

X-ray analyses of clusters of galaxies

- Imaging
- Spatially resolved spectroscopy
- Mass analysis

Cluster mass modeling

- Hydrostatic equilibrium

$$M_{tot}(<r) = -\frac{k}{\mu m_p G} T_g(r) r \left(\frac{d \ln \rho_g(r)}{d \ln r} + \frac{d \ln T_g(r)}{d \ln r} \right) \quad \text{Eq. 1}$$

- ρ_g from image analysis
- $T(r)$ from spatially resolved spectroscopy

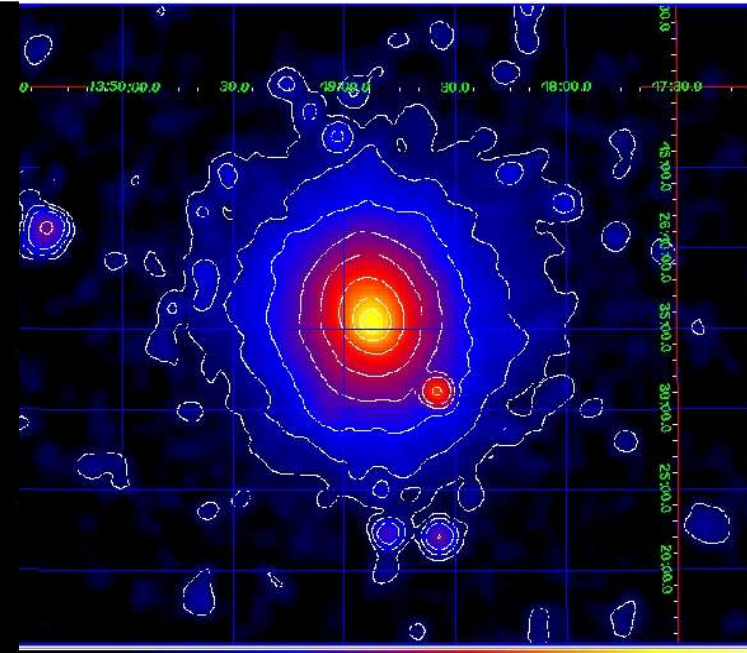
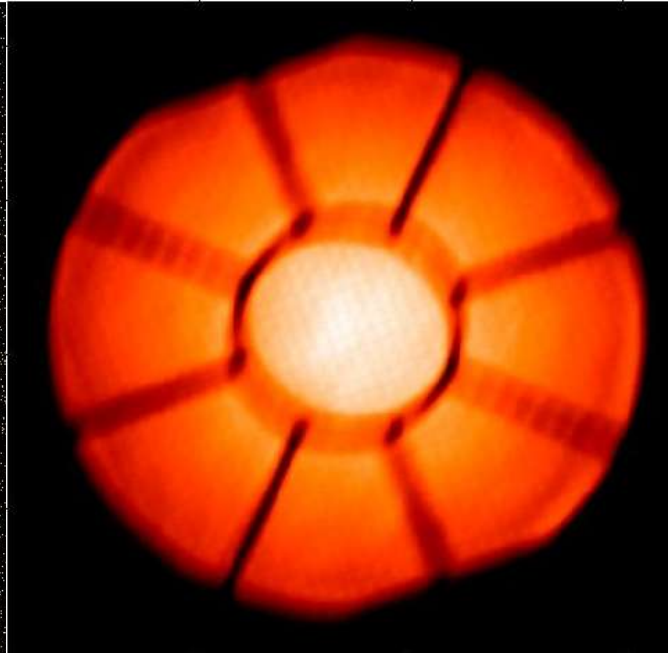
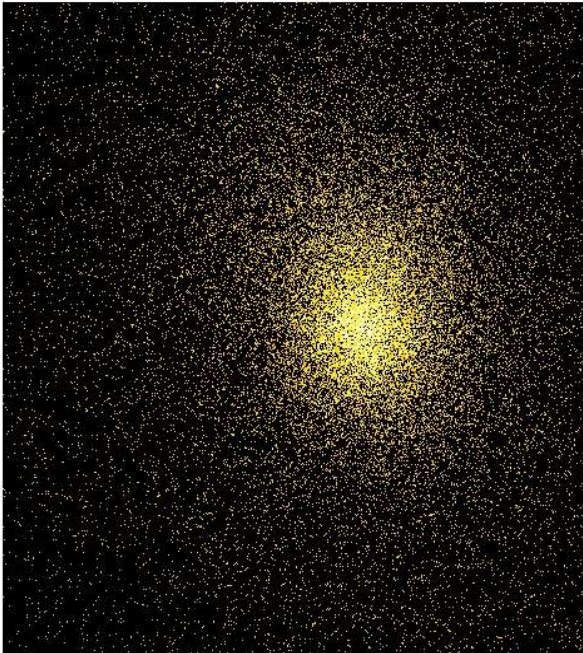
Imaging data

- FITS table recording 1) energy, 2) arrival time , 3) X and Y coordinate of each event (event list)
- Integrate data over time and energy for each pixel \rightarrow raw image [c pixel^{-1}]
- instrument background subtraction (sky bkg constant so leave it in)
- Vignetting correction by dividing with the exposure map [s] \rightarrow rate map [$\text{c s}^{-1} \text{ pixel}^{-1}$]

raw image

exposure map

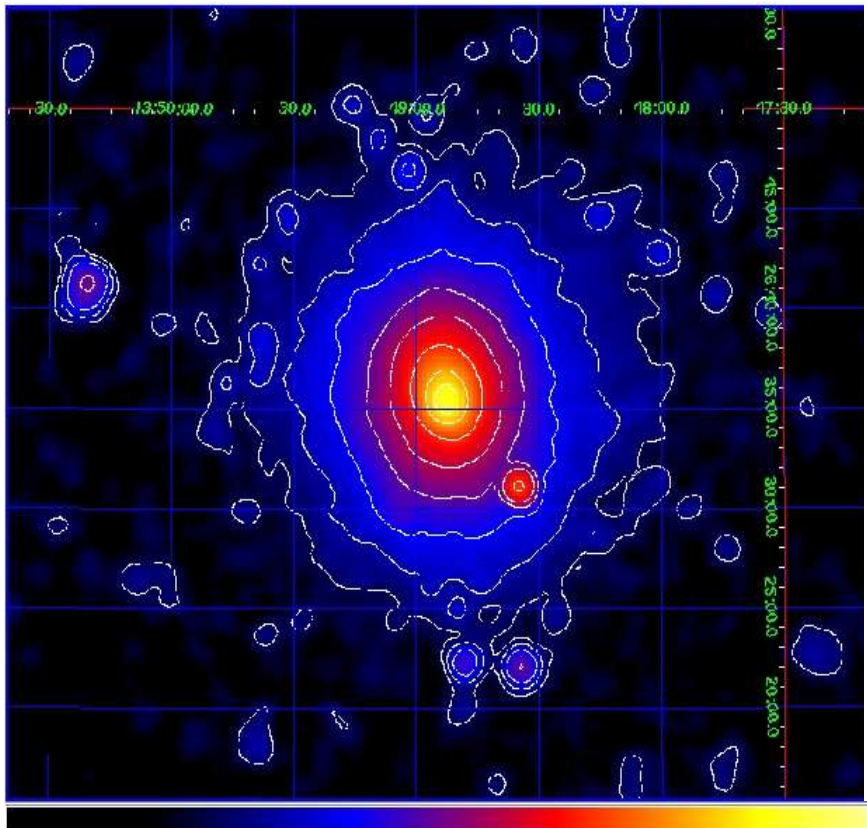
rate map



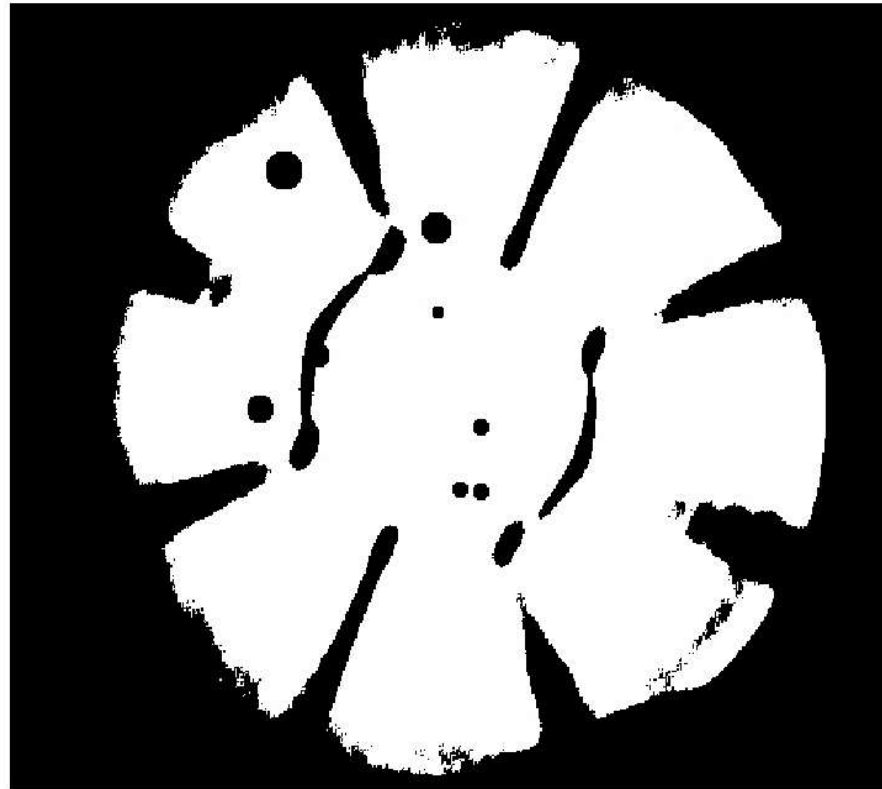
Imaging data

- mask out point sources and detector artifacts
- in the mask image, the excluded pixels have a value of 0, others 1

