Lecture 2 Notes for STT861

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1 Review

Def A **random experiment** is an action which will result in one of the many possible outcomes.

Def A **sample space** is the collection of all possible outcomes of a random experiment. We shall denote by **S**.

Def An **Event** is a subset of sample space **S** for which we can define probability.

2 Class Notes

Def **Intersection** A, B are two events.
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Def **Union** A, B are two events.
$A \cup B = \{x : x \in A \text{ or } x \in B \text{ or } both\}$

Def A and B are **disjoint** if $A \cap B = \emptyset$

Properties of set theory
Commutative
$A \cup B = B \cup A$ and $A \cap B = B \cap A$
Associative
$(A \cup B) \cup C = A \cup (B \cup C) =: A \cup B \cup C$
$(A \cap B) \cap C = A \cap (B \cap C) =: A \cap B \cap C$
Distributive
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
De Morgan's Law
$(A \cup B)^c = A^c \cap B^c$
$(A \cap B)^c = A^c \cup B^c$

Remark Proving Distributive Property:
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Proof. Prove left direction:
$x \in (A \cup B) \cap C \Rightarrow x \in A \cup B$ and $x \in C$
$\Rightarrow (x \in A$ or $x \in B)$ and $x \in C$
$\Rightarrow (x \in A$ and $x \in C)$ or $(x \in A$ and $x \in C)$ or both
$\Rightarrow x \in A \cap C$ or $x \in B \cap C$ or both
$\Rightarrow x \in (A \cap C) \cup (B \cap C)$
Right direction is reverse of above.

Def **Set Difference**
$A \setminus B = \{x : x \in A, \text{ but } x \notin B\}$

Def **Symmetric Difference**
$A \triangle B = \{x : x \in A \setminus B, \text{ or } x \in B \setminus A\}$

<u>Def</u> A set A is **finite** if there exists a 1-1 fn $A \mapsto \{1, 2, ..., n\}$ for some $n \in \mathbb{N}$

<u>Def</u> A set A is **countably infinite** if there exists a one-to-one function from $A \mapsto \mathbb{N}$.

<u>Def</u> A set is called **coutable** if it is either finite or countably infinite.

Consequences

a. $A \cap B \subset A$ and $A \cap B \subset B$

b. $A \cap A = A$

c. if $A \subset B$ then $A \cap B = A$

d. $A \subset A \cup B$ and $B \subset A \cup B$

e. $A \cup A = A$

f. if $A \subset B$ then $A \cup B = B$

g. $A \cup A^c = \mathbf{S}$ and $A \cap A^c = \emptyset$

h. if $A \subset C$ and $B \subset C$ then $A \cap B \subset A \cup B \subset C$

i. $\emptyset$ is disjoint to all events

j. if $a \cap B = \emptyset$ then $A \subset B^c$ and $B \subset A^c$

k. if $A \subset B$, then $B^c \subset A^c$

l. $A \setminus B = A \cap B^c$ and $B \setminus A = A^c \cap B$

m. $A \triangle B = (A \cup B) \setminus (A \cap B)$

Remark There is some stuff here about lambda that I don't fully grok