Class Notes for STT861

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1 Lecture One

Ex 1. Toss a fair coin; what is the probability of obtaining heads? $P(H) = \frac{1}{2}$

Ex 2. Throw a fair die; what is the probability of obtaining 6? What about obtaining an even number?
$P(6) = \frac{1}{6}$
$P(n = 2, 4, 6) = \frac{1}{2}$

The probability is not the realized result, but the convergence of results as the number of iterations approaches infinity. Review the Weak Law of Large Numbers.

<u>Def.</u> A **random experiment** is an action which will result in one of the many possible outcomes.

<u>Def.</u> A **sample space** is the collection of all possible outcomes of a random experiment. We shall denote by **S**.

<u>Def.</u> A **set** is a collection of some <u>well defined</u> objects.

<u>Def.</u> **Outcomes** are also called **sample points**.

<u>Def.</u> An **Event** is a subset of sample space **S** for which we can define probability.

<u>Def.</u> Suppose A and B are two sets. $A \subset B$ (A is a **subset** of B) if $x \in A$ implies $x \in B$. If $A \subset B$ and $B \subset A$ then $A = B$.

<u>Def.</u> A set is called an empty set (or **null set**) if it contains no elements.
Notation: $\{\emptyset\}$
Convention: $\emptyset \subset A$, for any set A
Corrolary: $\forall A, \emptyset \subset A \subset \mathbf{S}$

<u>Def.</u> **Complement** A^c is the set such that $x \in A^c \Rightarrow x \notin A$.
In other words, $A^c = \{x : x \notin A\}$
Notation: A^c or A' or $\overline{A}$

<u>Def.</u> **Intersection** A, B are two events.
$A \cap B = \{x : x \in A \, and \, x \in B\}$

<u>Def.</u> **Union** A, B are two events.
$A \cup B = \{x : x \in A \, or \, x \in B \, or \, both\}$

<u>Def.</u> A and B are **disjoint** if $A \cap B = \emptyset$

<u>Properties of set theory</u>
Commutative
$A \cup B = B \cup A$ and $A \cap B = B \cap A$
Associative
$(A \cup B) \cup C = A \cup (B \cup C) =: A \cup B \cup C$
$(A \cap B) \cap C = A \cap (B \cap C) =: A \cap B \cap C$
Distributive
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
De Morgan's Law
$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$

<u>Def.</u> Set Difference
$A \setminus B = \{x : x \in A, \text{ but } x \notin B\}$

<u>Def.</u> Symmetric Difference
$A \triangle B = \{x : x \in A \setminus B, \text{ or } x \in B \setminus A\}$

<u>Def.</u> A set A is **finite** if there exists a 1-1 fn $A \mapsto \{1, 2, ..., n\}$ for some $n \in \mathbb{N}$

<u>Def.</u> A set with only one outcome from **S** is called a **simple event**.

<u>Def.</u> A set with more than one outcome is known as a **composite event**.

<u>Def.</u> A **set function** is a function defined on a set.

<u>Def.</u> Probability is a set function which takes a real value.