THE EFFECT OF GRAVITATIONAL-WAVE RECOIL ON THE DEMOGRAPHY OF MASSIVE BLACK HOLES

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ABSTRACT

The coalescence of massive black hole (MBH) binaries following galaxy mergers is one of the main sources of low-frequency gravitational radiation. A higher-order relativistic phenomenon, the recoil as a result of the non-zero net linear momentum carried away by gravitational waves, may have interesting consequences for the demography of MBHs at the centers of galaxies. We study the dynamics of recoiling MBHs and its observational consequences. The "gravitational rocket" may: i) deplete MBHs from late-type spirals, dwarf galaxies, and stellar clusters; ii) produce off-nuclear quasars, including unusual radio morphologies during the recoil of a radio-loud source; and iii) give rise to a population of interstellar and intergalactic MBHs.

Subject headings: black hole physics – cosmology: theory – galaxies: nuclei – stellar dynamics

1. INTRODUCTION

The first massive black holes likely formed at high redshifts ($z \gtrsim 10$) at the centers of low-mass ($\sim 10^6 M_\odot$) dark matter concentrations. These black holes evolve into the population of bright quasars observed at $z \lesssim 6$ and eventually grow into the supermassive remnants that are ubiquitous at the centers of galaxies in the nearby universe. In popular cold dark matter (CDM) cosmogonies, dark matter halos and their associated galaxies undergo many mergers as mass is assembled from high redshift to the present. The merging – driven by dynamical friction against the dark matter – of two comparable-mass halo+MBH systems will drag in the satellite hole towards the center of the more massive progenitor, leading to the formation of a bound MBH binary with separation of $\sim$ pc. If stellar dynamical and/or gas processes drive the binary sufficiently close ($\lesssim 0.01$ pc), gravitational radiation will eventually dominate angular momentum and energy losses and cause the two MBHs to coalesce. Such catastrophic events are one of the primary motivations for low-frequency, space-based, gravitational wave detectors such as the planned *Laser Interferometer Space Antenna* (*LISA*). For unequal mass pairs, gravitational waves also remove net *linear momentum* from the binary and impart a "kick" velocity to the center of mass of the system. The dominant recoil effect arises from the interference between the mass-quadrupole and mass-octupole or current-quadrupole contributions (Peres 1962; Bekenstein 1973). To date, the outcome of this "gravitational rocket" remains uncertain, as fully general relativistic numerical computations of radiation reaction effects during the coalescence of two Kerr holes are not available. For sufficiently asymmetric configurations, recoil velocities may exceed a few hundred kilometers per second and lead to a significant displacement of the MBH from the center of its host galaxy. In the shallow potential wells of small-mass halos at high redshifts, recoil velocities may be so large in the late stage of black hole-black hole coalescence to reach galactic escape velocities (Madau et al. 2004). If it is not ejected from the host altogether (giving origin to a population of *intergalactic* MBHs), the hole will return to the nucleus via dynamical friction.

In this *Letter* we address the dynamics of recoiling holes in galaxy cores, discuss the implications of coalescence-induced kicks for the demography of MBHs, and consider the prospects for directly detecting the observational signatures of gravitational-wave recoil. While preparing this work for submission, we learned of an independent study by Merritt et al. (2004) of the consequences of the gravitational rocket.

2. DYNAMICS OF RECOILING BLACK HOLES

Gravitation radiation recoil is a strong field effect that depends on the lack of symmetry in the binary system. The lighter hole in a quasi-circular inspiraling orbit moves faster than the heavier one, and its gravitational radiation is more "forward beamed". This gives a net momentum ejection in the direction of motion of the lighter mass, and the binary recoils in the opposite direction (Wiseman 1992). According to quasi-Newtonian calculations (Fitchett 1983), at the transition from the inspiral to "plunge" phase, the center of mass of a compact binary of total mass $M = m_1 + m_2$ (with the convention $m_1 < m_2$) recoils with a velocity

$$v_{\rm CM} = 1480 \ {\rm km \, s^{-1}} \frac{f(q)}{f_{\max}} \left(\frac{2GM/c^2}{r_{\rm isco}} \right)^4, \qquad (1)$$

where the function $f(q \equiv m_1/m_2) = q^2(1 - q)/(1 + q)^5$ reaches a maximum value $f_{\max} = 0.0179$ for $q = 1/2.6$, and $r_{\rm isco}$ is the radius of the innermost stable circular orbit, which moves inwards for a comparable-mass system relative to its test-mass limit (e.g. Buonanno & Damour

