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Finance

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Finance

Kenneth J Boudreaux is Professor of Economics and Finance at the AB Freeman School of Business, Tulane University, New Orleans, US.

Professor Boudreaux is an eminent scholar in finance, known widely for his ability to combine cutting-edge knowledge of the field with understandable explanations to professionals. In addition to being a successful researcher and university professor, he has for the past two decades lectured extensively to executives on topics in finance, in all parts of the world. Professor Boudreaux is the co-author of *The Basic Theory of Corporate Finance*, a widely used graduate-level text, and has published significant scholarly research on issues in corporate finance, securities markets and corporate restructuring. His research is frequently cited in journals of finance and economics world wide.

Professor Boudreaux is an active consultant to the business world, and regularly performs analyses involving financial issues for firms across industries that include shipping, petroleum exploration and production, airlines, consumer products and computers. Included on this list are: Atlantic Container Lines, British Petroleum, Central Gulf Lines, Exxon, Hewlett-Packard, Petroleum Helicopters and Reckitt & Colman.

Notwithstanding Professor Boudreaux's wide experience, all organisations referred to in the worked examples are for illustrative purposes only and are entirely fictitious.

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Module 1

The Basic Ideas, Scope and Tools of Finance

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Learning Objectives

This module introduces the student to Finance as a subject area. It describes the participants in financial markets, the decisions they must make, and the basic processes that are common to all such financial decisions. The module discusses the roles of borrowers, lenders, equity, security issuers and purchasers, and the sources of value for each. Because finance is inherently a quantitative and economic subject, this introductory module devotes much effort to instructing the student in the essential quantitative techniques of financial valuation, including discounting, present valuation, determination of rates of return, and some important financial economics relating to interest rates and security valuation. The module introduces some specialised financial concepts such as 'yield to maturity' and the 'term structure' of interest rates. It includes the first of several perspectives on the important company decision tools 'net present value' and 'internal rate of return'. The module finishes with an illustration of the usefulness of even these basic financial techniques in understanding a market that remains mysterious to many financial practitioners: forward and futures markets for interest rates. In this module the student will learn the essence of the financial environment, along with the basic quantitative tools of financial valuation that are used throughout the course.

1.1 Introduction

In this first module of the finance course you will study the basic concepts and techniques of analysis in finance. We shall investigate ideas of market value, of investment decision-making, of interest rates, and of various kinds of financial markets. It is always good to have some general appreciation of a subject before its details are studied, and this is particularly true of finance, which is a very large and complex field of study. This module will supply you with that essential understanding, providing you with information on a number of fundamental concepts that can be applied again and again to the solution of real financial problems.

Finance is **the economics of allocating resources across time**. This definition, of course, is not particularly informative, but an example of a financial transaction that is governed by it may help you to understand the definition. Thus, suppose a new audio device, the digital audio tape player, has just come onto the market. Being an audiophile, you must have one. Economic logic says that if you had the resources to purchase it you would, because your satisfaction would increase by exchanging cash for the digital device. But suppose that you had neither the cash nor other assets that could be readily sold for enough cash to purchase the player. Would you be able to buy it?

The answer is that you may or may not be able to, depending upon whether you can convince someone to lend you the money. Whether you can is a function both of the tangible assets you have and the expectation of developing more assets in the future from which your creditor can expect to be paid. Because you do not have the tangible financial resources now to buy what you want, **by borrowing you can shift some of your future resources back to the present** so as to enable you to buy what you desire. You are buying the player with resources that you do not yet have in hand but are expected to get sometime in the future. And from the perspective of the lender, an exactly opposite transaction will take place: the lender gives up some present resources in exchange for those that you are expected to provide in the future as you pay off the loan. This shifting around, or **reallocation of resources in time**, is the essence of finance.

This example is useful because it can also help us to understand why finance is an important subject. Think how many transactions have this essence of shifting resources around in time. We must include not only the borrowing and lending of money by individuals, but by governments, corporations and other institutions. And the borrowing and lending of money are not the only ways in which resources are reallocated in time. When a company issues equity capital (in other words, raises money from its owners) it is undertaking a financial transaction similar to your borrowing to get the tape player; that is, it is accepting money now and giving in exchange a promise to return money in the future (in the form of corporate dividends). The owners of the company are engaging in a financial transaction with the company that is very similar to the one that occurs between you and the lender in the financing of your tape player.

Think how many purchases and sales of tangible assets are made possible by the ability to shift resources across time. All personal credit purchases, much of corporate asset acquisition, and a great deal of a government's providing of assets and services would not be possible without underlying financial transactions. Understanding the finance characteristic of these activities is an important part of being educated in business.

The above array of transactions that have important financial dimensions is impressive in its breadth, but it can be intimidating in the complexity it implies for the study of finance. We shall nowhere deny that finance is a large and complex field, but its study need not be terribly intricate, at least at the outset. In this course our approach will be first to create a very simple model of a financial market wherein participants (the individuals, companies and governments in the market) can engage in rudimentary financial transactions. That model is useful in acquainting ourselves with the basic ideas common to all financial transactions. Once these have been developed, we shall gradually include more and more realism in the model, until we can deal with the characteristics of financial markets and transactions that we see in actual practice.

1.2 Financial Markets and Participants

In a developed economy most people participate frequently in financial markets. Individuals borrow from and lend to financial institutions such as banks. Corporations similarly transact with banks, but they also use financial markets through other intermediaries such as investment bankers (companies that help raise money directly from other companies and individuals), and insurance companies (which lend your insurance premiums to other companies). Governments also borrow and lend to individuals, companies and financial institutions.

It is useful to have a general picture in our minds of why companies, individuals and governments use financial markets. We already have one example: you can use the financial market to facilitate your purchase of the tape player. That transaction shifts some of your future resources to the present (by borrowing), and increases your satisfaction. Other types of participants often engage in the same type of transaction. Governments regularly shift future resources to the present so as to allow greater present consumption by citizens. They do that by borrowing in the financial markets with the promise to repay the loans with future cash inflows of the type expected by governments (taxes, more borrowing, etc.). One of the most common motivations for participating in financial markets is to shift future resources to the present so as to increase present consumption, and thus satisfaction.

On the other hand, individuals, governments and companies also sometimes find themselves with more current resources than they wish to consume at present. They can shift present resources to the future by making them available to the financial markets. They can shift these resources by lending them, by buying ordinary (ownership) shares in a company, or by a number of other transactions. In exchange for giving up current resources, they get an expectation of increased future resources in the form, for example, of interest and principal payments from the amounts lent, and dividends and capital gains on the ordinary shares purchased. Individuals and institutions engaging in such transactions are happier with less present and more future resources, and that is their motivation for participating in financial markets. The money that they **provide** to the financial markets, of course, is the same money **borrowed** by others wishing to increase their present consumption by shifting resources from the future to the present. Depending upon a participant's resources and preferences for consuming them across time, that participant may at various times be a lender, a borrower, or both. Such financial transactions are motivated by a desire to increase satisfaction by changing the time allocation of resources.

Financial market participants borrow or otherwise raise money not only to alter their patterns of consumption but also to make investments in real assets. In finance, we distinguish between financial investments (such as borrowing, lending or buying ordinary shares), and real asset investments (such as building a new factory or buying a piece of equipment to be used in production). Whereas financial investments serve the purpose of reallocating resources across time, real asset investment can actually create new future resources that did not before exist. Real asset investment is obviously an important activity. Many economists feel that it may be the single most important activity in determining how wealthy people are.

Without financial markets, however, participants with ideas for good investments would find it difficult or impossible to get the money necessary to undertake those investments. Financial markets are the bridge between those willing to give up present consumption of resources in order to increase future consumption, and those in need of present resources in order to undertake real asset investment. This is another important function of financial markets.

The provision of funds for real asset investment is important, but just as important is the **allocative information** that financial markets provide to those interested in making real asset investments. Financial markets can help the investor tell whether a proposed real asset investment is worthwhile by comparing the returns from the investment with those available on competing uses of the resources. If the financial market did not do that, some other authority, such as the government, would. There often are significant differences between the decisions that would be made by a government and by competitive financial markets.

There is one other important service provided by financial markets to participants. We can describe this generally as **risk adjustment**. We are not yet ready to give a rigorous definition of risk in financial transactions, but your own intuitions about risk will serve as an acceptable definition for now. Financial market participants are **risk-averse**. That phrase means that their dislike for risk would, for example, cause them to choose the less risky of two otherwise identical investments. This implies not that participants reject risky transactions, but that the riskiness of an opportunity affects the price that they are willing to pay for it. Because financial markets have such wide variety of riskinesses available, participants can combine borrowings, lendings, the buying and selling of shares, and other transactions to shape the riskiness of their position to be whatever makes them most satisfied. Such decisions made by participants also influence the information that financial markets give to potential real asset investors, as above.

In sum, financial markets allow participants to reallocate resources across time, to decide correctly about – and make – real asset investments, and shape the riskiness of their holdings. All of these services are inherent in the transactions that participants make in those markets.

1.2.1 Market Interest Rates and Prices

When some participants wish to bring future resources to the present by borrowing, and others wish to shift present resources to the future by lending, the possibility of beneficial transactions is obvious (the potential lenders can provide current resources to the potential borrowers in exchange for the borrowers' promises to provide future resources to the lenders, making both groups happier). But they must decide on the amount of future resources to be exchanged for present ones. In other words, the lenders and borrowers must agree how many pounds of future resources it will be necessary to expect in exchange for each pound of current resources provided.

The financial market makes that decision for participants by setting the **market interest rate**. The market interest rate is the rate of exchange between present and future resources. It tells participants how many pounds are expected to be provided in the future for each pound of resources provided now. For example, if the market interest rate is 8 per cent per year, a lender can expect to receive 108 at the end of the year for each 100 lent at the beginning. The 108 comprises the 100 originally lent plus 8 as interest or compensation for lending. The relative demand and supply of resources to be borrowed and lent determines the market interest rate. (The market interest rate is always positive, because lenders have the alternative of simply keeping their money, and will therefore not agree to getting less in the future than they give up now.)

There is actually no such thing as *the* market interest rate. There are many market interest rates, all of which exist at the same time. The reason why there can simultaneously be many market interest rates is that interest rates can cover different lengths of time in the future, and different riskinesses of transactions. For example, it is entirely possible that the interest rate for borrowing across a two-year period is different from that for borrowing across a one-year period, because of the relative demand and supply of lendable resources over those times. And the interest rate that applies to borrowings by a risky company will be higher than that paid by the government (which controls the presses that can print currency to pay off its loans), because lenders are risk-averse and require greater expected compensation in future resources from risky borrowers.

From that perspective, there are even more 'interest rates' than we tend to regard as such. Suppose, for example, that you were to purchase ordinary shares from a company with the expectation of getting future dividends in return. We do not describe that transaction as lending money to the company, nor is there a quoted interest rate, but in general economic terms the transaction is very similar to lending. You are giving up present pounds for the expectation of future pounds. Financial markets do not quote an interest rate for your purchase of ordinary shares, but they do quote a price for the shares. And when you receive dividends or cash from selling the shares, you will earn a rate of return that is similar to an interest rate. In other words the market price is telling you how much you must 'lend' to the company in order to get the expected future dividends and increases in value. This information is almost the same as quoting a market interest rate for the transaction, as we shall now see.

1.3 A Simple Financial Market

1.3.1 Shifting Resources in Time

Financial markets are complicated when there are many different kinds of participants, when the transactions that they make are risky, and when those transactions cover several periods of time. Before we finish studying finance we shall deal with all of those things. First, however, we must develop the basic concepts inherent in all financial transactions. We shall do this with the simplest financial model that can accomplish that goal. The first financial market we shall examine therefore has the following characteristics:

1. We assume away all 'frictions' such as taxes, costs of transacting (brokerage fees) and costs of finding information.
2. We assume away all risk. Whenever a transaction is agreed upon, all of its terms will be kept by everyone.
3. Time is very simple in this market. There is only 'now' and 'later', with one period of time between. All financial transactions take place 'now' and have their resolution (for example, pay their interest and principal) 'later'.

In this kind of financial market we need not distinguish between types of participants, because individuals, governments and companies would all have the same risk (none), pay the same taxes (none), and last the same time (one period). That is not to say that all participants are exactly the same, however. As a matter of fact they will be different enough to present a surprisingly realistic picture of a quite diverse market.

Suppose in this market there is a participant who will receive both £1000 'now' and £1540 'later'. (You can regard this as a set of earnings for work that the participant expects to do, a set of inheritance payments from a deceased relative, or anything else that produces the two amounts. Because there are no taxes and receipt of the payments is certain, their source is irrelevant.) The participant could consume (spend) the £1000 immediately, wait until 'later', and then also consume the £1540. Actually, if there was no financial market, the participant would have no choice but to consume in that pattern, because he or she could not shift resources (the expected cash flows) around in time. Figure 1.1 shows how that set of cash flows, point E, appears on a graph with CF_1, the cash flow expected 'later' (we call 'later' t_1 for 'time period one') on the vertical axis and CF_0, the 'now' (t_0) cash flow on the horizontal.

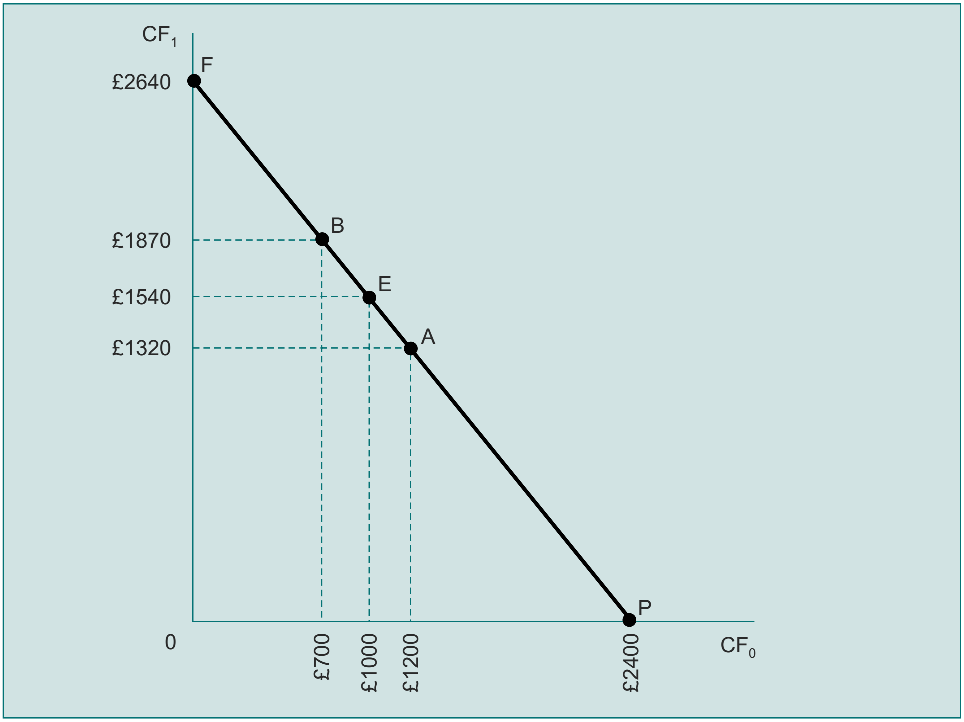

Figure 1.1 **The financial exchange line**

Suppose that the participant does not particularly like this pattern of consumption and prefers to consume somewhat more than £1000 at t_0. He can accomplish that by borrowing some money at t_0 with the promise to repay it with interest at t_1. Suppose further that the balance of potential borrowers and lenders has resulted in a market interest rate of 10 per cent. With such a market interest rate, the participant could, for example, increase his t_0 consumption to £1200 by borrowing £200 now and promising to repay that amount plus 10 per cent interest at t_1. He would owe £200 × (1 + 10%), or £220 at t_1, so at t_1 he could consume £1540 minus £220, or £1320. The move from his original pattern (point E) to this new pattern (point A) is shown in Figure 1.1.

The financial market also allows participants to shift resources into the future by deferring current consumption. If the participant originally decides that £1000 of present consumption is too much, he could, for example, lend £300 of his t_0 money and get in return an increase of £300 × (1.10) = £330 at t_1. That transaction is shown as a move from E to B in Figure 1.1.

You may have noticed already that if we were to join all of the points we have discussed in Figure 1.1, they would form a straight line (we shall henceforth call this a **financial exchange line**, and for convenience we shall also use the letter i to stand for the interest rate). Actually, any transaction that a participant with this initial endowment might take by borrowing or lending at the market rate of interest will produce a result that lies on that straight line between the two axes. For example, if all cash flows were transferred to t_1, there would be £[1540 + (1000 × 1.10)], or £2640 for t_1, and nothing for t_0 (point F).[1]

On the other hand, if all cash flows were shifted to t_0, the participant would have £1000 plus whatever he could borrow at t_0 with a promise to repay £1540 at t_1. How much is this? For each £1 we borrow at t_0, we must repay £1 × (1 + i) at t_1, so we can borrow (using CF_t to mean 'cash flow at time t') where

$$CF_1 = CF_0(1+i)$$

$$CF_0 = \frac{CF_1}{(1+i)}$$

Thus in our example:

$$CF_0 = \frac{£1540}{1.10} = £1400$$

The participant could borrow £1400 at t_0 with the promise to pay £1540 at t_1, the £1540 comprising £1400 of principal and £140 of interest. The maximum amount the participant could consume at t_0 is thus £2400, consisting of the original £1000 cash flow, plus the £1400 which can be borrowed at t_0 with the promise to repay £1540 at t_1. This is point P in Figure 1.1, £2400 at t_0 and £0 at t_1.

Believe it or not we have just done a calculation and arrived at a result that is of the greatest importance and underlies many of the ideas of finance. Discovering that £1540 at t_1 is worth £1400 at t_0 is called finding the **present value** of the £1540. **Present value is defined as the amount of money you must invest or lend at the present time so as to end up with a particular amount of money in the future**. Here, you would necessarily invest £1400 at t_0 at 10 per cent interest to end up with £1540 at t_1, so £1400 is the present (t_0) value of £1540 at t_1. The person or institution that was willing to lend the participant £1400 must have done just that type of calculation.

Finding the present value of a future cash flow is often called **discounting** the cash flow. In the above example, £1400 is the 'discounted value of the £1540', or the 'present value of the t_1 £1540, discounted at 10 per cent per period'.

From the calculations above you can see the type of information that the present value gives us about the future cash flow it represents. For example, should the participant's expectation of receiving t_1 cash flow increase to greater than £1540, he could borrow more than £1400 at t_0 (and vice versa for a smaller t_1 expectation). Or if the t_1 cash flow expectation becomes risky, the lender will require a return higher than 10 per cent to compensate for the risk being borne. (The risk here is that when t_1 arrives, the full amount of the £1540 expectation does not appear.) If the interest rate

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[1] The market interest rate is thus really an 'exchange rate' between present and future resources. It tells us the price of t_1 pounds in terms of t_0 pounds. The exchange line in Figure 1.1 and the market interest rate are thus giving us the same basic information. It should not surprise you to hear that the steepness or slope of the exchange line (the ratio of exchanging t_0 for t_1 pounds) is determined by the market interest rate. The higher is the market interest rate, the more steep is the slope of the exchange line. In plain words, this says imply that the higher is the interest rate, the more pounds you must promise at t_1 in order to borrow a pound at t_0 – which you doubtless knew already!

increases, you can see that the present value, and thus the amount that the participant can borrow against it, declines. So the present value of a future cash flow is the amount that a willing and informed lender would agree to lend, getting in return a claim upon the future cash amount. The amount of the present value will depend upon the expected size and risk of the cash flow, and when it is expected to occur.

How much you can borrow by promising to pay an expected future amount is an important interpretation of present value, but by no means is it the only, or even the most important, interpretation. **Present value is also an accurate representation of what the financial market does when it sets a price on a financial asset**. For example, suppose that our participant does not wish to borrow money, but instead wishes to sell outright the expectation of receiving cash at t_1. The participant can do this by issuing a **security** that endows its owner with the legal right to claim the t_1 cash flow. This security could be a simple piece of paper with the agreement written upon it, or could be a very formal contract of the type issued by companies when they borrow money or issue shares.

For how much do you think the participant might sell such a security? Everyone thinking of buying will of course examine alternatives to buying this security. (Economists call such alternatives **opportunity costs** because they represent the 'costs' of doing this instead of something else, in the sense of an opportunity forgone.) They will discover that for each t_0 £1 used to buy the security, £1.10 at t_1 will be returned by other same-risk investments in the financial market (for example, lending at 10 per cent). That being the case, the participant will be able to sell the security for no more than £1400 (the present value of £1540 at t_1 discounted at 10 per cent), because potential buyers need only lend £1400 to the financial market at t_0 in order to get £1540 at t_1, exactly what the security promises. And because of the competitive nature of financial markets, the security will not sell for less than £1400, because if it did it would provide a cash return the same as other alternatives, but at a lower present price. As potential buyers of the security begin bidding against each other, the security's price must increase or decline to the point where its expected future cash flow costs the same as that future cash flow acquired by any other means.

Present value is thus the market value of a security when market interest rates or opportunity rates of return are used as discount rates. This is perhaps the most important application of the notion of present value.

This brings us to another important application of the present-value idea. We have seen that the present value of all of our participant's present and future resources (cash flows) is £2400. This amount also has a special name in finance: it is known as **present wealth**. Present wealth is a useful concept in that it tells us the total value of a participant's entire time-specified resources with a single number. It is even more important because it can be used as a benchmark or standard to judge whether someone is going to be better or worse off because of a proposed financial decision. But we shall need to introduce a few more ideas before we can develop that point as completely as it deserves.

One thing we can now see readily from Figure 1.1 is that one cannot change present wealth merely by transacting (borrowing and lending at the market rate) in the financial market. Though borrowing and lending will move us up and down the financial exchange line, and thereby allow us to choose the time allocation of our present wealth that makes us most happy, such transactions cannot move the line and therefore cannot change our wealth. The reason is easy enough to deduce. In financial markets, by buying and selling securities (or borrowing and lending), the total amount of wealth that is associated with those securities is unchanged. So if one wishes to increase one's wealth by buying and selling in those markets, one would be forced to find another participant who, doubtless inadvertently, would allow his wealth to be reduced. As we shall see soon, the odds of doing so are low.

1.3.2 Investing

If we cannot expect to change our wealth by transacting in financial markets, how *can* we get richer? The answer is by **investing in real assets**. This type of financial activity can change our present wealth because it is not necessary to find someone else to give us part of their wealth in order for ours to increase. Investing in real assets such as productive machinery, new production facilities, research, or a new product line to be marketed, because it creates new future cash flows that did not previously exist, can generate new wealth that was not there before.

Of course not all real-asset investments are wealth-increasing. Investments are not free; we must give up some resources in order to undertake an investment. If the present value of the amounts we give up is greater than the present value of what we gain from the investment, the investment will decrease our present wealth. Because that will allow us to consume less across time, it is a bad investment. Of course a good investment would produce more wealth than it uses, and would therefore be desirable.

Figure 1.2 shows how real-asset investments work in our simple financial market. Suppose that our participant discovers an opportunity to invest £550 at t_0 in a real asset that is expected to return £770 at t_1. In Figure 1.2 this appears as a move from point E to point I. That investment would result in t_1 resources of £2310 and t_0 resources of £450. Should this opportunity be accepted or not? The answer is that it depends upon the effect on the participant's present wealth.

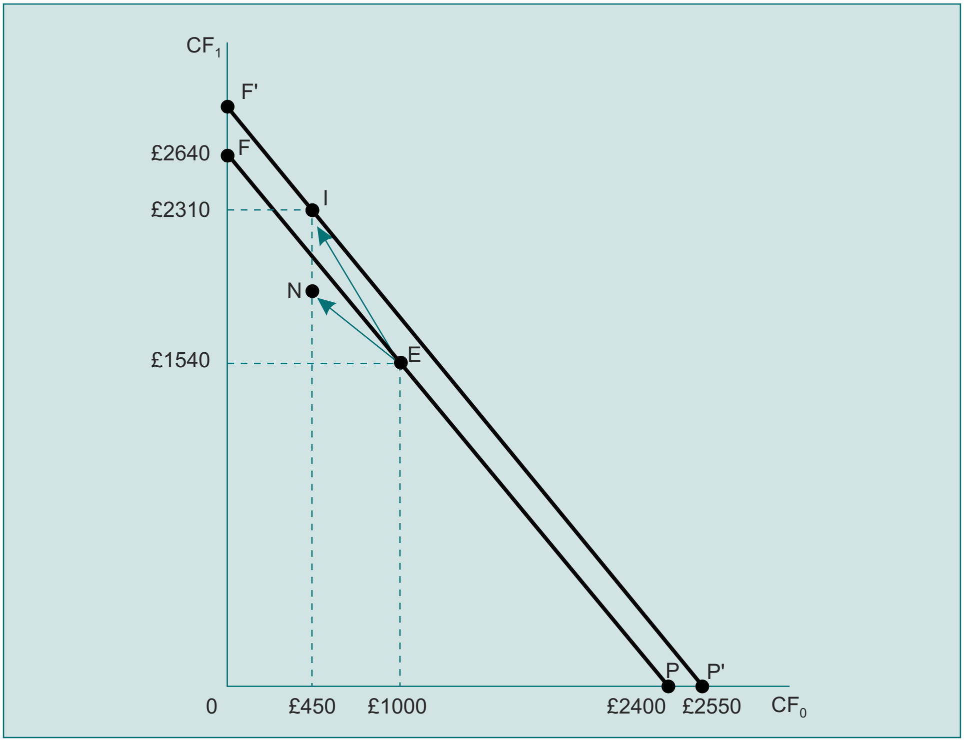

Figure 1.2 **Investment and the financial exchange line**

To see this, consider Figure 1.2 (point I) again. Looking at the cash-flow time pattern that results from the investment, you might be tempted to answer that the participant would be willing to accept the investment if he or she prefers this new pattern to the one without the investment (point E). But that would be the wrong answer, because it ignores the participant's additional opportunities to borrow and lend at the market interest rate, reallocating the new resources across time. We can see this by creating a new financial exchange line going through point I, the point at which the participant will end up if only the investment is undertaken. All of the points along the line through I are accessible to the participant if he or she both invests in the real asset and either borrows or lends.

The important thing to notice about this situation is that the participant must be better off than he or she was without the investment. As long as the participant prefers more capacity to consume, you can see easily that – regardless of location on the original exchange line – the participant can now find a location on the new exchange line that will allow more consumption at both t_0 and t_1. This is simply because the exchange line has been shifted outwards from the origin by the investment and is parallel to the original line. (It is parallel because the market interest rate, which determines the line's slope, has not changed.)

We can calculate the amount of this parallel shift by seeing how far the line's intercept has moved along the horizontal axis. As before, this means taking the present value of any position on the new line. Since we know point I already, we can use it:

$$\begin{aligned} PV &= \frac{CF_1}{(1+i)} \\ &= £450 + \frac{£2310}{(1.10)} \\ &= £2550 \end{aligned}$$

The outward shift in the exchange line is to £2550 at t_0. But this is also the horizontal intercept of the new exchange line, which (from what we know about the discounted value of future resources) is our participant's new present wealth. So we have also discovered that his or her present wealth will increase from its original level of £2400 to £2550 with the investment.

Remember that we are trying to connect the investment's desirability to the participant's change in present wealth. The last step in that process is easy: since any outward shift in the exchange line signals a good investment, and since any outward shift is an increase in present wealth, any investment that increases present wealth is a good one. That is simply another way of saying what we said earlier: investments are desirable when they produce more present value than they cost.

1.3.3 Net Present Value

Although you may have found it interesting to see how an investment is judged for desirability by calculating its effect on our participant's present wealth, the technique is somewhat cumbersome. Fortunately, there is a much more direct method of testing the desirability of an investment, which gives the same answers as the present wealth calculation. This approach deals directly with the investment's cash flows, and does not require that any particular participant's resources be used in the calculation. In finance this technique is called **net present value**; it is simply the present value of the difference between an investment's cash inflows and outflows.

Recall that our participant's investment requires an outlay of £550 at t_0, and returns £770 at t_1. If we calculate the present value of the t_1 cash inflow and subtract the (already present value of the) t_0 outflow we get:

$$\begin{aligned} \text{PV inflow} - \text{PV outflow} &= \frac{CF_1}{(1+i)} - CF_0 \\ &= \frac{£770}{(1.1)} - £550 \\ &= £700 - £550 \\ &= £150 \end{aligned}$$

The difference between the present values of the cash inflows and outflows of the investment is +£150. That number is the **net present value of the investment**.

Net present value, or **NPV** as it is commonly known, is a very important concept for a number of reasons. First, notice that the NPV of the investment, £150, is exactly equal to the change in the present wealth (£2550 − £2400) of our participant, were he or she to undertake the investment. That is no accident. It is generally true that correctly calculated NPVs are always equal to the changes in present wealths of participants who undertake the investments. The NPV is thus an excellent substitute for our original laborious technique of calculating the change in present wealth of an investing participant. NPV gives us that number directly.

Why does NPV equal the increase in present wealth? We could use some algebra to show you, but a more important economic point can be made by considering the NPV as a reflection of how much the investment differs from its opportunity cost. Remember that our investor's opportunity cost of undertaking the investment is the alternative of earning a 10 per cent return in the financial market. The investment costs £550 to undertake. If our participant had put that money in the financial market instead of the investment, he or she could have earned $£550 \times (1.1) = £605$ at t_1. Since the investment returned £770 at t_1 the earnings were $£770 - £605 = £165$ **more** at t_1 with the investment than with the next-best opportunity. The £165 is the **excess** return on the investment at t_1. If we take the present value of that amount,

$$PV = \frac{£165}{(1.1)} = £150$$

we produce a number that we have seen before: the investment's NPV. This gives us yet another important interpretation of NPV. It is the present value of the future amount by which the returns from the investment exceed the opportunity costs of the investor.

NPV is the most useful concept in finance. We shall encounter it in various important financial decisions throughout the course, so it is most important that you appreciate its conceptual underpinnings, its method of calculation, and its varied applications. To review briefly what we have discovered about NPV:

1. The NPV of an investment is the present value of all of its present and future cash flows, discounted at the opportunity cost of those cash flows. These opportunity costs reflect the returns available on investing in an alternative of equal timing and equal risk.
2. The NPV of an investment is the change in the present wealth of the wise investor who chooses a positive NPV investment, and also of the unfortunate investor who chooses a negative NPV investment.
3. The NPV of an investment is the discounted value of the amounts by which the investment's cash flows differ from those of its opportunity cost. When NPV is positive, the investment is expected to produce (in present value total) more cash across the future than the same amount of money invested in the comparable alternative.

1.3.4 Internal Rate of Return

Net present value is an excellent technique to use for investment decisions. But NPV is not the only investment decision technique that can allow us to make correct decisions. The **internal rate of return (IRR)** is another technique that can be used to make such decisions. It tells us how good or bad an investment is by calculating the **average per-period rate of return on the money invested**. Once the IRR has been calculated, we compare it to the rate of return that could be earned on a comparable financial market opportunity of equal timing and equal risk. If the investment earns a higher return than this opportunity cost, it is good and we accept it; if it earns a lower rate of return, we reject it.

A more specific definition of the IRR is that it is **the discount rate that equates the present values of an investment's cash inflows and outflows**. From our earlier discussion of NPV, this implies that IRR is **the discount rate that causes an investment's NPV to be zero**. We shall see shortly why the IRR can be defined this way, but aside from simply broadening our education in finance these definitions are useful in that they give us hints as to how we can calculate the IRR. In our one-period financial market, calculating the IRR is easy. Returning to the original example, and using the definitions immediately above, we have

$$\begin{aligned}
\text{NPV} &= 0 = -£550 + \frac{£770}{(1+\text{IRR})} \\
(1+\text{IRR}) &= \frac{£770}{£550} \\
(1+\text{IRR}) &= 1.4 \\
\text{IRR} &= 0.4 \textit{ or } 40\%
\end{aligned}$$

The internal rate of return of our participant's investment is 40 per cent. Since the opportunity cost as a rate of return is 10 per cent (from an investment of comparable risk and timing in the financial market), the investment has a higher average per-period earning rate than the best alternative, so it is acceptable.

By looking again at Figure 1.2 we can gain a valuable intuition about the things that IRR and NPV are telling us. Remembering that the slope of an exchange line on that graph reflects the interest or discount rate, we can interpret the line from point E to point I as an 'exchange line for this investment' (i.e. giving up £550 at t_0 for £770 at t_1). Notice that the slope of the exchange line for the investment is steeper than the exchange line for the financial market. This implies clearly that the rate of return or earning rate on the investment is higher than the financial market's. Notice also that if the investment's exchange line is steeper than the financial market's, the resulting resource location of the investment (point I) must lie outside the original market exchange line. This means, as we saw in our discussion of NPV, that our participant's wealth would increase were he or she to accept the investment.

These observations about IRR in Figure 1.2 also imply that when IRR is greater than the financial market rate, NPV is positive. So the two techniques are telling us very similar things about the investment, but with slightly different perspectives. NPV describes an investment by the amount of the wealth increase that would be experienced by the participant who accepts it, whereas IRR tells us how the average earning rate on the investment compares with the opportunity rate.[2]

---

2 We usually assume in finance that an investment at the financial market rate, as the opportunity cost of the investment, is 'the best alternative' for any particular investment even though that might not be strictly true. As we shall see eventually, if the investment decision is handled correctly and thoroughly, we get the same answer as we would by actually using the true 'best alternative'.

The IRR and NPV techniques usually give the same answers to the question of whether or not an investment is acceptable. But they often give **different** answers to the question of which of **two** acceptable investments is the better. This is one of the major problems in finance, not so much because we do not know which one is correct but because many people seem to like the technique that gives the wrong answer! Obviously this deserves discussion, but we shall postpone that until we make the financial market a more realistic place, so the reasons for the disagreements between IRR and NPV can be explored more fully.

To review what we have discovered about the IRR technique:

1. IRR is the average per-period rate of return on the money invested in an opportunity.
2. IRR is best calculated by finding the discount rate that would cause the NPV of the investment to be zero.
3. To use IRR, we compare it with the return available on an equal-risk investment of comparable cash-flow timing. If the IRR is greater than its opportunity cost, the investment is good, and we accept it; if it is not, we reject the investment.
4. IRR and NPV usually give us the same answer as to whether an investment is acceptable, but often different answers as to which of two investments is better.

As a review of your understanding of some of the points we have made so far, look at the investment N in Figure 1.2. It requires an outlay of £550 at t_0 and returns £594 at t_1. N's NPV is given as follows:

$$\begin{aligned} \text{NPV} &= -£550 + \frac{£594}{(1.10)} \\ &= -£10 \end{aligned}$$

Similarly, N has an IRR given by:

$$\begin{aligned} 0 &= -£550 + \frac{£594}{(1+\text{IRR})} \\ 1+\text{IRR} &= \frac{£594}{£550} \\ &= 1.08 \\ \text{IRR} &= 0.08 \textit{ or } 8\% \end{aligned}$$

The two techniques, NPV and IRR, give the same answer about N: it is not a good investment. The NPV is −£10, which implies that a participant in this market would lose £10 of present wealth if investment N was accepted. In Figure 1.2 the resulting exchange line would shift back toward the origin and intercept the horizontal axis at £2390 rather than £2400, making the participant less well off than without the investment. N's IRR is 8 per cent, which is a lower per-period earning rate than the 10 per cent generally available in the financial market for investments of equal risk and timing. Note that the exchange line for investment N in Figure 1.2 has a slope less steep than the market's exchange line, FEP. This is a visual statement that N's earning rate is less than that of the market; so, again, N should be rejected.

1.3.5 A Simple Corporate Example

Look at Figure 1.3. In it we have displayed the decision situation faced by a company that can undertake any of a number of investments. For example, investment G involves spending GP of CF_0 and getting in return GG' of CF_1. Notice also that we have stacked these investments one on top of another in decreasing order of desirability (e.g. G has a higher NPV and IRR than H, and so forth). The company must decide which of these to accept.

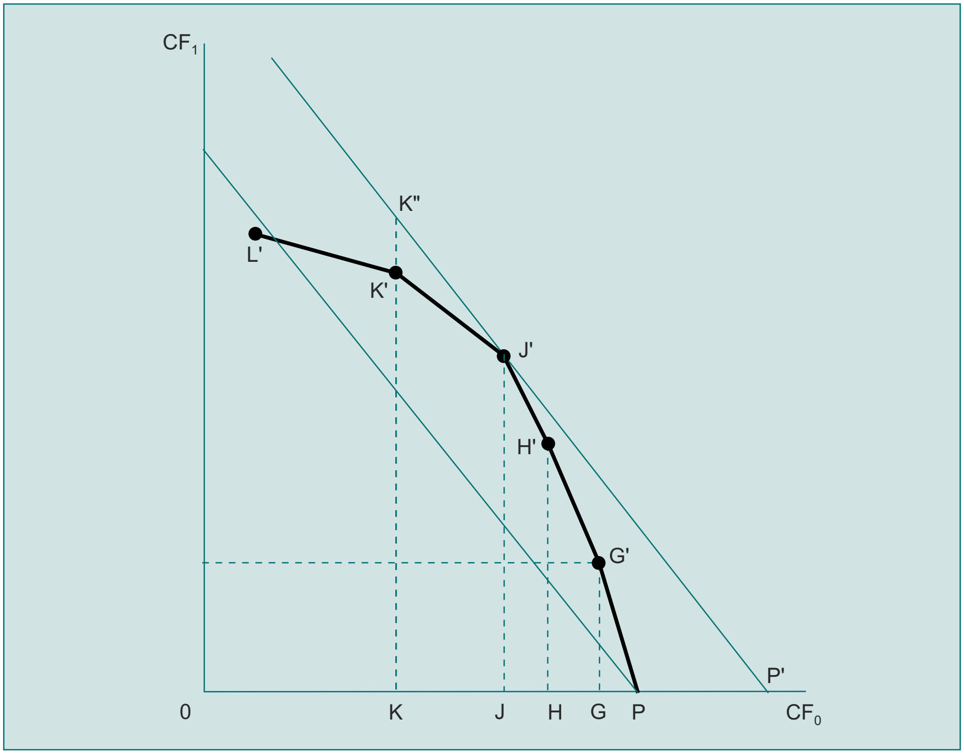

Figure 1.3 Multiple investments and the financial exchange line

How should it make this decision? We usually assume that companies decide such things by opting for the choice that makes their existing shareholders as wealthy as possible. Since this is a simple one-period world, and all investment outcomes will be resolved at t_1, the company would make its shareholders most wealthy by accepting the set of investments that produces the exchange line farthest to the right. As you can see from Figure 1.3, that would be the set of investments G, H and J.

Notice that this set is delineated by the tangency of the exchange line and the stacked investments. Since a tangency is the point where lines are parallel (having the same slope), the slopes of the exchange line and the investment stack must be equal at J'. But those slopes have economic meaning: the exchange line's slope is determined by the market interest rate, and the investment stack's slope at J' is determined by the IRR of J. All investments below J have IRRs greater than J's, so the decision to accept the investments up to and including the one tangential to the new financial exchange line is the same thing as accepting investments until the IRR of the last one is just equal to (or above) the market interest rate. This is the process that will create the most wealth for shareholders, because it causes the company to accept all investments with average per-period earnings rates (IRRs) greater than what the company's shareholders can earn on comparable investments in the financial market. (To accept investments until the next has a negative or zero NPV is, of course, to do the same thing.)

'Not so fast,' you say. 'Suppose I were the type of person who preferred t_0 to t_1 consumption. If I were a shareholder of the company I would be happier if they stopped at investment H or G or even made none at all. Then I would have the highest capacity to consume at t_0.'

That, of course, is not true. If the company undertakes no investment, your maximum t_0 consumption is P of CF_0. Whereas if the company accepts all of the investments up to and including J, you can consume up to P' of CF_0 simply by selling your shares at t_0 after the market discovers the astuteness of the company's investment decisions and adjusts the price of its shares. If you have an aversion to selling, nothing would prevent you from borrowing against those shares at t_0 in our frictionless market and getting the same amount P' by that mechanism instead.

'Fair enough,' you say. 'But my sister is also a shareholder of the same company, and her consumption preferences are exactly the reverse of mine. She likes nothing better than to increase her future consumption by reducing her current spending. How is the company going to solve the problem of pleasing both of us?'

The answer is that in a market such as this one, companies face no such problem because shareholders can easily solve it themselves. Your sister would simply avoid selling shares, and reinvest any dividends that the company paid her, either in more shares or in lending. The result would be that she delays present consumption until the future. In essence we are saying that a company in this market need not worry about its shareholders' consumption preferences; the financial market will allow them to make whatever transactions are necessary to be content with their time pattern of resources. Shareholders with quite different preferences for patterns of consumption can thus be content to own the same company's shares, and the company need not be concerned about the pattern it chooses in which to pay dividends. The sole task of the company is to **maximise the present wealth of its shareholders**. The shareholders can then adjust their individual patterns of resources by dealing in the financial market.

Suppose that the company mistakenly undertook to please your sister by investing a greater amount at t_0, and accepted all investments up to K'. From Figure 1.3 you can readily see that her t_0 cash flow would decrease and her t_1 increase, which is her preferred pattern. But notice also that were the company to maximise her present wealth by investing only to J', she could in fact retain the same t_0 consumption (0K) and increase her t_1 consumption to KK'', thereby increasing her satisfaction.

To review the important ideas we have discussed in this section:

1. We distinguished between financial and real asset investments, and argued that because of the competitiveness of financial markets it is (usually) necessary to choose real investments in order to expect wealth to increase by investing.
2. We developed the measure of investment desirability called net present value, as the present value of the amounts by which an investment's cash flows exceed those of its opportunity cost. We also showed that NPV is equal to the change in the wealth of the participant accepting the investment, and that NPV measures the change in market value of the investor's wealth.
3. We introduced the measure of investment desirability called internal rate of return, the average per-period earning rate of the money invested. When IRR exceeds the opportunity cost (as a rate) of an investment, the investment will have a positive NPV, and therefore be acceptable.
4. We illustrated how these ideas could apply to the investment decisions of a company in a simple financial market like the one described. The company would accept investments up to the point where the next investment would have a negative NPV or an IRR less than its opportunity cost. The company can ignore its shareholders' preferences for particular patterns of cash flow across time because the financial market allows shareholders to reallocate those resources by borrowing and lending as they see fit. This lets the company concentrate on maximising the present wealth of its shareholders, the result of adhering to the investment evaluation techniques of NPV and IRR in this market.

All of these ideas are important introductions to finance for people who will be dealing with these decisions. As important is the general appreciation that we have gained for what a financial market does:

1. It lets people reallocate resources across time, which provides money for real investment.
2. It gives very important signals, in terms of market rates of return or interest rates, about the opportunity costs faced by investors. These rates are used as discount rates for making the real asset investment decisions that are so important to an economy.

1.4 More Realistic Financial Markets

The simple financial market we have dealt with to this point has allowed us to discover many important characteristics common to all financial markets. You will probably be surprised by the general applicability to 'real world' financial decisions of much that you have already learned. It is nevertheless true that our simple financial market cannot portray some of the features of actual markets and financial decisions that are important to learning finance, and so we shall now begin adding those other features.

1.4.1 Multiple-Period Finance

The financial market until now has been limited to single-period transactions; whenever a financial action was taken at t_0, its final result occurred at t_1, one period later. Actual financial markets, however, contain real and financial assets with returns spanning more than a single period: you can leave your money in a bank for more than one interest period before taking it out; you can buy bonds that pay interest for decades before they stop (or 'mature'); and you can invest in corporate equities (ordinary shares) that are expected to continue paying dividends for an indeterminately long period into the future. (Some have been paying for a hundred years or so already.) We must be able to address the questions of how such securities are valued, and how financial decision makers deal with real asset choices when the returns from those real assets cover many periods.

With multiple-period assets generating returns across long periods of time, it must seem at first that real financial markets are terribly complex things with which to contend. And we would be telling less than the absolute truth if we said there are no complications introduced by multiple-period assets in the financial market. But it is true that these complexities introduce few new general concepts and are mostly involved in the calculations that are necessary to describe and value the returns that the assets produce.

Actually, there is one way of looking at multiple-period transactions in the financial market that is almost identical to the way we described the single-period market. When we shifted resources across time in the single-period market, we multiplied by $(1 + i)$ to move a period into the future (accruing interest), and divided by $(1 + i)$ to move one period into the past (discounting). The $(1 + i)$ is effectively an 'exchange rate' between t_0 and t_1 resources. In multiple-period transactions the same type of exchange rate applies to shifting resources between any two time points.

Picture the financial market now covering the time points t_0, t_1 and t_2. This means simply that we have introduced another period after t_1, the point at which our single-period market stopped:

$$t_0 \overset{\text{Period 1}}{\overline{\qquad\qquad}} t_1 \overset{\text{Period 2}}{\overline{\qquad\qquad}} t_2$$

The financial market will now allow us to shift resources not only between t_0 and t_1 but also between t_0 and t_2 (or any pair of time points). The rate of exchange between t_0 and t_1 resources is $(1 + i)$, but since we can now have another exchange rate between t_0 and t_2, we must be able to distinguish between that rate and the rate between t_0 and t_1. To do so we shall designate i_1 as the interest rate between t_0 and t_1, and i_2, as the rate between and t_0 and t_2. Thus $(1 + i_1)$ is the single-period exchange rate.

To be able to write the two-period exchange rate, we must now deal with one of the complexities of multi-period markets. Instead of writing the exchange rate between t_0 and t_2 as $(1 + i_2)$, we usually write it as $(1 + i_2)^2$. This may seem unnecessarily complicated, but it does serve a purpose: people are evidently more comfortable in talking about interest rates **per period** than exchange rates over more than one period, and this way of writing the exchange rate allows them to do that.

An example might be useful here. Suppose that the same per-period exchange rate existed between t_0 and t_2 as between t_0 and t_1, and that this rate **per period** was our familiar 10 per cent. To shift resources either backward or forward between t_0

and t_1, the exchange rate 1.10 applies. But to shift resources between t_0 and t_2, we must travel through **two** periods at the rate 1.10 **per period**.

Suppose that we wished to invest £100 in the financial market at t_0, and leave it there until t_2, so as to then have an amount CF_2. How much would CF_2 be?

$$\begin{aligned} CF_2 &= CF_0(1+i_2)(1+i_2) \\ &= CF0(1+i_2)^2 \\ &= £100(1.21) \\ &= £121 \end{aligned}$$

We would end up with £121 at t_2. That is the result of earning 10 per cent **per period** for two periods on an initial £100 investment. In finance, when we say that the two-period interest rate is 10 per cent, we mean that to shift resources between t_0 and t_2 the exchange rate is $(1 + 10\%)^2$, or 1.21.

Naturally, the present-value calculation works in exactly the opposite way. If we expected to receive £121 at t_2, and wished to know its present value (its market price right now) we would calculate:

$$\begin{aligned} PV &= \frac{CF_2}{(1+i_2)^2} \\ &= \frac{£121}{(1.10)^2} \\ &= £100 \end{aligned}$$

1.4.2 Compound Interest

These calculations allow us to introduce a few more important ideas that appear in financial markets. When we shifted the £100 at t_0 outwards to t_2, multiplying twice by $(1 + i_2)$ or once by $(1+ i_2)^2$, we **compounded** the interest rate i_2 for two periods. **Compounding** means that the exchange rate between two time points is such that you earn interest not only on your original investment but also (in subsequent periods) on interest you earned previously.

It is easiest to understand that idea by returning to our example. Another way of looking at the money you get at t_2 is:

$$\begin{aligned} CF_2 &= CF_0 + CF_0(i_1) + CF_0(i_2) + CF_0(i_1)(i_2) \\ £121 &= £100 + £100(10\%) + £100(10\%) + £100(10\%)(10\%) \end{aligned}$$

The way to read this statement is that the money you end up with at t_2, £121, is equal to the amount you invested at t_0, £100, plus interest on that for the first period, £100(10%), plus interest on that for the second period, £100(10%), plus interest for the second period on the first period interest, £100(10%)(10%). That, of course, is an unnecessarily complicated way of writing what can simply be written as

$$\begin{aligned} CF_2 &= CF_0(1+i_2)^2 \\ £121 &= £100(1+10\%)^2 \end{aligned}$$

but it may help you to understand how we end up with the amounts we do.

Compounding of interest (earning interest on interest) can be done as often as the borrower and lender agree that it be done. In our example, we compounded once per period. There would be nothing to prohibit an agreement to compound twice, three or even more times per period. If the interest rate stays the same, the money amounts would be different because of the number of times interest was compounded between time points.

The general arithmetic of interest compounding is not very complicated. The amount of money you end up with by investing CF_0 at compound interest is:

$$\mathrm{CF}_0[1 + (i/m)]^{mt}$$

where i is the interest rate, m is the number of times per period that compounding takes place, and t is the number of periods the investment covers. You can see that this formula becomes our familiar $CF_0(1 + i)^t$ when interest is compounded only once per period.

Using the formula, were we to compound £100 twice per period at 10 per cent interest, we would have at the end of the first period:

$$£100[1 + (0.10/2)]^2 = £110.25$$

and at the end of the second:

$$£100[1 + (0.10/2)]^4 = £121.55$$

continuing for as many periods as we choose.

You can see that these amounts are higher at each future time point than those we figured earlier when compounding only once per period at the 10 per cent rate. If your calculator can raise numbers to powers, see if you can use the compounding formula to demonstrate to yourself that £100 invested at 10 per cent interest for fifty years increases to £11 739.09 if compounded once per year, and £14 831.26 if compounded daily (365 times per year).

Compounding of interest can be even more frequent than daily. The most frequent type of compounding is called '**continuous**'. Continuous compounding means that interest is calculated and added to begin earning interest on itself **without any passage of time between compoundings**. In the general compounding formula above, that means m is infinitely large and, without belabouring the algebra, the formula reduces to:

$$CF_0(e^{it})$$

where $e = 2.718 \ldots$, the base of the natural logarithm system.

If interest is compounded continuously, with 10 per cent interest, £100 increases, for example, to £110.52 in one period, to £122.14 in two periods, and to £14 841.32 in fifty periods.

Financial institutions that borrow by accepting deposits from customers occasionally use interest compounding as a marketing tactic in an attempt to lure customers seeking high interest earnings. The advertisements are usually variants on the above examples and, with the exception of an occasional inadvertent arithmetic error, are correct in their implication that more frequent compounding produces higher final amounts. Customers should, however, exercise care in choosing among financial assets on the basis of compounding intervals. In a very competitive market for deposits it is unlikely that one bank can afford to offer consistently larger payments to customers than other banks. If the stated interest rate is only very slightly lower than one compounded less frequently, the difference may nevertheless offset any compounding benefit. Or some non-monetary dimension of service may be different.

We must always remember that financial market participants consume money resources, not interest rates or compounding intervals; they make their comparisons of desirability on the basis of money-measured values. They cannot be fooled into thinking, for example, that continuous compounding is necessarily better than no compounding, unless they are also informed of the interest rates to be compounded. Nor will they assume that lending money at a rate of 10.1 per cent is necessarily better than lending at 10 per cent, if the two rates are not identically compounded.

For the remainder of the course we shall adopt the common convention that, unless told otherwise, **interest is compounded once per period**.

1.4.3 Multiple-Period Cash Flows

Extending the financial market to cover any number of periods is now easily within our reach. Suppose that you expect to receive a cash flow at t_3 and are curious about its present value. If you know that your average three-period opportunity cost is i_3 per period, the present value of the t_3 cash flow is:

$$PV = \frac{CF_3}{(1 + i_3)^3}$$

The same general procedure will allow us to find the present value of a cash flow occurring at any future time point. Where t can be any time, the general method for finding the present value of any cash flow is:

$$PV = \frac{CF_t}{(1 + i_t)^t}$$

Moving in the other direction, the future value of a present amount invested at the rate i_t for t periods is, of course, the invested amount multiplied by $(1 + i_t)^t$.

The securities and assets in multiple-period financial markets often have more than a single cash flow expected for the future. Usually a corporate bond or an ordinary share (equity) is expected to pay several cash amounts as interest, principal or dividends at several times in the future. Similarly, a real asset investment, such as a new piece of machinery or going into a new line of business, almost always has cash flows expected for many periods. How does finance deal with valuing such cash flows?

