## Chapter 6: Electron Velocity Distribution

### 6.1 Motivation

In chapter 4, two atomic models were used to estimate the electron density and temperature. These models depend on the excitation cross sections for each transition, which is averaged over the velocity distribution of the electrons. It was assumed that the distribution was Maxwellian for a given temperature. However, the inelastic collisions that lead to excitation and ionization will change the electron distribution by taking waya energy.

Electrons in the plasma are created through ionization of neutrals. The energy to heat the electrons initially comes from RF power heating some electrons, which then collide with other electrons heating them up as well. The exact electron distribution would have to satisfy the full Boltzmann kinetic equation, which would be non-local and anisotropic. Solving the exact distribution would essentially entail a full simulation of the plasma. The ultimate goal, though, is to see how much the excitation and ionization rates change due to a deviation from pure Maxwellian distribution caused only by those collisions.

To simplify the solution, several assumptions were made. The first is that the distribution found is local with a specified effective temperature such that the average kinetic energy is . This allows easy comparison to the collision rates already tabulated in terms of . The second is that it is isotropic, since any anisotropy makes no difference to collision rates. Energy and particle balance also have to be satisfied, and this is done by using additional operators, which may not be entirely physical. These assumptions will hopefully still give some meaningful improvement over simply assuming a Maxwellian distribution, but a more sophisticated model could be a future improvement.

### 6.2 Fokker-Planck Scattering

The starting point is the Fokker-Planck Equation, which can be used to describe the behavior of the electron distribution undergoing collisions between electrons which would drive the distribution toward a Maxwellian. The Landau-Boltzmann collision operator, denoted here as in (6.1), is taken from The Framework of Plasma Physics[2] in the case of electron-electron collisions. Since both populations are electrons, the notation is used so that and .

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|  | [], [] (taken from [13]) |  |

The solution desired will be isotropic, which means it only depends on the magnitude of the . (6.1) needs to be re-written using this condition. The velocity can be expressed as the magnitude times its unit vector . By using spherical coordinates, and assuming only the radial derivative is non-zero, the divergence and gradient in velocity space can be written as (6.6) and (6.7), and (6.1) written as (6.8).

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The dot products can be reduced to two expressions involving , which only depends on direction.

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This form lends itself to introducing the coordinates and . The magnitude of , as well as (6.9) and (6.10), can be re-written using these coordinates.

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The distribution does not depend on the angular variables, so when the integral is performed over the angles, only the two forms involving are involved. Performing the integral with the expressions in (6.12) and (6.13) gives a simple result.

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Plugging this back into (6.8) gives the final form of the isotropic scattering operator.

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### 6.3 Inelastic Scattering

To account for inelastic scattering, inelastic collision operators must be constructed for these processes. Two issues arise from this. One is that inelastic collisions do not conserve energy. The other is that if the operator is to include ionization, the creation of electrons due to that ionization will not conserve particles. This means that with no other affects included, there cannot be an equilibrium solution. Additional operators must be included to find a steady state solution to maintain the target electron density and temperature, which is defined by the average energy .

The inelastic operator due to excitation collisions is simpler than ionization, as the energy of the incident electron is reduced by a fixed amount due to an inelastic collision. It consists of a loss term that is proportional to the target atom density , velocity of the incident electron, and the collisional cross section at the energy . The gain term should exactly balance the loss term at to conserve particles. Writing this operator for the distribution over energy first for a single excitation process j gives (6.17).

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Converting this operator to velocity space brings in a factor of due to the angular integrals of velocity, and also a factor of due to the conversion of the distribution to velocity from energy to give . The conversion of is the same, and so all factors cancel except the ratios of the velocity in (6.18). The total excitation operator is then the sum of all the individual excitation processes to be included.

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The ionization operator is a bit trickier because, in the absence of a differential cross-section, some assumption must be made about how the incident electron and the ejected electron share the total available kinetic energy. All that can be said for sure is that the total energy is conserved: . Here is the kinetic energy of the incident electron, is the energy of the ejected electron, and I is the ionization energy.

