Week 8, Session 2 Problems

GSI: Caleb Eades

10/9

1 Fields, Potential, and Energy

1.1 The Classic Assembly Problem

Three charges, $+5Q$,$-5Q$, and $+3Q$ are located on the y-axis at $y = +4a$, $y = 0$, and $y = -4a$, respectively. The point P is on the x-axis at $x = 3a$.

(a) Draw a picture of the situation.

(b) How much energy did it take to assemble these charges?

(c) What is the electric potential V at point P, taking $V = 0$ at infinity?

(d) What are the x, y, and z components of the electric field $\mathbf{E}$ at P?

(e) A fourth charge of $+Q$ is brought to P from infinity. What are the x, y, and z components of the force $\mathbf{F}$ that is exerted on it by the other three charges?

(f) How much work was done (by the external agent) in moving the fourth charge $+Q$ from infinity to P? This can be done without integrating anything!

(*Source: MIT 8.02 Course Notes 3.10.9*)

1.2 Ice Cream? I wish..

Suppose we have a charged surface that looks like an empty ice-cream cone. The height of the cone is h and the radius of the base is also h. The surface has a uniform surface charge density σ. Find the potential at the tip of the cone, taking the zero of potential to be an infinity. Note that *only* the sloped surface of the cone is charged, *not* the base.

(*Source: part of Griffiths Introduction to Electrodynamics 2.26*)

1.3 Impossible!

One of these is an impossible electrostatic field. Which one?

(a) $\mathbf{E} = k[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}]$;

(b) $\mathbf{E} = k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$;

For the *possible* one, find the potential, using the *origin* as your reference point. Check your answer by computing ∇V.

(*Source: Griffiths Introduction to Electrodynamics 2.20*)

1.4 Calculating Potential

Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct field. Sketch $V(r)$.

(*Source: Griffiths Introduction to Electrodynamics 2.21*)

1.5 Challenge: Madelung constants

Consider an infinite chain of point charges, $\pm q$ (with alternating signs), strung out along the x axis, each a distance a from its nearest neighbors. Find the work per particle required to assemble this system. [*Partial Answer*: $-\alpha q^2/(4\pi\epsilon_0 a)$, for some dimensionless number α; your problem is to determine α. It is known as the Madelung constant. Calculating the Madelung constant for $2-$ and $3-$dimensional arrays is much more subtle and difficult.]

(*Source: Griffiths Introduction to Electrodynamics 2.33*)