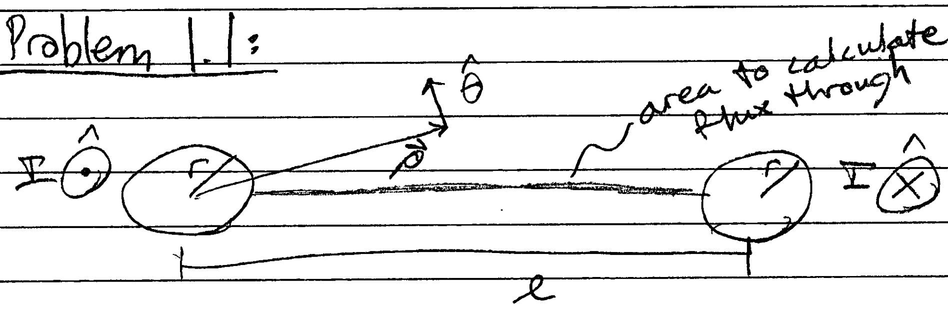
Problem 1.1:



This is like a single loop coil, so we want to find the magnetic flux between the wires along a length 'd'.
The field from the left wire is

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{\theta}$$

So our magnetic flux due to the left wire is

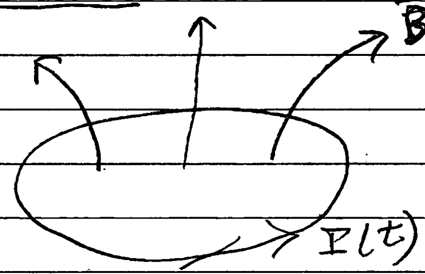
$$\Phi_B = \int \vec{B}\cdot d\vec{a}$$
$$= \int_r^{l-r} \frac{\mu_0 I}{2\pi\rho}\, d\, d\rho$$
$$= \frac{\mu_0 I d}{2\pi}\int_r^{l-r}\frac{1}{\rho}\,d\rho$$
$$= \frac{\mu_0 I d}{2\pi}\ln\frac{l-r}{r}$$

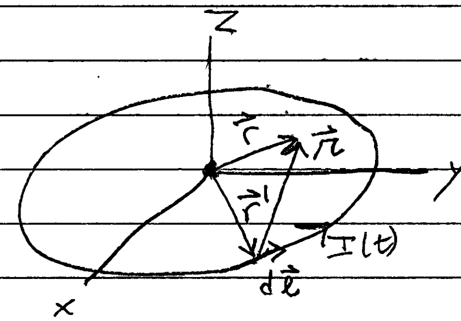
The total flux will be twice this (the right wire contributes the same amount) such that the inductance per unit length is

$$\frac{L}{d} = \frac{1}{d}\frac{N\Phi_B}{I}$$
$$= \frac{1}{d}\frac{(1)}{I}\left(\frac{\mu_0 I d}{\pi}\ln\frac{l-r}{r}\right)$$
$$= \frac{\mu_0}{\pi}\ln\frac{l-r}{r}$$

Problem 1.2:

(a)





(b) By the Biot-Savart Law,

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{wire} \frac{I\, d\vec{\ell} \times \vec{\mathscr{r}}}{|\vec{\mathscr{r}}|^3}$$

So the magnetic flux through the wire will be

$$\Phi_B = \int_{loop} \vec{B}(\vec{r}) \cdot d\vec{a}$$

$$= \frac{\mu_0 I(t)}{4\pi} \int_{loop}\int_{wire} \frac{d\vec{\ell} \times \vec{\mathscr{r}}}{|\vec{\mathscr{r}}|^3} \cdot d\vec{a}$$

$$= L\, I(t)$$

where $L = \frac{\mu_0}{4\pi}\int_{loop}\int_{wire} \frac{d\vec{\ell} \times \vec{\mathscr{r}}}{|\vec{\mathscr{r}}|^3} \cdot d\vec{a}$

