## Chapter 5: Neutral Density Profile

The question of the neutral density profile for Argon within the Helimak will try to be addressed theoretically. The value of the neutral density would change the calculated values of and in the procedure of chapter 4 if the neutral profile is not flat.

### 5.1 Motivation

Ionization rate data is available from the Bogaerts[4] CR model used in calculating density and temperature. Another source of Argon ionization rates is also referenced from Arnaud et al[12] as well. The velocity integrated cross section is plotted in figure 5.1 from both sources for neutral Argon, as well as the first two ionization states.

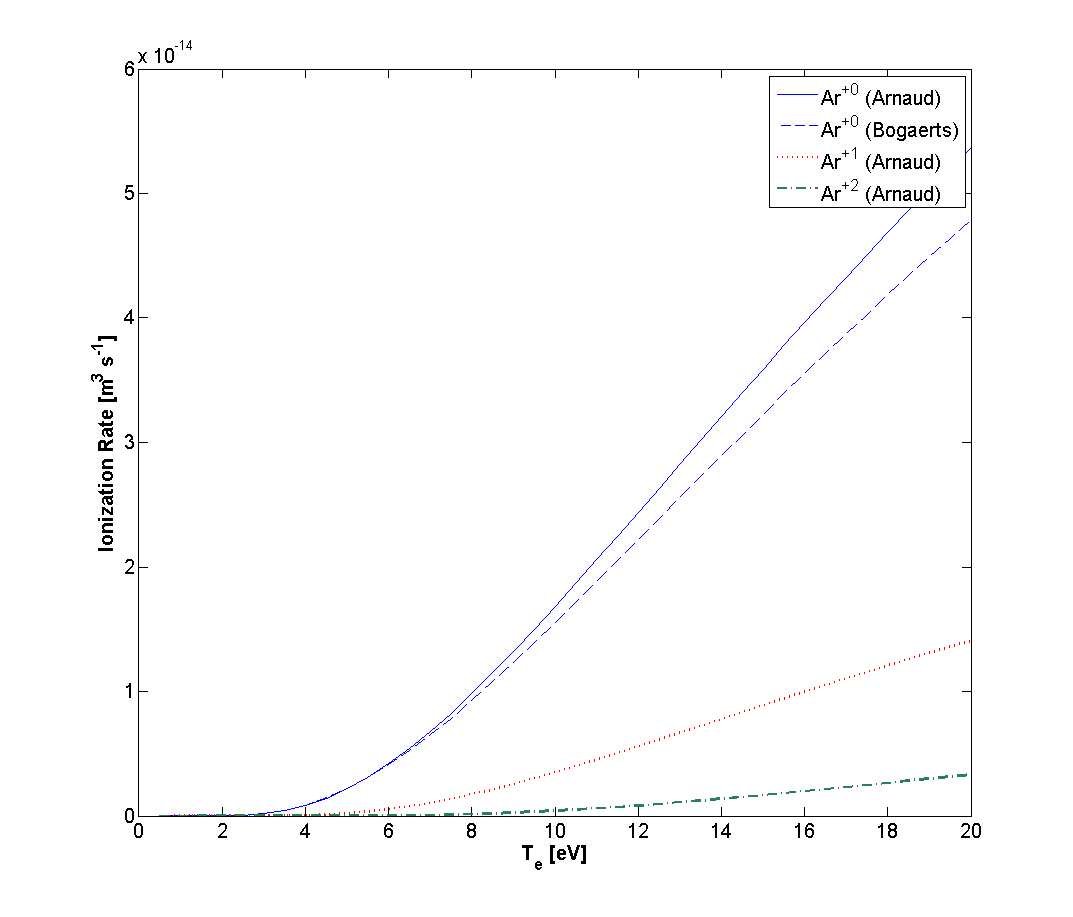


Figure 5.1: Ionization rates for Argon from Bogaerts[4] and Arnaud[12].

The ionization rate for neutral Argon at and is . Using (T=300K), this gives a mean free path of before an atom is ionized. The length scale of the Helimak is of order 1m, which implies that a significant fraction of the neutral atoms could be ionized while traversing the Helimak chamber. That is, it would be hard to justify that the profile must be flat.

