STT 861 Compendium - Part Two

Kenyon Cavender

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Expected Values

Def The **expected value** of a random variable $g(X)$:

$$\mathbb{E}(g(X)) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} g(x) f_X(x) dx & \text{if } X \text{ is continuous} \\ \displaystyle\sum_X g(x) f_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

Remark If $-\infty \leq \mathbb{E}(g(x)) \leq \infty$ we say the expectation of $g(x)$ exists. Else it does not exist.
notation: $|\mathbb{E}(g(x))| \leq \infty$
In particular, if $g(x) = x$, then we get the expected value of X:

$$\mathbb{E}(g(X)) = \begin{cases} \displaystyle\int x f_X(x) dx & \text{if } X \text{ is continuous} \\ \displaystyle\sum x f_X(x) & \text{if } X \text{ is discrete} \end{cases}$$

This is called the **mean** of r.v. X
Also denoted by μ or μ_X

Remark Theorem

a. $\mathbb{E}(ag(x) + b) = a\mathbb{E}(g(x)) + b a,b$ real constants

b. If$g(x) \geq 0$for all$x \in \mathbb{R}$, then$\mathbb{E}(g(x)) \geq 0$

c. If$g_1(x) \geq g_2(x)$for all$x \in \mathbb{R}$, then$\mathbb{E}(g_1(x)) \geq \mathbb{E}(g_2(x))$

d. For any real constants a,b if $a \leq X \leq b$ then $a \leq \mathbb{E}(X) \leq b$