(c) By Faraday's law, the induced emf in the wire is

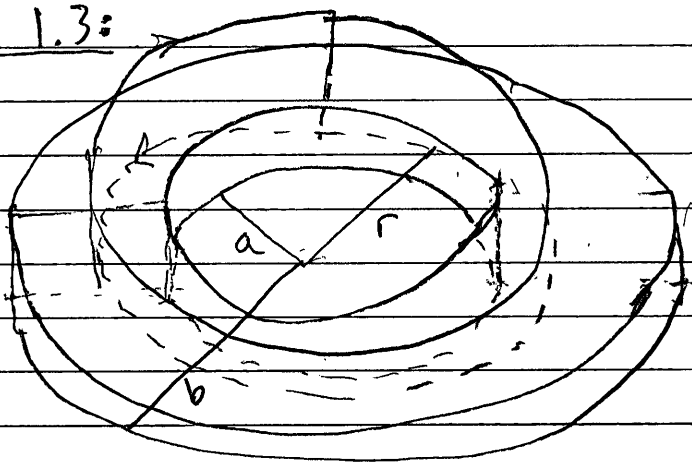
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$= -L\frac{dI}{dt}$$

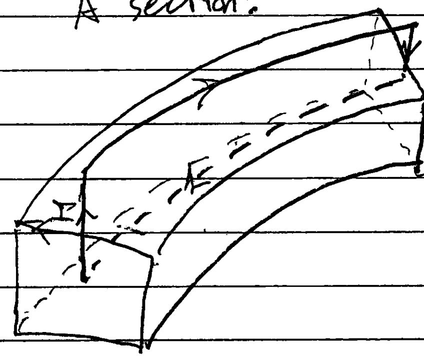
The minus sign is necessary because if the current is increasing and $L > 0$, the flux through the loop is increasing, so the induced emf will be opposed to this change, and vice versa.

Problem 1.3:

(a)



A section:



We will take the Amperian loop of a rectangle that is bent so that it nearly connects at the other edge.
By symmetry, any field that ~~will~~ exist will be on the inside of toroid. and could only depend on r, so by Ampere's law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$
$$\Rightarrow B(2\pi r) = \mu_0 I N$$
$$\Rightarrow B = \frac{\mu_0 I N}{2\pi r}$$

where we have used the fact that $\hat{B} = \hat{\theta}$ in our calculation such that

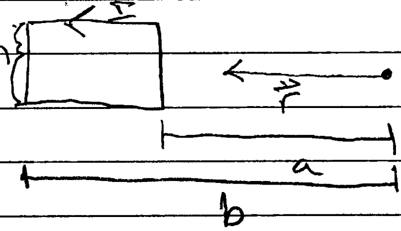
$$\vec{B} = \frac{\mu_0 I N}{2\pi r}\hat{\theta}$$

(b) The magnetic flux through one loop of the torus is

$$\Phi_B = \oint_{loop} \vec{B}\cdot d\vec{a}$$
$$= \int_a^b \frac{\mu_0 I N}{2\pi r} h\,dr$$
$$= \frac{\mu_0 I N h}{2\pi}\ln\frac{b}{a}$$

$\hat{A} = \hat{\theta}$

$\hat{B} = \hat{\theta}$



(c) The inductance is given by

$$\Phi_{B,total} = LI$$

The total magnetic flux through the toroid is N times the answer to part (b), so

$$L = N\Phi_B / I$$
$$= \frac{\mu_0 N^2 h}{2\pi}\ln\frac{b}{a}$$

(d) The energy stored in the inductor is found with

$$\mathcal{U} = \frac{1}{2\mu_0}\int_V B^2$$
$$= \frac{1}{2\mu_0}\int_a^b \frac{\mu_0^2 I^2 N^2}{4\pi^2 r^2} 2\pi r h \, dr$$
$$= \frac{\mu_0 I^2 N^2 h}{4\pi}\int_a^b r \, dr$$
$$= \frac{\mu_0 I^2 N^2 h}{2\pi} \ln\frac{b}{a}$$

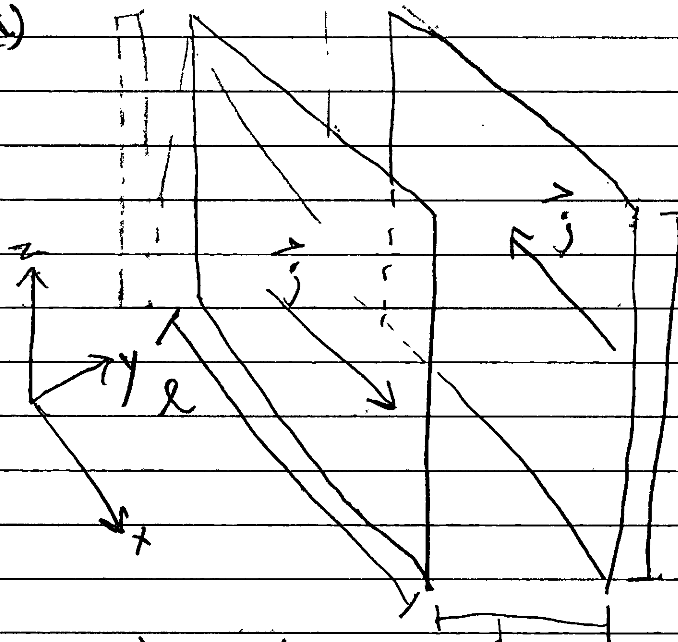
Note that we also could have computed this with