rate image



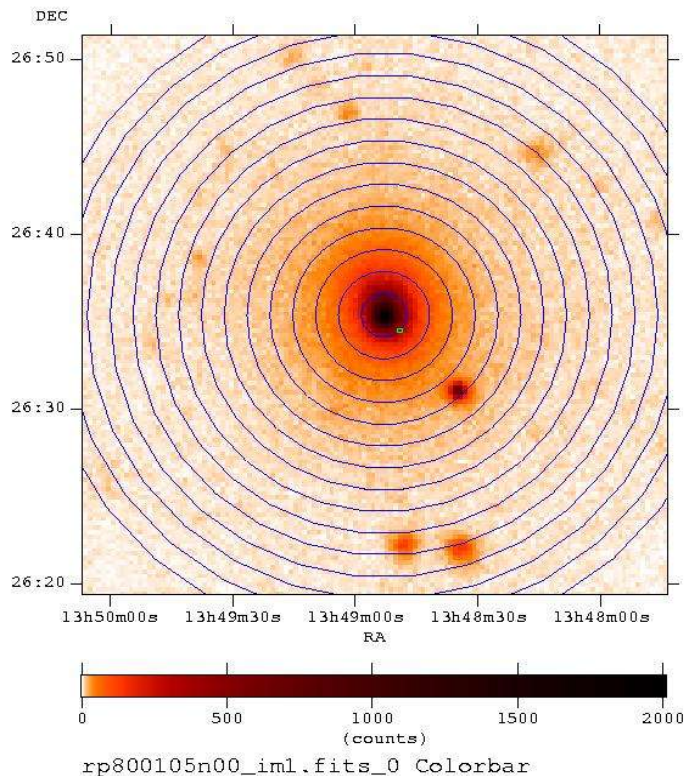
mask image



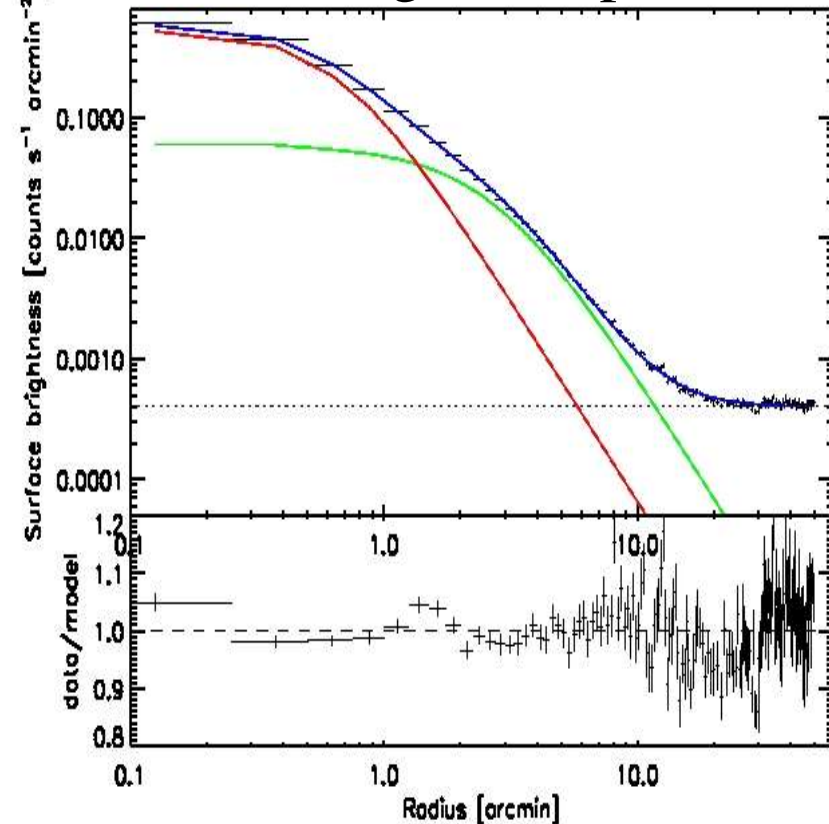
Surface brightness profile

- add up the count rates of each pixel of rate image (where mask value = 1) in concentric annuli
- divide by the (number \times size) of pixels in the annulus [arcmin^2] \rightarrow surface brightness [$\text{c s}^{-1} \text{ arcmin}^{-2}$]
- plot with distances of annuli \rightarrow surface brightness profile

rate image



surface brightness profile



Modeling the surface brightness profile

- X-ray imaging \rightarrow surface brightness $I(x,y)$ [$\text{c s}^{-1} \text{ arcmin}^{-2}$]
- azimuthal symmetry \rightarrow surface brightness profile $I(b)$

$$I(b) = I_0 \left[1 + \left(\frac{b}{r_{\text{core}}} \right)^2 \right]^{(-3\beta + 0.5)}$$

β profile (Cavaliere & Fusco-Femiano, 1976, A&A, 49, 137)

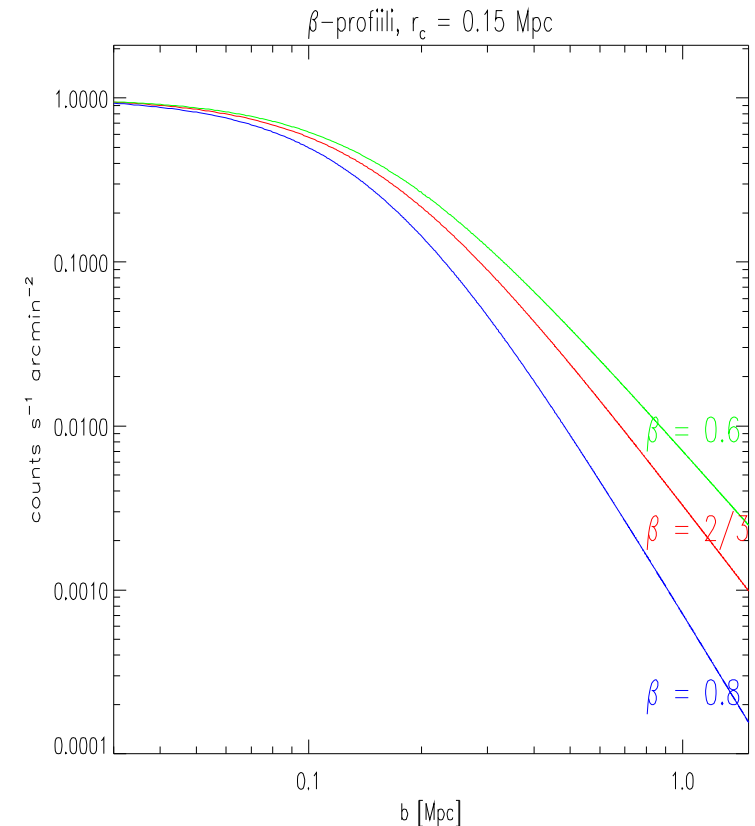
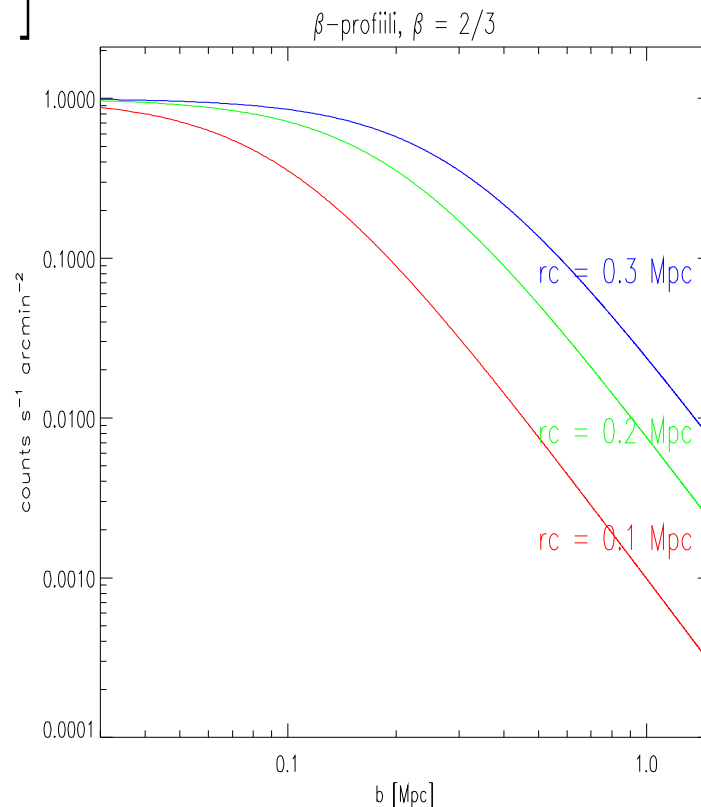
b = projected radius

β = index

r_{core} = core radius

typically: $\beta \sim 0.6-0.8$

$r_c \sim 100 \text{ kpc}$



Cooling Sarazin 5.3.1

- Clusters radiate energy away, so they cool
- Bremsstrahlung cooling time scale (over which the cluster loses a significant amount of thermal energy):

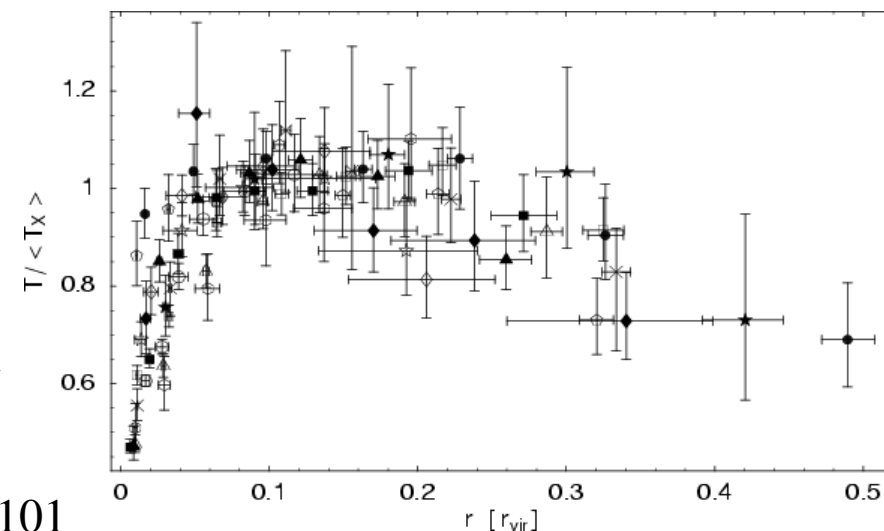
$$t_{cool} \equiv \left(\frac{d \ln T_g}{dt} \right)^{-1} \approx 8.5 \times 10^{10} \text{ yr} \left(\frac{T_g}{10^8 \text{ K}} \right)^{1/2} \left(\frac{n_p}{10^{-3} \text{ cm}^{-3}} \right)^{-1}$$

- Typically no problem $T \approx 10^8 \text{ K}, n_e \approx 10^{-3} \rightarrow t_{cool} \approx 10^{11} \text{ yr} > t_{Hubble}$

- Some cluster centers are so dense that

t_{cool} is shorter than the cluster age \rightarrow cooling happens

- Gas pressure ($P = knT$) decreases and the gas flows into the center \rightarrow gas density increases in the center \rightarrow brightness peak

