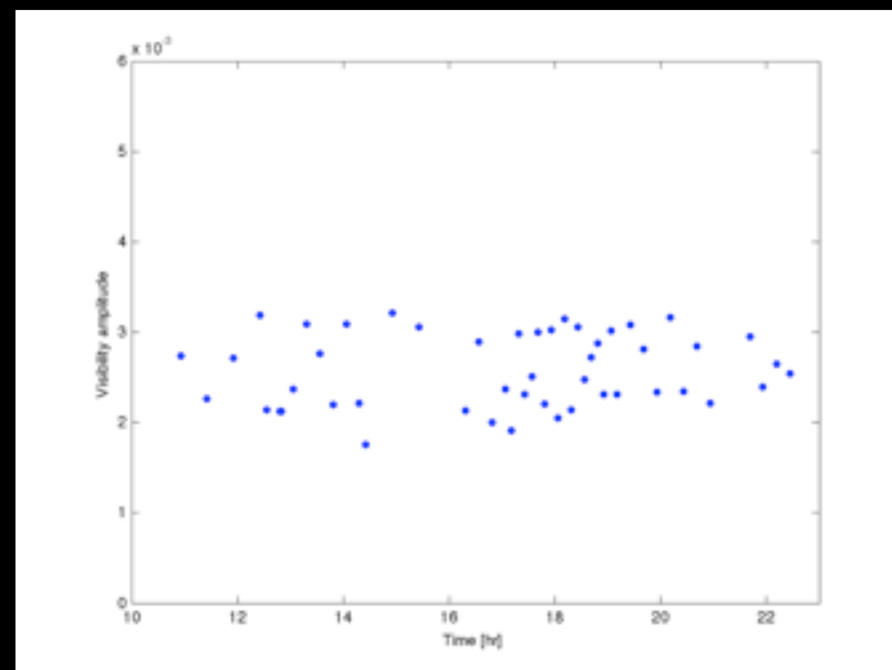

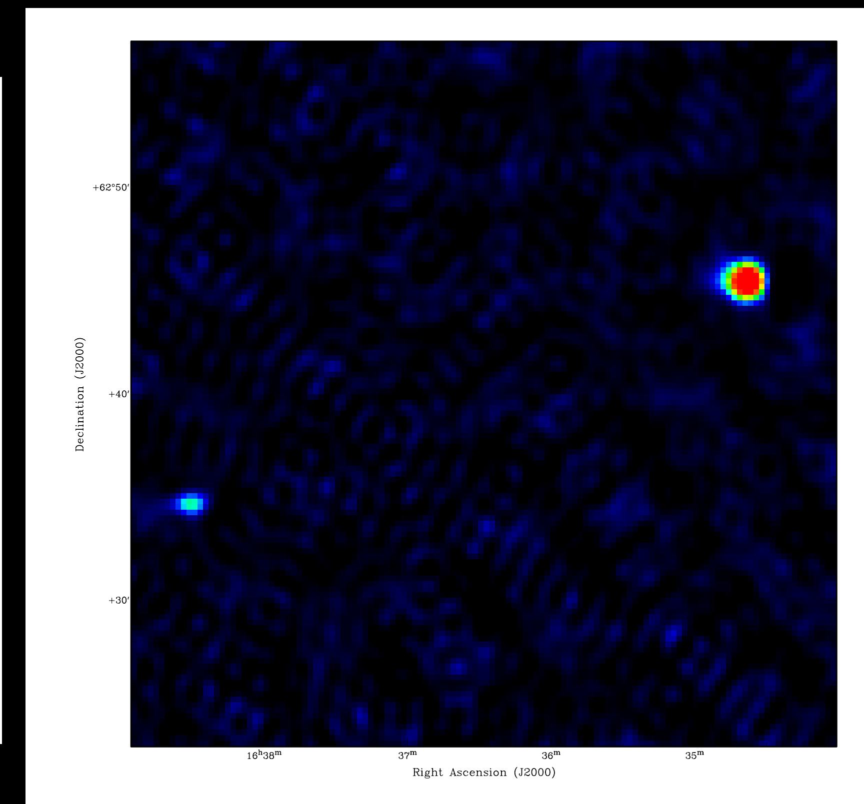

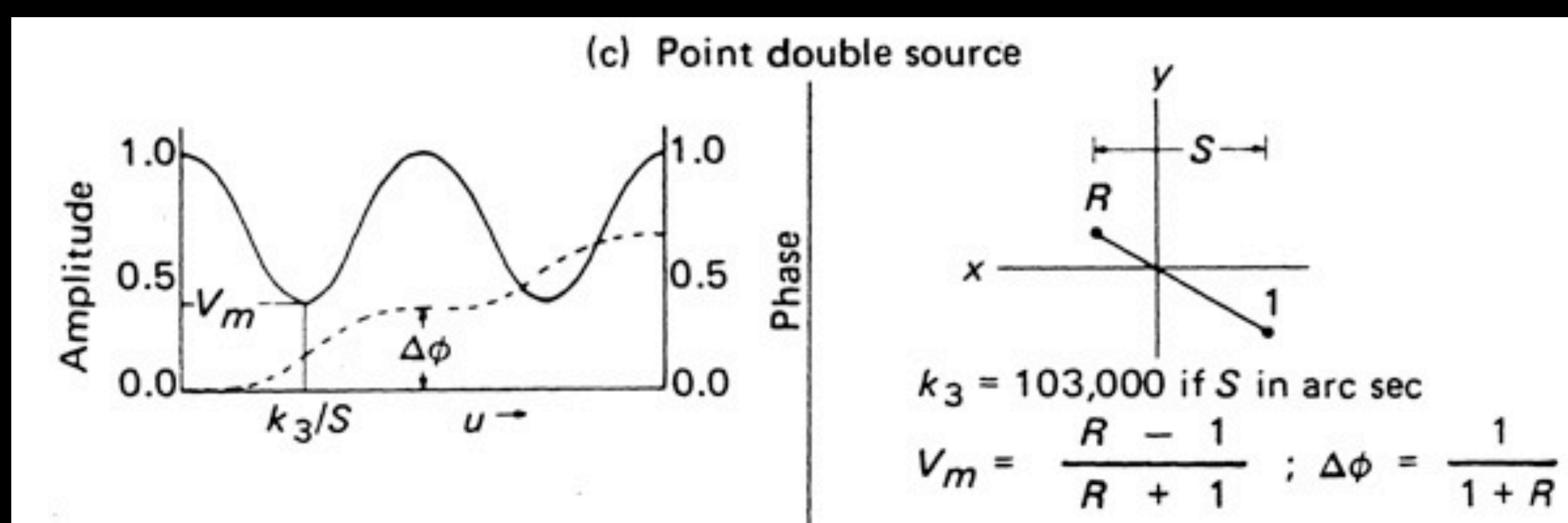

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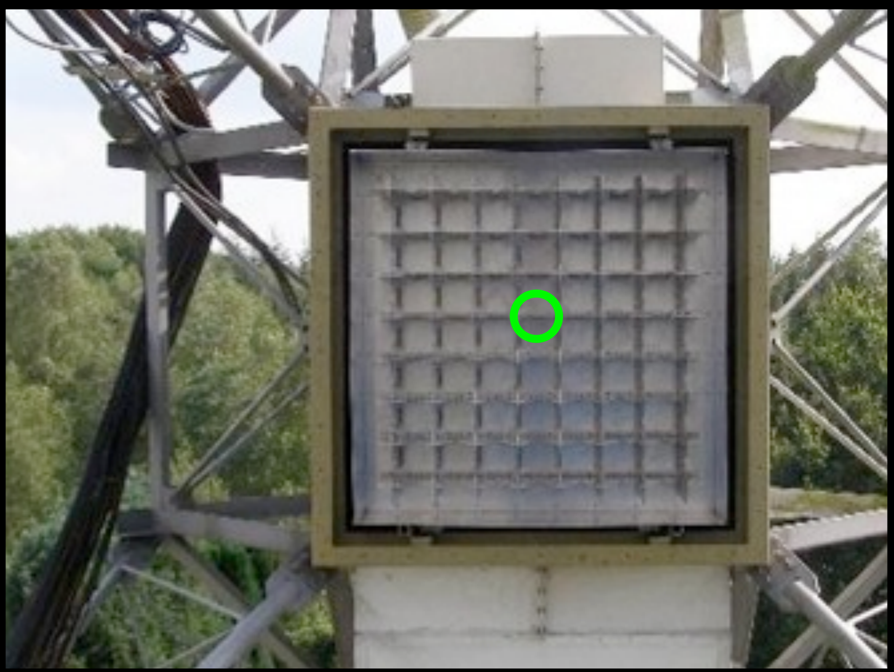