We follow the same rules that we have used thus far, while merely combining the cash flow present values. For example, suppose that we are interested in the present value of a set (we call this a '**stream**') of cash flows that comprises £100 at each of t_1, t_2 and t_3, and our opportunity costs are all 10 per cent per period:

$$
\begin{aligned}
PV &= \frac{CF_1}{(1+i_1)} + \frac{CF_2}{(1+i_2)^2} + \frac{CF_3}{(1+i_3)^3} \\
&= \frac{£100}{(1.10)} + \frac{£100}{(1.10)^2} + \frac{£100}{(1.10)^3} \\
&= \frac{£100}{(1.10)} + \frac{£100}{(1.21)} + \frac{£100}{(1.331)} \\
&= £90.91 + £82.65 + £75.13 \\
&= £248.69
\end{aligned}
$$

The present value of the stream of cash flows is £248.69, being the sum of the present values of the future cash flows in the stream. Though the arithmetic of this example is quite rudimentary, the economic lesson it portrays is important. It tells us that the correct way to view the value of an asset that generates a stream of future cash flows is as the **sum of the present values of each of the future cash flows associated with the asset**.

1.4.4 Multiple-Period Investment Decisions

Earlier we introduced two investment decision-making techniques that are consistent with present wealth maximisation in financial markets. We shall now illustrate the basics of how these techniques, NPV and IRR, operate in multiple-period asset evaluations.

Calculating NPV when the investment decision will affect several future cash flows is no more difficult than any multiple-period present value calculation. We need simply to remember that NPV must include **all** present and future cash flows associated with the investment. For example, suppose that the stream we valued in the section immediately above is the set of future cash flows of an investment with a t_0 cash outlay of £200. Combining all of the present values of the investment's cash flows produces an NPV of £48.69, which is the net of £248.69 (the present value of future cash flows) minus £200 (the present value of the present cash flow):

$$
\begin{aligned}
\text{NPV} &= CF_0 + \frac{CF_1}{(1+i_1)} + \frac{CF_2}{(1+i_2)^2} + \frac{CF_3}{(1+i_3)^3} \\
&= -£200 + \frac{£100}{(1.10)} + \frac{£100}{(1.10)^2} + \frac{£100}{(1.10)^3} \\
&= -£200 + \frac{£100}{(1.10)} + \frac{£100}{(1.21)} + \frac{£100}{(1.331)} \\
&= -£200 + £90.91 + £82.65 + £75.13 \\
&= +£48.69
\end{aligned}
$$

Finding the IRR of a set of cash flows that extends across several future periods is more complicated than calculating its NPV. Remember that IRR is the discount rate that causes the present value of all cash flows (NPV) to equal zero. In the example we are working on, the following equation for IRR must be solved:

$$
\begin{aligned}
0 &= CF_0 + \frac{CF_1}{(1+\text{IRR})} + \frac{CF_2}{(1+\text{IRR})^2} + \frac{CF_3}{(1+\text{IRR})^3} \\
&= -£200 + \frac{£100}{(1+\text{IRR})} + \frac{£100}{(1+\text{IRR})^2} + \frac{£100}{(1+\text{IRR})^3}
\end{aligned}
$$

In terms of the mathematics involved, there is no general formula that will allow us to solve all such IRR equations. Instead, the way we solve for the IRR of a multiple-period cash flow stream is with a technique called '**trial and error**'. This means we choose some arbitrary discount rate for IRR in the above equation, and calculate NPV. We then examine the result to decide whether the rate we used was too high or too low, choose another rate that appears to be better than the one we just used, and again calculate NPV. We continue this process until we find the IRR (or as close an approximation to it as seems necessary) that creates an NPV equal to zero.

Suppose that we first try 15 per cent as a potential IRR:

$$\begin{aligned} \text{NPV} &= -£200 + \frac{£100}{(1.15)} + \frac{£100}{(1.15)^2} + \frac{£100}{(1.15)^3} \\ &= +£28.32 \end{aligned}$$

A 15 per cent discount rate produces an NPV greater than zero, so 15 per cent is not the IRR of the cash flows. Since the NPV is too large, we should probably choose a higher discount rate (since increases in the discount rate, being in the denominator, will cause a lower NPV for these cash flows). Let us try 25 per cent:

$$\begin{aligned} \text{NPV} &= -£200 + \frac{£100}{(1.25)} + \frac{£100}{(1.25)^2} + \frac{£100}{(1.25)^3} \\ &= -£4.80 \end{aligned}$$

A 25 per cent discount rate yields a small but negative NPV, and so 25 per cent is too large. Nevertheless, we have discovered that the IRR of the cash flows is somewhere between 15 and 25 per cent because the former yields a positive and the latter a negative NPV. To find the true IRR, we continue this search process until we find the exact IRR, become convinced that the range we have found is sufficiently accurate for the decision at hand, or run out of patience.

The actual IRR for this example is 23.4 per cent per period, implying that, because IRR exceeds the 10 per cent opportunity cost, the investment is acceptable. Actually, we could have decided that as soon as we saw a positive NPV at a 15 per cent discount rate, since we knew thereby that the IRR was greater than 15 per cent. With an opportunity cost of 10 per cent and an IRR greater than 15 per cent we have enough information to decide that the investment is desirable.

Figure 1.4 should be helpful in visualising the method of finding the IRR. Note that the vertical axis of the figure records NPV, and the horizontal axis plots the discount rates used to calculate NPV. The curve indicates that, in the example we are examining, as the discount rate increases, NPV declines. (This is a very common relationship between NPV and its discount rates. As long as cash outflows tend to be closer to the present than cash inflows from an investment, we usually see a curve that looks like this one.) The search for an IRR is easy to visualise in Figure 1.4. If your first try uses a rate less than 23.4 per cent, the NPV will be positive; if more, it will be negative. If you get a positive NPV, you should try a rate higher than the one you have used; if you get a negative NPV, you should try a lower one. Eventually you will narrow the search to a rate that creates an NPV nearly equal to zero, and that rate will be the IRR.

The process for estimating an IRR can be troublesome because NPV must be recalculated in each pass of the search process. This can become quite tedious when an investment is expected to generate cash flows for many periods. Fortunately for those who like using the IRR, the wonders of modern technology have come to the rescue with commonly available pocket calculators that have this search process programmed into them. If you are faced with the prospect of calculating IRRs for long-lived investments, you should consider one of these instruments or appropriate software for your computer.

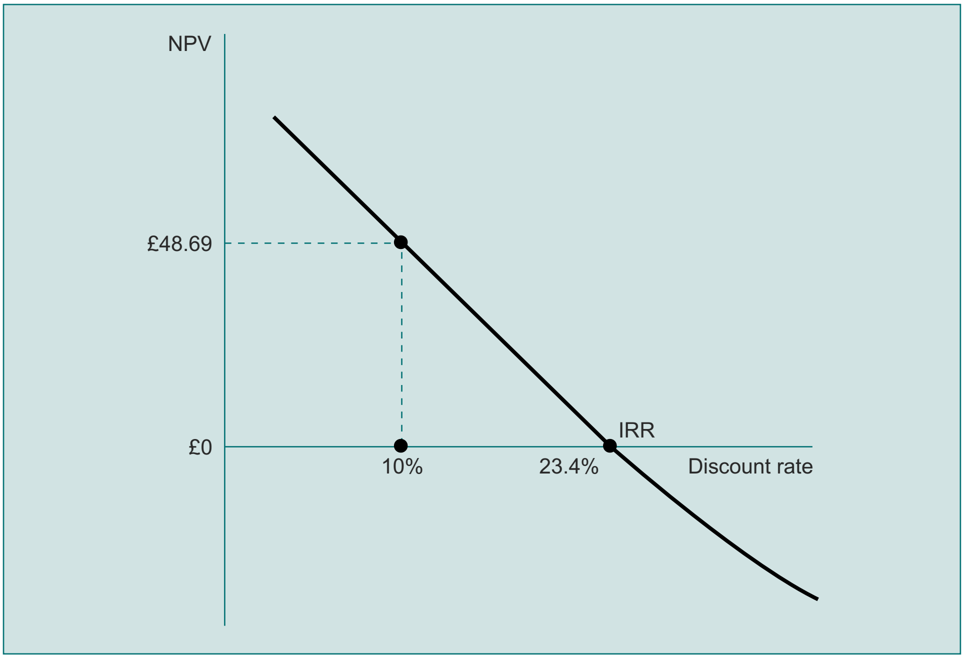

Figure 1.4 **The relationship between NPV and IRR**

1.4.5 Calculating Techniques and Short Cuts in Multiple-Period Analysis

The calculating of present values is so basic to finance that we should develop a good understanding of the various means at the disposal of the financial manager to perform the calculations. Before beginning this exploration, however, we should emphasise that you have already seen a technique that works in every situation where information about cash flow expectations and discount rates is available. As you know, the present value of any future cash flow can be found by:

$$PV = \frac{CF_t}{(1+i_t)^t} \tag{1.1}$$

When we wish to discount a stream of future cash flows to find its present value, we simply find the sum of the present values as calculated above. The way to write the instruction to calculate the present value of a stream of future cash flows is

$$PV = \sum_{t=1}^{T} \frac{CF_t}{(1 + i_t)^t} \tag{1.2}$$

Though this Equation 1.2 may appear forbidding, it is telling you simply to find the present value of each future cash flow and add up the results, which is exactly what we did in the last section. (The $\sum$ sign is a symbol that says to sum everything to the right of itself, beginning at t_1, until you exhaust the cash flows at t_T.) Occasionally we shall use this equation (or a near relative of it) in our discussions. For our purposes it is entirely appropriate to regard such equations as an efficient kind of shorthand for a set of instructions that is telling you simply to calculate the present value of a set of future cash flows.

When faced with a present-value calculation that has different cash flows across the future and different discount rates for these cash flows, we have no choice but to use the technique implied by Equation 1.2. Though that does happen, there are common situations where we can find correct present values more easily than by using that technique. One of the most often encountered simplifications is where discount rates are constant across the future. This is rarely an accurate reflection of what is really expected to happen, but it reduces the complexity of the calculations so much that it is widely used.

Where discount rates are taken to be the same for all cash flows, Equation 1.2 becomes

$$PV = \sum_{t=1}^{T} \frac{CF_t}{(1 + i)^t} \tag{1.3}$$

Equation 1.3 is the instruction to discount a stream of future cash flows to the present using the same per-period discount rate for all cash flows. (Note that the difference between Equation 1.2 and Equation 1.3 is that the discount rate i is not time-subscripted.)

There are at least two reasonably straightforward ways of following the instruction of Equation 1.3. To illustrate these, we can use the numerical example of £100 per period for three future periods at a 10 per cent discount rate. The first technique begins with the cash flow furthest into the future (in this case $CF_3 = £100$), and divides it by $(1 + i)$: $£100/(1.10) = £90.91$. That number is the t_2 value of the t_3 cash flow. To this is added the next-nearest cash flow, $CF_2 = £100$, and that sum, £190.91, is divided by $(1 + i)$: $£190.91/(1.10) = £173.55$. That step produces the t_1 value of the t_2 and t_3 cash flows. To that is added the t_1 cash flow itself, and that sum, £273.55, is again divided by $(1 + i)$: $£273.55/(1.10) = £248.69$. We have seen this last result before. It is the present value of the stream or, as we have seen from the foregoing explanation, it is the t_0 value of the t_1, t_2 and t_3 cash flows.

The above technique is clumsy to explain but actually works quite neatly, and it is better than anything else if you have a basic pocket calculator available. Put simply, you start with the cash flow furthest into the future, discount it one period closer to the present, add the cash flow from that closer time point, and discount that sum one period nearer to the present; you continue that process until all cash flows are included, and discounted back to t_0.

Another commonly used technique of finding present values relies on **present value tables**. Present value tables are simply lists of actual values of Equation 1.1 and Equation 1.3, with 1 as the cash flow in the numerators of the equations, and with present values calculated for a wide range of time points, stream lengths and discount rates. Present value tables therefore give the present values per pound of future cash flow, either for a single cash flow (Equation 1.1) or a stream of constant cash flows discounted at constant discount rates (Equation 1.3). We have included a set of these tables in Appendix 1.

To illustrate the use of the tables, turn to Table A1.1 in Appendix 1, which shows the present value of £1 to be received at time point t. Note that in the column for a 10 per cent discount, the factors for the first three time points are indicated as 0.9091, 0.8264, and 0.7513 respectively. To find the present value of our £100 per period for three periods, we multiply each of these factors by the £100 cash flow occurring at its time point, and add up the result. The answer is, of course, £248.69 (actually the result is £0.01 less because of rounding).

Using present value tables for such calculations is unnecessary since the calculator technique above has the same number of steps and does not use the tables. There are, however, instances where the tables are efficient. One obvious example is in calculating the present value of a single cash flow located far into the future when your calculator cannot perform exponentiation (i.e. raising numbers to powers). You would not enjoy dividing £100 by (1.10) twenty times if your calculator could not figure $(1.10)^{20}$ directly. Table A1.1 tells you that $£1/(1.10)^{20}$ is 0.1486, and therefore that the value of £100 to be received at t_{20} is £14.86 at t_0 with a discount rate of 10 per cent.

Another circumstance when the tables are useful is in finding the present value of **annuities**. A **constant** annuity is a set of cash flows that are the same amounts across future time points. Such a present value is calculated with our Equation 1.3, but with no subscript on the cash flows since they are all the same. Since Table A1.2 in Appendix 1 gives the present value of £1 per period for t periods, it implies a constant cash flow per period and is therefore an annuity table. To illustrate its use, note that under the column for 10 per cent discount, the three-period annuity factor is listed as 2.4869. It takes little effort to see that with £100 per period for three periods, we arrive at our familiar answer, £248.69. When annuities run for many periods, the use of this table rather than a calculator is wise (unless you own one of the sophisticated devices programmed to do such calculations directly).[3]

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[3] Cash flows can, of course, be invested at interest to produce *future* amounts of cash. Table A1.3 and Table A1.4 in Appendix 1 are the future value counterparts of Table A1.1 and Table A1.2 in the same Appendix. In addition to the tables and pocket calculators with built-in financial functions, all current personal computer spreadsheet programs have these (and many more) financial formulas able to be called up as automatic functions. You should not hesitate to use such spreadsheets instead of the tables or calculators if you feel comfortable doing so. Because of the increasingly common availability

We mentioned earlier that some of the assets commonly valued in finance have cash-flow expectations that extend very far into the future. When faced with cash flows of that nature, there is yet another technique of present value calculation that is often used in financial practice: the **perpetuity**. A perpetuity is a cash flow stream that is assumed to continue for ever. Perpetuity present values are used because they are so easy to calculate. The formula for the present value of a perpetuity is:

$$PV = \frac{CF}{i} \tag{1.4}$$

To find the present value of a perpetuity one merely divides the (constant) per-period cash flow by the (constant) per-period discount rate. This means, for example, that £100 per period for ever at a discount rate of 10 per cent has a present value of £100/0.10 = £1000. (It is intuitively easy to see why this formula works. Another way of saying the same thing is that if you put £1000 in the bank at 10 per cent interest, you can take out £100 per year for ever.)

The ease of calculation of perpetuities is attractive. Not so attractive is the assumption that the cash-flow stream will continue for ever. Obviously, no asset is actually likely to continue producing cash flows for ever, but before you become offended that financial practitioners use a technique that makes such a ridiculous assumption, let us test whether or not the **answers** it gives are ridiculous.

Suppose that you are faced with valuing a stream of £100 cash flows with 10 per cent opportunity costs, and the stream is expected to continue very far into the future but you are not exactly sure how far. Were you to use an admittedly incorrect perpetuity value, it would be £100/0.10 = £1000. How large an error are you likely to be making by using that value?

Suppose that the cash flows are unlikely to last much beyond forty periods. With a discount rate of 10 per cent, £100 in the fortieth year has a present value of only $£2.21 = £100/(1.10)^{40}$. As a matter of fact, the total present value of *all* cash flows from the fortieth year until the end of time is only $(£100/0.10)/(1.10)^{40} = £22.10$. This means that by using a perpetuity valuation, even if the stream actually ends in the fortieth year instead of continuing for ever, the value error is £22.10 out of £1000, or 2.21 per cent. As we shall see in our study of uncertainty, other errors in making cash-flow estimates are likely to be large enough so as to overshadow mistakes of this magnitude in present-value calculations. Of course, the size of the error is a function of when the actual cash flow would cease, and could be much larger than 2 per cent. For example, if the cash flow were to cease in the twentieth year, the error in using the £1000 perpetuity value would be $£1000/(1.10)^{20} = £148.64$, and a 14.9 per cent error may be too large to be tolerable.

Though perpetuity valuation can be convenient and may not err greatly in long-lived assets, Equation 1.4 also assumes that the cash flows will be constant for each future period. That is not very representative of actual patterns of cash flows that we see. Fortunately there is a slight modification of Equation 1.4 which can make it

of these spreadsheets (with the computers to use them) and their inherent power and flexibility, if you have not made the decision to begin using one, now might be a good time to consider doing so.

a bit more useful without significantly altering its simplicity. If we assume that the cash flows will continue for ever, but will grow or decline at a constant percentage rate during each period, the perpetuity formula becomes

$$PV = \frac{CF}{(i - g)} \tag{1.5}$$

where g is the constant per-period growth rate of the cash flow.

For example, suppose that we must value a cash-flow stream that begins at the end of this period with £100, but that will grow at a rate of 5 per cent per period every period thereafter (such that there will be a cash inflow of £105 at t_2, of £110.25 at t_3, and so forth, for ever). With a discount rate of 10 per cent, the value of the stream is:

$$PV = \frac{£100}{(0.10 - 0.05)} = £2000$$

(The 'bank account' intuition here is like that of the constant perpetuity, except that your withdrawals grow each year at 5 per cent.)

This 'growth perpetuity' present-value calculation is widely used for several financial applications, especially when investigating the values of long-lived organisations like large modern corporations. One note of caution, however: the equation obviously does not work when the discount rate i is less than or equal to the growth rate g. The implication that a cash flow, growing for ever at a rate nearly equal to its opportunity cost, has an infinitely high present value, is numerically correct, but not economically useful, because it could not reasonably be expected to happen.

Since this has been a rather long section, we should review the points we have developed in it. In discussing the various techniques that finance uses to perform discounting calculations we have discovered that:

1. There is a simple calculator-based technique that is very effective for valuing cash-flow streams that run for only a few periods.
2. When the stream continues for several periods and has the same cash flow for each period, annuity present-value tables (giving the present value of £1 per period) can be used.
3. When cash flows are well into the future, and the calculator being used cannot exponentiate, single-cash-flow present-value tables ('present value of £1') are useful.
4. Some financial pocket calculators can do all of the above with pre-programmed ease. The spreadsheet programs widely available on personal computers have financial functions that are even easier to use, more flexible and more powerful than sophisticated pocket calculators.
5. Perpetuities, either constant or growing (or even declining) by a constant percentage per period, can often be used as reasonable approximations for cash-flow streams from very long-lived assets.

With enough practice, some of which is provided by the exercises at the end of Module 1, you will quickly come to recognise the particular situation in which each technique is most efficient.

1.5 Interest Rates, Interest Rate Futures and Yields

In this section we shall develop more fully some ideas about interest rates that we introduced in earlier sections. You probably feel as if you have heard enough about interest rates to last you a lifetime, but there are a few additional concepts involved in their use that we have been holding back until our multiple-period framework was complete. The first set of these ideas concerns itself with **forward** or future interest rates, and something called the '**term structure**' of interest rates. In our discussion of these topics we shall also learn a few important things about the debt securities called **bonds**.

When we first discussed interest rates we said that the best way to understand them was as ratios or 'rates of exchange' used when shifting resources across time. We would now like to consider the idea that such exchanges or interest rates can take place not only between now and some future time point but also between any two time points, present or future. In other words, if there is an interest rate that applies between t_0 and t_2, there could also be one between t_1 and t_2, or between t_2 and t_6, or any other combination. Hearing this proposal, your reaction is probably that such rates may be conceptually fine, but (1) would be of little use because no one ever borrows or lends with such rates, and (2) like most academics we are unnecessarily trying to make an already complicated system even more complex.

We would be the first to admit that academics are often attracted to complexity for its own sake (it encourages the consumers of our product to think that we are uniquely able to produce it). But that is not the case with this interest-rate discussion. There are large and active markets today that do in fact effectively borrow and lend between future time points, and therefore cause such rates to exist and be observed. Such transactions are increasingly important in a wide range of financial decisions made by sophisticated modern organisations. Equally importantly, the concept of interest rates between future time points also allows us to understand much more about everyday securities (such as the bonds our governments issue) than we could without this idea.

To illustrate some of the important relationships in this market, suppose that five securities, A to E, are traded in this market and have future cash-flow expectations as listed in Table 1.1 for time points t_1 to t_3. Further, suppose that the securities' cash flows are riskless, and that the per-period interest rates that would apply are 5 per cent for the first period (between t_0 and t_1), 6 per cent for the two-period rate (between t_0 and t_2), and 7 per cent for the three-period rate (between t_0 and t_3). Incidentally, in financial markets, interest rates that begin at the present and run to some future time point are called **spot interest rates**. So another way of saying what we have just said is that the one-period spot rate is 5 per cent, the two-period spot rate is 6 per cent, and the three-period spot rate is 7 per cent.

The set of all spot rates in a financial market is called the **term structure** of interest rates. With those rates, we can easily calculate the present values (or market prices) of these securities, and they appear in the t_0 column of Table 1.1. You might find it a valuable exercise to see if you can arrive at the same result. (Remember that the market price of a security is the sum of the present values of the cash flows expected from it, discounted at the rates appropriate to those flows.)

Table 1.1 Bond cash flows and prices

	Price	Cash flows		
Security	t_0	t_1	t_2	t_3
A	£1029	£1080		
B	£1037	£80	£1080	
C	£1029	£80	£80	£1080
D	£923	£40	£40	£1040
E	£1136	£120	£120	£1120

Hint: In case you are having some difficulty in arriving at the same prices as shown in Table 1.1, the way we arrived at the price of security C is:

$$£1029 = \frac{£80}{(1.05)} + \frac{£80}{(1.06)^2} + \frac{£1080}{(1.07)^3}$$

Securities such as those in Table 1.1 are not at all uncommon in financial markets. We have designed the cash-flow pattern of A, B, C, D and E to be similar to that of **coupon bonds**, which are the type seen most often in bond markets. A coupon bond has a **face value** that is used, along with its **coupon rate**, to figure the pattern of cash flows promised by the bond. These cash flows comprise interest payments each period (which are given by the face value of the bond multiplied by its coupon rate). This continues until the final (maturity) period, when the face value itself, as a 'principal payment' plus a final interest payment, is promised. All of the bonds in Table 1.1 are £1000 face-value bonds; their coupon rates differ, however. Bond E has a 12 per cent coupon rate, which means that E promises to pay 12 per cent of £1000, or £120, each period (t_1 and t_2) until it matures, when it will pay 12 per cent **plus** £1000, or £1120 (at t_3). Bond A, of course, is an 8 per cent coupon bond maturing in one period. See if you can similarly describe the other bonds.[4]

1.5.1 The Yield to Maturity

In your daily newspaper, the business section may well regularly publish information about the market for bonds such as those described above. Table 1.2 is a reasonable approximation of the way that information usually appears. From our previous

[4] The face value of a coupon bond is usually called the '**principal**' and the coupon payment the '**interest**'. You should always keep in mind that, regardless of what they are called, these amounts are simply the cash-flow promises from the bond issuer; the coupon rate of interest bears no necessary relationship to market interest rates. The coupon rate is simply a contractual provision of the bond, which determines the amounts and timings of cash-flow promises. (The separation into 'interest' and 'principal' may be important for tax purposes, but we are not yet ready to worry about taxes.) The market uses the **bond's** cash-flow promises or expectations, and the market's own interest rates to set bond prices.

discussion you should now be able to examine a newspaper's table and see the correspondence between it and Table 1.2.

Table 1.2 **Government bonds**

Coupon rate	Maturity	Price	Yield	
8%	t_1	£1029	5.00%	(A)
8%	t_2	£1037	5.96%	(B)
8%	t_3	£1029	6.90%	(C)
4%	t_3	£923	6.94%	(D)
12%	t_3	£1136	6.85%	(E)

The only piece of information in Table 1.2 that may not be familiar to you is the column headed 'Yield'. That column presents the **yield to maturity** of the bonds; the yield to maturity (YTM) is something you have already seen in another guise. It is the IRR of the bonds' promised cash flows.[5] In other words, if you used the constant discount rate of 5.96 per cent on the cash flows of bond B in Table 1.1, you would get a present value (or market price) of £1037. The yield to maturity is the rate that discounts a bond's promised cash flows to equal its market price, and (from our knowledge of IRR) is the 'average per-period earning rate on the money invested in the bond' (which money, of course, is the market price).

Before embarking upon our investigation of forward interest rates, we can prepare ourselves by flexing our mental muscles briefly over the relationship between a bond's YTM and the set of spot rates that determine the bond's price. Look at bonds C, D and E in the two tables. All three have the same time until maturity, the same number of interest payments, the same cash-flow risk (none), and are subject to the same set of spot interest rates, which have produced the market prices we see. But the YTMs of the bonds differ. How can that be? If the same interest or discount rates have operated upon the bonds, how can their average per-period earning rates differ? The answer is that the **pattern** of a bond's cash flow across time influences its YTM, and these three bonds have different cash-flow patterns.

Compare the cash flows of bonds D and E in Table 1.1. Bond D, with a 4 per cent coupon, has interim interest (£40) payments that are smaller relative to its final payment than does bond E with its 12 per cent (£120) coupon. In effect, bond D has a greater proportion of its present value being generated by its t_3 cash flow than does bond E with its relatively larger interim interest payments. Remember that the spot interest rate for t_3 is 7 per cent, whereas the rates for t_2 and t_1 are lower at 6 per cent and 5 per cent respectively. So relatively more of bond D's value is being generated with the higher interest rate than is the case for bond E, and we therefore

[5] When seeing the word 'yield' in the newspaper you must be careful to read the newspaper's footnote to the table, because newspapers also use that term to stand for 'current yield' or 'dividend yield' instead of yield to maturity. The other 'yields' are simply **this period's** interest payment or dividend divided by the current price of the security. That ratio is of little use in bond markets.

see a higher per-period earning rate or YTM for D than for E. See if you can convince yourself that bond C's yield can be similarly explained.

The phenomenon we have described above has a name in finance; it is called the **coupon effect on the yield to maturity**. It gets this name because the size of the coupon of a bond determines the pattern of its cash flows, and thus how its YTM will reflect the set of spot rates that exists in the market. The YTM is mathematically a very complex average of the spot rates of interest, as weighted by the pattern of cash flows of a bond. Depending upon your attraction to matters mathematical, you will be either pleased or disappointed to hear that at this point it would not benefit us to pursue further this particular aspect of the YTM.

What we have learned about the yield to maturity should teach us to be very careful how we use it. For example, it would be most unwise to make comparisons among securities on the basis of their YTMs unless their patterns of cash flows (coupons, for bonds) were identical. The YTM, being a complex constant per-period average, might lead the uninitiated to think that the money invested in one bond was earning a better or worse rate than another bond in any given period, whereas it is clear that bonds of equal risk must earn the same rates during the same periods. The YTMs are expressing not only the earning rates but also the **amounts invested** in the bonds across time. In our example, bond E had a lower YTM than bond D because E's higher interim cash interest payments meant that relatively less was invested at the later periods (which had the higher interest rates).

Occasionally you may see the **yield curve** referenced as synonymous with the term structure of interest rates. The former is the set of YTMs that exists in the market, usually for government coupon bonds. The latter is, as we know, the set of spot interest rates. From what you now know about interest rates, you should be uncomfortable with using the yield curve as a substitute for the term structure.

1.5.2 Forward Interest Rates

We are now ready to discuss the set of interest rates that begin at some time point other than t_0 (now), and hence cannot be spot rates. As we mentioned earlier, these are called **forward rates** because of their location forward in time. Look at bond B in Table 1.1 and Table 1.2. The t_0 investor in bond B spends £1037 in order to get £80 at t_1 and £1080 at t_2. The spot rates tell us that the £1037 t_0 value of these cash flows can be regarded as:[6]

$$\begin{aligned} PV &= \frac{CF_1}{(1+i_1)} + \frac{CF_2}{(1+i_2)^2} \\ &= \frac{£80}{(1.05)} + \frac{£1080}{(1.06)^2} \\ &= £76 + £961 \\ &= £1037 \end{aligned}$$

[6] For simplicity of presentation we have selectively rounded off the cash flows, values and rates in this section. If you prefer numbers of higher accuracy, and if your calculator runs to several decimal places, we encourage you to investigate the more accurate results. To have the results consistent, you can assume that the spot rates, coupons and face amounts above are exact, and all other values and rates produced herein are rounded.

Of the £1037 invested in the bond at t_0, £76 of it produces £80 at t_1 for a 5 per cent return during the first period, and £961 of the t_0 investment produces £1080 at t_2, a 6 per cent return per period for two periods.

To this point we have done nothing more than we did when presenting the YTM in terms of invested amounts and earning rates, with a bit more numerical detail. But you recall that in pointing out the reasons why bond E had a lower YTM than bond D, we said that bond E had 'less invested in it' during the later time periods that contained the higher interest rates. Though that statement is entirely correct, we were not specific about what it means to have money invested in an asset at some future time. Understanding that will bring us a long way toward understanding forward interest rates.

Let us return to bond B and investigate the amounts invested in it across time. We know that there is £1037 invested at t_0, and that after the final payment is made at t_2, the investment must be zero. So the only question is the amount invested at t_1. If there is £1037 invested at t_0 and if the earning rate for the first period is 5 per cent, the amount invested at t_1 (before the t_1 interest payment) must be £1037 × 1.05 = £1089. After the £80 payment, the amount invested at t_1 is thus £1089 – £80 = £1009. We can find the amount invested in an asset across time by accruing past invested amounts outward at the same rates that we discounted cash flows backward in time.[7]

So £1009 is the amount invested in bond B at t_1. That information is important because it allows us to calculate an example of what we have been seeking: a forward interest rate. (A forward interest rate is usually noted with the letter f surrounded by a left subscript indicating the rate's beginning time point, and a right subscript indicating the rate's ending time point.) The £1009 invested in bond B at t_1 produces a payment of £1080 at t_2. The implied earning or interest rate for bond B between t_1 and t_2 (noted as ${}_1f_2$) must therefore be:

$$\begin{aligned} V_1(1+{}_1f_2) &= CF_2 \\ £1009(1+{}_1f_2) &= £1080 \\ {}_1f_2 &= 7\% \end{aligned}$$

Bond B is earning 7 per cent between t_1 and t_2: ${}_1f_2 = 7\%$. The **implied forward rate** for bond B in the second time period is 7 per cent.

We can continue with this example to illustrate another important relationship: that between spot rates and forward rates. We now know that the £1080 at t_2 is worth £1009 at t_1 (discounted for one period at ${}_1f_2$), and £961 at t_0 (discounted for two periods at i_2). But we have seen that it is also correct to think of the t_1 value of bond B as being generated by an investment earning 5 per cent during the first

[7] There is a financial market risk characteristic in addition to cash-flow risk that we are holding in abeyance for the moment: the uncertainty of future interest rates. The examples with which we are dealing in this module assume that the interest rates expected for future time periods will actually occur. This assumption, of course, has little meaning to you until we discuss what future or 'forward' interest rates are. So until we do, you can regard this as a gratuitous comment designed to appease picky academics who might be reading this.

period. Since an earning rate is just a discount rate in reverse, we can also think of the £1080 at t_2 being discounted to t_0 with the appropriate forward rates:

$$PV = \frac{CF_2}{(1+\ {}_0f_1)(1+\ {}_1f_2)}$$

And since ${}_0f_1$ is simply i_1:

$$PV = \frac{£1080}{(1.05)(1.07)} = £961$$

So it is entirely correct to think of the present value of the t_2 cash flow as being arrived at either by discounting with the spot rate for two periods or by discounting with the forward rates for one period each. This in turn implies that the relationship between the rates is:

$$(1+i_2)^2 = (1+\ {}_0f_1)(1+\ {}_1f_2)$$

Generally, this type of relationship will hold for all spot rates compared with the forward rates covering the same time. If the forward rates are known, the spot rate of interest can be found by multiplying together 1 plus each of the intervening forward rates, taking the *n*th root of that product (where *n* is the number of periods covered), and subtracting 1. If the spot rates are known, the forward rates can be found by a process of solving first for the forward rate nearest the present, and successively working to rates further in the future, exactly as we did with bond B. (Those of you with quantitative backgrounds will have recognised that (1 + spot rates) are merely the geometric means of (1 + forward rates).)

As an exercise, let us calculate ${}_2f_3$:

$$\begin{aligned}
(1+i_3)^3 &= (1+\ {}_0f_1)(1+\ {}_1f_2)(1+\ {}_2f_3) \\
(1.07)^3 &= (1.05)(1.07)(1+\ {}_2f_3) \\
(1+\ {}_2f_3) &= (1.07)^3/[(1.05)(1.07)] = 1.09 \\
{}_2f_3 &= 9\%
\end{aligned}$$

(Note that the rate i_3 = 7 per cent is, as we have shown, not the simple average of the 5 per cent, 7 per cent and 9 per cent forward rates, but is the result of subtracting 1 from the *n*th root of the product of 1 plus the intervening forward rates. This is a geometric mean.)

To test your conceptual understanding of the various interest rate ideas that we have produced, see if you can explain to yourself (or to anyone who is willing to listen) these three different methods of calculating bond D's present value, and how they are related to each other:

$$\begin{aligned}
£923 &= \frac{£40}{(1.05)} + \frac{£40}{(1.06)^2} + \frac{£1040}{(1.07)^3} \\
£923 &= \frac{£40}{(1.05)} + \frac{£40}{(1.05)(1.07)} + \frac{£1040}{(1.05)(1.07)(1.09)} \\
£923 &= \frac{£40}{(1.069)} + \frac{£40}{(1.069)^2} + \frac{£1040}{(1.069)^3}
\end{aligned}$$

Your explanation should be to the effect that all three methods of calculating the value of bond D are correct, the first using the spot rates, the second using the forward interest rates, and the last using the bond's YTM. The most accurate portrayals of the financial market's valuation process is given by the spot or forward rates.

1.5.3 Interest Rate Futures

To finish our discussion of forward rates we should point out why financial market participants might be interested in such a concept. Remember we discovered that bond B, after paying its t_1 interest, was worth £1009 at that time point. That amount can be regarded as bond B's **forward price** for t_1, given the information available as at t_0. You may have heard of 'forward' markets of one type or another (in exchange rates, commodities, and even in other financial assets much like the bonds we have been studying). In these markets, participants enter into contracts whose worths are determined by exactly the kinds of forward value and rate systems that we have been discussing. These markets are growing very quickly around the world and, as we shall see, they provide an important financial service to those participants sophisticated enough to use them wisely.

To illustrate their use requires that we introduce the final important characteristic of financial markets: **risk**. As we said earlier, it is not practical to discuss all of the manifestations of risk in financial decisions at one time, so we shall introduce them in gradual progression. The first one we shall mention is the risk that actual interest rates in the future may be different from the forward rates implied by the term structure of rates at an earlier time. For example, we found the forward rate for the second period in our example, ${}_1f_2$, to be 7 per cent, and that implied a t_1 forward price for bond B of £1009. Both the 7 per cent interest rate and the £1009 price are **expected** to exist at t_1 given the information available at t_0, and we have assumed that this information will turn out to be correct.

The truth of the matter is, however, that such expectations are almost never exactly borne out, and rather frequently are wildly incorrect. Financial markets often make very bad mistakes in the sense that implied forward rates and prices turn out not to have been correct expectations of the interest rates that actually appear in the future periods. (It is not at all clear, however, that we should regard this as a shortcoming of financial markets, especially without a demonstration that another entity could have made consistently better predictions with the information available at the time. This is a very important point and will come up again and again in various financial contexts.)

This potential difference between projection and reality implies the perhaps disturbing likelihood that by the time you actually get to t_1 the price or interest rate you expected, given what you knew at t_0, will not be available. The reason is simple: between the time the expectation is formed (t_0) and its realisation occurs (t_1) additional information will have appeared that causes the market to revise its cash-flow expectations, its opportunity costs, or both. (Since we are here discussing only the risk of interest rate changes, only opportunity costs would have changed in this example.)

The risk that interest rates might change unexpectedly is something that many market participants would like to avoid. If you had decided to undertake a real asset investment because it had a positive NPV and, after you had the investment well under way, interest rates increased so as to cause NPV to be negative, you would be disappointed. You need not have been: there are today available 'financial futures' markets that allow participants to guard against this kind of risk (and many other related types) by buying and selling commitments to transact in financial securities at future time points, **at prices fixed as at the present**. This would allow you, in the situation described above, to 'lock in' or guarantee a set of discount rates for your asset's NPV by agreeing to sell some financial securities at set prices across the life of the real asset. (You need not even own the securities you agree to sell, as long as you can convince the market that your credit is good by posting what is called a 'margin' or amount of money that would make up any likely losses on the transaction.)

To illustrate, suppose that you are about to undertake an investment that has a positive NPV using the current set of interest rates of the market but that also would be unlikely to be desirable if interest rates increased during the life of the project. One tactic to insure against the detrimental effects of interest rate increases would be to sell an **interest rate futures contract** in the approximate amounts and timings of the cash inflows of the project. Let us examine a simple example of such a transaction.

Suppose that you are about to undertake an investment with the following cash flows:

t_0	t_1	t_2
−£1700	£1000	£1000

Further, suppose that the term structure of interest rates is:

$$i_1 = 10\% \text{ and}$$
$$i_2 = 11\%$$

Thus the NPV of the investment is:

$$\begin{aligned} \text{NPV} &= -£1700 + \frac{£1000}{(1.10)} + \frac{£1000}{(1.11)^2} \\ &= +£20.71 \end{aligned}$$

and so the investment is acceptable. But suppose interest rates that were to occur in the future were not known for certain, and this manifested itself in the risk that the interest rate applicable between t_1 and t_2 (the forward rate ${}_1f_2$) might change from the one now implied by the current term structure of interest rates. You recall that we can find the ${}_1f_2$ implied by the current term structure by using the relationship between spot and forward rates:

$$\begin{aligned}(1+i_2)^2 &= (1+i_1)(1+{}_1f_2)\\(1+{}_1f_2) &= (1+i_2)^2/(1+i_1)\\(1+{}_1f_2) &= (1.11)^2/(1.10)\\{}_1f_2 &= 12.009\%\end{aligned}$$

So the forward rate between t_1 and t_2 implied by the current term structure is 12.009 per cent.

Now suppose there is risk that the ${}_1f_2$ rate would increase to 15 per cent. If this happens at t_0, there will be a new i_2 of:

$$\begin{aligned}(1+i_2)^2 &= (1+i_1)(1+{}_1f_2)\\(1+i_2)^2 &= (1.10)(1.15)\\(1+i_2)^2 &= (1.265)\\i_2 &= 12.4722\%\end{aligned}$$

And the present value of the investment becomes:

$$\begin{aligned}\text{NPV} &= -£1700+\frac{£1000}{(1.10)}+\frac{£1000}{(1.124722)^2}\\&= -£0.40\end{aligned}$$

The positive NPV of the investment has become negative due to the increased interest rate applicable to the t_2 cash flow. Were you to have committed any resources to the investment prior to the interest rate change, you would doubtless be distressed that an investment you expected to be good is now expected to be one that will decrease your present wealth.

The futures market in interest rates can enable you to avoid (or, to use the terminology of that market, **hedge**) the risk of such an occurrence. Suppose there were such a market, and you were faced with the same investment, original term structure and interest rate risk. This market would allow you to hedge the risk of a change in ${}_1f_2$ by selling an interest rate futures contract. The ${}_1f_2$ interest rate applies between t_1 and t_2, so your transaction will commit to sell at a fixed price a security at t_1 that has a single (£1000) cash flow at t_2. If ${}_1f_2$ increases, the price of such a security will decline. But since you will have a contract to sell that (now cheaper) security at a fixed higher price, the value of your contract will increase. The increase in the value of your contract will offset the decrease in the NPV of your investment, and you will have avoided the risk of interest rate changes. Let us examine the associated financial arithmetic.

Given the original term structure, the futures market will dictate a t_1 price for the one-period interest rate future of:

$$t_1 \text{ futures price} = \frac{£1000}{(1 + {}_1f_2)} = \frac{£1000}{(1.12009)} = £892.79$$

To hedge against interest rate risk you commit to sell a £1000 t_2 cash flow at t_1 for £892.79; that is the essence of your futures contract. Now, still at t_0, having undertaken the investment and sold the futures contract (no cash actually changing hands at t_0, the contract being 'sold' meaning only that you have made a commitment to sell the security at t_1), ${}_1f_2$ increases to 15 per cent. We have already seen the deleterious effect of that interest rate change upon the NPV of your investment: NPV declines from +£20.71 with the original term structure to −£0.40 with the new term structure, a wealth loss to you of £21.11. But what has happened to your futures contract? The increase in ${}_1f_2$ causes a £1000 cash flow at t_2 to decline in value. The new t_1 value of the t_2 £1000 is:

$$t_1 \text{ value of } t_2 \text{ cash flow} = \frac{£1000}{(1.15)} = £869.57$$

This decline in value is actually good news to you because you own a contract (the interest rate future you sold) that allows you to sell at t_1 for £892.79 a security that promises £1000 at t_2. Even though the t_2 cash flow is worth only £869.57 at the new interest rate, you can sell it for £892.79. That is a valuable capability; the value of your contract must therefore increase (remember that before the interest rate change the contract was without value, because it was promising the same interest rate as the market). Now the contract must obviously be worth £892.79 – £869.57 = £23.22 at t_1. That, however, is a t_1 amount. If we discount that value to t_0, with the unchanged i_1 = 10 per cent we get:

$$\text{Increase in contract value at } t_0 = \frac{£23.22}{(1.10)} = £21.11$$

We have seen that number before. In addition to being the **increase** in value of your interest rate futures contract at t_0, it is also the **decrease** in value of your investment's NPV. Thus the interest rate futures contract has hedged you against the risk that interest rates will change so as to decrease the NPV of your investment. Both the increase in futures contract value and the decrease in investment NPV are caused by the change in interest rate. Because you are to **receive** a cash flow from the investment that is discounted with that interest rate, the investment declines in value; but because you are to **sell** a cash flow discounted with that interest rate through the interest rate futures contract, the contract increases in value. Since the amounts and timings of the cash flow are the same, the value changes are also the same (and of course in opposite directions).

A word or two of caution about this illustration is necessary, however. First, although the financial economics of the illustration is accurate, we have simplified the contract and transaction somewhat for clarity of exposition (i.e. we have not worried about margins, brokerage fees, the difficulties of finding an exact contractual hedge, and other characteristics of real-life transactions). The analysis of hedging in actual financial markets is best left to more advanced texts, and to professionals in that marketplace.

Furthermore, you should keep in mind that hedging works in both directions to remove the effects of interest rate changes. For example, if ${}_1f_2$ were to decrease, the NPV of your investment would increase, but your futures contract value would decrease to offset the NPV increase. Hedging means you lose both the bad and the good surprises. Finally, note that the investment's cash flow itself was unchanged throughout the example. In real investments it is often the case that the same kinds of event that cause interest rates to increase and decrease (inflation, for example) will also cause cash-flow expectations to increase and decrease (in the same direction as interest rates). Were that to be a characteristic of your investment's cash flows, you would not be interested in hedging interest rate risks, because revisions in your cash-flow expectations would effectively do it for you. Nevertheless, there are many instances where such hedging is worthwhile, and it is ever becoming more popular with sophisticated financial market participants.

This is a useful discussion for us, because it is a rather advanced illustration of the ideas involved with forward interest rates, values, and the way financial markets operate. But, for all that, we have introduced no really new basic concepts. A careful review of the foregoing will show that we have essentially done nothing more than discount future cash flows at market rates.

Forward interest rates, forward prices and futures contracts with their associated transactions are some of the more sophisticated ideas that exist in finance. We do not expect that you feel expert enough to participate in those markets at this point; as a matter of fact, we would not be surprised if you were a little discouraged at the apparent complexity of all these financial manipulations. But take heart. Our only purpose in the illustrations above is to convince you that the interest rate ideas upon which we have spent so much time are not mere academic exercises but valuable concepts for the financial practitioner.

1.5.4 Interest Rate Risk and Duration

Our discussion of interest rate phenomena would not be complete without some deeper discussion of the nature of interest rate risk. As is clear from the arithmetic of discounting and valuation, as interest rates move up and down with time while other factors remain the same, values will move down and up. The variability of values due to changes in interest rates is the effect of interest rate risk.

There are a number of competing theories for the determination of interest rates, but for our purposes at this point it suffices for us to understand that interest rates change across time because of changes in a number of influences on the opportunity costs of investing (such as the effect of inflation on the purchasing power of eventual cash payments of interest and principal, or changes in the creditworthiness of bond issuers, or changes in the rates of return available on real asset investments). But for whatever reason, such changes in interest rates imply changes in value and therefore in wealth, which is important.

There is an interesting measure of the extent to which a particular bond with specified interest payments is subject to interest rate risk. This measure is called **duration**, and is a kind of index that tells us how much a particular bond value will go up and down as interest rates change. It measures the 'exposure' of the value of a bond to changes in interest rates. Rather than give a more detailed definition at this point, let us look at an example.

Return to Table 1.1, and consider bonds C and D. Their prices are indicated as £1029 and £923 respectively. Suppose that interest rates instantaneously increased, such that the spot rates were 6%, 7% and 8%, instead of the 5%, 6% and 7% that gave us the original values. The bonds' values must, of course, decline, and if you do your arithmetic correctly, you will now see that bond C is worth £1003 and bond D is worth £898. But notice that the decline in value of bond C is less in percentage terms than is the decline in the value of bond D – about 2.6% for bond C and 2.8% for bond D. The same tendency would appear for reductions in interest rates: bond D's value would react more than bond C's. Why is that? Why does bond D experience a greater percentage change in value than bond C? The answer is that bond D has a greater duration than bond C.

We are now ready to see a more rigorous definition of duration. **Duration** is the number of periods into the future where a bond's value, on average, is generated. The greater the duration of a bond, the farther into the future its average value is generated, and the more its value will react to changes in interest rates. The reason for this is not difficult to understand.

Consider two bonds, each of which have only one payment, but one of the bonds will receive its payment after one year, and the other will receive its payment after five years. (These types of bond are called **zero coupon** bonds, because in effect they have only a final principal payment and no interim interest payments.) No matter what the term structure of interest rates, the five-year bond's value will react more strongly to a given change in interest rates than will the one-year bond. If interest rates go up or down, the five-year zero coupon bond's value will decline or increase in percentage terms more than the one-year bond's. The reason of course is that the five-year bond's cash flow is discounted with an interest exponent of 5, whereas the one-year bond's cash flow is discounted with an interest exponent of only 1. Notice also that the duration of the five-year zero coupon bond is simply 5, because that is the time in the future that generates the bond's entire value. Similarly, the one-year zero coupon bond has a duration of 1. And so, here again, the longer-duration bond is associated with a greater reaction to changes in interest rates.

Let us return now to bonds C and D. Finding their durations is more complicated than finding durations for zero coupon bonds because bond C's and bond D's values come from more than a single future time. We can calculate their durations by 'weighting' the time points from which cash flows are generated, by the proportion of total value generated at each time. One way to calculate bond C's duration is:

$$\text{Duration C} \quad = \quad (1)\left[\left(\frac{80}{1.05}\right)/1029\right] + (2)\left[\left(\frac{80}{(1.06)^2}\right)/1029\right] + (3)\left[\left(\frac{1080}{(1.07)^3}\right)/1029\right]$$

$$\text{Duration C} \quad = \quad 2.78$$

Similarly, bond D's duration is:

$$\text{Duration D} \quad = \quad (1)\left[\left(\frac{40}{1.05}\right)/923\right] + (2)\left[\left(\frac{40}{(1.06)^2}\right)/923\right] + (3)\left[\left(\frac{1040}{(1.07)^3}\right)/923\right]$$

$$\text{Duration D} \quad = \quad 2.88$$

So bond D has a longer duration than does bond C. Bond D's average present value is generated 2.88 periods into the future, whereas bond C's average present value comes sooner, 2.78 periods into the future. And with this longer duration, bond D experiences a greater interest rate risk; its value will go up or down more than bond C's for a given change in interest rates.

The idea that duration is a measure of interest rate risk is particularly valuable for coupon bonds, where comparative riskinesses might not be obvious simply by inspection. For example, depending upon their coupon rates, a nine-year coupon bond could have a longer duration than a ten-year coupon bond and therefore be subject to greater interest rate risk. Further, duration is the starting point for an important aspect of professional bond investing called **immunisation** which allows certain portfolios of coupon bonds or other types of investments to be shielded against unexpected changes in interest rates. Teaching you these techniques is beyond the scope of this text, but it is important that you have heard that such a thing is possible.

Learning Summary

One final note is of a more philosophic nature on interest rate structures. Amidst all this discussion of complicated interest rate calculations, of forward rates, spot rates, yields to maturity and so forth, you should remember that financial markets really do only one thing: they set prices on financial securities. The array of interest rates of various types that we see is merely the attempt of practitioners to make some sense of the consistency among those market prices. As long as we keep in mind that the market is made up of many wealth-maximising participants, each aware of and concerned about opportunity costs, then the rest of the system almost invents itself.

The section on realistic financial markets has introduced and developed several important financial concepts and techniques, but at the same time it has been a long and occasionally arduous one. While extending the single-period financial market of the previous section to multiple periods, we studied present value tables, calculator techniques of finding present values, compounding of interest, various perpetuity formulations, yields to maturity, bond value ideas, the relationships among spot rates, forward rates and yields, forward prices, and the workings of futures markets in financial assets. This is a great deal of material, and it will take you some time and effort to digest. You should not hesitate to reread sections; much of the material will seem easier the second time around. Give very close attention also to the problems that follow, for they are excellent learning devices.

There is no substitute for a thorough knowledge of what we have been studying in this module: the ways a financial market can quote interest rates and prices. Any well-educated business person has some familiarity with these ideas. Though there is a large amount of detailed information to remember, it is also true that the financial market is a single system, and since all of these concepts operate within that system, there is an elegant consistency to everything it does. The best signal of a student's being well along the road toward conquering these materials is the recognition of the extent to which all of the complex terminology, arithmetic and technical detail of financial markets are manifestations of the same few basic ideas time and time again.

Review Questions

1.1 Suppose that you are a participant in the single-period financial market described in this module. Your certain expectation is to receive £3000 immediately, and an additional £5328 at the end of the period. If the market rate of interest for riskless borrowing and lending is 11 per cent, the maximum you can consume immediately is which of the following?

A. £8328.00
B. £7800.00
C. £51 436.36
D. £7843.64

Questions 1.2 to 1.11 use the information in Question 1.1. You may find it convenient to sketch a graph of your answers.

1.2 The maximum that you can consume at the end of the period is which of the following?

A. £8328.
B. £7800.
C. £5658.
D. £8658.

1.3 Suppose you wished to consume £5000 immediately. Which of the following amounts could you then consume at the end of the period?

A. £3108.
B. £3128.
C. £8658.
D. £7548.

1.4 If you wished to consume £7548 at the end of the period, which of the following could you consume now?

A. £3000.00
B. £1000.00
C. £981.82
D. £7800.00

1.5 Consider the present value of all four consumption combinations in Questions 1.1 to 1.4 above. You should expect them to be:

I. All different because the amounts consumed are all different.
II. All the same because they are all allocations of the same total wealth.
III. All the same because they are all available through borrowing and lending transactions from the same initial expectations.

Which of the following is correct?

A. I alone.
B. II alone.
C. III alone.
D. Both II and III.

1.6 Suppose that you prefer a pattern of consumption that tends to emphasise present consumption, while another participant with identical cash-flow expectations prefers one that delays consumption into the future. Which of you would be the wealthiest?

A. You, because the other participant will have made investments that do not produce income until the end of the period.
B. The other participant, because you will consume more of your income first.
C. Neither, because consumption patterns do not affect wealth.
D. Neither, because you will both eventually get to consume the same amounts.

1.7 Suppose that in the same financial market the following investments, also with riskless cash flows, are available:

Investment	t_0	t_1
1	−£1000	£1250
2	−£500	£650
3	−£1500	£1650

Using the NPV criterion, you would accept which of the following?

A. Investment 1 alone.
B. Investments 1 and 2.
C. Investments 1 and 3.
D. All three investments.

1.8 Using the IRR criterion for the investments in Question 1.7, you would accept which of the following?

A. Investment 1 alone.
B. Investments 1 and 2.
C. Investments 1 and 3.
D. All three investments.

1.9 Suppose that the investments above were undertaken correctly by a company of which you were the only shareholder. The change in your present wealth would be which of the following?

