Week 12, Session 2 Solutions

Problem 1.1:

This velocity will presumably be much larger than the drift velocity, so the magnetic field pushes the charges into separation until the electric field cancels it with

$$q\frac{V_0}{w} = qvB \Rightarrow v = \frac{V_0}{wB}$$

Problem 1.2:

(a) The current is equal to the number of charge carriers with the ~~drift~~ velocity:

$$I = \frac{dQ}{dt}$$
$$= ndhve$$

This is like the charge passing through a cross-section per unit time. Then, $v = I/ndhe$. So the the Hall voltage will be when

$$qE_H = qvB \Rightarrow E_H = vB$$
$$\Rightarrow V_H = hvB$$
$$= hB \frac{I}{ndhe}$$
$$= \frac{BI}{nde}$$

(b) $$K_H = \frac{V_H}{BI}$$
$$= 1/nde$$

(c) Based on these results, if you measure the current through the block as well as the magnetic field and the voltage difference (the Hall voltage) across the height, then the sign of the charge carriers is given by

$$\text{sign}(e) = \text{sign}\left(\frac{BI}{V_H}\right)$$

The density of charge carriers is given by

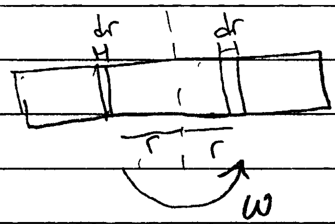
$$n = \frac{BI}{deV_H}$$

Problem 2.1:

(a) The magnetic moment is generally given by

$$\vec{\mu} = I\vec{A}$$

so $d\vec{\mu} = \vec{A}\,dI$. We split the rod into bits of charge, which, while rotating, generate a current.



In one period, a total charge $\frac{2dr}{L}Q$ passes through a given point, so the current is

$$dI = \frac{2Q\,dr/L}{T}$$

with $T = \frac{2\pi}{\omega}$, so

$$dI = \frac{\omega Q}{\pi L}dr$$

The area for each loop is

$$\vec{A} = \pi r^2 \hat{z}$$

Putting this together

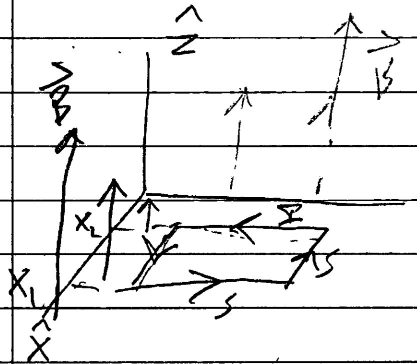
$$\begin{aligned}\vec{\mu} &= \int d\vec{\mu} \\ &= \int_0^{L/2} \vec{A}\,dI \\ &= \int_0^{L/2} (\pi r^2 \hat{z})\left(\frac{\omega Q}{\pi L}dr\right) \\ &= \frac{\omega Q}{L}\hat{z}\int_0^{L/2} r^2 dr \\ &= \frac{\omega Q}{L}\left(\frac{L}{2}\right)^3 \cdot \frac{1}{3}\hat{z} \\ &= \frac{\omega Q L^2}{24}\hat{z}\end{aligned}$$

(b) The torque and potential energy are respectively given by

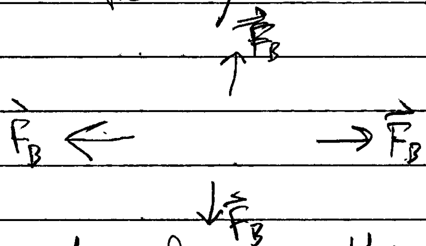
$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} \\ &= \frac{\omega Q L^2}{24}\hat{z} \times B_0\hat{z} \\ &= \vec{0}\end{aligned}$$

$$\begin{aligned}U &= -\vec{\mu}\cdot\vec{B} \\ &= -\frac{\omega Q L^2 B_0}{24}\end{aligned}$$

Problem 2.2:



The force on the right and left segments (running along $\hat{x}$) will cancel, at least from the magnetic forces:



Hence, the overall magnetic force on this square will be

$$\vec{F}_{net,m} = \left(B_0 \frac{x_1}{a}\right)(Is)\,\hat{x} - \left(B_0 \frac{x_2}{a}\right)(Is)\,\hat{x}$$
$$= B_0 I \frac{s}{a}(x_1 - x_2)\,\hat{x}$$
$$= B_0 I \frac{s^2}{a}\,\hat{x}$$

Friction can exert a maximum of $|\vec{F}_s| = \mu_s |\vec{n}|$ where $|\vec{n}| = mg$ here, so to prevent the loop from moving,

$$\mu_s mg = B_0 I \frac{s^2}{a} \Rightarrow \mu_s = \frac{B_0 I}{mg}\frac{s^2}{a}$$

Problem 2.3:

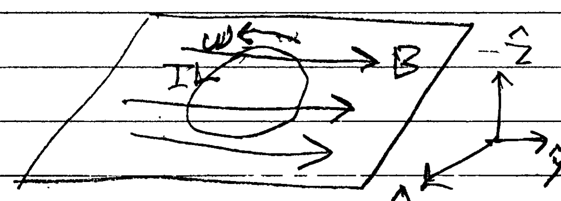
The total charge of the ring passes through a given point in one period, so

$$I = \frac{Q}{T}$$

and $\omega = \frac{2\pi}{T}$, so $T = \frac{2\pi}{\omega}$ and

$$I = \frac{Q\omega}{2\pi}$$

Now, our setup looks as to the right. The magnetic force on the rightmost edge of the ring will be in the $-\hat{z}$ direction with the strongest magnitude. On the leftmost edge, it will be the strongest in the $+\hat{z}$ direction. So for a little bit $d\ell$ of the ring there, there will be no normal force when



$$F_m = dmg \Rightarrow I\,d\ell\,B = M\frac{d\ell}{2\pi R}g$$
$$\Rightarrow \frac{Q\omega}{2\pi}B = \frac{Mg}{2\pi R}$$
$$\Rightarrow \omega = \frac{Mg}{QBR}$$