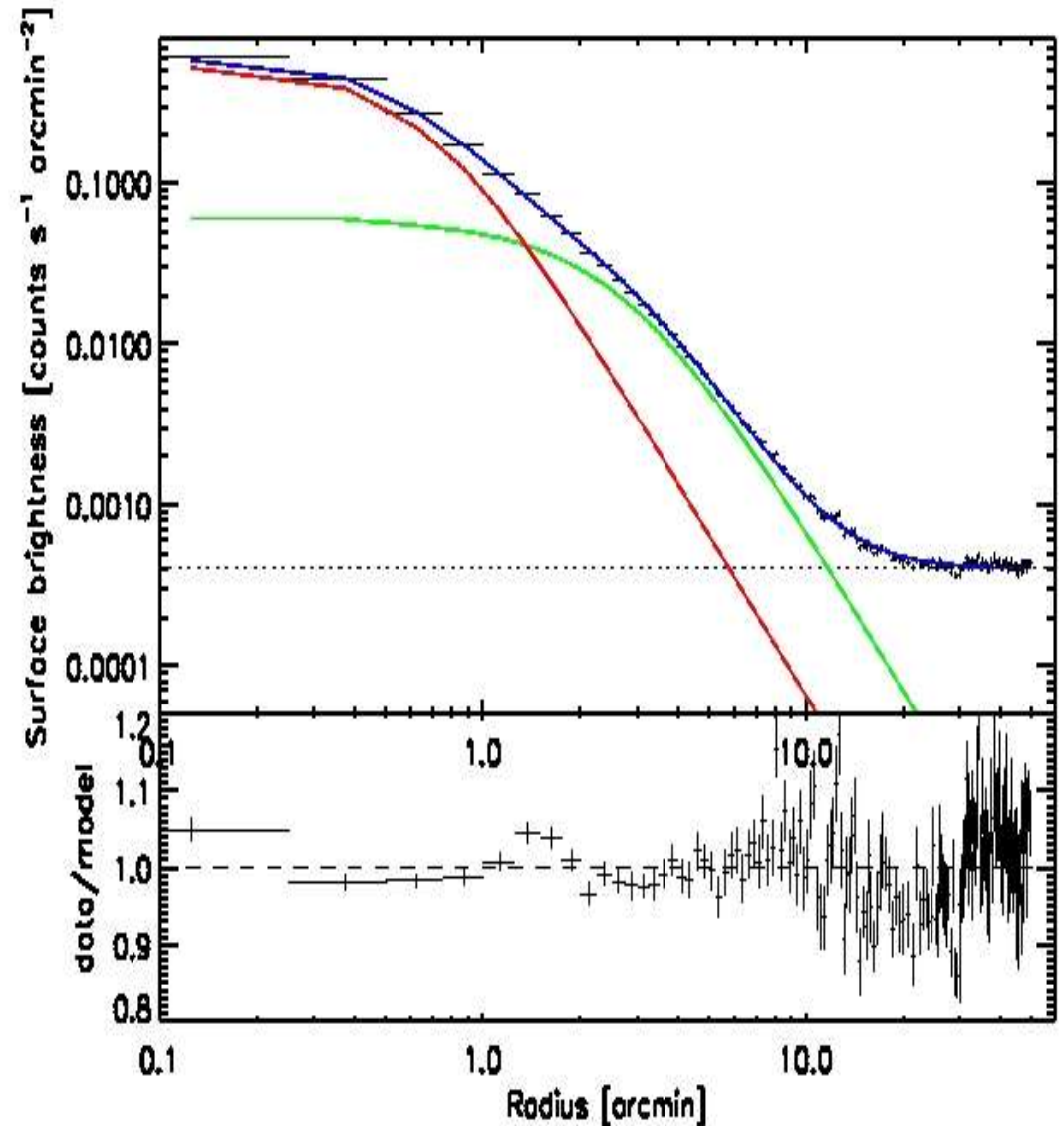


Modeling the surface brightness profile

double β profile

$$I(b) = I_{0,1} \left[1 + \left(\frac{b}{r_{core,1}} \right)^2 \right]^{(-3\beta + 0.5)} +$$

$$I_{0,2} \left[1 + \left(\frac{b}{r_{core,2}} \right)^2 \right]^{(-3\beta + 0.5)}$$



2D to 3D Sarazin 5.5.4

Bremsstrahlung emissivity $\varepsilon_v \propto n_e^2 \rightarrow I_v(b) = f(\varepsilon_v(r)) = f(n_e^2)$ (T dependence negligible)

$$I_v(b) = \text{const} \times \int_{b^2}^{\infty} \frac{\varepsilon_v(r) dr^2}{\sqrt{r^2 - b^2}} \quad (\text{spherical symmetry, const instrumental})$$

$$\rightarrow \varepsilon_v(r) \propto -\frac{1}{2\pi r} \frac{d}{dr} \int_{r^2}^{\infty} \frac{I_v(b) db^2}{\sqrt{b^2 - r^2}}$$

$$(\beta \text{ profile}) \quad I(b) = I_0 \left[1 + \left(\frac{b}{r_{\text{core}}} \right)^2 \right]^{(-3\beta + 0.5)} \rightarrow$$

$$n_e(r) = n_e(0) \left[1 + \left(\frac{r}{r_{\text{core}}} \right)^2 \right]^{(-\frac{3}{2}\beta)}$$

Eq. 2

r_{core} and β (= gas density profile shape) from surface brightness profile fit

Central density

$$n_e/n_p = 1.17 \text{ (for typical cluster mix, } n_p = \text{H density, } n_e \text{ contains He)}$$

Eq. 3

$$\rho_g = 1.35 m_p n_p$$

Eq. 4

$$EM (= \text{emission measure}) \equiv \int n_e n_p dV = \int n_e \frac{n_e}{1.17} dV = \frac{1}{1.17} \int n_e^2 dV \quad (\beta \text{ profile for } n_e \rightarrow)$$

$$EM = \frac{1}{1.17} n_e(0)^2 \int \left[1 + \left(\frac{r}{r_{core}} \right)^2 \right]^{-3\beta} dV \rightarrow$$

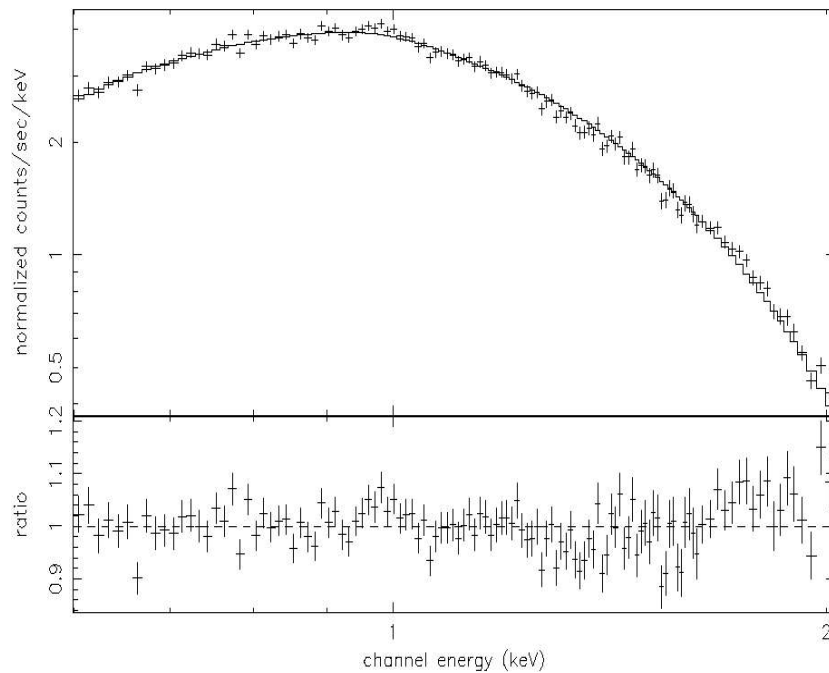
$$n_e(0) = \sqrt{1.17 \times EM / \int \left[1 + \left(\frac{r}{r_{core}} \right)^2 \right]^{-3\beta} dV}$$

Eq. 5

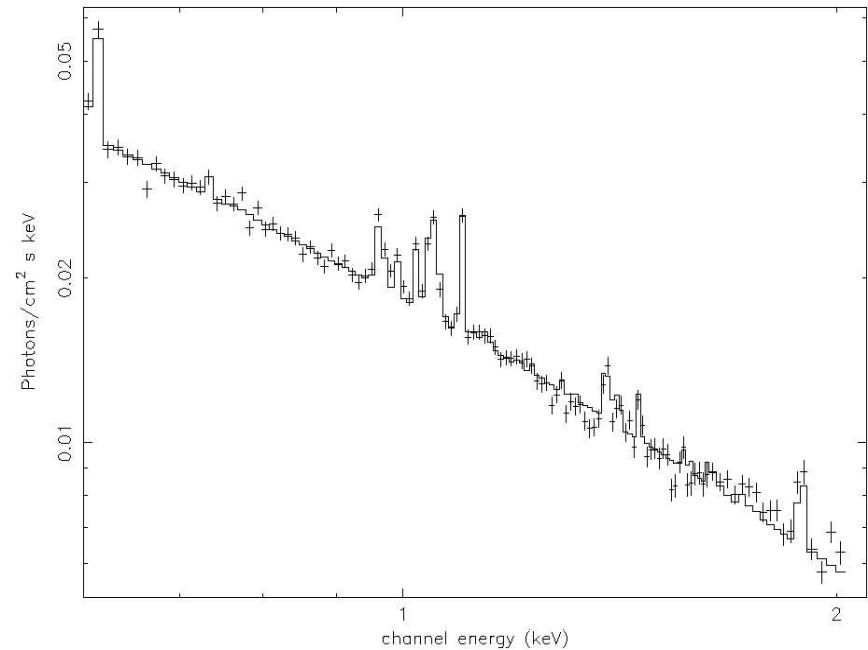
EM from spectral analysis

A1795 15 arcmin circle spectra

Data and folded model



unfolded model



Spectral fit yields: $T = 4.1$ keV, $\text{norm} = 8.21 \times 10^{-2}$

$$EM [cm^{-3}] = \text{norm}_{MEKAL} [cm^{-5}] \times 10^{14} \times 4\pi \left[D_A \times (1+z) \right]^2 [cm^2]$$

Eq. 6

EM $\rightarrow n_0$: example calculation

- A1795 is a cluster at $z = 0.062$, i.e. a distance $DA = 233 \text{ Mpc} (=7.20 \times 10^{26} \text{ cm})$
- Imaging has yielded: $r_{\text{core}} = 2.9 \text{ arcmin}$, $\beta = 0.7$
- Spectral analysis has yielded MEKAL norm = 8.21×10^{-2} within 15 arcmin
- What is the central electron density $n_e(0)$?

