Lecture 5 Notes for STT861

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1 Review

$\mathbf{S}(\neq \emptyset)$ Sample Space $\mathscr{A} = \sigma$ - field A probability $P : \mathscr{A} \mapsto \mathbb{R}$ satisfies $P(A) \geq 0 \, \forall A \in \mathscr{A}$
$P(\mathbf{S}) = 1$
If $A_n \in \mathscr{A}, n = 1, 2, ...$ and $A_n's$ are parwise disjoint then $P(\cup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} P(A_n)$

2 Class Notes

Ex 1 Let $\mathbf{S} = \{s_1, s_2, ...\}$ be a countable sample space
Let $\mathscr{A} = \mathscr{P}(\mathbf{S})$ Suppose $\{P_n : n \geq 1\}$ in a sequence satisfying:

a. $p_n \geq 0$

b. $\sum_{n=1}^{\infty} p_n = 1$

For any $A \in \mathscr{A}$ define: $P(A) = \sum_{\{j: s_j \in A\}} p_j$

Remark Suppose:
$A = \{s_3, s_{10}, s_{19}\}$
$P(A) = p_3 + p_{10} + p_{19}$

Suppose:
$A_1 = \{s_{11}, s_{12}, s_{13}...\}$
$A_2 = \{s_{21}, s_{22}, s_{23}...\}$
$A_3 = \{s_{31}, s_{32}, s_{33}...\}$ etc.
All A_n are pairwise disjoint and therefore, s_{ij} are distinct for all i, j
$P(\cup_{n=1}^{\infty} A_n) = P(\cup_{n-1}^{\infty} \{s_{n1}, s_{n2}...\})$
$= P(\cup_{n=1}^{\infty} \cup_{j=1}^{\infty} \{s_{nj}\}) = \sum_{n=1}^{\infty} (\sum_{j=1}^{\infty} p_{nj})$
As $A_n = \{s_{n1}, s_{n2}...\}$ therefore $P(A_n) = \sum_{j=1}^{\infty} p_{nj}$

Suppose:
$\mathbf{S} = \{1, 2, 3, 4...\}$ and $p_n = (frac12)^n n = 1, 2, 3...$
Then, $\sum_{n-1}^{\infty} = 1$ by geometric sum: $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ if $|a| < 1$
Therefore,

$$\sum_{n=1}^{\infty} (\frac{1}{2})^n = \sum_{n=0}^{\infty} (\frac{1}{2})^n - 1 = \frac{1}{1 - 1/2} - 1 = 1$$

Suppose for some set $A, P(A) = 1$ Does this imply $A = S$?
Does $P(B) = 0$ imply $B \neq \emptyset$?
Does $P(A \cap B) = 0$ imply A, B are disjoint?
Not necessarily for all above!

Suppose $S = \{1, 2, 3, 4, ...\}$
$p_n = 0$ for all odd n

$p_n = \frac{1}{2}^{\frac{n}{2}}$ for all even n
$P(\text{odd}) = 0$ and $P(\text{even}) = 1$
Let $A = \{1, 3, 5\}$, $B = \{3, 5\}$. $P(A \cap B) = 0$ and $A \cap B = \{3, 5\}$

2.1 Counting

	WOR	**WR**
Ordered	$\frac{n!}{(n-r)!}$	n^r
Unordered	$\frac{n!}{r!(n-r)!} = \binom{n}{r}$	$\binom{n+r-1}{r}$

<u>**Def**</u> **Multiplication Rule**