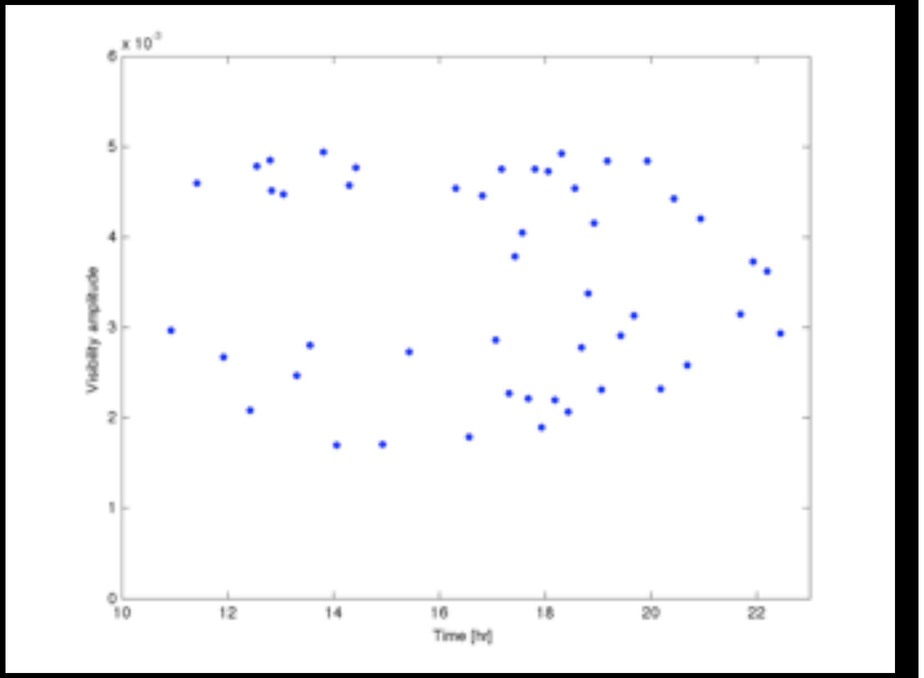

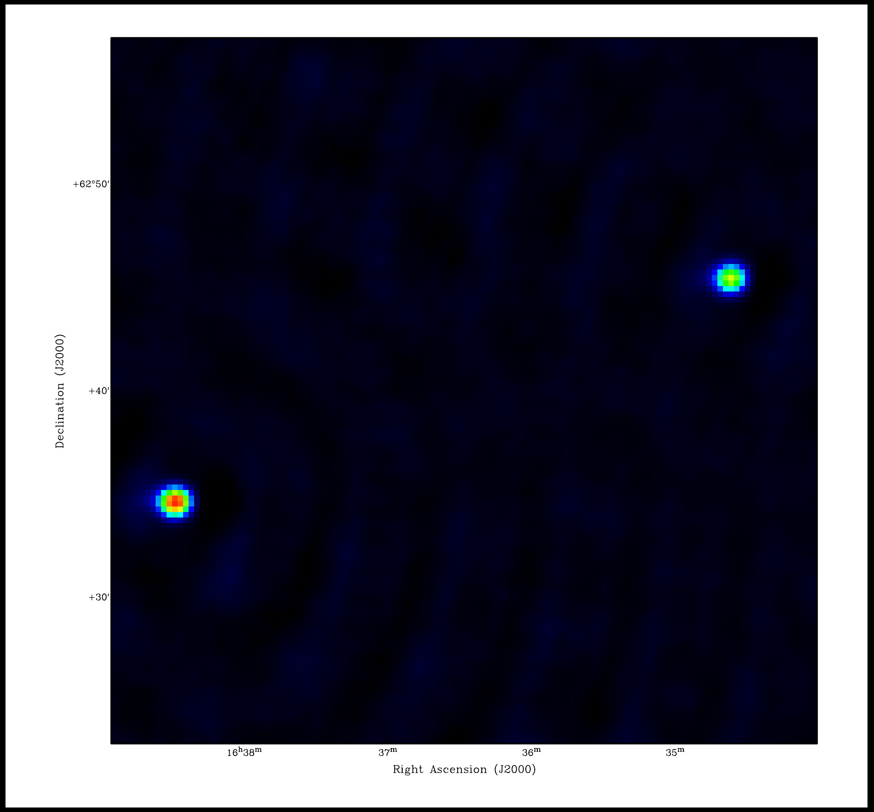

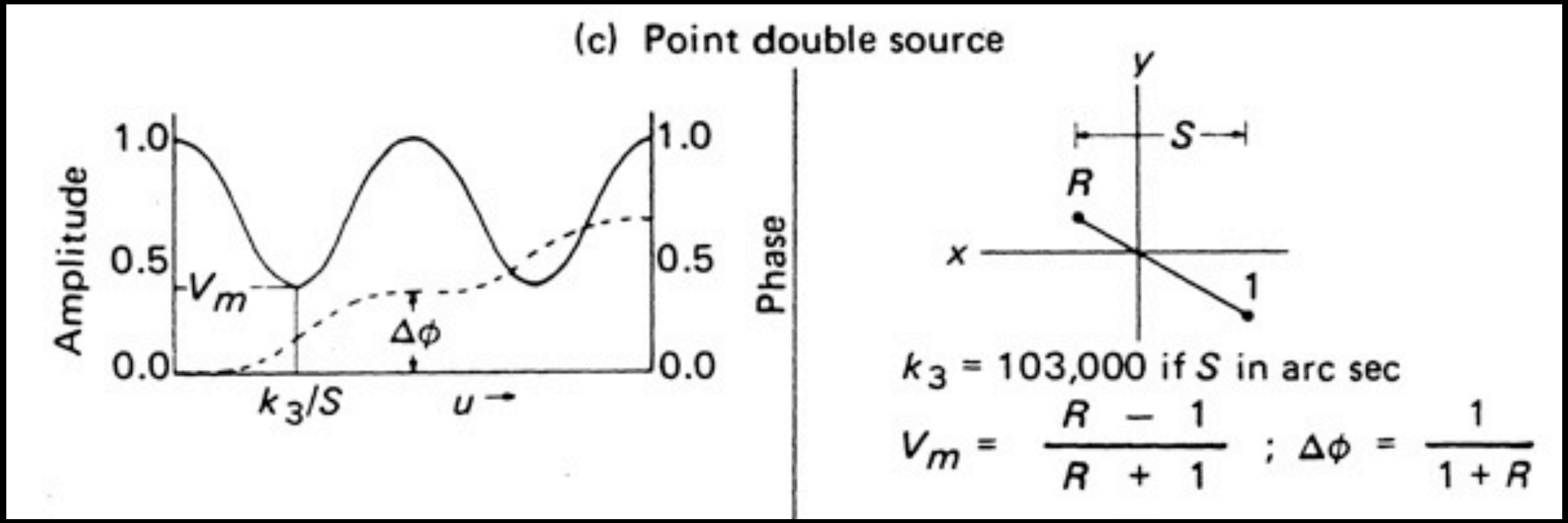

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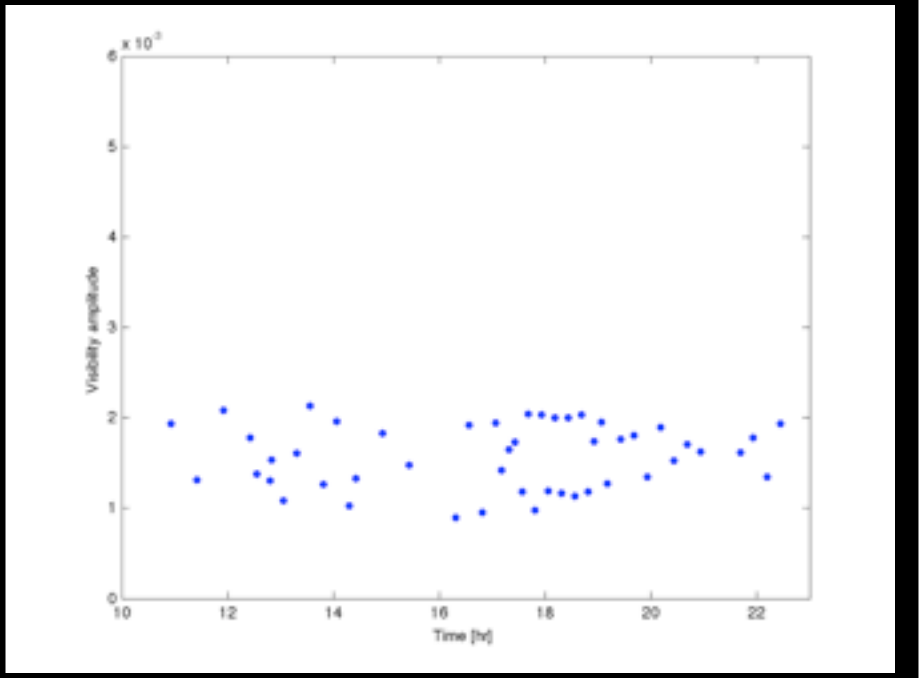
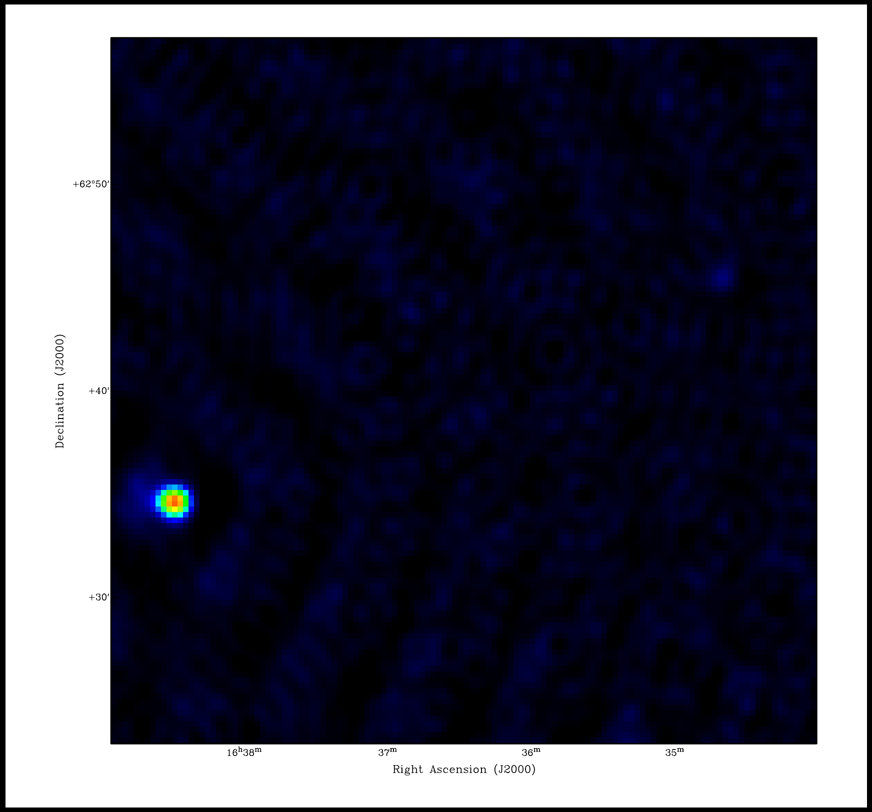
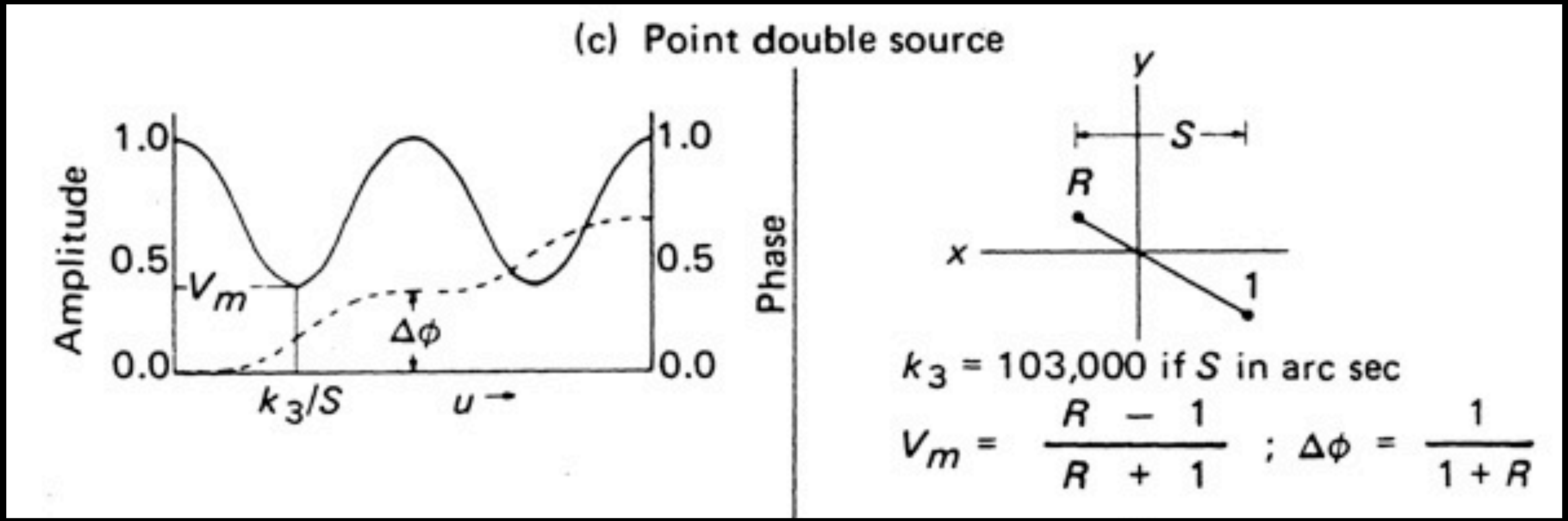


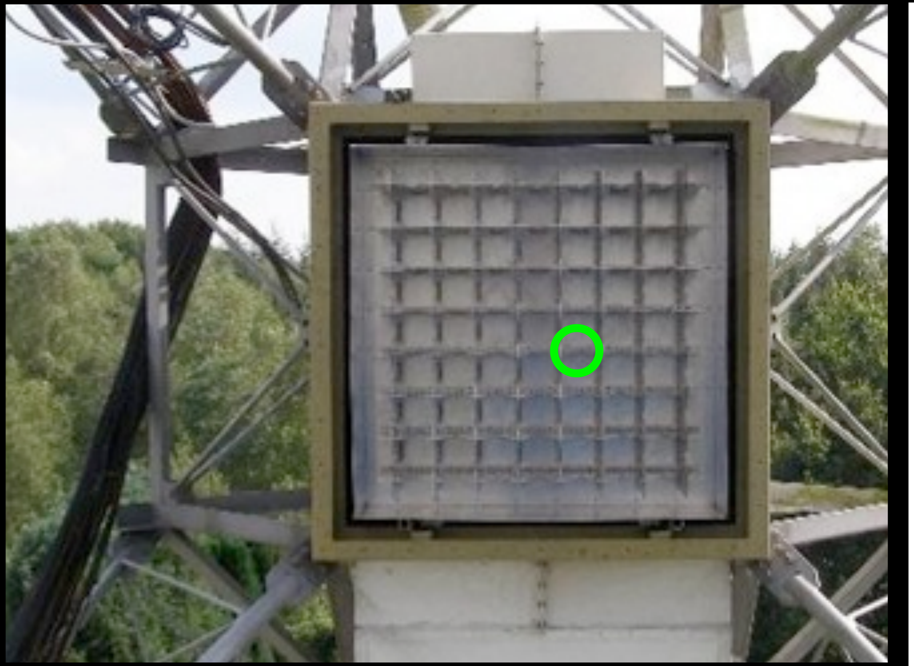


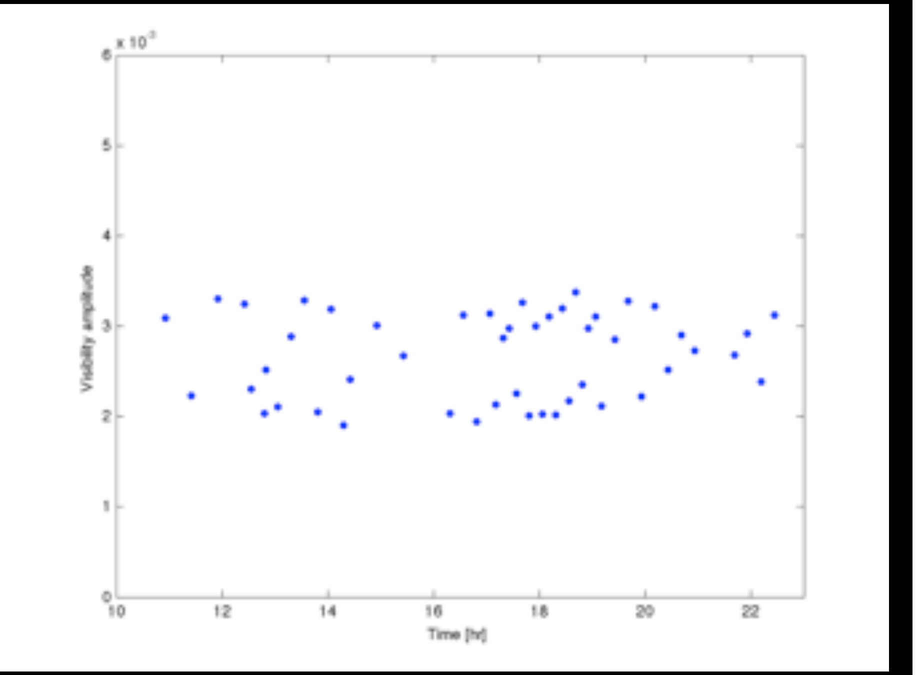


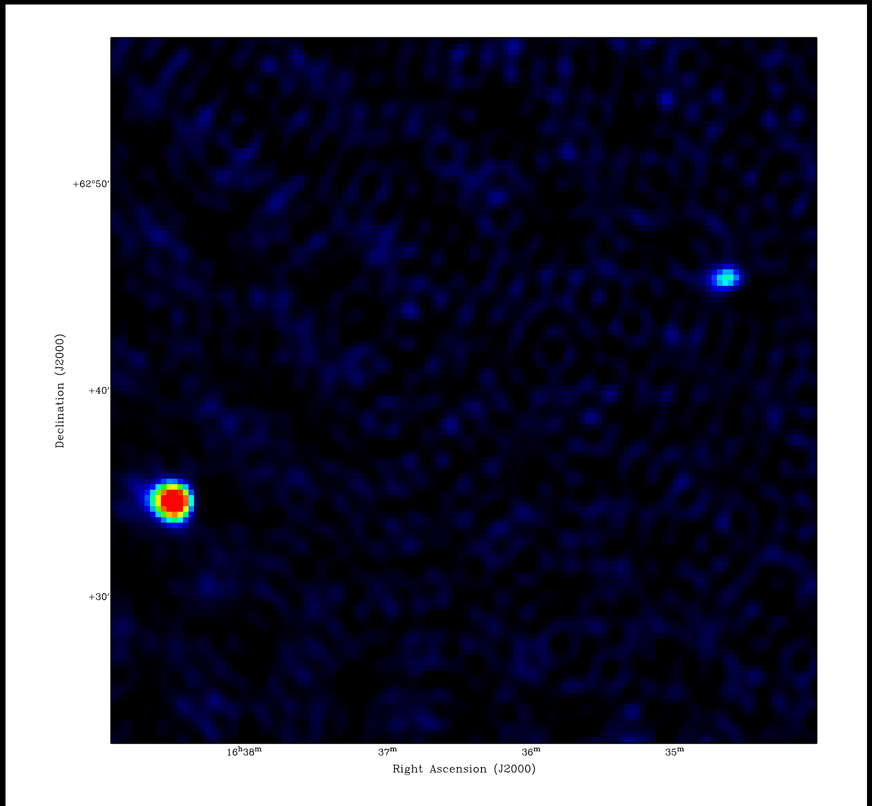
$$V(\vec{b}) = \int\int \sqrt{A'_\nu(\vec{s})}\sqrt{A''_\nu(\vec{s})}I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

