Class Notes for STT861

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2019 - 09 - 01

1 Lecture One

Ex 1. Toss a fair coin; what is the probability of obtaining heads? $P(H) = \frac{1}{2}$

Ex 2. Throw a fair die; what is the probability of obtaining 6? What about obtaining an even number?
$P(6) = \frac{1}{6}$
$P(n = 2, 4, 6) = \frac{1}{2}$

The probability is not the realized result, but the convergence of results as the number of iterations approaches infinity. Review the Weak Law of Large Numbers.

Def. A **random experiment** is an action which will result in one of the many possible outcomes.

Def. A **sample space** is the collection of all possible outcomes of a random experiment. We shall denote by **S**.

Def. A **set** is a collection of some well defined objects.

Def. **Outcomes** are also called **sample points**.

Def. An **Event** is a subset of sample space **S** for which we can define probability.

Def. Suppose A and B are two sets. $A \subset B$ (A is a **subset** of B) if $x \in A$ implies $x \in B$. If $A \subset B$ and $B \subset A$ then $A = B$.

Def. A set is called an empty set (or **null set**) if it contains no elements.
Notation: $\{\emptyset\}$
Convention: $\emptyset \subset A$, for any set A
Corrolary: $\forall A$, $\emptyset \subset A \subset \mathbf{S}$

Def. **Complement** A^c is the set such that $x \in A^c \Rightarrow x \notin A$.
In other words, $A^c = \{x : x \notin A\}$
Notation: A^c *or* A' *or* $\overline{A}$

Def. **Intersection** A, B are two events.
$A \cap B = \{x : x \in A and x \in B\}$

Def. **Union** A, B are two events.
$A \cup B = \{x : x \in A or x \in B or both\}$

Def. A and B are **disjoint** if $A \cap B = \emptyset$

Properties of set theory
Commutative
$A \cup B = B \cup A$ and $A \cap B = B \cap A$
Associative
$(A \cup B) \cup C = A \cup (B \cup C) =: A \cup B \cup C$
$(A \cap B) \cap C = A \cap (B \cap C) =: A \cap B \cap C$
Distributive
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
De Morgan's Law
$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$

Def. **Set Difference**
$A \setminus B = \{x : x \in A, \text{ but } x \notin B\}$

Def. **Symmetric Difference**
$A \triangle B = \{x : x \in A \setminus B, \text{ or } x \in B \setminus A\}$

Def. A set A is **finite** if there exists a 1-1 fn $A \mapsto \{1, 2, ..., n\}$ for some $n \in \mathbb{N}$