The ionization rate should have a peaked profile, since the electron density and temperature both have peaked profiles, and the neutral profile would be determined in a non-local manner. Since the neutral atoms’ velocities should have a thermal distribution, the mean free path per atom would depend on the exact velocity of that atom. To find the actual profile of the neutral density, the non-local and thermal distribution effects should be included.

The neutral-neutral collisional mean free path as defined by , with for Argon [ref. ??], which is larger than the dimension of the Helimak chamber. The gas is in the molecular flow, or ballistic, regime where collisions with the walls are more frequent than with other atoms. A simple fluid model may not be adequate to predict the neutral dynamics.

### 5.2 Kinetic Model

Going back to a fully kinetic description of the gas allows direct computation of the profile. Even though this approach was somewhat computationally intensive, it was straightforward to implement for the problem at hand. The evolution of the gas can be described by the Boltzmann equation, (5.1), which models the distribution of particles in velocity and spatial dimensions . For three spatial dimensions, the distribution function has six dimensions plus time.

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The first term on the left hand side, , is simply the time rate of change of the distribution at a particular velocity, position, and time. The second term, , represents the streaming of particles through space. The third term, , represents how forces move the particles through velocity space, where the acceleration vector can also be a function of both position and velocity . The first term on the RHS, , is the effect of collisions between particles in the distribution and with other species of particles, which also moves particles around in velocity space. This term can be complicated and would in reality be a functional of the distribution itself. The last term, , is the source term which can add or remove particles from the distribution, such as through ionization and recombination.

A neutral gas does not produce, nor interact with, electromagnetic fields. The interaction with the gravitational force can also be neglected since . This allows the dropping of all external forces. The neutral-neutral collision rate is very low, and electron-neutral collisions should not significantly alter the atom’s velocity during a transit through the chamber. This should allow the dropping of the collision term as well.

A steady state solution is desired setting , which leaves (5.2). The source term within the plasma has been replaced with , which describes only losses due to ionization. Recombination is much slower within the plasma by over an order of magnitude, and so it was dropped. The source term will need to include recombination at the wall somewhere, but was included in the boundary conditions.

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If a particular direction is chosen, , then the equation can be re-arranged into (5.3). The ionization rate can also be written as a function of position since the electron density and temperature are functions of position. This equation can be directly integrated in the form of (5.4) if the position is also parameterized in terms of to give (5.5), where is the distance to the wall from the test position along the direction .

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The position the path intersects the wall is . The electron density and temperature are assumed to be cylindrically symmetric, so the rate should only depend on the radial coordinate. The radius can be parameterized in terms of to give (5.6). Using spherical coordinates for the velocity, and and at the point where the solution is being computed.

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The next task is to define the function , which depends on the geometry of the Helimak. The path can be broken into two components, one perpendicular to the z-axis and one parallel. If the floor and ceiling of the Helimak are ignored for a moment, the perpendicular component has only two possible cases depicted in figure 5.2. One where the path originates from the outer wall, and one from the inner wall, depending on the value of .

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| (a) | (b) |

Figure 5.2: Component of path perpendicular to z-axis of Helimak. Can intersect either inner wall or outer wall depending on . (a) Path Intersects Inner Wall. (b) Path Intersects Outer Wall.

The law of cosines can be used to derive the expressions for . For the inner wall this gives , and for the outer wall , which can be solved using the quadratic formula. At the transition point the path is tangent to the inner wall giving . When the inner wall value is used, and when the outer wall is used giving (5.7).

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The parallel component is simply the distance to either the floor or the ceiling. However, there are additional restrictions now to match both the wall and the floor and ceiling conditions depending on depicted in figure 5.3. The solution is summarized in (5.9).

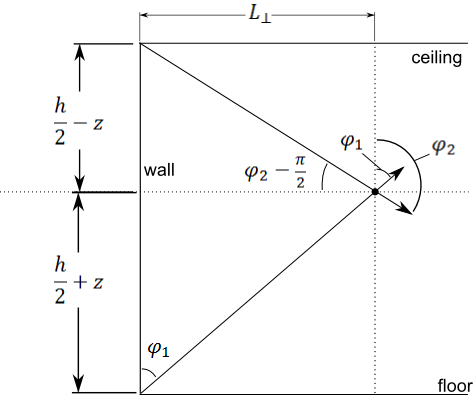


Figure 5.3: Matching wall condition with floor and ceiling.