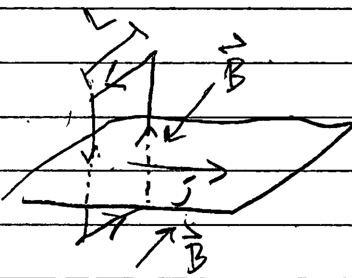
$$\mathcal{U} = \frac{1}{2} L I^2$$
$$= \frac{\mu_0 I^2 N^2 h}{4\pi} \ln\frac{b}{a}$$

so integrating the energy density over the volume and using $\mathcal{U} = \frac{1}{2} L I^2$ are equivalent for finding the total energy in an inductor.

Problem 2.1:

(a)





This is basically a wire loop, so we just want to find the magnetic flux in between the sheets. Since $d \ll w$ and $d \ll \ell$, these can be treated as infinite sheets. Using Ampère's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow 2BL = \mu_0 j L$$
$$\Rightarrow B = \frac{\mu_0 j}{2}$$

By the right hand rule, this will be in the $\hat{z}$ direction. With two plates we get

$$\vec{B} = \begin{cases} \mu_0 \vec{j}\ \hat{z}, & \text{in between the plates} \\ \vec{0}, & \text{elsewhere} \end{cases}$$

So the magnetic flux between the two is

$$\Phi_B = \mu_0 j \ell d$$

Writing $j = \frac{I}{w}$, the self inductance of this inductor is found with

$$L = \frac{\Phi_B}{I}$$
$$= \frac{\mu_0 I \ell d}{w} \frac{1}{I}$$
$$= \mu_0 \frac{\ell d}{w}$$

(b) The magnetic flux between the metal sheets is given by

$$\Phi_{B,\text{metal}} = \mu_0 j \ell x$$
$$= \mu_0 \frac{I \ell x}{w}$$

If $j = \frac{I}{w}$ changes with time, then there is an induced emf between the metal sheets with

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$
$$= -\mu_0 \frac{\ell x}{w} \frac{dI}{dt}$$

Comparing this to

$$\mathcal{E} = -M \frac{dI}{dt}$$

we see that

$$M = \mu_0 \frac{\ell x}{w}$$

Problem 2.2:

(a) The magnetic field inside this solenoid is found with Ampère's law with

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} \Rightarrow B\ell = \mu_0 N_p I$$
$$\Rightarrow B = \frac{\mu_0 N_p I}{\ell}$$

So the magnetic flux is

$$\Phi_B = BA$$
$$= \frac{\mu_0 N_p I}{\ell} A$$

through each tun. The self-inductance is then

$$L = \frac{N_p \Phi_B}{I}$$
$$= \frac{\mu_0 N_p^2 A}{\ell}$$

(b) We have

$$U = \frac{1}{2} L I^2$$
$$= \frac{\mu_0 N_p^2 I^2 A}{2\ell}$$

(c) The primary and secondary voltages are both related to the magnetic flux by Faraday's Law with

$$V_p = N_p \frac{d\Phi_B}{dt}$$
$$V_s = N_s \frac{d\Phi_B}{dt}$$

Dividing the two,

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

Now, with low resistivity, there is minimal power loss, so with $P = IV$ being a constant,

$$I_s V_s = I_p V_p \Rightarrow \frac{I_p}{I_s} = \frac{V_s}{V_p}$$
$$= \frac{N_s}{N_p}$$

(d) Using $\frac{V_p}{V_s} = \frac{N_p}{N_s}$, we have

$$N_s = N_p \frac{V_s}{V_p}$$
$$= 110 \frac{15 V}{110 V}$$
$$= 15$$