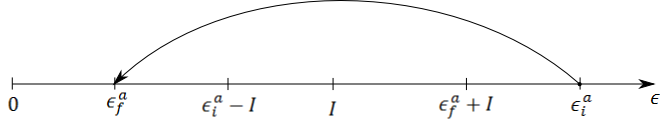


Figure 6.1 Energy diagram of inelastic scattering from ionization.

Tzeng and Kunhardt [24] discuss the effect of different sharing of energy between the two electrons. For ease of implementation, a symmetric sharing of energy was used, where the incident electron has an equal probability of ending with any possible energy, and the ejected electron has the remaining energy. That is, the probability of the incident electron landing in the range is , if the final energy is in the range , and zero otherwise.

Electrons landing at a particular can only come from energies in the range . The total rate of electrons landing at is found by integrating the collision rate, , over that entire valid range, weighted by the probability that it lands at . If the ejected electron has the same distribution, then the total gain rate is simply twice that due only to the incident electron, which is what gives rise to particle imbalance. The loss rate is equal in magnitude to the total ionization collision rate. The operator in energy space is given in (6.20), and converted to the velocity space in (6.21).

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### 6.4 Energy and Particle Balance

Both inelastic operators do not conserve energy, and the ionization operator does not conserve particles. In order to find some steady state solution, the energy loss and particle gain must be balanced somehow by adding energy to the distribution, and including some kind of particle loss. The particle gain from ionization is found from the 1st order moment of the ionization operator.

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To balance this, a particle loss term was added of the form , where L would be some constant with units of . A physical particle loss would come from streaming resulting from spatial gradients of the distribution. L could represent some average loss process that has been averaged over all directions and velocities. The magnitude of L though has to be chosen to balance particle gain from ionization. Using the 1st order moment of the ionization operator gives the required magnitude of L.

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The power loss density is found by taking a 2nd order moment of all the operators. The particle loss operator has to be included since it also does not conserve energy.

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To balance this energy loss, diffusion in velocity space was added to effectively heat the distribution: . The effect is similar to the existence of a shell of hot electrons surrounding the bulk distribution, which causes heating for the slower electrons. Instead of adding this second population at some arbitrary energy, a constant diffusion coefficient is used which would represent a hot electron population at infinite velocity. Similar to (6.23), the magnitude of has to be found to balance the energy loss in (6.24).

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This should fix both energy and particle balance, since the added diffusion still conserves particles. The full equation to be solved is the sum of all the operators, and equating (6.26) to zero.

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### 6.5 Numerical Solution for Argon I

The atomic data used comes from the CR model for Ar I by Bogaerts et al.[4] used in chapter 4. It provides the collision cross sections for excitations and ionization from the ground state. Transitions between levels are not included since the excited state population densities are many orders of magnitude lower than the ground state. The density for Ar II is also much lower than that of Ar I by about one order of magnitude.

The solution was computed numerically. Velocity was discretized up to a maximum velocity of , with 500 divisions. This value was determined empirically. A low value of was found to cause the tail of the distribution to be numerically unstable. Also, a larger number of divisions per unit velocity required shorter time-steps to be numerically stable. A time step of was used with this value of .

The distribution was initially set to a Maxwellian of the desired temperature. It was then stepped in time using the total collision operator that was derived. The characteristic time of change of the distribution was defined as . When this timescale reached , the resulting distribution was taken as the solution. This time-scale is much longer than the expected confinement time of the Helimak of order , and so should be close to the distribution attainable by the electrons.

Each time-step was computed using a linear explicit step (6.27). However, the integrals in the operators were computed every 10-100 steps to speed calculation, as only the final distribution was needed.

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Integrals were computed using a three-point Simpson’s method (6.28). First derivatives were computed using a four-point method (6.29). Second derivatives were computed using a five-point method (6.30).

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An example solution is plotted in figures 6.2 and 6.3 in comparison to a Maxwellian distribution of the same temperature. A high neutral density was used for this example to exaggerate any differences. The solution distribution has a much higher value near . All electrons born of ionization appear there, and those that lose energy from due to inelastic collisions land there as well.

The depression of the distribution at higher velocities is not as obvious. In the log plot of the distribution, a depression can be seen between about 10eV and 50eV. Above 50eV it appears to develop a higher tail, which is due to the heating mechanism introduced to balance power loss.