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The solution in (5.5) still depends on the value of the distribution at the boundary . The boundary condition is that the total flux at the walls, minus the source, must be zero. The total particle flux is the first velocity moment of the distribution. For velocities going into the wall, the distribution is found using (5.5), where is the value at some other wall. The distribution coming from the wall must be found to match the boundary conditions.

To simplify the process of solving the boundary condition, it will be assumed that the part of the distribution not determined by (5.5) is Maxwellian in shape. The justification for this is that particles will scatter off the walls changing their direction and energy, and would eventually come into thermal equilibrium with the walls after scattering many times.

The distribution is divided into two components with and defined as the particle density and flux from the part of the distribution from (5.5) going into the wall, and and from the rest of the distribution that is Maxwellian like. The total density and flux at the wall is then and . For the walls the condition is , where is the inward directed normal vector of the wall surface. For a flat wall surface, is found by integrating over the half Maxwellian distribution, and results in (5.11), where . is computed from integrating over the outward directed half of the solution at the wall as in (5.12).

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The boundary was then discretized into a finite number of flat wall segments. The center of each segment was used as a test position to calculate the contribution to at that point from every other boundary segment using (5.5) and (5.12). The total value of at the test point is then found by summing over all of the contributions as in (5.13), where the are the computed relative contributions from the boundary segment j to the segment i. The addition of is the source term from recombination of ions at the wall in that segment. The inward directed density at the wall can then found by solving (5.13) by iteration.

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Assuming a Maxwellian distribution at the boundary has another important feature. Since the inward half of the distribution can then be written as (5.14), integrating (5.5) over all velocity space to get the first two moments of the distribution can be simplified by expressing the integral over the magnitude of velocity as a pre-computed function. The integral in the exponent of (5.5) can be defined as , a characteristic velocity below which the distribution is highly attenuated by ionization. Using the dimensionless parameters and , the first two moments can be written as (5.15) and (5.16).

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The two functions and , plotted in figure 5.4, can be pre-computed and placed in a lookup table. The integrals over and must still be calculated at every location since and are complicated functions of , , and .

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Figure 5.4: Integrals and plotted over a range of the parameter.

There are several obvious symmetries that were used to minimize the amount of computation required. If the solution is axially symmetric, it needs only be calculated for and . This would also imply a reflection symmetry in velocity space along the azimuthal direction of the Helimak, so that only half of the velocity directions need to be computed. This assumption may well not be valid because even though the plasma is mostly axially symmetric, the source profile due to recombination may not be. The plasma terminates on two sets of plates located on the top and bottom and separated by , which means recombination would occur mostly there. However, going from a 2D problem to a 3D problem is not a trivial step.

If there is an up-down symmetry to the solution, then only the +z positions need to be computed. This may not be valid either, again due to asymmetric recombination profiles. However, including this asymmetry is not difficult.

The integral over and must be computed numerically for a discrete set of points on the unit sphere, which represents all possible velocity directions. If the points are placed at evenly spaced increments of and , as depicted in figure 5.5, then bilinear interpolation (5.19) can be used to interpolate any function, , between the points.

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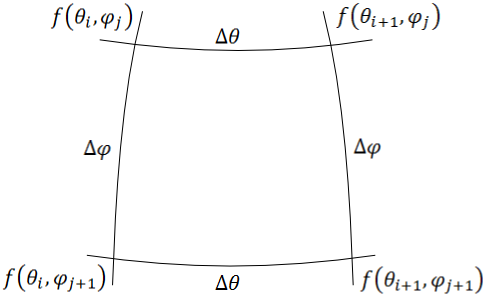


Figure 5.5: Discrete segment on the unit sphere.

The interpolated function (5.19) can be integrated over its domain giving (5.20), as would have to be done for (5.15) or (5.16). All of the discrete integrals can be summed to give the total integral over the sphere. The total integral simplifies to (5.21).

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### 5.3 Results

For the numerical runs, each coordinate was discretized into 100 divisions for r, z, , and . Values of , , and were used. There is some uncertainty about h since the plasma does not extend uniformly past the bias plates, which extend about 28cm from the top and bottom of the chamber. However, as was discovered after running the simulation for several values of , from 1.44m up to 2m, it doesn’t seem to change the shape of the neutral profile very much.