2000). By symmetry, the recoil vanishes for equal-mass nonrotating holes. Fitchett & Detweiler (1984) extended Fitchett's work to perturbation theory and estimated kick velocities $\gtrsim 100 \text{ km s}^{-1}$. More recent perturbation theory calculations by Favata, Hughes, & Holz (2004) find that the recoil velocity can readily reach $\approx 100 - 200 \text{ km s}^{-1}$, but is unlikely to exceed $\approx 500 \text{ km s}^{-1}$. Numerical relativistic calculations of radiation recoil from highly distorted Schwarzschild holes yields maximum kick velocities in excess of 400 km s^{-1} (Brandt & Anninos 1999). Because of the strong dependence of the rocket effect on the radius of the final prior-to-plunging orbit, larger kicks are expected for prograde inspiral into rapidly rotating MBHs. Moreover, in the case of Kerr holes, recoil is significant even for holes of equal mass (Favata et al. 2004) and can be directed out of the plane of the orbit (Redmount & Rees 1989).

What is the dynamics of a recoiling MBH in the gravitational potential of its host galaxy? As the MBH (with mass M_{BH}) recoils from the nucleus, stars bound to the hole are displaced with it, so that the total ejected mass is $M_{\text{tot}} = M_{\text{BH}} + M_{\text{cusp}}$, where the bound stellar cusp has a mass $M_{\text{cusp}} \approx M_{\text{BH}}$ for $v_{\text{CM}} \approx \sigma$, while M_{cusp} rapidly decreases for $v_{\text{CM}} \gtrsim \sigma$. As a first approximation let us assume that the hole + stellar cusp is on a radial orbit in a spherical potential (e.g., an early-type galaxy or the bulge of a spiral). Its orbit is then governed by

$$\frac{d^2 \vec{r}}{dt^2} = -\frac{GM(r)}{r^2}\hat{r}$$
$$- \frac{4\pi G^2 \ln \Lambda \, \rho M_{\text{tot}}}{v^2} \left(\text{erf}(x) - \frac{2x}{\sqrt{\pi}} e^{-x^2} \right) \hat{v}, \quad (2)$$

where $x = v/\sqrt{2}\sigma$, $M(r)$ describes the mass profile of the galaxy, $\rho(r)$ is the density profile of stars with 1D velocity dispersion σ, and the second term represents dynamical friction against the stellar background (e.g. Binney & Tremaine 1987; hereafter BT). We approximate the stellar density as an isothermal sphere, in which case the dynamical friction time scales with radius as $t_{\text{DF}} \propto r^2$, while the 'residence' time for a radial orbit scales as $t \approx r/v \propto r$. Thus most of the decay of the MBH's orbit by friction takes place at small radii in the galactic nucleus, where it is reasonable to assume that the gravitational potential is dominated by stars. Stars within the gravitational sphere of influence of the hole, $R_{\text{BH}} \approx GM_{\text{BH}}/\sigma^2$, are bound to it and do not contribute to dynamical friction. We therefore truncate the stellar density inside a core radius $\approx R_{\text{BH}}$, so that $\rho(r) = \sigma^2/[2\pi G(r^2 + R_{\text{BH}}^2)]$ and $M(r) = 2\sigma^2 r[1 - (R_{\text{BH}}/r)\arctan(r/R_{\text{BH}})]/G$. Flattening of the inner stellar density profile is also produced physically during the decay of the initial MBH binary, when dynamical friction and three-body interactions transfer energy from the binary to stars in the nucleus of the galaxy (e.g., Milosavljevic & Merritt 2001).