A. £211.72
B. £198.21
C. −£2500.00
D. £227.27

1.10 Suppose that the company in Question 1.9 above informs you as its owner that it does not have enough money to undertake the chosen investments, and requests that you provide the necessary funds. Suppose also that your preferences for consuming your wealth are such that you wish to spend £3000 at the present (t_0), which is equal to your initial resources at that time point (*see* Question 1.1). Your options are to:

I. Refuse the investment appeal because you would not be able to consume as you wished at t_0 if you provided the company with the money.
II. Provide the money as requested at t_0 and borrow enough to consume as you wish.
III. Provide the money as requested at t_0 and sell your shares so as to consume as you wish.
IV. Refuse to invest further and suggest that the company borrows the money elsewhere.
V. Refuse to invest further and suggest that the company sells shares to others so as to raise the money.

A correct decision is to choose which of the following?

A. I.
B. Either of II or III.
C. Either of IV or V.
D. Either of II and III or IV and V.

1.11 Suppose that the three investments shown in Question 1.7 above are mutually exclusive; that is, you could only accept one of the three. Which of the following should you choose?

A. Accept the one with the highest NPV because it increases your wealth the most.
B. Accept the one with the highest IRR because it increases your wealth the most.
C. Accept the one with the highest NPV because it earns the greatest return per period.
D. Accept the one with the highest IRR because it earns the greatest return per period.

Questions 1.12 to 1.18 refer to Module 1, Section 1.4 and Section 1.5.

1.12 You intend to open an ice cream stand and must choose a location for it. There are two sites available, each of which requires that you make a present (t_0) cash outlay of £2500. You expect that the locations' net cash inflows for the three-period life of the stand will be:

	t_1	t_2	t_3
Location 1	£1200	£1300	£1450
Location 2	£1300	£1300	£1300

If your opportunity costs are constant at 10 per cent per period, you would, using the NPV criterion, choose which of the following?

A. Location 1.
B. Location 2.
C. Either one, because you are as well off with either.
D. Neither, because they are both undesirable investments.

1.13 Suppose that your opportunity costs were 25 per cent per period rather than 10 per cent in the scenario of Question 1.12. Would that change your answer?

A. Yes, you would choose the other location.
B. No, you would choose the same location.
C. No, you would still be indifferent between them.
D. No, you would still reject both of them.

1.14 You are faced with a choice between two investments that require the same outlay. The first is expected to provide a stream of perpetual cash flows equal to £1000 per period for ever. The second investment is also a perpetuity, and has a t_1 cash flow of £800, which will grow at a constant rate of increase each period for ever. If your opportunity costs are constant at 10 per cent per period, what rate of increase in the second investment's cash flows is necessary to make it as desirable as the first?

A. 10%.
B. 2%.
C. 0%.
D. There is no rate which will make the second investment as desirable as the first.

1.15 Suppose that the first investment in Question 1.14 was not a perpetuity, and the second investment's cash flows did not grow. For how many periods would you necessarily expect the first investment to run so as to render it as desirable as the second?

A. About ten years.
B. About thirteen years.
C. About seventeen years.
D. About twenty years.

1.16 *(To be completed easily, this problem requires that you be able to exponentiate. If your calculator cannot do that,* ***describe*** *how you would solve the problem, and check your answer against that provided.)*

Suppose that you wish to purchase the digital tape device mentioned in the text, but instead of borrowing money to buy it, you prefer to put aside enough in an interest-bearing account to be able to pay cash for the machine. If the device costs £800, you are willing to wait a year to get it, and the bank pays 10 per cent annual interest compounded monthly, how much money must you put aside each month, beginning at the end of this one, to purchase the machine at the end of the year?

A. £66.00.
B. £60.60.
C. £63.67.
D. £60.32.

1.17 Suppose the bank in Question 1.16 compounded interest continuously. How much would you necessarily deposit in the bank at the beginning of the year in order to end the year with £800? (Continue to assume a 10 per cent annual interest rate.)

A. £727.27
B. £723.87
C. £724.17
D. £738.16

1.18 You are now considering an investment opportunity that has the following certain cash-flow expectations:

t_0	t_1	t_2
−£15 000	+£7000	+£11 000

Market interest rates, and thus your opportunity costs, are 10 per cent per period. You are concerned that, in the event that you decide to undertake the investment, there is a chance that the actual interest rate occurring between t_1 and t_2 will not be 10 per cent but 20 per cent instead. You should do which of the following?

A. Not undertake the investment because it has a negative NPV or an IRR less than your opportunity cost.
B. Undertake the investment because its NPV is positive or IRR exceeds opportunity costs, and would continue to do so even with the increase in interest rate described.
C. Undertake the investment and simultaneously sell an interest rate future for t_1 in the amount of the value currently expected for the t_2 cash flow at t_1.
D. Undertake the investment and simultaneously buy an interest rate future for t_1 in the amount of the value currently expected for the t_2 cash flow at t_1.

Case Study 1.1: Bond and Interest Rate Arithmetic

The following bonds, all of which have risk-free cash-flow expectations, £1000 face values, and that pay interest once per period, are available in the market:

A. A 4 per cent coupon bond maturing at t_2 sells now (t_0) for £919.97.
B. A 10 per cent coupon bond maturing at t_2 has a YTM of 8.5595 per cent.
C. An 8 per cent coupon bond maturing at t_3 sells now for £1014.59.

If the current one-period spot rate of interest is 10 per cent:

1 What is the current price of bond (B)?

2 What is the current two-period spot rate of interest (i_2)?

3 What is the one-period forward rate of interest for the second period (${}_1f_2$)?

4 What is the one-period forward rate of interest for the third period (${}_2f_3$)?

5 What is the current three-period spot rate of interest (i_3)?

6 Without actually performing the calculation, would you expect the YTM of bond (A) to be greater or smaller than that of bond (B)? Explain.

7 After its interest is paid at t_1, what is the current expectation for the price of bond (B) at t_1 (its forward price at t_1)?

8 Some investment bankers are now selling securities that they form by purchasing coupon bonds and separately offering the coupons and principal payments to the financial markets. In other words, it is now possible for you to purchase a security that is a future claim upon a single interest payment from a coupon bond. If we assume that interest rates in the future are known for certain, what would be the current price of the t_2 interest payment from bond (C)?

9 Suppose now that interest rates expected to occur in the future are uncertain – in other words we can, if we wish, calculate a rate such as ${}_2f_3$, but there is no guarantee that when t_2 actually arrives the existing i_2 will equal that rate. As a matter of fact, there is no guarantee that **any** forward rate will be the same in even the next instant of time. If you were now about to undertake an investment that had cash-flow expectations extending for the next few periods, outline the general characteristics of a strategy for eliminating the risk that interest rates (and thus your NPV) would change during that time. You may assume that any financial markets necessary to that strategy do exist.

10 Now assume that you are considering an investment, one of whose cash inflows is £1000 at t_3. Illustrate, with a quantitative example based upon your answer to Question 9 above, how you can hedge the risk that ${}_2f_3$ will change.

Case Study 1.2: A Multiple-Period Resource Reallocation

Assume that you expect with certainty to receive the following cash amounts at the times indicated:

t_0	t_1	t_2	t_3
£12 000	£13 000	£14 000	£15 000

The rates of interest in the market are constant across the future at 8 per cent per period.

1 What is your present wealth?

2 At what current prices would you be able to sell each of your future cash flows?

3 At what price would you expect to be able to sell your t_2 cash flow at t_1?

4 What do you think your t_3 cash flow will be worth at t_1?

5 Suppose that you wished to consume a constant amount at each time point, beginning now. How much would you be able to consume as a maximum at each time? Demonstrate with a specific set of financial market transactions (borrowing and lending) how you could arrive at that consumption pattern.

Assume now, still at t_0, market interest rates change such that the one-period spot rate is 6 per cent, the two-period spot rate is 8 per cent, and the three-period spot rate is 9 per cent.

6 What has happened to your present wealth?

7 Describe how that effect occurred, with reference to the present value of each of your expected cash flows. Have they all changed in the same direction as your present wealth? Explain.

8 Is it still possible to consume in the pattern that your answer to Question 5 said was your choice?

Assume now that interest rates have returned to their original levels (constant at 8 per cent per period). An investment becomes available that requires a t_0 outlay of £5060, and returns £1500 at t_1, £2000 at t_2 and £2480 at t_3.

9 Would you accept the investment?

10 Suppose that the interest rate structure was the one that applied to Question 6 immediately above. Would your answer to Question 9 be the same?

11 Suppose that the cash flows for the investment at t_1 and t_3 were swapped. Would the relative desirability of the investment with the two interest rate structures be the same? Explain.

Module 2

Fundamentals of Company Investment Decisions

Contents

Learning Objectives

Module 2 focuses the discussion of financial decision making upon the incorporated firm, the 'corporation' or plc. It begins with a discussion of the corporate form of economic organisation, with its important distinctions between debt and equity capital and how these securities form a hierarchy of claims against the cash flows of the firm. Next are the specific sources of value, based upon cash-flow expectations, of those capital claims. The module then illustrates how real asset investment and financing decisions taken by companies affect the wealth of shareholders and bondholders, and this is shown by associating the internal financial decisions with changes in values placed on the securities by stock and bond exchanges. The NPV and IRR again form an important part of the analysis. Finally, this module provides the student with some real market context in a discussion of the popular financial measurement called the 'price/earnings ratio' and its correct application. Module 2 introduces students to the important distinctions between making financial decisions as individuals compared with making such decisions in complex organisations such as the modern corporation.

2.1 Introduction

In this module we begin to apply the ideas and techniques of the first module within a familiar context: the organisations in which most of us work. Although this course is a general introduction to finance, which is applicable to many different situations, the richest and most varied applications are those that occur in complex organisations, especially the modern company organised as a corporation (we shall use the terms 'company' and 'corporation' synonymously). The things we learn about finance in companies will, for example, be transferable to the decisions that are made in other organisations (such as governmental agencies) and to the level of our own personal financial concerns.

Because we shall be studying finance from the company viewpoint, it is important that we understand from the perspective of finance exactly what a company organised in the corporate form is. For our purposes we are interested in the characteristics of corporations that distinguish them from other participants in financial markets. In this context:

> The corporation (plc) is an organisation that raises money from capital suppliers by issuing contracts (we call them securities), invests that money in productive assets, operates those assets (perhaps hiring other resources such as management and labour), and distributes the money proceeds from operating those assets to all that have claims on those proceeds.

Those having claims upon the company's cash flows include everyone holding contracts, formal and informal, with the company. They include workers and management (receiving cash flows as compensation for services), government (receiving cash flows as taxes and fees), suppliers of raw materials (being paid for materials used in the productive process), and finally, capital suppliers (being paid their capital returns in the forms of interest, principal and dividends). This latter group, the capital suppliers, are those who have purchased the securities that the company has issued; the company issues these securities so as to raise money to acquire productive assets. We usually know this group of capital suppliers as those who own the **equity** and **debt** claims on a company.

Corporations are 'creatures of the law' in the countries within which they operate. They may be called plcs, SAs, Incs, AGs, or any number of other titles depending upon their country of location, but they will all have the essential characteristics mentioned above and will all face the same basic financial decisions. These decisions, as confronted by companies, are in economic dimension no different than those that are faced by the simple participants in our Module 1, except that corporations are subject to many complicated taxation and other 'real world' phenomena that make their analyses more challenging.

In this module we shall concentrate upon the investment decisions that companies make. In a sense, we shall be adding realism to the investment ideas that we studied in the prior module. We shall show some of the elaborations that are necessary for those techniques to work with real companies, and why those techniques work well for those organisations. We shall connect the esoteric 'textbook' methods of finance to familiar measures of corporate performance such as earnings per share, price/earnings ratios and share prices.

2.2 Investment Decisions and Shareholder Wealth

This section will demonstrate that the basic investment decision procedure that we studied in Module 1 is directly applicable to the investment decisions of companies. In order to prove that, we show that a corporate investment NPV translates directly into a wealth increase of that amount for the shareholders of the company. The first step in doing that is to learn more about the equity of corporations.

2.2.1 Corporate Equity

When a company issues shares to raise money from the capital market, it creates a security known variously as 'ordinary shares', 'common stock', 'equity', 'common shares', 'ownership capital' and a number of other names. The important things to remember about this type of capital claim are as follows.

1. It is a **residual claim**. That is, equity has no specific contract with the company that requires any particular amounts of money to be paid to the shareholders at any particular time. The shareholders are entitled only to vote for the directors of the company (which have the power to set policy and hire and fire management), and to expect that the company's directors and management make decisions in the best interests of the shareholders. This means that the shareholders of the company have effectively delegated almost all of their ownership decision-making authority to the directors and management of the company. The directors and management are thus the 'agents' of the shareholders, and make decisions on their behalf. For the time being we shall assume that the company makes decisions with the purpose of maximising the wealth of its current shareholders. This is not always an automatically correct assumption, but for now it will serve well our purpose of studying corporate investment decisions.
2. Equity has **limited liability**. This means that the possible losses that a shareholder (not participating in the decisions of the company) can incur are limited to the value of the shares that the shareholder owns. For example, if the company makes a bad decision and ends up having more fixed contractual claims against its resources than it has resources to make good on those claims, the shareholders will get nothing because they are the residual or 'last-in-the-queue' claimants of the company (see item 1 above). That is bad, but not nearly so bad as it would have been if the shareholders had been forced to use their own personal resources to make good on the corporate contractual claims, as the owners of most other types of organisations do. Limited liability is a nice attribute of corporations, but like most nice things it is not free. You can bet that the company would be able to borrow money more cheaply (and therefore pay higher dividends to its shareholders) if its creditors also got a claim on the personal resources of the company's shareholders. Exactly why the limited liability company has become the predominant form of private capital organisation is a matter of some interest and debate in finance and economics, but that need not concern us here, so long as we understand the rules by which the game is played.
3. The net result of these characteristics is that the shareholders of a company do not have a contract to which they can refer for developing an idea of how much money they can expect to receive or when they may receive it (in the way, for example, that bondholders do). Until all other contracts that the company has entered into have been fulfilled, the shareholders are entitled to nothing. But once those contracts have been met, the shareholders have an ownership claim on **all** of the remaining corporate resources. In a sense, shareholders have agreed

to stand last in queue for the corporate largesse, in exchange for everything that is left over.

This module is concerned primarily about the investment decisions that companies make. We have already learned one important thing about those decisions: companies seek to make investments that are in the best interests of their existing shareholders. And we know from Module 1, shareholders are best off when they are wealthiest. Consequently, companies, in making investment decisions, are attempting to make their shareholders as wealthy as possible. We also know from Module 1 that individual participants can maximise their wealth by choosing investments that have the highest NPVs. We shall now demonstrate that companies maximise their shareholders' wealth by choosing corporate investments with the highest NPVs.

First we must become more specific about exactly what 'shareholder wealth' is. Shareholder wealth is the market value of the common shares or equity that the shareholders own. So if the company wishes to maximise its share-holders' wealth, it should seek to maximise the market value of the shareholders' ordinary shares.

2.2.2 The Market Value of Common Shares

But what is the source of that market value? What exactly determines the value of a company's equity? The answer is that the market value of a company's common shares is the discounted value of all of the future dividends that the current shareholders are expected to receive.

'Hold on a minute!' you say. 'For the most part the things you have said until now make some sense, but I have real problems with this last point. I can go along with the idea that equity market value must be the discounted present value of *something* in the future (because of everything we did in Module 1), but why dividends?' you ask. 'Dividends are the amounts of money that the company pays to its shareholders, so they are not irrelevant, but most people I know who own common shares are at least as much interested in the **price increases** that they may get in their shares. Haven't you ignored that part of the present value of owning corporate equity by looking only at dividends?'

A good, though predictable, question. It is true that the return that shareholders expect to earn on their common shares often comprises both dividends and price increases (or **capital gains** as they are called). But, believe it or not, it is still **dividends alone** that are the basis for the value of a company's common shares in the financial markets. To see this, suppose that you own the common shares of International Eugenics (an imaginary company), and you plan to hold those shares for only one period, at which point you intend to sell them. Using the basic value ideas from Module 1, you would indeed be interested in the end-of-period price at which you could expect to sell your shares, and would view those shares as having a value to you given by:

$$\text{Value}_0 = \frac{\text{Dividend}_1 + \text{Value}_1}{(1 + \text{Equity discount rate})}$$

If we use the abbreviations E_t to mean equity value at time t, Div_t for expected dividends, and *re* for the equity discount rate, we can write that idea in shorthand as:

$$E_0 = \frac{Div_1 + E_1}{(1 + re)}$$

This seems to answer your question positively: the value of your common shares in International Eugenics depends upon the price at which you can expect to sell at t_1. But there is something incomplete about this idea. What about the person to whom you intend to sell the shares? What would induce that person to purchase the shares from you for a price of E_1? Naturally, he or she would be willing to pay only the present value of what they expect to get back in the future, which is perhaps one period of dividends, and then the market value of the shares when they sell them. So they would be willing to pay, and you can reasonably expect to receive at t_1:

$$E_1 = \frac{Div_2 + E_2}{(1 + re)}$$

This means that the value of your shares at t_0 is really:

$$E_0 = \frac{Div_1}{(1 + re)} + \frac{Div_2 + E_2}{(1 + re)^2}$$

Compare this formula to the earlier one for E_0 and notice the difference. The price at which you can sell your shares at the end of the first period (E_1 in the first formula) has disappeared from the equation for E_0 and has been replaced by the dividend at the end of the next period plus the price at which the purchaser can expect to sell the shares of International Eugenics at the end of that period.

You can probably see where this argument is heading. The person to whom you sell your shares must in turn sell them to someone who expects to get some dividends, and themselves sell to someone expecting dividends, and so forth. When all is said and done, and all of the reasonable expectations of returns from holding the shares are included, we are left with only future dividends as the basis for the market value of a company's equity.

A simple numerical example might be useful at this point. Suppose that International Eugenics has expectations of paying £100 in dividends for ever, and the appropriate discount rate for the dividends is 10 per cent per period:

$$E_0 = \frac{£100}{0.10} = £1000$$

Your shares are thus worth £1000, based upon the set of future dividends expected.

You, however, having a finite life, cannot expect to receive all of those dividends. In fact, you expect to sell your shares at the end of this period. At that time the shares will still be expected to pay dividends of £100 per period for ever, and with the same discount rate will still be worth £1000. So your expectation of cash flow from holding the shares of International Eugenics is £100 of dividends and a £1000 sale price, both at the end of the period:

$$E_0 = \frac{£100 + £1000}{(1.10)} = £1000$$

At a 10 per cent discount rate, the present value is £1000, the same price that the market is quoting for your shares based upon an infinite stream of £100 dividends. So even though you feel that the value of your shares is due in major part to the price at which you can sell them, the market price actually depends upon all of the future dividends that those shares are expected to receive.

This idea should actually be of some comfort to those of us who prefer the world to be a place where, in most instances, intuitive common sense is a reasonable guide. Most basically, this argument is saying that the value of the claims against a company's future cash flows must finally depend upon the amounts of cash that the **company** is expected to pay to the claimholders, not upon what claimholders are expected to pay each other when they buy and sell shares. The idea that the market value of the shares of a company is the present value of its future dividends is simply a manifestation of that common sense.

To review where our investigation has taken us to this point:

1. Corporations in making financial decisions attempt to maximise the wealth of their existing shareholders.
2. Shareholder wealth consists essentially of the market value of the common shares or equity of the company.
3. So the company, in maximising the wealth of its shareholders, must attempt to maximise the market value of its common shares.
4. The market value of the common shares of a company is the present value of the future dividends expected to be paid to the currently existing shares of the company.[1]

2.2.3 Investment and Shareholder Wealth

We are now ready to address the question that is the subject of this section: how the investment decisions of a company affect the wealth of its shareholders. As promised, we can now show that the use of a properly calculated investment NPV by the company does in fact result in an increase in the present wealth of its shareholders, and is consistent with the desire to maximise shareholder wealth.

2.3 Investment Decisions in All-Equity Corporations

Suppose that the Simple Corporation (another made-up organisation) is financed only with ordinary shares and that the company has no other capital claims outstanding (e.g. it does not owe any money to creditors). The Simple Corporation undertakes an investment that costs £100 000 now and returns £50 000 net per year in perpetuity. The riskiness of the future cash flows of the investment indicates that a 10 per cent discount rate is appropriate, so the investment has a corporate net present value of:

[1] You might be curious about our insistence upon always referring to the dividends expected for the 'currently existing' shares of the company as the basis for share value. The reason we talk about only those dividends as the source of the value of the equity of a company is that the company will probably issue some new shares at some time in the future, and will, naturally, pay dividends to those shares. Since the dividends paid to new shares will not, of course, be paid to the current shares, those other dividends cannot be part of the value of the company's equity at the present time.

$$\text{NPV} = -£100\,000 + \frac{£50\,000}{0.10} = £400\,000$$

What has happened to the wealth of the shareholders? First, there will be an increase in the market value of the company's shares due to the increased future amounts of cash that the Simple Corporation will be able to pay to its shareholders. The £50 000 per year has a present value, with a 10 per cent discount rate, of £500 000. The market value of the company's shares will increase by that amount. But that market value increase is larger than the NPV we calculated for the investment. If shareholder wealth increased by as much as the increase in the value of the shares (£500 000), the corporate NPV (£400 000) would be less than the shareholder wealth increase, and thus NPV would not be a good indicator of investment desirability. What we have ignored to this point, and what is causing the difference, is the £100 000 present cash outlay that the Simple Corporation makes when it undertakes the investment.

Suppose that the Simple Corporation made the £100 000 investment outlay from cash that would have otherwise been used to pay a shareholder dividend. Were that the case, it is easy to see that the investment would increase the shareholders' wealth by £400 000, not £500 000, because the £500 000 increase in the market value of the shares is partially offset by the £100 000 cash dividend that shareholders did not get. The net difference is £400 000, as the increase in shareholder wealth and as the corporate NPV.

'Fine,' you say. 'But the Simple Corporation did not have to undertake the investment that way, by using the cash that would have otherwise been paid as a dividend. Suppose the company paid the £100 000 dividend and at the same time raised £100 000 from new equityholders for it, to be used for the investment outlay. That sounds better for shareholders since they get both the current £100 000 cash dividend **and** the market value increase in their shares.'

Clever, but wrong. The error is in underestimating the intelligence of the **new** equityholders. The new equityholders are being asked to contribute £100 000 cash to the Simple Corporation, and will naturally require something in exchange that has a value at least as high. That something is, of course, shares. So the new shareholders must end up with at least £100 000 of the company's share value or they would not contribute that amount of cash. (And if the new shareholders end up with significantly more than £100 000 of the value of the shares, the financial managers of the Simple Corporation should be fired for not protecting the **old** shareholders' interests.)

You can see where this leads us. If the Simple Corporation issues new equity to raise the money for the investment, the market value of the company's shares increases by £500 000. But £100 000 of that must go to the new shareholders, which leaves £400 000 for the original shareholders. Relative to the old financing plan that used the dividend to make the investment, the original shareholders here do get the £100 000 cash dividend, but lose £100 000 of the share value increase to the new equityholders, so the original shareholder wealth increase remains at £400 000, the corporate NPV of the investment. (Where does the £100 000 of value for the new shares come from? Naturally, from the future dividends that old shareholders now must share with the new shareholders – and that the old shareholders would not have been forced to share had they contributed the £100 000 of cash themselves for the investment.) So it does not matter from where the money for the investment comes; from either our forgone dividends or outside equity, the investment's corporate NPV is the wealth increase of the existing or 'old' shareholders. Table 2.1 provides a useful and coherent display of the above ideas.

Table 2.1 The Simple Corporation: changes in shareholder wealth due to investment

	Cash	Value	Net
If dividend is not paid:			
Old shareholders	−£100 000	+£500 000	+£400 000
New shareholders	0	0	0
If dividend is paid:			
Old shareholders	0	+£400 000	+£400 000
New shareholders	−£100 000	+£100 000	0

The important things to notice about this example, and the things that will be essentially true about all corporate NPVs, are:

1. The total value of the Simple Corporation increases by the NPV of the investment **plus** what it cost to undertake it. The market value of the company increases £500 000, the investment costs £100 000, and the NPV is £400 000.
2. The shareholders of Simple Corporation, when the financial market discovers that the company is going to undertake the investment, experience a wealth increase equal to the investment's NPV. Corporate NPV is defined as the difference between how much the value of the company increases in the market as a result of an investment, and how much its investment costs.
3. The existing, or 'old', shareholders of the company get a wealth increase equal to the investment's NPV regardless of who contributes the money necessary to undertake the investment. We saw that the old shareholders get the same £400 000 wealth increase whether the £100 000 investment cost comes from new equityholders (with whom future dividends must be shared, and who therefore share market value), or from a dividend that would have been paid to the old shareholders but is instead used to make the investment.

Note in Table 2.1 that the amounts listed are the **changes** that will occur when the investment is accepted. This concentration upon only changes induced by the decision is a very important attribute of investment analyses. One manifestation in Table 2.1 of this reliance upon only changes that are caused by the investment is the indication that there is no change in the cash position of the old shareholders if the dividend is paid to them (see the 0 in the cash column in that part of the table). One might be tempted to argue that a +£100 000 amount is appropriate there since that is the amount of cash that the shareholders will receive as the dividend. Remember, however, that in the absence of the investment, the old shareholders would have received that dividend anyway. Thus if they continue to receive the dividend, there is **no change** in their cash flow as a result of the investment. It is only when the dividend is used instead to finance the investment that there is a change in shareholder cash, and that is the −£100 000 in the upper half of the table.

Let us try another example to see how well we have learned the lesson of NPV and shareholder wealth. Suppose that you own one share of common equity in Cheetah Autos Ltd, the manufacturer of a luxury line of cars. Like the Simple Corporation, Cheetah only has equity capital claims outstanding.

Cheetah has evaluated its prospective return to selling a convertible model as having a positive NPV of £3 500 000. The necessary cash outlays associated with assembly-line and other changes associated with the new model will total £10 000 000. The company can either raise that money from new equityholders outside the corporation or use the cash that it was going to pay as a dividend to its old shareholders in this period. Cheetah currently has 1 000 000 shares outstanding, and their market price immediately before the project is announced is £100 apiece. Thus Cheetah before (or 'without') the investment is worth £100 000 000, and the investment will add £13 500 000 to the market value of the company, but will cost £10 000 000, producing an NPV of £3 500 000. If Cheetah undertakes the project and if the market agrees with the opinion of profitability that Cheetah managers hold, the share value of Cheetah will immediately go up by an amount that will produce a total wealth increase of £3 500 000 for the existing shareholders of Cheetah, and thus of £3.50 for you who own one of Cheetah's 1 000 000 shares.

Suppose now that the company decides to raise the money for the project by cancelling this period's dividend. That means you will be losing the £10 per share that you would have received as your dividend this period (the £10 000 000 cost of the project divided by the 1 000 000 shares outstanding). But the market will value Cheetah as being worth £113 500 000, £13 500 000 more than the £100 000 000 it was worth without the project. And each of the 1 000 000 shares will therefore increase in value from £100 to £113.50. Thus if the project is accepted and financed this way, you lose £10 per share in dividends, but gain £13.50 in market value, a net gain of £3.50 per share. With 1 000 000 shares, the £3 500 000 NPV of the project is attained.

Should Cheetah decide to raise the money from new equityholders, the company would sell them £10 000 000 of newly issued shares upon the announcement of the project. All of the shares of the company will at that point have a total worth of £113 500 000, so the new shareholders' £10 000 000 comprises that amount of total value. This means that the original shareholders will own the rest, which is £113 500 000 − £10 000 000 = £103 500 000. With 1 000 000 old shares outstanding, the share price of the old equity is £103.50, so the old equityholders again get a wealth increase of £3.50 per share, which is NPV per share. (Since new equityholders provide the cash for the investment, the old equityholders simply receive the dividend that they would get without the project. Thus though your share price is £10 lower than had you forgone the dividend, your cash in hand is £10 greater, so you are exactly as well off.) Again, regardless of the way the company gets the money to do the project, existing shareholder wealth increases by an amount equal to the project NPV.

Table 2.2 illustrates the attributes of this investment from the shareholder viewpoint, as did Table 2.1 for the Simple Corporation.

Table 2.2 Cheetah Autos Ltd: changes in shareholder wealth due to investment

	Cash	Value	Net
If dividend is not paid:			
Old shareholders	−£10 000 000	+£13 500 000	+£3 500 000
New shareholders	0	0	0
If dividend is paid:			
Old shareholders	0	+£3 500 000	+£3 500 000
New shareholders	−£10 000 000	+£10 000 000	0

The Cheetah example is admittedly slightly more complicated than the one immediately before it, but it is useful because it shows that these ideas can also be used in realistic contexts. To test your understanding of this example, suppose that Cheetah decided to pay the dividend but instead of raising the cash for the project from new shareholders it intended to raise it from the old shareholders. We can assert in these circumstances that the old shareholders would again get the £3 500 000 NPV, but here they would receive the dividend with one hand and with the other immediately pay into the company the same amount of cash, so that their change in cash is −£10 000 000 but they now own the entire value increase of £13 500 000, so that NPV is again £3 500 000. If this assertion troubles you, Table 2.3 may be helpful.

Table 2.3 Cheetah Autos Ltd: changes in shareholder wealth due to investment

	Cash	Value	Net
If dividend is paid, but investment financed by old shareholders:			
Old shareholders	−£10 000 000	+£13 500 000	+£3 500 000
New shareholders	0	0	0

Here the old shareholders would simply hold the shares that new shareholders would have held, and thus have the values of those shares, but would have paid £10 000 000 for them. Again, however, the lesson is the same: the old shareholders get the project's NPV.

2.4 Investment Decisions in Borrowing Corporations

The examples of corporate investment decisions in the previous section dealt with companies that did not borrow money. We shall now see that the general lessons about corporate investment decisions and shareholder wealth that we found for all-equity corporations also hold for companies that, in addition to having equityholders, borrow money from debt capital suppliers.

Suppose that Lynx Autos plc is a (fictitious) corporation identical to Cheetah except that Lynx occasionally borrows money instead of raising all necessary cash for investments from equityholders. Suppose, further, that Lynx faces an investment identical to Cheetah's in that the Lynx investment requires an outlay of £10 000 000 and, if accepted, will increase the total value of the company by £13 500 000 for a corporate NPV of £3 500 000. But here the financing of the investment will be a bit different from how it was with Cheetah: Lynx intends to borrow £5 400 000 of the £10 000 000 necessary for the investment, and reduce its dividend by £4 600 000 to get the rest of the cash.

What will happen to the wealth of the shareholders of Lynx if the company undertakes the investment by borrowing in addition to delaying a dividend? The shareholders will get the same wealth increase regardless. The only requirement is that the new bondholders or creditors of Lynx are as intelligent as were the new equityholders of Cheetah. You remember that Cheetah's new shareholders would give the £10 000 000 to the company only if they got £10 000 000 in share value as compensation. The same idea applies to the new bondholders of Lynx: they will provide the £5 400 000 in cash to the company if they, in return, receive £5 400 000 in bond value from Lynx.

Consequently, when the market value of Lynx increases by £13 500 000 as it undertakes the investment, £5 400 000 of that market value increase is the value of the **new bonds** that Lynx has issued to finance the investment. If the total value increase of the company is £13 500 000, and the company's debt value has increased by £5 400 000, Lynx's equity value must have increased by the difference, £8 100 000. Thus the equityholders of Lynx are richer by the £8 100 000 share value increase, but they have lost £4 600 000 in dividends that they would have had were the project to have been rejected. The net of those two amounts is the change in the wealth of Lynx's equityholders. Of course that wealth increase is equal to £3 500 000, the corporate NPV.

Table 2.4 illustrates these value and NPV results for Lynx. So again, corporate NPV is equal to the change in the wealth of existing shareholders – even if some of the money for the investment comes from creditors instead of equityholders.[2]

[2] There is actually one other important condition for the Lynx example to work correctly, but we did not wish to distract you with it during the discussion. If Lynx has old bonds outstanding, the investment must not be allowed to increase **their** value. The reason is not too hard to see. If £5 400 000 of the £13 500 000 value increase goes to new bondholders and £4 600 000 of equity dividends are invested, any increase at all in the value of old bonds means that there will not be enough corporate value left to generate £3 500 000 of NPV for the equity holders of Lynx. How could an investment cause the value of old borrowings of the company to increase? Lots of ways: for example, if the investment made Lynx a less risky company as a whole, the discount rates on the old bonds would decline and the market prices of the old bonds would increase.

Table 2.4 Lynx Autos plc: changes in shareholder wealth due to investment

	Cash	Value	Net
New bondholders	−£5 400 000	+£5 400 000	0
Old shareholders	−£4 600 000	+£8 100 000	+£3 500 000

Suppose now that Lynx borrowed the money as planned, but raised the remaining £4 600 000 necessary for the investment by issuing new shares instead of reducing its dividend. You should be able to work out that the old shareholders will still gain the £3 500 000 NPV because the total value of the company would increase by £13 500 000, of which £5 400 000 would belong to new bondholders, £4 600 000 to new shareholders, and thus £3 500 000 to old shareholders, who themselves put up none of the cash for the investment. Thus their wealth has increased by the full £3 500 000 NPV of the investment.

So the Cheetah and Lynx examples have shown us that equityholders get the NPV of an investment regardless of the source of the money to undertake the investment, be it forgone dividends, the selling of new shares, or the borrowing of money from bondholders. The economics is easy to see: an investment will generate £X of market value, and costs £ Y. If we raise £Y to undertake the investment, the new capital suppliers will get £Y of the £ X value increase, leaving £X − £ Y = NPV for the equityholders of the company. Why do the equityholders get to keep the difference? Because, as we saw above, they are the **residual** capital claimants of the company; they get to keep whatever is left after all other claimants' contracts have been honoured.

The lessons of Module 2 to this point are important ones because they allow us to connect the actions of the company to the wealth of its shareholders. We have found that:

1. The corporation is an organisation that has many potential claimants against its future cash flows, all of these claimants being protected with contracts that specify timing and amounts of money to be paid, except for the claimants called equity or shareholders. These equity capital suppliers have delegated their ownership authority in the company to the managers and board of directors, retaining only a residual claim and the capacity to vote for the directors. A residual claim is simply the right to claim ownership of the resources of the company after all other claims – capital and otherwise – have been satisfied.
2. Shareholders in a corporation usually have limited liability. This means that the shareholders are at risk only for the value of their holdings in the company, and even though they are owners in one sense, they are not personally liable for the actions of the corporation.
3. When a company undertakes an investment, the investment will have an NPV that can be calculated with a technique very similar to the one we used in Module 1. Because of the residual nature of their claim, the shareholders of the company will experience the corporate investment NPV as their increase in wealth. Shareholders, being residual claimants, get nothing until the other capital claimants' contracts are fulfilled. But after those contracts are fulfilled, share-

holders get everything. A corporate investment NPV is the present value of the amounts that are left over after all such contracts associated with the investment have been fulfilled, and that NPV is thus the increase in wealth of the company's shareholders.

As a final note, you may have noticed we illustrated the investment analyses and examples in this section with NPV instead of IRR techniques. Our reason for doing that was not that we have forgotten about IRR; the simple truth is that the connections between investment decisions and shareholder wealth are easier to demonstrate with NPV than with IRR. At this point we stand on the conclusion that we can infer from our discussions in Module 1: a properly performed IRR analysis will usually produce the same answers as a properly performed NPV analysis, and so a company correctly using IRR to evaluate and accept an investment will produce a wealth increase for its shareholders identical to that in a correctly formulated NPV analysis.

2.5 Share Values and Price/Earnings Ratios

The lessons about equity market value that we learned in the previous section can be used conveniently here to clear up a few points of common misunderstanding about share prices and what determines them. In particular, whenever conversation drifts to the stock market and discussion centres upon a company, one often hears what its 'multiplier' is. The multiplier of a company's shares is the ratio of its market value to its earnings, usually on a per-share basis. In other words, if Cheetah Auto Ltd's earnings for this year are expected to be £10 per share, and its shares are selling for £100, it would be said to have a multiplier or **price/earnings (P/E) ratio** of 10. In this section we shall investigate this measure not only because it is so widely used, but also because we can learn a good bit more about how the market places values on companies by studying this P/E ratio.

In conversations about companies and their shares, great attention is often lavished upon companies' P/E ratios. The ratios are variously offered as signals that the market is providing as to the companies' future earning prospects, growth rates, riskinesses, and any number of other attributes. A high P/E ratio is often taken as a signal of the market's high opinion of a company's shares, and also as a sign that the company's share prices may be too high. So what is the P/E ratio really telling us about a company?

The obvious thing about the P/E ratio is that it seems to be performing a function very much like our discounting of future dividends to find the market price of a company's shares. Is multiplying a company's earnings by its P/E ratio the same thing as discounting its future dividends? The answer is both yes and no.

Let us return to Cheetah Autos Ltd. As we saw in the previous section, Cheetah's market value (before the investment) is £100 per share, which we can take to be the discounted present value of a perpetual stream of future cash dividends of £10. If we assume that Cheetah's earnings are also £10 per share, its earnings and its dividends are the same. This implies that Cheetah is expected to pay out all of its earnings as dividends. Using the Module 1 constant perpetuity formula for varying Cheetah on a per-share basis, and re once again as the equity discount rate, we have:

$$\begin{aligned}\text{Price per share} &= \frac{\text{Dividend per share}}{re}\\ &= \frac{£10}{0.10}\\ &= £100\end{aligned}$$

If dividends are equal to earnings, we can manipulate these a bit to see that:

$$\begin{aligned}\text{Price per share} &= \frac{\text{Earnings per share}}{re}\\ \frac{\text{Price per share}}{\text{Earnings per share}} &= \frac{1}{re}\end{aligned}$$

For Cheetah, the P/E ratio is the reciprocal of the equity discount rate. So there is some type of relationship between the P/E ratio and the discounting procedure that the market is using. But so far we have found this relationship only for companies like Cheetah, that are expected to pay out all of earnings as dividends and produce a constant, perpetual stream of future dividends. What if we are interested in a company that does not fit this mould? What does its P/E ratio tell us? Does it bear a nice relationship to the company's equity discount rate or not?

We can begin to answer these questions by supposing that conversation turns to Ocelot Autos Ltd, a manufacturer of a broad range of medium-priced cars and trucks. Ocelot is expected also to earn £10 per share this year, and has, like Cheetah, a market price of £100 per share. Ocelot, however, is going to pay out only £7 of its earnings in dividends. Ocelot thus has the same P/E ratio as Cheetah (Ocelot's earnings and market price are the same as Cheetah's), but the market must be valuing a different stream of future dividends for Ocelot, because it is paying out only £7 per share in contrast to Cheetah's £10 per share. Suppose that the market expects Ocelot's future dividends to grow at a constant rate of 3 per cent per period for ever, and also considers Ocelot to have the same risk as Cheetah (so the same value of re would apply). Remembering our lesson on growth perpetuities from Module 1, we can state that Ocelot's market valuation is being generated thus:

$$\begin{aligned}\text{Price per share} &= \frac{\text{Dividend per share}}{(re - g)}\\ &= \frac{£7}{(0.10 - 0.03)}\\ &= £100\end{aligned}$$

What about the relationship between Ocelot's P/E ratio and its discount rate? Since the discount rate applies only to dividends, and the P/E ratio to earnings, and here the two are not the same, we must first connect earnings and dividends in some manner. The way we usually do this in finance is by something called the **payout ratio**. The payout ratio is simply the proportion of a company's earnings that it pays as dividends. If Ocelot is expected to pay a constant 70 per cent of its earnings as dividends, we can find the relationship between the price/earnings ratio and the discount rate by remembering that dividends are the result of multiplying the earnings by the payout ratio:

$$\begin{aligned}\text{Price per share} &= \frac{\text{Dividend per share}}{(re-g)}\\ &= \frac{\text{Earnings per share} \times \text{Payout ratio}}{(re-g)}\\ \frac{\text{Price per share}}{\text{Earnings per share}} &= \frac{1}{(re-g)} \times \text{Payout ratio}\end{aligned}$$

As the above formula shows, the interpretation of Ocelot's P/E ratio is not as simple as the one for Cheetah. The P/E ratio here is influenced not only by Ocelot's discount rate, but also by the rate of growth expected for the company's future dividends and the proportion of its earnings that it will pay as dividends. (Of course Cheetah's P/E ratio is subject to the same general relationship, and happens to be reflecting an expectation of a 100 per cent payout ratio and a dividend growth rate of 0 per cent.)

Though Cheetah and Ocelot have the same initial earnings (£10), the same market price per share (£100) and thus the same P/E ratio, they are quite different in terms of the stream of dividends, associated rates of growth in dividends and earnings, and the extent to which they pay out earnings as dividends. Obviously, one could make a significant mistake about the implied discount rates, rates of growth in income or dividends, or payout ratios of these two companies by examining only their price/earnings ratios.

Even so, Cheetah and Ocelot are fairly similar companies compared with those we might pick in a random draw among all companies. They are both perpetuities, and they are equally risky, and thus have the same equity discount rates.

Suppose that the conversation now turned to the shares of Win-or-Lose Mining Ltd. Win-or-Lose is expected to last for only a few more years, at which time its ore will have been exhausted. Win-or-Lose is also substantially more risky than Cheetah or Ocelot, such that the market requires a 20 per cent per period discount rate on Win-or-Lose's equity. Suppose that Win-or-Lose was expected to pay all of its earnings as dividends of £10, £107, and £30 per share for the next three periods, and then disappear. The market's valuation of Win-or-Lose is thus:

$$\text{Price per share} = \frac{£10}{(1.20)} + \frac{£107}{(1.20)^2} + \frac{£30}{(1.20)^3} = £100$$

Win-or-Lose has a market price of £100 per share, the same as Cheetah and Ocelot, and Win-or-Lose also has the same P/E ratio, 10, as the two car companies. But Win-or-Lose otherwise bears almost no similarity to the others. It is more risky, it is expected to last only a few more periods, and it is a mining company rather than a car manufacturer. Its P/E ratio has no simple interpretation and is a very strange function of the discount rate, the number of periods in the remaining life of the company, and the pattern of dividends that the market is expecting Win-or-Lose to pay.

What are we to make of all this? The lesson is one of caution in using or attaching much importance to the P/E ratio of a company. If the company is a constant perpetuity, the interpretation of its P/E ratio is straightforward. But how many companies do we encounter that are expected to pay the same dividend each year for ever? Precious few. We do not see many constant growth perpetuity companies either.

Does this mean that we can find no use for the P/E ratio of a company? No, there are some uses to which that number can be put. For example, suppose that a question arises as to which of two companies' shares are riskier, and you are reasonably sure that the companies are very similar in all other attributes. Take the Lose-or-Win Mining Co., for instance. If it has the same dividend expectations as Win-or-Lose but a P/E ratio of 5 instead of 10, that tells us that the market is assigning a much higher discount rate to Lose-or-Win, and thus the company must be more risky than Win-or-Lose. Or if Leopard Motors was seemingly identical to Cheetah in risk and length of dividend stream and paid the same initial dividend as Cheetah, but had a lower P/E ratio, the implication is that Leopard's dividends are expected to decline in the future.

The basic difficulty with the P/E ratio is that one of its components, the market price of a company's shares, is the net result of a great number of factors, only one of which is the initial period's earnings that appear in the numerator of the ratio. Thus, unless we can feel comfortable that all but one of the factors influencing market price are reasonably similar among companies being compared, the P/E ratio can give little conclusive or useful information about the market's relative opinions about the prospects for such companies.

To summarise what we have learned about P/E ratios:

1. The P/E ratio is nothing more nor less than the ratio between the present value of all the company's future dividends (its market price) and its expected earnings during the first period.
2. A company's P/E ratio is a very complex number in terms of the information that can influence it. It is affected by the pattern of dividends that a company pays, its payout ratio, the riskiness of the company as evidenced by the discount rate of its equity, and the stream of earnings that the company is expected to be able to generate across the future.
3. For certain companies with very simple cash-flow patterns (such as constant or constant growth perpetuities), we can derive a specific relationship between P/E ratios and equity discount rates. But for most companies the relationship is much too complex to make numerical estimation worthwhile.
4. If we are careful to limit the comparisons that we make (for example, staying within the same industry so that the risk differences are minimised), we may be able to examine P/E ratios so as to gain some qualitative information as to the

market's opinion about the companies whose ratios we examine. For example, we might infer that future growth rates expected for a company's dividends and earnings are above average if its P/E ratio is higher than companies in the same industry (which companies therefore have the same risk and perhaps similar payout ratios).

Learning Summary

This module has introduced you to the company investment decision process in the following ways:

1. It has presented the general economic and legal basis for the corporate form of organisation, the types of capital claims (debt and equity) which these entities can issue, and some of the important characteristics of these claims.
2. It has shown that the prime purpose for financial decisions is the wealth of currently existing shareholders, measured as the market value of their equity claims.
3. It has argued that the best way to think of the market value of equity is as the discounted value of all future dividends that current shareholders can expect to receive.
4. It has demonstrated that calculating the NPV of an investment project is a good way to estimate the effect upon shareholder wealth of the company accepting the investment, regardless of the source of the funding for the investment.
5. It has discussed the weaknesses and strengths of the price/earnings ratio as an indicator of the capital market's opinion about the prospects for a company's future performance.

These ideas are important groundwork for the more applied investment analyses of company projects that we shall present in following modules, and so the ideas should be studied carefully. You will find that the review questions that follow are a significant aid in your developing a good understanding of Module 2.

Review Questions

2.1 Financial decisions of companies should attempt to maximise the wealth of shareholders because of which of the following?

A. All other claimants are protected by formal or informal contracts requiring specific performance by the company.
B. Company financial managers usually own shares themselves, and would thereby make themselves wealthy.
C. It is a way for companies to avoid their social responsibilities of looking to the interests of consumers, the poor and the environment.
D. They are constrained to do so by the equity contract.

2.2 You are considering the purchase of a common share of Perpetual Payments plc which is expected to continue to pay dividends for the indeterminate future. Those dividends are expected to be £10 per share at the (end of the) first year, and you are convinced that you will be able to sell the shares at that time for £210. If the company is expected to pay a steadily increasing dividend each year, and its equity discount rate, re, is 10 per cent, what rate of annual increase are you imputing to the dividends of Perpetual Payments?

A. 10%.
B. No particular rate, simply the capacity to sell the shares at that price.
C. 5%.
D. The question is impossible to answer with the information given.

2.3 Suppose that Consumer Products SA is considering the introduction of a new line of floor polish. The project will cost the company £2 000 000 in the first year, but is expected to increase the total value of all of its capital claims by £3 000 000 at the same time, as a result of the market's evaluation of the alterations thereby caused in the future cash flows of the company. Then:

I. The NPV of the project is £1 000 000 only if the project is financed totally by a dividend reduction.
II. The NPV of the project is £1 000 000 if the project is financed either with new equity or a dividend reduction.
III. The NPV of the project is £1 000 000 regardless of how it is financed, even with debt in whole or part.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both II and III.

2.4 'I notice that the market must have a very good opinion of our future prospects,' says your boss, glancing at the Stock Exchange quotations one morning. 'Our price/earnings ratio was 10 yesterday, whereas the average for all companies on the exchange was only 7. We must be doing something right – right?'

The correct response to the question is:

A. 'Yes, because the ratio implies that our rate of increase in future cash flow is above average.'
B. 'No, because a higher ratio implies that the discount rate applicable to our future cash flows is higher and thus riskier than the average.'
C. 'Yes, because the higher ratio implies that the market thinks we are either less risky or have higher increases expected for cash flow.'
D. 'No, because such comparisons must be made with companies in essentially the same line of business with ourselves, not market averages.'

Module 3

Earnings, Profit and Cash Flow

Contents

Learning Objectives

Modules 1 and 2 have demonstrated that value calculations are the most basic and important of processes in financial decision making, and that cash-flow expectations are the most vital part of such value calculations. The corporate form of organisation presents complexities in estimation of cash flows for valuation processes. This module instructs the student in the relationships between accounting measures of performance (such as 'profits') and the financial cash flows necessary for valuation exercises. Students learn here also the correct treatment and interpretation of certain non-cash accounting numbers (such as 'depreciation') and the effect of such numbers on company cash flows. The module contains several examples of the correct interpretation of financial statement results of companies investigating important investment decisions. Again, the context of these discussions is that for company investment and financing decisions.

3.1 Introduction

We now return to the main concern, the investment-decision process of companies. We have thus far learned certain things about this process:

1. Corporations should attempt to maximise the wealth of their existing shareholders.
2. Corporate investment NPV is the change that occurs in the wealth of existing shareholders upon the corporation accepting an investment.
3. Corporate investment NPV is the difference between the increase in the market value of all capital claims of the corporation (both equity and debt) minus the present cost of the investment.

This section begins our more detailed study of exactly how corporations should calculate investment NPVs (and IRRs) so as to make them consistent with these points. And learning point (3) immediately above is an excellent clue to the proper manner of calculating an NPV for a company's investment. We must first calculate the **total corporate value change** (defined as the increase or decrease in the market value of all of a corporation's capital claims) that would take place if the investment were accepted. Next we subtract the cost of the investment. The result is the investment NPV.

How do we arrive at the total value change in the capital claims of the corporation? However we do it, we must be consistent with the principles of value in financial markets that we learned about in Module 1. You recall we established there that market value is the discounted value of all future cash flows expected to be paid to the security. The idea that the market value of a company's equity is the present value of its future dividends is merely an application of that principle.

The clear implication is that in order to find the changes that will occur in the market values of all capital claims of a corporation as a result of an investment, we must discount to the present all of the investment-induced changes in the cash that capital suppliers can expect to get from the corporation in the future. In this module we shall discuss and illustrate the cash flows that should be of interest in the corporate investment decision.

3.2 Corporate Cash Flows

Figure 3.1 illustrates one view of the corporation and its cash flows, and this from the perspective of an outsider not intimately acquainted with the internal operations of the corporation. Actually, the company is shown as the proverbial 'black box' (although white in our figure!) because at this point we are less interested in the internal mechanisms of the company and more interested in the transactions that the company undertakes with outside entities.

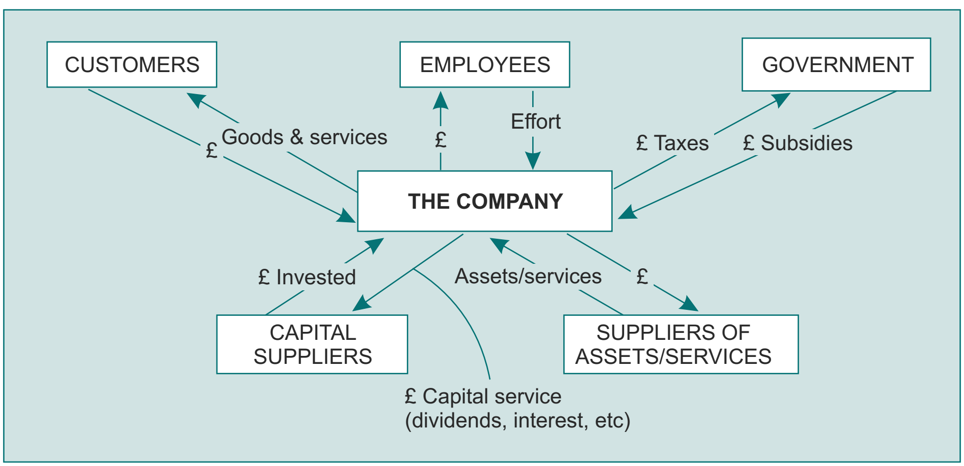

Figure 3.1 **Cash flows between a corporation and others**

Figure 3.1 is an attempt to display all of the important outside cash transactions which a corporation regularly undertakes. Represented are the **customers**, who pay cash into the corporation in return for goods or services provided by the company; **employees**, who are paid for the effort they sell to the firm; the **government**, which is paid taxes and renders subsidies in the opposite direction; the **suppliers of**

assets and services to the corporation (i.e. those who sell the raw materials, machines, land, expert advice, etc., which the company uses in its activities); and the **capital suppliers** of the company who, as we know, give cash to the company in exchange for dividends, interest and other future cash distributions.

You might question why we go to the trouble to show all of these other entities when we should be interested only in the cash flows to the capital suppliers of the corporation. The answer is that capital-supplier cash flows are residual to the other groups. We would know very little about capital-supplier cash flows were we to ignore those to whom the debt and equityholders must defer when it comes time to distribute cash.

At this point the corporation should appear in your mind as an entity accepting assets and services from employees and other vendors and paying for those with cash, somehow or other operating those assets to produce goods or services that it provides to customers who in turn pay for them with cash, paying taxes and receiving subsidies from government, and raising cash from capital suppliers to acquire productive capacity while servicing those capital claims with cash interest payments, principal payments and dividends. The corporation relies upon the capital suppliers to provide the cash necessary for it to accomplish its productive plans, such as investment in assets to expand, enter a new line of business, or even simply to keep its head above water for one more period.