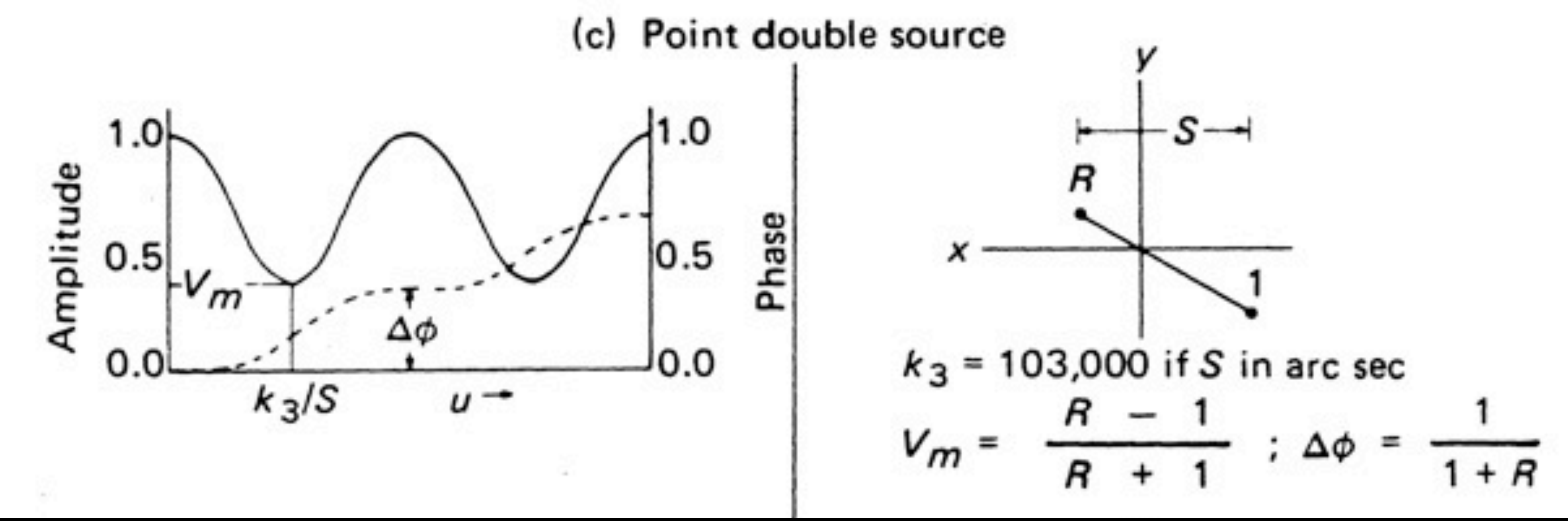
Example: DIGESTIF

- DIGESTIF interferometric observation of 3C343









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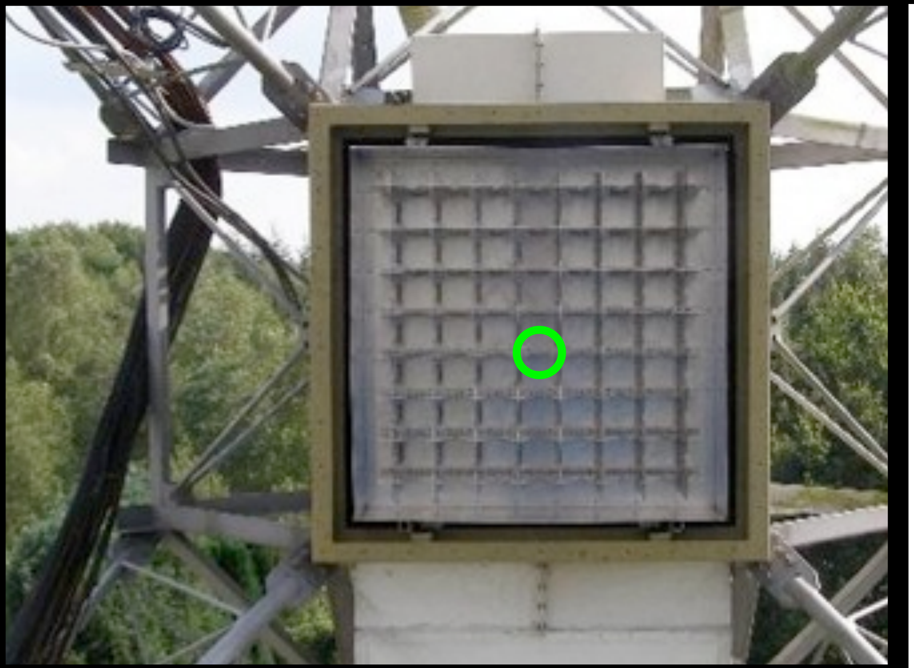

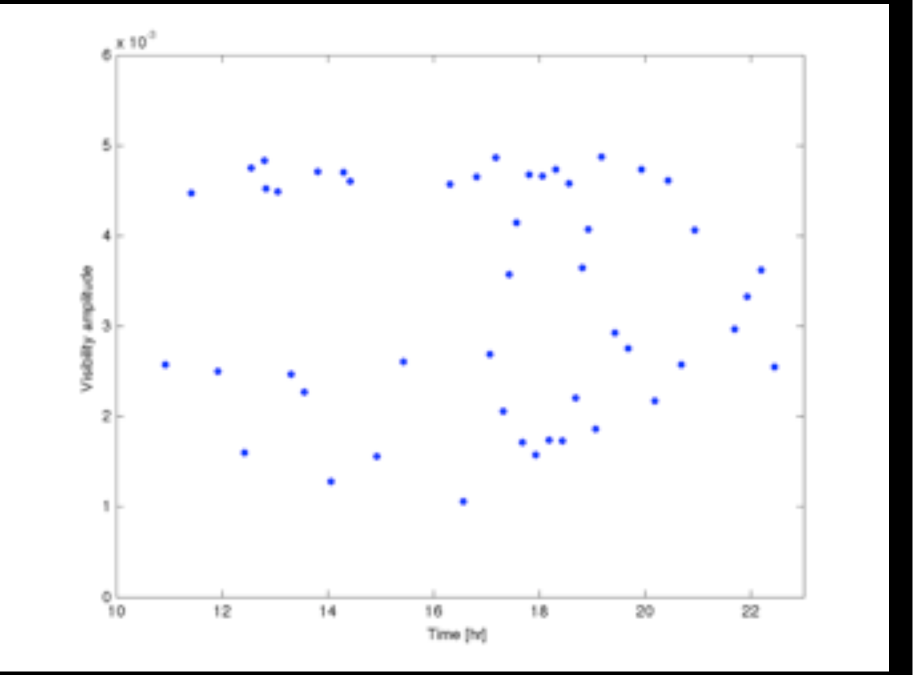

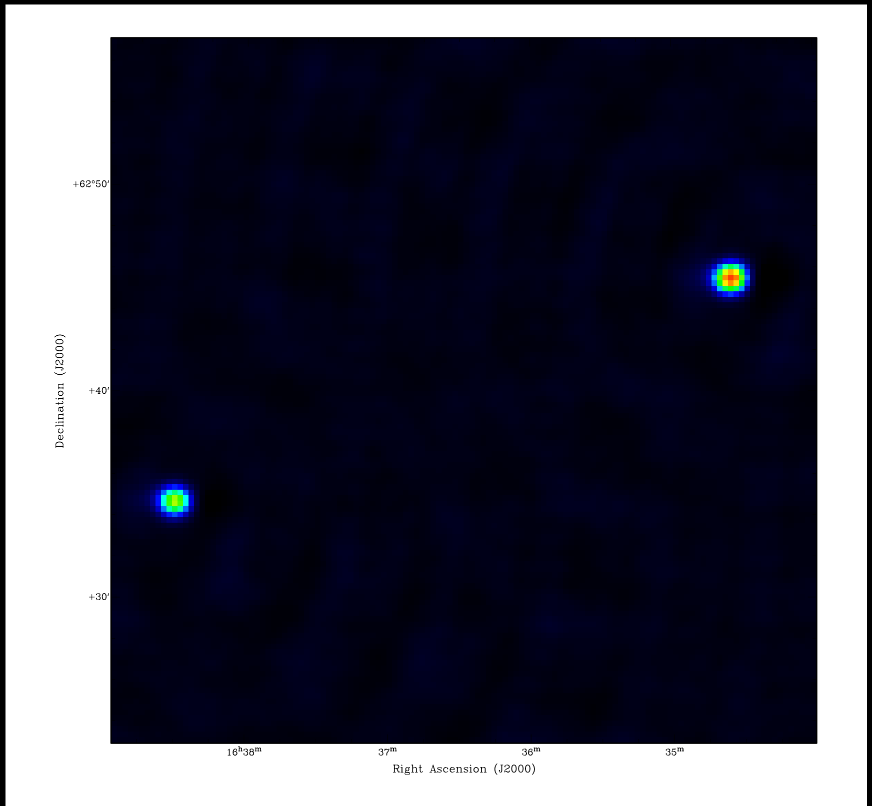

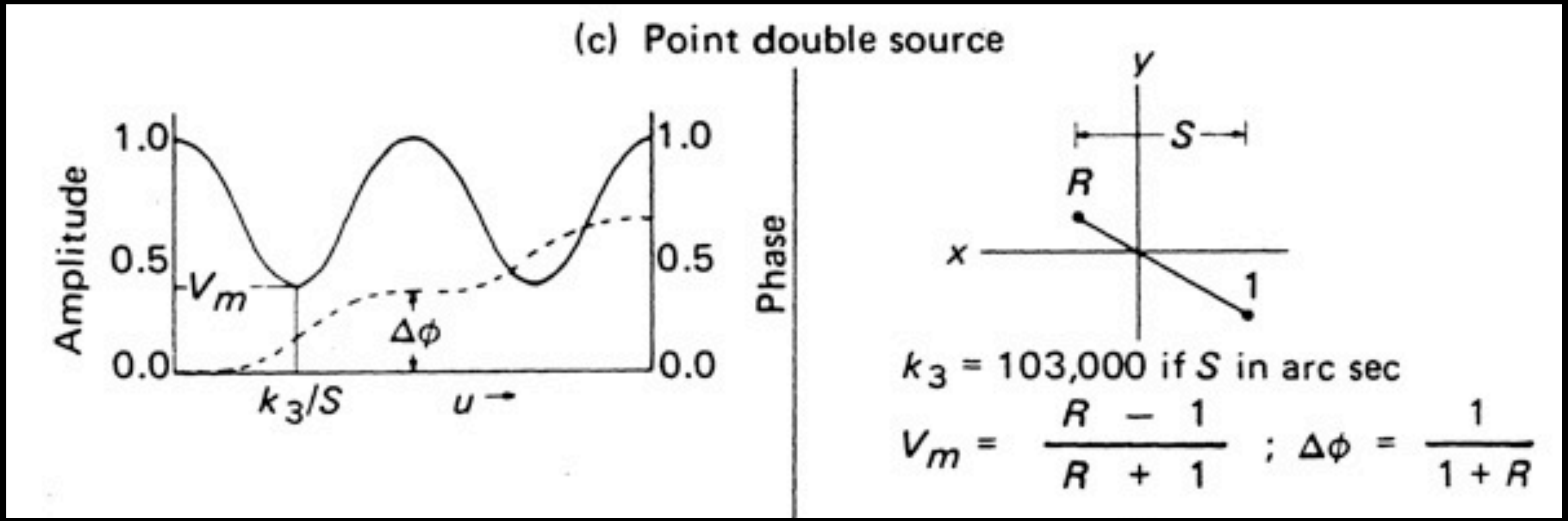

$$V(\vec{b}) = \int\int \sqrt{A'_\nu(\vec{s})}\sqrt{A''_\nu(\vec{s})}I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

