

Non-Ideal Gases, Solutions

GSI: Caleb Eades

9/4

1 Gases: Ideal, Real and van der Waals

1.1 Conceptual Questions

- (a) Average velocity is zero but speed looks at the magnitude (some move left and some move right, for example). So overall while average speed is nonzero, average velocity will be.
- (b) We can infer that there is less inter-molecular bonding since there is a lower “escape velocity” for the alcohol.

1.2 Escape Velocities

First off, in a Maxwell Distribution,

$$\bar{v} = \sqrt{\frac{8}{\pi} \frac{kT}{m}} \approx 1.60 \sqrt{\frac{kT}{m}} \quad (1)$$

- (a) Solving for T ,

$$\begin{aligned} T &= \frac{m\pi}{8k} \bar{v}^2 \\ &= \frac{(2 * 16 * 1.66 \times 10^{-27} \text{ kg})\pi}{8(1.38 \times 10^{-23} \text{ J/K})} (1.12 \times 10^4 \text{ m/s})^2 \\ &\approx 1.9 \times 10^5 \text{ K} \end{aligned}$$

- (b) $m_{He} = \frac{1}{8} m_{O_2}$, so

$$T_{He} = \frac{1}{8} T_{O_2} \approx 2.4 \times 10^4 \text{ K} \quad (2)$$

- (c) At the same temperature, He will go much faster than O_2 , so more of it will escape, leaving the O_2 concentration much higher than He .

1.3 Fermi-ish Validation of Ideal Gas Law

Starting with the Ideal Gas Law,

$$\begin{aligned}
 PV &= nRT \\
 n &= \frac{PV}{RT} \\
 &\approx \frac{(1 \times 10^5 Pa)(100 m^3)}{(10 J mol^{-1} K^{-1})(300 K)} \\
 &\approx \frac{1}{3} \times 10^4 mol \\
 &\approx 3 \times 10^3 mol
 \end{aligned}$$

Now, looking at the volume for the gas,

$$\begin{aligned}
 V_{gas} &= \frac{4}{3} \pi (3 \times 10^{-10} m)^3 (3 \times 10^3 mol) (6.022 \times 10^{23}) \\
 &\approx 0.03 m^3
 \end{aligned}$$

Hence, the ratio of the volume of a gas particle to the volume of the room is

$$\frac{V_{gas}}{V_{room}} \approx \frac{0.03}{10^3} = 3 \times 10^{-5} \quad (3)$$

As we can see, the gas molecules take up less than three parts per hundred thousand in volume. So the Ideal Gas Law is a decent approximation.

1.4 Reformulation of Pressure

From the Ideal Gas Law and the average kinetic energy of a molecule, $PV = NkT$ and $\frac{1}{2}mv^2 = \frac{3}{2}kT$, respectively. Identifying $\langle v^2 \rangle = v_{rms}^2$,

$$v_{rms}^2 = \frac{3kT}{m} = 3 \frac{PV/N}{m} \quad (4)$$

At this point, it is useful to recall that m is the mass of a single molecule, so $mN = M$ is the total mass of the gas and hence $\frac{mN}{V} = \frac{M}{V} = \rho$, the density. Putting this together,

$$v_{rms}^2 = 3 \frac{P}{\rho} \implies v_{rms} = \sqrt{3P/\rho} \quad (5)$$

1.5 Pressure from Other Things

Assuming they collide elastically with the window, the total change in momentum is $2mv_x$, with $v_x = \sqrt{2}v/2$. Then,

$$\begin{aligned} P &= F/A \\ &= \frac{\Delta p}{\Delta t} / A \\ &= \frac{2\sqrt{2}vm}{2} \times \left(30 \frac{1}{\text{second}} \right) / A \end{aligned}$$

where we have identified $\frac{1}{\Delta t}$ as the number of molecules that hit the window per second. So

$$\begin{aligned} P &= \frac{2 * 15 * 2 \times 10^{-3}}{\sqrt{2}} \times 30 / (0.5) \\ &\approx 2.6 Nm^{-2} \end{aligned}$$

This is about five orders of magnitude weaker than atmospheric pressure of 1 atm $\approx 1 \times 10^5 Nm^{-2}$.