STT 861 Compendium - Part Two

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Expected Values

<u>**Def**</u> The **expected value** of a random variable $g(X)$:

$$\mathbb{E}(g(X)) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} g(x) f_X(x) dx & \text{if } X \text{ is continuous} \\ \displaystyle\sum_X g(x) f_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

Remark If $-\infty \leq \mathbb{E}(g(x)) \leq \infty$ we say the expectation of $g(x)$ exists. Else it does not exist.
notation: $|\mathbb{E}(g(x))| \leq \infty$
In particular, if $g(x) = x$, then we get the expected value of X:

$$\mathbb{E}(g(X)) = \begin{cases} \displaystyle\int x f_X(x) dx & \text{if } X \text{ is continuous} \\ \displaystyle\sum x f_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

This is called the **mean** of r.v. X
Also denoted by μ or μ_X

Theorem Expectation

a. $\mathbb{E}(ag(x) + b) = a\mathbb{E}(g(x)) + b a,b$ real constants

b. If$g(x) \geq 0$for all$x \in \mathbb{R}$, then$\mathbb{E}(g(x)) \geq 0$

c. If$g_1(x) \geq g_2(x)$for all$x \in \mathbb{R}$, then$\mathbb{E}(g_1(x)) \geq \mathbb{E}(g_2(x))$

d. For any real constants a,b if $a \leq X \leq b$ then $a < \mathbb{E}(X) < b$

Remark Expectation

a. If g is a linear fn, $\mathbb{E}[g(x)] = g[\mathbb{E}(x)]$

b. $g(x)$ has finite expectation if $0 \leq \mathbb{E}[|g(x)|] \leq \infty$

Moments

<u>**Def**</u> **Moments** For a r.v. X, we define the r^{th} raw moments by

$$\mu_r' = \begin{cases} \displaystyle\int x' f_X(x) dx & \text{if } X \text{ is continuous} \\ \displaystyle\sum x' f_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

$\mu_1' = \mathbb{E}(x') = \mathbb{E}(x) = \mu$

<u>**Def**</u> The r^{th} central moment is defined as $\mu_r = \mathbb{E}[(x-\mu)^r]$

Theorem 2.3.11
Let $F_X(x)$ and $F_Y(y)$ be two cdfs all of whose moments exist. If X, Y have bounded support
(i.e. Support$(f_X) = \{x : f_X(x) = 0\}$)
Then $F_X(u) = F_Y(u) \, \forall \, u$ iff $\mathbb{E}(x') = \mathbb{E}(y') \, \forall \, r \in \mathbb{Z}$
Example: P(Heads) for a coin flip and P(Even) for a die roll have the same distribution, but are different variables.

<u>**Def**</u> **Moment Generating Function**: For a r.v. X, the mgf is defined as: $M_X(t) = \mathbb{E}(e^{tx})$ provided the expectation exists for all t in a neighborhood of 0. I.e. $\mathbb{E} < \infty \, \forall \, t \in (-h, h)$for some$h > 0$