A radial profile for electron density and temperature are needed to define the ionization rate function in (5.5). Also needed is the recombination source profile on the walls. For and , a fitting function is used to approximate their profile based on probe data. The fitting function used is (5.22), where A is the maximum value of either or , is the ratio of the value at the outer wall to the maximum value, , (where is the location of the profile peak), and . A value of and was used for , and and for . An example fit is shown in figure 5.6.

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Figure 5.6: Fitting function for and profiles.

The recombination profile is a bit trickier to know. Ions should mostly flow vertically along field lines, which means that recombination will occur at the ceiling and floor, and not on the inner and outer walls. It was assumed that the recombination rate is proportional to the vertical component of the parallel flow rate, which would be proportional to the sound speed, the electron density, and the radius: . The ion velocity along the field lines should scale with the sound speed, and the ion density should scale with the electron density. The pitch of the field lines also changes with radius due to the 1/R scaling of the toroidal field. So the vertical component of the ion velocity should scale with R.

If the plasma is mostly convected out through established flows, instead of through parallel flow along field lines, the recombination profile would be slightly different. It would be proportional to the radial derivative of , since The plasma potential should scale with the temperature due to the sheath potentials generated at the top and bottom termination plates,, and the electric field would be minus the derivative of the plasma potential. Also, since changes sign due to the peaked profile of , the could be downwardly directed for some radii and upward for others, which would break the up-down symmetry of the solution.

However, these profiles would only give a shape for the recombination profile, but not the magnitude. Given a fixed ionization rate profile, the neutral density at any point should be proportional to the recombination rate at the wall. Once the solution is computed using some arbitrary magnitude for the recombination rate, the whole solution can be re-scaled so that the average neutral density matches the estimated average neutral density (4.7). Since the initial scale of the solution is then completely arbitrary, only the shape of the solution matters, it only needs to be computed in normalized units.

For an example solution, the plasma parameters used for the fitting functions are peak values for and . The peak used is higher than seen in probe data and are taken from the example calculation in chapter 4 using spectroscopic measurements. The streaming loss, up/down symmetric recombination source profile is used for this example.

The neutral density is plotted in figure 5.7a, which is only the upper half of the chamber, and has been normalized to the average density. The solution shows variation in the density in both r and z. The spectroscopic measurements are a chord integral through the z-direction, and so what is important is the average density along each vertical chord, which is plotted in fig 5.7b. A profile like this could be used now to make the next correction to the spectroscopically measured values of and in chapter 4.

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Figure 5.7 The first moment of plotted in upper half of Helimak chamber. (a) [left] , normalized density. (b) [right] z-average of normalized .

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| (a) | (b) |

Figure 5.8 The second moment of in the upper half of chamber. (a) streamlines of flux density . (b) magnitude of .

Also plotted in figure 5.8a is a streamline plot of the neutral flux density, which is the second moment of the distribution. The neutrals originate at the top and bottom from recombination, and on average flow into the region of highest ionization. This is only an average process since the individual atoms are simply bouncing from wall to wall until they are ionized. The peak averaged speed of the neutrals is around 150m/s near the top and bottom.

The effect of the up-down symmetry was tested by extending the solution to the -z coordinates as well. The same radial recombination source profile was used as for the symmetric run, except it was only defined on the top of the chamber. The bottom had no source at all, which should give the maximum possible effect of up/down asymmetry to the problem.

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Figure 5.9 Solution for asymmetric source profile. (a) normalized density, (b) flux streamlines.

Figure 5.9a shows the resulting asymmetric solution. The solution is no longer the same in the upper and lower half. However the z-averaged density ends up being virtually identical to the symmetric solution. This means that in terms of using the solution for the procedure in chapter 4, the breaking of up/down symmetry may not make much difference after all.

The reason for this may be in the boundary condition . A side effect of the symmetric solution is that at z=0, the only difference from a boundary being there was no assumption about the distribution at z=0. This means the asymmetric solution should be nearly the same as a symmetric solution for a chamber that was extended to twice the height. Since the entire solution is rescaled to match the estimated average density anyway, the up/down asymmetry of the source, or even the height used for the chamber, would not alter the z-average of the solution very much.