The Coulomb logarithm $\ln \Lambda$ in equation (2) is approximately given by $\ln[b_{\text{max}}/b_{\text{min}}]$, where b_{max} and b_{min} are the maximum and minimum impact parameters for stars that contribute to dynamical friction (BT); b_{min} is typically taken to be $\sim R_{\text{BH}}$ (BT; Maoz 1993). In the case of a uniform medium, b_{max} is comparable to the size of the system (galaxy), so that $b_{\text{max}} \gg b_{\text{min}}$. For our problem, however, the decay of the hole's orbit occurs in the inner nuclear re-

gions where the stellar density peaks, i.e. at radius $\sim R_{\text{BH}}$, so that $b_{\text{max}} \sim R_{\text{BH}} \sim b_{\text{min}}$ and $\ln \Lambda \sim 1$. Maoz (1993) has given a more careful derivation of dynamical friction in an inhomogeneous medium; the Coulomb logarithm is replaced (see his eq. 4.4) by $\ln \Lambda \to \int_{R_{\text{BH}}}^{R_{\text{max}}} \rho(r) dr/(\rho_0 r)$, where R_{max} is the size of the stellar system and ρ_0 is the central stellar density. For our problem this integral is ~ 1, confirming the above argument that $\ln \Lambda \approx 1$ is appropriate.
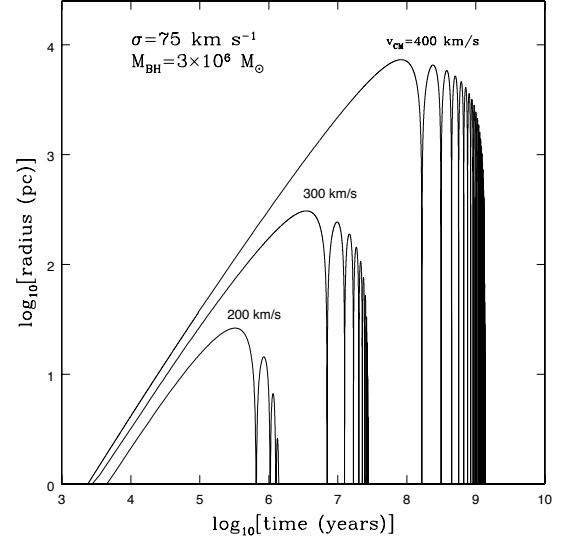

FIG. 1.— The response of a $3 \times 10^6 M_\odot$ MBH to a given recoil velocity v_{CM} in an isothermal potential with dispersion $\sigma = 75 \text{ km s}^{-1}$; radial orbits are assumed. As explained in the text, the curves depend primarily on the dimensionless number v_{CM}/σ and so can be scaled to other parameters.

Figure 1 shows the orbit of a $3 \times 10^6 M_\odot$ MBH moving with initial recoil velocity of $v_{\text{CM}} = 200, 300, \text{ and } 400 \text{ km s}^{-1}$ in an isothermal sphere of dispersion $\sigma = 75 \text{ km s}^{-1}$ (reasonable for a Milky Way-type host) and radius $R_{\text{max}} \gg R_{\text{BH}}$. The escape speed from such a sphere is $v_{\text{esc}}(0) = \sqrt{2|\phi|(r=0)} = 2\sigma \ln^{1/2}(R_{\text{max}}/R_{\text{BH}})$. We fix the hole mass using the $M_{\text{BH}} - \sigma$ relation of Tremaine et al. (2002): $M_{\text{BH}} = 1.5 \times 10^8 \sigma_{200}^4 \, M_\odot$, so that $R_{\text{BH}} \approx 15 \sigma_{200}^2 \text{ pc}$, where σ_{200} is the velocity dispersion in units of 200 km s^{-1}. As Figure 1 shows, the timescale for the recoiling hole to return to the center of its host galaxy depends very sensitively on the magnitude of the kick it receives, ranging from $\sim 10^6$ years for $v_{\text{CM}} = 200 \text{ km s}^{-1}$ to more than 10^9 years for $v_{\text{CM}} = 400 \text{ km s}^{-1}$. Because the friction takes place primarily at small radii, the decay time is sensitive to the inner stellar density. For example, if we increase the core radius of the isothermal sphere by a factor of 2, the decay times increase by a factor of ≈ 3. Given uncertainties in the recoil velocity, this uncertainty in the decay time will not significantly modify our conclusions.