'Very nice,' you say. 'Figure 3.1 seems to have covered pretty well all of a company's transactions affecting the net amounts of cash that capital suppliers might expect to get. But I do have a significant problem with your description of these transactions being done for **cash**,' you complain. 'Take, for example, the company buying raw materials for something it intends to produce. In my experience, companies rarely or never actually pay cash when they get those materials. The usual practice is to take some time to pay. And customers! Good heavens, in my company we wait for what seems interminable periods to get cash for what we sell. Your model of the company seems to be something less than realistic in that dimension.'

It is true that companies rarely pay cash 'on the spot' for the assets or services that they buy, and customers often take some time to pay their bills. But that is not a problem with the model of the corporation in Figure 3.1. Keep in mind that these transactions are an ongoing process for corporations, and are continuously taking place. Figure 3.1 should be taken to represent **activity across time** rather than at a single time point.

Let us see how this works in terms of the concerns raised in your question. You argue that something the company sells right now does not result in a cash payment from a customer until next period. Fine. But remember that customers will be paying **this** period for some of the things they bought **last** period. Similarly, although the company may not be paying this period for its purchases this period, it almost certainly will be expecting to be paying for what it bought earlier. Figure 3.1 makes no claim to be associating at a single time point particular cash flows with particular services or assets provided. It simply says that companies receive cash from, and pay cash out to, various groups during each period. And since our basic concern is how much cash capital suppliers can expect to get across time, associations between particular flows in and out are at this point irrelevant.

'I still do not feel comfortable with that explanation of corporate cash flows,' you declare. 'I know for a fact that companies go to a lot of trouble to make exactly the kind of associations between what they sell, who bought it, and when it is paid for, that you say are irrelevant. Companies even have an asset called "accounts receivable" or "debtors" that appears in their books to record just such things. Are you saying that companies are wasting their efforts in keeping track of such things?' Not at all. (As a matter of fact, companies even have an account for the services and assets they have received but not yet paid for, called 'creditors' or 'accounts payable'.) One of the very important managerial tasks of the company is to ensure efficient handling of what it is owed and what it owes. Actually, the very information in those accounts is one of the important inputs to estimating the amounts of cash that a corporation can expect to pay and be paid across time.

But it would be wrong to record a **cash** transaction as taking place before the cash was actually expected to change hands. Suppose that Cheetah Autos Ltd recorded a cash inflow every time it sold a car to a dealer, even though the dealers took a few time periods to remit the cash. If Cheetah tried to use that 'cash' to pay interest or dividends to its capital suppliers it would likely soon run into trouble. Similarly, if Cheetah recorded cash as having been paid whenever it bought a machine or raw materials, even though the company was not expected to pay the cash for some time, there would likely be money lying unspent that could have been used to service capital claims (and increase the wealth of equity) that was not. So financial cash flows are the **cash amounts** that are expected to occur at the times for which the expectations are recorded. They are not to be confused with the **accounting numbers** that are generated for quite different purposes.

It is now time for us to construct an example that illustrates these points about corporate cash flows. Let us return to Cheetah Autos Ltd. Recall that in our discussions of that company in Module 2 we mentioned an investment consisting of Cheetah's return to selling a convertible model. The investment cash outlay was to be £10 000 000 and the investment had an NPV of £3 500 000. We shall now take a closer look at that investment, paying particular attention to the cash flows that go to make up its NPV.

Suppose that you are the managing director of Cheetah, and you ask to see more detail from the finance department about the convertible project. In particular, you have just seen Figure 3.1, and therefore ask that the data be presented in a format consistent with that view of the corporation. After a bit of computer reprogramming, the finance department sends you the information in Table 3.1.

Table 3.1 **Cheetah Autos Ltd: cash flows (£000s) for the convertible project**

	Now	Yr 1	Yr 2	Yr 3
Customers	0	+£17 500	+£23 500	+£4 000
Operations	0	−£7 000	−£3 830	−£5 200
Assets	−£10 000	−£4 000	−£2 000	0
Government	0	−£4 000	−£8 085	+£5 600
Capital*	−£10 000	+£2 500	+£9 585	+£4 400

* Free cash flow

$$\begin{aligned}\text{NPV} &= -\$10\,000\,000 + \frac{\$2\,500\,000}{(1.10)} + \frac{\$9\,585\,000}{(1.10)^2} + \frac{\$4\,400\,000}{(1.10)^3}\\ &= +\$3\,500\,000\end{aligned}$$

'Very nice,' you say. 'But I have a few questions about the numbers here. The first line says "Customers", is that what I usually call "Sales" or "Revenues"?'

The finance department responds, 'No, it is not. The amounts you see recorded in the first line are the actual amounts of cash that we expect to take in from the sales of cars in the first two years, and the third year includes both cash receipts from car sales and also the selling of some of the machinery that would at that time no longer be useful. The "Revenue" figures that our accountants will record for tax and reporting purposes are a bit different because they recognise the sale of a car when we ship it to a dealer, even though it has not yet been paid for.'

'OK,' you agree, 'I remember having earlier heard something about that. How about the duration of the cash flows across time? Why do they stop at the third year?'

'Well, Sir,' the analysts say, 'Our marketing department tells us that the convertible will be quite popular when we first introduce it, but because it is a speciality car its appeal will be quite finite, and by the end of the third year its market will have all but disappeared. At that point we are planning to get into something else. That is another reason why the third year's cash flow includes some money from the sale of machines in addition to cars.'

'Let's move on to the line called "Operations". Can I assume that the cash outlays listed there are the expenses that the company will incur in the production and sale of cars?' you enquire.

'In a sense that is true,' reply your finance people. 'But you must be careful about the word "expenses". That term has a very specific meaning to accountants and tax experts. What we call "Operations" cash outlays is not exactly the same thing as what the accountants call "expenses". Actually, everything in our "Operations" line would also be an expense to an accountant. There we record employee costs, fuel and electricity, and so forth. Our rule for "Operations" cash flows is that they be:

1. paid in cash that year, and
2. deductible for taxes that year, and

3. not a payment to a capital supplier.

This means that everything in "Operations" would also be an "expense", but not all "expenses" would appear in our "Operations" line. For example, when we buy the £10 000 000 of assets to undertake the investment we will be allowed for tax purposes to reduce our taxable income in the future by something called "Depreciation" on those assets. Depreciation is not a cash flow, so we do not include it in our "Operations" line. But accountants and tax people regard depreciation as an expense, for reasons of their own, which are irrelevant to finance. Another example is interest payments. Interest is, of course, a cash flow, and it is an accounting "expense" deductible for tax purposes. But it is also a payment to a capital supplier. So by our definition of cash for "Operations", interest does not fit there.'

'Interesting,' you observe. 'But something about this bothers me. Take depreciation, for example. I agree that depreciation is not a cash flow and does not therefore belong in "Operations" cash, given your definition. What causes me difficulty is the statement that depreciation is irrelevant. If depreciation is an expense for tax purposes, it must affect the cash flow that you list as "Government", which I take to be taxes. How can you say that depreciation is irrelevant from that viewpoint?'

'We did not mean to give you the impression that depreciation is irrelevant; it certainly is not,' the analysts answer. 'What we said is that the **reasons** accountants and tax people treat depreciation as an expense are irrelevant to us. Depreciation has very much affected the cash flow we listed as "Government" (which you are correct in interpreting as the taxes that Cheetah will be paying, due to this investment). We did not list the details of our tax calculations because we understood that you wanted to see the net cash flows between Cheetah and these other entities with whom we transact.'

'Alright,' you concede, 'I shall have to think about that. Maybe it would help if you explained why you use the particular definition of "Operations" cash that you do. I can understand your insistence upon cash alone (because of cash flows being our concern in NPV analyses), which means that expenses like depreciation (not being cash flows) are omitted. Why do you require that a cash flow, in order to be in "Operations", be deductible in that period; and why do you exclude cash payments (such as interest) to capital suppliers?'

'Those are straightforward to answer,' say your financial experts. 'We require that "Operations" cash flows be deductible in that period to distinguish them from the report's next cash flow, "Assets". Actually both the "Operations" and "Assets" cash flows are outlays that Cheetah will make across time because of the convertible project. The "Operations" cash flow is deductible in the period under which it is listed. The "Assets" cash flow, while made at the time listed, is not deductible at that time point, but must be "capitalised" (as the accountants say) and depreciated across time. So our taxes are reduced **this** period by the "Operations" cash flow (which are expenses), and in the **future** by the "Assets" cash flow through its depreciation charges across time. Both the "Operations" and "Assets" cash outlays occur at the same times, but the tax laws treat them differently and so we do too. If that bothers you, feel free to add them together. The final answer will be the same, as long as the taxes in "Government" have been figured correctly.

'To answer your second question, the reason we do not include interest payments in "Operations" cash flow is because interest is part of what we are trying to show as a net result: the amounts of cash that capital suppliers can extract without disturbing Cheetah's plans. If we were to include interest in "Operations" cash flow, it would be subtracted from the total but in fact should be part of it. (As you are aware, Cheetah currently has no borrowings, and we plan none for this project, so we actually had no interest amounts to worry about anyway. But if we had, they would not have been part of our "Operations" cash flow.)'

'Tell me about the line called "Capital",' you ask. 'It is obviously the net total of the lines above it for each time point, but what is its meaning beyond that?'

'The "Capital" line is the amounts of cash that could be taken out of the corporation by its capital suppliers and still have the convertible project run as planned,' a finance analyst responds. 'It is the amounts of cash that are expected to be left over after all necessary cash amounts have been paid. Note that it is negative at the first time point when the big cash outlay for "Assets" is made, and then becomes positive in future years. The negative £10 000 000 is the recognition that we plan to cut our equity dividend by that amount at the point we begin the project, to pay for the initial "Assets". Because we are financed only by equity at Cheetah, you can regard the "Capital" cash flows in future years as the **increases** in dividends that we expect the project to allow.'

'I think I am beginning to understand,' you say. 'Am I correct in interpreting the NPV you have calculated in the report as the present value of all increases and decreases in dividends that we expect to happen if we accept the convertible project?'

'Exactly!' exclaim your finance people. 'You can see that this is a good measure of how that project will affect our equityholders' wealth, because they will lose the £10 000 000 dividend this period but at the same time their share values will increase by the present value of the amounts that their **future** dividends are expected to increase. The net of those two things is the investment's NPV. The only caution that we would give about this interpretation is that it works only for companies like Cheetah, which are financed totally with equity and have issued no debt capital. For companies with debt in their capital structure a similar, but not identical, explanation of the genesis of NPV applies.'

'It is nice to hear that I finally got something right,' you declare. 'Now if you tell me that the discount rate of 10 per cent is our estimate of the rate of return that would be required on the stream of "Capital" cash flows due to their risk, I shall be doubly pleased.'

'Correct again!' the finance people congratulate. 'As a matter of fact you can see how we estimated that rate by reading Section 2.5 of Module 2 entitled "Share Values and Price/Earnings Ratios" in a certain finance text that we shall provide you. We figured the convertible project to have about the same risk as Cheetah in general, so we used the rate that would apply to the company's capital as a whole.'

'I am feeling a lot better about what goes on in the finance department,' you offer. 'Incidentally, what is that asterisk doing on the word "Capital" in the report?'

'Well,' an analyst replies, 'you can see that it refers to a term entitled "**Free cash flow**" as being synonymous with the "Capital" cash flow as we have defined and calculated it. We put that reference there because it has a great deal of currency right now, having been "discovered" by the investment bankers and security analysts who make recommendations about companies' share values. They think free cash flow is some kind of new idea they have invented, but in fact we have been using that calculation, without that name, for years. It is, as I have just said, the **amount of cash that can be taken by capital suppliers from the company as a result of the investment while leaving all of the plans of the company unchanged**. "Free" in that sense means "unfettered" rather than "costless"; the free cash-flow amounts do cost money (specifically, the negative free cash flow in the first period is the "cost" of the positive ones that follow). The "free cash flow" term should stand you in good stead in your many cocktail conversations with other managing directors, and with the investment bankers, analysts and others who are always seeking information about Cheetah. It stands **us** in good stead as the basis for calculating NPVs of investment projects.'

'Thank you,' you congratulate. 'You have helped me understand the convertible project, and corporate investment analysis in general, much more deeply. I have one more request. Would you send me another report on the convertible project, this one arriving at the same free cash-flow figures but with the detail of taxes, depreciation and so forth? As long as I am being educated in finance, I suppose I should be thorough.'

'Right away, Sir,' say your finance people as, with relief, they begin to exit.

'Oh, and by the way,' you catch them, 'come up to my office about an hour after you send the reports so that we can discuss them.'

'We shall be delighted to,' they lie.

3.3 Cash Flows and Profits

The next day the reports given in Table 3.2 and Table 3.3 arrive on your desk from the finance department as they respond to your request for more information on the convertible project. Of course, the format of the income statements for the convertible project is more familiar to you than the cash-flow statements that the finance group provided earlier. Expenses are subtracted from revenues, taxes are calculated and subtracted, and the result is the ubiquitous 'bottom line' or profit after tax.

Table 3.2 Cheetah Autos Ltd: income statements (£000s) for the convertible project

	Yr 1	Yr 2	Yr 3
Revenues	£18 000	£25 000	£2 000
Expenses:			
Operations	7 000	3 830	5 200
Depreciation	3 000	5 000	8 000
Interest	0	0	0
Total expenses	10 000	8 830	13 200
Profit before taxes	8 000	16 170	−11 200
Taxes (50%)	4 000	8 085	−5 600
Profit after taxes	4 000	8 085	−5 600

Table 3.3 Cheetah Autos Ltd: depreciation schedules (£000s) for the convertible project

	Yr 1	Yr 2	Yr 3
Due to asset outlay of:			
£10 000 Yr 1	£3 000	£3 000	£4 000
£4 000 Yr 2		£2 000	£2 000
£2 000 Yr 3			£2 000
Total depreciation expense	£3 000	£5 000	£8 000

To see where the items in the cash-flow statement came from, you begin comparing the items in the income and cash-flow statements. First, the differences between the Revenue (Table 3.2) and Customer (Table 3.1) lines catch your attention. The Revenues in the first two years exceed the reported cash inflows from customers in those years, but the opposite happens in the last year. Further, the total of all Revenues equals the total of all Customer cash inflows. What you figure out is this: in the first year, Cheetah expects to deliver £18 000 000 of cars to dealers but to be paid for only £17 500 000 of the cars. The following year Cheetah sees itself selling £25 000 000 of cars, and getting £23 500 000 of cash. Some of that cash will be coming from the dealers who bought cars in the first year but are paying the cash in year 2. Finally, in year 3, Cheetah sells £1 000 000 of cars, collects that plus the £2 000 000 from prior-year sales that is still owed, and sells the assets of the convertible project for £1 000 000, collecting the cash on the spot. For Cheetah, the Revenue figures are not an inaccurate representation of the **total** amounts of cash that the company will receive from the project (they would be inaccurate if, for example, some dealers never did pay for their cars), but the Revenues do give a false picture of the **timings** of the cash coming in from customers. And cash timing is important to present values, and therefore to the present wealth of equityholders, as you know.

Total expenses exceed total cash outlays for Operations by the amounts of Depreciation expense, and they would also do so by the amount of interest if any were expected to be paid. Total expenses are thus not themselves a cash measure, though they do include some items of operating cash flow (those that are deductible).

Revenues and expenses are therefore different from cash flows, but by no means are these accounting numbers unimportant. The source of their importance is that Government uses the difference between them, called 'taxable income' or 'profit before tax' to calculate the amount of tax that Cheetah must pay. And tax, of course, **is** a cash flow. Thus you notice that the Government cash flow was figured by your finance people as Cheetah's corporate income tax rate (50 per cent) multiplied by the amounts of profit before tax on the income statements for the project. Profit after tax is simply the difference between profit before tax and the tax amount.

Table 3.3 of depreciation calculations supplied by finance for the project next draws your attention. You notice that the sum of all the depreciation expenses for each asset is equal to the original cash outlay for the asset, as your discussion with the analysts had predicted. A fast telephone conversation with the accounting department informs you that for assets like these, Cheetah depreciates 'straight line' with 'salvage value' if applicable. This means that the original cash outlay is written off as a depreciation expense in even amounts over the remaining life of the project, except for the expected amounts that the asset may be sold for at the end of project life. For the assets bought first, the original £10 000 000 is split into £9 000 000 of 'depreciable value' and £1 000 000 of 'salvage value'. The £9 000 000 is written off at £3 000 000 per year for three years, and the year-3 depreciation expense includes that amount plus the writing-off of salvage value when the asset is expected to be sold. The depreciation expenses for the other assets are easy to figure because they have no salvage value. And the total depreciation expenses in each year are simply the sum of those for each asset during that year.

Just then the finance group shows up for its meeting with you.

'Welcome,' you say. 'This should be brief because, believe it or not, I seem to understand almost everything about the genesis of the cash-flow figures for the convertible project. I do have just a couple of questions, however. First, I notice that in year 3 we are showing negative profits but positive cash flow. Now I can understand how such a thing can happen if, as you predict, some of our customers are not paying until that year even though the Revenue was recorded earlier; and I do understand about Depreciation not being a cash flow even though it reduces profits. But can you explain to me how we can convince the government to give us £5 600 000 as **negative taxes** that year? I can see how the number is calculated, as 50 per cent of the negative "profit before taxes" number. But it is one thing to have a number on a piece of paper, and quite another to get £5 600 000 in cash from the government.'

'The answer to that question points out one of the most important characteristics of investment project cash flows,' respond the finance people. 'When we estimate the cash flows of a proposal, we seek to include **all of the changes** that will take place in the cash flows of the corporation **as a whole**. In this situation the cash flow that we call "Government" is therefore the **changes** that we expect will occur in the taxes that Cheetah will pay, should we accept the convertible project. The correct interpretation of the negative tax amount in year 3 is that the **company as a whole** will pay £5 600 000 **less** in corporate income taxes when the £11 200 000 negative income number from the convertible project is **combined with the other income** amounts that we expect Cheetah to earn that year.

'In simpler words, if we accept the convertible project, Cheetah's income for tax purposes in year 3 will be lower by £11 200 000 than it would be if we reject the project. That means that, at a 50 per cent tax rate, our corporate income taxes will be less by one half the "loss", or £5 600 000. This assumes that the company will have enough taxable profits from other operations, or can take advantage of things such as "carry-backs" of taxes (which set the negative income against some past positive income) so as actually to get cash back from the government as a reduction of taxes we paid earlier. There are all sorts of ways to get the "negative tax" cash flow, but the important thing is the general idea about investment cash flows that it illustrates: include all **changes** in the cash flows of the corporation as a whole were it to accept the project, and include only those changes.'

'That makes sense,' you say. 'Let me ask one last thing. I infer from the general tone of your comments about accounting numbers such as depreciation expenses and profits that you do not think very highly of the ideas that underlie accounting estimates. Yet our entire information and management control system at Cheetah (and most other companies) is based on accounting data. Is this something that I should be concerned about?'

'Now that,' complains the senior finance analyst present, '**is** a tough question'. It is undeniably true that for the types of decisions that we face in finance, cash flows are more important than accounting numbers, which are not cash flows. But we must always remember that particular cash flows are themselves functions of accounting numbers. Our discussion of taxes a moment ago is a good example of this. Taxes, a cash flow, are a function of "income" or "earnings" or "profit", which are accounting numbers. So, like it or not, we must use the data that the system of "generally accepted accounting principles" produces.

'That, however, is not the whole story. Without going into all of the reasoning (and continuing argument within that profession), the information that accountants produce in financial statements is by itself useful for certain other decisions that companies make. Furthermore, accountants take very seriously their responsibility of reliably reporting corporate results to the public and to government. That leads them to use some measures that we in finance, not having to let outsiders see our work, would not use. We are content to rely on our own "guesses" more than accountants are.

'You can see one illustration of this distinction between finance and accounting by comparing our "free cash flow" figures for the convertible project to the "profit after taxes" numbers in the income statements. The cash flows, as you know, are the estimates of the amounts of money that the market will use to place a value on the capital claims of Cheetah. And those amounts are significantly different at each time point than the accounting profits. But notice that if we separately add up the free cash flows and the profits after tax for all years of the project, they come to the same amount. So the accountants are doing something similar to our cash-flow calculation as they produce an earnings number, but the procedures they use cause timing deviations from actual cash flows. In other situations there may also be differences in the total amounts (particularly when companies have borrowed money). You can doubtless figure the causes for the differences each period between the accounting and cash-flow figures, but that is beside the point. Everyone well educated in finance and accounting knows about those things and uses the appropriate numbers for the decision at hand. For company investment decisions, the correct numbers are the cash flows, not the accounting figures.

'In sum, our answer to your question is yes, we do feel that accounting numbers are not correct to use as substitutes for rigorous cash-flow estimates. However, that is not to say that accounting numbers are useless. They serve an important function in estimating particular cash flows, and are the primary way that corporations communicate with the outside world. So in our earlier discussions we probably gave you a misleadingly negative view of how finance people regard accounting numbers. But make no mistake about our conviction that for the great range of financial decisions it is cash flows, and only cash flows, that matter.'

'Thank you again,' you conclude, 'You have, as usual, given me a lot to think about.'

Learning Summary

When the finance group has exited, you summarise the major points that came up in your discussion:

1. Investment NPV is correctly calculated by discounting to the present all of the changes that will occur in the cash flows of the corporation as a whole, were it to accept a project. The net of these amounts is called the **free cash flow**.
2. These corporate cash flows can be conveniently depicted at one level of generality by listing all of the outside groups with which the corporation transacts, as is done in Figure 3.1.
3. Free cash flow is defined as the net amounts of cash that the company could pay to its capital suppliers from the proceeds of a project at each time point without upsetting the expectations associated with the project. This essentially means that free cash flow is the amount that is expected to be left over (or 'residual') after all commitments and contracts other than those to capital suppliers have been satisfied according to the market's expectations.
4. Cash flows are not the same as the numbers that appear in the financial statements of corporations. Included in items such as Revenues and Expenses are non-cash items that cause accounting numbers to differ from cash flows (changes in debtors or accounts receivable, and creditors or accounts payable, and depreciation are examples). There are also certain cash flows that are not included in accounting figures at the times when they occur (such as outlays for depreciable assets, and collections of receivables and payment of payables). If accounting data are to be used as the basis for cash-flow estimates, all necessary

adjustments must be made to result in cash-flow estimates at each time point of the investment.

Review Questions

3.1 Figure 3.1 is a depiction of the basic idea underlying the calculation of what we have called a company's 'free cash flow'. The transactions there closely resemble those presented in a company's profit and loss account or income statement generated by its accountants. The income statement, however, is not generally applicable to the types of financial decisions for which the 'free cash flow' concept is used. The reasons for that are:

I. The numbers appearing in company income statements are not necessarily cash flows, and the concept of present values (with associated market value and wealth measures) requires that cash flows alone be used.
II. The numbers appearing in company income statements may be incorrectly located with respect to the time occurrence of the actual cash flows associated with the recorded transactions.
III. The above are too generous. The numbers appearing in company income statements are wrong, and are developed under incorrect assumptions on the part of accountants as to what actually goes on in companies.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both I and II.

3.2 'Our company sells office supplies, and it is often several weeks before we are paid by customers for our sales to them. If, as you suggest in Module 3, we wait to record these sales until the cash is received, we would, during that interim, be ignoring the fact that we have a valuable asset, namely what our customers owe us.' Which of the following is correct in relation to this statement?

A. It misapplies the point of Module 3, that finance is more concerned with cash flows than the recording of accounting numbers. Finance is unconcerned whether you keep track of the amounts of money owed the company by customers; it cares only about accurate recording of cash flows.
B. It mis-states the point of Module 3, that finance is more concerned with cash flows than the recording of accounting numbers. Finance recognises that there are quite legitimate reasons for keeping track of debtors or accounts receivable, and only insists that for financial decision-making purposes, the timings and amounts of cash flows are the important measures.
C. It is a correct application of the point of Module 3.
D. It is an incorrect indictment of the point of Module 3, that cash flows are important. Because financial cash flows will be presented simultaneously for several periods of time, the eventual payment of receivables by customers will appear in the cash-flow statements. Therefore it is unnecessary for the company to worry about accounting numbers such as the assets mentioned.

3.3 'Your point about depreciation expenses in Module 3 continues to disturb me. I have grown fond of the argument used by accountants to validate the use of depreciation expenses, namely that the charging of such expenses across time, instead of instantaneously, recognises appropriately the deterioration in usefulness or value of the asset in question. Your insistence in finance that all of the cash outlay be charged at the outset of the investment seems to defeat this worthwhile perspective of depreciation expenses.' As a result:

I. It is true that the cash-flow approach of finance does not attempt to recognise the period-by-period deterioration of assets over time, but simply what is paid for them.
II. It is true that the cash-flow approach of finance does not attempt to recognise the period-by-period deterioration of assets over time, but the complete set of cash flows used by finance would include the future sale and any future cash maintenance costs of the asset and thus accomplish the same effect, but in cash-flow terms, which accountants produce with their depreciation expenses.
III. It is true that the cash-flow approach of finance does not attempt to recognise the period-by-period deterioration of assets over time, because that is irrelevant to any foreseeable financial decision.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both II and III.

3.4 Suppose that your company was contemplating the introduction of a new product. Furthermore, suppose sale of that new product would likely not produce positive accounting income for a number of periods. During that time, what is the correct treatment for cash-flow purposes of the taxes that will be associated with the new product?

A. Taxes should be ignored, because the company will not be earning any profits from the new product and therefore will not be paying any new taxes.
B. Taxes should be levied against the product, because even though it is causing no tax liabilities now, it will in the future when it turns profitable. We must therefore recognise the need to cover those taxes in the future.
C. Taxes should not be levied against the product, because the accounting losses reduce the taxes the company must pay, thus offsetting the future tax liabilities that the product will generate when it turns profitable.
D. Negative taxes should be recorded as cash inflows to the company during the time when the product will be generating accounting losses.

3.5 Which of the following is the correct interpretation of the concept of 'free cash flow'?

A. The amounts of money that the capital suppliers of the company can extract from the company across time, without disturbing any of the plans or constraints under which the company is expected to operate.

B. The maximum amounts of money that the company can pay to its capital suppliers at each future time.

C. The amounts of cash flow that can be considered to be free of legal encumbrances.

D. The sum of expected interest, principal and dividends that the company is expected to pay.

Module 4

Company Investment Decisions Using the Weighted Average Cost of Capital

Contents

Learning Objectives

Module 4 extends the introduction to company investment decision making of Module 3 to the more realistic context of companies with complex capital structures (companies financed with both debt and equity capital). This is a two-step process: first, the module describes necessary cash-flow estimates for such organisations (and therein further explains the concept of 'free cash flow', the concept that has gained such notoriety in merchant and investment banking realms). With complex capital structures, good financial outcomes for companies imply adequate servicing of more than a single claim, and the criterion by which financial judgments are made must therefore be consistent with servicing all outstanding capital claims (both debt and equity). The module next develops the weighted average cost of capital (WACC) as such a criterion. Students will also encounter an important alternative to the WACC, the adjusted present value (APV), and learn the applicabilities of, and relationships between, APV and NPV.

4.1 Introduction

In this module we shall explore one of the more important concepts in finance, that of a company's **weighted average cost of capital**. It is important that you understand this concept because it is a major component of the most widely used techniques of investment decision making.

The weighted average cost of capital as an idea is straightforward. It is simply a discount rate that combines the capital costs of all of the various types of capital claims that a company issues. Thus it may include the costs of equity capital, debt capital, and any other capital claims outstanding. The weighted average cost of capital is a convenient measure for a company to use, because it captures in a single discount rate all of the returns necessary to service the company's capital claims.

This module will first develop the basic idea of weighted average rates of return to capital suppliers of companies as legitimate valuation rates, and then show the specifics of the calculation of the weighted average cost of capital.

4.2 Free Cash Flow and Profits for Borrowing Corporations

The example that Cheetah Autos Ltd provided (in Module 2) was a good introduction to corporate cash flows and value, and their relation to more commonly seen measures such as earnings and profits. But the Cheetah example, though fairly realistic, does lack one dimension of significant importance that would apply in many real corporations: the existence of corporate borrowing. The cash flows of a company must service all of its outstanding capital claims including the claims of those from whom the company has borrowed. Our first step in understanding the weighted average cost of capital is to investigate more deeply the cash flows of a company that has more than one type of capital claim: typically, both debt and equity.

When a company borrows money it creates a capital claim different from equity. The creditors or debt capital suppliers of corporations have, as we know, a contract with the company that requires the company to pay them particular amounts of money at particular times. (There are usually several other requirements, called '**covenants**' or '**indenture provisions**' in the contract, but right now we are concerned with the money amounts that the company commits itself to pay.) Financial markets place values on corporate debt claims by discounting with risk-adjusted rates the amounts of cash that the company is expected to pay to those claims. We are further aware that the debt of a company has a higher-priority claim than does the equity: whatever cash is available to service capital claims will go first toward fulfilling contracts of debt, and the residue then belongs to equity.

Our concern in this section is the cash flows that corporate investments provide to capital suppliers. We have previously been introduced to those cash flows for a company that has no debt in its capital structure; let us now examine the cash-flow expectations of a company that is essentially identical to Cheetah Autos but borrows money in order to undertake an investment.

Fortunately we have already had a brief acquaintance with just such a company: Lynx Autos plc (in Module 2). We were told there that Lynx has the opportunity to undertake an investment similar to Cheetah's, one that requires a £10 000 000 outlay and generates a £13 500 000 increase in the total market value of all capital claims of Lynx. Lynx intends to finance this investment by borrowing £5 400 000 and by reducing its dividends by £4 600 000. Thus equityholders will give up £4 600 000 in cash dividends, but in return get an increase in their share prices. That increase is £8 100 000, the difference between the £13 500 000 total value increase in all claims and the £5 400 000 increase in debt value from the project borrowing. We therefore found that the £3 500 000 NPV of the Lynx project comes to equity as an £8 100 000 increase in share value and a £4 600 000 decrease in dividends.

But upon exactly what cash flows is the financial market depending for its placing of such values? Consistent with the principles we have developed, the market must be discounting to the present the changes that will occur to the cash flows of all of the capital suppliers of Lynx were it to accept the project.

As it turns out, we discover that the Lynx investment is also in a new model of car, this one having a 'dickey seat' and intended to appeal to nostalgia buffs and wealthy adolescents. The dickey-seat project is expected to produce exactly the same cash flows and accounting results for Lynx as the convertible project does for Cheetah, with the exception that Lynx must service its debt by paying interest. Though the **expected** amounts are the same, the dickey-seat project's cash-flow expectations are riskier than the convertible project's because of the uncertainty of demand for the dickey seat's anachronistic car-seating arrangement.

The borrowing that Lynx intends to use in the project financing has the following characteristics:

1. Lynx borrows £5 400 000 now, and repays in instalments of £460 000 at the end of the first year, £3 340 000 at the end of the second year and £1 600 000 at the end of the third. This means that the amounts of debt outstanding during the three periods are £5 400 000 for the first year, £4 940 000 for the second year, and £1 600 000 during the third.
2. Lynx expects to pay 8 per cent interest on the amounts outstanding in each year. Thus the interest payments are £432 000 for the first year, £395 200 for the second and £128 000 for the third.

With this basic financing plan (along with the immediate one-time dividend reduction of £4 600 000 to make up the necessary £10 000 000 outlay), the income statements for the project for Lynx appear in Table 4.1, along with the associated cash flows to and from all entities with which Lynx is expected to transact in Table 4.2.

Table 4.1 **Lynx Autos plc: income statements (£000s) for the dickey-seat project**

	Yr 1	Yr 2	Yr 3
Revenues	£18 000	£25 000	£2 000
Expenses:			
Operations	7 000	3 830	5 200
Depreciation	3 000	5 000	8 000
Interest	432	395.2	128
Total expenses	10 432	9 255.2	13 328
Profit before taxes	7 568	15 774.8	−11 328
Taxes (50%)	3 784	7 887.4	−5 664
Profit after taxes	3 784	7 887.4	−5 664

Table 4.2 Lynx Autos plc: cash flows (£000s) for the dickey-seat project

	Now	Yr 1	Yr 2	Yr 3
Customers	0	+£17 500	+£23 500	+£4 000
Operations	0	−£7 000	−£3 830	−£5 200
Assets	−£10 000	−£4 000	−£2 000	0
Government	0	−£3 784	−£7 887.4	+£5 664
Capital*	−£10 000	+£2 716	+£9 782.6	+£4 464

* Free cash flow

If we compare these statements with those of Cheetah in Table 3.1 and Table 3.2 in Module 3 you can see that the Revenue item on the income statements and the Customer item in the cash flows hold no mystery because Lynx's are identical to those we discussed in Cheetah. The same holds true in comparing the Operations items and Depreciation and Assets figures in the statements. But at that point Lynx's results begin to differ from those we saw for Cheetah.

Lynx's income statements show an expense in each year for the interest that the company expects to pay. This interest is deductible for taxes, so Lynx's Profit before taxes is less than Cheetah's by the amount of the interest expectation in each year, as are Lynx's Tax item and Profit after tax item less than Cheetah's. So, from the perspective of Lynx's income statements, the dickey-seat project shows lower profits and taxes and higher expenses than does the convertible project for Cheetah, and this is due solely to the interest payments that Lynx is expected to make.

The cash-flow statements for the two projects also begin to differ after the Customer, Operations and Assets items. Those statements show that the essential cash-flow difference between the convertible and dickey-seat projects is that Lynx is expected to pay less taxes than Cheetah, and thus have more free cash flow to distribute to capital suppliers. The reason for all of these differences is easy to see. Tax laws allow corporations to reduce their taxes by deducting interest expense, even though that expense is a payment to a capital supplier, just as are dividends. But dividends are not a deductible expense. So companies that pay dividends to equityholders instead of interest to debt suppliers pay more taxes than companies that do pay interest.[1]

The Cheetah and Lynx projects are good examples of this. The two projects' cash-flow amounts are identical to each other except that Lynx will distribute some of its payment to capital as interest, and Cheetah, having no debt, will pay only dividends to equityholders. Because Lynx's interest is deductible. Lynx is expected to have more cash to distribute to its capital suppliers than will Cheetah.

[1] Some countries allow partial deductibility of dividends, but these deductibilities are generally not as generous as those that are allowed for interest payments.

Table 4.3 Cheetah and Lynx projects: free cash flow (£000s)

	Now	Yr 1	Yr 2	Yr 3
Cheetah	−£10 000	+£2 500	+£9 585	+£4 400
Lynx	−£10 000	+£2 716	+£9 782.6	+£4 464
Difference	0	£216	£197.6	£64

The amounts by which Lynx's free cash flow is expected to exceed Cheetah's are given in Table 4.3. These differences, as we have seen, are due to the lower amount of taxes that Lynx is expected to pay. But these amounts also have a special name in finance, signifying the source of the differences: the reduction in income taxes that a corporation receives because of interest deductibility is called its '**interest tax shield**'. Look at the set of Lynx's interest tax shields in Table 4.4. We calculate these shields by figuring the amounts of tax that Lynx would pay if it had no debt (like Cheetah), and subtracting the amount of tax that Lynx will pay with the interest deduction.

Table 4.4 Cheetah and Lynx projects: taxes (£000s)

	Yr 1	Yr 2	Yr 3
Cheetah	£4 000	£8 085	−£5 600
Lynx	£3 784	£7 887.4	−£5 664
Lynx's interest tax shield	£216	£197.6	£64

There is another way to calculate interest tax shields that is less cumbersome and more to the point of the economic source of those shields. Since interest expense is a reduction in taxable income, and since taxes are figured by taking a percentage of the income number, interest tax shields can be found by simply multiplying the expected interest payment in each period by the corporate income tax rate. For example, Lynx is expected to pay £432 000 of interest in year 1. To find the interest tax shield for that year we merely multiply £432 000 by the corporate income tax rate of 50 per cent to arrive at the shield of £216 000. By multiplying Lynx's interest expenses from its income statement in Table 4.1 by the 50 per cent tax rate, you can prove to yourself that the shields in Table 4.4 are correct.

'All of that is most interesting, we suppose,' you say. 'But what significance does it have? We already had the cash flows for the Lynx project in enough detail to find its free cash flow in Table 4.2. And from what you told us before, that should be enough to calculate the NPV of the project, which is our only concern. Why do we need to be concerned about interest tax shields?'

The discussion of interest tax shields may at this point seem esoteric but its importance to real-world financial decision making is undeniable. This concept is intimately involved with the weighted average cost of capital.

4.3 Investment Value for Borrowing Corporations

We have now calculated all of the cash flows that are necessary to illustrate a corporate investment decision when the company borrows money. Our current task is to see how those cash flows are valued by capital markets.

Table 4.5 **Lynx Autos plc: cash flows to debt and equity (£000s) for the dickey-seat project**

	Now	Yr 1	Yr 2	Yr 3
Free Cash Flow	−£10 000	+£2 716	+£9 782.6	+£4 464
Interest		432	395.2	128
Principal	−5 400	460	3 340.0	1 600
Equity Claim	−4 600	£1 824	£6 047.4	£2 736

Unlike the cash flows for Cheetah, which belonged to equity alone, Lynx's free cash flow is claimed by two types of capital suppliers: debt and equity. Since we have been informed of the expectations for interest and principal payments to debt suppliers, we can calculate the amounts of cash that equity can claim, as shown in Table 4.5. With these cash flows, it is a simple matter to see the effect of the project upon the wealth of the two types of claimholders. For debt, we know that an 8 per cent return is required and so, by combining the interest and principal payments from Table 4.5:

$$\begin{aligned}\text{Debt value}_0 &= \frac{£892000}{(1.08)} + \frac{£3735200}{(1.08)^2} + \frac{£1728000}{(1.08)^3} \\ &= £5400000\end{aligned}$$

The cash that debt suppliers expect to get in the future is equal in value to the amount that is raised from them at present, namely £5 400 000, and so the debt suppliers' wealth is unaffected by the investment. From our study of the workings of efficient financial markets in Module 1, this should not surprise you.

But how about equity? In order to figure its wealth change, we must compare the investment-induced change in equity value to the forgone £4 600 000 dividend. To calculate the value change we must have cash-flow expectations, which are given in Table 4.5, and an appropriate discount rate, which is not. We obviously cannot use the Cheetah convertible project's equity rate, because we know that that project is of lower risk and is purely equity financed. Let us assume that we refer the question to our capital market experts, and they tell us that the operating and financing risks of the dickey-seat project are such that the equity market would require a 14 per cent return per period. That being the case, we can arrive at the change in Lynx's equity value due to the project:

$$\begin{aligned}\text{Equity value}_0 &= \frac{£1824000}{(1.14)} + \frac{£6047400}{(1.14)^2} + \frac{£2736000}{(1.14)^3} \\ &= £8100000\end{aligned}$$

Thus Lynx's shareholders will forego a £4 600 000 dividend and in return receive an £8 100 000 increase in the market value of their shares as a result of the project. The net of those two effects is the £3 500 000 NPV of the project.

Finding a project's NPV by this process (estimating the changes in the amounts of cash that equity will claim as a result of a corporate investment, and discounting those to arrive at the expected present change in equity value) is a perfectly proper technique. However, it is not one of the more commonly used techniques in the modern company, for corporate financial decision makers are evidently more comfortable with techniques that do not separate the cash flows into those going to the various capital claims, but instead merely deal with the company's cash flows as a whole. These too can be perfectly acceptable techniques of investment analysis, and will bring us finally to the notion of the weighted average cost of capital. We shall now examine them.

4.3.1 Overall Corporate Cash Flows and Investment Value

The most commonly used techniques for finding the NPVs and IRRs of investment projects use the corporate cash flows as a whole, rather than separating those for debt and equity as we did above for Lynx. To introduce this manner of NPV calculation recall that the free cash flow of the dickey-seat project is expected to be (£000s):

Now	Yr 1	Yr 2	Yr 3
−£10 000	+£2716	+£9782.6	+£4464

We wish to find the NPV of these cash flows. In order to do that correctly, we must discount them with a rate that is commensurate with their risk. How do we find such a rate? We want a rate that the market would apply to these cash flows were they to be sold as a combined expectation, in other words, if the company sold a security that had these cash flows as expectations, what rate would the market apply to discount those flows?

We already know the discount rates that are necessary on these same cash flows when they are separated into debt (8 per cent) and equity (14 per cent). To find the discount rate appropriate for the combined cash flows we must therefore combine the equity and debt discount rates. But careful here. We must combine the equity and debt rates in the correct proportions, or the resulting corporate discount rate will be incorrect.

What should the proportional combination be? We are seeking a market discount rate for the total corporate cash flows by combining market discount rates for debt and equity. Those rates for debt and equity are based on their proportional claims upon the corporate cash flow. So we must combine the equity and debt rates in the ratio of their claims on the corporate cash flow, and that ratio is given by the market values of the two claims.

Think of the overall company rate this way: it is an average of the rates that all capital claims require on the amounts they have invested in the project. And the amounts they have invested are not necessarily the amounts of cash they paid into the project but the market prices at which they could sell those claims. So the overall project rate is a market-value weighted average of the rates required by the various capital claims upon the investment.

For the Lynx project the rate is given as follows:

$$
\begin{aligned}
\text{Overall rate} &= \frac{\text{Debt market value}}{\text{Total market value}} \times \text{Debt required rate} \\
&\quad + \frac{\text{Equity market value}}{\text{Total market value}} \times \text{Equity required rate} \\
&= \frac{£5\,400\,000}{£13\,500\,000}(0.08) + \frac{£8\,100\,000}{£13\,500\,000}(014) \\
&= 0.116 or 11.6\%
\end{aligned}
$$

This tells us that the market would require an 11.6 per cent return on the dickey-seat project's free cash flow were it to be offered to the market as a single-security expectation.

What would such a security be worth? Since we have its discount rate and its cash flows, finding its value is child's play:

$$
\begin{aligned}
\text{Value of the project} &= \sum_{t=1}^{n} \frac{\text{Free cash flow}}{(1 + \text{Overall required rate})^t} \\
&= \frac{£2\,716\,000}{(1.116)} + \frac{£9\,782\,600}{(1.116)^2} + \frac{£4\,464\,000}{(1.116)^3} \\
&= £13\,500\,000
\end{aligned}
$$

The investment is worth £13 500 000 if regarded as a single capital claim. It should not surprise you that the project's equity and debt values added together also equal this amount. (If a participant for some reason wished to own such a claim as this overall one, and it was not available, he could effectively create the claim by buying both the debt and equity of the project. Therefore the total value of the project **must** equal the sum of its constituent capital claim values.)

Of course the NPV of the investment is simply the difference between the market value created by undertaking it (£13 500 000) and what it cost (£10 000 000), or £3 500 000. So we now have another technique for finding the NPV of an investment, namely one that discounts the free cash flow of the project at a rate commensurate with the risks of those cash flows.

4.4 Investment NPV and the Weighted Average Cost of Capital

We have now successfully developed a technique that gives us the correct investment NPV by discounting a project's free cash flow with the market-value weighted average of the discount rates of the capital claims that will be generated by the project. For convenience we shall refer to this technique as the 'overall' NPV method. It would seem that we have now accomplished the task we set for ourselves in the previous section: to learn how companies find NPVs by using projects' cash flows, not separated into debt and equity flows, but discounted as if they were single claims. The overall NPV method accomplishes that, but it is not the technique used by most companies. The preferred technique will require that we make one small adjustment in the 'overall' technique we have just developed. Once that is done, you will know as much (or more) about the NPV techniques that companies use as do most of the corporate decision makers who use them.

The revision of the 'overall' technique of finding the NPV that we shall now study has a long and distinguished history in finance. We should forewarn you, however, that it will seem at first that we are taking some liberties with the basic principles that we have emphasised so often in our discussions up to this point. The truth is that we shall not be breaking any rules but rather at first merely extending some basic relationships to be a bit more complex, and in the process we shall produce a technique that is simpler but as accurate as the 'overall' one we already have. Remember as you work through the scenario below that the net result will be an easier, not a more difficult, method of finding a corporate NPV, and it will involve the use of our long-sought weighted average cost of capital.

Remember, too, that you analysed the Cheetah investment from the perspective of the managing director of that company. Suppose that Lynx has lured you away from your position at Cheetah, and you are now the head of Lynx. As you did at Cheetah, you ask to see all the details of pending investment proposals. If Lynx's financial people have been educated in reasonably modern times, they will send you Lynx project data that looks like Table 4.6.

Table 4.6 **Lynx Autos plc: cash flows (£000s) for the dickey-seat project**

	Now	Yr 1	Yr 2	Yr 3
Customers	0	+£17 500	+£23 500	+£4 000
Operations	0	−£7 000	−£3 830	−£5 200
Assets	−£10 000	−£4 000	−£2 000	0
Government	0	−£4 000	−£8 085	+£5 600
Capital*	−£10 000	+£2 500	+£9 585	+£4 400

* Free cash flow

'Now wait just one moment!' you exclaim. 'That is not fair. Those numbers are not the same ones that we have just developed for Lynx following all of the principles of value that you led us to believe were so important (Table 4.2). What happened? Are Lynx's financial managers stupid, or have you been leading us down the wrong paths?'

Fortunately neither of the possibilities you fear is true. Lynx's managers have calculated the project's cash flows in the way that most financial analysts would, and this is not necessarily inconsistent with the principles we have laboured so hard to develop. First, let us see what these new numbers are.

Even a brief examination of Table 4.6 shows that these Lynx project cash flows differ from those in Table 4.2 only by the amounts of Lynx's interest tax shields, as shown in Table 4.7. So Lynx's financial analysts have produced figures that are exactly the same as we did in our earlier analyses for the Lynx project, except that interest tax shields are not included. The common practice in financial analysis of corporate investment is that, when estimating the cash flows of a project, its interest tax shields are **not** included in the cash flows. In other words, the cash flows are calculated as if the project will be financed totally with equity, even if the financing plan is actually to use debt.

Table 4.7 **Lynx Autos plc: free cash flows (£000s) for the dickey-seat project**

	Now	Yr 1	Yr 2	Yr 3
Table 4.2:				
Free cash flow	−£10 000	+£2 716	+£9 782.6	+£4 464
Interest tax shields	0	£216	£197.6	64
Table 4.6:				
Free cash flow	−£10 000	+£2 500	+£9 585	+£4 400

'That cannot be correct,' you argue. 'You have told us consistently that in doing present-value calculations we must discount the cash flows that a capital claim expects to receive by the rate appropriate to those cash flows. Yet here the cash flows are not the ones that the capital claimants of Lynx expect to receive. By omitting the interest tax shields, we are ignoring money that the equity and debt suppliers can expect to get, and therefore our answers will be wrong.'

You are entirely correct that the cash flows in Table 4.6 for the Lynx project are not the ones that the capital suppliers of the company expect to receive. And you are further correct in the implication of your comment that if we discounted these cash flows with the overall rate for the project (11.6 per cent) as calculated in the last section, we would get the wrong answer. But let us take the final step and see how Lynx's financial analysts would calculate the investment's NPV:

$$\begin{aligned} \text{NPV} &= -£10\,000\,000 + \frac{£2\,500\,000}{(1.10)} + \frac{£9\,585\,000}{(1.10)^2} + \frac{£4\,400\,000}{(1.10)^3} \\ &= +£3\,500\,000 \end{aligned}$$

'Well, we suppose that nothing should surprise us now,' you say. 'Would it be impolite of us to ask the rationale for discounting these all-equity cash flows at a rate that is neither the equity rate for Lynx (14 per cent), nor the overall rate (11.6 per cent), but is yet a third rate (10 per cent) whose origin is not at all clear. You obviously get the correct NPV (£3 500 000), but it is not obvious why you chose to get it that way.'

You are again entirely correct that we have not yet demonstrated how the 10 per cent discount rate was derived, or the rationale behind it. That is exactly what we shall now do. However, before we do so, let us point out that the 10 per cent rate is in fact the goal of our long search in this module: it is the weighted average cost of capital for the Lynx project.

How did Lynx's financial managers arrive at this rate? The answer is in the cash-flow alteration that took place when we moved from the overall flows to those that ignore the tax shield benefits of debt. Think of the situation this way: for this calculation to produce a correct NPV using the cash flows that exclude interest tax shields, we must instead include the effect of interest tax shields in the discount rate. And that is exactly what the weighted average cost of capital does. The weighted average cost of capital that the Lynx financial analysts are using is 10 per cent instead of the 11.6 per cent overall rate because this new rate is adjusted downward for the deductibility of interest.

Intuitively speaking, the weighted average cost of capital (WACC) is the discount rate that:

1. reflects the operating risks of the project;
2. reflects the project's proportional debt and equity financing with attendant financial risks; and
3. reflects the effect of interest deductibility for the debt-financed portion of the project.

You can see that this list of attributes asks quite a lot of the WACC. It must account for the basic operating risk of the project, much as the rate for the all-equity financed Cheetah project did. It must be consistent with the proportional claims of debt and equity, as does Lynx's overall rate. And finally it must include the effect of interest deductibility since the cash flows upon which it operates do not. That sounds like a terribly complex number, and in a sense it is. But, fortunately, it is not hard to calculate. (Actually, one of the main reasons why it is so widely used is that it is relatively easy to derive.)

To see how to find a WACC, remember that Lynx's overall (11.6 per cent) rate has two of the three attributes of the WACC mentioned above; the overall rate is adjusted for the operating risk of the project, and reflects its proportional debt and equity financing. So the easiest way to find the WACC would be to alter the overall rate to reflect interest deductibility on the investment's debt-financed portion. The overall rate, you recall is found by:

$$\begin{aligned}\text{Overall rate} &= \frac{\text{Debt market value}}{\text{Total market value}} \times \text{Debt required rate} \\ &\quad + \frac{\text{Equity market value}}{\text{Total market value}} \times \text{Equity required rate} \\ &= \frac{£5\,400\,000}{£13\,500\,000}(0.08) + \frac{£8\,100\,000}{£13\,500\,000}(014) \\ &= 0.116 \textit{ or } 11.6\%\end{aligned}$$

To find the WACC we must include the effect of interest deductibility in the overall project discount rate. Interest deductibility can be regarded as an effective reduction in the cost of borrowing for the investment. In other words, though the debt suppliers of Lynx are requiring an 8 per cent return on their investment, Lynx expects debt to cost less than 8 per cent. The reason is that the government, by allowing interest deductibility, subsidises debt capital relative to other types whose payments are not deductible. If interest is deductible at a 50 per cent tax rate, the government is effectively subsidising 50 per cent of the interest payment that the company makes. (Lynx's taxes are lower by 50 per cent of the amount of its interest payments because the payments are deductible at the 50 per cent tax rate.) This appears, for example, as Lynx's interest tax shield cash flows (which can be regarded as governmental subsidies) being 50 per cent of its interest payments (*see* Table 4.1, Table 4.3 and Table 4.4). So if we wish to reflect interest deductibility in the discount rate (WACC) rather than the cash flows, the weighted average must use the company's **after-tax cost of debt** rather than the debt suppliers' required rate.

The cost of debt to a company with deductible interest is simply debt's required return multiplied by the complement of the corporate income tax rate:

$$\text{Debt cost} = \text{Debt required return} \times (1 - \text{Corporate income tax rate})$$

For the Lynx project:

$$\text{Debt cost} = 0.08(1 - 0.5) = 0.04 \text{ } or \text{ } 4\%$$

So the debt that Lynx will issue actually costs the company 4 per cent and not the 8 per cent return that the debt suppliers expect to get. The reason is that in our scenario the government will provide a subsidy to the interest payment equal to the reduction in taxes due to debt being in the company's capital structure.[2]

Lynx's WACC calculation is therefore exactly like its overall rate calculation, except that we use the project's debt cost instead of its debt required rate:

$$\begin{aligned}\text{WACC} &= \frac{\text{Debt market value}}{\text{Total market value}} \times \text{Debt cost rate} \\ &\quad + \frac{\text{Equity market value}}{\text{Total market value}} \times \text{Equity required rate} \\ &= \frac{£5\,400\,000}{£13\,500\,000}(0.04) + \frac{£8\,100\,000}{£13\,500\,000}(014) \\ &= 0.10 \text{ or } 10\%\end{aligned}$$

So the financial analysts of Lynx were not trying to fool their new managing director. The WACC–NPV analysis of an investment project is performed by discounting the project's free cash flow **not including the interest tax shields that the project's financing will generate** (these free cash flows are the same as would be expected if the investment were financed totally by equity). The discount rate used is the project's weighted average cost of capital, defined as the market-value weighted average of the project's equity required rate and debt cost. Debt cost is the debt suppliers' required rate multiplied by the complement of the project's corporate income tax rate. This method adjusts for the deductibility of corporate interest in the discount rate as opposed to the cash flows of the project, but yields NPV answers identical to those of both the overall method and the method which calculates the separate values of debt and equity generated by the investment.