- Use Eq. 5 (repeat)
$$n_e(0) = \sqrt{1.17 \times EM / \int \left[1 + \left(\frac{r}{r_{\text{core}}} \right)^2 \right]^{-3\beta} dV}$$

- EM using Eq. 6:

$$EM [cm^{-3}] = norm_{MEKAL} [cm^{-5}] \times 10^{14} \times 4\pi \left[D_A \times (1+z) \right]^2 [cm^2] =$$

$$8.21 \times 10^{-2} cm^{-5} \times 10^{14} \times 4 \times \pi \times (7.20 \times 10^{26} cm)^2 \times (1+0.062)^2 = \underline{6.0 \times 10^{67} cm^{-3}}$$

EM \rightarrow n_0 : example calculation

- The integral in Eq. 5:

$$\int \left[1 + \left(\frac{r}{r_{core}} \right)^2 \right]^{-3\beta} dV \rightarrow 4\pi \int_0^{15 \text{ arcmin}} \left[1 + \left(\frac{r}{2.9} \right)^2 \right]^{-2.1} r^2 dr = \dots \text{numerically} \dots 1.60 \times 10^{72} \text{ cm}^3 \rightarrow$$

$$n_e(0) = \sqrt{\frac{1.17 \times 6.0 \times 10^{67} \text{ cm}^{-3}}{1.60 \times 10^{72} \text{ cm}^3}} = 6.6 \times 10^{-3} \text{ cm}^{-3} \rightarrow$$

- Using Eq. 3 $n_p(0) = n_e(0)/1.17 = 5.6 \times 10^{-3} \text{ cm}^{-3}$

- Using Eq. 4

$$Mpc = 3.086 \times 10^{24} \text{ cm} \quad \rho_g(0) = 1.35 \times m_p \times n_p(0) = 1.35 \times 1.6725 \times 10^{-24} \text{ g} \times 5.6 \times 10^{-3} \text{ cm}^{-3} = 1.3 \times 10^{-26} \text{ g cm}^{-3}$$

$$M_\odot = 1.989 \times 10^{33} \text{ g}$$

\rightarrow

$$\rho_{gas}(0) = \underline{1.9 \times 10^{14} M_\odot Mpc^{-3}}$$

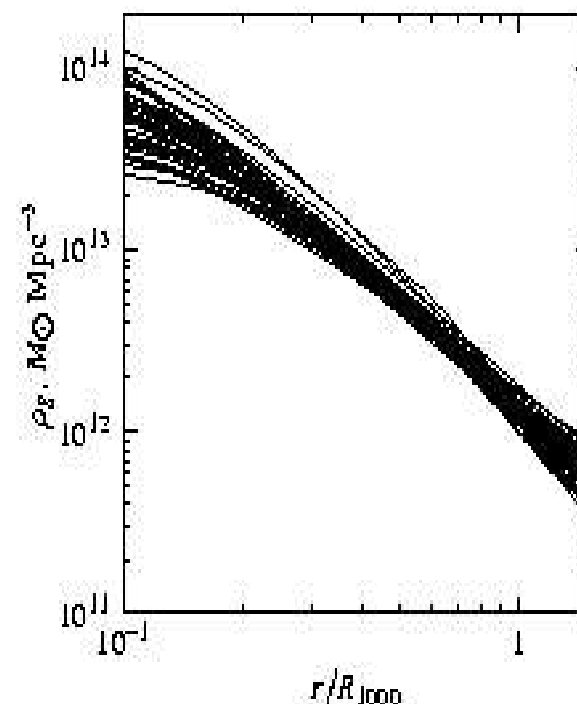
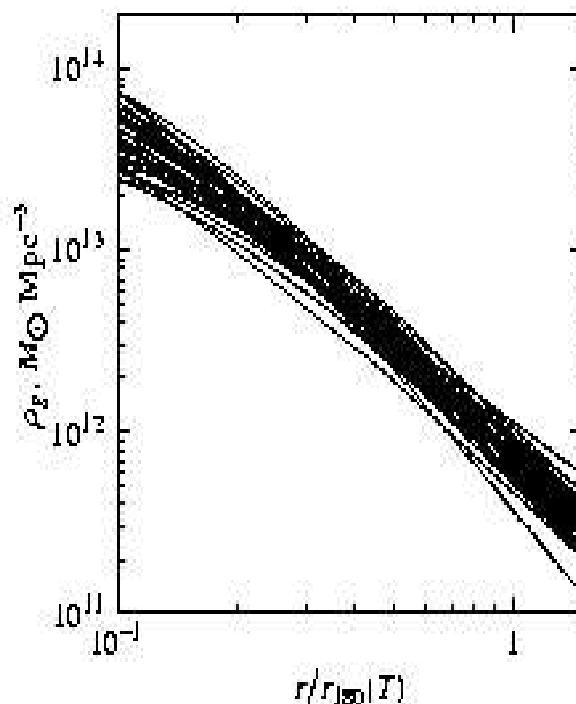
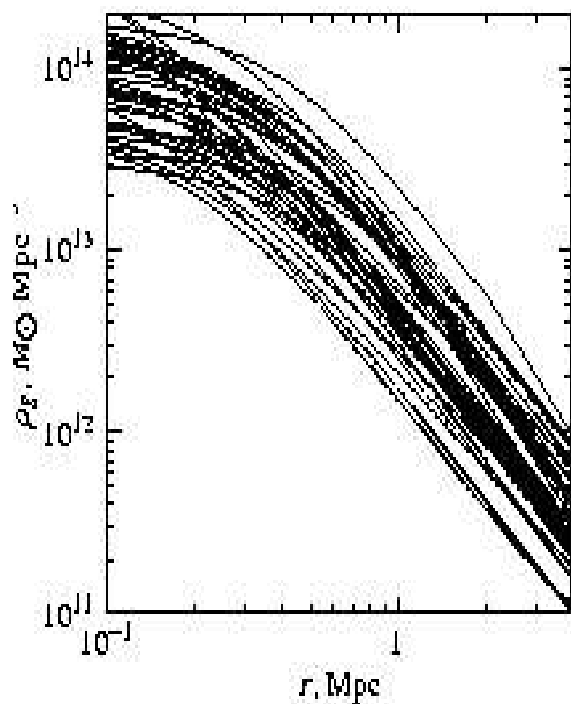
- Now you know the gas density at any radius by Eq. 2

Gas density profiles

- $\rho_{g,0} \sim 10^{14} \text{ M}_{\odot} \text{ Mpc}^{-3} \sim 10^{-3} \text{ cm}^{-3}$ $\beta \sim 0.6 - 0.7 \rightarrow n_e(r) \propto r^{-2}$, when $r \rightarrow \infty$
- $M_{\text{gas}}(< 1 \text{ Mpc}) \sim 10^{14} \text{ M}_{\odot}$