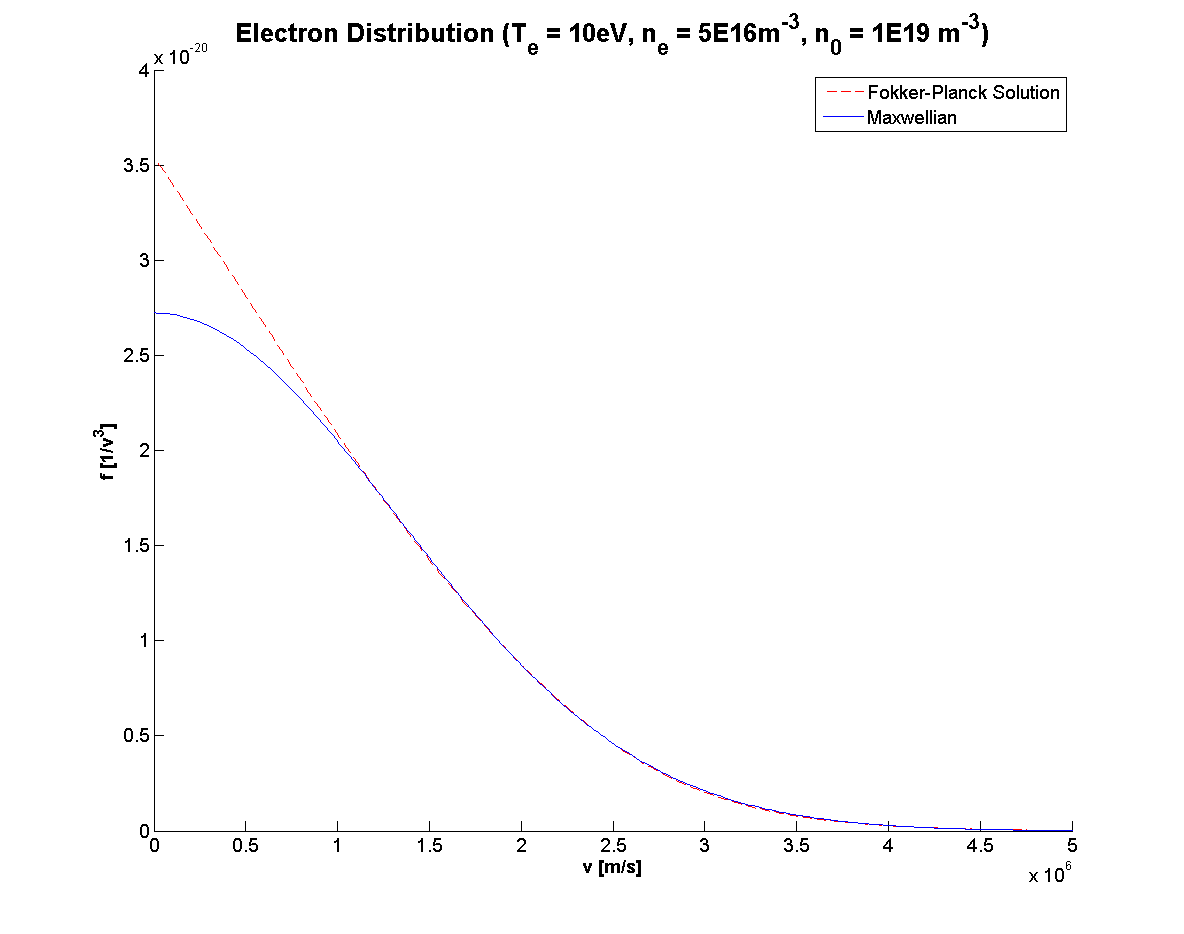


Figure 6.2 Solution electron velocity distribution compared to a Maxwellian at the effective temperature.