Example: DIGESTIF

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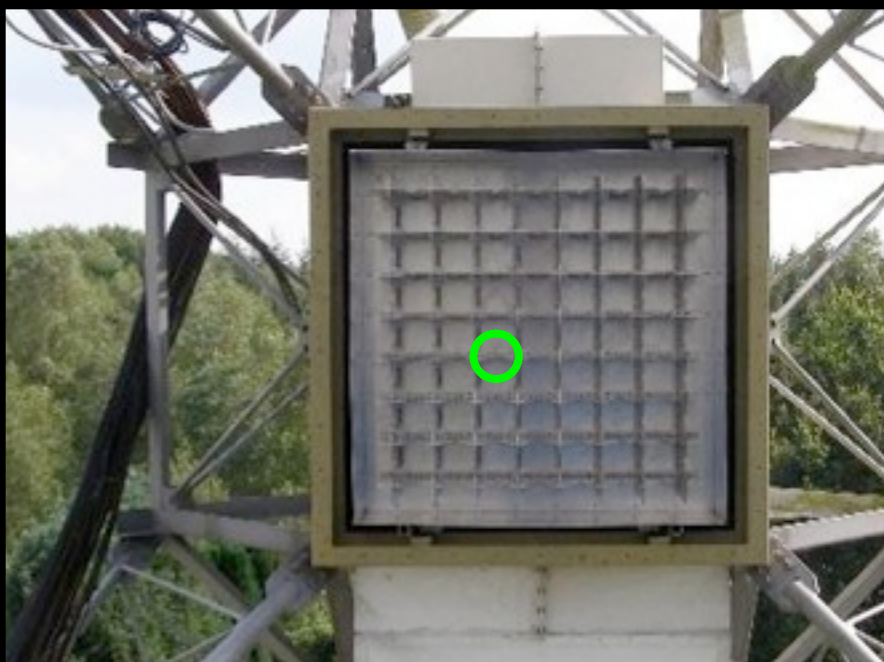
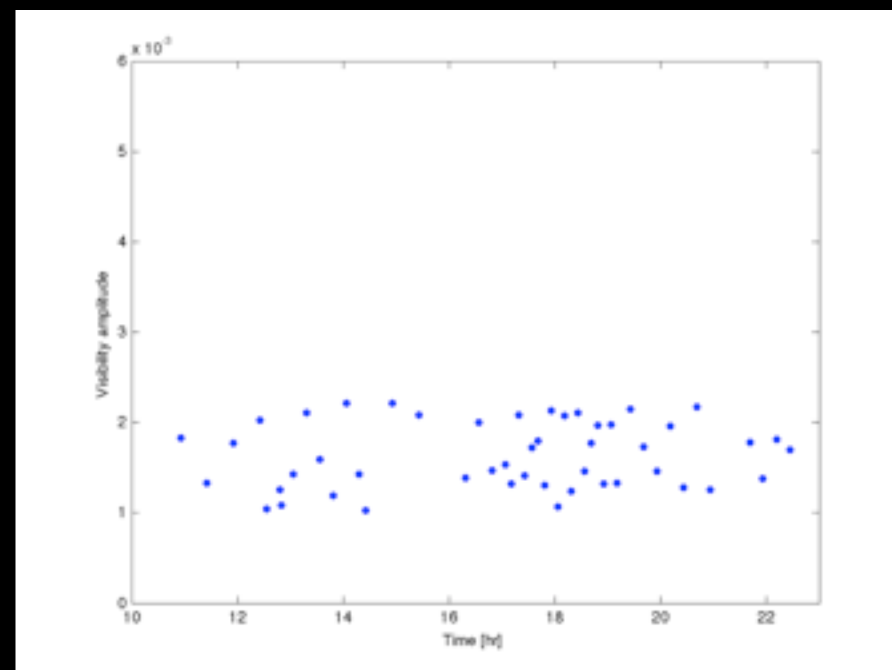
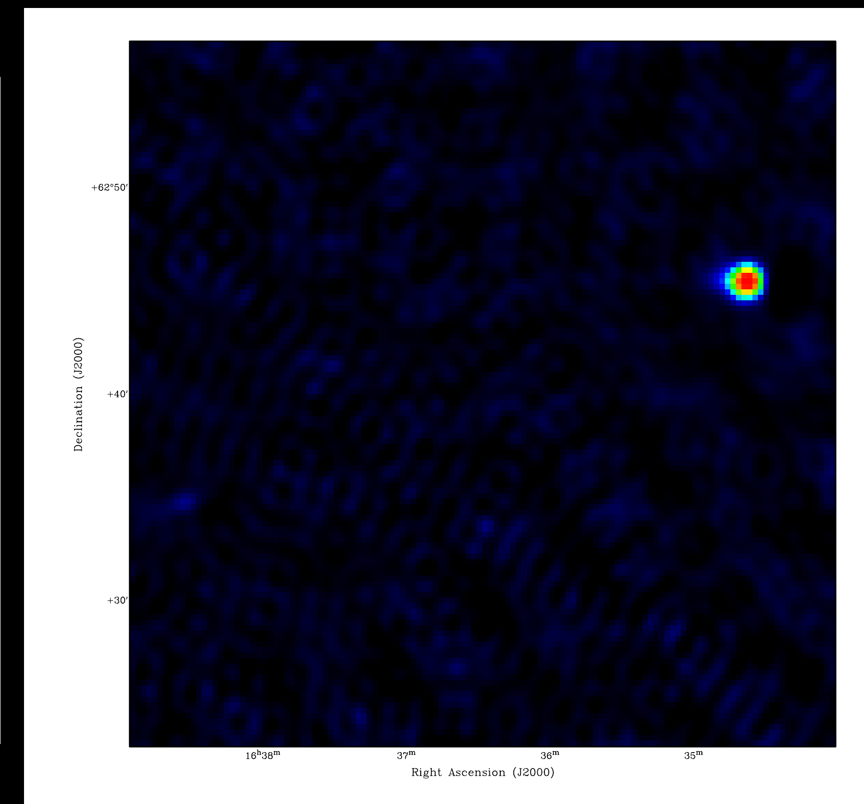
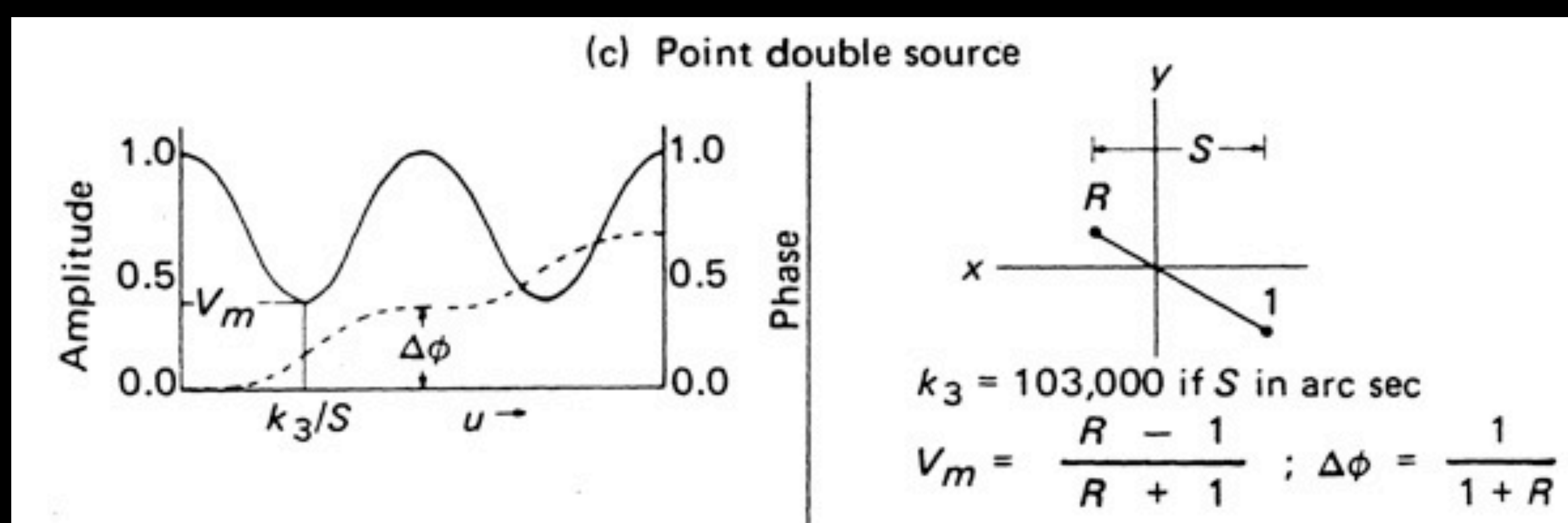




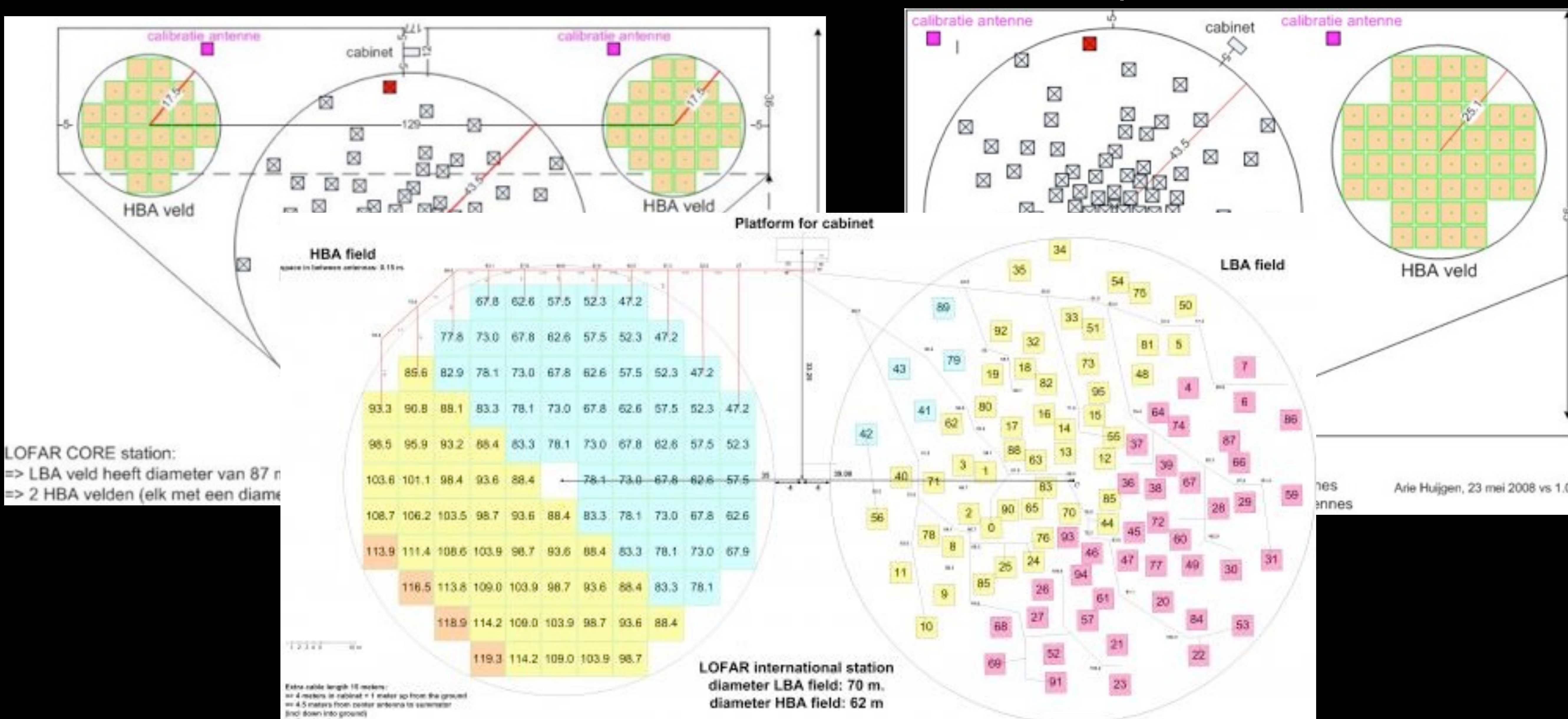




$$V(\vec{b}) = \int \int \sqrt{A'_\nu(\vec{s})}\sqrt{A''_\nu(\vec{s})} I_\nu(\vec{s}) e^{-2\pi i \nu \vec{b}\cdot\vec{s}/c} d\Omega$$

Example: LOFAR

- LOFAR High Band Antenna (HBA) stations have different sizes: core stations are 2x24 tiles, but remote stations are 1x48 tiles. And international stations have 1x96 tiles. This leads to three different antenna patterns!



