

# Week 14, Session 1 Problems

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## 1 Faraday's Law

### 1.1 Alternating Current through Rotations

Alternating current generator. A rectangular loop of  $N$  turns with length  $a$  and width  $b$  is rotated at an angular frequency  $\omega$  in a uniform field of induction  $\vec{B}$ . Show that there is an induced EMF  $\epsilon = \omega N b a B \sin \omega t = \epsilon_0 \sin \omega t$ .

(Source: Halliday and Resnick, Problem 35.9)

### 1.2 Falling Rails

A rod with length  $l$ , mass  $m$ , and resistance  $R$  slides without friction down parallel conducting rails of negligible resistance, as shown below. The plane of the rails makes an angle  $\theta$  with the horizontal, and a uniform vertical magnetic field  $\vec{B}$  exists throughout the region.

- (a) Find the steady-state terminal velocity of the sliding rod.
- (b) Show that the rate at which the internal energy of the rod is increasing is equal to the rate at which the rod is losing gravitational potential energy.
- (c) Discuss the situation if  $\vec{B}$  were directed down instead of up.

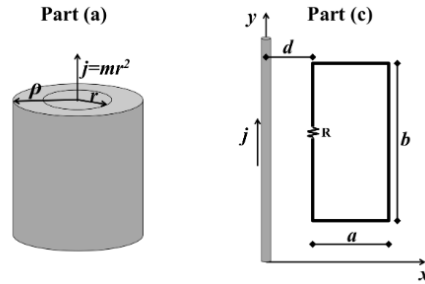
(Source: Halliday, Resnick, Krane 34.9)

## 2 Previous Exam Problems

A long straight wire of radius  $\rho$  carries a current along its axis with a non-uniform current density  $j(r) = mr^2$  ( $m=\text{constant}$ ),  $r$  being the radial distance measured from the symmetry axis of the wire, as shown below.

- (a) Calculate the magnitude of the magnetic field produced inside and outside the wire.
- (b) Draw some field lines to show qualitatively how the magnitude and direction of the magnetic field vary. Specify the direction of the current in your drawing.

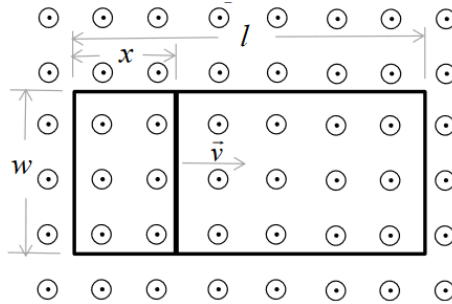
- (c) A rectangular loop of sides  $a$  and  $b$  and resistance  $R$  is placed a distance  $d$  ( $d > \rho$ ) from the center of the current-carrying wire, as shown below. What is the induced current in the loop if it is (i) translated along the  $y$ -axis at constant speed  $v$ ? (ii) translated along the  $x$ -axis at speed  $v$ ?



(Source: Birgeneau, Fall 2015 Final Exam, Problem 4)

## 2.1 Loops and Forces

The figure shows a wire loop, which has a length,  $l$ , and width,  $w$ , inside a magnetic field,  $B$ . The loop is made of a wire with resistivity,  $\rho$ , and cross-sectional area,  $A$ . A crossbar made of a conducting metal with negligible resistance is in contact with the loop, and at the instant shown in the figure, it has velocity,  $v$ , and there are currents on both sides of the bar. What force,  $F$ , (magnitude and direction) must be placed on the rod when it is at the position shown so that the velocity of the rod is constant?



(Source: Speliotopoulos, Fall 2012 Final Exam, Problem 3)