Linear Expansion Solutions

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1 Thermal Expansions

1.1 Three Dimensions

The initial volume:

$$V_0 = L_0 H_0 W_0 \tag{1}$$

The volume after isotropic expansion:

$$\begin{aligned} V &= L_0(1+\alpha\Delta T)H_0(1+\alpha\Delta T)W_0(1+\alpha\Delta T) \\ &= L_0 H_0 W_0 (1+\alpha\Delta T)^3 \\ &= V_0(1+3\alpha\Delta T+3\alpha^2\Delta T^2+\alpha^3\Delta T^3) \\ &\approx V_0(1+3\alpha\Delta T) \end{aligned}$$

for small ΔT. So then $\beta = 3\alpha$ is the volume expansion coefficient.

Similarly, for anisotropic expansion,

$$\begin{aligned} V &= V_0(1+\alpha_L\Delta T)(1+\alpha_{HW}\Delta T)^2 \\ &= V_0(1+\alpha_L\Delta T)(1+2\alpha_{HW}\Delta T+\alpha_{HW}^2\Delta T^2) \\ &= V_0(1+(\alpha_L+2\alpha_{HW}\Delta T+...) \\ &\approx V_0(1+(\alpha_L+2\alpha_{HW})\Delta T) \end{aligned}$$

for small ΔT. So the volume expansion coefficient is $\beta = \alpha_L + 2\alpha_{HW}$.

1.2 Expanding/contracting holes

The hole gets larger! There are two ways to view this.

View 1:

Concentric thin circles are like lines. So a circle of radius r and circumference $C = 2\pi r$ will expand to $C = 2\pi r(1+\alpha\Delta T)$. Every small circle gets bigger so the hole of the annulus grows larger!

$$\begin{aligned} a &= a_0(1+\alpha\Delta T) \\ b-a &= (b_0-a_0)(1+\alpha\Delta T) \\ \pi(b^2-a^2) &\approx \pi(b_0^2-a_0^2)(1+2\alpha\Delta T) \end{aligned}$$

View 2:

Radial distance is just a length dimension that should expand linearly as $r = r_0(1 + \alpha\Delta T)$. The other results follow.

1.3 Becoming an experimenter

Have fun!