This then accomplishes the primary goal of this module: displaying the WACC–NPV technique used by companies as they evaluate investment projects. We have,

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2 We multiply by the complement of the tax rate instead of the tax rate itself because the corporate benefit is the reduction in taxes rather than the taxes themselves. We multiplied by the tax rate rather than its complement in the cash-flow form of the benefit (the interest tax shields) because interest payments are expenses that reduce taxes. Remember our example uses a 50 per cent tax rate and we therefore get the same answer for debt cost whether we multiply the required rate by the tax rate itself or by its complement. That, of course, would not be the case for any other tax rate, and the **complement** of the tax rate is the only generally correct multiplier.

however, in a sense begged the question that you raised when we began discussion of the WACC–NPV method. That question was **why** companies used this evaluation technique instead of any of the others that we have discussed. The answer lies in examining the information each of the methods requires to work correctly.

The overall technique and the one we discussed first, namely the separated debt and equity value process, both require that we know exactly how much debt financing will be issued by the company for the project. That is because those methods use the actual cash taxes expected for the project in their free cash-flow estimates, including the interest tax shields in the project's cash flows. The WACC–NPV, on the other hand, does not require that the exact amount of debt to be used in the project be known in order to calculate the project's NPV. The cash flows used in the WACC–NPV are those that would be expected if the investment were all-equity financed. (The interest tax shields are not included in the free cash flows of the project.) So in terms of the data necessary to estimate free cash flow, the WACC–NPV technique is less demanding.

'Aha!' you exclaim. 'We have caught you now. How seriously can you expect us to take all of this talk about real applications and information requirements of these techniques when your own example does not make sense from that perspective? Lynx's financial analysts came up with the correct answer using the WACC method as you described it, and the rationale of WACC–NPV seems to make sense. But look at the calculation that they did to arrive at the WACC:

$$\begin{aligned}\text{WACC} &= \frac{£5\,400\,000}{£13\,500\,000}(0.04)+\frac{£8\,100\,000}{£13\,500\,000}(014)\\ &= 0.10 or 10\%\end{aligned}$$

'The thing that disturbs us about this calculation is that the project's total present value (£13 500 000) is used to find the proportions of debt and equity financing. Now, we agree that the proportions are necessary for the technique to work correctly. But if you have the £13 500 000 value for the project as a whole, why go further? At that point, merely subtract the present cash cost of the investment (£10 000 000), and you get the NPV directly! The whole example makes no sense if we can get the final answer that easily.'

We wish it were that simple. Remember where we got the £13 500 000 value: from the earlier and quite exhausting NPV calculations before we arrived at the WACC method. So the number was not arrived at by any 'easy' shortcut. But your question does raise another important point about why and how companies use the WACC–NPV method. It is true that the analysis we presented used the total market value of the project as one of the pieces of information in the analysis. And if that piece of information were required in order to complete a WACC–NPV calculation, the method **would** be redundant. But it is not required. The £13 500 000 was used to conclude that the debt financed proportion of the project is expected to be:

$$\begin{aligned}\text{Debt financing ratio} &= \frac{£5400000}{£13500000}\\ &= 40\%\end{aligned}$$

The equity ratio is, of course, 60 per cent. This implies that information about the market values of the capital claims of a project are not necessary to find its

WACC, as long as the expected ratios or proportions of those financings are known. Companies using the WACC–NPV are those willing to specify the expected **proportions** of debt and equity in terms of their market values, but they do not know exactly what the claims will be worth until after the analysis is complete.

You can now see why many corporations prefer the WACC–NPV method over the others we have seen. It does not require that the amounts of debt to be issued are known, nor does it require that the market values of any of the resulting claims be estimated beforehand. The only information necessary comprises estimates of the required rate for equity, debt's after-tax cost rate, the all-equity free cash flows, and the proportions intended for debt and equity financing. This is less demanding an information set than the other techniques require, and is probably the reason that companies prefer this NPV method.

4.5 The Adjusted Present Value Technique

The WACC–NPV method is the most widely used NPV technique, but there is at least one other that has a significant following. This one is called the '**adjusted present value**' (APV) technique, and our discussion of NPV techniques would be incomplete without it. Before you become depressed at the prospect of yet another tedious derivation of NPV, take heart. APV is easy to master once you know the WACC–NPV method.

Immediately above we made the point that WACC–NPV is preferred by many analysts because of the relative simplicity of the information it requires. The APV is also preferred because of certain of its information attributes, but these are different from those of the WACC–NPV.

You recall that WACC–NPV requires that the proportions of debt and equity market values be known, but that knowledge of the cash amounts of such financing is not necessary for the final result. The APV, on the other hand, does not require that the proportions of debt and equity be known, but does require that the interest tax shields of the project's debt be estimated. Otherwise the information required by the two techniques is basically the same.

The APV technique is therefore preferred for corporations comfortable in estimating the **amounts** of debt their projects will use, but not the value proportions that will be generated, while the WACC–NPV method is best for companies that are willing to estimate market value **proportions** of financing for investments, but not the actual amounts that will be generated. Each side has its adherents, but one aspect of the comparison of the techniques is necessary to mention at this point: if performed correctly, the WACC–NPV and the APV methods give exactly the same answers.

Let us return to the Lynx example and see how the APV technique works. The APV relies upon the observation that one effect of financing upon the value of a project is to add interest tax shield cash flows as the proportion of debt financing increases. If a project is financed only with equity, its value can be found by discounting the all-equity free cash flows with an all-equity discount rate, as we did with the Cheetah project. If a project is to be financed partially with debt, one financing effect on the value of the project is simply to add the present value of the interest tax shields to the all-equity financed value. So the APV finds the NPV by first finding the value of an investment as if it were financed only by equity, and then adds the present value of the project's interest tax shields.

Suppose that the Lynx corporation used the APV method and wished to analyse the dickey-seat project. The APV technique begins by finding the value of a project as if it were financed wholly through equity and so our first task is to find that value for the dickey-seat project. The all-equity cash flows have, as we know, already been estimated, because those are the ones we used in the WACC–NPV analysis (Table 4.6 and Table 4.7). The only question is what discount rate to use.

'That is also easy,' you say. 'We already have an equity rate for the dickey-seat project, the 14 per cent rate that we used to value the equity claim in the "separated" NPV that we calculated first of all for Lynx.' Careful here. If you were to use that 14 per cent equity rate you would be making a mistake. Remember that this calculation is to be as if the project were financed wholly through equity. And the 14 per cent rate, though an equity rate, is not an 'all-equity' rate. The 14 per cent equity discount rate is associated with a project that is partly financed with debt. Since that equity claim is residual to the debt claim, the 14 per cent rate is in part recognising the risk of the higher-priority debt claim upon the cash flows of the equityholders. So you cannot use the 14 per cent rate; it is too high for all-equity financing.

'Well, then, how about the 10 per cent true "all-equity" rate that we used for the Cheetah project?' you ask. That too would be wrong. The 10 per cent rate is an all-equity rate, but it applies to a different investment, namely one that we have been told has a lower operating risk than the dickey-seat project. So the 10 per cent rate is too low.

'We give up. How *do* we find the correct all-equity rate for this investment?' you sigh. Since it will be a rate we have not seen before, we must confer with our capital market experts, who consider the cash-flow operating risks of the project and advise us that the all-equity rate for the Lynx project is approximately 11.69 per cent. (If you happen to be checking our numbers as you go along, use 11.690475 per cent as the exact all-equity rate necessary to reproduce our results.) You can see that this rate is greater than the lower-risk all-equity Cheetah project rate of 10 per cent, and is lower than the 14 per cent residual equity rate that this investment requires when £5 400 000 of debt (40 per cent of the project value) is used. So the Lynx project, if totally equity-financed, would be worth:

$$\begin{aligned}\text{All-equity value} &= \frac{\pounds 2500000}{(1.11690475)} + \frac{\pounds 9585000}{(1.11690475)^2} + \frac{\pounds 4400000}{(1.11690475)^3} \\ &= +\pounds 13079785\end{aligned}$$

But of course the Lynx project will not be financed only with equity. The debt that will be used with the project will reduce its taxes, and cause more cash to be available to service its capital claims. The amounts of cash that will be added to the all-equity flows are the interest tax shields, and they will cause the actual value of the investment to be higher than its all-equity value. The APV technique adds the value of the interest tax shields to the all-equity value to find the investment's total value.

What is the value of the interest tax shields? To find it, we follow the same value rules that we always have: discount the cash flows at the appropriately risk-adjusted discount rate. And what rate will thereby apply to the interest tax shields of the investment? Since the interest tax shields are contingent upon the interest payments, their risk must be the same as the debt cash flows, which require an 8 per cent return. So 8 per cent is the correct risk-adjusted rate to use for valuing the interest tax shields of the project,[3] and the interest tax shields (*see* Table 4.7) for the Lynx project are thus worth:

$$\begin{aligned}\text{Interest tax shield value} &= \frac{£216\,000}{(1.08)} + \frac{£197\,600}{(1.08)^2} + \frac{£64\,000}{(1.08)^3} \\ &= £420\,215\end{aligned}$$

So the APV technique says that the Lynx project's present value comprises £420 215 of interest tax shield value and £13 079 785 of 'all-equity' value. The APV calculation of the dickey-seat project's effect upon Lynx shareholder wealth is thus:

$$\begin{aligned}\text{APV} &= (\text{All-equity value}) + (\text{Interest tax shield value}) - \text{Present cost} \\ &= £13\,079\,785 + £420\,215 - £10\,000\,000 \\ &= £13\,500\,000 - £10\,000\,000 \\ &= £3\,500\,000\end{aligned}$$

As we promised, the APV method gives us exactly the same result as did the several NPV techniques applied earlier to this example. Note particularly that the APV method tells us that the sum of the all-equity and interest tax shield present values is £13 500 000, which is the value that we earlier discovered for the capital claims on the project. The APV arrives at the capital claim value not by discounting the claims' cash flows separately, as we did at first, nor does it combine them as did the overall and WACC–NPV methods. The APV finds present values by splitting up the cash flows into a basic operating set (which we have called the all-equity cash flows) and a set of cash flows caused by the way the project is financed (the interest tax shield cash flows). These are each then discounted separately at rates appropriate to their individual risks.[4]

4.6 The Choice of NPV Techniques

As we discussed each of the means by which a project's NPV can be calculated, we mentioned some of the strengths and weaknesses of the methods. The WACC–

[3] Actually, there are a few more hidden assumptions in this statement which the picky will appreciate. For example, since the interest tax shields are tax reductions, this rate implicitly assumes that if interest is paid, taxes will be reduced. In situations where accounting income is nil and taxes are as well, interest obviously cannot reduce taxes. Companies recognising the potential value of these reductions use tax 'carry-backs' and 'carry-forwards', leasing agreements, mergers, and all manner of other tactics to reduce **someone's** taxes with their interest deduction (and of course be compensated for doing that).

[4] This demonstration of the equality of valuations using WACC and APV is actually labouring under some restrictive conditions that are probably not obvious to the first-time reader. Amongst these conditions are that the project's debt repayment schedule is designed to maintain a constant market value captial structure over the life of the project. For more sophisticated extensions of this idea, see J. Miles and R. Ezzell, 'The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification' in the *Journal of Financial and Quantitative Analysis* (September 1980).

NPV method seemed to be justifiably the most popular when compared with the 'separated claim' and 'overall' techniques, because it required either the same or less information, was a bit easier to calculate, and gave exactly the same answers as the other two. Making the same comparisons between the WACC–NPV and the APV, it is not clear that one would automatically be preferred to the other.

The reason for this ambiguity is that they require different information for their result. Because it does not require that actual amounts of debt to be issued are known, the WACC–NPV is less demanding than APV, which does. On the other hand, the APV does not require that the market value proportions of debt and equity claims resulting from the investment be known, as does the WACC–NPV. Though both, possessed of full information about a project, would arrive at the same result, on many occasions companies simply do not have all of the information necessary to use one or the other of the techniques. Or companies may be of the opinion that they are not comfortable in forecasting the type of information that one or other of the techniques requires. In those situations the choice of WACC–NPV versus APV is easier.

There is one characteristic of the APV method that fosters its particular use in complex investment situations. We have concentrated on the financing cash-flow effect called interest tax shields to illustrate the value effect of how a project is financed. Interest deductibility, however, is not the only effect that a project's financing may have upon its value. For example, some governments give tax credits of various types; sometimes the deductibilities for interest may conflict with other deductions that the company may take for tax purposes; and it is also possible that borrowing may create cash costs in addition to interest and its deductions (for example, those costs involved in bankruptcy proceedings). When complexities such as these appear relevant to a company's financial decisions, the APV approach to investment analysis may be easier to use than WACC–NPV.

The reason for APV's desirability where there are several financing-induced cash-flow effects is the way that APV treats each of these separately, by estimating the cash flows of each, and discounting them at rates appropriate to their unique risks. In other words, an APV analysis in a real company may produce a final calculation that includes separate values for interest tax shields, investment tax credits, litigation costs with creditors, etc. The APV method is set up to handle each of these separately, and they are best understood in that way. The WACC–NPV, on the other hand, seeks to include no financing-induced cash flows in the analysis, but to capture their effect by altering the discount rate (the WACC). It would perhaps be possible to adjust the WACC to include several effects in addition to interest deductibility, but because of its complexity the task has never been seriously attempted.

On the negative side of APV, it does not have the automatic characteristic of being consistent with maintaining an intended ratio between the various kinds of financing a company uses in its investments. If it is important to a company, for example, to have debt claims to be about *x* per cent of the corporation's market value, the WACC, because it is a function of these ratios, can display such a result.

The APV, however, being determined by an amount rather than a ratio of debt, cannot easily do this.

Learning Summary

The information that you should have gleaned from this module is some of the most important in the finance course. The investment evaluation techniques used by corporations, and the reasons why they use the ones they do, is central to a good education in finance.

At this point it is time to bring together in a convenient summary all of the basic ideas we have introduced about corporate investments. Rather than give you a long and tedious narrative of the concepts you have covered in this module we have chosen to kill two birds with one stone and structure the summary in a shorthand system of notation that should help you to remember these ideas efficiently. In other words, we are going to show you the formulas for the important forms of NPV and APV that we have discussed above. Because we did not wish you to be distracted by these formulas when we first introduced the ideas behind them, we have delayed their introduction until now. Actually there was no good reason to produce them for our earlier purposes, because the best uses to which such formulas can be put are as aids in remembering to include all of the things that should be included, and to exclude all that should be excluded in investment analyses.

Henceforth we shall use the notation that follows to signify these ideas, all of which you have already studied.

Cash flows

FCF_t = Free cash flow: the amount of cash that the corporation can distribute to its capital suppliers at time t due to an investment, consistent with the company's contractual and operating expectations. This can be a negative amount if the investment is expected to raise more cash than it pays to capital suppliers at a given time. FCF is the **net amount** of the **cash** amounts to be transacted with customers (as cash receipts), government (as taxes and subsidies), and suppliers of labour and assets (as their cash costs).

I_t = Interest cash flow at time t.

T_c = Corporate income tax rate.

ITS_t = Interest tax shield cash flow: the reduction in corporate income taxes at time t caused by interest deductibility of the debt issued for the investment; equal to $I_t \times T_c$.

FCF_t^* = Unleveraged (ungeared) free cash flow: the amount of free cash flow that the company is expected to generate at time t due to a project, **not including interest tax shields**; equal to $FCF_t - ITS_t$.

Market values

E_t = Market value of the equity of the investment at time t.

D_t = Market value of the debt of the investment at time t.

V_t = Market value of the investment at time t; equal to $E_t + D_t$.

Discount rates

re = Required return on the equity of the investment; required returns are not usually time-subscripted, but can be.

rd = Required return on the debt of the investment.

rd^* = Cost of debt as a rate to the investment; equal to $rd \times (1 - T_c)$.

rv = Overall weighted average return on the capital claims of the investment; equal to

$$\frac{D}{V}(rd) + \frac{E}{V}(re)$$

rv^* = The weighted average cost of capital (WACC) of the investment; equal to:

$$\frac{D}{V}(rd^*) + \frac{E}{V}(re)$$

ru = All-equity or unleveraged (ungeared) required return on the investment; the rate that would be required on the investment were it to be financed purely with equity.

Investment evaluation techniques

WACC–NPV:

$$\mathrm{NPV}_0 = \sum_{t=0}^{n} \frac{\mathrm{FCF}_t^*}{(1 + rv^*)^t}$$

APV:

$$\mathrm{APV}_0 = \sum_{t=0}^{n} \left[\frac{\mathrm{FCF}_t^*}{(1 + ru)^t} + \frac{\mathrm{ITS}_t}{(1 + rd)^t}\right]$$

At first glance these definitions, notations and equations can be intimidating to those who do not regularly deal with formally stated systems. The reason why we have delayed presenting them is just that. It is important, however, that you understand the purpose for which we are now presenting these formal definitions and formulas: they are intended as a convenient shorthand method for remembering the financial relationships and techniques we have been discussing, and will save us a lot of time and effort in the remainder of the course as we discuss applications and extensions of these ideas. But they, in and of themselves, are not intended to help you learn and understand finance. That must come from your study of our verbal and numerical text, examples, and the problems and cases following the module.

Do not attempt to **memorise** anything in the summation given above. Not only would that not be particularly helpful to your finance education, but it might distract you from what you should be doing, namely learning how to use the relationships and techniques to make financial decisions. If it makes you feel better, we promise never to ask you to reproduce any formula given above in an examination. You will surely, however, find it useful to refer to that compilation as you continue this course in order to refresh your memory about specific definitions. Believe it or not, before long you will have internalised the formulas and definitions enough so that it is unnecessary.

To review briefly the summary above, you can see that it has four main sections: Cash flows, Market values, Discount rates, and Investment evaluation techniques. The section on cash-flow definitions lists those for free cash flow, interest tax shield, and FCF* (the notation that we use for the cash flow necessary for the WACC–NPV method), along with brief definitions of each.

In the market value section, we simply introduce notation for debt and equity market values, and define the total value of the investment as the sum of its debt and equity market values. The discount rate section lists the notation for debt and equity required rates of return, shows how the overall investment required rate, *rv*, is calculated, defines the cost of debt, rd^*, and shows how its use generates the WACC, rv^*. That section also lists the all-equity discount rate, *ru*, for the investment (the rate that would be required were it to be financed totally by equity). The *ru* rate is used in the APV calculation.

Finally, the investment evaluation section shows the WACC–NPV and APV techniques in the form of equations. Though these equations are correct, and the mathematician–economists among you might be able to master the techniques by simply studying the formulas and related definitions, we remind you that our major purpose in showing them to you is so that you will recognise them as we use them in the rest of the course as a convenient method of referring to the components of the calculations.

Notice that we have not included the formulas for finding investment values with either the 'overall' or 'separated' techniques. The reason we have not included those methods is simply that they are not regularly used by corporations in making investment decisions. They were useful in developing the ideas behind WACC–NPV and APV in the text, but once the latter are understood, the others are unnecessary. As a matter of fact, the others are so little used that from now on, whenever we refer to NPV, you should take that to indicate that we are talking about the NPV found by the WACC–NPV method, or the method itself. That is the usual convention in corporate finance.

We must also say that you **should** be able to recognise, for example, that the NPV formula is telling you to do exactly the same thing that earlier took several pages of text to accomplish: discount the FCF* cash flow with the rv^*, or WACC. Note also for example that, unlike the text discussion, there is no separate indication in the NPV formula above to subtract the present cost of the investment (e.g. the £10 000 000 for the Lynx project). The reason for that is that the formula begins its summation of FCF* present values at $t = 0$, which would automatically include initial costs as part of the FCF* summation (e.g. if you follow exactly the instructions of the formula, you will have an $\mathrm{FCF}_0^* = -£10\,000\,000$ for the Lynx project, which is discounted at $(1 + rv^*)^0 = 1$, just what you wanted to do). See if you can similarly decipher the APV formula given above.

In following modules we shall investigate some of the important uses, information sources, competitors, and interpretations of these techniques. Before you plunge into that material you should feel reasonably comfortable with the material in this and the preceding module. We suggest not only thorough reading but also a significant attempt at the problems given at the ends of these modules.

Review Questions

All of the Review Questions for this module are based on the following data for an investment proposal being considered by Weir Fishing Tackle plc to purchase and place into service a robot reel fabricator. The investment is expected to have only a three-year life because of technological advances. The basic marketing, operational, taxation and asset cash flows of the project (the **changes expected in the company's cash flows were it to accept the investment**) are as set out in Table 4.8.

Table 4.8 Weir Fishing Tackle plc: robot reel fabricator (£s)

	Now	Yr 1	Yr 2	Yr 3
Customers		+10 000	+14 000	+9 000
Operations		−3 000	−6 000	−4 000
Assets	−7 000			+1 000
Government	nil	−4 122.47	−5 095.84	−1 065.22

(*Note*: Do **not** attempt to derive the taxes shown by applying Weir's income tax rate, which is 50 per cent, to the displayed cash-flow estimates. As you are aware, taxes are based upon accounting 'income', numbers which are likely to be quite different from cash flows.)

Weir's finance department has decided to finance the project with both debt and equity capital. The company's intention is to keep Weir's market value ratios of debt and equity at 25 per cent debt and 75 per cent equity. If this is done, Weir thinks that equity's required rate *(re)* will be 16 per cent whilst debt's (*(rd)* will be 12 per cent.

4.1 Using the appropriate formulations for the WACC–NPV (see the cash flows and discount rates in Learning Summary above), make a recommendation to Weir as to the desirability of the robot reel fabricator project.

4.2 Is it proper in answering Question 4.1 to include the cash-flow effects upon the company of the income tax reductions produced by the debt used to finance the investment?

A. Yes, these must be included, because they are legitimate cash flows caused by the project, as we have defined such cash flows.
B. No, because the effect of interest deductibility has been captured in the formulation of the WACC for the project, and to include it in the cash flows would be double counting.
C. Yes and no, because you can get the same answers for NPV if you appropriately adjust the cash flows to include the effect of interest deductibility while excluding that effect from the discount rate, or vice versa.
D. No, because we have not yet been told how much interest Weir will be paying, and thus interest tax shields cannot be known for the purposes of Question 4.1.

4.3 Suppose that you were asked to explain to the uninitiated in general terms what economic information is contained within the WACC. Choose from among the responses below the one that which you consider to be the best:

A. The WACC is the discount rate that will tell you how much an investment will be worth.
B. The WACC tells you the minimum acceptable return that must be earned on the funds invested in a project.
C. The WACC takes account of the returns required by capital suppliers, and the tax effects of financing.
D. The WACC applies to the return required by shareholders were the investment to be all-equity financed, and separately discounts the tax effects of interest deductibility of debt (interest tax shields).

4.4 A positive NPV is:

I. The present value of the amounts by which the investment's cash flows exceed the returns required by the capital suppliers of the firm undertaking the investment.
II. The net amount of the increase in the market values of capital claims outstanding, less the outlay necessary to undertake the project.
III. The increase in wealth of the shareholders of the company.

Which of the following is correct?

A. I and II.
B. II and III.
C. I and III.
D. I, II and III.

4.5 Suppose that you are now informed that the interest and principal payments for the debt of the robot reel fabricator are expected to be (£s):

	Now	Yr 1	Yr 2	Yr 3
Interest	–	244.94	191.68	130.44
Principal	–	443.83	510.39	1086.96

If Weir were to finance this project totally with equity, the project's required return (ru) would be 15.07 per cent.

With this information, calculate the APV of the investment, and compare it with the NPV calculated for Question 4.1.

4.6 The APV is preferred to the NPV by some financial practitioners because:

A. The APV is more easily amenable to inclusion of financing-induced effects upon shareholder wealth (other than interest deductibility) than is NPV.
B. They do not know any better.
C. The APV requires only that proportions as opposed to actual amounts of borrowing be pre-specified.
D. There is a seemingly psycho-social bias toward presenting numbers as rates of return as opposed to cash-stated wealth changes.

Module 5

Estimating Cash Flows for Investment Projects

Contents

Learning Objectives

This module is devoted entirely to a detailed example of how a company would go about deciding on a major proposal to enter a new line of business. In it, a firm is considering introducing a new product line, similar to its current business, but with major implications for the future cash flows of the company. The central issues in the example deal with which of a company's cash flows are relevant to an investment decision and which are not. In particular, the module gives significant attention to the issues of estimating appropriate cash flows based upon the types of accounting data normally available within companies for decisions of this type. The student should pay special attention to solving the case at the end of the module, which gives the opportunity for a full analysis.

5.1 Introduction

In Module 3 and Module 4 our main concern was to study the basic techniques of company investment decision making, to understand why they take the forms that they do, and how they are consistent with the primary corporate goal of maximising shareholder wealth. Now that you have a good appreciation of these important characteristics of the techniques, we shall begin studying an equally important set of topics: how these techniques are implemented in real companies. This module will concentrate upon the process of estimating the cash flows to be evaluated by the techniques.

You recall that we introduced the cash flows to be included in a corporate investment analysis by examining Figure 3.1 in Module 3, which shows the general outline of the transactions that corporations undertake with outside entities. Those transactions are: with customers; with suppliers of labour, materials, services and assets; with government; and with suppliers of capital. In the Cheetah and Lynx projects we examined examples of these cash flows, and how they related to some of the most commonly encountered accounting measures such as profits and depreciation. Now we shall go 'behind the scenes' in the cash-flow estimation process to see exactly where those numbers came from, and some of the pitfalls and errors that financial managers must avoid in order to make good investment decisions.

Our goal is to arrive at a correct set of estimates for FCF_t^*, the free cash-flow measure necessary for either the NPV or APV (or IRR) investment evaluation techniques. The basic idea to keep in mind is that we must include all of the changes that are expected to take place in the corporation's cash flows if the project is accepted, and include **nothing but** cash flows. (When an NPV analysis is undertaken, we estimate the cash flows as if the project is to be 'all-equity' financed; if APV is the technique of choice, we also estimate the interest tax shields that any debt issued for the project would generate.)[1]

Here is a list of some of the main ideas that financial managers must keep in mind when they estimate investment cash flows:

1. 'Inclusion of all corporate cash flows affected by the investment' sometimes means that financial analysts must invoke the idea of **economic opportunity costs**. For example, suppose that a company is considering a new product line that could be produced in a plant that the company already owns. The correct cash flow to include as the cost of the plant for the project is the amount for which the plant could be sold, or the amounts of cash that the plant would generate if it were used in the best alternative to the project in question. Naturally, if the project is to be charged with a cash flow for the plant's value at the project's outset, the project should be given a cash inflow for the time point in the future when the project no longer needs the plant, at which time it can be sold. The correct inflow at that future time is the expectation of the price that can be obtained for the plant at the time it is expected to be sold, or the value of its best use. There are other important examples of opportunity costs, such as management's time.
2. 'Inclusion of all relevant cash flows' means that analysts must include cash flows from interactions of the investment with other activities of the corporation. For example, suppose a company intends to begin selling a new product that it has not marketed before. Further suppose that this product, if introduced, will likely cause some of the corporation's customers to shift their demand to the new product from another product that the company also produces. In such a situation, the reduction in the cash flows from the company's other product must be included as a cash outflow for the new product, because this is a change that will be induced in the **company's** cash flows if the new product is introduced. Analysts must be careful here, however, to be realistic about the displaced sales of

[1] Note that we use the term 'expectations' or 'expected' to characterise the cash-flow estimates used in evaluating projects. Our use of such terms is a signal that the actual amounts of these cash flows that will occur in the future are not known for certain. Thus you should recognise that such estimates are not deterministic but are more in the nature of 'best guesses' at the time such estimates are made. We shall have more to say about these estimates and their uncertainties in a succeeding module of the course.

the other product. Often, when a company is about to introduce a new product its competitors are not far behind; if the other product's cash flows are about to decline anyway because of competitive pressures, it would not be correct to charge those declines to the new product. In estimating investment cash flows, analysts must ask the right questions of the marketing people in a company, and listen to their answers very carefully. Remember, the correct cash flows are the **changes** in the **company's** cash flows if this project is accepted.

3. 'Inclusion of all relevant cash flows' also means that analysts must know what things should be omitted from the investment's cash flows. For example, one must never forget the doctrine that 'sunk costs' are to be ignored. 'Sunk costs' are cash outlays that have already been made. The fact that a company has millions 'invested' in a project is often argued as a reason that the project should be continued or even augmented. This is simply bad economics; whether or not an investment deserves to be increased or continued must depend totally upon whether it will generate enough future cash flow to make any necessary additional investment worthwhile. Said another way, an investment should be discontinued if its future cash flows' present value is less than what the company would obtain by selling or even abandoning the project, now or later.
4. 'Inclusion of all relevant cash flows' means, thirdly, that analysts must be very careful that the accounting numbers provided for a project are interpreted correctly. For example, analysts are usually given figures by the accounting department for 'overhead' that a project will incur. It would almost certainly be a mistake to regard those amounts as cash outflows that the company will experience if the project is accepted. There are two main reasons for this. The first is that overhead can include non-cash expenses, such as depreciation, in addition to overall corporate costs for things such as the electricity and gas bills, and head office functions such as financial analysts' salaries. The second reason is that accountants 'allocate' overhead expenses on the basis of arbitrary rules which have no necessary correspondence to the additional or **incremental cash flows** that a project will require. Accountants perform many such overhead allocations on the basis of measures such as the amount of plant floorspace devoted to the manufacture of the product, or to some arbitrary activity measure. Neither of these will generate an accurate estimate of the changes in the company's 'overhead' cash flows that will occur if the project is accepted.

   By no means should you interpret this to say that overhead expenses are irrelevant to corporations; accountants doubtless have reasons for figuring the numbers the way they do. Our point is that we are interested only in cash flows, and only in those that will be caused by the investment. If the project will cause the company to have higher electricity bills, it is correct to include as cash outflows the **increments** to overall corporate electricity bills caused by the acceptance of the project. If new managerial talent will be hired because of the project, its cost is appropriately included (or if old managers' effectiveness in their other efforts deteriorates because of the demands of the new project, the cost reckoned for that deterioration is an incremental cash outflow of the new project). These costs will probably be quite different from the overhead allocations that accountants will charge against the project, but are nevertheless the correct ones for an investment analysis.

This list is not, of course, exhaustive of all the cash-flow estimates about which analysts must be careful, but it is instructive. Note that all of the examples have a common thread: the only amounts of relevance to an investment analysis are the **changes or increments** that are expected to take place in the **cash flows** of the corporation were it to accept the investment. We urge you to take a most literal interpretation of this instruction: if an amount is offered as 'relevant to an investment', it must pass that test to be included as a cash flow for analytical purposes.

There are many corporate cash flows that should not be included because they are not incremental; they would not be affected by the project's acceptance or rejection (managerial salaries often fall into this category). There are also many expenses and revenues that should not be included because they are not cash amounts (depreciation, inventory cost allocations, and changes in receivables or payables are all examples). There are other cash flows that are relevant but are always in danger of being overlooked (like interactive effects of a project with the other activities of the company, or increments to the amounts of cash that the company will need to keep in the bank to facilitate the transactions that the investment will generate across its lifetime). As you can see, it would be impossible for us to give you a complete checklist or template of cash flows to accept as relevant or to reject as irrelevant. The best analysts are those who understand the basic principles of cash-flow estimation, and apply those consistently to the projects they are evaluating.

5.2 A Cash-Flow Estimation Example

Imagine yourself as a fresh young MBA having just finished your finance course and found a lucrative job with Remote Systems plc, a successful corporation known for its willingness to give significant responsibility to fledgling financial analysts. During your first day in the job your boss calls you in and assigns you the task of evaluating a new product-line proposal that the company is considering, namely the WalkPhone, a telephone that has no cord and can be carried around the house. The boss tells you that in response to a request for information about the proposal, the Marketing, Accounting and Production Departments have put their heads together and come up with the financial statements given in Table 5.1 and Table 5.2 for the WalkPhone.

Table 5.1 WalkPhone project: pro forma balance sheets (£000s)

	t_0	t_1	t_2	t_3
Current assets (working capital):				
Cash and marketable securities	600	600	600	0
Accounts receivable (debtors)	0	500	500	150
Inventory (stocks)	500	650	220	0
Total current assets	1 100	1 750	1 320	150

	t_0	t_1	t_2	t_3
Fixed assets:				
Gross plant and equipment	1 300	1 500	1 500	1 500
Accumulated depreciation	0	300	600	1 500
Net fixed assets	1 300	1 200	900	0
Total assets	£2 400	£2 950	£2 220	£150
Current liabilities (working capital):				
Accounts payable (creditors)	0	300	800	150
Total current liabilities	0	300	800	150
Long-term liabilities and equity:				
Debt	1 300	1 000	800	0
Equity:				
Paid-in equity capital	1 100	1 475	620	0
Retained earnings	0	175	0	0
Total equity	1 100	1 650	620	0
Total long-term liabilities and equity	2 400	2 650	1 420	0
Total liabilities and equity	£2 400	£2 950	£2 220	£150

Though your boss trusts that these are good estimates, she is informed enough about modern finance to know that the accounting numbers in these financial statements cannot be directly applied to the decision at hand, namely whether Remote Systems should embark upon the WalkPhone project. She would like you to manipulate the above information, collecting more as necessary, to produce a correct analysis.

Table 5.2 WalkPhone project: pro forma income statements (£000s)

	t_0	t_1	t_2	t_3
Revenues:				
Gross operating revenues	0	1 500	2 000	1 750
Less returns	0	0	−30	−100
Net operating revenues	0	1 500	1 970	1 650
Operating expenses	0	750	1 350	1 250
Operating profit	0	750	620	400
Other revenues:				
Sales of fixed assets	0	0	0	950
Total other revenues	0	0	0	950
Other expenses:				
Depreciation	0	300	300	900
Interest	0	130	100	80
Total other expenses	0	430	400	980

	t_0	t_1	t_2	t_3
Profit before tax	0	320	220	370
Income tax	0	160	110	185
Profit after tax*	0	160	110	185

* Not shown is a t_4 negative £60 000 profit before tax due to an expected bad debts write-off from the project at that time.

The first data that claim your attention are the estimates of operating revenues made by the Marketing Department (*see* Table 5.2). You are fully aware that operating revenues are not necessarily the same thing as cash inflows from product sales, and so you decide to grapple with that problem first. You go to the Marketing Department to enquire about the extent to which the revenue figures for WalkPhone are really cash amounts.

'Our responsibility is to sell, not to be collection agents. Go and ask the Credit Department,' responds the Marketing Department head, indignantly. Tail between legs, you hustle to the Credit Department. The credit manager indicates to you that cash receipts can be figured by adjusting the operating revenue numbers for the changes in accounts receivable (debtors). Any increase in an account receivable from one period to another means that net operating revenues overestimate cash receipts by that amount, and vice versa.

'May I ask a stupid question?' you enquire of the credit manager. 'I assume that the accounts receivable in the balance sheets are the total amounts of money that Remote Systems' customers will owe on their WalkPhone purchases at the times indicated. If that is so, I do not understand why we show £150 000 as accounts receivable as at t_3 [*see* Table 5.1] when all of the other asset estimates are finished at that point. The amount owed at t_3 implies that there will be another cash receipt at t_4 when those people finally pay up.'

'Very perceptive,' offers the credit manager. 'It is in fact true that some of the sales made in the last period that we intend to sell the WalkPhone will not result in receipt of cash until the next period. The only thing I would add is that, of the £150 000 in accounts receivable at t_3, we will probably never collect £60 000. That amount is what we expect in bad debts coming from sales in **all** periods. We probably should have included a negative profit figure for t_4 in the pro forma income statements, but we put it in as a footnote instead. You might also notice that we expect to owe £150 000 to our suppliers at t_3 from the WalkPhone project [*see* Accounts payable in Table 5.1]. Of course we intend to pay that in full by t_4.'

Returning to your office, you recall that operating revenues overestimate receipts in any period by the amount of the increase in accounts receivable during that period, and vice versa. With that idea in mind, you start to do the necessary adjustments to the sales revenues estimates of WalkPhone, but immediately another thought strikes you: if revenues are potentially misestimating cash inflows, will not expenses also do the same for cash outflows? So you return to the Accounting Department.

'Can you answer a few questions about the expense figures for the WalkPhone project?' you ask. 'I am particularly interested in the numbers for operating expenses.'

'Certainly,' answer the accountants. 'We estimate operating expenses as follows.' (At this point the accountant launches into a completely confusing lecture about how these expenses are estimated, the interaction with inventory (stocks) valuation methods called 'LIFO' and 'FIFO', the crediting and debiting of accounts payable, and the company's overhead allocation procedures.)

You listen politely to the explanation, and ask what you should have asked in the first place: 'Suppose that I wanted to change each of your expense numbers into the actual amounts of cash that the company was going to spend for operations due to the WalkPhone project, and would not otherwise have spent. How would I do that?'

'Oh,' say the accountants. 'Why didn't you ask that in the first place? To take care of the payable and inventory adjustments, we would suggest that you subtract from our expense figure the increase in payables, and add to it the increase in inventories and cash, and vice versa for decreases in payables and decreases in inventories and cash. For overhead, you can subtract the overhead expenses, and add estimates of cash payments in the company's overhead accounts that are caused uniquely by the project. We have all of the necessary information for those adjustments right here, but we would not recommend that you make the adjustments that way.'

'Why not?' you say. 'It sounds like a pretty complicated process, but I suppose that I must go through all of those adjustments if I wish to estimate actual cash flows.'

'It would indeed be a very complicated process separately to adjust each operating expense (and revenue) item to be a cash flow, and it would also be unnecessary,' says the Accounting Department representative. 'There is a much more straightforward process, which accomplishes the same thing. Recall that the balance sheet for the WalkPhone project [Table 5.1] contains current asset and liability items which sum as follows (£000s):

Total current assets	1100	1750	1320	150
Total current liabilities	0	300	800	150

The difference between those has a name: "net working capital", and is:

Net working capital	1100	1450	520	0

If you simply examine the **changes** that occur in net working capital from period to period, you will adjust for almost everything about which you are concerned [*see* Table 5.3].

Table 5.3 WalkPhone project: net cash from sales and after outlays other than fixed assets, taxes and capital (easy calculation: £000s)

	t_0	t_1	t_2	t_3	t_4
Operating profit	0	750	620	400	−60
Less change in					
net working capital	−1 100	−350	+930	+520	0
Plus overhead changes	0	+70	+80	+80	0
Operating cash flow	−£1 100	+£470	+£1 630	+£1 000	−£60

'This is indeed a correct and much simpler method of estimating operating cash flow. The reason it works is that almost all of the adjustments you would have to make so laboriously to the separate operating revenue and expense items on the income statement in fact come from changes in working capital accounts (receivables, payables, inventory and cash). Well, by going directly to the total change in **all** of those amounts, the change in net working capital, we are simply saving a lot of detailed work, and accomplishing the same thing. The reason why we subtract that from the operating profits is that operating profits is the difference between net operating revenue and operating expenses, the two numbers upon which you began making all of the adjustments in the first place.

'Finally, the reason why we must add the overhead adjustment separately is that the overhead problem was not a working-capital consideration, so it would not have appeared in the changes in those accounts. The numbers you see as cash flows for changes in the company's overhead due to the project are figures that we got from our records. You must trust us to have estimated those correctly.'

'What you say has a ring of truth,' you agree. 'But I shall have to think about it for a while. Meantime, I take it that the negative £60 000 operating profit figure at t_4 is the write-off of bad debts that the Credit Department mentioned to me earlier?'

'Yes, and if you had read the footnote to the income statements you would have seen that for yourself,' scolds the accountant.

At this point you pause for a moment to consider what you have discovered and what has yet to be done. You have arrived at good figures for the cash flows that the company will experience from operating the WalkPhone project, including cash inflows from sales, and outflows for operating expenditures, inventory and cash and marketable securities. The only items yet to be included are the taxes that Remote Systems will pay due to the project, and the cash flows from originally purchasing and subsequently selling the necessary fixed assets.

Taking a deep breath, you begin to consider the cash flows for fixed assets. A glance at the project's balance sheets (Table 5.1) indicates that the company would invest £1 300 000 in such assets at t_0, and that this investment would increase to £1 500 000 at t_1. A quick call to the Production Department verifies that these amounts are indeed a cash outlay of £1 300 000 at t_0 and an additional £200 000 at t_1. The Production people indicate that these are the only such outlays for the

WalkPhone project. In response to your question about what will happen to those assets at the end of the WalkPhone, they say that the plan is to sell them, and that a £950 000 inflow is expected at t_3 from that source. You refer to the income statements (Table 5.2) and spot an item of other revenue for t_3 that corresponds to that cash inflow.

So it turns out that the fixed-asset cash flows are quite straightforward.[2] The fixed-asset cash flows you calculate are given in Table 5.4.

Table 5.4 WalkPhone project: fixed assets cash flows (£000s)

	t_0	t_1	t_2	t_3	t_4
Purchases	1 300	200	0	0	0
Sales	0	0	0	950	0
Fixed asset cash flow	−£1 300	−£200	0	+£950	0

The only remaining cash flow to be dealt with is taxes. And that one is the easiest yet. WalkPhone's income statements (Table 5.2) indicate that the project's taxes are expected to be as in Table 5.5 (remembering the £60 000 t_4 bad debt write-off and tax at 50 per cent).

Table 5.5 WalkPhone project: taxes (£000s)

	t_0	t_1	t_2	t_3	t_4
Income tax	0	£160	£110	£185	−£30

Because they have been so helpful in the past, you call the Accounting Department with the only question that comes to mind about the tax calculation. 'What about these fixed asset sales that we plan at t_3? I see that we have included the cash amounts as other revenues, but are such things taxed in the same way as any operational revenues? I thought that asset sales were somehow different. What is the term "capital gains" that I hear so often? Does it have anything to do with asset sales?'

'You've asked a lot of questions,' respond the accountants. 'To be absolutely honest, the best answer we can give you in general about tax questions is simply to trust we have figured them correctly, and go on to some more productive pursuit. The reason is that tax laws vary so much, depending upon which government is doing the taxing, and they also change too much across time, so that it takes many full-time tax experts to keep up with the tax rules. We do have some tax people (they are part of the overhead that you were so concerned about earlier) who do nothing but that.

2 Actually they were straightforward anyway because the accumulated depreciation item on the balance sheet was shown separately, before the net fixed assets accounting number was calculated. Many companies show just the latter, especially on their publicly released statements. If the income statements are not detailed enough to show the depreciation expense as a separate item, figuring out the fixed-asset cash flows can be a nightmare.

'The specific question that you have asked is about the taxation of fixed-asset sales. The way our laws work right now is that we are allowed to write off as an expense anything that is still on the books (undepreciated book value) for an asset when we sell it. The only time that capital gains is important is when we sell an asset for more than our original purchase price, and we do not plan that here. So the other revenue from asset sales is taxed at the normal corporate income tax rate, and we are allowed to reduce our taxes on that to some extent by a "shield" created by the remaining book value of the asset sold.

'WalkPhone assets are to be depreciated "straight line" over the life of the project, should it proceed, and the original £1 300 000 t_0 outlay has a £400 000 "salvage value" for depreciation purposes, leaving £900 000 to be depreciated. A depreciation-based salvage value is not really a value in the sense that you understand the word; it is an amount that we keep on the books for an asset and do not expense as depreciation until we sell the asset in question. The £200 000 t_1 asset outlay is not regularly depreciable at all, and is to be simply written off against other revenue at t_3. You can thus see the origin of the depreciation expenses in the income statements [Table 5.2]. The £300 000 expenses at t_1 and t_2 are the regular straight-line charges for a three-period depreciation of the £900 000 of depreciable value of the t_0 asset purchase. The £900 000 t_3 depreciation is the sum of the last £300 000 of that expense, the £400 000 book salvage value of that asset, and the £200 000 on the books for the t_1 asset outlay. Actually, we plan to sell the t_0 asset for £800 000 (more than its book value at that time), and the t_1 asset for £150 000 (less than its book value), but that makes no difference. They will both be subject to the same income tax rates (our taxes will be a bit higher because of the first asset's sale, and a bit reduced because of the "loss" on the sale of the second asset), but it makes no difference if we combine the revenues and depreciations from them or not. The taxes will be the same.

'Now if you really want to hear something complicated, let us tell you about what would happen if we had taken an investment tax credit on either of those assets, and had sold them before the prescribed holding period!' chuckles the accountant.

'I would love to, but I have a bit of a headache,' you reply. 'I shall be back soon to hear your story about that. It sounds enthralling,' you lie.

Again returning to your office, you consider what you now know about the expected cash flows of the WalkPhone project. With the information about the taxes that Remote Systems will experience from the investment, you have covered all of WalkPhone's transactions other than capital cash flows. You know what the project is expected to generate in terms of cash flows, both inflows and outflows, with labour and asset suppliers, and with government. The summary of those cash flows is given in Table 5.6.

Table 5.6 WalkPhone project: summary of cash flows (£000s)

	t_0	t_1	t_2	t_3	t_4
Operating cash flow	−1 100	+470	+1 630	+1 000	−60
Fixed-asset cash flow	−1 300	−200	0	+950	0
Less income tax	0	160	110	185	−30
FCF	−£2 400	+£110	+£1 520	+£1 765	−£30

Proudly you carry the results of your labours to your boss, who eagerly peruses the numbers.

'Well, I shall say this for you. You certainly came up with results that look different from the profit figures that appeared in the pro forma financial statements for the project. Without going into all of the detail of where you got them, can you explain to me what these figures are, and how we should use them?' she asks.

'I shall be delighted to,' you respond. 'Let me call your attention first to the "bottom line" number in this type of analysis, namely the figures labelled FCF in the summary of cash flows [Table 5.6]. The amounts that you see listed are our expectations for the changes that will occur in Remote Systems's cash flows if we accept the WalkPhone project. The FCF cash flows are the amounts of cash that the company can distribute to the debt and equity capital suppliers of WalkPhone without disturbing the currently held expectations and plans for the project.'

'Interesting, and well explained,' says your boss. 'But I know all that. What I really would like is for you to explain the general steps by which you arrived at the numerical results you have in the summary.'

'OK,' you answer. 'You see that the first line in the summary is the "Operating cash flow". These are the amounts of cash that we will take in and pay out from operating the assets that we use in the project. It includes the cash from selling WalkPhones, and the costs of producing them including the amounts of cash that we use as working capital. The easiest way to come up with this number is to subtract the changes in net working capital from operating profit, and then include any non-working capital adjustments to cash flows that are necessary to turn the operating profit number into cash. One example is the difference between overhead allocations and the actual changes in cash overhead that the project will induce at Remote Systems.

'The next line is "Fixed-asset cash flow". That is simply the amounts of cash that we expect to spend on assets such as machines, plant and equipment, and the amounts of cash that we expect to get when we sell them. Or if we do not expect to sell the assets at the end of the project, we would include the benefit that the company would expect to receive from using the assets in whatever operations they are intended for (and that, of course, will be more than what we could sell them for, or we would have sold them).

'The cash flows for taxes appear next. Those amounts come from the taxes that will be due to be paid, based upon the income statements for WalkPhone. The taxes are based on the classic accounting definitions of income or profit, and cannot generally be deduced without having calculated those numbers. Naturally, taxes include not only those from the operations of WalkPhone, but also the effect of asset depreciation and sales of assets. FCF is simply the result of adding operating cash flow to fixed asset cash flow, and subtracting taxes.'

5.3 Calculating the NPV, APV and IRR of the Example

'Good,' says your boss. 'That is exactly what I wanted. Now we should see if the project is worthwhile. I have contacted our experts on the matter, and they have informed me that the appropriate "cost of capital" for the WalkPhone is 18 per cent per period. I guess all we need to do now is discount your cash flows at that rate to find the project's NPV.'

'Before we do, may I see the report that shows the cost of capital?' you ask. You establish that the report says: 'In our opinion the appropriate cost of capital (rv^*) for the WalkPhone project is 18 per cent per period.'

'Aha!' you exclaim. 'I thought so. If we wish to use the 18 per cent discount rate, we must perform one other adjustment to the FCF cash flow in the summary. From my recent Finance course I know that the cost of capital, rv^*, has been reduced to reflect the deductibility of the project's interest payments. But the FCF, by using the actual taxes that WalkPhone will pay, also has included the tax deductibilities of interest. We must adjust the FCF to be FCF*, by taking out the interest tax shields (i.e. the interest payments from Table 5.2 multiplied by the income tax rate[3]).' The adjusted amounts are shown in Table 5.7.

Table 5.7 WalkPhone project: calculation of FCF* (all amounts in £000s)

	t_0	t_1	t_2	t_3	t_4
FCF	−2 400	+110	+1 520	+1 765	−30
Less interest tax shields	0	65	50	40	0
FCF*	−£2 400	+£45	+£1 470	+£1 725	−£30

'**Now** can we find the NPV of WalkPhone?' your boss asks.

'Yes, we finally can,' you respond (amounts are again in £000s):

$$\begin{aligned} \text{NPV}_0 &= \sum_{t=0}^{n} \frac{\text{FCF}_t^*}{(1+rv*)^t} \\ &= -£2400 + \frac{+£45}{(1.18)} + \frac{£1470}{(1.18)^2} + \frac{£1725}{(1.18)^3} + \frac{-£30}{(1.18)^4} \\ &= -£272 \end{aligned}$$

'The WalkPhone project is no good,' you say. 'If we accept it our shareholders' wealth will decrease by £272 000.'

'Well, if that is the case, we should certainly not accept the project,' declares your boss. 'But I know there are a lot of people around the company that will be disap-

[3] If taxes had originally been calculated ignoring the interest deduction, this step would of course be unnecessary. Many companies estimate project taxes in just that manner.

pointed at that outcome. So we had better have all of our arguments in line for the questions that will be asked. I have heard of a couple of alternative techniques of making investment decisions that are supposedly consistent with modern finance. I believe they were called the adjusted present value and the internal rate of return. Is it a lot of trouble to see how the WalkPhone would look under them?'

'Not at all,' you say. 'Let me see if that report from your capital market experts says anything helpful.' As it turns out, it does. The report says: 'In our opinion the unleveraged required return *(ru)* for the WalkPhone project is 21.08 per cent, and the required return to debtholders *(rd)* is 10 per cent.

With this information you can calculate the APV for WalkPhone:

$$
\begin{aligned}
\text{APV} &= \sum_{t=0}^{n}\left[\frac{\text{FCF}_t^*}{(1+ru)^t}+\frac{\text{ITS}_t}{(1+rd)^t}\right] \\
&= -£2400+\frac{+£45}{(1.2108)}+\frac{£1470}{(1.2108)^2}+\frac{£1725}{(1.2108)^3}+\frac{-£30}{(1.2108)^4} \\
&\quad +\frac{+£65}{(1.1)}+\frac{£50}{(1.1)^2}+\frac{£40}{(1.1)^3} \\
&= -£402+£130 \\
&= -£272
\end{aligned}
$$

So the NPV and APV techniques give the same result, as they should. (Note the two different discount rates in the APV analysis.)

The calculation of an investment's internal rate of return, you recall from the first module of your Finance course, requires that you find the discount rate that will cause NPV to be zero. (Or the IRR can be calculated by finding the rate that causes the present value of an investment's cash outflows to be equal to the present value of its inflows, which is to say the same thing.) By trial and error, you solve the following for the IRR:

$$
\begin{aligned}
0 &= -£2400+\frac{+£45}{(1+\text{IRR})}+\frac{£1470}{(1+\text{IRR})^2}+\frac{£1725}{(1+\text{IRR})^3}+\frac{-£30}{(1+\text{IRR})^4} \\
\text{IRR} &= 12.39\%
\end{aligned}
$$

The WalkPhone project has an internal rate of return of 12.39 per cent, which can be interpreted as the average per-period rate of return on the resources that would be invested. Since the financial market consultants have advised that the project's cost of capital is 18 per cent, our comparison of the IRR with the returns that capital suppliers can earn on equal-risk investments is:

IRR = 12.39% (return on WalkPhone project)

rv^* = 18% (return available on equal-risk investments)

So the WalkPhone, if accepted, would yield a return substantially less than that which is available on comparable investments in the financial market. (This is often called the 'hurdle rate', a graphic description of what the IRR must do to be acceptable; the rv^* is the IRR's hurdle rate if the cash flows used to calculate the IRR are those that would have been used for an NPV analysis.)

Here the IRR tells us the same thing as the NPV and APV do: reject the WalkPhone project. The NPV and APV tell us exactly how much wealth will be lost by the shareholders of Remote Systems if the project is accepted, whereas the IRR simply says the project's return is too low.

'Excellent,' says your boss. 'The message is clear. We should recommend that the WalkPhone be rejected.'

