Week 3, Meeting 1 Problems

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9/6

1 More Statistics

1.1 Continuous vs Discrete

Consider a system of $N >> 1$ ideal gas particles moving with speeds determined by a distribution, either continuous or discrete. For the continuous distribution, let $f(v) = Asin(\pi v)$ for $v \in (0, 1)$ m/s. For the discrete distribution, the particles can have velocity $v = 0$ m/s or $v = 1$ m/s with probability of 0, $v = 1/4$ m/s or $v = 3/4$ m/s with probability $B/\sqrt{2}$, or $v = 1/2$ m/s with probability of B.

(a) Find A and B such that the distributions, individually, are each valid probability distributions. Do A and B have units associated with them? What about π in $f(v)$?

(b) For each distribution, calculate $< v >= \bar{v}$, v_{rms}, σ, and v_p.

(c) What is the temperature of the gas?

(d) Comment on any discrepancies (or lack thereof) between the statistics for this discrete and continuous distribution. (e.g., Why did (or didn't) these discrepancies arise?)

1.2 Fun with Maxwell

Show that the total energy of a gas governed by the Maxwell distribution is given by

$$E = \left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}k_B T. \tag{1}$$

(Challenge: do this two different ways.)

2 Gases: Ideal, Real and van der Waals

2.1 Conceptual Questions

(a) If a container of gas is at rest, the average velocity of molecules must be zero. Yet the average speed is not zero. Explain. (*Source: Giancoli Problem 18.11*)

(b) Alcohol evaporates more quickly than water at room temperature. What can you infer about the molecular properties of one relative to the other? (*Source: Giancoli Problem 18.15*)

2.2 Escape Velocities

The escape speed from the Earth is 1.12×10^4 m/s, so that a gas molecule traveling away from Earth near the outer boundary of the Earth's atmosphere would, at this speed, be able to escape from the Earth's gravitational field and be lost to the atmosphere. At what temperature is the average speed of (a) oxygen molecules, and (b) helium atoms equal to 1.12×10^4 m/s? (c) Can you explain why our atmosphere contains oxygen but not helium? (*Source: Giancoli Problem 18.60*)

2.3 Fermi-ish Validation of Ideal Gas Law

Assuming a typical nitrogen or oxygen molecule is about 0.3 nm in diameter, what percent of the room you are sitting in is taken up by the volume of the molecules themselves? (*Source: Giancoli Problem 18.65*)

2.4 Reformulation of Pressure

Show that the rms speed of molecules in a gas is given by $v_{\text{rms}} = \sqrt{3P/\rho}$, where P is the pressure of the gas and ρ is the gas density. (*Source: Prof. Adrian Lee, Fall 2009 Phys. 7B 1st Midterm*)

2.5 Pressure from Other Things

During a hailstorm, hailstones with an average mass of 2g and a speed of 15 m/s strike a window pane at a 45^o angle. The area of the window is 0.5m^2 and the hailstones hit it at a rate of 30 per second. What average pressure do they exert on the window? How does this compare to the pressure of the atmosphere? (*Source: Schroeder - Thermal Physics Problem 1.21*)