Week 15, Session 1 Problems

GSI: Caleb Eades

11/27

1 Inductors

1.1 Mutual Inductance

A pair of straight parallel thin wires, such as a lamp cord, each of radius r, are a distance l apart and carry current to a circuit some distance away. Ignoring the field within each wire, show that the inductance per unit length is $(\mu_0/\pi)ln[(l-r)/r]$.

(*Source: Giancoli 30.75*)

1.2 Introduction to Inductors

The simplest possible example of an inductor is a loop of wire. Specifically, supposer there is a circular loop of wire with a time-varying current $I(t)$.

(a) Sketch the magnetic field in the plane of the loop.

(b) Let's think about this situation. We have a current running through the loop, which gives us a field. The field goes through the loop, giving a non-zero magnetic flux. Using the definition of magnetic flux and then the Biot-Savart Law to find the $\vec{B}$ field, write the magnetic flux through the loop in the form $\Phi_B = LI(t)$ where L a quantity is in terms of several nasty integrals (don't actually do the integrals). The point is that the magnetic flux is *proportional* to the current.

(c) Use Faraday's Law to show that the inductance is related to the induced emf $\mathcal{E}$ in the wire by

$$\mathcal{E} = -L\frac{dI}{dt} \text{ or } L = -\frac{\mathcal{E}}{\frac{dI}{dt}}. \tag{1}$$

If L is greater than zero, then why does this equation need the minus sign on the right-hand side? (This is the equivalent of $C = Q/V$.)

(*Source: Dan and Vetri*)

1.3 Toroid

Suppose there is a toroid with a rectangular cross-section whose inner radius is a, outer radius is b, and height is h. Suppose that N turns of wire wrapped around the torus with current I going through them.

(a) Use Ampère's Law to show that the magnetic field is

$$\vec{B} = \frac{\mu_0 I N}{2\pi r}\hat{\theta}.$$

(b) Find the magnetic flux through one loop of the torus.

(c) Show that the inductance of the toroid is

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$$

using $\Phi_B = LI$

(d) Calculate the energy stored in the inductor and show this satisfies $U = \frac{1}{2}LI^2$.

(*Source: Dan and Vetri*)

2 Previous Exam Problems

2.1 Mutual Inductance

Figure A to the right shows an inductor made from two sheets of current each with width, w, and length, l, and they are separated by a distance, d. The left sheet has a current per unit length, j, flowing out of the page, while the right has the same, j, flowing into the page. The thickness of the sheets is negligible, and $d << l$ and $d << w$ so that the sheets can be treated as infinite.

(a) What is the self inductance, L, of the inductor? Express it in terms of d, w, l, and μ_0.

(b) In Fig B, two sheets of metal with negligible thickness are placed in the inductor. If j changes with time, an emf is measured between the two metal sheets. What is the mutual inductance, M, of the system? Express it in terms of x, w, l, and μ_0.

(*Source: Speliotopoulos Fall 2014 Final, Problem 6*)

2.2 Transformers: the less-cool kind

A solenoid of length l and cross-sectional area A is made of N_p turns of a weakly resistive conducting wire.

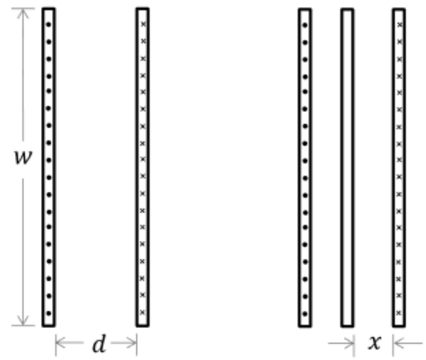


Figure A

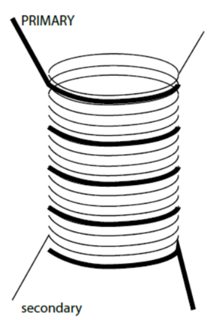
Figure B

(a) Calculate the self-inductance L of the solenoid.

(b) Calculate the energy U stored in this inductor when a current I passes through it.

(c) If this solenoid is used as the primary coil of an ideal transformer (as shown), what is the ratio of the currents I_p and I_s passing in the 2 coils?

(d) What is the number of turns N_s required on the secondary coil in order to transform 110 V into 15 V? Use $N_p = 110$.



(*Source: Bordel Fall 2012 Final, Problem 7*)