Week 2, Meeting 1 Solutions

1.1

$$V_0 = L_0 H_0 W_0$$

$$\begin{aligned} V &= L_0(1+\alpha\Delta T)H_0(1+\alpha\Delta T)W_0(1+\alpha\Delta T) \\ &= L_0 H_0 W_0 (1+\alpha\Delta T)^3 \\ &= V_0(1+3\alpha\Delta T+3\alpha^2\Delta T^2+\alpha^3\Delta T^3) \\ &\approx V_0(1+3\alpha\Delta T) \end{aligned}$$

for small ΔT. So then $\beta = 3\alpha$ is the volume expansion coefficient. Similarly,

$$\begin{aligned} V &= V_0(1+\alpha_L\Delta T)(1+\alpha_{HW}\Delta T)^2 \\ &= V_0(1+\alpha_L\Delta T)(1+2\alpha_{HW}\Delta T+\alpha_{HW}^2\Delta T^2) \\ &= V_0(1+(\alpha_L+2\alpha_{HW})\Delta T+\cdots) \\ &\approx V_0(1+(\alpha_L+2\alpha_{HW})\Delta T) \end{aligned}$$

for small ΔT. So the $\beta = \alpha_L + 2\alpha_{HW}$ for the anisotropic case.

1.2

It gets larger! There are two views for this:

view 1	view 2
Concentric thin circles like lines:	Radial distance is just a length dimension that should expand linearly as $r = r_0(1+\alpha\Delta T)$, so
$C_0 = 2\pi r$	$a = a_0(1+\alpha\Delta T)$
$C = 2\pi r(1+\alpha\Delta T)$	$b = b_0(1+\alpha\Delta T)$
Every small circle gets bigger so the hole of the annulus grows larger!	etc.
$a = a_0(1+\alpha\Delta T)$	
$b-a = (b_0-a_0)(1+\alpha\Delta T)$	
$\pi(b^2-a^2) \approx \pi(b_0^2-a_0^2)(1+2\alpha\Delta T)$	

1.3

Tell me what you come up with! Have fun!

2.1

The lake adds a pressure $\rho g h$ where h is the ~~de~~ height beneath the surface. So letting D be the depth of the lake,

$$PV = nRT$$

$$P' = nRT'/V'$$

$$P_{balloon} + P_{depth} = nRT'/V'$$

$$nRT/V + \rho g h = nRT'/V'$$

$$\rho g h = nR\left(T'/V' - T/V\right)$$

$$h = \frac{nR}{\rho g}\left(T'/V' - T/V\right)$$

$$= \frac{P}{\rho g}\frac{V}{T}\left(T'/V' - T/V\right)$$

$$= \frac{P}{\rho g}\left(\frac{VT'}{TV'} - 1\right)$$

2.2

Again have fun with it and tell me what you come up with!

2.3

$P \approx 1$ atm, $T \approx 300$ K, $R \approx 8 \frac{J}{mol \cdot K}$, $V \approx (5\ m)^2 \cdot (4\ m) = 100\ m^3$, $1\ \text{atm} \approx 10^5 \frac{N}{m^2}$:

$$n = \frac{PV}{RT} \approx \frac{(10^5 \frac{N}{m^2})(100\ m^3)}{(8 \frac{N \cdot m}{mol \cdot K})(300\ K)} \approx \frac{100}{25} \cdot 10^3 = 4000 \text{ moles}$$

Multiplying by Avogadro's number (6.022×10^{23}),
$N \cong 2.5 \times 10^{26}$ molecules

2.4

(a) The gas expands into the remainder of the box that used to be a vacuum. So $\frac{V}{2} \to V$ and by the Ideal Gas Law,

$$P_0 \frac{V}{2} = nRT_0$$

$$P_f V = nRT_f$$

The temperature remains unaltered, however, because nothing has been done to increase or decrease the kinetic energy of the molecules, so

$T_0 \to T_0 = T_f$ and $P_0 \to P_0/2 = P_f$.

(b) This is trickier because along the way something has to hold the wall in place, which does negative work on the gas (can also think about the gas doing work on the wall to expand and push it outwards), so the temperature no longer remains constant. Hence,

T decreases

V increases ($V/2 \to V$ as before)

P decreases (by more than half now)