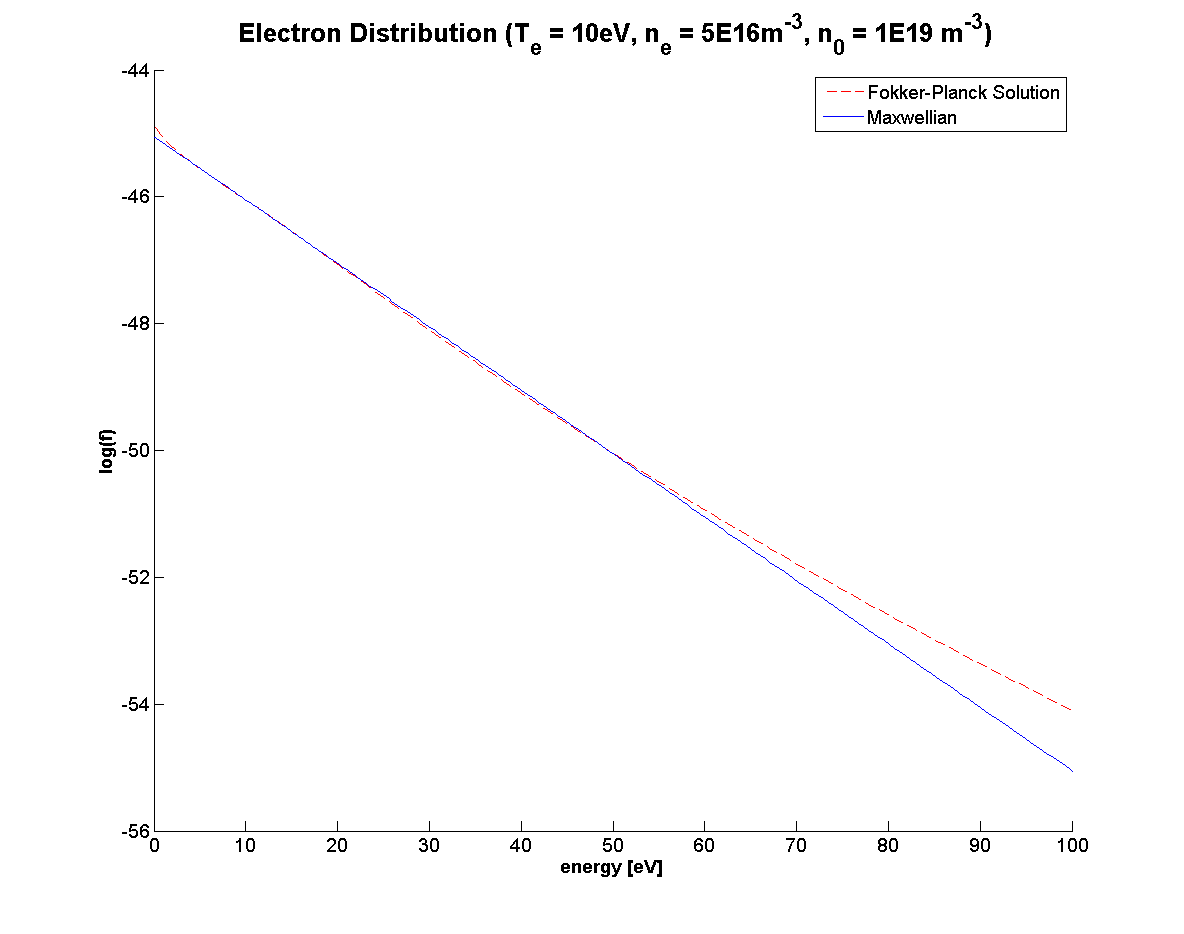


Figure 6.3 The distribution in versus

### 6.6 Correction Factors

The difference in excitation rates due to the deviation from Maxwellian should be proportional to the difference in the number of electrons above the threshold energy of the collisional process in question. If there are an equal number of electrons above 15.8eV, for example, then the ionization rate of Argon I should be unaffected. A correction factor was defined by (6.31) by taking the ratio of the total number of electrons above a certain energy, where is the solution to the model including inelastic collisions and is a Maxwellian distribution.

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These coefficients were tabulated for three difference energies. Excitation rates for the observed lines of Argon I would be corrected using as the threshold energy. Ionization rates of Argon I would use . Excitation rates for the lines of Argon II would have a threshold energy of . Solutions were calculated for electron temperatures of 5, 10 and 20eV, except for the ArI rates, where only 10 and 20eV are given. Electron densities used are 1, 2, 5, 10, 20, 50, 100 and 200 , which relate to the tables 4.1 and 4.2. Neutral density values of 1, 2, 5, and 10 are used. These results are given in tables 6.1, 6.2 and 6.3.