The numerical results in Figure 1 can be understood analytically as follows. In the absence of dynamical friction, energy is conserved, and an ejected MBH will reach its apocenter at

$$R_a \sim R_{\text{BH}} \exp\left[(v_{\text{CM}}/2\sigma)^2\right], \quad (3)$$

where the exponential dependence arises because the potential is logarithmic. During each passage through the galactic nucleus, however, the hole loses a fraction f of its orbital energy because of dynamical friction; this fraction can be estimated by comparing the DF timescale at $\sim R_{\rm BH}$ to the transit time across this region, $\sim R_{\rm BH}/v_{\rm CM}$, which yields $f \sim (\sigma/v_{\rm CM})^4$. It takes $\sim f^{-1}$ orbits through the nucleus for the MBH to return to the center of the galaxy, and so the decay time can be roughly estimated by $t_{\rm decay} \sim f^{-1} R_{\rm a}/v_{\rm CM}$. Since both f and $R_{\rm a}$ depend primarily on $v_{\rm CM}/\sigma$, this estimate shows that the decay time of the hole back to the center is determined primarily by $v_{\rm CM}/\sigma$ for radial orbits. Thus, the results in Figure 1 can be scaled to other parameters, e.g. the decay times in Figure 1 apply reasonably accurately to $\sigma = 50\,{\rm km\,s^{-1}}$, so long as $v_{\rm CM}$ is rescaled to $v_{\rm CM} = 266, 200,$ and $133\,{\rm km\,s^{-1}}$ (from top to bottom).[1]

The above analysis assumes that the MBH remains on a purely radial orbit. It is, however, likely that the recoiling hole will gain angular momentum upon leaving the nucleus (e.g. if the galactic potential is triaxial): this will modify the timescale for the orbital decay. Detailed estimates of the decay time in this case are uncertain. We can roughly bracket the effect by considering the MBH on a circular orbit with an energy $(1/2)M_{\rm BH}v_{\rm CM}^2$; for an isothermal potential, the radius of such an orbit is $\approx R_a/\sqrt{e}$, where R_a is the apocenter radius for a radial orbit (shown in Figure 1 and estimated in eq. [3]). The decay time is then given by the standard dynamical friction timescale (BT)

$$t_{\rm DF} \approx \frac{10^{10}{\rm yrs}}{\ln\Lambda} \left(\frac{R_a}{1.5\,{\rm kpc}}\right)^2 \left(\frac{\sigma}{75\,{\rm km\,s^{-1}}}\right)^{-3}, \quad (4)$$

where we have used the $M_{\rm BH} - \sigma$ relation to eliminate the hole mass in favor of σ. Note that the decay time is now determined by dynamical friction at large radii, rather than small radii, and so $\ln\Lambda \sim 10$ is probably more appropriate. For $v_{\rm CM} \approx \sigma$ the decay time predicted by equation (4) is comparable to that for radial orbits (a few crossing times; see Fig. 1) while it can be significantly longer for large $v_{\rm CM}/\sigma$ (large apocenter distances).

3. MBH BINARIES IN HIERARCHICAL CLUSTERING COSMOLOGIES

In hierarchical clustering scenarios, MBH-MBH binaries are likely products of galaxy major mergers only. When two halo+MBH systems of (total) mass M and $M_s < M$ merge, the 'satellite' (less massive) progenitor will sink to the center of the more massive pre-existing system by dynamical friction against the dark matter; for an isothermal sphere the Chandrasekhar dynamical friction timescale is

$$t_{\rm DF} = 1.65 \frac{1 + M_s/M}{M_s/M} \frac{1}{H\sqrt{\Delta_{\rm vir}}\ln\Lambda} \Theta \quad (5)$$

(Lacey & Cole 1993), where $\Delta_{\rm vir}$ is the density contrast at virialization, H is the Hubble parameter, and the term Θ contains the dependence of this timescale on the orbital parameters. After including the increase in the orbital decay timescale due to tidal stripping of the satellite (Colpi et al. 1999), it is possible to show that satellites will merge with the central galaxy on timescales shorter than the Hubble

time only in the case of major mergers, $M_s/M \gtrsim 0.3-0.5$. In minor mergers tidal stripping may leave the satellite MBH wandering in the halo, too far from the center of the remnant for the formation of a black hole binary.