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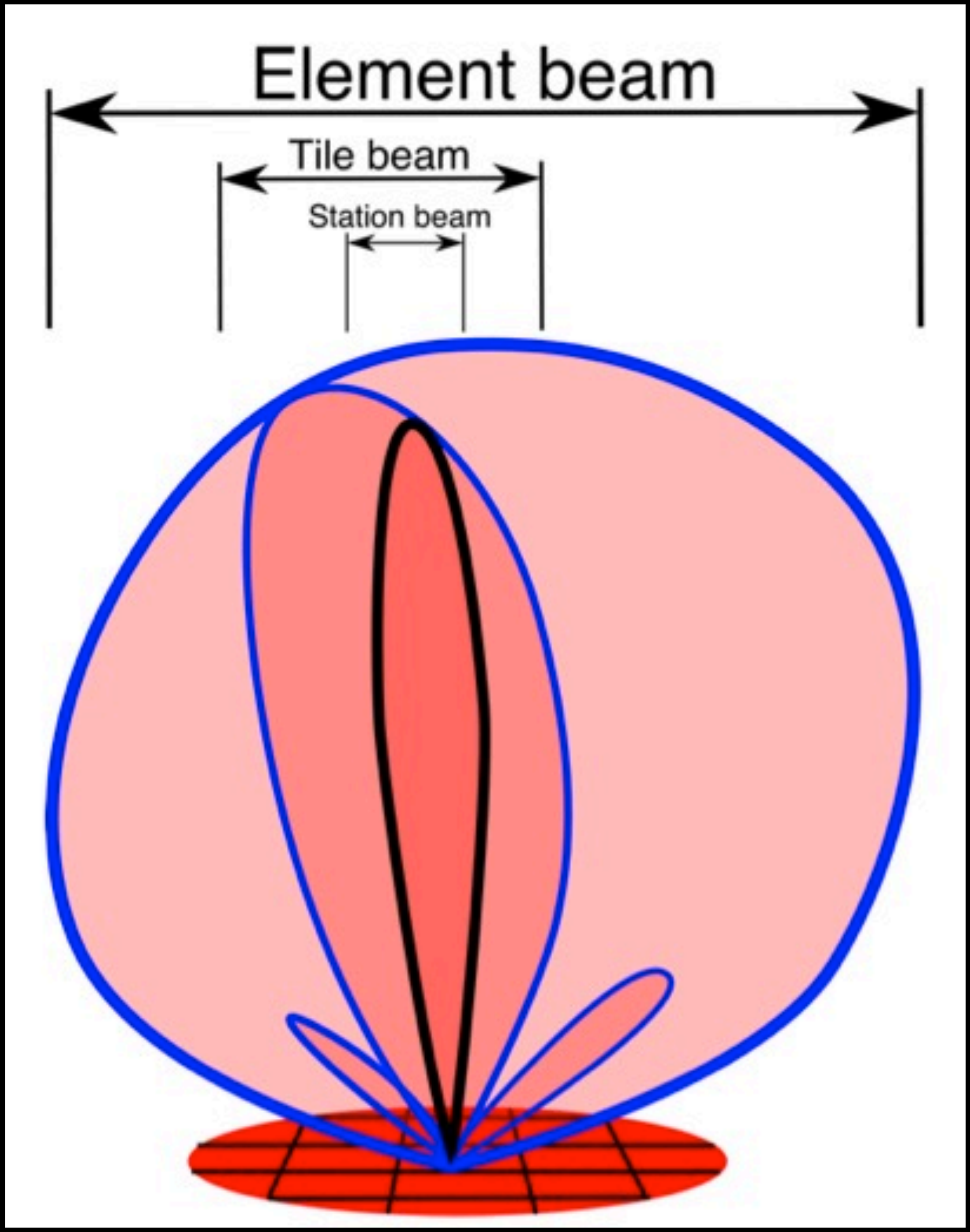
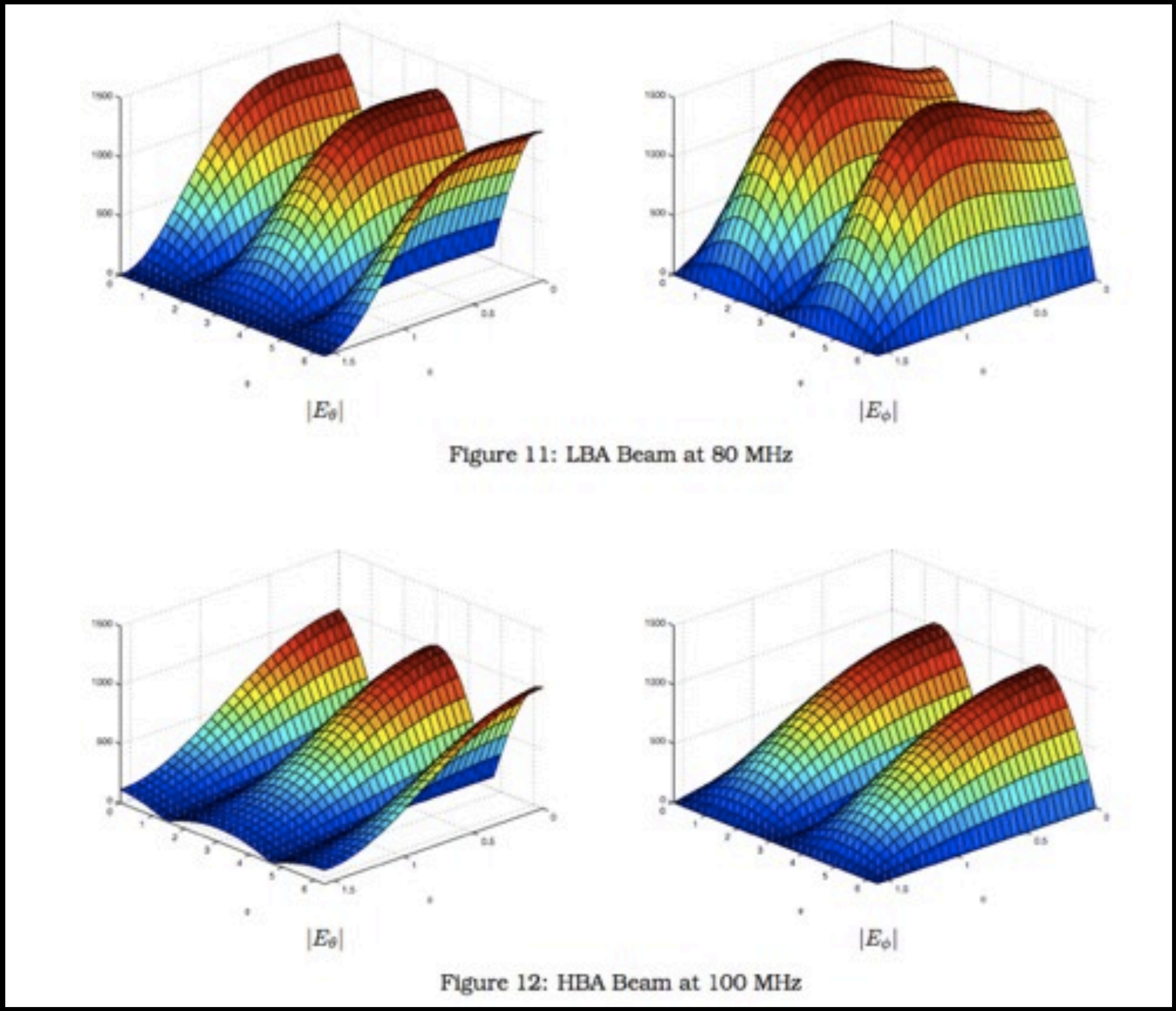
Example: LOFAR

- Antenna response pattern can change the polarization properties of the sky



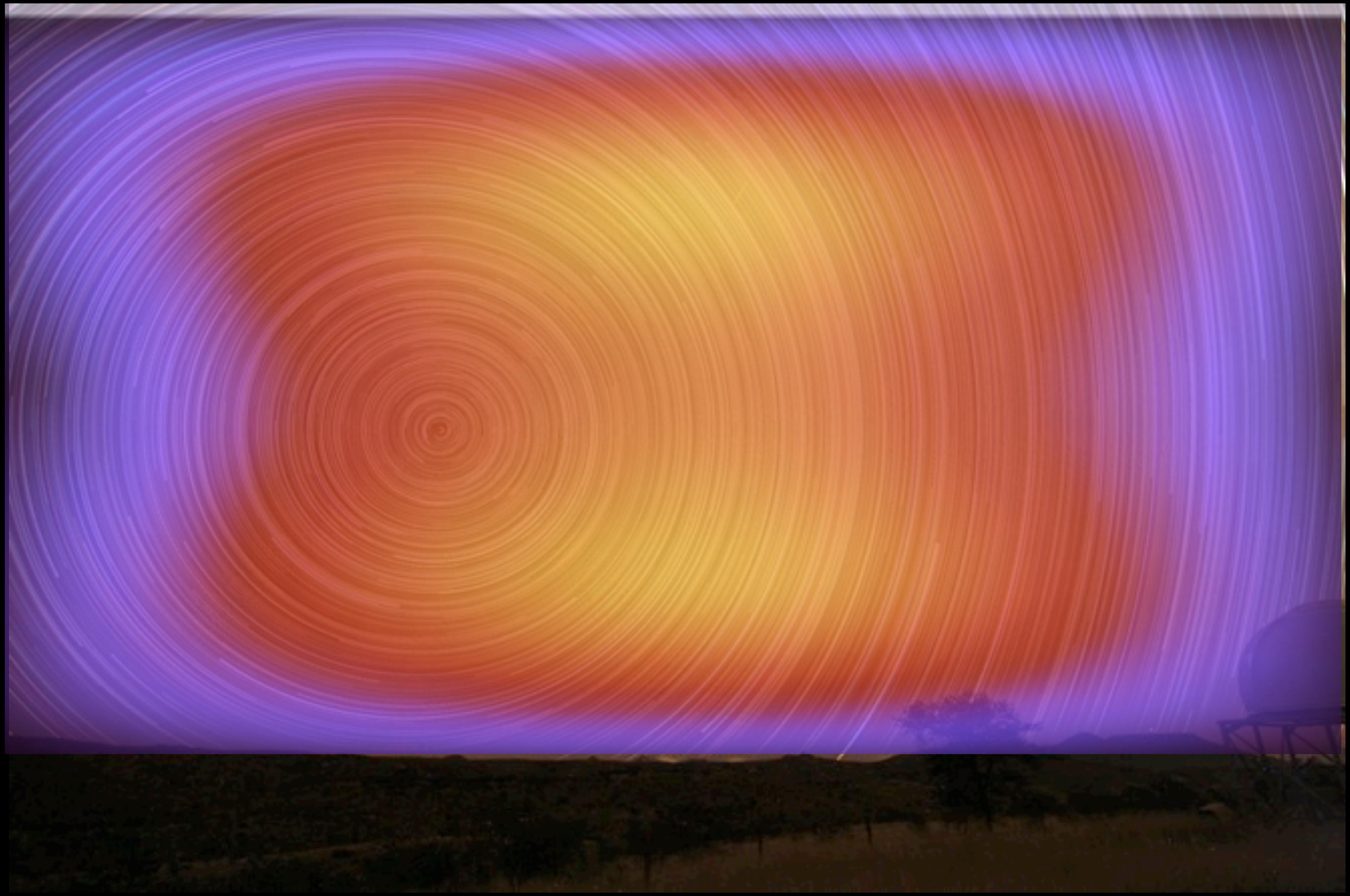
Picture from Michiel Brentjens



LOFAR dipole beams derived by Sarod Yatawatta

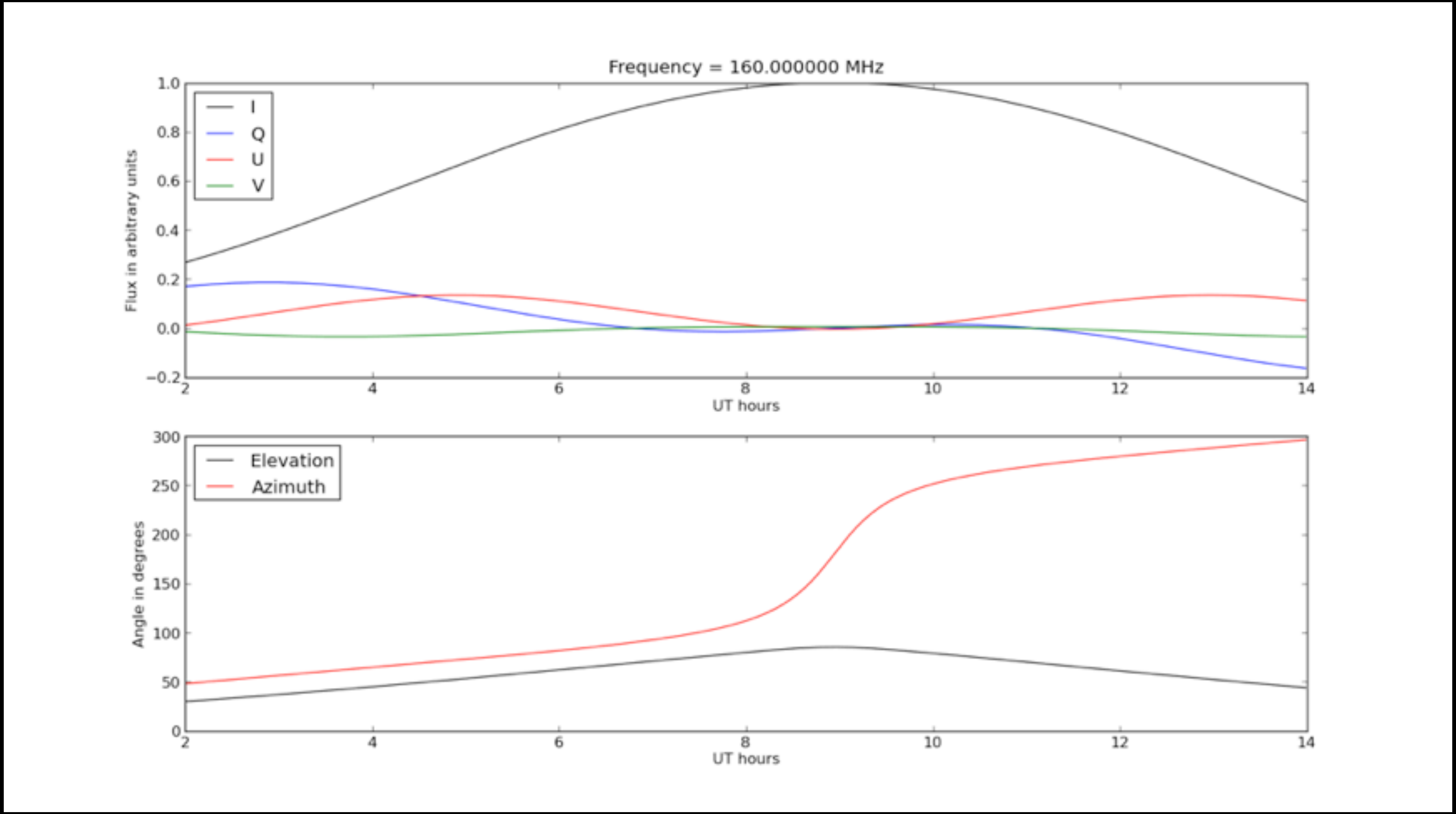
Example: LOFAR

- LOFAR dipoles are fixed on the ground, so their response pattern is fixed on the sky.
- As a source passes through the sky, it sees the dipoles at constantly changing projections - the X and Y responses change with time.



