Probability Distributions and Gases

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1 Ideal Gas Law

1.1 Pressured balloons

A balloon is filled at ta pressure and temperature P and T, respectively, to a volume of V. Then, it is taken to the bottom of a lake, where the temperature is T'. The balloon's volume is measured to be V'. How deep is the lake? Suppose that the density of water is a constant value ρ.

1.2 Pressure gauges

How could you measure the pressure of a gas?

1.3 Estimations with the ideal gas law

Estimate the number of air molecules in an average-sized room.

1.4 Partitioned boxes

Suppose we have a box with total volume V and we have a partition in the middle with a monoatomic ideal gas at pressure P_0 and temperature T_0 on the left with a vacuum on the right.

(a) If we suddenly remove the partition, what happens? What is the final temperature and pressure of the gas?

(b) (Challenge) If we instead slowly move the partition to the right until it joins the rightmost wall, what happens to the gas along the way? (Qualitatively) What is the final temperature and pressure of the gas?

2 Probability Distributions

2.1 Statistics on distributions

Consider the following probability distribution for velocities of atoms in a gas:

$$f(v) = C(a - bv^2) \tag{1}$$

for $-\sqrt{a/b} < v < \sqrt{a/b}$ and

$$f(v) = 0 \tag{2}$$

otherwise, where a and b are fixed positive constants and C is an unknown constant.

(a) Find the value fo C which ensures that all atoms in the gas have some velocity.

(b) Find the average velocity v_{avg} of atoms in this gas.

(c) Find the average value of v^2 of atoms in this gas.

(d) What is the number distribution of this gas (a distribution of hte number of atoms with some velocity)?

2.2 Concrete numbers

You have 10 gas molecules in a box. At one moment, two have a speed of 10 m/s, four have a speed of 12 m/s, two have a speed of 14 m/s, one has a speed of 15 m/s and one has a speed of 17 m/s. The gas molecules each have a mass m.

(a) Calculate the average speed and the rms speed.

(b) Using its strict definition, what would the "temperature" be for this theoretical distribution? Leave your answer in terms of m and k_B.

2.3 Setting up expressions

At extremely low temperatures, the usual methods of cooling a gas do not work. Instead, a form of evaporative cooling is used, and it is based on the fact that for a Maxwell speed distribution the $p_N = 10\%$ of the gas molecules with the highest speed carry $p_E = 28\%$ of the total energy of the gas. By removing these atoms and waiting for the gas to come back gas to come back to equilibrium, you can cool the gas a fixed amount in each evaporation cycle.

(a) The fastest $p_N = 10\%$ of molecules will correspond to molecules with speeds larger than some value . Write down (but do not solve) the equation that you would use to determine α.

This is simple to write down, but hard to solve so do not try to solve it!

(b) Write down (but do not evaluate) the expression to determine what percentage of the energy the molecules with speeds larger than α carry. That is, relate the $p_E = 28\%$ above to the Maxwell distribution.

(c) Suppose a gas has initial temperature T_0. What is the temperature T_1 of the gas afer one cycle?

(d) Suppose you wanted to cool the gas to below $T_0/2$. What is the minimum number of cycles it will take?

2.4 Puddles!

Why do puddles evaporate, even if the temperature is much colder than the boiling point of water? Why do sealed jars never evaporate?

2.5 Building intuition

Plot a typical Maxwell Distribution for some value of N and T. What would it look like if you increased the termpature, keeping N constant? What would it look like if you increased the number of molecules, but kept T constant?