Learning Summary

The example in the previous section should serve at least two purposes in your Finance education. First, the example shows several 'tricks of the trade' that are valuable for financial analysts to know when attempting to estimate project cash flows. Below we remind you of the highlights of those procedures. As important – perhaps more so – is the example's consistent application of the **general principles** of investment cash-flow estimation. By this we mean that the major importance of an illustration like the WalkPhone example is not that you learn specific procedures (though they are useful, indeed necessary), but that a correct investment analysis requires adherence to certain principles of cash-flow estimation. In their most succinct form, these can be distilled to the following:

1. Include all changes that will occur in the cash flows of the corporation were it to accept the project, at the time points when those cash-flow changes are expected to take place. Those cash flows comprise all transactions that the corporation would undertake with suppliers of labour and management skills, with suppliers of materials, services and assets, and with government. Operational opportunity costs are legitimately included in a project's cash flows, but cash flows to and from capital suppliers are not.
2. When the cash flows are being estimated for an NPV or IRR analysis, it is not necessary to estimate the interest tax shields for the project; the cash flows are all-equity flows. When an APV analysis is to be done, interest tax shields must also be estimated, based upon the debt issued to finance the project.

These principles seem easy enough to comprehend, but as with so many straightforward ideas, implementation is more complex. The WalkPhone example is reasonably rich in applying the general principles of cash-flow estimation to the specifics of a realistic investment decision. In the WalkPhone example, the following cash-flow considerations were important:

1. The project's cash inflows from selling WalkPhones to customers were expected to occur across time in a pattern different from the way revenues were recorded on the accounting records of the company. Use of revenues as the cash inflows from that source would produce erroneous results.
2. Similarly, the costs of the project in terms of the cash outlays for operational activities (labour costs, management salaries, utilities costs, raw materials costs, and so forth) also had a pattern different from the accounting costs registered across time as operational expenses.
3. The accounting expenses for overhead, depreciation and interest generally differed from cash flows, though the transactions that they represented may have had cash-flow importance. For example, we found depreciation to be the accountant's way of recognising the original cash outlay for acquiring the asset,

whereas in the cash flows of the investment it was important to recognise the cash outlay for the asset at the time it was expected to occur. Overhead accounting expenses allocated to the project were not the actual changes in the overhead-type costs that the company was expected to experience should the project be accepted. The accounting overhead numbers needed to be thrown out and the correct cash-flow changes substituted. Interest, of course, was not a cash outflow at all if the purpose was to estimate the cash that is to be **available** to service capital claims. Interest goes to a capital supplier, and therefore was not subtracted from the total that we sought.

In the WalkPhone example we presented these complications in the context of a situation where the only data available were from accounting pro forma (expected) income statements and balance sheets. In that example we adjusted operating profits to be the cash flows that they represented by subtracting the changes in working capital and adding the overhead adjustments. We found the timing and amounts of cash that the company would be paying for fixed assets, and the money that would be received upon their sale by examining the changes in book values of the fixed asset accounts and other revenue items.

All of this was reasonable to do in the context of the WalkPhone problem in order to get as close as possible to actual cash-flow expectations. Your accounting friends, however, in reading the WalkPhone example, will doubtless point out to you that we were making several optimistic assumptions about the way the accounting numbers were formed, so as to be able to claim that our adjustments created cash flows from accounting data. We would be the first to agree, but also to argue that, if the question at hand is whether or not the WalkPhone project should be undertaken and the only data available are financial statements, the adjustments we made are by far preferable to using the unadjusted accounting numbers to decide about the project.

One corollary is that financial analysts find their jobs much easier to perform correctly if the company's information system is designed to produce cash-flow estimates in addition to accounting data. In other words, the pro forma financial statements themselves are developed from the same basic information that would be used to form cash-flow figures. The basic information in the WalkPhone example was first 'coded' into accounting numbers, and then 'decoded' into cash flows. Imagine how much easier the analysis would have been had Remote Systems' information system been set up, when queried about the WalkPhone proposal, to produce a report such as Table 5.8. There is nothing that would prohibit such a system being implemented in a company; indeed, many sophisticated corporations have done just that.

Table 5.8 WalkPhone project: cash flows (£000s)

	t_0	t_1	t_2	t_3	t_4
Operating cash flow	−1 100	+470	+1 630	+1 000	−60
Fixed asset cash flow	−1 300	−200	0	+950	0
Less income tax (ignoring interest)	0	225	160	225	−30
FCF*	−£2 400	+£45	+£1 470	+£1 725	−£30

Even if your company has such a system, it is nevertheless quite important that you have the capacity to translate between accounting numbers and cash flows. The main reason is that accounting numbers are still the primary 'language' in which the financial results and expectations of companies are displayed. Not only must you be able to function with cash flows as your decision-making vehicle, but you must convince those who are interested in 'profits' that your numbers are meaningful. Often that will mean performing the type of translations that we showed in the WalkPhone example.

Though our example is fairly rich in realistic detail, there are several common cash-flow considerations that did not appear in it. We have mentioned most of these in the introduction to this module, but let us try to put them in the context of WalkPhone. Suppose, during your investigations of the cash flows, you discovered that:

1. Remote Systems manufactures another telephone product, the StayPhone, which is similar to the WalkPhone but must remain connected by a cord to a telephone outlet. The marketing people are unanimous in their opinion that the introduction of the WalkPhone would cause the sales of StayPhone to decline.
2. The WalkPhone manufacturing process will use a piece of equipment that Remote Systems already owns, and thus is not forced to purchase. The accounting system of the company has not included that equipment in the assets of WalkPhone's balance sheets.
3. One of the assets that Remote Systems will purchase if it accepts the WalkPhone project is actually a replacement for an asset that the company is currently using in other operations. If the project is rejected, the company will stick with the old asset. If, however, the WalkPhone is accepted, the increased volume of WalkPhone production will make replacing the old asset worthwhile, and the efficiency of its other applications will improve.

We could make the list as long as our imaginations permitted, but the above are sufficient to make the point: there are often indirect effects of an investment, which are as legitimate in their effect on corporate cash flows as the direct inflows and outflows from the sales of the product.

The StayPhone consideration cannot be ignored when judging the desirability of the WalkPhone. As a first approximation of the cash-flow effect, we might subtract from the WalkPhone cash flows the loss in cash inflows (and outflows) that would be caused in StayPhone. A more sophisticated approach would be to question whether StayPhone's sales might decline regardless of WalkPhone's introduction, if other companies are considering the same technology. Were that to be true, there is less to be said toward penalising WalkPhone for StayPhone's problems.

The piece of equipment that is already owned by the company, and would be used in WalkPhone production, may also have cash-flow implications for the project. If on rejecting the WalkPhone the company would have simply let the asset sit idle for the duration of the project, and if using it does not cause a decline in its eventual sale value, then it can be ignored in WalkPhone's cash flows. If, however, the asset could have been used productively elsewhere in the company, the loss of the cash flows associated with that use must be included in those of accepting WalkPhone, and thus using the asset there as opposed to its next best use. Or, if the asset were to be sold instead of being used, the forgone cash from the asset sale is an outflow for WalkPhone, and the eventual sale at a perhaps different price is an inflow for the project at the time point at which it is expected to occur. In addition, there may be income tax alterations due to the asset sales, including the effects of any associated depreciation changes that would result.

Any replaced asset must also be considered. If Remote Systems accepts the WalkPhone, it will receive cash from selling the replaced asset immediately, but will lose the cash from selling the replaced asset at the time in the future that it would have, were the WalkPhone to be rejected. Those cash flows must be included in WalkPhone's project analysis. Similarly, there will probably be income tax cash-flow effects from the interaction of the changes in other revenues, and depreciation on the income statements of Remote Systems.

These ideas seem to be overwhelming in their complexity and scope if approached as discrete concerns that must be uncovered and dealt with individually. However, it is most important to remember that these are simply examples of adherence to the basic principles of corporate investment cash-flow estimation with which we introduced this section:

1. **All changes** that would be caused in the **cash flows** of a corporation by its accepting a project *must* be included in the analysis of the project.
2. **Only cash flows** are to be included.

When dealing with the direct cash flows of a project, the implementation of these principles seems obvious; when dealing with the no-less-important indirect effects, one must take care to include and exclude cash flows in strict adherence to these principles.

Review Questions

Case Study 5.1: PC Problems plc

PC Problems plc is a (fictitious) company engaged in the servicing of personal computers on a carry-in basis. The company started out several years ago as a small shop working on household appliances. During the course of normal business, several customers owning small computers asked the shop whether they had the capacity to fix problems with these devices. Technical manuals for such computers, a few simple testing devices, and a small parts inventory (consisting mostly of commonly used memory chips, as well as electronic and mechanical parts) were acquired, and the shop started the rather straightforward process of fixing computers.

Within a short period of time word had spread that quick service was available for PC problems, and demand exploded. New, larger quarters were acquired, a counter service was added, the catchy name invented, and serious growth began. Since that time PC Problems plc has become one of the leading service firms in PC repairs, being eclipsed only by the manufacturers and retailers offering warranty servicing.

PC Problems is now faced with a major decision about its line of business. The company has developed a secure reputation for quality, speed and price among individual users of PCs whose machines are out of warranty and who are willing to bring their computers in for repairs. Though this is a good and profitable market, there are others in which PC Problems could probably compete. One such is the on-site repair of PCs in large businesses.

On-site repair of PCs has the same technological knowledge requirements as bring-in repair, but the business itself is substantially different. Among the important new dimensions of the business for PC Problems are the need to set service contract terms for customers, the acquisition of necessary operating assets to make on-site service efficient (equipped service vans, remote communications devices, etc.), and the marketing of such a service to customers familiar with manufacturers' names but not of independent service companies. In addition, to be practical, PC Problems would doubtless be required to offer warranty service to customers who have a mix of old and new machines, which means negotiating terms with manufacturers offering warranties.

PC Problems have put a study team together to evaluate the desirability of going into the on-site repair business, and they have produced the set of information below.

Customers

The increase in revenues for PC Problems would begin gradually, with £75 000 in the first year, increasing to £150 000 in the second, £250 000 in the third, and thence 10 per cent per annum thereafter. Because the customers are to be most creditworthy, there is little likelihood of bad debts or eventual non-collection. The company is convinced that projections beyond five years are useless, because of technological change inherent in the industry.

Operations

Because the customer base has changed, PC Problems foresees that accounts receivable (debtors) will increase. The company thinks that approximately 20 per cent of each year's revenues will be paid the following year. The normal increases in other working capital accounts are likely to accompany the expansion implied by the new activity, with cash on hand, and inventories amounting to 10 per cent of revenues each year, and accounts payable (creditors) being itself 10 per cent of revenues. Cash on hand and inventories will necessarily be increased at the beginning of the years in which sales based upon them are expected to occur.

Additional repairmen will cost £20 000 in the first year, £40 000 in the second, and will increase at 15 per cent per year each year thereafter. Other direct costs of operating the service (fuel, insurance, training, etc.) are expected to amount to 15 per cent of revenues each year.

PC Problems' accountants have informed the study team that administrative overhead for the project (consisting of management salaries allocated on the basis of revenues) will be 20 per cent of revenues each year. Independent of overhead considerations, it will be necessary to hire a full-time counter-service manager at an annual salary of £10 000 to replace the person who will now be devoted full-time to on-site servicing. The latter's remuneration is £15 000 per annum. Management remuneration is expected to increase by 10 per cent per annum.

Marketing outlays consist of a £25 000 market survey recently completed by the company, and annual expenses of £15 000 in the first year and £10 000 in the second and following years. No increase in marketing outlays is predicted past that point.

Assets

PC Problems must acquire service vans, which will cost a total of £54 000. These vans will be depreciated by the straight-line method over a three-year life with a book (accounting) salvage value of £9000. It is expected that the vans could be sold in year five for £15 000. The communications equipment of choice appears to be the new portable cellular telephones, which will cost £4000 in total (their operating charges being included in the direct costs described above). The phones will be depreciated by the straight-line method on a four-year life with no book salvage value. They will likely become obsolete by the end of the five-year project life.

It is assumed that any other assets (or liabilities) held by PC Problems as a result of this project can be liquidated at the end of year five for their book values.

Government

PC Problems pays taxes on accounting income at the flat rate of 52 per cent. Interest and depreciation are deductible expenses, as of course are all typically deductible operating costs.

Capital suppliers

The company is planning to borrow enough money to pay for the vans and cellular telephones. The interest and principal payment expectations for these are (£000s):

	t_1	t_2	t_3	t_4	t_5
Interest	6.96	5.22	3.48	1.74	0
Principal	14.50	14.50	14.50	14.50	0

PC Problems generally has a policy of borrowing so as to maintain its capital structure at 25 per cent debt on a market-value basis; that is, the company prefers to have 75 per cent of its total market value in equity and 25 per cent in debt. When it adheres to this policy, its borrowing rate is expected to be 12 per cent per period, and its shareholders require a return of 20 per cent per period. The company is expected to continue with this policy, and its shareholders regard this new line of business as risky, but no more so than the current line.

Topics for review and action

1 In order to undertake an investment analysis, the managers of PC Problems understand that they must estimate the changes that are expected to occur in the cash flows of the company were it to accept the project, but they are not confident of their ability to do that. Suppose you have been hired as a financial consultant to advise the company as to the desirability of the on-site service project. Construct the set of relevant cash flows implied by the above information.

(*Note*: This is the most difficult part of the case study. You will have most success by attempting to duplicate the process described earlier in the text of the module. Your first step must be to translate the information provided into a format that is useful. We suggest that a set of financial statements (balance sheets and income statements) for each period is the best place to start.)

2 PC Problems also must decide whether the adoption of the cash flows you have estimated above is desirable. Please advise the company about this, using the WACC–NPV criterion.

3 What do you think of the borrowing plans described in the information above? Is this useful information? What relevance does it have for this investment analysis? Is there any particular technique of investment analysis that would utilise that information differently from the way that you have?

(*Hint*: A company's WACC is in part a function of the **proportion** of debt it uses in financing itself. To test this for PC Problems, recall that the new **value** increment will be NPV + FCF_0^* and the borrowing is £58 000.)

4 Upon the presentation of your report, a marketing manager of the company raises the point that the analysis has not considered the possible erosion of carry-in service revenues due to the new service being offered. How should you respond to such a question?

Module 6

Applications of Company Investment Analysis

Contents

Learning Objectives

Module 6 presents a wide range of tools for companies making real-asset investment decisions (called 'capital budgeting'). The module begins with a review of the set of techniques that are commonly used, and this set includes, in addition to the NPV and IRR techniques already studied, the 'payback period', 'return on investment', 'profitability index' and 'cost–benefit ratio'. The module discusses each of these, pointing out situations when each is worthwhile and when use could produce incorrect results. Module 6 also illustrates investment mutual-exclusivity situations where IRR is potentially misleading as an evaluation technique, and develops a variant of IRR that corrects that difficulty. Other topics include deciding upon investments when capital is in limited supply (i.e. rationed), and dealing with causal economic interrelatedness among investment cash flows, with 'repeatable' investments, and with inflation. Students will emerge from studying this module with a much better 'toolkit' of specific techniques to apply in various realistic environments for company decision making.

6.1 Introduction

In the preceding modules we have dealt at some length with the NPV technique, and its close cousin APV, in corporate investment decisions. Our extensive attention to these two approaches is warranted because they are the methods of investment analysis that hold the greatest general promise of producing shareholder wealth-increasing decisions. But by no means are NPV and APV the only techniques used by companies in their investment decisions. The most widely used competitors of NPV and APV are:

1. Payback period
2. Average (accounting) return on investment (AROI)
3. Internal rate of return (IRR)
4. Cost–Benefit Ratio (CBR) and Profitability index (PI)

We shall examine each of these investment evaluation methods, contrasting them with our preferred techniques. (Henceforth in this section we shall refer to NPV and APV collectively as NPV because the comparisons apply to both, and it saves space.) The end result of these comparisons will be that all of the alternatives to NPV have significant shortcomings, and should **not** generally be used unless an NPV analysis is performed concurrently.

6.2 The Payback Period

The payback period method of investment analysis is quite straightforward. It is simply the number of periods until a project's cash flows accumulate positively to equal its initial outlay. It is a measure of the length of time expected before the investment outlay is recouped. Companies use this method by picking a maximum period of time beyond which an investment's payback will be unacceptable, and rejecting all investment proposals that do not promise to recoup their initial outlays in that time or less.

To see how this technique works, look at the relevant FCF* cash flows of investments A and B:

	t_0	t_1	t_2	t_3	t_4
A	−£1000	+£600	+£500	+£300	+£200
B	−£1000	+£500	+£400	+£500	+£400

Suppose that the company had decided that a two-period payback period was the maximum acceptable for a project. A glance at the above shows that investment A is acceptable and B is not. But suppose also that the appropriate discount rate to apply to the above projects is 10 per cent. Project A's NPV is £321, while B's is £434. So it is clearly possible that payback period and NPV can yield different answers. And if NPV decisions maximise shareholder wealth, payback period must be doing something other than that.

Upon seeing the above example, you might be tempted to argue that we have designed it to make payback look bad, by choosing a minimum time that conflicts with the 10 per cent discount rate. Actually, we did. But that is not to say that payback period can generally be made to work correctly (i.e. to give the same decisions as NPV) by carefully choosing a minimum time period. Suppose, for instance, that we had specified a four-period payback. Then project B would have been allowed under payback, and preferred under NPV. But A nevertheless still has the shorter payback, and if the technique is applied both relatively and absolutely, as it often is, A remains the investment of choice regardless of the fact that shareholders would be better off with B.

We could continue with examples, but to little gain. The problems with payback period are:

1. It ignores all cash flows beyond the minimum acceptable payback period, even though there are often likely to be cash flows of importance beyond that time.
2. It does not discount the cash flows within the minimum acceptable period, thereby effectively giving equal weight to all cash flows within those periods. This is inconsistent with shareholder opportunity costs on the monies invested.

These characteristics of the technique will in many cases cause investments to be erroneously accepted and rejected.

In recent years, bowing to the pressure of modern times, some companies have altered their payback period techniques to be 'discounted payback period'. This procedure calculates the present values of cash flows, and dictates a minimum acceptable period until the discounted cash flows accumulate to equal the initial outlay. This alteration does away with our second complaint about payback period, but leaves the fact that cash flows beyond the accumulation point are ignored. In case you might be interested, the discounted payback periods of A and B are 2.18 and 2.57 periods respectively. (Both projects require all of their first two discounted cash inflows plus the indicated percentage – 18 per cent for A and 57 per cent for B – of the **discounted** third period inflow to total the initial outlay.)

Why do some companies still use payback period with all of its faults? A survey of companies using this investment technique would probably indicate their preference for it depends upon its ease of calculation, simplicity and 'understandability', along with the capacity to adjust for project riskiness by decreasing the required payback period. It is difficult to take seriously statements about ease of computation with the aids available to that end today; and simplicity alone is also difficult to accept because there are lots of even simpler techniques (like flipping a coin, or consulting a fortune teller). We think that the payback-period method is an artefact of the times before discounting techniques were well understood, and when concepts of risk were quite rudimentary. If you think about it a little, you can see that the shorter the minimum payback period, the 'safer' will be the acceptable projects, because of the bias toward near-term cash payout. This attribute of payback period is of course not generally to be desired above the ability to discover the wealth-changing promise of investments, but was probably used to ensure that some minimum standard of cash-flow return safety resided in investments. There are superior techniques available today to accomplish this end, as we shall see in a later module.

Should a company feel it must use a payback criterion, the maximum allowable period itself should be set with some basis in reason. To this end, a rudimentary estimate of such a maximum can be found by:

$$\text{Payback} = \frac{1}{rv^*} - \frac{1}{rv^*(1+rv^*)^n}$$

where n is the number of periods in the project's total lifetime. Among other serious restrictions, this payback is accurate only for projects with fairly constant cash flows each period. Nevertheless, within these restrictions, the above will indicate the number of periods across which, if the original outlay is not returned (in FCF*), the investment will have a negative NPV.

6.3 The Average (Accounting) Return on Investment

The average (accounting) return on investment (AROI) is even worse than the payback period. This method calculates a rate of return on the investment in each period by dividing expected accounting profits by the **net book value** (i.e. depreciated value) of the investment's assets. These numbers are then added together and divided by the number of periods for which rates of return have been calculated. The result is then compared with some standard in the form of a minimum acceptable return (often an industry or company historical average). From our discussions of accounting numbers and cash flows in Module 5, you should have no trouble seeing that this technique would yield the same answers as NPV only by the wildest accident. Not only does AROI not discount cash flows but it also uses the wrong numbers to begin with! Examples of the errors that can be made with AROI decisions should be almost superfluous. Nevertheless most good finance texts still offer an example of the errors, and so we shall too (though we can tell that these other texts' hearts are not in it, because they all use exactly the same example, as shall we).

Suppose that there are three investments (C, D and E), all of which have a £9000 initial outlay, last for three periods, and depreciate the initial outlay evenly over the three periods. (This means that the average depreciated book value of each investment is £4500.[1]) Further, suppose that the investments generate profits and free cash flows as follows:

[1] If something begins at a value of £9000 and steadily declines to £0, its average must be £4500 across time.

Investment	t_1	t_2	t_3
		Profits	
C	£3000	£2000	£1000
D	£2000	£2000	£2000
E	£1000	£2000	£3000
		Free cash flow	
C	£6000	£5000	£4000
D	£5000	£5000	£5000
E	£4000	£5000	£6000

It is (or should be) clear to you that regardless of the discount rate used with NPV as the criterion (as long as the rate is greater than zero) project C is the best project. But AROI would rank all projects as equally acceptable, since the AROI of each is their average profit divided by average assets: £2000/£4500 = 0.44444 … Whether or not these investments would be acceptable under the AROI method depends upon the standard chosen. It is not generally possible to choose a standard minimum AROI that will generate the same answers as NPV.

Why do many companies continue to use the AROI technique? The answer is probably that the habit of judging the result of an endeavour by looking at its accounting profits has been erroneously carried over to prospective investments. Nevertheless, we can also report that the use of AROI as an investment evaluative method is very much on the decline, and justifiably so. It is the rare company that relies solely upon AROI to decide on the projects it will accept and reject.

AROI is, however, often used as an evaluative or control device to check the progress of an ongoing project on a period-by-period basis. This use of AROI is more defensible in that such single-period deviations from targets are less subject to the time-based errors of AROI than are decisions about prospective investments.

6.4 Internal Rate of Return vs. Net Present Value

In this section we address one of the most common conflicts that occur in the investment decision-making process of modern corporations: the choice between NPV and IRR as the evaluative technique. That a complete section in a finance course must be devoted to this topic is one of the curiosities of professional education because the answer in a rigorous sense is obvious: we shall show you that without question the NPV is superior as an investment decision criterion. The strange thing is that more companies continue to prefer IRR to NPV. We have no good explanation for this. The managers of those companies are well educated and understand logical argument, but they continue to feel more comfortable in deciding upon corporate investments on the basis of a technique that is inferior in many instances to an equally accessible alternative. The only explanation that holds any appeal at the moment is that there is a psycho-sociological bias that causes people to prefer making investment decisions by comparing internal rates of return (IRRs) as opposed to the wealth changes that will be produced for equityholders (NPVs).

This situation produces a necessity for well educated finance students to be conversant with both techniques. So another purpose of this section is to show you how to use IRR in a manner that is most likely to produce ‘correct’ investment decisions (that is, decisions identical with those via NPV).

In Module 1 you learned certain things about IRR that we have no intention of reteaching you here. These introductory matters are covered in Section 1.3.4 and Section 1.4.4 of that module, if you need a review. Further comparison between IRR and NPV occurred in our worked example of the WalkPhone in Section 5.3.

In those sections:

1. The IRR was defined as the average per-period earnings rate on the amounts (market values) invested in the project across time.
2. The IRR calculation process was described as finding the discount rate that causes NPV to equal zero, and we said that the only way to do that in most cases is by a ‘trial and error’ search method of using various discount rates until you find one that creates a zero NPV.
3. We indicated that IRR is comparable with a ‘hurdle rate’ or minimally acceptable rate of return to judge an investment’s desirability. In the case of a corporate investment project wherein the FCF* free cash flows are used, the appropriate hurdle rate is naturally the project’s cost of capital, *rv**.

If you are not comfortable with these ideas, you should review the appropriate sections of Module 1 and Module 5.

Before beginning our discussion of IRR vs. NPV, we should mention one bad argument that is often made in favour of using IRR instead of NPV. This assertion is that the IRR is preferable because it requires less information: in order to calculate an NPV there needs to be known both a discount rate and the relevant cash flows, whereas in order to calculate an IRR only cash flows are necessary. This of course is true in one sense, but begs the question of what is done with the IRR once it is calculated: the IRR must be compared with a hurdle rate – and the correct hurdle rate to use for the IRR is the discount rate that an NPV analysis would use. So the information necessary to accomplish the two analyses is identical, and not a basis upon which to choose either.

In a sense our introductory remarks about IRR vs. NPV at the beginning of this section were a bit overstated, because in many instances the two techniques will yield the same decision. But it is important that you are able to recognise the situations in which that is **not** true, because using the IRR in those decisions can be a serious error. We shall now start our discussion of those instances where NPV and IRR do indeed give different answers.

Most of the time, when the decision to be made is simply whether or not a particular project should be accepted (so that the choice does not involve finding the best among a set of competing projects), IRR gives the correct answers. There is, however, one exception to this rule.

Our earlier discussions of IRR in Module 1 ended with an example that generated Figure 1.4, which we reproduce here in a more generalised form as Figure 6.1. You recall that the outcome of the discussion in Module 1 was to show that IRR and NPV give the same answer when the relationship between a project's NPV and its discount rate is a smooth, negatively sloped curve like the one in Figure 6.1. As long as the IRR is greater than the hurdle rate (the discount rate used to find NPV), the NPV will be positive. And we accept projects when the NPV is positive, or when the IRR is greater than its hurdle rate. We have good news and bad news about the idea shown in Figure 6.1.

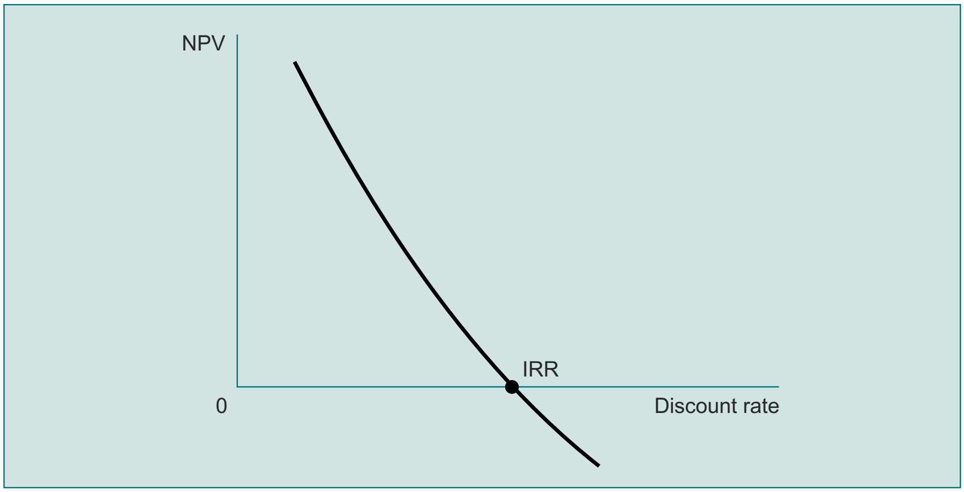

Figure 6.1 **Typical project cash flows: IRR and NPV**

First the bad news. Suppose you own a gold mine with a production process that works tolerably well but is rather slow. Along comes a new machine that would speed up the mining of gold, so that the ore is used up sooner. The machine costs £12 000, and using it would have the effect of shifting £75 000 of FCF* to the next period from one period further in the future, at which time the mine would be exhausted. In numbers, the cash flows of the project are:

t_0	t_1	t_2
−£12 000	+£75 000	−£75 000

If you were to calculate the IRR of this project you would find that it had no less than **two** IRRs, 25 per cent and 400 per cent. Figure 6.2 shows the relationship among discount rates, IRR and NPV for this project.

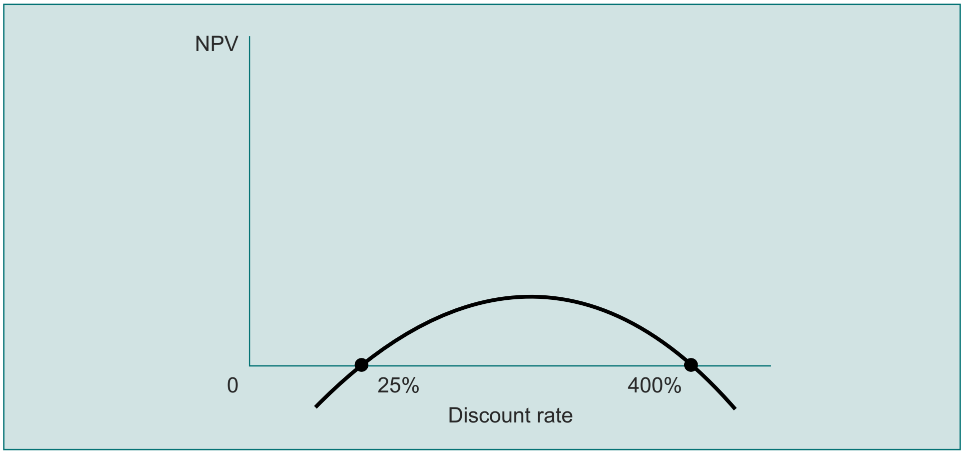

Figure 6.2 **Multiple IRRs**

The figure shows clearly what the problem is. The relationship between discount rates and NPV is not a smooth negative relationship. At times it is positive, at other times it is negative, and it produces two points of intersection with the horizontal axis – two IRRs. With this situation, two questions immediately come to mind. First, why is this happening? And, secondly, how can we interpret the signals that IRR is giving us about the project? We shall answer these questions in order.

The reason the graph looks the way it does is because the pattern of cash flows in the project is unusual. There is an outlay, an inflow, and another outlay (in the form of an opportunity cost). Because of the mathematics of the IRR calculation, this type of pattern, where there are **sign changes** in the cash flows across time, can produce **multiple IRRs** for a project. It would not be worthwhile for us to worry in depth about why that happens; it has to do with the number of sign changes and the relative sizes of the cash flows. But depending upon those characteristics of a cash-flow stream, you can have as many IRRs as sign changes. A pattern of sign changes in cash flows can also produce a project that has a totally **upward sloping** relationship between NPV and IRR, and has but a single IRR. To get a correct answer using IRR, you would have to know to accept the project if its IRR were **less** than the appropriate hurdle rate, not more. Or you can encounter a set of cash flows that has **no** IRR at all.

To answer the second question, these things are serious problems with IRR as a decision criterion. There are various means that can be used to make the IRR come up with a correct answer in a particular situation (for example, in the gold mine problem, you must ignore **both** IRRs and find a new one using the amounts of cash that investing the interim t_1 £75 000 for one period would yield as an inflow at t_2). The difficulty with those solutions is that you must know **beforehand** that you are going to have a problem with the IRR, and what the solution to that problem is. Thus, it will always be easier to use the NPV, which will always give you the correct answer, and has no such problems of interpretation.

The lesson of the above is that if you are faced with an investment project whose cash flows contain changes of sign, it would be best to abandon the IRR technique and turn to NPV. The danger with IRR is that even though you may get answers, the economic interpretation of them is either difficult or impossible. It is within the capacity of finance to find specific 'fixes' for the IRR in such situations, but not within the scope of an introductory course. If you are in a company that uses IRR, and you are occasionally faced with such projects, you can make a valuable contribution by making certain that everyone is aware of these attributes of the IRR technique.

What is the good news? It is simply that there are not too many individual projects that exhibit cash-flow patterns that produce anomalous IRRs. So the initial statement we made about the IRR's usefulness still holds: when the only question is whether an individual investment should be accepted or rejected, the IRR usually works well. But watch out if the cash-flow pattern of the project is not our classic 'early outlays followed by subsequent inflows' – if you use IRR in such situations you will sooner or later make a bad decision.

There is one other situation in which IRR can cause problems not encountered by NPV. We have been assuming throughout that multiple-period cash-flow investments can be evaluated using the same discount rate for every cash flow across time. If that is a correct procedure for NPV, it must be acceptable for IRR, because the discount rate for NPV is the hurdle rate for IRR. But suppose that a single rate could not be correctly applied, and different rates were necessary for each cash flow (as we illustrated with bonds in Module 1). This presents no problem for NPV; we merely use the different rates on each cash flow, as we did in Module 1. But IRR has a problem here. IRR is by its nature a **single** rate calculated from a group of cash flows. You can find the cash flows' IRR, but to which of their discount rates would you compare the IRR? It would not be correct to compare it to **any** of their rates. To find a correct hurdle rate, you would have to find the yield to maturity of a security with the same risk and cash-flow pattern as the investment in question. The likelihood of regularly finding such securities when necessary is low. We do not at this time, however, wish to make too much of this indictment of IRR. The reason is that almost all companies use only a single discount rate for the cash flows of a project when calculating its NPV. That being the case, until companies start using the term structure of interest rates to estimate individual period discount rates for NPVs, it is not really fair to criticise IRR alone for this shortcoming.

6.4.1 IRR vs. NPV in Mutually Exclusive Investment Decisions

We now come to the more serious confrontation between IRR and NPV. Many investment decisions of corporations do not involve simply deciding whether an investment is acceptable or not, but require that a decision be made as to which of a number of available projects is best. In such situations we ask of a technique not merely an absolute 'yes' or 'no' but a statement of relative desirability or '**ranking**' of investments. This request is managed handily by the NPV technique. Projects that must be chosen among, or are **mutually exclusive** to, each other can be ranked by the NPV technique in order of the size of their (positive) NPVs. The project with the highest NPV will increase shareholder wealth the most, and is therefore preferred to the others. Unfortunately the same test will not in general work for the IRR.

Consider the investments between which you decided in Review Questions 1.12 and 1.13 at the end of Module 1. The cash flows of the projects are:

	t_0	t_1	t_2	t_3
No. 1	−£2500	+£1200	+£1300	+£1450
No. 2	−£2500	+£1300	+£1300	+£1300

If you did the problems correctly, you discovered that investment in location No. 1 has a higher NPV than investment in location No. 2 at a discount rate of 10 per cent, but No. 2's NPV is greater than No. 1's at a discount rate of 25 per cent. How can such a thing happen? In the answers given we show how through Figure A4.3, which we reproduce here in slightly modified form as Figure 6.3.

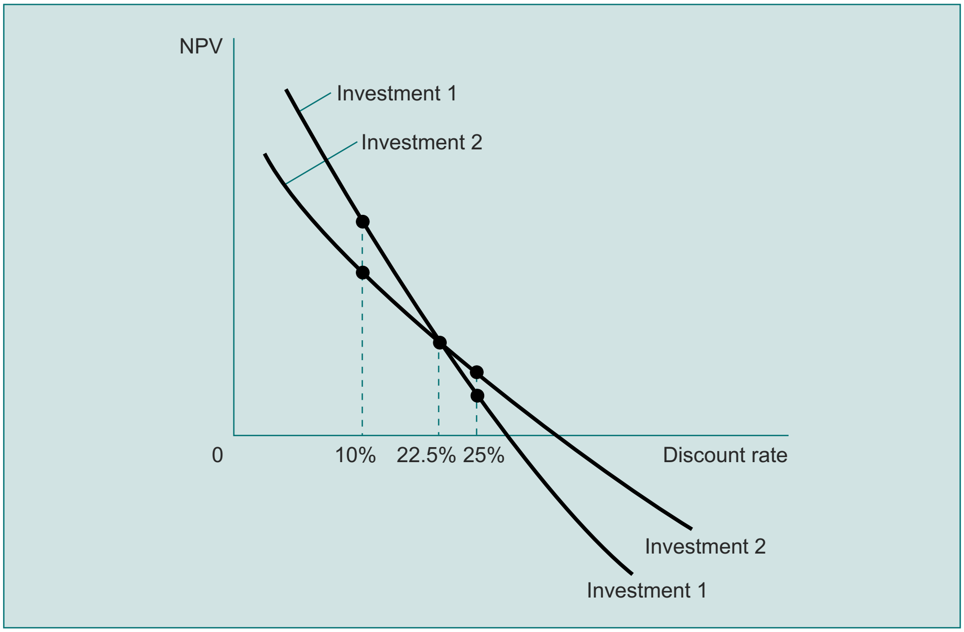

Figure 6.3 **Mutually exclusive investments with a crossover rate**

You can see that the culprit is again the relationship between discount rates and NPVs. Here the projects' cash flows are such that at relatively low discount rates investment No. 1 is better, while at relatively high rates, No. 2 wins. This causes no particular problem for NPV. Given an appropriate discount rate it will tell you which of No.1 or No. 2 will generate the most shareholder wealth.

IRR, however, has a problem here. Suppose that the correct discount rate is 10 per cent. It is clear from Figure 6.3 that No. 1 is the best investment (Figure 6.3 shows No. 2's IRR as occurring further along the discount rate axis), but it is also clear that IRR would rank No. 2 better than No. 1. If you decided to use the IRR criterion, you would not be maximising your shareholders' wealths. This is a very serious problem with IRR, because the situation we illustrate here is not at all uncommon. Competing projects' cash-flow patterns are not always such that they 'cross-over' at some discount rate less than the IRR as do No. 1 and No. 2, but it happens often enough that you should be concerned about it. And particularly insidious is that, unlike the multiple-rates problem with IRR (when sign changes signal a problem with IRR use), there is no simple way to examine the projects' cash flows and know beforehand that this is going to happen.

What should be done? In the best of all worlds, our advice is to forget the IRR and use the NPV for such comparisons because it is not subject to this problem. But sometimes you do not have that luxury; your company may have as a policy that IRR is the technique of choice, or your boss may not want to hear anything but comparisons of rates of return. What then? We shall now teach you a technique that can be used in such situations (indeed **should** be used in all comparisons of investments where IRR is the evaluation method of choice).

This addition to the IRR technique is known in finance as **incremental cash-flow analysis**. As we said above, it should be used whenever attempting to choose among investments on the basis of their IRRs. The technique is not very complicated, but there are enough steps in it so that it is easier to remember if they are in a list. (A list of steps to take is called an **algorithm**, so we are giving you an algorithm to use when choosing the best of mutually exclusive projects on the basis of IRR.) Here are the steps:

1. If you have a lot of projects to decide among, think of them as in a pot, and pick any two from the pot.
2. Find the one that has the highest net positive cash-flow total (the simple sum of all of its FCF*s, which can be thought of as an NPV calculated with a discount rate of zero). Call the investment with the highest net positive cash-flow total the 'defender', and the other one the 'challenger'.
3. At each time point, subtract the cash flows of the challenger from those of the defender, and call the resulting stream the 'incremental cash flows'.
4. Find the IRR of the incremental cash flows.
5. If the IRR of the incremental cash flows is greater than the appropriate hurdle rate, keep the defender, and throw out the challenger. If the IRR of the incremental cash flows is less than the appropriate hurdle rate, keep the challenger, and throw out the defender.
6. Omitting the thrown-out project(s) and keeping the survivor of the procedure to this point, return to the pot, pick another investment, and begin the algorithm at step 2. When all available projects have been tested, you will have a single survivor that has not been thrown out. Call that one the 'winner'.
7. Calculate the IRR of the winner. If it exceeds the hurdle rate, accept the winner; if the winner's IRR is lower than the hurdle rate, reject the winner and all other projects that were in the pot.

We shall explain to you why this works in just a moment. First, you should see how to use the technique with an example, and the cash flows of the investments in locations No. 1 and No. 2 used earlier will serve that purpose well. Let us assume that the appropriate discount (hurdle) rate for these projects is 10 per cent. Taking the steps of the algorithm in order, we have the following:

1. We choose No. 1 and No. 2 because they are the only ones in the pot.
2. Adding up the cash flows of each, we see that No. 1's equal £1450, and No. 2's are £1400, so No. 1 is the defender and No. 2 is the challenger.
3. Subtracting the challenger's cash flows from the defender's, we arrive at the incremental cash flows:

	t_0	t_1	t_2	t_3
No. 1	−£2500	+£1200	+£1300	+£1450
No. 2	+£2500	−£1300	−£1300	−£1300
Incremental cash flows	0	−£100	0	+£150

4. Through trial and error, we discover that the IRR of the incremental cash flows is about 22.5 per cent.
5. Since the IRR of the incremental cash flows is greater than the hurdle rate (22.5% > 10%), we keep the defender (No. 1) and throw out the challenger (No. 2).
6. We return to the pot, and discover that we have exhausted its contents, so the last defender (No. 1) becomes the winner.
7. We calculate, by trial and error, that the winner's IRR is greater than the hurdle rate (No. 1's IRR is slightly less than 25.9 per cent, and the hurdle rate is 10 per cent), so we accept the winner, No. 1, as the better of the two projects.

After working through the algorithm in this way you might be tempted to think that this is a great deal of trouble to go to when we could have simply calculated the IRR of No. 1 in the first place, and found that it was acceptable. A glance at Figure 6.3 reminds us how wrong that attitude would be. Since our task is to choose between No. 1 and No. 2 on the basis of IRR, No. 2 (with an IRR of slightly less than 26 per cent) having the higher IRR seems to be the logical choice. But with a discount rate of 10 per cent, No. 1 is the better project. Only by dealing with incremental cash flows are we able to use the IRR to choose No. 1, the correct answer.

Why does this algorithm work? One way to see why it does is to consider the economic interpretation of the IRRs of incremental cash flows, since that is the main decision variable in the algorithm. Look again at steps 3, 4 and 5 of the calculation above. The incremental cash flows (ICF) are the differences between the cash flows of No. 1 and No. 2, or:

$$\mathrm{ICF} = \mathrm{CF(No.1)} - \mathrm{CF(No.2)}$$

By calculating the IRR of the incremental cash flows we are testing whether investing in the **difference** between No. 1 and No. 2 is worthwhile. If the IRR of the incremental cash flows is greater than the hurdle rate, that tells us it would be worthwhile to invest in the cash-flow **differences** between the two investments, which is to say that it would be worthwhile to **switch** from No. 2 to No. 1. If it is worthwhile to switch from No. 2 to No. 1, we can discard No. 2, which is exactly what we do.

The algorithm works because instead of comparing investments' IRRs (which we know is a problem), it looks at the IRR of choosing one instead of the other. This should also help you to understand why we need step 7, in which we test whether the winner is acceptable on the basis of its own IRR. This is simply because it is entirely possible that an entire group of projects under consideration is no good. If you were to adhere to algorithm steps 1–6 alone, one of such a bad group is bound to turn out to be the winner. Were you to accept the winner, you would be taking a bad investment. So you must see if the winner itself is any good (via algorithm step 7). If it is bad, since it is better than its mutually exclusive alternatives, every investment in the pot is bad.

For those of you who enjoy graphical explanation, Figure 6.3 can be helpful in seeing how the algorithm works. Remember that the IRR is the discount rate that causes a set of cash flows to have an NPV equal to zero. Since the incremental cash flows are the differences between those of No. 1 and No. 2, the IRR of the incremental cash flows is the discount rate that causes the NPV of the **differences** between No. 1 and No. 2 to be equal to zero. Saying the same thing with different words, the IRR of the incremental cash flows is the rate at which the NPVs of defender and challenger are equal. Graphically, this is the discount rate directly below the point where the curves for No. 1 and No. 2 intersect in Figure 6.3. For this reason the IRR of the incremental cash flows is sometimes called the '**crossover rate**'.

Envisage now in Figure 6.3 how the algorithm works. In steps 1 to 4 we calculate the IRR of the incremental cash flows (or the crossover rate), which is 22.5 per cent. Step 5 compares the crossover rate to the hurdle rate of 10 per cent and finds that the crossover rate exceeds the hurdle rate. This causes us to choose the defender (No. 1) over the challenger (No. 2). Graphically, the reason why No. 1 is better than No. 2 is as follows. The defender, with the greatest positive cash-flow total, is always the project that will have the higher NPV at discount rates less than (to the left of) the crossover rate. You can see in Figure 6.3 that this is No. 1. By comparing the crossover rate to the hurdle rate, we are in effect testing whether the defender or the challenger has the higher NPV. If the crossover rate is greater than the hurdle rate, the defender's NPV will be higher; if the crossover rate is less than the hurdle rate (in Figure 6.3, if the hurdle rate is greater than 22.5 per cent), the challenger's NPV will be higher.

Neither of the above explanations is the 'reason' that IRR and NPV produce results you see in the example and in Figure 6.3. The reason has to do with something called '**reinvesting interim cash flows**'. When we compare investments on the basis of their IRRs we are making an implicit assumption that the money the investments produce during the time before they finish (their interim cash flows) can itself be invested at the IRR rate. Look again at the example. If we compare investments numbered 1 and 2 on the basis of their IRRs, we would choose No. 2, and that would be the correct choice **if** all of both No. 1's and No. 2's interim cash flows could be reinvested at the IRRs of the respective projects. In other words, if we could expect to invest the stream of No. 1's interim cash flows at a rate of slightly less than 25.9 per cent, and those of No. 2 at slightly less than 26.0 per cent (their IRRs), No. 2 would have a higher NPV than No. 1, including the reinvestment outflows and inflows.

On the other hand, using the NPV technique, if interim reinvestments are **not** explicitly included (which is the usual procedure) this implicitly assumes that interim cash flows can be invested at the discount rate used in the NPV analysis. Most finance people who worry about this question think that the NPV assumption of investing interim cash flows at the opportunity costs of capital suppliers (rv^*) is more viable than the IRR's assumption of interim investments being available that return the same rate as the one being analysed. We agree with them. Of course, because it gives the same answers as NPV, the incremental cash-flow algorithm causes the IRR to make the same interim cash-flow investment assumption as the NPV.

With the incremental cash-flow algorithm, the IRR technique can be made to operate in many corporate investment situations where a ranking or choice among alternatives must be made. Without the use of the algorithm or an equivalent method, the IRR will not as a rule give correct indications of **relative** investment desirability. Even with the incremental cash-flow method of IRR, however, there can be problems. These are sufficiently significant that we recommend that, should you encounter any of the situations listed below, you abandon the IRR method and use only the NPV technique. Educating your boss on the reason why you think this is the correct thing to do will give you some feeling for the challenges in writing a finance course such as this.

The situations in which the incremental cash-flow method of choosing among investments on the basis of IRR should *never* be used are:

1. When the incremental cash flows that you calculate have more than one change of sign across time. This is the same problem as we discussed in the first part of this section. The algorithm does not fix it.
2. When the projects differ in risk or financing, so that they require different hurdle rates. Rereading the paragraph above on the graphical explanation of the algorithm will show why this presents problems.

Even when the above situations appear, the algorithm can be made to work correctly. But the task of doing so will be ridiculously complex in comparison simply to using the NPV, which always works well.

6.4.2 Summary of IRR vs. NPV

We have discovered certain important characteristics of using the internal rate of return as opposed to the net present value in corporate investment decisions:

1. Both techniques are 'discounted cash flow' methods, with the IRR comparing the average per-period earnings rate of an investment to the minimum rate necessary to maintain shareholder wealth, and the NPV using the latter rate as the

discount rate in a present-value investment cash-flow calculation, which results in the change in shareholder wealth caused by the investment.

2. The IRR and NPV will usually give the same indication of investment desirability if the question is simply whether or not to accept a particular project. The IRR, however, will not generally give correct rankings of several alternative (mutually exclusive) projects. When such decisions must be analysed with IRR, the incremental rate algorithm should be used.
3. The IRR has significant problems in application when the cash flows from which it is to be calculated have more than one change of sign across time. When that occurs, the IRR should not be used.
4. The IRR is unlikely to be useful if each cash flow must be considered as having a unique discount rate.

Is there an overall statement we can make about using the IRR as opposed to the NPV in corporate investment decisions? The answer is both 'yes' and 'no'. It is impossible to make a decent case that the IRR is ever better, and it is easy to show that it is often worse than the NPV. This argues that we should never use the IRR and always use the NPV. In a purely financial–analytical world, that is our advice. But we are also aware that both techniques have been around for a long time, most financial managers are at least generally aware of the concerns we raise in this section, and yet they and their companies continue to use the IRR. It is only reasonable for us to conclude that there must be some non-financial–analytical reason why they do, and that reason, though we have no idea what it is, may be perfectly valid. So our final word on this matter is: if you like IRR, go ahead and use it. It produces acceptable results much of the time. But if you are concerned about making correct decisions **all** of the time, you must be aware of and adjust to the limitations of IRR.

6.5 The Cost–Benefit Ratio and the Profitability Index

In this section we shall briefly examine our last two investment evaluation methods, the **cost–benefit ratio** and the **profitability index**. We treat these together because they are fairly closely related to each other, though their correct applications differ.

6.5.1 The Cost–Benefit Ratio

The cost–benefit ratio (CBR) is defined as the ratio between the present value of the cash inflows and the cash outflows of an investment:

$$\text{CBR} = \frac{\sum_{t=0}^{n} \frac{\text{Inflows}_t}{(1+rv^*)^t}}{\sum_{t=0}^{n} \frac{\text{Outflows}_t}{(1+rv^*)^t}}$$

where n is the number of time periods applicable for the investment and where Inflows_t plus Outflows_t equals FCF_t^*. To use the CBR, an investment is accepted if the ratio is greater than 1 and rejected if the CBR is less than 1.

Though the CBR formula appears menacing, a closer inspection reveals that it is really nothing more than the NPV formula with the cash inflows in the numerator and the cash outflows in the denominator. If the NPV would have been positive, the CBR will be greater than 1; a negative NPV is the same as a CBR less than 1. It is clear that the NPV and CBR will produce the same investment recommendations if the same cash flow and discount rate information is used in each, when faced with the question of an investment's desirability.

The CBR, however, has some of the same problems as the IRR when asked to choose among mutually exclusive investments. Just as the IRR errs in choosing high-return investments over those with high wealth-increasing expectations, the CBR is attracted to those investments that have the greatest **ratio** differences between inflow and outflow present values instead of actual cash or value differences.

Consider the two investments, M and N, below:

	t_0	t_1	t_2
M	−£100	+£200	+£400
N	−£1000	+£1500	+£3500

At a discount rate of 10 per cent, the CBRs of both are greater than 1, so both are acceptable. If they are mutually exclusive, however, the CBR indicates that M is preferred (CBR = 5.12) to N (CBR = 4.26). The NPV of M is 412 whereas the NPV of N is £3256.

The CBR is giving an incorrect signal as to which of the two investments should be preferred if they are mutually exclusive. The reason is that it is a **ratio** of values rather than a value measure itself. Capital claimants are concerned about their absolute wealth – which in the final analysis is measured in value amounts not ratios. If the NPV is available, it should always be used in preference to the CBR.

6.5.2 The Profitability Index

The profitability index (PI) has the form:

$$PI = \frac{\sum_{t=1}^{n} \frac{\mathrm{FCF}_t^*}{(1+rv^*)^t}}{-\mathrm{FCF}_0^*}$$

A PI greater than 1 signals an acceptable investment. The PI is only used when the present cash flow (FCF_0^*) is a **net outlay**.

The PI formula also looks formidable when first encountered. As with the CBR, the PI ratio upon inspection reveals itself to be simply another variant of the NPV calculation. The PI is the ratio of the accumulated present values of future cash flows (the numerator, which is a sum from time 1 forward) to the present cash flow (the denominator, which is the time 0 or present cash flow) of an investment. Like the CBR, the PI uses all of the same cash flows and discount rates as the NPV, and also like the CBR, the PI is a ratio measure. Because of that, the PI has almost exactly the same problems as the CBR.

Consider the two investments:

	t_0	t_1	t_2	t_3
No. 3	−£1000	+£500	+£600	+£700
No. 4	−£2000	+£900	+£1000	+£1300

The PI of No. 3, using a 10 per cent discount rate, is 1.476, whereas the PI of No. 4 is 1.311 at the same discount. The PI tells us (correctly) that both investments are acceptable. But suppose that the two are mutually exclusive. The PI would seem to say that No. 3 is better than No. 4 because the former has the higher PI ratio. Were you to choose No. 3 over No. 4 on that basis, you would be making a mistake. The NPV of No. 3 is £476.33, while No. 4's is £621.34. No. 4 is the better choice if the projects are mutually exclusive.

The reason that the PI is unsuitable for ranking investments is because it displays another **relative** measure, namely the wealth increase per pound of initial outlay instead of the wealth increase itself. One can easily run into a situation where the investment that would promise the greatest wealth increase has the lower wealth increase per pound of initial outlay (or PI) because its initial outlay is higher. Though you may not earn as high a return on each pound invested in No. 4, it has the dual virtues of returning more than its opportunity cost, and also allowing you to invest a greater amount of money at that desirable rate than does the alternative investment. (Naturally, if you could have invested £2000 in No. 3, and thereby doubled its future cash-flow expectations, it would be the more desirable. We would, however, maintain that such opportunities to expand outlay without reducing rate of return are rare to the point of non existence in real asset investments.)