Major mergers are frequent at early times, so a significant number of binary MBH systems is expected to form then. It is still unclear whether halo major mergers necessarily lead to the coalescence of their MBHs, or whether the binaries "stall" before the backreaction from gravitational waves becomes important (Begelman, Blandford, & Rees 1980). A number of plausible mechanisms that may help avoid such stalling have been suggested in the recent literature (e.g., Gould & Rix 2000; Zhao, Haehnelt, & Rees 2001; Armitage & Natarajan 2002; Yu 2002; Chatterjee, Hernquist, & Loeb 2003).
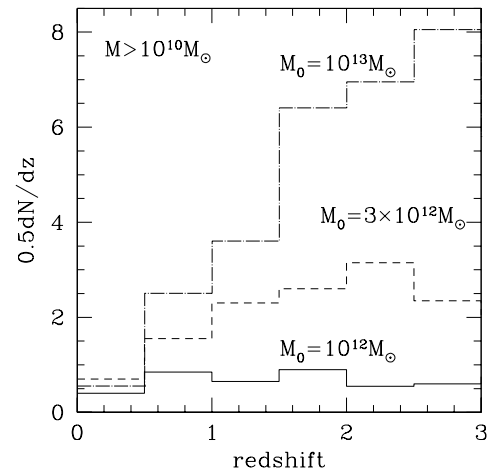

Fig. 2.— Mean number of major mergers (with mass ratios > 0.5) experienced per unit redshift by halos with masses $> 10^{10}$ M$_\odot$. *Solid line:* progenitors of a $M_0 = 10^{12}$ M$_\odot$ halo at $z = 0$. *Dashed line:* same for $M_0 = 3 \times 10^{12}$ M$_\odot$. *Dot-dashed line:* same for $M_0 = 10^{13}$ M$_\odot$. The rates have been computed assuming the currently favoured ΛCDM world model with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $\Omega_b = 0.045$, $\sigma_8 = 0.93$, and $n = 1$.

Figure 2 shows the mean number of major mergers per unit redshift bin experienced by all halos with masses $> 10^{10}$ M$_\odot$ that are progenitors of a $z = 0$ parent halo of mass $M_0 = 10^{12}, 3 \times 10^{12},$ and 10^{13} M$_\odot$. We have tracked backwards in time the merger history of parent halos with a Monte Carlo algorithm based on the extended Press-Schechter formalism (Volonteri, Haardt, & Madau 2003). For the most massive parent halos this quantity peaks in the redshift range 2-3, the epoch when the observed space density of optically-selected quasar also reaches a maximum (Kauffmann & Haehnelt 2000). Hydrodynamic simulations of major mergers have shown that a significant fraction of the gas in interacting galaxies falls to the center of the merged system (Mihos & Hernquist 1994): the cold gas may be eventually driven into the very inner regions, fueling an accretion episode and the growth of the nuclear MBH. We will discuss below the possibility that gravitational-wave recoil may thus give rise to off-nuclear active galactic nuclei (AGN).

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[1]More accurately, at fixed $v_{\rm CM}/\sigma$, there remains a weak dependence on σ via $t_{\rm decay} \propto R_{\rm a}/v_{\rm CM} \propto R_{\rm BH}/v_{\rm CM} \propto \sigma$.

4. IMPLICATIONS

For galaxies with $\sigma \lesssim 50 - 75$ km s^{-1}, typical kick velocities of a few hundred km s^{-1} are sufficient to unbind the hole or displace it sufficiently from the nucleus that the decay time due to dynamical friction is comparable to the Hubble time (Fig. 1). This implies that MBHs with masses $M_{\rm BH} \lesssim 10^6 M_\odot$ (using the $M_{\rm BH} - \sigma$ relation) may be comparably rare in their late-type spiral or dwarf-galaxy hosts. Interestingly, there are very few observational constraints on the presence of MBHs in such galaxies, though Filippenko & Ho (2003) argued for a $\sim 10^5 M_\odot$ MBH for the Seyfert galaxy in the late type (bulgeless) spiral NGC 4395, and Barth et al. (2004) reached a similar conclusion for the dwarf Seyfert 1 Galaxy POX 52. It is important to stress that even if galaxies do not currently harbor a central MBH, they may have done so in the past. Thus the absence of MBHs in shallow potential wells would not necessarily imply inefficient MBH formation (e.g., Haiman, Madau, & Loeb 1999), but could instead be due to recoil during MBH coalescence. Note also that the low-mass black holes that are preferentially affected by gravitational recoil are also those that are expected to dominate the *LISA* gravitational wave signal from MBH-MBH coalescence. If ejection of such MBHs is common, this may decrease the number of sources detected by *LISA*.

