Cycles Problems

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1 Looking Forward to Spring(s)?

1.1 Springs and Expansions

Suppose N molecules of a monoatomic ideal gas at (P_1, T_1) are held inside a container by a piston with cross-sectional area A. The motion of the piston is resisted by a spring with spring constant k. The spring exerts no force in the initial state. The helium is then heated until $T_2 = 3T_1$. Express your answers in terms of the variables given in the problem and fundamental constants. Do not neglect atmospheric pressure — $P_1 \neq 0$!. Determine:

(a) the final volume of the system.

(b) the total work done by the gas.

(c) how much heat was added to the gas.

(d) Some answers are *very messy*. It is a good strategy in cases like this to define your own new variables and use those to keep everything looking nicer. Which *one* new variable should you chose to make your final answers look nice? Write them out in terms of this new variable.

Source: Former GSI Lenny Evans.

2 Test-like and other Interesting Problems

2.1 Interlude on Entropy

Suppose a 10 kg piece of iron ($c = 450$ J/kg*K) at initial temperature 1000 K is placed in a 100 kg water bath initially at temperature 300 K.

(a) Estimate the change in entropy for the iron, water, and system.

(b) Now actually calculate it. Compare your answer to part (a).

2.2 Derivations

Derive the efficiency of:

(a) a Carnot cycle. This consists of an isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression. (See Figure 20-7 on pg. 533 of the text, but try not to just copy their derivation. After all, this is for you, as a frequently seen test problem.)

(b) an Otto cycle. This approximates a car engine and consists of an adiabatic compression, isovolumetric heat increase, adiabatic expansion, and isovolumetric heat loss. (See Figure 20-8 on pg. 535 as well as the accompanying explanation.)

2.3 Net Efficiency of Two Engines

Consider two heat engines, Engine A and Engine B, with efficiencies e_A and e_B. We will create a composite engine, Engine C, by letting the heat output from Engine A be the heat input Engine B, as shown schematically on pg. 30 of the workbook.

(a) If a heat $Q_{in,A}$ is fed into Engine A, what is the net work output and the total heat output from Engine A, W_A and $Q_{out,A}$ in terms of $Q_{in,A}$ and e_A?

(b) If the heat input for Engine B is equal to the heat output of Engine A ($Q_{out,A} = Q_{in,B}$), what is the net work output and the total heat output from Engine B, W_B and $Q_{out,B}$ in terms of $Q_{in,A}$ and e_A?

(c) What is the total work that is output from both engines as a result of feeding the engines the heat $Q_{in,A}$?

(d) What is the net efficiency, e_C, of the combined engine system?

(e) Show that if both $e_A < 1$ and $e_B < 1$, then $e_C < 1$.

(f) Suppose Engine A is a Carnot engine operating between temperatures T_H and T_M and Engine B is a Carnot engine operating between temperatures T_M and T_C ($T_H > T_M > T_C$). Show that the net efficiency, e_C, is just the efficiency of a Carnot engine operating between temperatures T_H and T_C.

Source: Workbook pg. 30

2.4 Preparing for the Test?

Just for a note, there are quite a few decent problems in the workbook, specifically between pgs. 19-43 for cycles, processes on gases, entropy, etc. Taking a look at these might be useful in preparing for the exam.