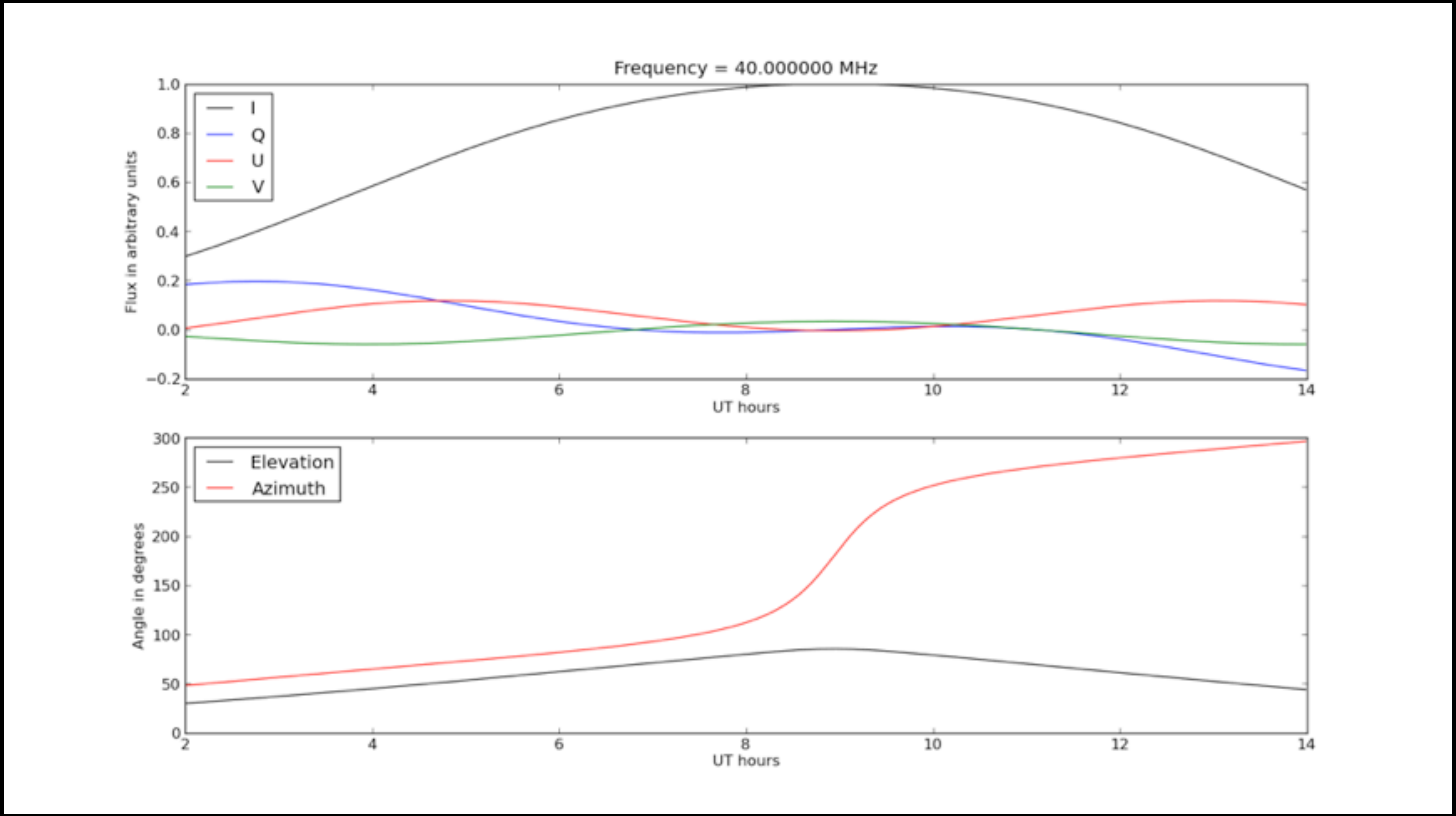
Example: LOFAR

- Simulation: unpolarized source (3C196), observed from the LOFAR core today (HBA @ 160 MHz)



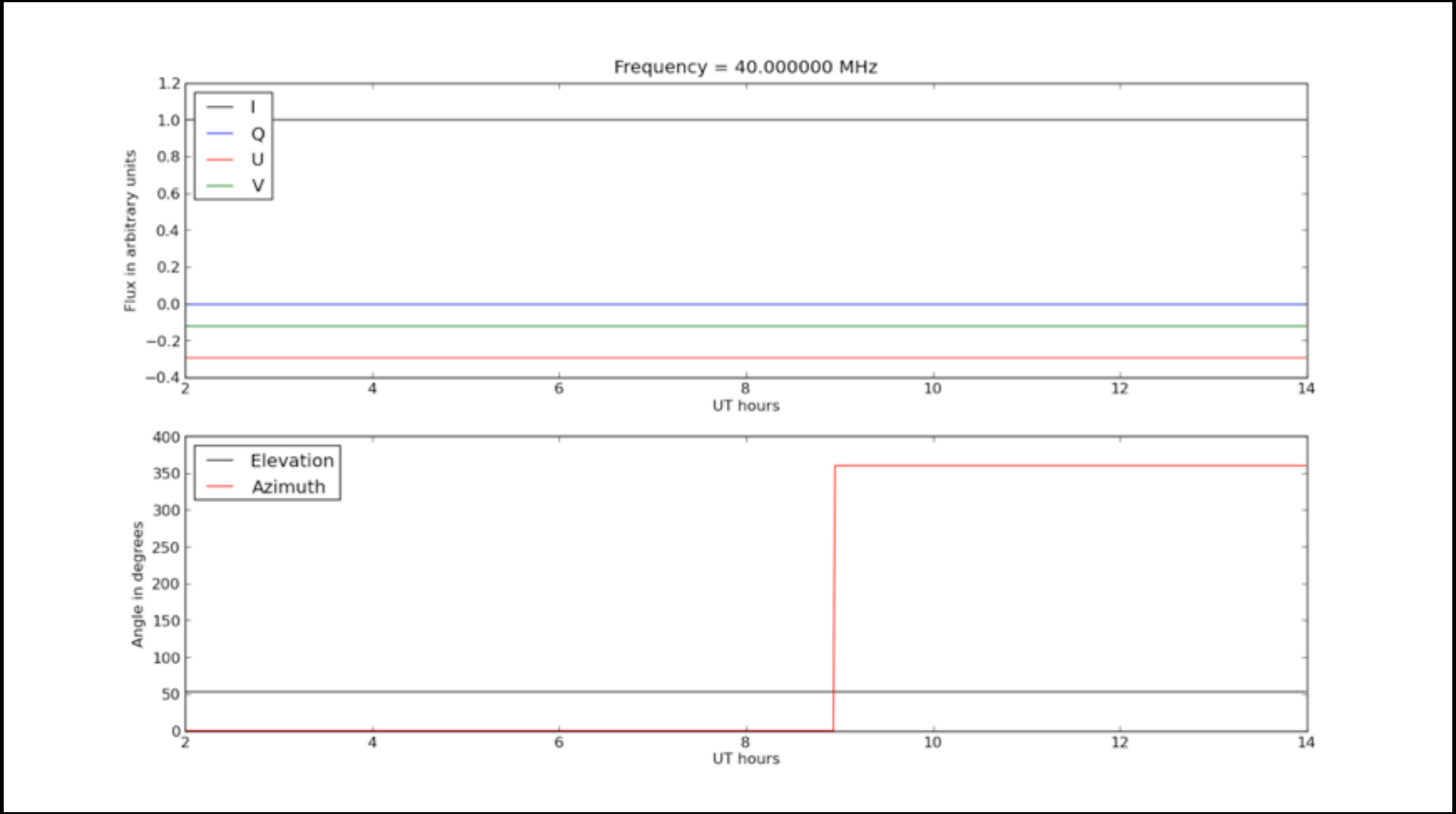
Example: LOFAR

- Simulation: unpolarized source (3C196), observed from the LOFAR core today (LBA @ 40 MHz)



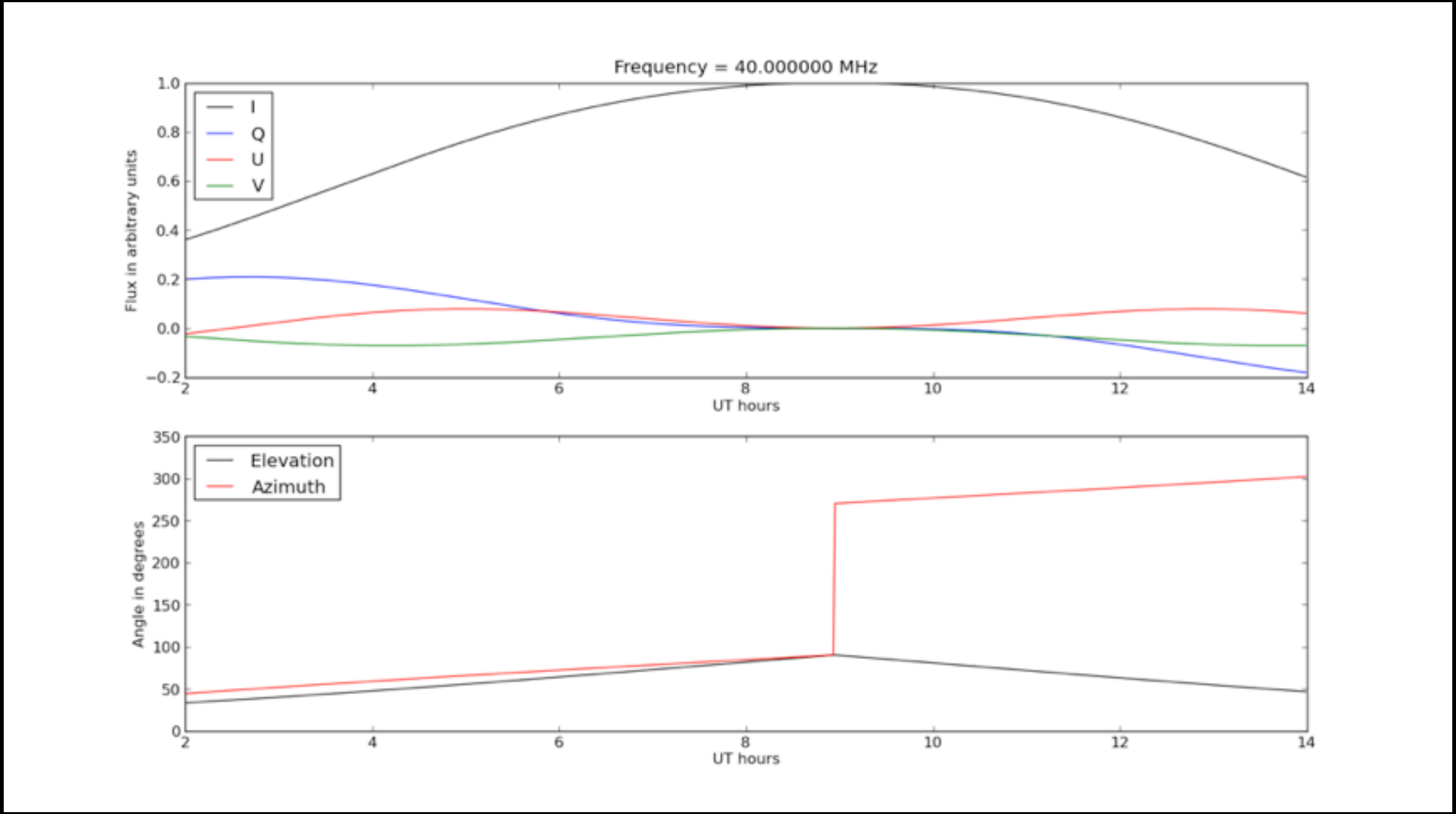
Example: LOFAR

- Simulation: unpolarized source (3C196) moved to dec=90, observed from the LOFAR core today (LBA @ 40 MHz)