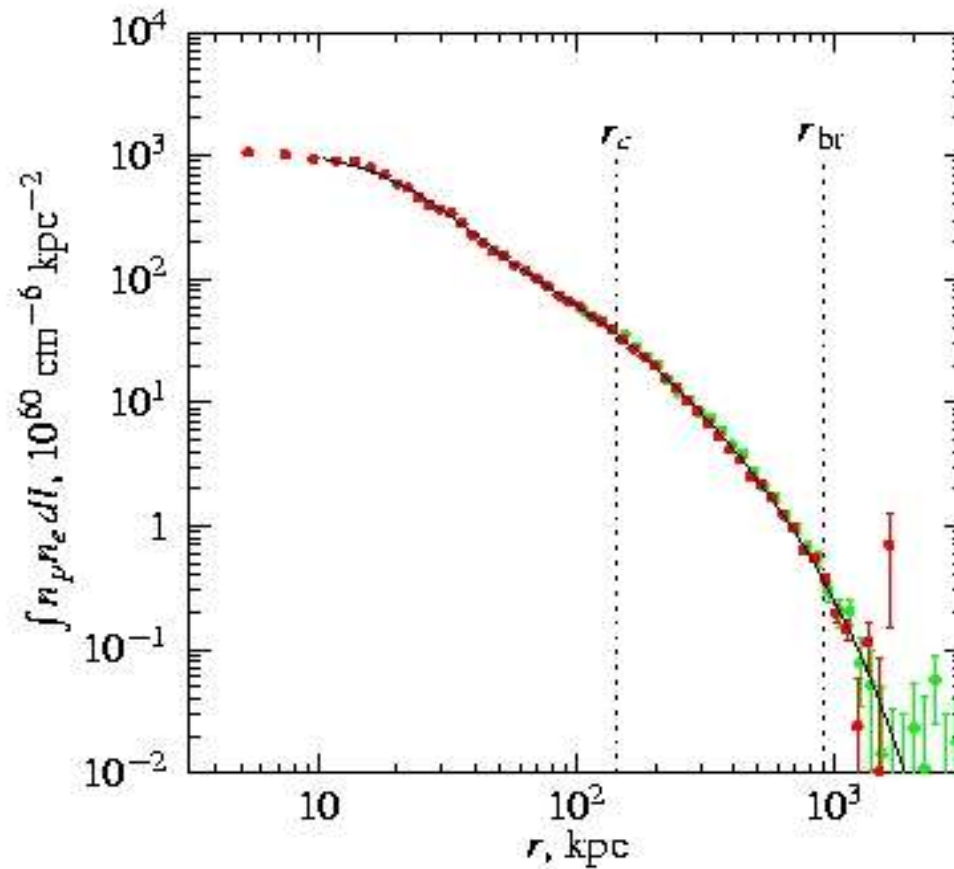
Vikhlinin et al., 1999, ApJ, 525, 47

ROSAT PSPC data



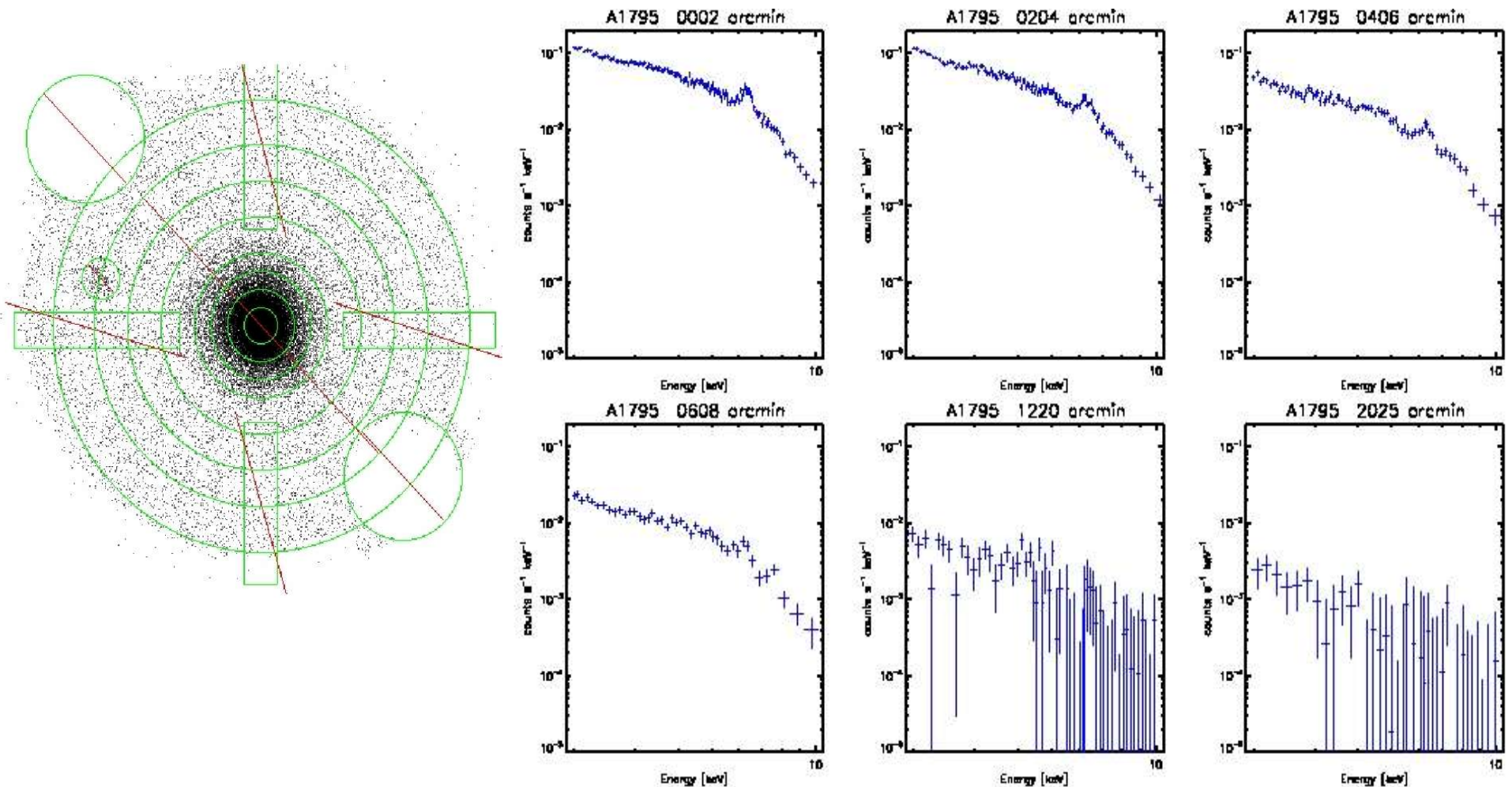
Gas density profiles

- surface brightness steeper than beta model beyond r_{br} (Vikhlinin et al, 2006, ApJ, 640, 691)



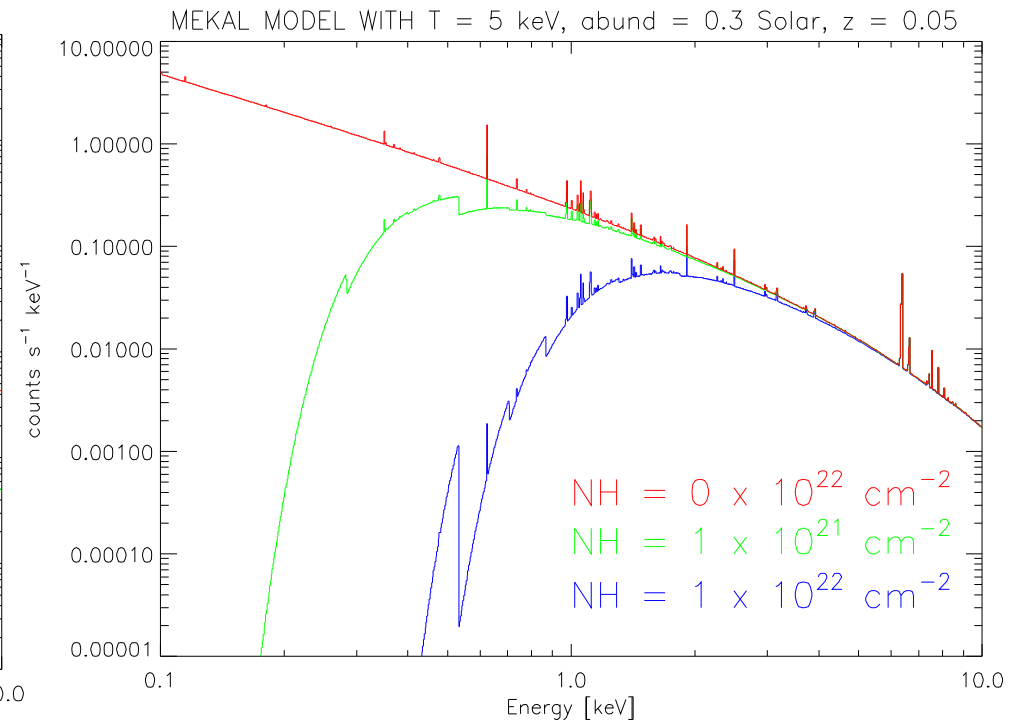
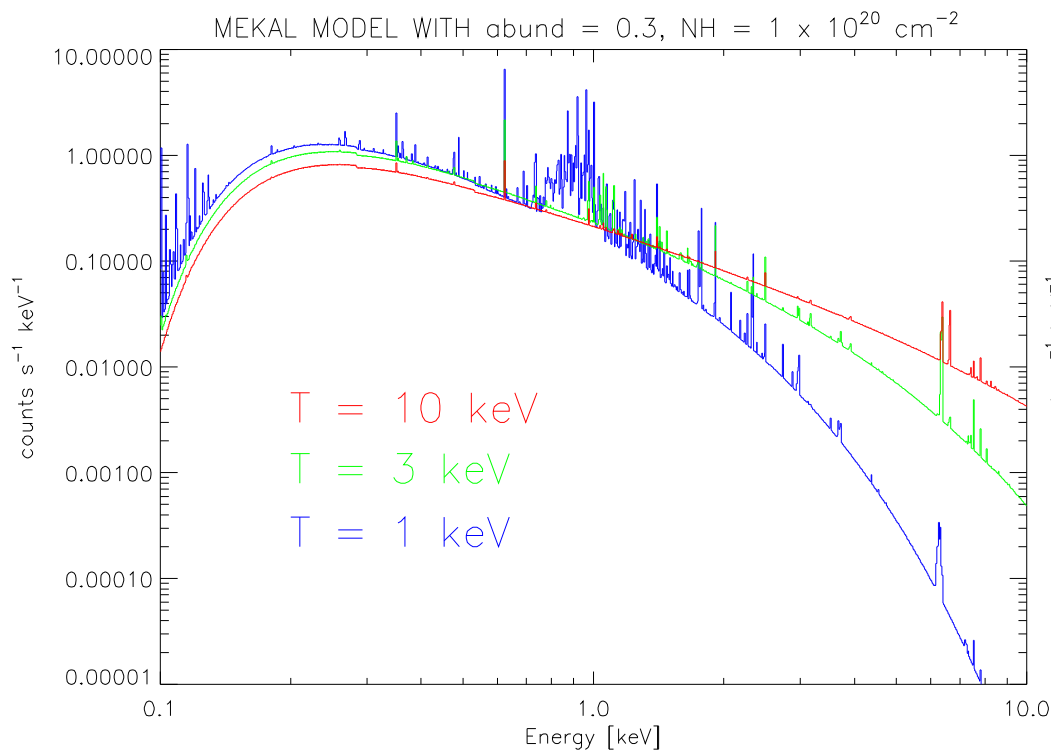
Spectroscopy

- extract spectra in concentric annuli (= integrate events over time, chosen area (X,Y) and sort by energy bins)



Spectral modeling

- MEKAL model (T, abund, redshift) for emission
- WABS model (NH) for Galactic absorption



Spectral fit

- **RMF** (= Redistribution Matrix File):
 - contains dE/E (energy resolution) information of the instrument
 - convolution of the model $\mathbf{S}_{\text{model}}(E)$ with RMF approximates the spreading of counts
- **ARF** (= Auxiliary Response File)
 - contains mirror effective area, filter transmission and detector quantum efficiency
 - $\mathbf{S}_{\text{model}}(E) \times \mathbf{ARF}(E) : [\text{photons s}^{-1} \text{ cm}^{-2} \text{ keV}^{-1}] \rightarrow [\text{counts / channel}]$
- \mathbf{S}_{pred} (= model prediction) = $[\mathbf{S}_{\text{model}}(E) \otimes \mathbf{RMF}(E, \text{channel})] \times \mathbf{ARF}(E)$
- **bkg** (= background = CXRB + Galactic emission + CR induced detector background)
 - must be removed from the total observed signal : $\mathbf{S}_{\text{data}}(E) = \mathbf{S}_{\text{total obs}}(E) - \mathbf{bkg}(E)$

Spectral fit

- background subtracted data is compared with the model prediction
- model parameters varied until best match between background subtracted data and model prediction found, i.e χ^2 minimisation

$$\chi^2 = \sum \frac{\left(S_{data}(E) - S_{pred}(E) \right)^2}{\sigma(E)^2}$$

where

$$S_{data}(E) = S_{total\,obs}(E) - bkg(E)$$

$$S_{pred}(E) = \left[S_{model}(E) \otimes RMF(E, channel) \right] \times ARF(E)$$

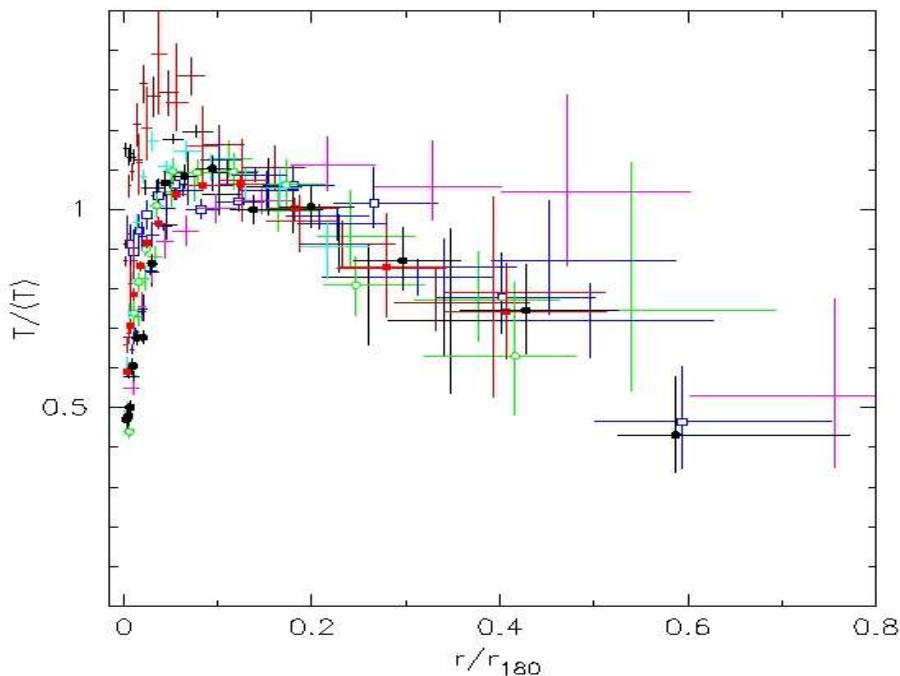
$\sigma(E)$ are the statistical uncertainties

