STT 861 (Fall 2019): Homework 5

This homework will be collected at the start of the lecture of **Monday Nov 18, 2019**.

Question 1. Suppose (X, Y) are continuous random variables with joint pdf

$$f(x,y) = \begin{cases} k(x+y), & \text{if } 0 < x < 1, \quad 0 < y < 1, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find k.

(b) Find marginal pdf of X, i.e. $f_X(x)$.

(c) Find marginal pdf of Y, i.e. $f_Y(y)$.

(d) Find conditional pdf of X given $Y = y$, i.e. $f_{X|Y}(x|y)$.

(e) Find conditional pdf of Y given $X = x$, i.e. $f_{Y|X}(y|x)$.

(f) Compute $P(X + Y \leq 1)$.

(g) Find $P(Y < 0.5|X = 0.4)$.

(h) Compute $\mathbf{Cov}(X, Y)$.

Question 2. Suppose (X, Y) are continuous random variables with joint pdf

$$f(x,y) = \begin{cases} 2e^{-x-y}, & \text{if } 0 < y < x < \infty, \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the joint cdf $F(x, y)$ of (X, Y).

Hint: Consider three cases: (i) $0 < y < x$, (ii) $0 < x \leq y$, (iii) otherwise.

(b) Find conditional pdf of X given $Y = y$, i.e. $f_{X|Y}(x|y)$.

(c) Compute $\mathbb{E}(X|Y = 5)$.

(d) Compute $\mathbf{Var}(X|Y = 5)$.