More Gas Math, Problems

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1 More Statistics

1.1 Continuous vs Discrete

Consider a system of $N >> 1$ ideal gas particles moving with speeds determined by a distribution, either continuous or discrete. For the continuous distribution, let $f(v) = Asin(\pi v)$ for $v \in (0, 1)$ m/s. For the discrete distribution, the particles can have velocity $v = 0$ m/s or $v = 1$ m/s with probability of 0, $v = 1/4$ m/s or $v = 3/4$ m/s with probability $B/\sqrt{2}$, or $v = 1/2$ m/s with probability of B.

(a) Find A and B such that the distributions, individually, are each valid probability distributions. Do A and B have units associated with them? What about π in $f(v)$?

(b) For each distribution, calculate $< v >= \bar{v}$, v_{rms}, σ, and v_p.

(c) What is the temperature of the gas?

(d) Comment on any discrepancies (or lack thereof) between the statistics for this discrete and continuous distribution. (e.g., Why did (or didn't) these discrepancies arise?)

1.2 Fun with Maxwell

Show that the total energy of a gas governed by the Maxwell distribution is given by

$$E = \left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}k_B T. \tag{1}$$

(Challenge: do this two different ways.)