T-profile results

- similar shape with scaled radius: factor of 2 variations inside a given cluster
- Cool core = relaxed cluster, merger destroys cool core
- T gradient at large radii

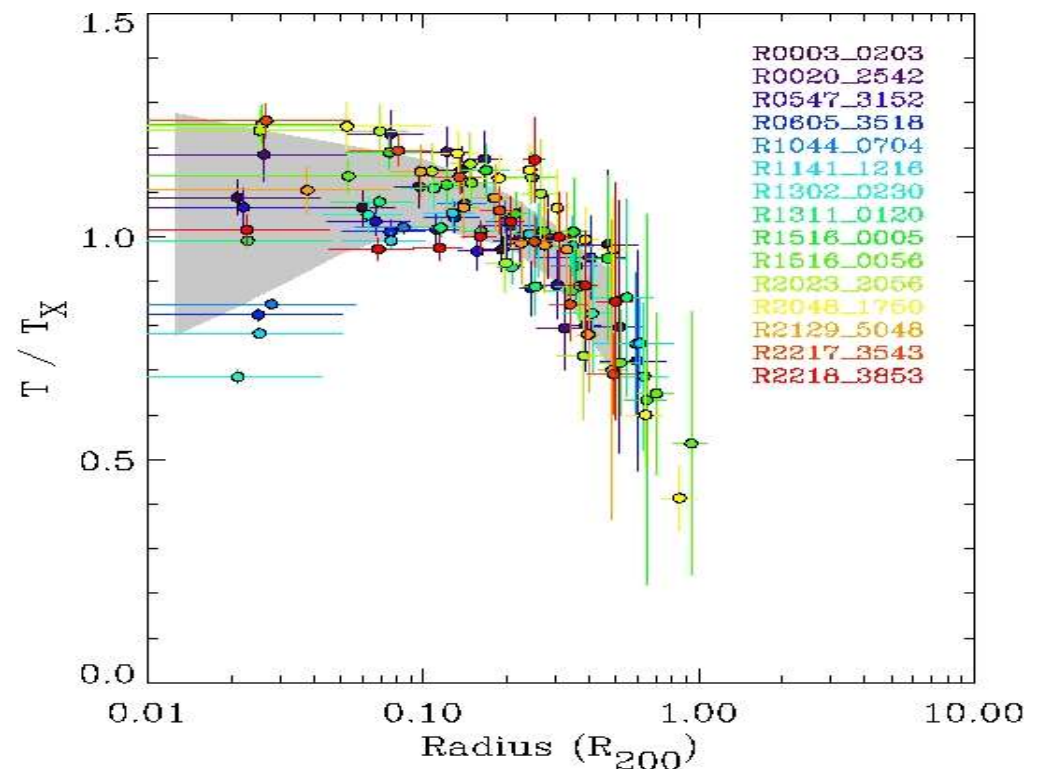
Chandra

Vikhlinin et al., 2005, ApJ 628, 655



XMM-Newton

Pratt et al, 2006, A&A, 446, 429



3D temperature profile models T(r)

- 2D observations → 3D physics
- use 3D T(r) model and project along the line-of-sight

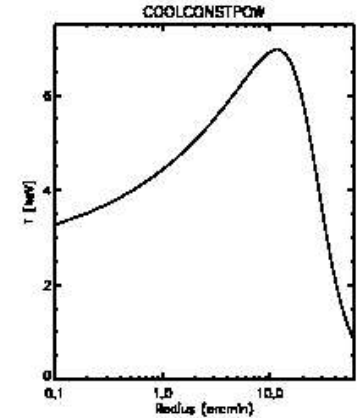
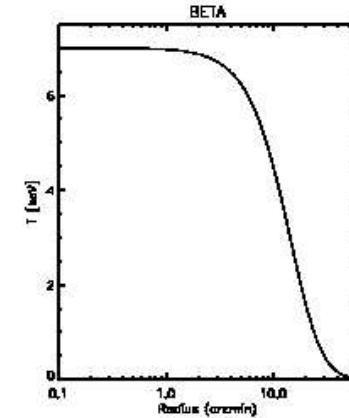
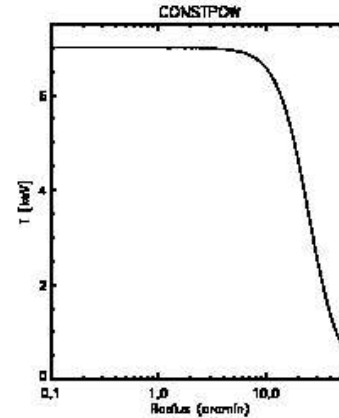
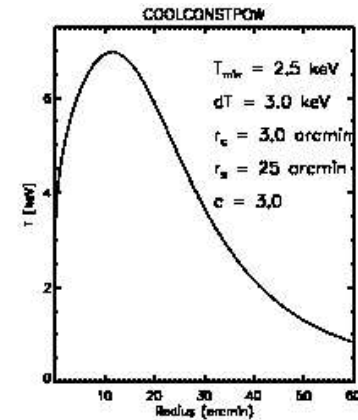
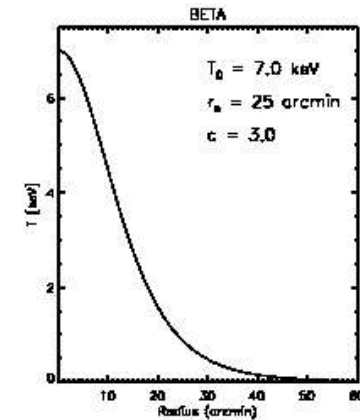
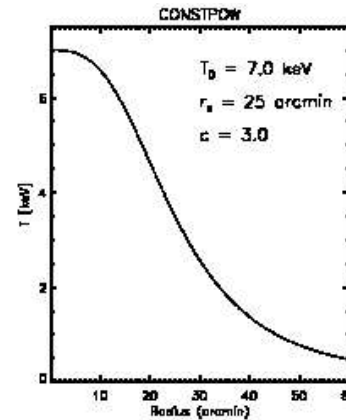
CONSTPOW:

$$T(r) = T_0 \times \left[1 + \left(\frac{r}{r_s} \right)^c \right]^{-1}$$

BETA:

$$T(r) = T_0 \times \left[1 + \left(\frac{r}{r_s} \right)^2 \right]^{-c}$$

$$\text{COOLCONSTPOW: } T(r) = \left[T_{\min} + dt \times \left(\frac{r}{r_c} \right)^{0.4} \right] \times \left[1 + \left(\frac{r}{r_s} \right)^c \right]^{-1}$$



T profile projection and fit

- $T_{\text{mod}}(r)$ = 3D T profile model
- $T_{\text{mod,proj}}(b)$ = model T projection to 2D detector plane

$$T_{\text{mod,proj}}(b) = \frac{\int_V \rho(r)^2 \times T_{\text{mod}}(r) dV}{\int_V \rho(r)^2 dV} \quad (\text{Emission weighted})$$

$$T_{\text{mod,proj}}(b) = \frac{\int_V \rho(r) \times T_{\text{mod}}(r) dV}{\int_V \rho(r) dV} \quad (\text{Mass weighted})$$

$$\chi^2 = \sum \frac{\left(T_{\text{data}}(b) - T_{\text{mod,proj}}(b) \right)^2}{\sigma_T^2}$$

$T_{\text{mod}}(r)$ parameters varied, χ^2 minimum gives best fit parameters

T profile projection and fit

- COOLCONSTPOW model (simulated data)

$$T_{\min} = 1.8 \text{ keV}$$

$$dT = 3.1 \text{ keV}$$

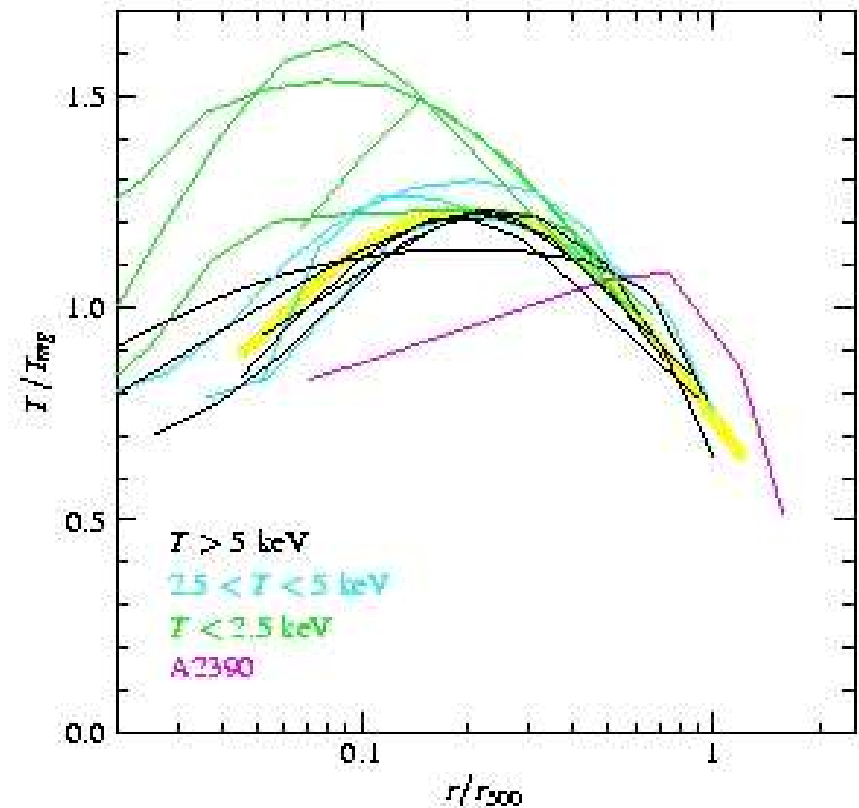
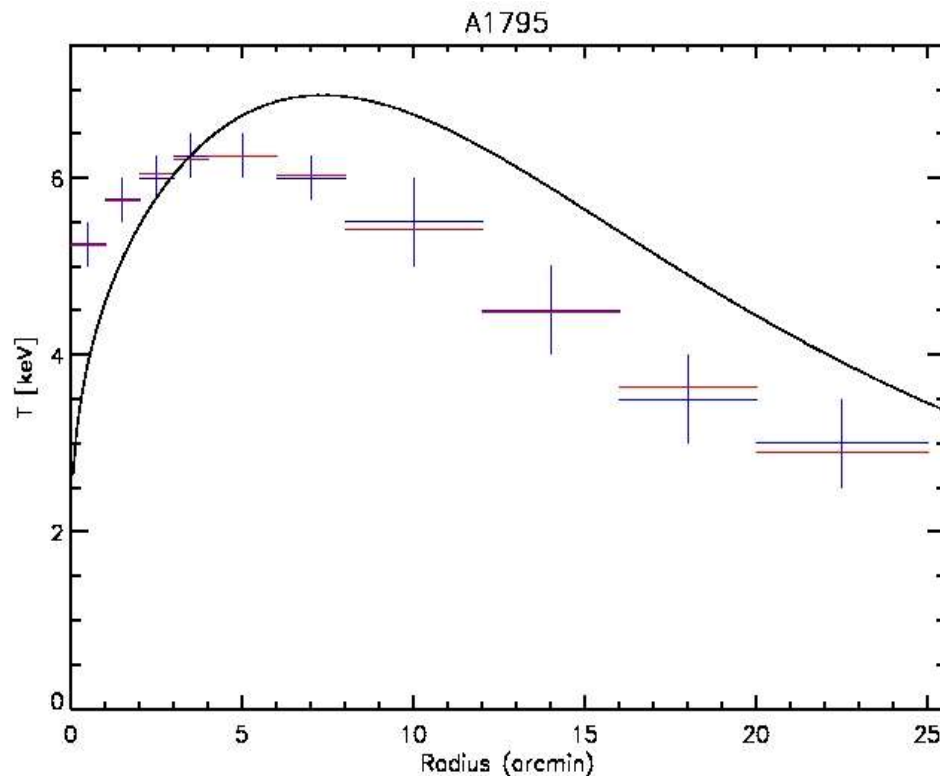
$$r_c = 1.3 \text{ arcmin}$$

$$r_s = 16.6 \text{ arcmin}$$

$$c = 2.2$$

Results:

