Class Notes for STT861

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1 Review

Definitions

$\mathbf{S}(\neq \emptyset)$ is the sample space.
$\mathscr{A} : \alpha$ - field on $\mathbf{S}$
A set function $P : \mathscr{A} \mapsto \mathbb{R}$ is a probability if it satisfied
  i) $P(A) \geq 0 \ \forall \ A \in \mathscr{A}$
  ii) $P(S) = 1$
  iii) if $A_1, A_2, ... \in \mathscr{A}$ are pairwise disjoint, then
    $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{infty} P(A_i)$

Consequences

a) $P(\emptyset) = 0$
b) if A and B are pairwise disjoint, then $P(A \cup B) = P(A) + P(B)$
  General Form: if A_1, ... , A_n are pairwise disjoint, then
    $P(\cup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$
c) $P(A^c) = 1 - P(A)$
d) if $A \subset B$, then $P(A) \geq P(B)$ and $P(B \setminus A) = P(B) - P(A)$
e) $P(A) \geq 1$, $\forall \ A \subset (A)$

2 Class Notes

Def. A collection of sets $\{E_1, E_2, ...\}$ is called a **partition** of event A if:
  i) $E_i \cap E_j = \emptyset$, $\forall \ i \neq j$ *(pairwise disjoint)*
  ii) $\cup_{i=1}^{\infty} E_i = A$ *(Exhaustive)*

Remark. Partition $\{E_1,$ ..., $E_n\}$ is a finite partition

Remark. For $\mathbf{S}$, $\{A, A^c\}$ is a partition

Remark. If $E_n : n \geq 1$ is a partition of A, then $P(A) = \sum_{i=1}^{\infty} P(E_i)$

More Consequences

f) Suppose A and B are two events. Then $\{A \cap B, A^c \cap B\}$ is a partition of B. Also, $P(A^c \cap B) = P(B) - P(A \cap B)$
g) A and B are two events. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  (This is the generalized version of b))
h) if $\{C_1, C_2, ...\}$ is a partition of $\mathbf{S}$, then
  $P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

Def. i) **Boole's Inequality**
  $P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i)$

Def. j) **Bonferroni's Inequality**
$P(\cap_{i=1}^{\infty} A_i) \geq 1 - \sum_{i=1}^{\infty} P(A_i^c)$

Def. A sequence of events $\{A_1, A_2 ...\}$ is **increasing to event** A if:
$A_1 \subset A_2 \subset ...$
and $A = \cup_{n=1}^{\infty} A_n$
Notation: $A_n \uparrow A$

Def. Similarly, $B_n \downarrow B$ if $B_1 \supset B_2 \supset ...$ and $B = \cap_{n=1}^{\infty} B_n$

k) If $A_n \uparrow A$, then $P(A) = \lim_{n \to \infty} P(A_n)$
l) If $B_n \downarrow B$ then $P(B) = \lim_{n \to \infty} P(B_n)$