Lecture 3 Notes for STT861

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1 Review

Def A set with only one outcome from **S** is called a **simple event**.

Def A set with more than one outcome is known as a **composite event**.

Def A **set function** is a function defined on a set.

Def **Probability** is a set function which takes a real value.

Def The **Power Set** $\mathscr{P}$ = collection of all subset of **S**

2 Class Notes

Remark Consider $\mathbf{S} = \{H, T\}$ and $\mathscr{P}(\mathbf{S}) = \{\emptyset, \{H\}, \{T\}, \mathbf{S}\}$
If **S** is countable, we can take $\mathscr{P}(\mathbf{S})$ as the domain for probability function P.
However, if **S** is uncountable, then **S** is too large, and it is not possible to define a function for $\mathscr{P}(\mathbf{S})$

Def $\mathscr{A}$ is a collection of subsets of $\mathbf{S}[\neq \emptyset]$ satisfying:
i) $\mathbf{S} \in \mathscr{A}$
ii) if $A \subset \mathscr{A}$, then $A^c \in \mathscr{A}$
iii) if $A_1, A_2, ... \in \mathscr{A}$ then $\cup_{i=1}^{\infty} A_i \in \mathscr{A}$
We call $\mathscr{A}$ a σ - algebra (or σ - field)
Any domain should be a σ - field

Ex 1 For $\mathbf{S} = \{H, T\}$ let's define $\mathscr{A} = \{\emptyset, \mathbf{S}\}$ (the trivial σ - field)

Ex 2 $\mathbf{S} = \{a, b, c\}$ let's deffine $\mathscr{A} = \{\emptyset, \{a\}, \{b, c\}, \mathbf{S}\}$ (a σ - field)

Def $(\mathbf{S}, \mathscr{A})$ is a measurable space

Def $(\mathbf{S}, \mathscr{A}, P)$ is a (probability) measure space

Ex 3 If $A, B \in \mathscr{A}$, is $A \cup B$ in $\mathscr{A}$?
Yes. By req. iii) if $A_1 = A, A_2 = B, A_3 = \emptyset = A_4 = A_5 ...$

Ex 4 If $A, B \in \mathscr{A}$ is $A \cap B in \mathscr{A}$?
Yes. By req ii) $A^c, B^c \in \mathscr{A} \Rightarrow A^c \cup B^c \in \mathscr{A}$
Therefore, $(A^c \cup B^c)^c \in \mathscr{A}$
By De Morgan's law, $(A^c \cup B^c)^c = A \cap B$

Def Given sample space $\mathbf{S}(\neq \emptyset)$, and the measurable space $(\mathbf{S}, \mathscr{A})$
A function $P : \mathscr{A} \mapsto \mathbb{R}$ is called probability if it satisfies:

a. $P(A) \geq 0$ for any $A \in mathscrA$

b. $P(\mathbf{S}) = 1$

c. if $A_1, A_2 ...$ are disjoint sets from $\mathscr{A}$, then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Desired Properties of P(.)

a. $P(\emptyset) = 0$

b. If A and B are disjoint then $P(A \cup B) = P(A) + P(B)$

c. $P(A^c) = 1 - P(A)$

d. If $A \subset B$ then $P(A) \leq P(B)$

e. $P(A) \leq 1$