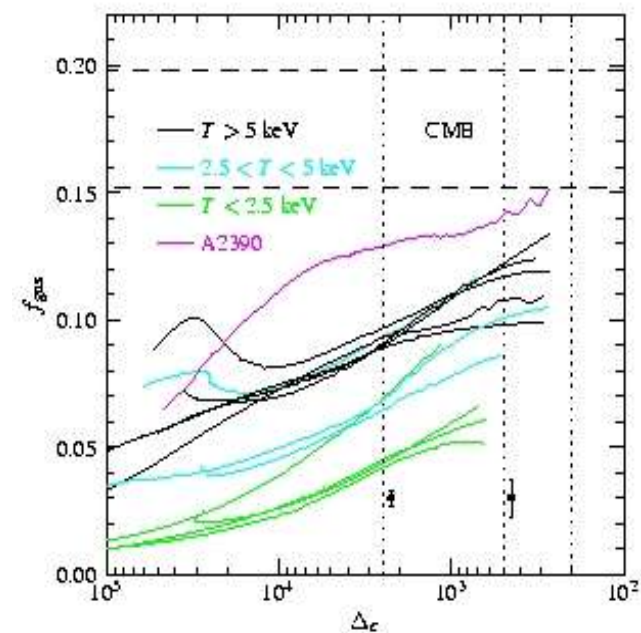
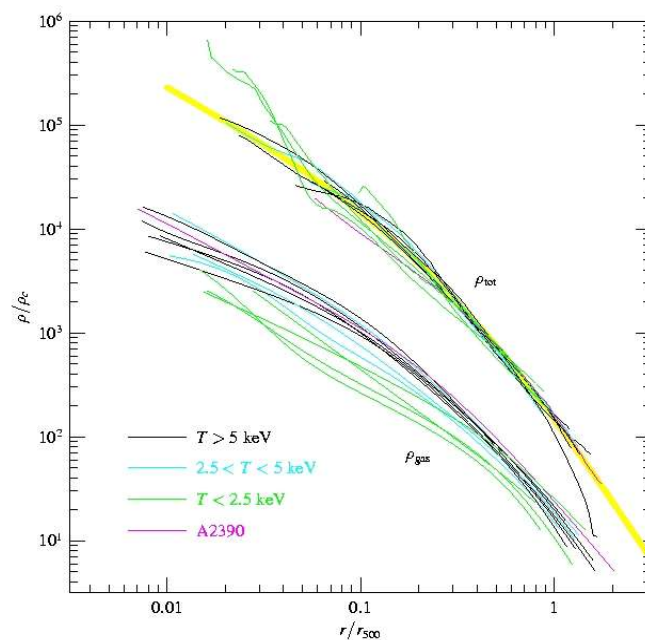
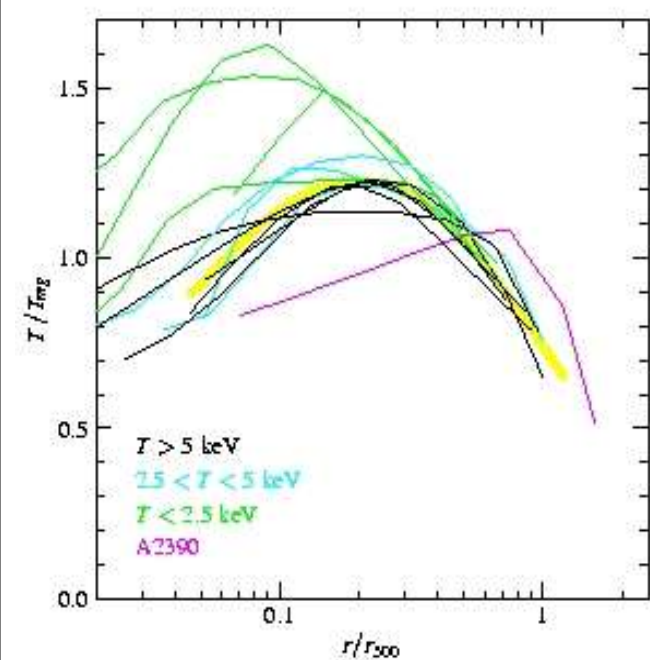
Chandra data (Vikhlinin et al., 2005, ApJ 628, 655)



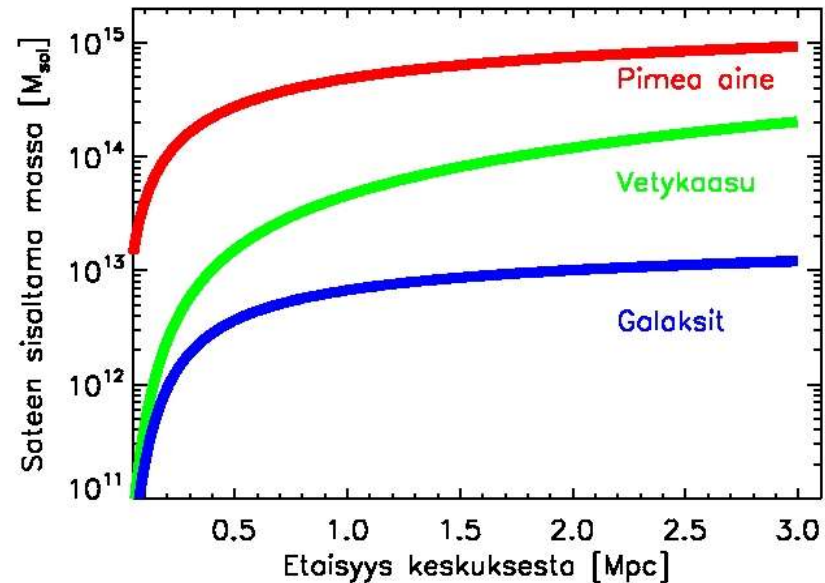
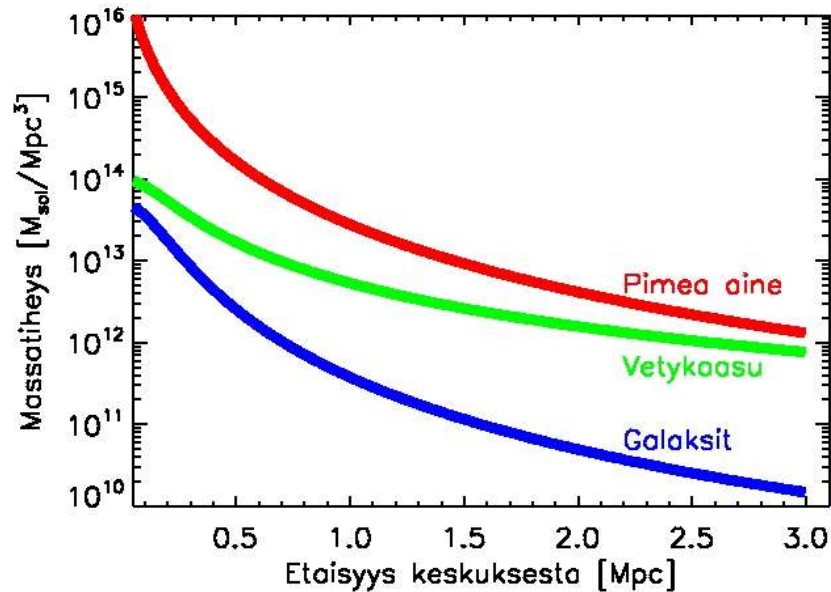
3D profile results

- Now we have $\rho_g(r)$ and $T(r)$ for the evaluation of hydrostatic equation (Eq. 1) \rightarrow total mass profile
- similar T profiles (and similar gas density profiles) \rightarrow similar DM profiles

Chandra data (Vikhlinin et al., 2005, ApJ 628, 655)



Mass components



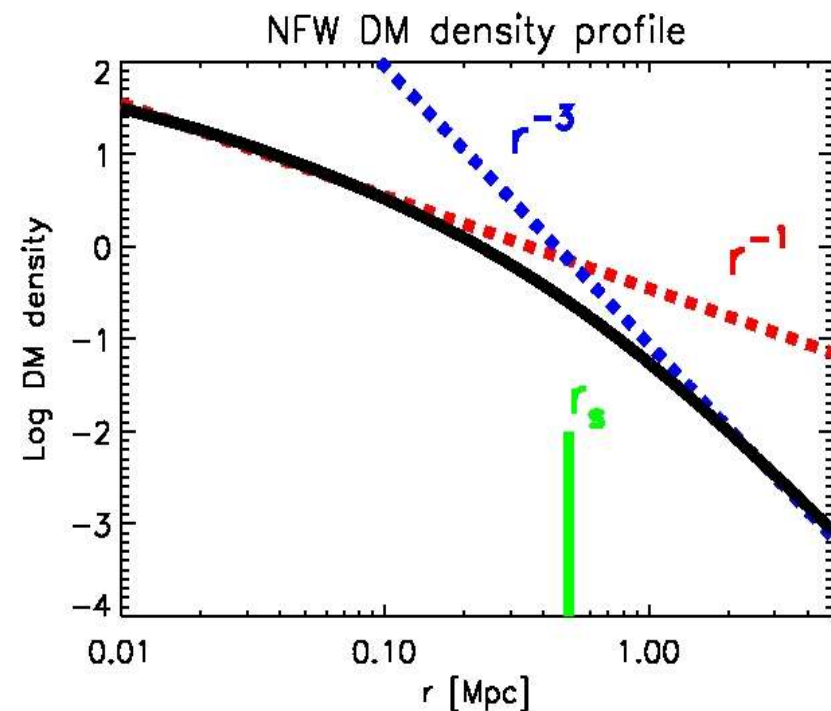
- gas density $\rho_{\text{gas}} \propto r^{-2}$
- galaxy mass $M_{\text{gal}} \sim 10^{13} M_{\odot} \sim 1 \%$
- gas mass $M_{\text{gas}} \sim 10^{14} M_{\odot} \sim 10\%$
- dark matter $M_{\text{DM}} \sim 10^{15} M_{\odot} \sim 90\%$

