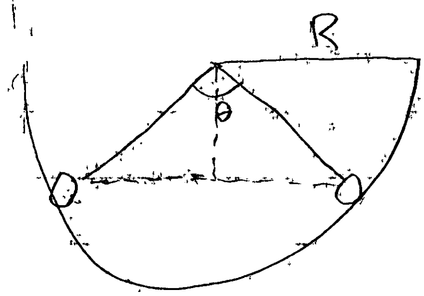
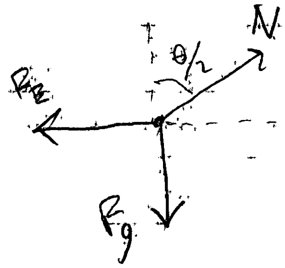
**P.1**




If the charges are separated by some angle $\theta$, then the physical distance separating them is

$$d = 2R\sin\left(\frac{\theta}{2}\right)$$




From this free-body diagram and Newton's second law we garner

$$N\cos\frac{\theta}{2} = mg$$

$$N\sin\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4R^2\sin^2\frac{\theta}{2}}$$

From the first equation, $N = mg/\cos\theta/2$. From the second,

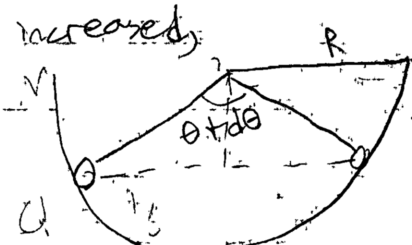
$$Q^2 = 16\pi\varepsilon_0 R^2 N\sin^3\frac{\theta}{2}$$

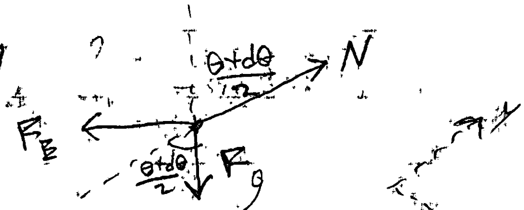
$$= 16\pi\varepsilon_0 mgR^2\tan\frac{\theta}{2}\sin^2\frac{\theta}{2}$$

Hence,

$$Q = 4R\sin\frac{\theta}{2}\sqrt{\pi\varepsilon_0 mg\tan\frac{\theta}{2}}$$

(b) Yes, this is a stable equilibrium. For a small change $d\theta$ increased,



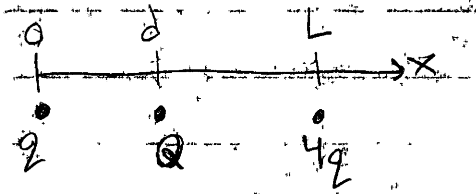

$$mg\sin\left(\frac{\theta}{2}+\frac{d\theta}{2}\right) - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4R^2\sin^2\left(\frac{\theta}{2}+\frac{d\theta}{2}\right)}\cos\left(\frac{\theta}{2}+\frac{d\theta}{2}\right) = m\ddot{x}$$

Note that $\sin\frac{\theta}{2}+\frac{d\theta}{2} = \sin\frac{\theta}{2}\cos\frac{d\theta}{2} + \cos\frac{\theta}{2}\sin\frac{d\theta}{2} \approx \sin\frac{\theta}{2} + \frac{d\theta}{2}\cos\frac{\theta}{2}$

and $\cos\frac{\theta}{2}+\frac{d\theta}{2} = \cos\frac{\theta}{2}\cos\frac{d\theta}{2} - \sin\frac{\theta}{2}\sin\frac{d\theta}{2} \approx \cos\frac{\theta}{2} - \frac{d\theta}{2}\sin\frac{\theta}{2}$

$$m\ddot{x} = mg\sin\frac{\theta}{2} + mg\frac{d\theta}{2}\cos\frac{\theta}{2} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4R^2\left(\sin^2\frac{\theta}{2} + \frac{1}{2}d\theta\sin\theta\right)}\left(\cos\frac{\theta}{2} - \frac{d\theta}{2}\sin\frac{\theta}{2}\right)$$