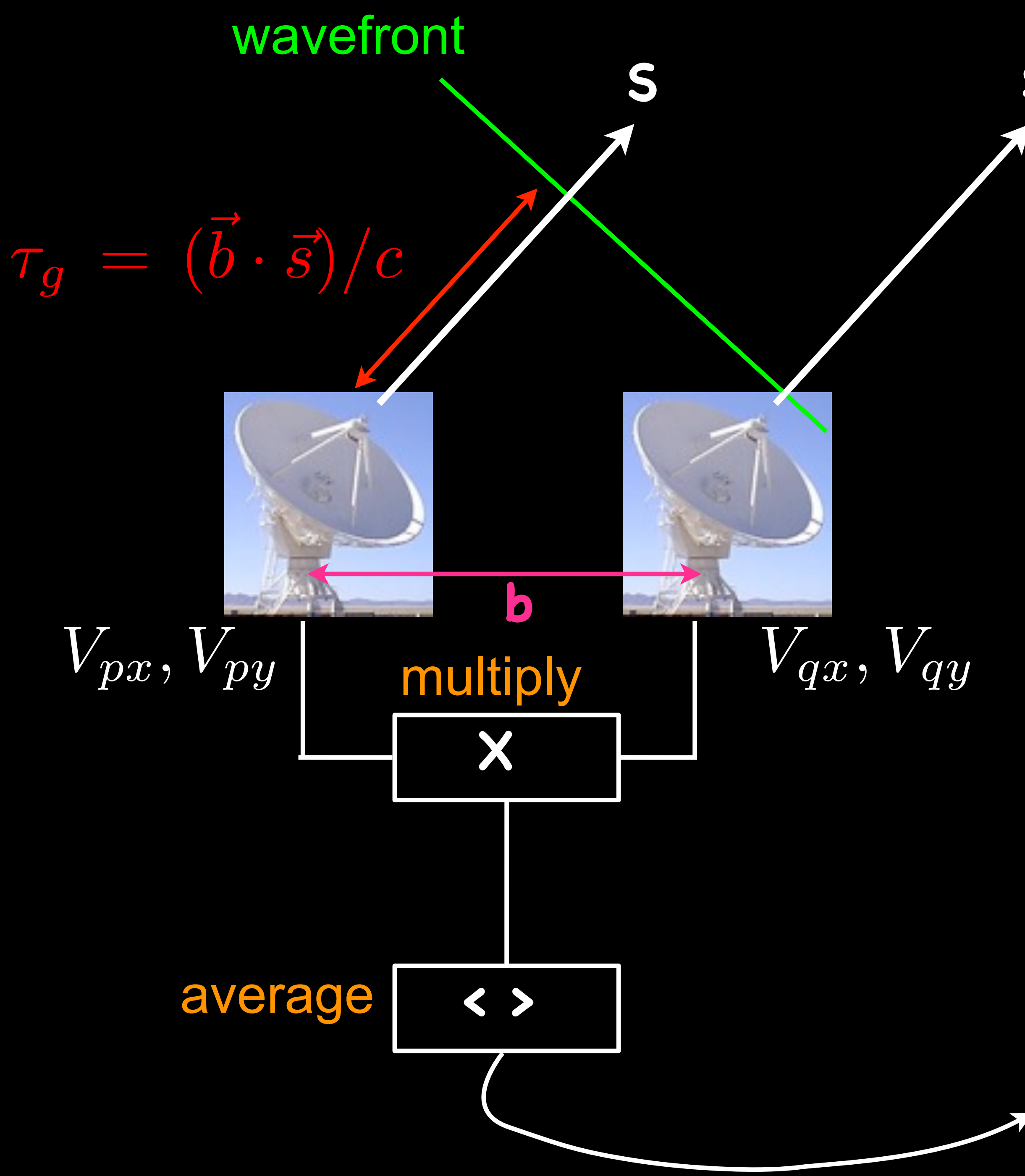
Example: LOFAR

- Simulation: unpolarized source (3C196) moved to dec = latitude, observed from the LOFAR core today (LBA @ 40 MHz)



The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a Müller-matrix formalism for expressing the response of an interferometer



$$V_{xx} = < V_{px} V_{qx}^* >$$

$$V_{xy} = < V_{px} V_{qy}^* >$$

$$V_{yx} = < V_{py} V_{qx}^* >$$

$$V_{yy} = < V_{py} V_{qy}^* >$$

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a Müller-matrix formalism for expressing the response of an interferometer

$$\boxed{\begin{aligned} V_{xx} &= < V_{px} V_{qx}^* > \\ V_{xy} &= < V_{px} V_{qy}^* > \\ V_{yx} &= < V_{py} V_{qx}^* > \\ V_{yy} &= < V_{py} V_{qy}^* > \end{aligned}}$$

can be more easily and elegantly expressed in matrix form, as:

$$\mathbf{V}_{pq} = \langle \vec{V}_p \vec{V}_q^{\dagger} \rangle = \langle \begin{pmatrix} V_{px} \\ V_{py} \end{pmatrix} \begin{pmatrix} V_{qx}^* & V_{qy}^* \end{pmatrix} \rangle = \begin{pmatrix} V_{xx} & V_{xy} \\ V_{yx} & V_{yy} \end{pmatrix}.$$

The Measurement Equation

- The measurement equation (Hamaker, Bregman & Sault) is a Müller-matrix formalism for expressing the response of an interferometer
- Introducing the coherency matrix C, which describes the intensity distribution:

$$\mathbf{C} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} = \langle \vec{E}\vec{E}^{\dagger} \rangle$$

- and the "Jones matrix" J which contains all of the information about what happens to (corrupts) the signal, from the source to the correlator,

$$\mathbf{V}_p = \mathbf{J}_p \vec{E} \qquad \mathbf{V}_q = \mathbf{J}_q \vec{E}$$

- then with a bit of math we can write down the measurement equation:

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{C} \mathbf{J}_q^{\dagger}$$

The Measurement Equation

- The Jones matrices contain all of the stuff that we have to calibrate

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{C} \mathbf{J}_q^\dagger$$

- For example,
  - G: the (complex) antenna gain
  - B: bandpass
  - F: Faraday rotation
  - E: antenna response pattern

$$\mathbf{V}_{pq} = \mathbf{G}_p \mathbf{B}_p \mathbf{E}_p \mathbf{F}_p \mathbf{C} \mathbf{F}_q^\dagger \mathbf{E}_q^\dagger \mathbf{B}_q^\dagger \mathbf{G}_q^\dagger$$

- "Calibration" is the process of determining the values of G,B,F,E in this case

The Measurement Equation

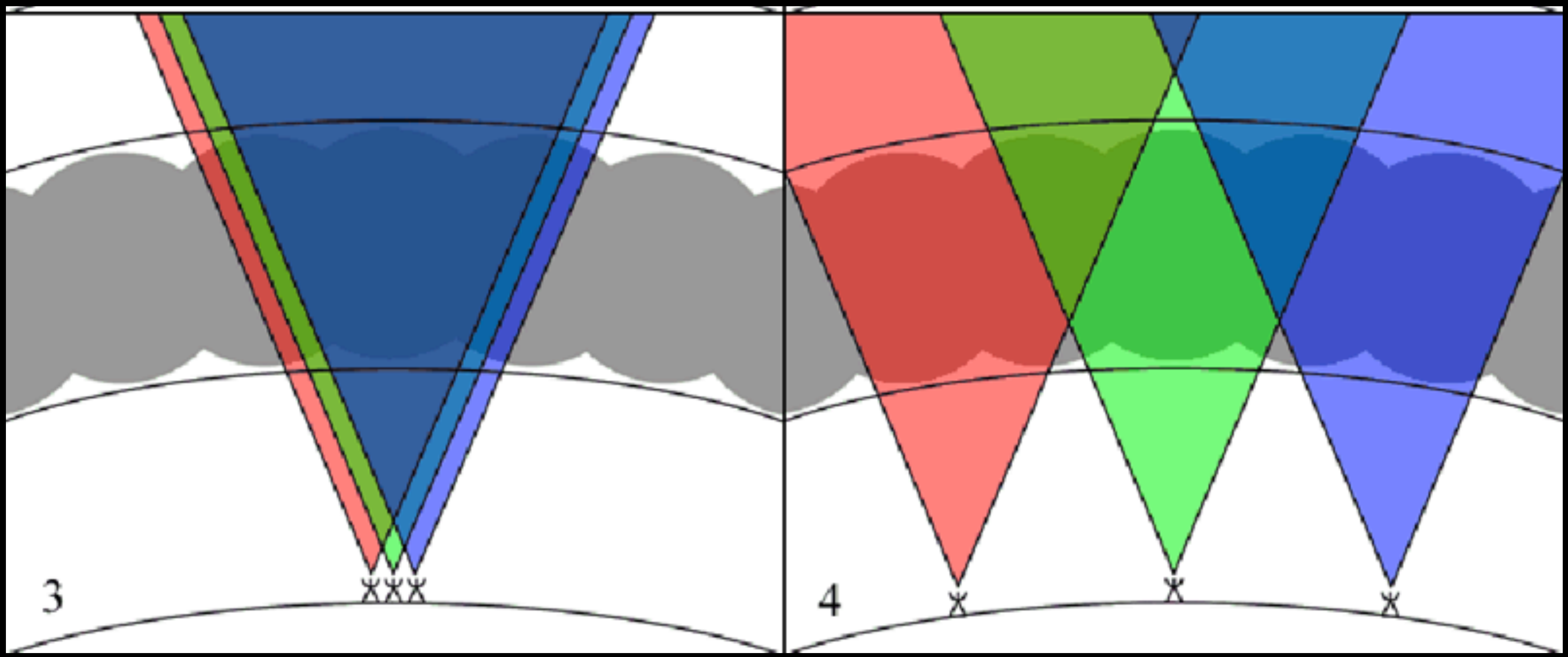
- So what?

$$\mathbf{V}_{pq} = \mathbf{J}_p \mathbf{C} \mathbf{J}_q^\dagger$$

- The measurement equation makes it more straightforward to handle polarization calibration, and direction dependent effects
- Explicit separation of dependencies (for example, G(t) and B(v) are the antenna gain and bandpass)
- Arguably necessary in order to understand LOFAR calibration!

Direction dependence

- A major complication for low frequency radio synthesis is that the Jones matrices are direction dependent



LOFAR core stations

LOFAR remote/intl stations

Sparse uv plane sampling

- In practice, V is only sampled in particular locations S(u,v) in the uv plane, so instead of

  $$I_\nu(l,m) = \int\int V_\nu(u,v)\, e^{2i\pi(ul+vm)}\, du\, dv$$

  we get a "corrupted" or "dirty" image:

  $$I_\nu^D(l,m) = \int\int S(u,v) V_\nu(u,v)\, e^{2i\pi(ul+vm)}\, du\, dv$$

  This amounts to

  $$I_\nu^D = I_\nu \star B_\nu$$

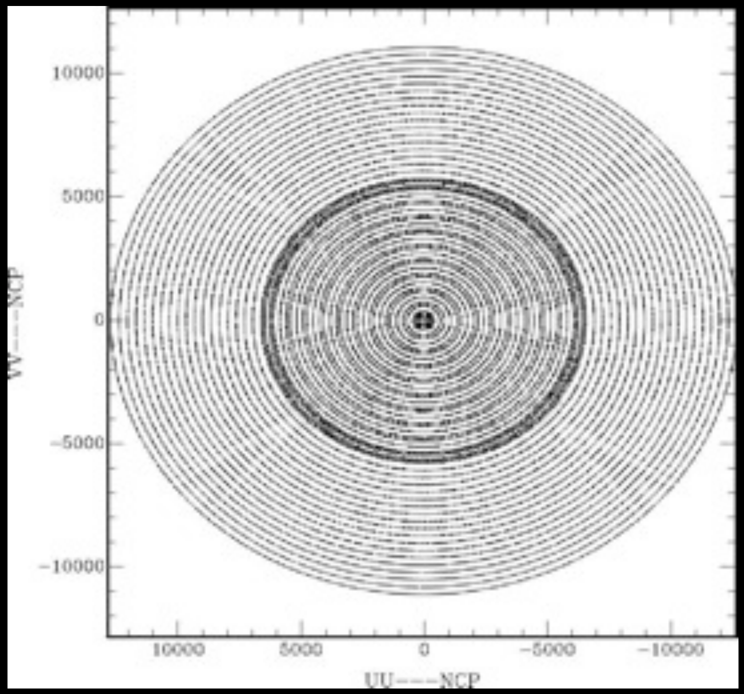
  where we have defined the "dirty beam"

  $$B(l,m) = \int\int S(u,v)\, e^{2i\pi(ul+vm)}\, du\, dv$$

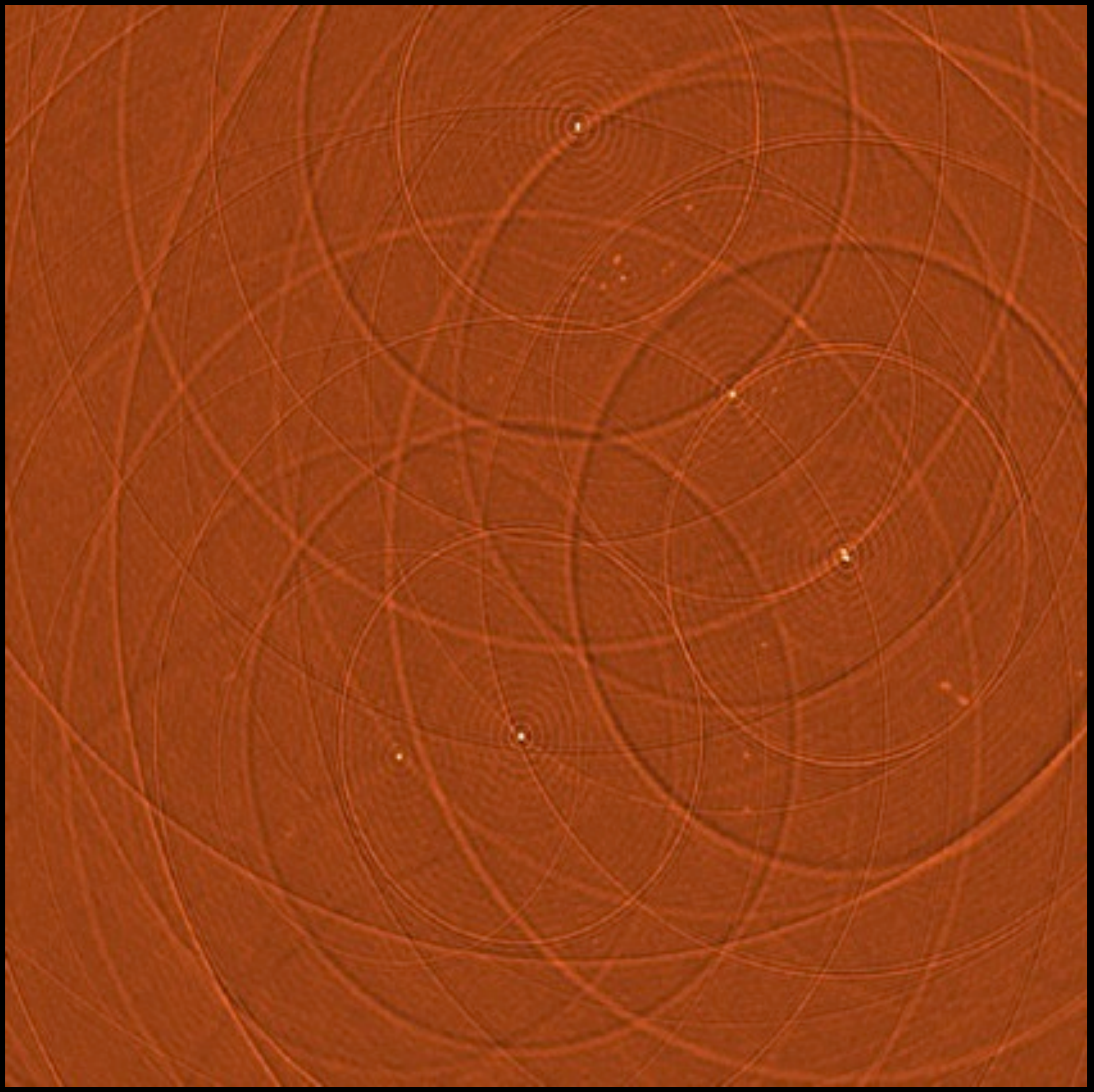
  In other words, we get a dirty image I^D, which is the source brightness distribution I convolved with the dirty beam B (the FT of uv sampling).
- To get nice images, we need to deconvolve the dirty images!

