

# Non-Ideal Gasses, Solutions

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## 1 Gasses: Ideal, Real and van der Waals

### 1.1 Conceptual Questions

- (a) Average velocity is zero but speed looks at the magnitude (some move left and some move right, for example). So overall while average speed is nonzero, average velocity will be.
- (b) We can infer that there is less inter-molecular bonding since there is a lower “escape velocity” for the alcohol.

### 1.2 Escape Velocities

First off, in a Maxwell Distribution,

$$\bar{v} = \sqrt{\frac{8}{\pi} \frac{kT}{m}} \approx 1.60 \sqrt{\frac{kT}{m}} \quad (1)$$

- (a) Solving for  $T$ ,

$$\begin{aligned} T &= \frac{m\pi}{8k} \bar{v}^2 \\ &= \frac{(2 * 16 * 1.66 \times 10^{-27} kg)\pi}{8(1.38 \times 10^{-23} J/K)} (1.12 \times 10^4 m/s)^2 \\ &\approx 1.9 \times 10^5 K \end{aligned}$$

- (b)  $m_{He} = \frac{1}{8} m_{O_2}$ , so

$$T_{He} = \frac{1}{8} T_{O_2} \approx 2.4 \times 10^4 K \quad (2)$$

- (c) At the same temperature,  $He$  will go much faster than  $O_2$ , so more of it will escape, leaving the  $O_2$  concentration much higher than  $He$ .

### 1.3 Fermi-ish Validation of Ideal Gas Law

Starting with the Ideal Gas Law,

$$\begin{aligned}
 PV &= nRT \\
 n &= \frac{PV}{RT} \\
 &\approx \frac{(1 \times 10^5 Pa)(100 m^3)}{(10 J mol^{-1} K^{-1})(300 K)} \\
 &\approx \frac{1}{3} \times 10^4 mol \\
 &\approx 3 \times 10^3 mol
 \end{aligned}$$

Now, looking at the volume for the gas,

$$\begin{aligned}
 V_{gas} &= \frac{4}{3} \pi (3 \times 10^{-10} m)^3 (3 \times 10^3 mol) (6.022 \times 10^{23}) \\
 &\approx 0.03 m^3
 \end{aligned}$$

Hence, the ratio of the volume of a gas particle to the volume of the room is

$$\frac{V_{gas}}{V_{room}} \approx \frac{0.03}{10^3} = 3 \times 10^{-5} \quad (3)$$

As we can see, the gas molecules take up less than three parts per hundred thousand in volume. So the Ideal Gas Law is a decent approximation.

### 1.4 Reformulation of Pressure

From the Ideal Gas Law and the average kinetic energy of a molecule,  $PV = NkT$  and  $\frac{1}{2}mv^2 = \frac{3}{2}kT$ , respectively. Identifying  $\langle v^2 \rangle = v_{rms}^2$ ,

$$v_{rms}^2 = \frac{3kT}{m} = 3 \frac{PV/N}{m} \quad (4)$$

At this point, it is useful to recall that  $m$  is the mass of a single molecule, so  $mN = M$  is the total mass of the gas and hence  $\frac{mN}{V} = \frac{M}{V} = \rho$ , the density. Putting this together,

$$v_{rms}^2 = 3 \frac{P}{\rho} \implies v_{rms} = \sqrt{3P/\rho} \quad (5)$$

## 1.5 Pressure from Other Things

Assuming they collide elastically with the window, the total change in momentum is  $2mv_x$ , with  $v_x = \sqrt{2}v/2$ . Then,

$$\begin{aligned} P &= F/A \\ &= \frac{\Delta p}{\Delta t} / A \\ &= \frac{2\sqrt{2}vm}{2} \times \left( 30 \frac{1}{\text{second}} \right) / A \end{aligned}$$

where we have identified  $\frac{1}{\Delta t}$  as the number of molecules that hit the window per second. So

$$\begin{aligned} P &= \frac{2 * 15 * 2 \times 10^{-3}}{\sqrt{2}} \times 30 / (0.5) \\ &\approx 2.6 Nm^{-2} \end{aligned}$$

This is about five orders of magnitude weaker than atmospheric pressure of 1 atm  $\approx 1 \times 10^5 Nm^{-2}$ .