

Lecture 1 Notes for STT861

Kenyon Cavender

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1 Lecture One

Ex 1 Toss a fair coin; what is the probability of obtaining heads? $P(H) = \frac{1}{2}$

Ex 2 Throw a fair die; what is the probability of obtaining 6? What about obtaining an even number?

$$P(6) = \frac{1}{6}$$

$$P(n = 2, 4, 6) = \frac{1}{2}$$

The probability is not the realized result, but the convergence of results as the number of iterations approaches infinity. Review the Weak Law of Large Numbers.

Def A **random experiment** is an action which will result in one of the many possible outcomes.

Def A **sample space** is the collection of all possible outcomes of a random experiment. We shall denote by **S**.

Def A **set** is a collection of some well defined objects.

Def **Outcomes** are also called **sample points**.

Def An **Event** is a subset of sample space **S** for which we can define probability.

Def Suppose A and B are two sets. $A \subset B$ (A is a **subset** of B) if $x \in A$ implies $x \in B$. If $A \subset B$ and $B \subset A$ then $A = B$.

Def A set is called an empty set (or **null set**) if it contains no elements.

Notation: $\{\emptyset\}$

Convention: $\emptyset \subset A$, for any set A

Corrolary: $\forall A, \emptyset \subset A \subset \mathbf{S}$

Def **Complement** A^c is the set such that $x \in A^c \Rightarrow x \notin A$.

In other words, $A^c = \{x : x \notin A\}$

Notation: A^c or A' or \bar{A}

Results

i) $(A^c)^c = A$

ii) $\mathbf{S}^c = \emptyset$

iii) $\emptyset^c = \mathbf{S}$

iv) if $A \subset B$, then $B^c \subset A^c$

Def **Intersection** A, B are two events.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Def **Union** A, B are two events.

$$A \cup B = \{x : x \in A \text{ or } x \in B \text{ or both}\}$$

Def A and B are **disjoint** if $A \cap B = \emptyset$

Properties of set theory

Commutative

$$A \cup B = B \cup A \text{ and } A \cap B = B \cap A$$

Associative

$$(A \cup B) \cup C = A \cup (B \cup C) =: A \cup B \cup C$$

$$(A \cap B) \cap C = A \cap (B \cap C) =: A \cap B \cap C$$

Distributive

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

De Morgan's Law

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Def Set Difference

$$A \setminus B = \{x : x \in A, \text{ but } x \notin B\}$$

Def Symmetric Difference

$$A \triangle B = \{x : x \in A \setminus B, \text{ or } x \in B \setminus A\}$$

Def A set **A** is **finite** if there exists a 1-1 fn $A \mapsto \{1, 2, \dots, n\}$ for some $n \in \mathbb{N}$

Def A set with only one outcome from **S** is called a **simple event**.

Def A set with more than one outcome is known as a **composite event**.

Def A **set function** is a function defined on a set.

Def Probability is a set function which takes a real value.