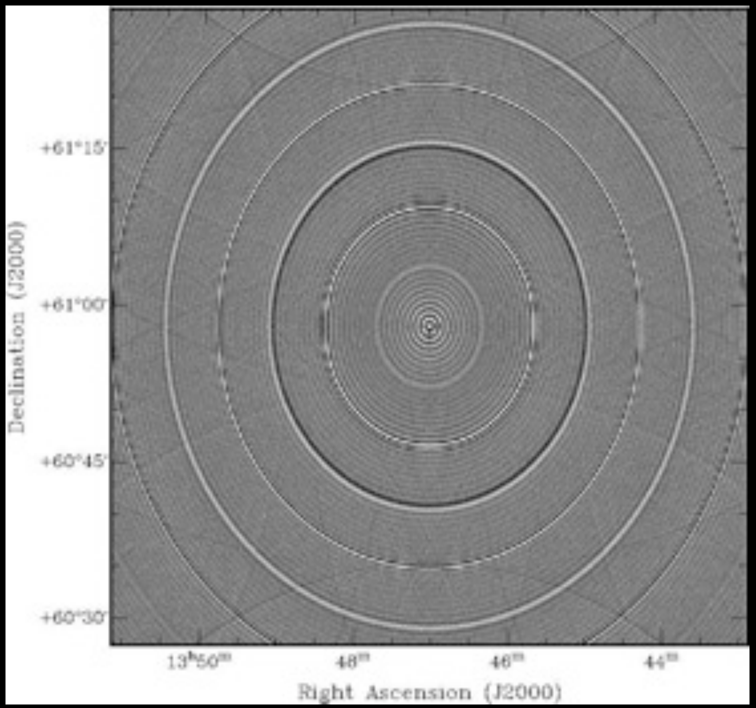
Examples of the dirty beam

- Here is what a dirty beam looks like for the WSRT...



WSRT uv sampling



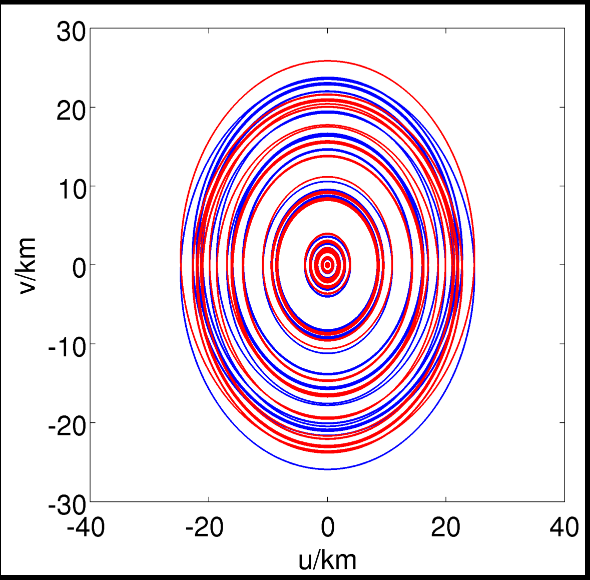


Dirty beam
(size of rings determined by antenna spacings)

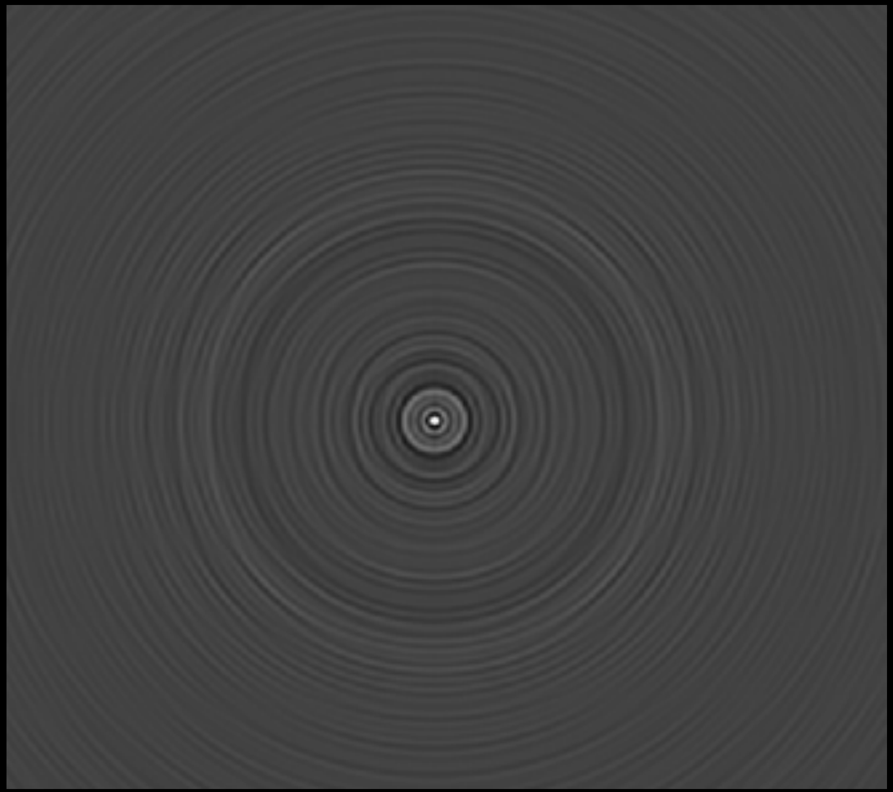
Dirty image

Examples of the dirty beam

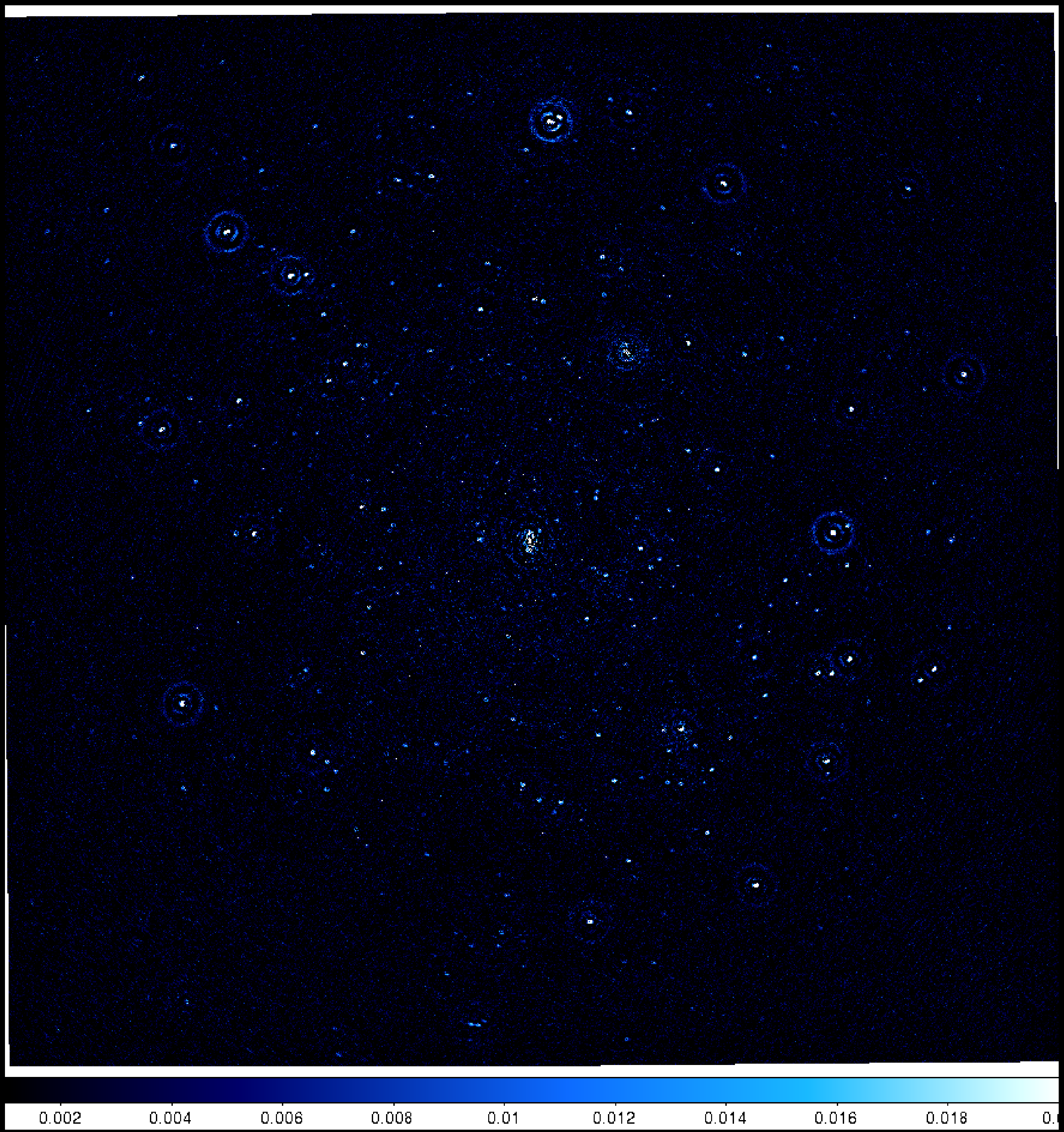
- ...and for LOFAR (field surrounding 3C61.1, image by Sarod Yatawatta)



LOFAR uv coverage



dirty beam



Deconvolution

- What we can assume about the sky emission:
  1. The sky does not look like cosine waves
  2. The sky brightness is usually positive (not Q,U,V and absorption lines)
  3. The sky is usually a collection of point sources (weak assertion)
  4. The sky could be smooth
  5. The sky is mostly blank (sometimes justifies “boxed” deconvolution)
- Non-linear deconvolution algorithms search for a model image I^M such that the residual visibilities $V^R=V^D-V^M$ are minimized, subject to the constraints given by the (assumed) prior knowledge.

Classic CLEAN algorithm

- Prior knowledge:
  - sky is composed of point sources
  - mostly blank
- Algorithm:
  1. Search for the peak in the dirty image
  2. Add a fraction g (loop gain) of the peak value to I^M (add delta function)
  3. Subtract a scaled version of the PSF (dirty beam) from the position of the peak in the dirty image

$$I^R_{i+1} = I^R_i - [g \cdot B \cdot \max(I^R_i)]$$

  4. If residuals are not "noise-like" then goto step 1
  5. Smooth I^M by an estimate of the main lobe of the dirty beam (this is called the clean beam) and add the residuals to make the "restored image".

More about CLEAN

- Model image is a collection of delta functions - a scale insensitive algorithm
- Stabilized by keeping a small loop gain (usually, g=0.1-0.2)
- Stopping criteria: either the maximum number of iterations, or the maximum of the residuals is a multiple of the expected peak noise.
- Search space constrained by user defined windows
- Ignores coupling between pixels (extended emission)

- Other imaging techniques (not addressed here):
  - MEM (Maximum Entropy Method)
  - Multi-resolution CLEAN

CLEAN in action

Key concepts

- Low frequency radio astronomy provides a unique view on the sky
- Aperture synthesis is used to increase angular resolution with small antennas
- Correlation takes place by multiplying and time-averaging antenna voltages
- Each baseline instantaneously measures the visibility function at a single location in the uv plane
- Earth rotation is exploited to fill the uv plane azimuthally, and bandwidth is exploited to fill the uv plane radially
- The visibility function is related to the intensity distribution on the sky via a Fourier transform relation
- The Measurement Equation is a useful tool for understanding the instrumental connection between visibilities and the sky brightness
- Aperture synthesis at low frequencies involves extra complications
- Sparse uv sampling causes image artifacts, which can be corrected (to a certain extent) with deconvolution routines