This explanation sounds very much like the one offered for the CBR above. In fact, with minor substitutions, we could have simply duplicated the CBR explanation, and saved some space. The astute reader may have already noticed that the CBR measures for No. 3 and No. 4 are identical to their PIs and the same thing holds for M and N. See if you can figure out why, and when that would not happen.

6.6 Summary of Alternatives to the NPV

In Section 6.2 to Section 6.5 we have examined several methods that corporations use to make investment decisions as alternatives to the NPV. We have found that none is better than – indeed even as good as – the NPV, which remains the technique of choice. Some of the alternative methods (the IRR, CBR and PI), if used carefully and selectively, can be made to function so as to yield correct decisions in a limited set of circumstances. Others (the payback period and the accounting return on investment) are not likely to give answers generally consistent with shareholder wealth maximisation.

We do not know why *any* of these alternative techniques are being used by companies, but they are. The detailed discussions of them provided in this section should allow you, if faced with a requirement to use such a technique in your company, either to show how the alternative technique can be made to work properly or to argue convincingly that the method should be abandoned in favour of the NPV.

A final postscript to our discussion of alternatives to the NPV must deal with nomenclature. We have used names for the various techniques that we believe are the most commonly accepted ones. You should be aware, however, that these same techniques are regularly found lurking in corporate finance departments under a wide range of aliases. The IRR is often called the 'discounted cash-flow rate of return'; the PI is sometimes called the CBR, and vice versa; the accounting return on investment on occasions goes under the name of the 'average return on book value', the 'accounting rate of return', or one of a number of other titles. Because one can rarely be certain of exactly what a particular name signifies, one should without embarrassment ask for the particular definition of any investment technique offered for consideration.

6.7 Capital Rationing

Up to this point we have been assuming, without mentioning it, that a company would accept all available investments having positive NPVs as long as the investments were not mutually exclusive. In other words, our approach has pictured companies as having unlimited access to investment funding for all legitimately acceptable investments. Said that way, you may have some hesitation about our assumption of unlimited funds. Can it be realistic to assume that a company can always get as much money for investments as it wants from the capital market? Actually, that is not a bad representation of reality.

Remember that an investment will be 'acceptable' when it returns more than a company's shareholders are able to earn by owning financial assets in the capital market. From that perspective it seems reasonable that the market would be delighted to fund real investments that return more than financial alternatives. And as long as the demands from any particular firm are not a significant portion of the total resources of the market (there are precious few companies in the world that would comprise any significant portion of the total capital market value of all companies, so the odds that any one company's investment opportunities would overwhelm the market are infinitesimal), there will be enough money available.

There are instances, however, when there is not enough money available for all profitable investments a corporation wishes to undertake. These instances are mostly variants upon one of the following themes:

1. The company has a number of divisions that are semi-autonomous and evaluate investments independently but that depend upon the centralised headquarters' financial operation for the funding of those investments. The headquarters of the company, being in charge of finding the money for investments, sometimes does not agree with the division as to the desirability of the division's investments. Rather than argue about that regularly, the headquarters decides in general how much money is likely to generate acceptable investment in each division, and simply tells each how much money it can have in that period's 'capital budget'. In addition, the company may have other constraints upon resources – most often, managerial talent – that dictate limits upon divisional investment and its attendant growth in demand for overall corporate attention.

2. The external capital markets disagree with the corporation as to the desirability of the investment. In this situation the company may find itself unable to raise the amounts of cash at the rates of return or interest that its analyses of projects indicate should be available from the market. When that happens, and if the market cannot be convinced of the company's opinion, there will be an effective limit on the money that can be invested in 'acceptable' projects.

One can doubtless uncover other situations in which a corporation with desirable projects cannot find the money to undertake all of them, but the above are common enough to require that financial analysts have some method for choosing the group of projects that will maximise shareholder wealth while using no more than the funding available.

In finance we use the term **capital rationing techniques** for the set of methods designed to deal with this situation. The goal of the financial manager is, as ever, to maximise shareholder wealth. But now the constraint on the total funds available to finance desirable investments requires that we accept the set of desirable projects that both is within the 'budget constraint' and yields the highest NPV. There are many techniques that financial analysts use to accomplish this goal, ranging from the quite straightforward to the very sophisticated. We shall illustrate the former and briefly discuss how you can gain access to the latter.

Suppose that your company had the following opportunities to invest in assets, with the associated expected results:

	Outlay at t_0	**NPV**
E	£1000	+£300
F	£2000	+£560
G	£5000	+£870

Of course, your preference would be to accept all of these investments, because they all exhibit positive NPVs. But suppose that you are informed that there will only be £5000 available at t_0 to undertake investments, no matter how many look good. Facing this problem, the answer to the capital-rationing problem can be found by simple inspection of the available investments, their NPVs and t_0 outlays. It is clear that investment G is the best choice. It should be accepted, and E and F rejected, for G has an outlay of £5000, which is equal to the budget constraint, and returns an NPV of +£870, higher than the alternative, which is to accept both E and F with a combined NPV of +£860.

There may nevertheless be something about the above example that bothers you. You may even express the difficulty as follows: 'That example does not make good sense. We are told to accept a project that costs £5000 and has an NPV of £870, and reject a set of projects that costs only £3000 and would generate an NPV of £860, only £10 less than the one we accept. Why are we paying an incremental £2000 to gain only £10?'

Though the question can only arise within a capital-rationing situation, the answer has less to do with the fact that outlays are constrained than it does with the simple economics of the NPV calculation. Anyone who is troubled by the above solution needs a brief refresher course in NPV: remember that the NPV is the present value of the amounts by which the future cash flows of a project **exceed** those that a comparable-risk outlay in the capital market would yield. This means that the additional £2000 invested in G is expected to return not only the £10 greater NPV, but also the required return on the £2000 itself. To see this, suppose that the three investments above were single-period ones and that the appropriate discount rate for all was 10 per cent. The cash flows of the investments can then be deduced to be:

	Outlay at t_0	**Inflow at t_1**	**NPV**
E	£1000	+£1430	+£300
F	£2000	+£2816	+£560
G	£5000	+£6457	+£870

With this data we can see that an additional £2000 invested in G produces an additional (£6457 – £2816 – £1430 =) £2211, a return in excess of its opportunity cost. Note that accepting E and F along with investing the unused £2000 at the opportunity cost of 10 per cent is not as good as accepting G, because the return on this £2000 investment would be £2200 whereas G returns £2211 on the 'extra' £2000 invested in it relative to E and F. Were these investments to be multiple-period, perhaps even of different number of time periods, the illustration would be more complex but the economics of the argument identical.

In the illustration we have seen to this point, the capital-rationing situation presents no particular analytical problem. We simply look at all possible combinations of investments that lie within the budget constraint and choose the package that has the greatest NPV. That kind of procedure is called 'exhaustive enumeration', and works well as long as there are not too many potentially acceptable projects. But it is extremely cumbersome when there are many projects to be considered. In that situation, exhaustive enumeration, if you remember your elementary school arithmetic of combinations, can be exhausting indeed. When there are many projects to be considered, in order to find which combination fits within the budget constraint and has the highest NPV, finance turns to more sophisticated search techniques.

Suppose that you were the financial analyst in a company that had the investment opportunities below:

	Outlay at t_0	NPV	PI
H	£5 000	+£3 000	1.60
I	£10 000	+£5 000	1.50
J	£20 000	+£6 000	1.30
K	£1 000	+£200	1.20
L	£1 000	+£150	1.15
M	£4 000	+£560	1.14
N	£1 000	+£100	1.10
O	£3 000	+£150	1.05

One of the commonly used methods of dealing with the capital-rationing problem when there are many projects is to calculate the profitability indices for the investments, and to list them in declining order of PI. The investments are then accepted in order of PI, until the budget has been exhausted. A project may be 'skipped' because its outlay is too large, and the next one having a small enough outlay taken as you work down the list.

You can see that this works nicely in the list above for budgets of £35 000 (choose H, I and J), £36 000 (choose H, I, J and K), £37 000 (choose H, I, J, K and L) and £38 000 (choose H, I, J, K, L and N). But look what happens with a budget of £39 000. The company must abandon the PI procedure and choose H, I, J and M, the last one having a PI less than the rejected projects K and L. So the PI technique must be used with some caution in ranking investments to be chosen under capital rationing. The problems tend to occur when the highest PI projects do not use up the entire budget (such as H, I, J, K, L and N under a £39 000 constraint).

For most companies of even modest size today, access to computer facilities (personal computers included) means that they need not rely upon hit-or-miss methods like the PI procedure when faced with a capital-rationing problem. There exist inexpensive software packages that contain the sophisticated techniques necessary to solve these problems correctly and quickly. (Look for 'integer programming' as the key words, and ask someone who understands optimisation techniques – an 'operational research' or 'management science' person – to tell you how to use it, if you cannot understand the instructions.) These sophisticated programs are little more than efficient search methods that go on within the computer to find the package of investments that has the highest total NPV, and that is within the budget constraint.

Capital-rationing situations do occur and require that appropriate analytical procedures be brought to bear in order to arrive at the best solutions. But you should remember that being under capital rationing in the first place is an undesirable situation. It means either that you have not been able to solve your internal organisation and communication problems, or that you have been unconvincing to the capital market about your prospects – both failures of some significance. They imply that you will be forced to forego investments that would have increased the wealth of your shareholders.

One last warning. By no means should you ever interpret the existence of high market required rates as a capital-rationing situation. When interest rates are high, it is simply a signal that your capital costs are also high. The market stands ready to provide capital, but at a rate that may cause you to arrive at a negative NPV for projects under consideration. The capital-rationing situation implies that financing beyond the budget constraint carries not a high but an **infinite** cost. The distinction is important, especially when arguing the politics of capital budgets within companies.

6.8 Investment Interrelatedness

All of the investment decisions we have studied to this point have been either 'yes' or 'no' questions about a single project, or choosing the best among a set of mutually exclusive projects. Financial analysts, however, often encounter more complicated choices than these. When the acceptance or rejection of one investment affects the cash flows that can be expected on some other investment, we say the investments are **economically interrelated**.

We have already been introduced to one form of economic interrelatedness among investments, which we have called **mutual exclusivity**. By the definition above, mutually exclusive investments are economically interrelated because only one can be accepted. The acceptance of one thus implies the complete disappearance of the cash flows of the other. This is one of the extreme types of investment interrelatedness, but it is nevertheless an important one, as we have seen in many previous examples. The capital-rationing situation we examined in the previous section is another example of interrelatedness of a mutually exclusive nature, though perhaps not so obviously so. Because of capital constraints, the acceptance of any one project decreases the remaining capital available to all others, and that thus lowers the likelihood of their acceptance and thus of obtaining their cash flows. Actually, one valuable way of looking at the capital-rationing decision is that it forces an analyst to choose among all of the possible **combinations** of investment projects that can be accepted within the budget constraint, and each such combination is thus mutually exclusive of all others.

Aside from mutual exclusivity, there are many nuances of relatedness among investments with which financial decision makers must deal. Suppose, for example, that Global Conglomerate Ltd owns a large plot of land in the suburbs, and is considering its development. The land can be split into two parcels, and there are two projects under consideration, a funeral parlour and an outside arena for rock-concerts. The marketing, engineering, etc. estimates for each project are collected by the analysts of Global, and indicate:

	NPV
Funeral parlour	£265 000
Rock-concert arena	£327 000

Clearly, both investments, having positive NPVs, should be accepted. Or should they? Suppose the decision makers discover that the marketing, engineering, etc. estimates for each project did not make allowances for the other – for example, when the marketing staff estimated the cash flows for the funeral parlour, they did not factor into the cash-flow estimates the possibility of a rock-concert arena next door. Asked to include that possibility, the results are:

	NPV
Funeral parlour, no rock-concert arena	£265 000
Rock-concert arena, no funeral parlour	£327 000
Funeral parlour, rock-concert arena next door	−£126 000
Rock-concert arena, funeral parlour next door	£327 000

Not surprisingly, there is a significant negative economic relatedness appearing in the cash flows of the funeral parlour when a rock-concert arena is placed next door. With this information, the decision must be that Global's shareholders are better off if the company builds the rock-concert arena alone (NPV = £327 000) than if it builds both (NPV = £327 000 − £126 000 = £201 000).

The analysts of Global would actually structure the decision as requiring a choice among four mutually exclusive alternatives, only one of which can be chosen:

	NPV
1. Build nothing	0
2. Funeral parlour	£265 000
3. Rock-concert arena	£327 000
4. Funeral parlour and rock-concert arena	£201 000

The Global Conglomerate example is one of **negative economic relatedness**. Financial decision makers also encounter the opposite, or **positive economic relatedness**. For example, Global may have rejected a record shop on the plot of land because a record shop, if built alone, would have a negative NPV. But when the positive interactions between the rock-concert arena and the record shop are included, the record shop may become viable.

Economic relatedness can be thought of as being somewhere along a continuum that runs from the very negative to the very positive, as in Figure 6.4.

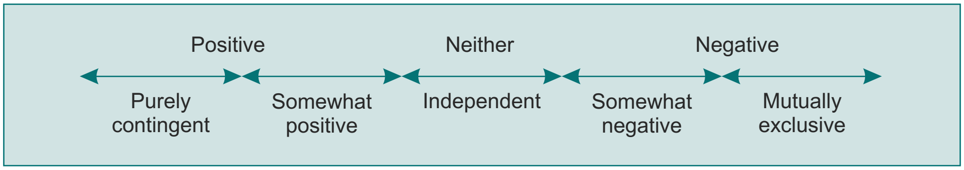

Figure 6.4 **Economic relatedness of investment cash flows**

Let us briefly examine this continuum of economic relatedness. The perfect form of positive economic relatedness occurs when the cash flows of a particular project cannot exist unless those associated with another investment are accepted. In finance we call that a situation of **pure contingency**. An example is the decision faced by a shipbuilder when evaluating the choice of power units to run a proposed ship. Unless the ship is accepted, the power-unit choice has no cash flows; the cash flows of any power unit are purely contingent upon the acceptance of the ship.

At the opposite end of this continuum is the perfectly negative relatedness which we call **mutual exclusivity**. Here the acceptance of a particular alternative implies the rejection of the cash flows of all that are mutually exclusive of it. Take the same ship example as above, and look within the decision as to which of a number of power units to use. Obviously only one can be accepted, and will be mutually exclusive of all other choices.

Within the extreme boundaries of the continuum are the 'somewhat positive' and 'somewhat negative' relatednesses. The decisions that Global faced with the funeral parlour, the rock-concert arena and the record shop are examples of both types. Notice that the acceptance or rejection of any of the three does not **necessarily** imply the acceptance or rejection of any other. But there are significant alterations in the cash flows among the projects depending upon the mix of those accepted.

Finally, you have no doubt noticed that the centre of the continuum contains the term 'Independent'. We include there the economic relationship of neutrality among the cash flows of investment projects. Consistent with our other definitions, **economic independence** implies that the acceptance or rejection of one project has no effect upon the cash flows of the projects with which it is economically independent. The decisions to invest in a new piece of efficient machinery and investing in government bonds is an example of economic independence; there is no causal relationship between the machine's and the bonds' cash flows.

How should companies deal with economic relatedness among investment proposals? If investments are economically independent, they may be considered individually and judged on the basis of their cash flows alone. But if a group of projects exhibits any shading of interrelatedness of the types described above, the analysis must be performed differently. First, all possible combinations of interrelated investments must be specified, along with their unique cash flows and NPVs. In the initial Global Conglomerates example, that means setting out the four combinations:

	NPV
1. Build nothing	0
2. Funeral parlour	£265 000
3. Rock-concert arena	£327 000
4. Funeral parlour and rock-concert arena	£201 000

Notice that if all possible combinations have been listed, the combinations must be mutually exclusive; in other words, if Global accepts any one of the above four, it automatically rejects the other three. Whether one has successfully listed all possible combinations can be tested by their being mutually exclusive. As an exercise, see if you can list the combinations that Global must decide among, were the record shop to be included in the analysis. Your answer should be:

- build nothing
- funeral parlour
- rock-concert arena
- record shop
- funeral parlour and rock-concert arena
- funeral parlour and record shop
- rock-concert arena and record shop
- funeral parlour, record shop and rock-concert arena.

Next, once all possible combinations of interrelated investments have been specified, being mutually exclusive, the combination with the highest NPV can be chosen. (In the initial Global example, it is the rock-concert arena alone. In the expanded group, it is probably the arena and the record shop.)

More generally, when facing a set of investments whose cash flows are interdependent, the financial manager would do well to consider the various **combinations** that are available to be separate investments themselves. In other words, as far as the Global analysis is concerned, undertaking the funeral parlour **and** the rock-concert arena is a separate investment that has characteristics that can be very different from those implied by undertaking either of the two alone.

6.9 Renewable Investments

Corporations must often choose among investments in real assets where the initial asset purchased will eventually wear out and be replaced by another of the same type. If all of the assets that could be chosen would be expected to wear out at approximately the same times in the future, the decision would present no problem for financial analysts. The correct approach to deciding among such assets would be simply to find the NPV for investing in each **for the first time**, and unless subsequent investments in the same assets would generate different cash flows as they were renewed, the choice would be of the asset with the most positive NPV. (Or if the investment is a 'cost minimising' one, the choice would be of the one with the least negative NPV.)

Very often, however, the alternative assets cannot realistically be expected to require renewal at the same intervals. When that happens, we must be very careful to design an analysis that recognises the disparate timings of replacements, or an incorrect decision will likely be made. If investment alternatives have unequal lives, and will be renewed, we cannot simply calculate the NPV of the first incarnation of the assets so as to judge their relative desirabilities. This problem is a special case of the 'reinvestment' concern that we discussed in an earlier section of this module. There we said that in certain situations the analyst must make explicit any special cash-flow expectations that would apply in reinvesting the interim cash flows of a project if the simple opportunity cost of the project is inapplicable. The situation wherein the investments will each be renewed is such a case.

To see how such a problem can arise, assume that Bayou Software plc must purchase a new machine to reproduce the program diskettes that it sells. Once the commitment has been made to a particular machine, because of other longer-term adjustments the company will make in its operations the machine will likely be replaced with another of the same model when the old one wears out. Bayou is considering two competing machines; first, the Largemouth Special, which has a £14 000 cash outflow associated with its purchase, and will last for three years, generating £3000 of cash costs each year during that time; and secondly, a Lunker Elite, which costs only £7000 each time it is purchased but generates annual costs of £5000 and must be replaced every two years. There are no other differences between the machines.

Not knowing any better, the uninitiated analyst might be content to set up the initial cash flows from the alternatives as (£000s):

	t_0	t_1	t_2	t_3
Largemouth	−14	−3	−3	−3
Lunker	−7	−5	−5	

If 10 per cent is an appropriate discount rate for these cash flows, the Largemouth's NPV is −£21 461, while the Lunker's is −£15 678. Since these are 'cost minimising' investments, the Lunker would seem the best, because it has the least negative NPV.

But the cash flows listed above for the two machines are not really descriptive of those that the company expects to incur from choosing one over the other. Choosing the Lunker implies purchasing another Lunker at t_2, t_4, and so forth. The Largemouth must be replaced at t_3, t_6 etc. Of course, each continues to incur its annual cash outlays during those times. Actually, Bayou Software has no particular plans ever to stop using whichever machine it adopts, so the stream of cash flows for each can be regarded as continuing indeterminately far into the future.

Thus the actual expectations for the machines are quite complex, being very long, non-constant cash flows across the future. This would seem to present a formidable problem in present-value calculation, and can in fact be exactly that. The most obvious logical answer to setting up a manageable analysis is to extend the cash flows as described above far enough into the future until the machines require replacement at the same time point. If we then calculate the NPVs, the numbers will not be the present values of the entire stream of costs that the machines will incur but will instead be of the same relative magnitudes of the entire stream's NPVs. This is because all cash flows to the future of the point where their replacements coincide will be multiples of those up to that time point.

Since the Largemouth's life is three years and the Lunker's is two, we need a total of six years until their replacements will coincide (again in £000s):

	t_0	t_1	t_2	t_3	t_4	t_5	t_6
Largemouth	−14	−3	−3	−17	−3	−3	−3
Lunker	−7	−5	−12	−5	−12	−5	−5

This display is the cash flows of the two machines up to the point that they first expire at the same time. (Note that the £17 000 outflow for the Largemouth at t_3 and the £12 000 outflows for the Lunker at t_2 and t_4 represent the sum of a new machine's purchase cost and an operating outlay, as would be the case if Bayou wished to avoid downtime in replacing machines.) We can now calculate NPVs that have some comparability. Under the replacement cycle above and with a 10 per cent discount rate, the Largemouth's NPV is −£37 584, and the Lunker's is −£39 343.

This result contrasts with the NPVs for a single lifetime for each machine, which indicated that the Lunker was the lower-cost alternative. In fact, if the machines are to replace themselves over time, the Largemouth is cheaper, because its somewhat higher initial outlays are spread further apart than the Lunker's, and the Largemouth's lower annual operating costs are now compared to the Lunker's for an equal number of periods, as opposed to the single-life procedure, which used fewer operating periods for the Lunker than for the Largemouth. So Bayou has narrowly missed accepting the wrong machine, and will correctly choose the Largemouth Special diskette copier.

The procedure above will work for all problems of this type, but can itself often become cumbersome. That happens when the common replacement of alternatives will happen very far into the future. For example, suppose that the Largemouth had a seven-year and the Lunker a nine-year interval. One must shudder at the sixty-three-year discounting period that would be necessary in that situation! There are other types of asset investments that can generate even worse combinations (try a new building that could go for either twenty-one or thirty-one years). Fortunately, finance has discovered a technique that can deal with such problems, and which we can also recommend for cases like our original Largemouth–Lunker example. This procedure is a rather elegant application of an idea with which we are already familiar: **annuities**, as discussed in Module 1.

An annuity, you will recall, is a stream of cash flows which is constant across time (these are the '£1 per period' present value tables in Appendix 1). Think of the situation this way: if the alternatives each had absolutely constant cash flows across time for ever, there would be no necessity to find their present values. We could decide which was preferred by simple inspection of any period's cash flow. In the case of cost-minimising investments, we would simply choose the alternative that had the lesser constant outflow in perpetuity, and we could be confident that we had chosen the least negative NPV.

The application of that idea to the Largemouth–Lunker type of problem is not obvious, however, because the investments do not have constant cash flows over time. But with a little effort we can make the analysis behave as if they do. The idea is to turn the non-constant perpetual streams into constant perpetual streams of the same present values. What we do is find the NPV of a **single cycle** for each asset, and divide that number by the annuity present-value factor for the number of years in each asset's replacement cycle at the appropriate discount rate. The result is the constant annuity outlay per period that has the same NPV as the asset. We then need only compare the per-period equivalent annuity outlay for each asset, and choose the one with the (equivalent constant) lower cost per period.

This sounds more difficult than it really is. Let us return to Bayou Software and apply this new procedure. First, recall that we have already calculated the single-cycle NPVs for the alternatives; the Largemouth's is −£21 461, and the Lunker has a single-cycle NPV of −£15 678. The present value tables in Appendix 1 (Table A1.2) indicate that the annuity present value factor for the Largemouth is 2.4869 (three periods, 10 per cent), and for the Lunker is 1.7355 (two periods, 10 per cent). Following the instructions above, we divide the single-cycle NPVs by the annuity factors, and arrive at constant annual equivalent cash flows of −£8630 for the Largemouth and −£9034 for the Lunker. This tells us that if we accept the Largemouth it is the same thing (i.e. has the same NPV) as accepting a constant cash outlay of £8630 per period, whereas the Lunker implies the equivalent of a £9034 outflow per period. So we would be led by this technique to accept the Largemouth and reject the Lunker, which we already know is the correct decision.

The application of this **equivalent annual cost** technique to this particular example requires about the same amount of effort as simply stringing out the cash flows until they end at the same time point. But that is not the case for most such comparisons – we designed the Largemouth–Lunker comparison to finish in only six periods. If you are not convinced that this is an easier way, try doing the example again for the seven- and nine-year cycles mentioned above. You will quickly come to appreciate this approach.

In summary, when a company faces a decision between renewable assets that are of different lifetimes, the simplest way to make a correct decision is to find the constant periodic cash flow having the same present value as a single lifetime or cycle for each asset. Analysts can then simply compare the periodic amounts and be certain that the correctly calculated NPVs would have the same rank-ordering. It is generally not appropriate to compare NPVs of single cycles for alternative assets in this situation.

6.10 Inflation and Company Investment Decisions

The reasonably recent experience of relatively high and variable rates of inflation in well developed countries has reminded financial managers that it is not only economies in developing countries that are subject to the effects of significant inflation, and that investment analyses, to be done correctly, must account for this phenomenon.

Our first task is to understand enough about the economics of inflation to manage its corporate financial aspects. Fortunately, this will not require that we become macroeconomists, not only because it is not clear how much **they** really understand about inflation, but also because much of their concern is why inflation occurs and how to control it. We cannot be concerned with those issues in this context. Our perspective is simply: given that inflation is an important dimension of investment decisions, how is it best treated?

Inflation is an increase in price unaccompanied by any other changes (such as quantity, quality, and so forth). With inflation, we must pay more money across time to acquire the same goods and services. If the inflation rate is 5 per cent in a period, something that cost £100 at the beginning of the period will cost £105 at the end. The prices in this example are stated in what economists call nominal terms. 'Nominal' means the actual number of £s (or whatever currency) that would change hands at the time the purchase is made.

There is another way to measure the amounts that are paid in inflationary times. Economists call this alternative measure a **real** price. A real price is the number of £s (or whatever currency) that **would have been exchanged** to purchase something **before** the inflation took place. In the above example the **real** price of the item, if inflation is 5 per cent, has not changed between the beginning and the end of the period. In terms of 'beginning-of-period-money', the **real price** is still £100, even though the nominal price is £105.

In corporate finance we are interested in the cash flows that investments will generate across the future. One of the important decisions that analysts must make is whether to use **nominal** or **real** measures of cash flow in their estimates for investments. Before we can answer which is best, we must deal with the influence of inflation on the other part of the investment analysis, the discount rates.

Economists are more or less unanimous in their opinion that inflation influences the rates of return that people require on their investments. In a very simple world where inflation is perfectly predictable, inflation affects required rates by merely adding enough to an inflation-free required return to compensate investors for their loss of purchasing power during the time of the investment. For example, suppose that when there is no inflation a particular investment requires a return of 10 per cent per period. If inflation is expected to be 5 per cent per period, the project would have to return enough to offset both the required 'inflation-free' return and the inflation rate itself in order to be acceptable. We call the 'inflation-free' return the **real rate**, and the return that compensates for both that and inflation, the **nominal rate**. The nominal return is thus given by:

$$\begin{aligned}(1+\text{Nominal return}) &= (1+\text{Real return})(1+\text{Inflation})\\ (1+15.5\%) &= (1+10\%)(1+5\%)\end{aligned}$$

In other words, in our example, the investment must return £15.50 each period for each £100 invested in order to be acceptable. If £100 is invested in a single-period project at t_0, it will be required to produce at least £115.50 at t_1. That is its **nominal** return. To find the **real** value of that t_1 nominal amount, we divide by (1 plus) the inflation rate expected, £115.50/(1.05) = £110, the required **real** cash return based upon a 10 per cent real required rate.

If we assume that the information for each is available, correct investment decisions can be made using either real or nominal cash flows and discount rates. For example, suppose that Channel Petroleum is evaluating a proposal to begin operating at a new location, and the expected **real** free cash flows are (in £millions):

t_0	t_1	t_2	t_3
−2000	+500	+900	+1300

If the real cost of capital of the project is 10 per cent, the NPV is (in £millions):

$$\begin{aligned}\text{NPV} &= -£2000 + \frac{+£500}{(1.10)} + \frac{+£900}{(1.10)^2} + \frac{+£1300}{(1.10)^3} \\ &= £175\end{aligned}$$

Suppose that the expectations are that inflation will certainly be 5 per cent per period. The nominal cash-flow expectations, if they are uniformly affected by that rate, will thus be (in £millions):

t_0	t_1	t_2	t_3
−2000	+525	+992	+1505

The nominal cash flows are simply the real cash flows for each time point multiplied by $(1 + \text{inflation rate})^t$, where t is the time point at which the cash flow is expected. The result is the actual (nominal) amount of cash that is expected to be available at that time, as opposed to the real cash flow, which is the t_0 purchasing power that the investment is expected to provide at that time point.

Recalculating the NPV of Channel Petroleum's project using the nominal cash flows and a nominal capital cost of 15.5 per cent (we know that to be the nominal rate because we have already found such a rate with a 10 per cent real return and 5 per cent inflation earlier in this section), we have the following (in £millions):

$$\begin{aligned}\text{NPV} &= -£2000 + \frac{+£525}{(1.155)} + \frac{+£992}{(1.155)^2} + \frac{+£1505}{(1.155)^3} \\ &= £175\end{aligned}$$

The NPV of the Channel Petroluem project is the same regardless of whether we use real or nominal calculations.

'Fine,' you say. 'That makes some sense, though it is not clear yet what the importance of all this is. But we have been dealing with cash flows and required returns all through the text to this point. What were those, real or nominal?'

Good question. You are correct in the inference that we have not been explicit as to which type of measure we were using; the reason was because it was not necessary until the whole question of inflation was raised. The values, cash flows and required rates we have been using all along have been **nominal** rates, and for a very good reason: nominal measures are the only ones that we can have much confidence in estimating. The prices and interest rates for financial securities that you see quoted in the financial press are nominal amounts, as are almost all of the financial results that companies release to the public in their financial statements.

The truth of the matter is that no one has ever really seen a real required rate. The reason is that such measures are economic concepts that are designed to help understand inflation's influence upon nominal rates, and not things that can be looked up in the newspaper. Remember that the real rate is the difference between the nominal rate and the **expected** influence of inflation on required rates for some time in the future. Because there is no way to measure such expectations' effects, the real rate is not measurable. Economists have attempted to estimate the real rate that actually occurred over past periods, but have not really reached agreement over the right way to do even that. One of the big problems in measurement is that inflation itself is not predictable. Many economists feel that nominal rates include not only the effects of expected inflation but also the uncertainty about it. So far, no one has been able to separate the two in a way that is widely accepted.

Does this mean that real measures of cash flow, value and required return are not useful? The answer is, as usual, both 'yes' and 'no'. For making financial decisions and calculating investment values, the answer is that real measures are not typically useful. Almost no companies attempt to estimate and use real rates and cash flows in their decisions, for the reasons of estimation difficulty mentioned above. But the real measures can be useful in helping us understand how inflation is correctly included in investment decisions, as we are now in the process of discovering.

What, then, is the answer to the best way to deal with inflation in corporate investment decisions? The answer is to use nominal cash flows (the actual amounts of money that are expected, at the time points that they are expected) and nominal discount rates. If the analysis is performed carefully, the impact of inflation on the investment, and on shareholder wealth, will appear in the NPV.

The most common error that investment analysts make in dealing with inflation appears in estimating cash flows. It is quite unusual for analysts to specify in their requests for cash-flow estimates that the estimates be 'inflated' by the estimator's belief about the effect of inflation on that amount by the time it is to be actually paid or received. Not being told otherwise, the marketing, production or other department is likely to estimate cash flows in the amounts that they are most comfortable with, namely those based upon the prices that exist today. The result is something like **real** cash flows being used in the analysis. If nominal discount rates (the 'observable' ones, which include inflation's influence and are therefore higher than the associated real rates) are used with real cash flows, there will be a bias toward rejecting good projects along with the bad.

To see how this can happen, suppose that Channel Petroleum's policy is to calculate NPVs using nominal rates and cash flows. If the company's analysts are not explicit in their requests for inflated future cash-flow estimates, they are likely to get the **uninflated** or real cash flows from the operating divisions in charge of coming up with the estimates. The NPV that they find is thus likely to be (in £millions):

$$\begin{aligned}\text{NPV} &= -£2000 + \frac{+£500}{(1.155)} + \frac{+£900}{(1.155)^2} + \frac{+£1300}{(1.155)^3} \\ &= -£49\end{aligned}$$

This is a disturbing result. Channel Petroleum is likely to reject this project, not because it is bad but because real cash flows were discounted with nominal discount rates, producing an NPV that is biased sufficiently downward to render it negative.

'Well,' you say. 'The solution to that problem is obvious. The analysts at Channel should simply inflate all of the cash flows that they get from those submitting estimates.'

That solution works in the Channel Petroleum problem, but is unlikely to be good in general for two reasons. First, unless the analysts can be certain that all cash flows submitted are real, they will probably end up inflating at least some already inflated cash flows. The result will be errors in the opposite direction.

The second problem is something that we assumed away in the Channel Petroleum example. You recall that we assumed that the real cash flows of the project will be uniformly affected by the inflation rate. This means that if inflation is 5 per cent, the cash flows of the project will all be higher by 5 per cent per period. The automatic adjustment you recommend will only work accurately if that is the case. Unfortunately it is not. Inflation rarely affects all cash flows identically; costs may rise by more or by less, as will eventual revenues. The only people likely to be able to make intelligent estimates of characteristic relationships with inflation are those who deal with the cash flows on a day-to-day operational basis, not the financial analysts.

The lesson now is more explicit: financial decision makers in corporations should explicitly request cash-flow estimates that **include** the effect of inflation on the specific cash flow in question. The analysts may wish to provide an estimate of an overall inflation rate to the operating estimators, but the latter should be the ones to specify inflation's effect on the particular cash flow to be estimated.

There is an additional reason why inflation would not be expected to result in a simple one-for-one effect on corporate cash flows. This has nothing to do with estimation errors but is a result of tax laws. In most countries, as we have already illustrated, government allows corporations to reduce taxable income by the amounts spent for productive assets only according to a defined schedule across time (called depreciation). When inflation occurs, the costs of productive assets will increase across time more or less with other cash flows, but depreciation expenses for assets bought in the past will not. The net effect can be to make taxes increase across time faster than the inflation rate, and thereby to make corporate free cash flow (FCF*) increase at less than the inflation rate. Governments are not unaware of this phenomenon, but are torn between the nice automatic increases in real government revenues generated thereby and the relative disincentives for corporate investments implied by that situation. The many and various schemes to allow 'accelerated' depreciation expenses by companies are at least in part attempts to treat that concern.

Accelerated depreciation schedules appear in almost infinite variety. Their common characteristic is to allow write-offs at a pace faster than that implied by the straight-line depreciation method, which simply expense an equal proportion of depreciable value each period. The **double-declining balance** method allows a deduction equal to twice what the straight-line amount would be if calculated on the depreciable amount and time remaining in each period. The **sum-of-the-years'-digits** method allows a deduction equal to the proportion found by dividing the remaining depreciable life of the asset by the sum of the years of the asset's total life. We shall not concentrate on the details of the methods allowed because they are constantly in flux; if you want to know the latest rules that apply for depreciation, investment tax credits, and so forth, you should depend upon a good accountant or tax specialist.

Occasionally a corporate financial analyst will be asked to render an opinion as to the depreciation technique that would be optimal to adopt for a particular asset. Since taxes are the only cash flow influenced by depreciation, the easiest way to perform that analysis is to regard the depreciation expenses as generating 'tax shields' in the same way that interest does, and compare the competing methods' depreciation tax shields' present values. If there is no other effect of the depreciation choice (such as an interaction with a tax credit or some other deductibility), the discount rate used should not be much above a risk-free rate.

There is one final implication of inflation for corporate investment decisions that we must mention. Here we are concerned not with the cash flows of the investment but how they are to be financed. We argued above that nominal required rates reflect in part the compensation that investors require for bearing the expectation and risk of inflation. There is one type of investor that could be particularly concerned about inflation and required rates: the debt suppliers of corporations. The reason that debt suppliers might be unusually concerned about inflation is the type of contract they get when they invest in a company.

Debt contracts promise specific amounts of **nominal** cash at particular times in the future, regardless of the inflation rate that actually occurs in the interim. Thus if the **nominal** interest rate that debt suppliers get at the inception of their investment turns out to be a poor estimator of actual inflation, debt suppliers will achieve a **real** return significantly different from their initial expectation. If inflation is higher than expected, debt suppliers will be hurt (wealth will shift from the company's debt suppliers to its equityholders, who are now paying off the debt promises with unanticipatedly cheaper nominal amounts). The reverse is true when inflation turns out to be less than comprehended by interest rates at the outset.

It would seem that this is a chance for equityholders to increase their wealth at the expense of debt, or vice versa. If one examines the historical evidence, it is obvious that this has happened often, and at times in important magnitudes. Market interest rates have not been impressive in their capacity to predict actual future rates of inflation. But what is interesting in this finding is that the errors are not biased in one direction or the other; sometimes equity wins, and at other times debt does. As a matter of fact, though the inflation predictions of interest rates are bad, economists have not been able to find any alternative predictor that is better – or even as good! So it would not be wise to enter a debt contract at market interest rates **as either a borrower or lender** with the expectation of benefiting from the effects of inflation. You are likely to win or lose something, depending on the side you choose, but it is impossible to know beforehand whether that will happen to the borrower or the lender.

That situation is exactly what we would expect to see in a very competitive market (such as the financial market) attempting to predict the unpredictable. No one can reasonably expect to win something from someone else possessed of the same information.

What have we learned about inflation and corporate investment decisions? Actually, the lesson itself is quite simple: discount estimated nominal cash flows with nominal market discount rates. Do not concern yourself with real rates and cash flows; they are the grist of theoretical economists. While the economists are observable, real rates are not. Be certain that all involved in the estimation of cash flows understand that the analysis requires cash flows **including** the effect of inflation. And do not choose to finance your investment in any particular way due to inflation, unless you can find someone foolish enough to accept a rate less than the market rate. In reasonably efficient economies, the odds of finding such an investor (other than governments) is somewhat less than finding a wallet full of £50 notes on the pavement.

6.11 Leasing

Our discussion of applications in company investment analysis would not be complete without some exposure to **leasing**. 'Leasing' is a contractual arrangement between an asset owner (the **lessor**) and a company that will actually operate the asset without owning it (the **lessee**); in essential economic terms the lessee rents the asset from the lessor. In exchange for allowing the lessee to operate the asset, the lessee must pay the lessor **lease payments** during the period that the lessee has the use of the asset.

There are many types of lease contracts that companies can pursue. The most commonly encountered one, and the one that is described above, is a **financial** or **capital lease**. In these kinds of leases the lessor is usually in the business of leasing assets, and not in the business of actually operating the asset that is leased. There are many large companies around the world that are in the business of leasing assets to other companies that actually operate the assets. Such assets include aeroplanes, ships, shipping containers, computers, buildings, and many others (some reasonably exotic, such as sewerage treatment facilities, jet fighters and nuclear fuel).

6.11.1 The Economics of Leasing

In the context of the ideas of investing and financial markets that we have covered in the text to this point, one could raise the reasonable question, 'Why do leases exist?' What is there about leasing that is superior to the lessee simply buying the asset itself? Why go to the trouble of the lessor buying the asset and then renting it to the lessee? As we shall see, there are good economic reasons for leases to exist, but these reasons must be related to the lease accomplishing something valuable to the lessee that the lessee could not accomplish by simply buying the asset.

First, here are some things that are often mistakenly taken to be benefits of leasing:

- *Leasing saves money for the lessee because the lessor is paying for the asset and the lessee therefore does not have to make the large initial purchase outlay for the asset.*
  Wrong. It is true that the lessee does not have to make a large initial outlay as the lessee would do if it purchased the asset, but the lessee of course must contract to make a stream in lease payments, which carry the same essential promise as a debt claim. On the face of it, there is no reason to think that the present value of the lease payments is either larger or smaller than the present value of the purchase price of the asset.
- *Leasing helps the lessee's balance sheet, because the lessee does not need to borrow money to buy the asset, and so the lessee's debt capacity is higher than it would otherwise be and therefore the company is less risky.*
  Wrong. Financial market participants who are interested in a company's balance sheet relationships and financing abilities are unlikely to be fooled into thinking that a lease contract is not roughly similar to a debt contract (most lease contracts have to be at least described in footnotes to the lessee's financial statements, and many actually appear as liabilities on the financing side of the balance sheet).

If the above are not good reasons for leasing to exist, what are the valid reasons for this to be such a popular financing or investment vehicle? Among the correct reasons are:

1. *Leasing allows for higher tax benefits than the alternative of borrowing and purchasing an asset.*
   Right. Consider the situation of a lessee firm having low or uncertain taxable income, and the wish to operate some particular asset. As you are aware, the deductions for depreciation and interest on such a transaction are potentially valuable to the firm operating the asset. But these benefits are contingent on the company having enough taxable income so that its taxes would be reduced. If the lessee does not have the prospect of such income over the life of the asset, it makes economic sense for another firm with such income tax liabilities (the lessor) to purchase the asset, take advantage of the tax benefits of depreciation and interest on money borrowed to purchase the asset, and set lease terms with the lessee such that these benefits are shared between the two. A similar benefit derives from a lease transaction if the lessor's income tax rate is simply higher than the lessee's.
2. *'Information asymmetries' exist on certain types of assets, and leasing can serve to lower the costs of such information problems.*
   Right. Certain assets, particularly expensive high-tech equipment such as large computers and complex medical devices, are subject to the frequent and uncertain timing of obsolescence. (Who among us has not purchased the 'latest and greatest' personal computer, only to hear within a very short time that its successor is twice as good and half the cost?) Sellers of such assets usually know more about the expected schedule of obsolescence than do buyers. A lease can be written in such a manner as to allow the lessee to take advantage of 'new and

improved' or upgraded assets, and thus avoid the risk of costly obsolescence. And the lessor/seller can probably purvey more of the asset through a leasing mechanism, given the high uncertainties of outright purchase by users.

3. *There are 'economies of scale' in the management of specialised asset leasing.*
   Right. As we have seen, leasing from the perspective of the lessee is economically similar to purchasing an asset and borrowing the money to do so. The transaction of 'borrow and purchase' requires that the lender evaluate the lessee, the asset, and other dimensions of the transaction. This can be a costly procedure, particularly if it is done only sporadically. A company in the business of leasing a particular kind of asset (say, airliners) can be more efficient than a potential lender (say, a commercial bank) because the leasing company is intimately familiar with the asset to be leased, and may well do several such transactions per day.

There are other good reasons for leasing, but these are sufficient to give you the flavour of why this transaction exists. These reasons have the common characteristic of providing a benefit to one or other party to the transaction, which benefit would not be available without the lease.

6.11.2 Evaluating Leases

The investment decision involved in leasing is somewhat different from those we have seen thus far in this module. The question to be decided is whether the lessee would be better off obtaining the asset through leasing as opposed to purchasing the asset with borrowed funds. Thus there is an implicit assumption that the decision to obtain the use of the asset has already been made, and the only remaining issue is to lease or to borrow-and-purchase.[2]

Consider the following example. Topsy-Turvy Airline has decided it requires another twin-engined propeller aeroplane for its expanding markets. Topsy-Turvy would pay £1 000 000 for the aeroplane if it were purchased, the aeroplane would be depreciated over four years by the straight-line method and then scrapped for no net value (it is not a new aeroplane even now). Brigadier Electric Financial, a large leasing company, has offered to purchase the plane itself and lease it to Topsy-Turvy for five annual lease payments (beginning now) of £270 000. Topsy-Turvy is subject to a 40% corporate income tax rate, and is otherwise profitable enough to use all deductions.

The cash flows relevant to the decision as to whether to purchase with borrowed funds or to lease are (£000s):

[2] The cash flows from operating the asset are typically not any different when the asset is leased as when it is purchased, although some leases provide for asset maintenance. You can consider the situation as being one where the potential lessee has calculated a positive NPV for the asset if purchased, and is now considering a lease alternative. It is possible also that an asset would not be an acceptable investment if purchased, but would be if leased. This implies that companies, when evaluating the acquisition of assets, should always investigate the alternative of leasing.

	Year 0	Year 1	Year 2	Year 3	Year 4
Cost of aeroplane	1 000				
Lost depreciation tax shields		(100)	(100)	(100)	(100)
Lease payment	(270)	(270)	(270)	(270)	(270)
Lease payment tax shields	108	108	108	108	108
Lease cash flows	838	(262)	(262)	(262)	(262)

Notice that:

- There are no operating cash flows from the aeroplane in the analysis because there is no *difference* for Topsy-Turvy in either alternative.
- Initially, Topsy-Turvy actually gains £838 000, through avoiding the £1 000 000 purchase outlay but paying the £270 000 lease payment and getting the deduction for making that payment, which is a tax saving of 0.4 × £270 000 = £108 000.
- Topsy-Turvy, after the initial cash flow, continues making the lease payment, and loses the depreciation tax shields of the aeroplane (which are taken by Brigadier Electric and are worth £100 000 per year, the £250 000 annual depreciation multiplied by Topsy-Turvy's tax rate of 40%).[3]

In essence, leasing compared with purchasing gives Topsy-Turvy an initial £838 000, but requires that the company make effective payments of £262 000 per year for four years.

What then, should Topsy-Turvy do? The answer of course depends on the NPV of the lease vs. borrow-and-purchase cash flows. The only issue that remains is the appropriate discount rate to use. What rate is appropriate? We are now familiar with the idea that the discount rate must be consistent with the riskiness of the cash flows upon which it operates. And the riskiness of the cash flows at issue here, because of the contractual nature of the lease, is essentially similar to those of Topsy-Turvy making comparable payments of interest and principal. So the correct discount rate is the after-tax interest rate (or what we have earlier called rd^*). Suppose that Topsy-Turvy can borrow at 14%. The after-tax rate is thus 14% × (1 – 0.4), or 8.4%.

Applying that discount rate to the lease cash flows above produces −£22 106.68, a negative NPV. Topsy-Turvy would be better off purchasing the aeroplane than leasing it. The present value of after-tax cash flows from leasing are more costly than those from purchasing.

Notice that if Topsy-Turvy can negotiate a lease payment less than approximately £255 000 per annum, the lease is better than purchasing. This is often a valuable

[3] These are designed to be very straightforward cash flows. More realistic ones might have a sale of the asset for some positive value and associated tax deduction in the purchase option, and perhaps a purchase option in the lease alternative. The analysis is nevertheless similar, though more complex.

piece of information for a potential lessee to have, because many such leases are individual negotiations between lessee and lessor.[4]

6.12 Managing the Investment Process

It is one thing to know the processes (such as those we have discussed in this Module) that must be used so as to produce correct investment decisions, but it is often quite another thing to get these to work correctly in the environment of modern business organisations.

Consider, for example, that all of the techniques we have illustrated have assumed implicitly that the cash flow forecasts they use are of 'high quality'. This does not mean that such forecasts always turn out to be accurate, but rather that there are no systematic biases that have crept into them, such as being overly optimistic. Clearly if the cash flow projections used in NPV-based analytical techniques tend to paint too rosy a picture of the future, the company will make poor investment decisions. Over the long term, cash flow forecasts should *on the average* turn out to be correct. This happy condition is, however, not particularly easy to achieve in complex organisations where incentives facing middle-level managers may encourage them to take on ever larger projects, rather than higher NPV projects, due to how their compensation is configured.

We have already seen one means by which this type of incentive conflict is managed: capital rationing. But we have also seen that relying on capital rationing to solve this problem is not a particularly attractive alternative because of the inherent economic shortcomings of that method. And the generation of overly optimistic forecasts is not the only problem that arises in the implementation of our NPV-based methods in complex organisations. There are several other biases that can creep into a company's investment evaluation processes, such as favoring projects that particular management groups have self-perceived expertise, misestimating riskiness of projects, and several others.

Is there a solution to these implementation problems? If there is a solution, the above discussion indicates that it will probably come from changing the way that managers' incentives interact with the investment decision-making process. Such incentives can be either positive (such as tying managers' compensation to how well shareholders do) or negative (such as imposing more stringent monitoring systems on management performance), or some combination of these. Of course methods that accomplish some of this have been around for some time: employee stock options, and outright dependence of management salaries and bonuses on the performance of the company's equity on the stock market. The threat of hostile 'takeovers' of one firm by another (where shareholders choose alternative management) is also an effective incentive for managers to make investment decisions with shareholders' interests in mind.

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[4] The lessor's cash flows are essentially the mirror-image of the lessee's, so the negative present value to Topsy-Turvy is a positive NPV to Brigadier Electric. However, if Brigadier Electric's tax rate is higher than Topsy-Turvy's, both can potentially have positive NPVs (which is, as we know, one of the reasons for leases to exist).

These methods are increasingly common, and because they directly relate management compensation to shareholder wealth outcomes, do work effectively at higher levels of management where decisions are company-wide and therefore have effects that are more obviously associated with stock market valuations of the company. But these methods too have shortcomings. For one thing, many factors other than the decisions made by its managers influence a company's stock price. For example, changes in overall economic activity and in the fortunes of related industries can all have very large effects on the value of a particular company's shares, and are essentially outside of the control of management.

Highly paid compensation consultants can perhaps design schemes for a company to pay managers so as to filter out such 'noise' in the value of the company's shares. But even this does not solve the problem of evaluating and creating appropriate incentives for middle and lower level managers whose decisions affect only one part of a larger firm, and therefore are not clearly connected to the company's overall share value. The decisions of these managers are in effect bound together in the company's share value, which likely makes that value an inappropriate vehicle to use for compensating individual managers, some of whose decisions may be good and some bad.

6.12.1 Using Economic Income in Performance Measurement (EVA, EP, etc.)

The application of 'economic income' concepts has become an increasingly popular approach to measuring the economic performance of management.[5] These methods are attempts to reproduce the benefits of market valuation measures as performance indicators, but at the same time make them useable at the middle management (or individual project) level of a company.

In concept these methods are straightforward. All have the attribute of charging a capital cost against the net cash flows of a company division in a given period, and seeing if there is anything left over. This cost is typically calculated by multiplying a WACC by an invested amount to produce a £-stated capital cost. If there is a positive residual amount left over, the division's revenues have covered not only the costs that are typical of those on its accounting income statement, but also the opportunity costs of its capital suppliers, leaving a 'economic profit' for its shareholders. If there is a negative residual, the division has not done as well as shareholders could have done in a comparable-risk investment elsewhere.

For example, suppose that the Slacker division of General Appliances, PLC has the following financial statement data for a given year:

[5] These methods have been refined and marketed by consulting firms such as Stern-Stewart (of Economic Value Added or 'EVA©' fame) and McKinsey & Company (selling Economic Profit as 'EP').

Revenues	£1 000
Expenses (other than interest)	500
Profit Before Tax	500
Taxes	200
Profit After Tax	£300
Net Assets (including working capital, and after depreciation)	£2 000

Now suppose that the Slacker division's WACC is 18%. This implies that the net investment of £2000 requires a break-even economic opportunity cost return of 18% × £2000 = £360. Yet Slacker has produced a return of only £300, so shareholders have actually experienced a *negative* £60 outcome, despite the division's showing £300 of accounting profit.

Though the actual implementations of EVA© and similar methods are much more sophisticated than this illustration (for example they use more refined measures of investment, and more cash-flow oriented definitions of income) they are consistent with its essence. And the appeal of such measures to those schooled in WACC–NPV techniques should be obvious. In a sense these economic profit measures are nothing more than 'period-by-period' applications of WACC–NPV. This tells us much about both their strengths and weaknesses.

The strength of economic profit measures is that they explicitly recognise that all capital suppliers, not just creditors, require adequate returns: the total of invested capital must be compensated for its opportunity costs. And as we know, the WACC is the best measure of the rate of such necessary compensation. Economic profit measures have the strength of uncovering company operations that are profitable in an accounting sense, but not in an economic sense. The company's activities that produce positive economic profit have positive effects on share value, the activities that do not generate positive economic profit cause share value to decline.

That economic profit is tied directly to shareholder wealth makes it an excellent candidate for use as a management performance measure. And it has become very popular for that reason.

This is not, however, to say that economic profit measures are without significant impediments to being used effectively. From our study of the differences between accounting numbers and cash flows, you know of some potential pitfalls in the application of economic profit. For example, how should the differences between accounting measures of depreciation and economic depreciation be dealt with in determining 'investment?' And even more importantly, recall that the economic profit performance measure is applied 'period-by-period.' How can this be used to evaluate a new division that can reasonably be expected to produce negative cash flows in its earlier years, which are only compensated later? Economic income measures are not present values of streams of income, as is NPV.

These reservations having been expressed, it is only fair to observe that economic profit performance measures continue to be implemented at a very fast pace at companies around the world, and have secured many devoted adherents. That the details of their implementations remain commercial secrets makes 'evaluating the evaluations' inherently difficult. A consolation is that markets for the companies' securities will of course offer the ultimate evaluation.

Learning Summary

In this module we have examined several miscellaneous issues in applying the investment-decision techniques developed in earlier modules. We have looked at the potential competitors to NPV, discussed their weaknesses and strengths, why many companies continue to use them, and some ways in which certain of them can be made to work correctly (that is, give the same answers as NPV would).

