

# Electric Fields Exam, Problems

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10/4

## 1 Finding Electric Fields

### 1.1 Progressive Derivations

(Note: this is the last problem from Week 7, Session 1 Problems in case you didn't get around to it. It is a good problem to know.)

This problem asks you to find the electric field from progressively more complex charge distributions. Indeed, part (c) seems very hard to solve for. [Hint: use the result of part (a) for part (b). Similarly, use the result of part (b) for part (c).]

- (a) Suppose there is a ring of charge of radius  $r$  centered at the origin in the  $xy$ -plane with linear charge density  $\lambda$ . Calculate the field at a point  $P$  that is at  $(x, y, z) = (0, 0, d)$ .
- (b) Suppose there is a disk of radius  $R$  centered at the origin in the  $xy$ -plane with area charge density  $\sigma$ . Calculate the field at the same point  $P$ .
- (c) Suppose there is a cylinder of radius  $R$  centered at the origin in the  $xy$ -plane with its bottom surface in that plane, extending a height  $h$  upwards along its axis, the  $z$ -axis. If this cylinder has charge density  $\rho$ , calculate the field at the same point  $P$ , assuming  $h < d$ .

## 2 Previous Midterm Problems

### 2.1 Non-Uniform Charge Distributions?!

A non-uniformly charged ring of radius  $R$  carries a linear charge density  $\lambda(\theta) = \lambda_0 \cos(\theta)$ , with  $\lambda_0 > 0$ . The ring lies in the  $(x, y)$  plane centered at the origin.

- (a) Determine the direction of the electric field created at  $(0, 0, z)$ , and explain your reasoning.
- (b) Calculate the magnitude of the field.

- (c) A similar ring of linear charge density  $\lambda(\theta) = -\lambda_0 \cos(\theta)$  is placed at distance  $2R$  from the first one along the symmetry axis. Determine the new field on the symmetry axis.
- (d) Make a qualitative plot of the magnitude on axis of the electric field between the two rings.

(Source: Bordel Spring 2013 Midterm 2)

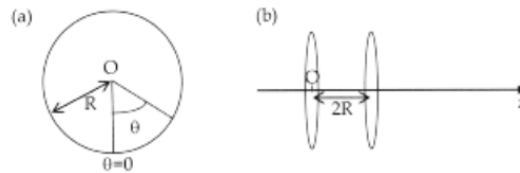


Figure 1: Setup for problem 2.1

## 2.2 Fields from Rings

A flat ring of inner radius  $R_1$  and outer radius  $R_2$  carries a non-uniform surface charge density  $\sigma(r) = \frac{\beta}{r}$ , where  $\beta$  is a positive constant, and  $r$  is the radial distance measured from the center of the ring. Calculate the electric field produced by such a charge distribution at any point on the symmetry axis.

(Source: Bordel Spring 2014 Midterm 2)

## 2.3 Dipoles with Lines

Figure ?? shows two equal and opposite charges,  $q$ , separated by a fixed distance,  $d$ . The rod connecting the two charges is at an angle,  $\theta$ , from the horizontal axis, and at a distance  $r \gg d$ , from an infinite line of charge with charge per unit length,  $\lambda > 0$ . ***This line of charge points out of the page.*** Find the force  $\vec{F}$ , on the charges to the lowest, nontrivial order in  $d/r$  when  $\theta = 0$  and when  $\theta = 90^\circ$ . Express them in terms of any of the variables given and  $\epsilon_0$ .

(Source: Speliotopoulos Spring 2014 Midterm 2)

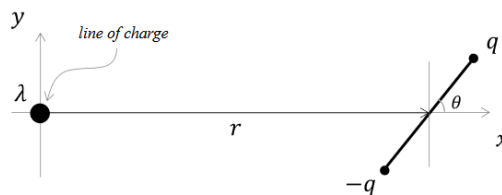


Figure 2: Setup for problem 2.3