1.2




We must have

$$\frac{qQ}{4\pi\varepsilon_0 d^2} = \frac{4qQ}{4\pi\varepsilon_0 (L-d)^2} \Rightarrow \frac{(L-d)^2}{d^2} = 4$$

$$\Rightarrow (L/d - 1)^2 = 4$$

$$\Rightarrow L/d = 1 \pm 2$$

$$\Rightarrow L = 3d$$

where we have chosen the positive answer since d must be between 0 and L. Now, to find Q,

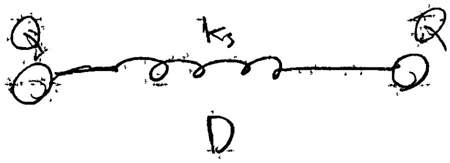
$$\frac{4q^2}{4\pi\varepsilon_0 L^2} + \frac{4qQ}{4\pi\varepsilon_0 (L-d)^2} = 0$$

$$\Rightarrow 4q/L^2 + 4Q/\left(\tfrac{4}{9}L^2\right) = 0$$

$$\Rightarrow Q = -\frac{4}{9}q$$

1.3

(a)




The net force on each charge must be zero:

$$\frac{Q^2}{4\pi\varepsilon_0 D^2} = k_s (D-L)$$

(b) Again,

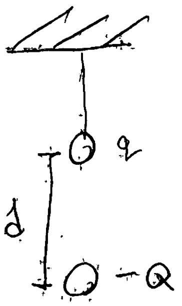
$$\frac{Q^2}{4\pi\varepsilon_0 D^2} = k_s (D-L)$$

since two negative charges act the same on each other as two positive charges.

(c) If they have opposite signs but the same magnitude $Q = |Q_1| = |Q_2|$,

$$\frac{Q^2}{4\pi\varepsilon_0 D^2} = k_s (L-D)$$

1.4

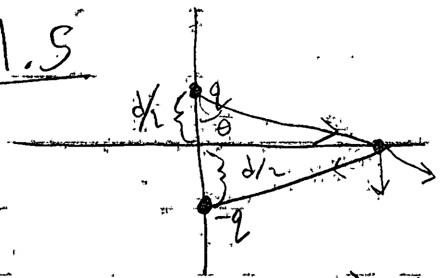



(a) We must have

$$\frac{qQ}{4\pi\varepsilon_0 d^2} = mg \Rightarrow d = \sqrt{\frac{qQ}{4\pi\varepsilon_0 mg}}$$

(b) No. If we go a little higher, the charges will attract stronger than gravity and it will keep rising. If we go below, the gravitational pull will be stronger than electric forces and it will keep falling.

1.5




$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= -2\,\frac{q}{4\pi\varepsilon_0 r^2}\cos\theta\,\hat{y}$$

$$= -\frac{qd}{4\pi\varepsilon_0 r^3}\,\hat{y}$$

$$= \frac{-p}{4\pi\varepsilon_0 r^3}\,\hat{y}$$

Taking the limit as $d \to 0$ while $p =$ constant, the field from a pure dipole in the plane perpendicular to the moment is

$$\vec{E} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$$

Along the axis of the moment,

$$\vec{E} = \left(\frac{q}{4\pi\varepsilon_0 (r - d/2)^2} - \frac{q}{4\pi\varepsilon_0 (r + d/2)^2}\right)\hat{y}$$

$$= kq\left[(r - d/2)^{-2} - (r + d/2)^{-2}\right]\hat{y}$$

Again taking the limit as $d \to 0$ while $p =$ constant,

$$\vec{E} = \frac{kq}{r^2}\left[(1 - d/2r)^{-2} - (1 + d/2r)^{-2}\right]\hat{y}$$

$$\approx \frac{kq}{r^2}\left[1 + d/r - 1 + d/r\right]\hat{y}$$

$$= \frac{2k\vec{p}}{r^3}$$

Hence, the field from a pure dipole along the dipole moment is given by

$$\vec{E} = \frac{\vec{p}}{2\pi\varepsilon_0 r^3}$$