Galaxy mergers are a leading mechanism for supplying fuel to MBHs (e.g., Barnes & Hernquist 1991), and so a natural implication of gravitational recoil is the possibility of off-nuclear quasar activity.[2] Since the lifetime of merger-driven activity is of order the Salpeter timescale $t_S = 4.5 \times 10^7$ yr, off-nuclear AGN are most likely to be found in relatively small potential wells. This is because, for (say) a $\sim 10^8 M_\odot$ MBH in a $\sigma = 200$ km s^{-1} galaxy, the decay timescale due to dynamical friction is $\lesssim 10^6$ yr $\ll t_S$ even for a kick velocity of $v_{\rm CM} = 300$ km s^{-1}, and the displacement from the nucleus is quite small, $\lesssim R_{\rm BH} \approx 10$ pc. By contrast a decay timescale comparable to $\sim t_S$ is plausible for MBHs with $M_{\rm BH} \approx 10^6 - 10^7 M_\odot$ and $v_{\rm CM} \approx 200$ km s^{-1}. Since the merger rate for progenitor halos hosting such MBHs peaks at $z \approx 2 - 3$ (Fig. 2), we predict that a significant fraction of *moderate- to high-redshift, low mass* AGN could be off-nuclear. Most currently detected AGN at high redshift are the rare $\sim 10^8 - 10^{10} M_\odot$ holes hosted by massive halos, for which recoil is probably not important. At early times, however, theoretical models of the evolution of the quasar population in hierarchical structure formation scenarios predict a large number of fainter, low-$M_{\rm BH}$ sources (e.g., Haiman & Loeb 1998; Haiman et al. 1999; Volonteri et al. 2003).

The effects of gravitational recoil may be particularly prominent in radio observations of AGN. Merritt & Ekers (2002) argued that MBH coalescence might imprint itself on the morphology of radio galaxies by changing the spin axis of the MBH and thus the direction of the radio jet. Gravitational recoil may have a comparably important effect by displacing the radio-loud AGN from the nucleus of the galaxy. This could manifest itself as a flat spectrum radio core displaced from the optical nucleus of the galaxy, along with a jet or lobe still symmetric about the nucleus. Radio observations also have the obvious advantages that contamination from the host galaxy is less important (relative to optical quasars) and VLBI interferometry allows precise localization of the radio emission.

Gravitational recoil may also be relevant for understanding the origin of off-nuclear ultraluminous X-ray sources in nearby galaxies (see, e.g., Colbert & Mushotzky 1999). One interpretation of such sources is that they are intermediate-mass black holes (IMBHs) with $M \sim 10^3$ M$_\odot$ accreting near the Eddington limit. Miller & Hamilton (2002) suggest that such intermediate-mass holes could form in globular clusters by repeated black hole mergers. The gravitational rocket will, however, probably prevent substantial growth via mergers in the shallow potential well of a globular cluster, since even mergers with mass ratios as small as ≈ 0.1 can lead to kick velocities in excess of the escape velocity of the cluster. It should be noted that an IMBH could still form via stellar (rather than compact object) mergers during the core-collapse of young star clusters (e.g., Gürkan et al. 2004).

Finally, we draw attention to the fact that the gravitational rocket does not necessarily limit the ability of MBHs to grow via gas accretion from rare less massive seeds, such as IMBHs produced by the collapse of Population III stars at $z \sim 20$ (Madau & Rees 2001). This is because seed holes that are as rare as (say) the 3.5σ peaks of the primordial density field will evolve largely in isolation, as the merging of two (mini)halos both hosting a black hole is a rare event at these very early epochs. A significant number of MBH binary systems may form only later, when the fraction of halos hosting MBHs is larger. By then the typical host will be further down the merger hierarchy (more massive) and the effect of radiation recoil less disruptive (Madau et al. 2004).

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[2]One complication is that the dominant episode of accretion onto MBHs during galaxy mergers could happen when the satellite galaxy is still sinking in towards the nucleus of the more massive galaxy. In this case the AGN activity could be completed before the binary actually coalesces.

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