3.3 Normalizing the surface brightness profile I (theory)

According to bremsstrahlung emission model, the emissivity and thus surface brightness (I = flux / area) is proportional to the product of electron and proton number density, n_e n_p. At typical cluster temperatures and metal abundances, n_e / n_p = 1.17 and the gas density ρ_g = 1.35 m_p n_p (see e.g. Vikhlinin et al, 1999, ApJ, 525, 47). We concentrate here first on the electron density n_e.

In general, the conversion of the 2D surface brightness profile into 3D gas density profile involves at least Abell inversion (see Sarazin Eq. 5.80, 5.81 and 5.82), and the information on the instrument response and the emission model. In the case of a simple analytical surface brightness model, such as the beta profile (Eq. 3.1) the Abell inversion is simple: the shape parameters of the surface brightness (r_{core} and β) determine the shape of the gas density profile with 3D radius,

$$n_e(r) = n_e(0)\left(1+\left(\frac{b}{r_{core}}\right)^2\right)^{\left(-\frac{3}{2}\beta\right)} \tag{3.4}$$

and all there is left to do is to normalize the gas density profile (i.e to determine the central density $n_{gas}(0)$).

In the case of double β profile the normalization is a bit more complicated. The 3D density is

$$n_e(r) = n_e(0,1)\left(1+\left(\frac{b}{r_{core_1}}\right)^2\right)^{\left(-\frac{3}{2}\beta\right)} + n_e(0,2)\left(1+\left(\frac{b}{r_{core_2}}\right)^2\right)^{\left(-\frac{3}{2}\beta\right)} \tag{3.5}$$

Let's define

$$f_{\beta_1} \equiv \left(1+\left(\frac{b}{r_{core_1}}\right)^2\right)^{\left(-\frac{3}{2}\beta\right)} \tag{3.6}$$

and

$$f_{\beta_2} \equiv \left(1+\left(\frac{b}{r_{core_2}}\right)^2\right)^{\left(-\frac{3}{2}\beta\right)} \tag{3.7}$$

so that Eq. 3.5 becomes

$$n_e(r) = n_e(0,1) \times f_{\beta_1} + n_e(0,2) \times f_{\beta_2} \tag{3.8}$$

Since I $\propto$ n_e n_p and n_e / n_p is a constant $\Rightarrow$

$$\frac{n_e(0,2)}{n_e(0,1)} = \sqrt{I_{0,2}/I_{0,1}} \equiv f_I. \tag{3.9}$$

Combining Eqs. 3.8 and 3.9, the total electron number density is given by

$$n_e(r) = n_e(0,1) \times (f_{\beta_1} + f_I \times f_{\beta_2}) \tag{3.10}$$

Thus, with the best-fit surface brightness profile parameters, Equation 3.4 (for single β) or 3.10 (for double β) yields the electron density profile, once the central density $n_e(0)$ or $n_e(0,1)$ is found. This can be found using the cluster emission spectrum. The flux is proportional to the emission measure, i.e. the integral of the density squared in the line of sight, or

$$EM = \int n_e(r) n_p(r) dV. \tag{3.11}$$

Combining the value of EM given by the spectral analysis with Eq. s 3.11 and 3.4 for the single β model, and using n_e / n_p = 1.17 the central density $n_e(0)$ can be calculated as

$$n_e(0) = \sqrt{1.17EM / \int \left(1 + \left(\frac{b}{r_{core}}\right)^2\right)^{(-3\beta)} dV} \tag{3.12}$$

In case of double β model, the Equation 3.11 can be written as

$$EM = n_e(0,1) n_p(0,1) \int (f_{\beta_1} + f_I \times f_{\beta_2})^2 \tag{3.13}$$

thus yielding

$$n_e(0,1) = \sqrt{1.17EM / \int (f_{\beta_1} + f_I \times f_{\beta_2})^2 dV} \tag{3.14}$$

$$n_e(0,2) = n_e(0,1) \times f_I \tag{3.15}$$

. The central proton density is then

$$n_p(0) = n_e(0)/1.17 \tag{3.16}$$

for single β model and

$$n_p(0,1) = n_e(0,1)/1.17 \tag{3.17}$$

$$n_p(0,2) = n_e(0,2)/1.17 \tag{3.18}$$

for the double β model.

3.4 Emission measure from X-ray spectroscopy

For the evaluation of the densities, we still need to find the value of the emission measure EM in a given region of the cluster. Inserting the value of EM into Eqs. 3.12 or 14 , and integrating the equations using the best fit β parameters you finally obtain the value for the central electron number density. Using Eq. 3.16 and 3.17 you can compute the central proton density. The number densities must be converted into gas mass densities. Remembering that ρ_g = 1.35 m_p n_p, the conversion is simple.

In order to obtain the emission measure needed above, one needs to obtain a global X-ray spectrum of the cluster from a large circular or annular region ($\sim$ 10 arcmin) around the cluster center. The normalisation parameter of the best-fit MEKAL model ($norm_{MEKAL}$) is linked to the emission measure by

$$norm_{MEKAL} = EM \times 10^{-14} / \left[4\pi (D_A(1+z))^2\right] \tag{3.19}$$

where D_A is the angular size distance to the source (cm), z is the redshift of the cluster and EM is the emission measure in units of cm^{-3}.

For the details of spectrum extraction, see the notes

`http://www.helsinki.fi/~junevala/kurssi2008/xmm_reduction4.pdf`

The spectrum should be modeled and analysed with MEKAL model in the XSPEC package, see the info at

`http://www.helsinki.fi/~junevala/kurssi2008/XSPEC_info.pdf`

3.5 Normalizing the surface brightness profile II (practise)

To perform the calculations of Section 3.3, go first to directory /**wrk**/*yourname*/**clusteranalysis/data/OBJNAME/INSTRUMENT/OBSID**, and edit the parameter file **gasmass_par.pro**. From the best fit spectral model above, take the MEKAL normalization (parameter **norm**) into file **gasmass_par.pro**. Also, change the values of **r_em_det_min_arcmin** and **r_em_det_max_arcmin** to correspond the radii of your extraction region above. Specify the result file from the parameter error analysis above as **errfile**. Start IDL and execute commands **.run idl_imaging_setup.pro**, **.run gasmass_par.pro** and **.run gasmass.pro**. This will normalize the given β profile according to formulae in the previous Section. Program also computes the gas density profile using Eq. 3.4 or 3.5 and integrates it over the volume to obtain enclosed gas mass profile. The results are plotted in the file defined with the parameter **plotfile**.

The above execution also estimates the gas mass uncertainties due to the β model parameter uncertainties. This is done by generating a large number (a parameter **nogm** in the file **gasmass_par.pro**) of profile model parameter sets, with the best-fit parameters as the centroid of the Gaussian and the statistical uncertainties of the parameters as the standard deviation. Each parameter set corresponds to a different gas density profile and thus we obtain a distribution of gas density and mass values at each radius. Using this distribution, we determine the 1 σ (= 68.5%) confidence interval by excluding 16% of the highest and lowest gas mass values at each radius. The best-fit gas mass and the uncertainties are saved in the file defined as **gasmassfile** and it is plotted in the file defined as **gasplotfile** in the file **gasmass_par.pro**.