NFW

- Universal NFW (Navarro-Frenk-White) dark matter density profile from simulations (Navarro et al, 1997, ApJ, 490, 493)

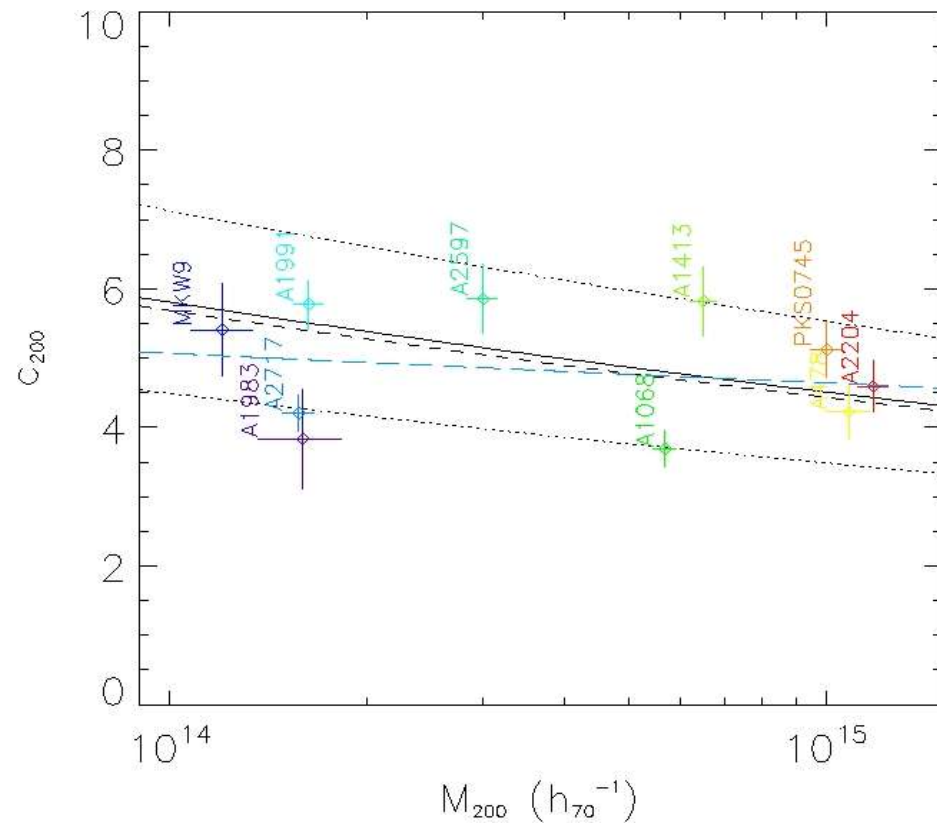
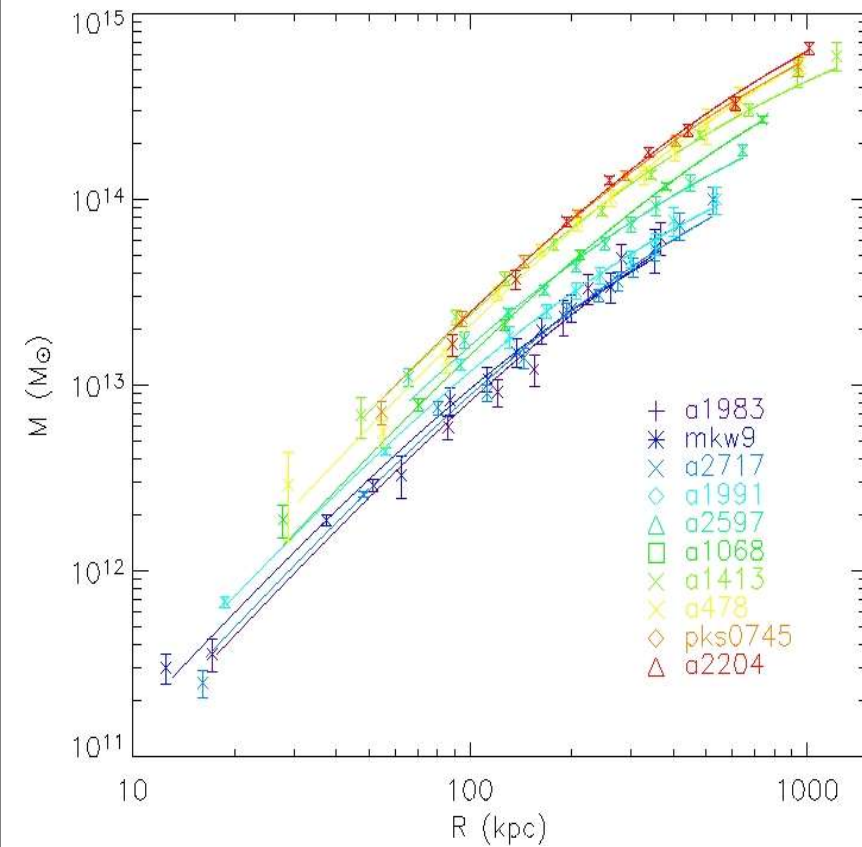
$$\rho_{DM}(r) \propto \left(\frac{r}{r_s} \right)^{-1} \times \left(1 + \frac{r}{r_s} \right)^{-2}$$

- central cusp $\rho_{DM}(r) \propto r^{-1}$
- steepening towards $\rho_{DM}(r) \propto r^{-3}$ beyond r_s



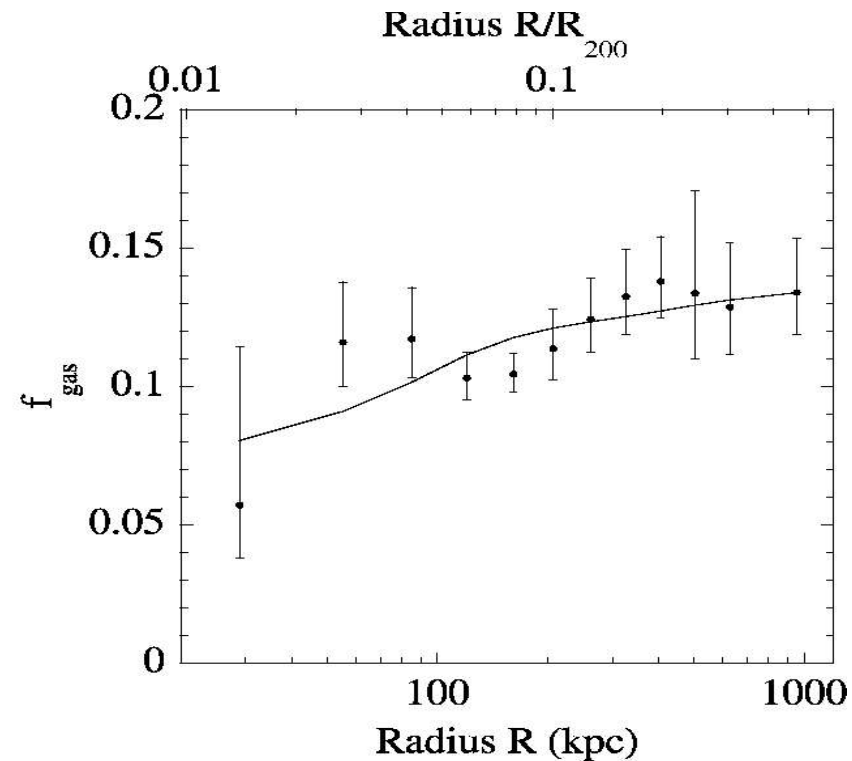
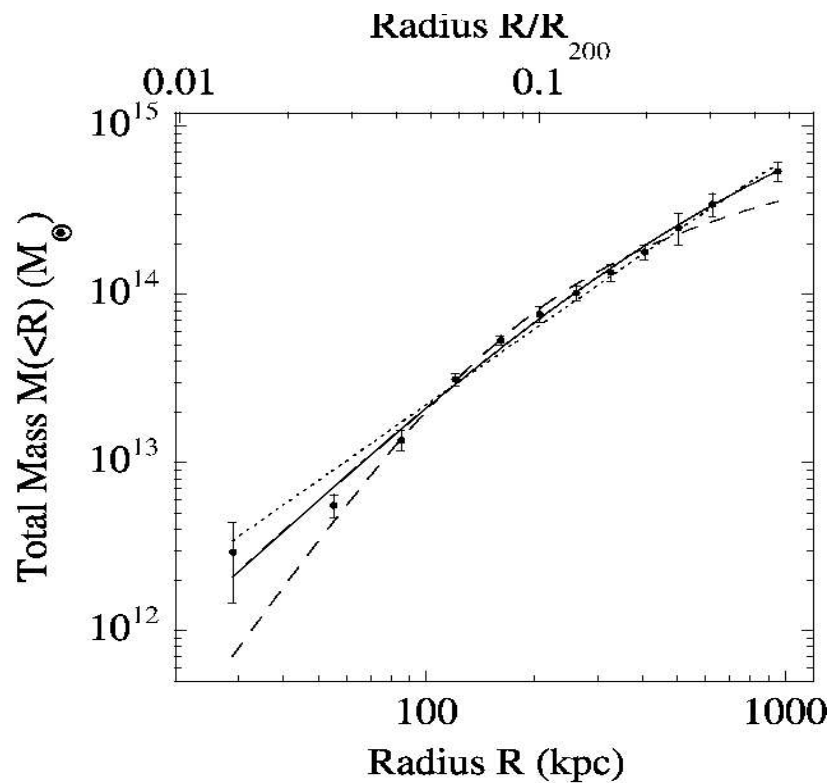
NFW modeling

- Pointecouteau, 2005 A&A...435: XMM



NFW modeling

- A&A., 2002, 423, 33: A478 with XMM-Newton



Gas mass fraction results

- with gas and total mass profile, we get the gas mass fraction profile:

$$f_{\text{gas}}(r) = M_{\text{gas}}(r) / M_{\text{tot}}(r)$$

- recent analyses yield: $f_{\text{gas}}(r_{\text{vir}}) \sim 15\%$ ($H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$)

Vikhlinin et al., 2006, ApJ, 640, 691

$$f_{\text{gas}}(r < r_{500})$$