Temperatures of 0.5, 1, and 2 eV are not given, and 5eV for ArI, because their correction factors did not deviate from unity. This may be because the tail of the distribution is being sampled at these low temperatures. The heating operator and the inelastic collision operators seem to balance the other’s effect for these energies.

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|  | 1.00E+16 | 2.00E+16 | 5.00E+16 | 1.00E+17 | 2.00E+17 | 5.00E+17 | 1.00E+18 | 2.00E+18 |
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| 10 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 | 1 |
| 20 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 0.99 | 1 | 1 |
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| 10 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 20 | 0.95 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 0.99 | 1 |
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| 10 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 |
| 20 | 0.94 | 0.95 | 0.95 | 0.96 | 0.96 | 0.98 | 0.98 | 0.99 |
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| 10 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 1 | 1 |
| 20 | 0.94 | 0.94 | 0.95 | 0.95 | 0.96 | 0.97 | 0.98 | 0.98 |

Table 6.1 Argon I Excitation Rate Corrections ()

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|  | 1.00E+16 | 2.00E+16 | 5.00E+16 | 1.00E+17 | 2.00E+17 | 5.00E+17 | 1.00E+18 | 2.00E+18 |
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| 5 | 0.96 | 0.98 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 20 | 0.93 | 0.94 | 0.95 | 0.96 | 0.98 | 0.99 | 0.99 | 1 |
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| 5 | 0.93 | 0.96 | 0.98 | 0.99 | 0.99 | 1 | 1 | 1 |
| 10 | 0.98 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 |
| 20 | 0.92 | 0.93 | 0.94 | 0.95 | 0.96 | 0.98 | 0.99 | 0.99 |
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| 5 | 0.86 | 0.91 | 0.95 | 0.97 | 0.99 | 0.99 | 1 | 1 |
| 10 | 0.98 | 0.98 | 0.98 | 0.99 | 0.99 | 1 | 1 | 1 |
| 20 | 0.92 | 0.92 | 0.93 | 0.93 | 0.95 | 0.96 | 0.98 | 0.99 |
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| 5 | 0.8 | 0.86 | 0.92 | 0.95 | 0.97 | 0.99 | 0.99 | 1 |
| 10 | 0.98 | 0.98 | 0.98 | 0.98 | 0.99 | 0.99 | 1 | 1 |
| 20 | 0.92 | 0.92 | 0.92 | 0.93 | 0.93 | 0.95 | 0.96 | 0.97 |

Table 6.2 Argon II Excitation Rate Corrections ()

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|  | 1.00E+16 | 2.00E+16 | 5.00E+16 | 1.00E+17 | 2.00E+17 | 5.00E+17 | 1.00E+18 | 2.00E+18 |
|  | | | | | | | | |
| 5 | 0.98 | 0.99 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 20 | 0.94 | 0.94 | 0.96 | 0.97 | 0.98 | 0.99 | 0.99 | 1 |
|  | | | | | | | | |
| 5 | 0.97 | 0.98 | 0.99 | 1 | 1 | 1 | 1 | 1 |
| 10 | 0.99 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 | 1 |
| 20 | 0.93 | 0.94 | 0.95 | 0.96 | 0.97 | 0.98 | 0.99 | 0.99 |
|  | | | | | | | | |
| 5 | 0.95 | 0.97 | 0.98 | 0.99 | 1 | 1 | 1 | 1 |
| 10 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 1 | 1 | 1 |
| 20 | 0.93 | 0.93 | 0.94 | 0.94 | 0.95 | 0.97 | 0.98 | 0.99 |
|  | | | | | | | | |
| 5 | 0.92 | 0.94 | 0.97 | 0.98 | 0.99 | 1 | 1 | 1 |
| 10 | 0.98 | 0.98 | 0.98 | 0.99 | 0.99 | 0.99 | 1 | 1 |
| 20 | 0.93 | 0.93 | 0.93 | 0.93 | 0.94 | 0.95 | 0.97 | 0.98 |

Table 6.3 Argon I Ionization Rate Corrections ()