In addition, we have dealt with several complexities of company investment analysis, including capital rationing, investment relatedness, renewable investments, inflation, leasing and the implementation of economic profit performance measures. As with so many topics in an introductory course such as this, the list of complexities is not exhaustive of all that an analyst may encounter. But the list is instructive of many such complexities and – as important – is illustrative of the economic logic that underlies finance's approach to all situations of that type. Remember that investment analyses attempt to calculate the effects upon shareholder wealth promised by accepting the cash flows of a project. These cash flows are best regarded as the changes that are expected to occur in the cash flows of the company as a whole if the project is accepted. The goal of such analyses is to accept the project or group of projects that has the highest possible increase in shareholder wealth. This is best measured by calculating the NPVs of the various investment proposals at hand.

Application of these principles cannot be one of strict rigidity in all situations, particularly ones that the analyst has not before encountered. For example, suppose that you are asked to evaluate a decision to abandon or liquidate an investment. At first glance it would seem that the correct analytical approach would be to estimate the changes that would take place in the company's cash flows were the investment to be ended, then calculate the NPV implied by those cash flows, and abandon the investment if the NPV so found is positive.

Actually, however, that procedure is not a good one for abandonment decisions. What it ignores is that, although abandonment **this** period is superior to operating the investment over the remainder of its expected lifetime, abandonment of the investment at some **future** time may be superior to both. This means that the astute analyst will produce NPVs for abandoning the investment not only in this period but also for all future periods during which it is expected to continue operating. If **any** of those NPVs is greater than that for abandoning now, the project is not abandoned now. (Actually, it may not be abandoned even at the future time with the currently highest NPV, because after the passage of time expectations as to the abandonment's desirability may change. For example, next period's analysis may indicate that abandonment is best then, at some other point, or never.)

These lessons in investment applications will not make you an instant expert in financial analysis, but sufficient exposure to the basic logic of the approach will go a long way toward developing the correct intuitions within you when you are faced with previously unexperienced analytical situations. Pay close attention to the Review Questions that follow, and the explanations associated with each in the answers provided.

Review Questions

6.1 Payback period as an investment evaluation technique for companies:

I. Is not a preferred technique because it is inconsistent with the discounting processes involved in valuing cash flows.

II. Is reasonably good most of the time because it can be made to work if used carefully in most situations.

III. Can be used to produce correct answers, but only in limited situations.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both I and III.

6.2 The average return on investment (AROI or ROI) as an investment evaluation technique can best be described through which of the following?

A. It is not preferred because, like the payback-period method, it does not discount, but can be made to work correctly in limited circumstances.
B. It will work essentially correctly because the accounting numbers' accrual basis does essentially the same thing as a discounting process.
C. It will occasionally produce different answers from using NPV because of certain mathematical complexities of the discounting in the AROI technique.
D. It is essentially unsuitable because it uses accounting numbers instead of cash flows and does not discount.

6.3 Suppose that you must choose at most one of the two investments whose free cash flows are listed below, and the appropriate discount rate to be used for both is 8 per cent (this is the investments' rv^*). Calculate the NPV and IRR for each investment, and decide which you would choose.

	t_0	t_1	t_2
Investment 1	−£1500	+£180	+£1680
Investment 2	−£1000	+£130	+£1130

6.4 Suppose your company requires that all investment analyses should be performed with the IRR technique. Return to the examples of Question 6.3 and produce an analysis that is both correct and in keeping with your company's policy.

6.5 Consider these statements:

I. The cost–benefit ratio (CBR) is essentially the same as the NPV, but with the inflows in the numerator and outflows in the denominator. So it should work the same as the NPV.

II. The cost–benefit ratio is more like the IRR than the NPV, in that it gives correct answers for a single investment but not necessarily for rankings of investments.

III. The cost–benefit ratio is really a combination of the NPV and the IRR, but presented in ratio terms. It thus avoids the problems of the IRR, but is in terms of ratios rather than wealth numbers, so people tend to understand it better.

Which of the following is correct?

A. I only.

B. II only.

C. III only.

D. Both I and III.

6.6 Consider these statements:

I. The profitability index (PI) is essentially the same as the NPV, but with the inflows in the numerator and outflows in the denominator. So it should work the same as the NPV.

II. The profitability index is more like the IRR than the NPV, in that it gives correct answers for a single investment but not necessarily for rankings of investments.

III. The profitability index is really a combination of the NPV and the IRR, but presented in ratio terms. It thus avoids the problems of the IRR, but is in terms of ratios rather than wealth numbers, so people tend to understand it better.

Which of the following is correct?

A. I only.

B. II only.

C. III only.

D. Both I and III.

6.7 In capital-rationing situations the object is, as ever, to maximise the wealth of currently existing shareholders. In order to do this the decision maker must do which of the following?

A. Use the IRR to rank investments in order so as to find which give the highest return per £ invested, and accept in order of declining IRR until the budget is exhausted.

B. Use the PI to rank investments in order so as to find which give the highest NPV per £ of outlay, and accept in order of declining PI until the budget is exhausted.

C. Use the CBR to rank investments in order so as to find which give the highest ratio of present-value inflow to outflow, and accept in order of declining CBR until the budget is exhausted.

D. By whatever method find the set of investments that has the highest NPV and is within the budget constraint.

6.8 The Happy Pooch Dog Food Company is considering three projects.

Project S is a proposal to increase the output of Happy Pooch Dog Food by investing in some new equipment at a cost of £100 000 and some plant modifications that are expected to cost £40 000. Because Happy Pooch has been quite successful, its managers expect that the company's future cash flows will increase as a result of this project in the amount of £210 000 on a present-value basis, thus creating a £210 000 − £100 000 − £40 000 = £70 000 NPV for the proposal.

Project H is the introduction of a new Healthy Pooch Dog Food line, with a somewhat higher vegetable and fibre mix, which can be marketed to dog owners greatly concerned about their pet's health. Project H will require an outlay of only £25 000 for some fancier labelling equipment, and is expected to produce present value future FCF* of £55 000, thus creating an NPV of £30 000.

Project F is a proposal for Happy Pooch to begin marketing a line of dog 'accessories' such as leashes, coats, playtoys and so forth, in order to capitalise upon the company's good name with dog owners. The equipment necessary to manufacture, package and market these items will cost £210 000, but the expected future net cash flows have a present value of £305 000, yielding an NPV of £95 000.

It is clear that all three proposals have positive NPVs. However, these numbers have been presented under the assumption that each project is undertaken independently of the others. Actually, there are several interactions among the projects of which Happy Pooch is aware:

I. If S and H are both undertaken (but F not), because of the volumes of output, there is the chance to buy a somewhat more efficient production process, which would add £15 000 to their total combined initial costs but would increase the present values of their combined future net cash flows by £30 000.

II. If H and F are both undertaken (but S not), the combined total initial outlay will be £15 000 less than their independently considered outlays, because a particular piece of labelling machinery can be used in both product lines and has enough capacity to service both.

III. If S and F are both undertaken (but H not), there are no foreseeable interactions of outlay or future cash flow.

IV. If all three projects are undertaken, all of the above interactions are expected to be present; and, in addition, there will necessarily be a plant expansion that will cost £100 000 but will alleviate the plant modification outlays necessary for S were it undertaken alone or with either of the other two projects.

Advise Happy Pooch what it should do, and why.

6.9 Icky Fishing Supplies must get a new aerator for its maggot and worm production facility. There are two models being considered by the financial managers of Icky. The Echh Aerator can be bought for £53 000, will have operating costs of £13 250 per year, and will require replacement at the end of year three. The Ugh Aerator costs £66 300 to purchase, £10 600 per year to operate, and should last a year longer than the Echh before needing replacement. Once purchased, the installation requirements dictate that a machine be replaced with itself rather than another model. Icky believes that its capital costs for these machines is 12 per cent.

Which of the following should Icky undertake?

A. Purchase the Echh because the NPV of its cash flows is less than those of the Ugh.

B. Purchase the Ugh because the NPV of its cash flows is greater than the Echh.

C. Purchase the Ugh because the company will end up paying less in present value considering future replacements.

D. Purchase the Echh because its lower NPV costs in the first cycle of ownership means that it will always be the cheapest alternative.

6.10 'Inflation is no problem for us to handle in our analyses,' says your financial department. 'We are aware that market interest rates and other capital rates already include compensation for expected future inflation, and also that we have no way of knowing exactly what part of such rates is due to inflation. So the use of real or uninflated discount rates and cash flows is not a good option for our analyses. Instead we accept the market rates with their inflation premium, and apply our own inflation estimates to the cash flows given us by operating units as they submit proposals.'

Which of the following should encapsulate your opinion of this statement?

A. It sounds correct.

B. It appears as if the finance department does not fully understand the workings of inflation and its interaction with capital rates and project cash flows. If they inflate the proposal cash flows instead of deflating the market rates, they are simply exchanging one set of uncertainties for another, to no benefit.

C. The finance department is definitely incorrect in its approach to adjusting NPV analyses for inflation. The problem is that the manner of accepting the market rates, and in addition adjusting the cash flows upward for inflation, actually double-counts the influence of inflation in the analyses: once in the discount rate, and then again in the cash flows.

D. The finance department seems to be doing the correct thing, given all dimensions of the problem, but it should be certain that the cash flows coming from the operating units are not already inflated, and that better inflation estimates might not be available from the operating units themselves.

Module 7

Risk and Company Investment Decisions

Contents

Learning Objectives

Module 7 is the first rigorous exposure for you to modern ideas of risk measurement in finance. These concepts are important because they underlie the set of returns and prices that exist in real financial markets for securities and assets of differing risk. The module begins with illustrations of appropriate risk measures for an individual investor's entire wealth position, and departs from that basis to notions of portfolios of assets, diversification and risk of individual assets within portfolios. The culmination of this process is the module's presentation of the capital asset pricing model (CAPM), which depicts the market relationship between the relevant risk of an individual asset and the rates of return or discount rates applied to the cash flows of that asset. In other words, the CAPM tells how securities markets set prices on risky securities. The module then illustrates how companies can use the CAPM's risky-discount-rate estimation process to make better investment decisions. Several detailed applications illustrate how these ideas are best applied. In this module you will discover one of the most important sets of ideas in modern finance, namely the correct treatment of risk in cash-flow expectations, and how that translates into required returns for decision making. This notion is one of the cornerstones of the course.

7.1 Introduction

Through the finance course to this point we have mentioned the concept of risk with some regularity, and have emphasised that the correct treatment of risk in financial decisions is most important. But we have not yet attempted to deal with risk in a measurable way. It has been sufficient to our purposes until now to describe risk in merely an ordinal manner, e.g. that an investment, a security or a cash flow is of greater or lesser risk than another, and thus requires a greater or lesser return.

It is now time to introduce the notion of risk as a measurable or quantitative characteristic and illustrate how financial managers can make good decisions using this risk measure.

What we seek is reasonably easy to describe. Consider Figure 7.1. The graph plots risk on the horizontal axis and required return on the vertical. You can see that the line (called the **security market line** or SML) describes the relationship between risk and return as being positive: the higher the risk, the higher the required return.

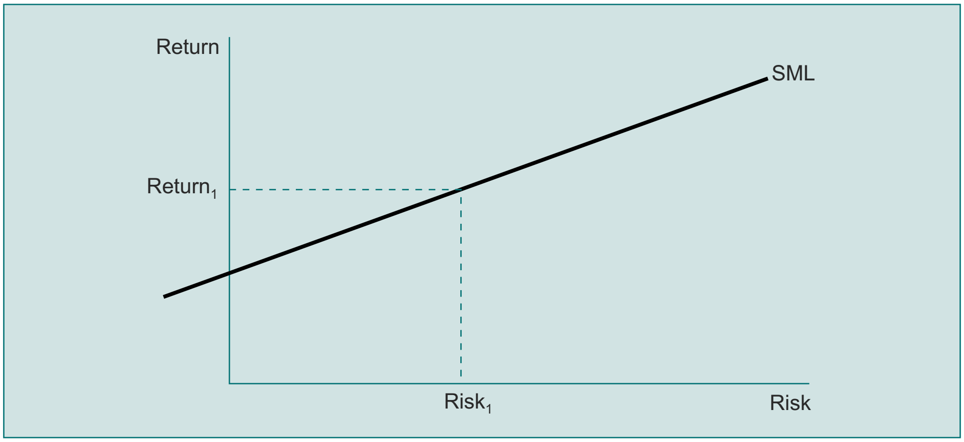

Figure 7.1 **Relationship between risk and return – the security market line**

Figure 7.1 also indicates how financial managers would be able to use a quantitative risk–return relationship to make financial decisions. Suppose that we were facing a decision to undertake an investment I, and all of the necessary cash flows had been estimated but the appropriate discount rate to apply to those cash flows was not known. If the risk of the cash flows could be measured and found along the horizontal axis (*see* ***$Risk_I$*** in Figure 7.1), then one could find the appropriate discount rate simply by using the SML (*see* ***$Return_I$*** in Figure 7.1).

Figure 7.1 indicates to us exactly what our main task will be in this module: to specify in a measurable fashion exactly what the financial markets regard as 'risk' (the horizontal axis of Figure 7.1), and to specify the location of the SML so that it can be used to generate risk-adjusted discount rates to be used in financial decisions. Before we finish the module we shall have redrawn Figure 7.1 with the appropriate quantitatively useful risk and return measures.

7.2 Risk and Individuals

The first step in developing a quantitative measure of risk is to describe what the capital suppliers of a company regard as risk. This is obviously important because we know that such individuals effectively set the required returns of the company. The greater the risk that they perceive, the greater are the returns that they require.

Economists have long studied the question of risk, and though it is by no means a globally settled issue, most financial thinkers have agreed that, to an individual capital supplier, risk is best measured by the **standard deviation of rates of return on the entire portfolio of assets**. This is a measure of the extent to which the portfolio's possible outcomes are likely to be different from its mean or expected average outcome. Without arguing the philosophical intricacies of this definition, let us see exactly how it is calculated.

Suppose that you are curious about the riskiness of the set of securities that you hold, and want to measure this riskiness as consistent with the above definition. In order to do so, you must have certain pieces of information about the returns you expect to earn from holding the securities. Of course, you do not know for certain what your returns will be (if you did, the securities would not be risky), but are able to quote likelihoods in the form of **probabilities** about the rates of return that you expect from your holdings. Such information about your holdings are called **probability distributions of returns**.

Table 7.1 Portfolio probability distribution

Rate of return	Probability
8.5%	35%
11.0%	10%
13.5%	30%
16.0%	25%

Let us assume that the probability distribution of rates of return on your entire asset holdings are as shown in Table 7.1. This information can be used to find the riskiness of this set of holdings by calculating the standard deviation of its probability distribution of returns. The calculation is not very difficult; it requires that we find the '**mean**' or average expected return. To calculate the mean or average expected return we multiply the rates of potential return by the probability of their occurrence. The sum of these products provides the mean. We then subtract each potential outcome from the mean return, square those differences, multiply these differences by the likelihoods of their occurring, add them up and take the square root. That probably sounds more intricate than it really is; here is the maths for our example:

$$
\begin{aligned}
\text{Mean return: } 0.085 \times 0.35 &= 0.02975 \\
\text{plus } 0.11 \times 0.10 &= 0.01100 \\
\text{plus } 0.135 \times 0.30 &= 0.04050 \\
\text{plus } 0.16 \times 0.25 &= 0.04000 \\
\text{Sum} &= 0.12125
\end{aligned}
$$

$$\text{Mean expected return} = 12.125\%$$

Following the rest of the instructions, we have:

$$(0.085 - 0.12125)^2 \times 0.35 = 0.00045992$$
$$(0.11 - 0.012125)^2 \times 0.10 = 0.00001266$$
$$(0.135 - 0.12125)^2 \times 0.30 = 0.00005672$$
$$(0.16 - 0.12125)^2 \times 0.25 = 0.00037539$$
$$\text{Sum} = 0.00090469$$

The square root of 0.00090469 = 0.03008 or 3.008 per cent, and this is by definition the standard deviation of the portfolio of your holdings.

So the standard deviation of your asset portfolio is slightly over 3 per cent, and the mean return is 12.125 per cent. The 3 per cent is a reflection of the risk inherent in your portfolio. Again, as you can now interpret from the above calculation, standard deviation is a measure of the extent to which the possible outcomes are likely to be different from the mean outcome, and finance assumes that this is the 'risk' as far as capital suppliers are concerned.

'Well, that's easier than I thought it would be,' you say with relief. 'We now know what must be plotted along the horizontal axis of Figure 7.1: the standard deviation of returns. We have solved the problem of the appropriate risk measure for capital suppliers, and thus for the company considering using capital suppliers' funds for investment.'

Not so fast. Though few in finance would disagree that the standard deviation of return is a decent measure of risk,[1] researchers have been unsuccessful in finding any reasonable relationship between securities' **risk**, as measured by their standard deviation, and their **return** as measured by the actual returns earned. Most studies have ended with results like those in Figure 7.2. One need not be a sophisticated statistician to see that the empirical relationship between risk (measured as standard deviation of return) and the level of actual return earned is not good.

[1] Actually, there is quite a bit of intricate economic and statistical thought of a theoretical nature behind the standard deviation as a risk measure, going all the way back to Daniel Bernoulli in the eighteenth century (and purists will not be pleased with the 'non-normality' of the distribution example above, but for our purposes at this point the illustration will serve well).

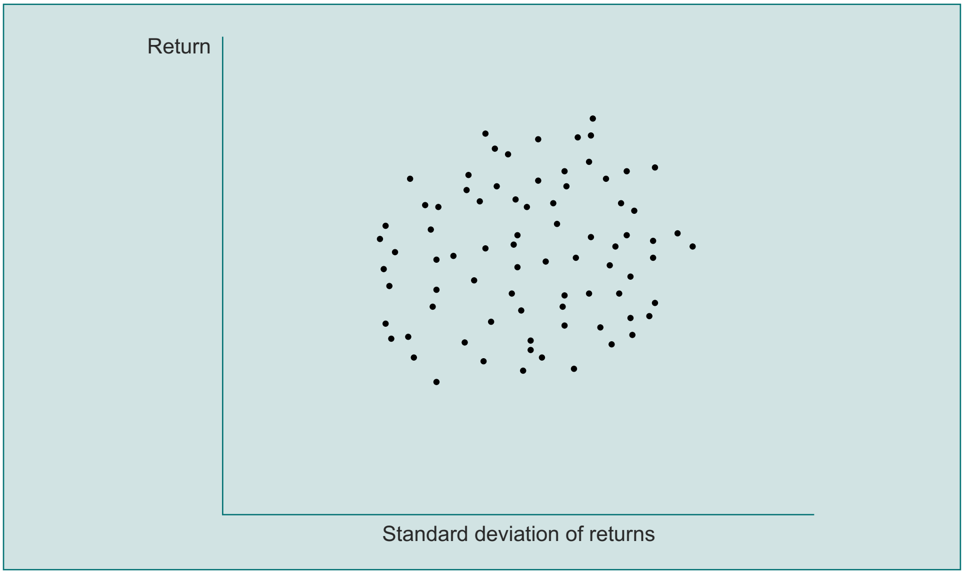

Figure 7.2 **Empirical relationship: standard deviation of return and rate of return**

This finding was, to say the least, of some concern to financial theoreticians. On the one hand, standard deviation has desirable theoretical attributes as a risk measure, but on the other it seems to have little empirical validity in financial markets. The situation for risk measures in finance was downright embarrassing – until two important breakthroughs occurred.

In the 1950s a man named Harry Markowitz invented a detailed model of how capital suppliers make decisions as to which assets to hold based upon their risk and return attributes.[2] And in the 1960s William Sharpe and John Lintner, independently and almost simultaneously, extended Markowitz's idea to the market as a whole to solve the problem of risk measures for assets. (Many financial theorists would argue – probably correctly – that the problem is not 'solved'. But it is so much better understood now than before these discoveries that use of the word 'solved' is not unwarranted.)

We shall look at each of these ideas in turn, but before we do so it will be useful for you to know where our arguments will eventually take us. The contribution of Markowitz was to show that company security holders are indeed risk-averse, and require higher returns when the risk (measured as standard deviation of return) is higher. However, he showed that the resulting positive relationship between return and standard deviation of return would be true only for the **entire** portfolio of the security holder, and not for the individual assets within it. The reason for this is that part of the standard deviation of return for individual assets is 'diversified away' when they are included in a portfolio with other assets. Because of its importance, we shall shortly spend a bit of effort to understand this financial diversification phenomenon.

[2] Markowitz, H.M. (1952) 'Portfolio selection', *Journal of Finances*, 7 (March), 77–91.

Sharpe and Lintner then offered a simple but elegant solution to the problem of a risk measure for individual assets by merely asking and answering the question of the relationship between risk and return for such assets when financial market participants understand the phenomenon of diversification in Markowitz's sense. The result is a measure of risk for individual assets that bears the requisite positive empirical relationship with returns earned, and that measures the risk that is not diversified away when individual assets are held in portfolios.[3]

7.2.1 Risk, Return and Diversification

Everyone appreciates intuitively that diversification implies reducing risk through combining different assets. We shall now explore what it means to diversify in a financial sense.

Recall the numerical example from the previous section. We saw there how to find the expected return and standard deviation of return for a portfolio of assets when we know its probability distribution of returns. Let us now go inside that portfolio to examine the individual assets within it.

Table 7.2 Individual security (assets A and B) probability distributions

	Return outcome	Probability
Asset A:	10%	45%
	20%	55%
Asset B:	7%	65%
	12%	35%

Suppose that the portfolio we studied in Section 7.2 comprises equal investments in two individual assets with probability distributions of return as given in Table 7.2. The expected returns for the two assets are:

Asset A: $(0.10 \times 0.45) + (0.20 \times 0.55) \quad = \quad 0.155 \; or \; 15.5\%$

Asset B: $(0.07 \times 0.65) + (0.12 \times 0.35) \quad = \quad 0.0875 \; or \; 8.75\%$

and the standard deviations of return are:

Asset A: $(0.10 - 0.155)^2 \times 0.45 \quad = \quad 0.00136$

$(0.20 - 0.155)^2 \times 0.55 \quad = \quad 0.00111$

$= \quad 0.00247$

the square root of which is 0.0497 or 4.97%, and:

Asset B: $(0.07 - 0.0875)^2 \times 0.65 \quad = \quad 0.00020$

$(0.12 - 0.875)^2 \times 0.35 \quad = \quad 0.00037$

$= \quad 0.00057$

[3] There are a couple of promising new advances in theory, which may supersede Sharpe's and Lintner's work. The conclusion to the module outlines the most prominent.

the square root of which is 0.0239 or 2.39%.

Since you have equal investments in each asset, the logical way to find the risks and returns of the portfolio formed therefrom seems to be to take the average of the returns and standard deviations of return for the two individual assets:

$$\text{Average return, assets A and B:}$$

$$(0.5 \times 0.1550) + (0.5 \times 0.0875) = 0.12125 \text{ or } 12.125\%$$

This is a nice finding. We knew from Section 7.2 that the portfolio's expected rate of return is 12.125 per cent, and have now found that to be the (weighted) average rate of return of the individual assets within the portfolio.

We turn to the calculation of the average standard deviation of the assets within the portfolio:

$$\text{Average standard deviation of return, assets A and B:}$$

$$(0.5 \times 0.0497) + (0.5 \times 0.0239) = 0.0368 \text{ or } 3.68\%$$

This is not a good finding. We already know from Section 7.2 that the standard deviation of return of this portfolio must be 3.008 per cent, and thus our average of 3.68 per cent is far too high. The (weighted) average standard deviation of return of the individual assets in the portfolio is **not** a correct way to calculate the standard deviation of return of the portfolio.

What then is correct? You recall that to find the correct standard deviation of return for the portfolio we worked directly with the information in Table 7.1, the probability distribution of returns for the portfolio as a whole. Perhaps investigating that data will lead us to an answer.

In order to derive the portfolio's return probability distribution, we need to have more information than just the individual return distributions for the assets in the portfolio. We must know how those returns interact. This information comes in the form of a **joint probability distribution**. Assume that our investigations indicate that the joint probability distribution for the returns of these two assets is as shown in Table 7.3.

Table 7.3 **Joint probability distribution of assets A and B**

		Asset B returns		
		7%	12%	**Probability**
Asset A returns	10%	0.35	0.10	0.45
	20%	0.30	0.25	0.55
Probability		0.65	0.35	1.00

A joint probability distribution is simply more detail about the individual asset probability distributions (as given in Table 7.2) with which we are already familiar. To understand the joint distribution in the table, note that there are four cells inside the box, and that each cell describes the probability of a particular set of returns being simultaneously earned by both Assets A and B. This is called a joint probability. For example, the upper left-hand cell says that the joint probability of asset A producing a 10 per cent return while asset B produces a 7 per cent return is 35 per cent. You can see that to be fully specified (to cover all of the eventualities) the total of the joint probabilities of a 10 per cent return occurring in asset A must be the 45 per cent we saw for this outcome earlier. The joint probability distribution now is telling us that the 45 per cent chance of a 10 per cent return on asset A is itself associated with a 35 per cent probability of it simultaneously occurring with asset B's 7 per cent return, and a 10 per cent chance of it occurring with asset B's 12 per cent return. The other cells are interpreted similarly, and the interior (joint) probabilities must sum in rows and columns to equal the original probabilities of the individual security returns, while the sum of all of the cells must equal a 100 per cent (1.0) probability.

Though displaying it may seem like a lot of trouble, the information contained in a joint probability distribution is necessary to find the likelihoods of various returns being earned on a portfolio. Without the probabilities of joint events being known, there would be no way to construct the portfolio's return distribution, because it is obviously determined by the simultaneous occurrences of various returns on the assets in the portfolio.

Let us see what all of this means for the portfolio formed from equal parts of your assets A and B. The joint probability distribution indicates that there are four possible events that can happen with the portfolio (*see* Table 7.4).

Table 7.4 **Portfolio events and probabilities**

	Returns			
	Asset A	**Asset B**	**Portfolio**	**Probability**
Event 1:	10%	7%	8.5%	.35
Event 2:	10%	12%	11.0%	.10
Event 3:	20%	7%	13.5%	.30
Event 4:	20%	12%	16.0%	.25

The separate events in Table 7.4 are the joint occurrences of the pairs of returns from the joint probability distribution for the two assets, and the column headed 'Portfolio' is simply the equally weighted average of the returns indicated for the assets in the respective events. The returns are equally weighted because the portfolio is formed of equal parts of assets A and B. The probabilities in the last column are merely those for the particular event, taken from the joint probability distribution.

Note the last two columns of Table 7.4. That information is identical to that in Table 7.1, the probability distribution of returns originally provided for the portfolio. So we now know the source of portfolio return probability distributions. Such a distribution is the specification of the likelihoods for joint events from the joint probability distribution of the assets in the portfolio.

We now also know where we went wrong in trying to calculate the standard deviation of the portfolio's return by merely taking the average standard deviation of the securities in the portfolio. The problem with that procedure is that it ignores the interactions of returns represented by the joint probability distribution. When we take those into account, we end up with the distribution for the portfolio in either Table 7.1 or Table 7.4, and we already have discovered that the standard deviation of that distribution is 3.008 per cent as opposed to the individual security average of 3.68 per cent.

Why does the whole portfolio have less risk than the average risk of the securities in it? The answer is **diversification**. Take another look at Table 7.3. Note that there are significant chances that when asset A is generating a high (20 per cent) return, asset B will be generating a low (7 per cent) return: a 30 per cent chance. And the reverse also holds: there is a 10 per cent chance of a low return on A while B provides a high return. This means that the risks of the two securities, not being perfectly positively related to each other, will to some extent cancel each other out. Much of the time things are good or bad for both simultaneously. But fully 40 per cent (30 per cent plus 10 per cent) of the time one security is expected to do relatively well while the other does relatively poorly. The reason why their joint risk (3.008 per cent) is less than their average risk (3.68 per cent) is just that.

'What a lot of trouble!' you exclaim. 'Does that mean we shall be forced to deal with complicated joint probability distributions whenever we attempt to find the risk of a portfolio?'

No, fortunately you will not. One of the important contributions of Markowitz was to inform us about easier ways to find portfolio risks, by dealing directly with the interrelatednesses of asset returns. One of statisticians' favourite measures of the extent to which two things are related to each other is called a **correlation coefficient**. This is just a number that can take on values from −1 to +1, with the former indicating perfect negative relatedness and the latter indicating perfect positive relatedness, while values in between are various degrees of positive or negative relatedness, short of perfection. For example, the joint probability distribution of returns in Table 7.3 produces a correlation coefficient between the returns for assets A and B of +0.2423. A correlation coefficient of that magnitude and sign indicates that the two securities' returns are positively related, but not very strongly so.

Though it is unnecessary for us to reproduce it here, there is a complex formula embodying the correlation coefficient (or its statistical counterpart, the **covariance**) which can be used to find a portfolio's risk directly, without the onerous specification of the joint probability distribution as we did it.[4] The important point for our purposes, however, is not the best way to find a portfolio's risk. The main lesson to be learned from this discussion is that portfolio risk is not the same as average individual security risk, and that risk reduction through diversification is the reason. The more positively related are securities' returns within a portfolio, the less there

4 For example, the answer to the portfolio risk above would be:

$$(0.03008)^2 = (0.5)^2\,(0.0497)^2 + (0.5)^2\,(0.0239)^2 + 2(0.5)(0.5)(0.0497)(0.0239)(0.2423)$$

which is more generally given by the expression:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} = \sigma_i \sigma_j \beta i j X_i X_j$$

will be to gain from diversification. The less positively related are the returns from assets in a portfolio, the greater will be the reduction in risk relative to average individual asset risk.

Figure 7.3 shows this argument graphically. The lines emanating from assets C and D show the various risk and return combinations available from combining C and D in various proportions. Each line is associated with an assumed correlation of returns between C and D. Note that as the correlation between the two assets declines, the potential for reducing risk through diversification increases.

We have now finished discussing what we need to know about portfolios and their risks. We have found that:

1. When individual assets are held in a portfolio, the risk associated with the portfolio is not likely to be simply the average risk of the assets of the portfolio.
2. The detailed interactions of individual asset returns, their return distributions, and the return distributions of a portfolio formed from them can be seen by developing the joint probability distribution of the assets in question.
3. The risk of individual assets, depending upon how their returns are related, will to some extent cancel each other out when a portfolio of these assets is formed.
4. The extent to which that risk will cancel depends upon how positively the returns of the constituent individual assets are related. One way to measure that relatedness is by the correlation coefficient of paired returns of the assets or securities within the portfolio.

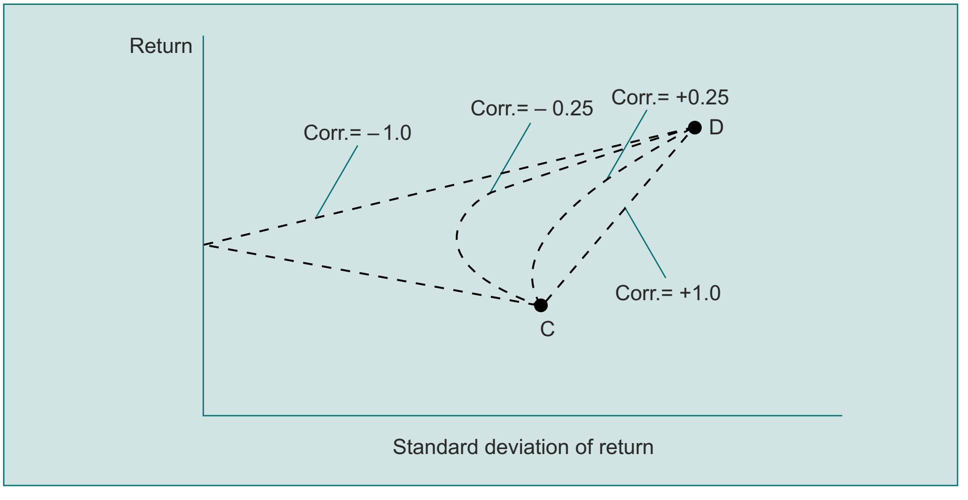

Figure 7.3 How correlation affects risk and return of portfolios

7.3 The Market Model and Individual Asset Risk

The foregoing discussion concerning the benefits of diversification and the determinants of portfolio risk is interesting and valuable in its own right, but it does not take us all of the distance to our final goal, namely the specification of an appropriate risk measure for individual assets. Portfolio analysis has taught us that the standard deviation of return on an individual asset is only an appropriate risk measure for that asset when it is held by itself, and not in a portfolio. When included in a portfolio, it is likely that some of that asset's (standard deviation) risk will be diversified away.

The next great step in understanding the riskiness of individual assets – and therefore the capacity to estimate their risk-adjusted discount rates – was, as with so many important discoveries, a simple one. William Sharpe and John Lintner simply asked (and answered) the question: 'What is an appropriate risk measure for individual assets when the market behaves as if everyone understands the benefits of diversification?'[5]

The answer, as you could probably have guessed yourself, is that when the benefits of diversification are understood by the market and its participants, the only risk that is relevant is the risk that cannot be diversified away, that which remains after diversification has taken place. In finance that is called the **undiversifiable** or **systematic** risk of an asset. We shall investigate how to measure the systematic risk of assets. Once we have understood it, we can easily specify exactly how to use the security market line relationship to find discount rates for company investment decisions.

Our first task is to understand the notion of undiversifiable or systematic risk. Perhaps the best way to do this is to examine the results of an experiment that has been performed several times in many different financial marketplaces. The researchers set up the following experiment: they collected data on the returns, standard deviation of returns and correlations of returns for many (usually over a thousand) securities in a certain stock market for a reasonable period of time (a number of years). Then they formed many hypothetical portfolios of those securities, gradually increasing the number of securities in them, and measured the riskiness (standard deviation of return) of the portfolios as more and more securities were added (i.e. as diversification became greater and greater). A summary of the results in all these studies would appear as the graph in Figure 7.4.

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[5] Sharpe, W.F. (1964) 'Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk', *Journal of Finance*, 19 (September), 425–42 and Lintner, J. (1965) 'The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets', *Review of Economics and Statistics*, 47 (February), 13–37.

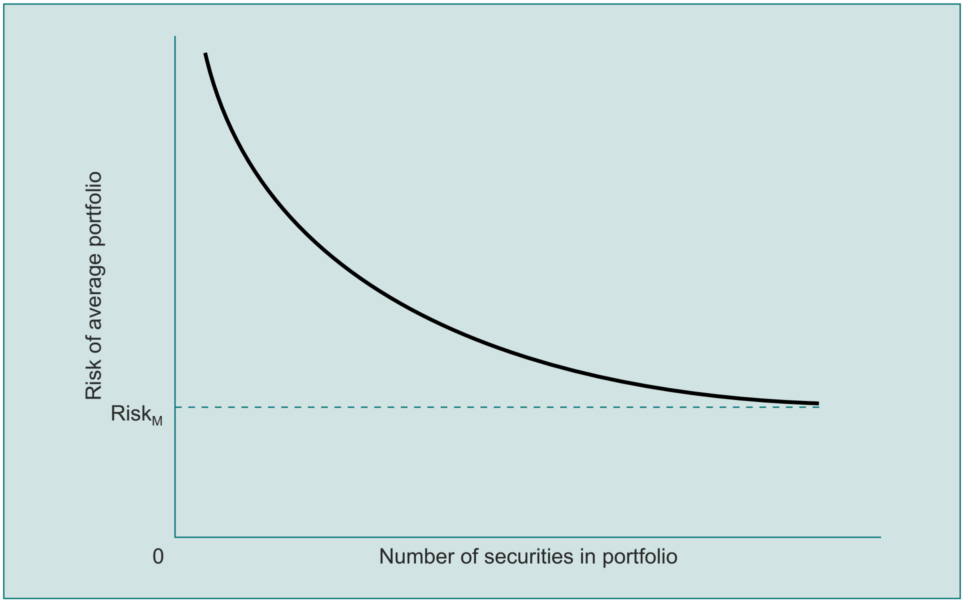

Figure 7.4 **Diversifiable and undiversifiable risk**

The horizontal axis displays how many securities are in the portfolios, and the vertical axis shows their average risk. As we would expect, there is a dramatic decrease in the riskiness of the portfolios as the number of securities in them, and thus their amount of diversification, increases.

But notice another characteristic of Figure 7.4. The reduction of risk due to diversification seems to have a **limit**. Diversification definitely reduces risk, but there is evidently a level of risk (indicated as $Risk_M$) below which portfolios do not go, regardless of how well diversified. How can this be explained?

Remember from our discussion of portfolio analysis that the higher the level of correlation among the assets in a portfolio, the less will be the benefits of diversification. Thus the reason for a minimum level of risk even in well diversified portfolios must be that there is a certain common correlation present in all securities, and this limits the amount of diversification possible. This common factor has a name in finance: we call it the **market factor**, or simply the **market**. (You can now see why the $Risk_M$ in Figure 7.4 is so named.)

Why does this market factor exist? At a very basic level we are not certain why it does, but there are some good guesses. The odds are that, in a given country, much of the basic economic activity that generates returns in securities markets is common to all companies whose shares and bonds are traded on those markets. You can think of this as 'overall economic activity'. When things are generally good economically, this effect will be felt by all companies to some degree or other. When things are bad, the same is true. It is not necessary that all companies feel such effects in the same degree or even in the same direction for this idea to be effective. As a matter of fact, it is just this lack of uniformity in companies' response to overall economic conditions that produces the potential for risk reduction from diversification. (If there were no such diversity, risks would be perfectly correlated across all companies, and no benefits of diversification would exist.) Evidently there is a significant common overall market effect found in the returns of securities in general to cause the results shown in Figure 7.4.

The systematic risk of securities is thus based upon the extent to which their returns are influenced by this common factor, the 'market'. The actual measure of the undiversifiable risk of a security *j* is:

$$\text{Systematic risk}_j = \text{Standard deviation of return}_j \times \text{Correlation of } j \text{ with market}$$

You can see the intuition behind this as a measure of the risk of an individual asset or security: the risk that cannot be diversified away is the total risk of the security as if it were held alone (standard deviation of return) multiplied by the extent to which its returns are correlated with the 'market', the common factor. Thus a security closely related to the market will have a correlation with the market close to +1, and will have systematic risk close to its standard deviation. Not much of its risk will be diversifiable. On the other hand, a security with low correlation to the market will have much of its risk diversified away when held in a portfolio with other securities, and thus has low systematic risk.

The systematic risk measure offered above is perfectly correct, and is actually the one most intuitively consistent with portfolio and market effect arguments. However, it is not the one seen most often in use. There is a simple manipulation done to the above systematic risk measure to arrive at the form most financial practitioners use today. If we divide the measure above by the standard deviation of returns on the 'market', we get:

$$\text{Beta}_j(\beta_j) = \frac{\text{Standard deviation of return}_j \times \text{Correlation of } j \text{ with market}}{\text{Standard deviation of market return}}$$

or,

$$\beta_j = \frac{\sigma_j \rho_{jm}}{\sigma_m}$$

which is the same as:

$$\beta_j = \frac{\sigma_j \sigma_m \rho_{jm}}{\sigma_m^2}$$

and because by statistical definition $\sigma_j\sigma_m\rho_{jm}$ is the "covariance" or σ_{jm} of security *j* with the market, can be expressed as:

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2}$$

where β_j is the beta coefficient for *j*; σ_{jm} is the covariance of *j* and the market; and σ_m^2 is the variance of the market. This expression for a 'beta' coefficient is also known as a 'regression' coefficient. It is telling us the same information as the previous systematic risk measure did, but scaled to the risk of the market as a whole. For example, a security having a β equal to 1.0 implies that the market's influence on that security is such that an *x* per cent increase or decrease in the return on the 'market' is associated with an *x* per cent increase or decrease in the return on that security. On the other hand, a β of 1.5 signals that an *x* per cent change in market return implies a 1.5x per cent change in the return on that security. Numerically, a security currently expected to have a 15 per cent return would, with a β of 1.3 and with the market return increasing from 12 per cent to 14 per cent, experience an increase in its expected return from 15 per cent to (15% + 1.3 (14% − 12%)) = 17.6 per cent.

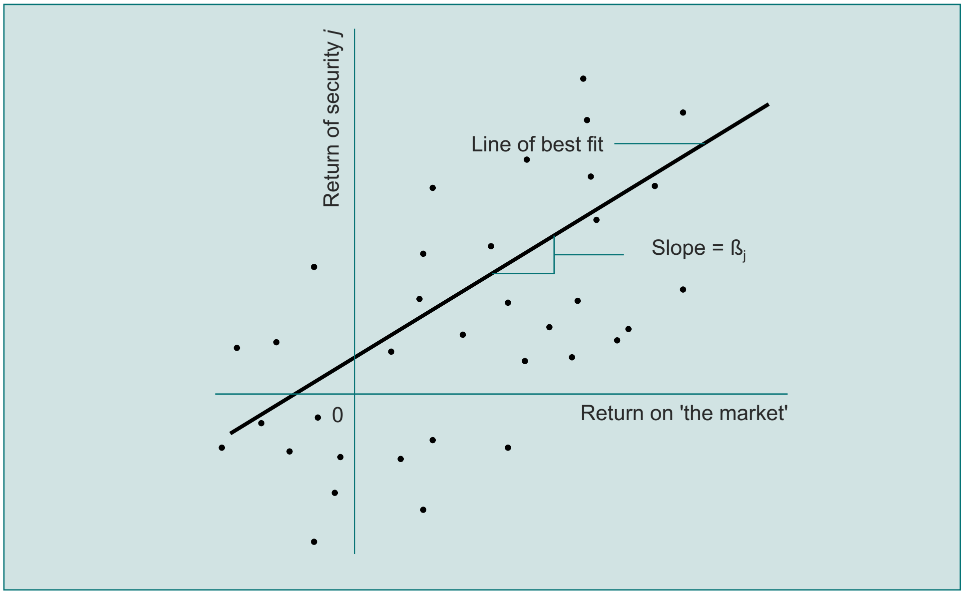

Figure 7.5 The beta coefficient

Thus a β coefficient expresses the relationship between the return expected from holding a security and that expected from the market as a whole (or any well diversified portfolio having a β of 1.0). Figure 7.5 shows that the β coefficient of a security *j* is simply the slope of a line of 'best fit' relating the returns of security *j* to those of the market. The graph indicates that the steeper the slope (the higher the β coefficient), the greater will the returns on the security *j* amplify or gear upward (or downward) the returns on the market portfolio.

Figure 7.5 is also useful in reminding us that the β coefficient does not express a perfect relationship between the returns of an individual security and the market as a whole. Graphically this appears as the distances away from the line of best fit that the actual returns for security *j* lie. One cannot predict exactly (or even necessarily very closely) what will happen to the returns on security *j* if it is held by itself and not in a well diversified portfolio, even if one knew exactly what the 'market' would do. But if *j* **is** held in such a portfolio, and if the β coefficient is well estimated, we can be fairly certain that the effect **on the portfolio** of security *j* is well described by its β coefficient. When held in a well diversified portfolio the 'distance away from the line', or the unsystematic risk of security *j*, is diversified away, leaving only security *j*'s market-based risk as its influence upon the returns of the portfolio.

7.3.1 The Market Model or Security Market Line

We are now ready to specify how the market sets required returns on risky assets. The argument is intuitively quite straightforward: if participants in financial markets understand the principles of diversification, and if market prices of securities reflect this, the only risk that will receive compensation in the form of higher expected returns is undiversifiable or systematic risk. All of the rest is diversified away, and in a competitive marketplace such diversifiable risk will not affect the returns of individual securities. The market sets required returns (and therefore securities' prices) as if securities are held in well diversified portfolios, and thus the correct measure of risk is that risk still existing after such diversification. We have seen that the β coefficient is an appropriate measure of this undiversifiable or systematic risk.

The only issue to be resolved is exactly how much return compensation the market provides for systematic risk. Once we know that, we shall have reached our goal of having a quantitative mechanism for setting the risk-adjusted returns necessary for company investment decisions.

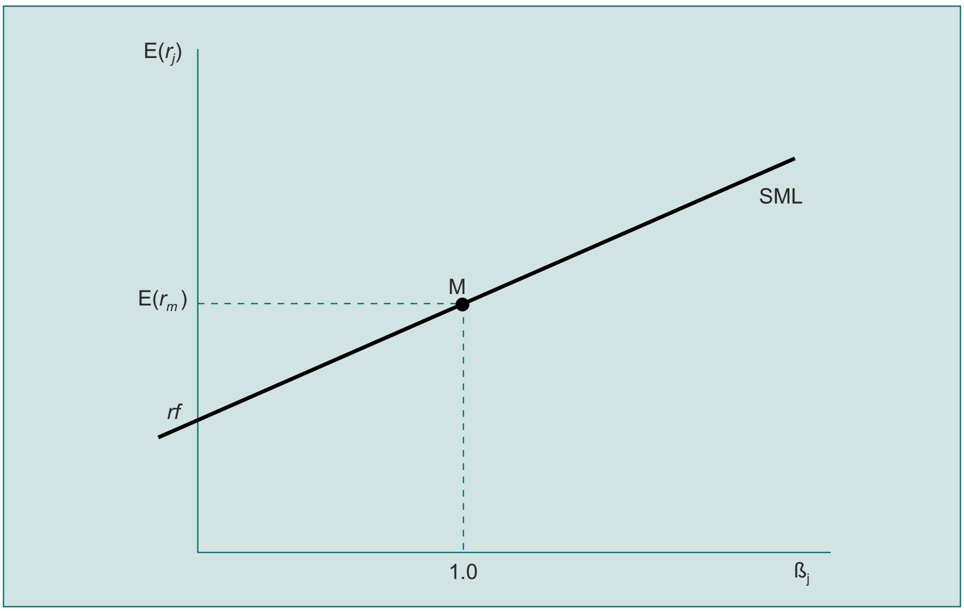

Figure 7.6 The security market line and market portfolio

Figure 7.6 resolves the issue. In it we have plotted the graph for the quantitative security market line consistent with the ideas we have discussed above. (The only addition is the assumption that financial market participants can borrow and lend at a risk-free return, *rf* (like the return from holding a government bond until it matures). Note carefully the characteristics of the SML. It relates the amount of systematic risk inherent in the returns of a security (its β coefficient) to the returns required on that security by the market. The relationship is, as we would expect, positive, in that the higher the systematic risk of security *j*, the higher its required return. The SML is located with respect to two important points on it, the risk-free rate rf and the market portfolio's risk–return location, M. The latter, as you can see, has a return of $E(r_m)$ and (by definition) a β coefficient of 1.0.

We can use the graphical presentation of the SML in Figure 7.6 to specify the quantitative relationship between risk and return in terms of the now famous equation:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j \tag{7.1}$$

This formula is simply the graphical SML expressed in terms of the line's intercept (rf) and its slope [E(r_m)-rf]. The economies of the SML expressed this way indicate that the required return on a particular security $E(r_j)$ comprises a compensation for the passage of time (rf), plus compensation for risk-bearing, [E(r_m)-rf]β_j. This latter is formed of a 'market price of risk', [E(r_m)-rf], multiplied by an amount of risk β_j. As we now know, that amount of risk is the part of the security's total risk that is undiversifiable.

Equation 7.1 is a very important one. It will allow us to estimate discount rates and opportunity costs for company investment projects in a manner consistent with the way that the company's capital suppliers actually hold the company's securities. In effect, by using the SML-based discount rates, we will be judging whether the company's capital suppliers would be willing to hold the proposed investments in their portfolios.

Though debate continues in both academic and practitioner circles about the correct specification of the SML, exactly how to measure systematic risk, and several other important issues surrounding its use, the SML has proved itself to be a very powerful idea with significant capacity to explain how rates of return align themselves in the market. Recall that we mentioned that researchers in finance found that their ability to explain the set of returns offered by the market, based upon total (standard deviation) risk was embarrassingly absent (*see* Figure 7.2, and the discussion of it). Although there continues to be debate as to the validity of the findings, a similar plot of actual returns and systematic risk, i.e. an empirical test of the SML, would show a tight fit of actual returns and systematic risk, with a positive linear slope, much like the theoretical Figure 7.6.

Before embarking upon an exploration of how financial managers can actually use the SML rates as investment criteria, we should consolidate the many ideas we have developed in this section to this point:

1. The total risk of an individual asset or security can be separated into two types of risk, that which can be diversified away, and that which cannot.
2. The risk that cannot be diversified away is related to an underlying 'market factor' that is common to all assets and securities, and is thus a common correlation limiting the amount of risk reduction through diversification that is possible by including a security in a portfolio.
3. This undiversifiable or systematic risk can be measured by the β coefficient (standard deviation times correlation with the market) of the security in question.

4. If the financial market sets securities' returns based upon their risks when held in well diversified portfolios, systematic risk will be the appropriate measure of risk for individual assets and securities, and the SML as depicted in Figure 7.6 and Equation 7.1 will dictate the set of risk-adjusted returns available in the market.
5. These SML-based returns are the opportunity costs of capital suppliers of companies, and thus can form the basis for evaluating internal company investments. These investments must offer returns in excess of capital suppliers' opportunity costs in order to be acceptable.

7.4 Using the Capital Asset Pricing Model in Evaluating Company Investment Decisions

We have now introduced the **capital asset pricing model (CAPM)** or security market line, the system that generates required rates of return based upon the riskinesses of assets. What remains is to show how this system can be used in everyday business practice to improve the investment decisions of companies.

The first question that we shall address is how such SML rates can be estimated in practice. Secondly, we must then investigate certain important adjustments to market-based rates that are necessary in order to expect them to work correctly.

Remember that the task we are embarked upon is one of great importance to companies, for the specification of appropriate risk-adjusted required rates for investment decisions is critical to the process of correctly estimating project NPVs. To illustrate this, look at Figure 7.7. On that graph we have plotted both a weighted average cost of capital of a certain (but unspecified) company, and the SML. From the discussions earlier in this section, we can now appreciate that the WACC of any company is in fact an average of the risk-adjusted rates of return of the company's various endeavours, including its asset types and associated future cash-flow expectations. In other words, the WACC of a company is based upon the company's mix of asset risks, and doubtless comprises assets of both relatively high and relatively low risk. Further, in order to be correctly acceptable, an investment must offer an expected return in excess of the return depicted on the SML for the investment's systematic risk level. This means that 'good' investments' expected returns would plot above the SML, perpendicularly above their systematic risks.

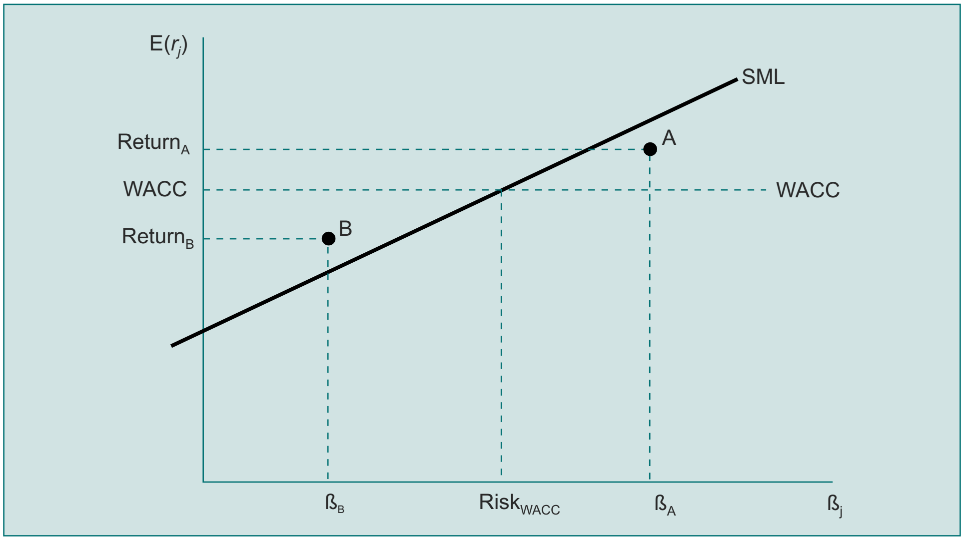

Figure 7.7 **The WACC and the SML**

Now consider the situation of the company whose WACC is depicted in Figure 7.7. Suppose that the company is considering whether or not to accept investment A. You can see that A's systematic risk is greater than that which would be implied by the company's WACC. Note that if the company decides about the desirability of A by using the company's WACC, it will make an **incorrect** decision. A's actual return is above the WACC, so that criterion would indicate the investment being acceptable, but the SML shows clearly that the investment does not offer a return high enough to compensate for its risk. The problem is that A's systematic risk exceeds that of the company as a whole, reflected in the WACC. Were A to be accepted, the company would be undertaking a high-risk investment whose expected return exceeds that of the company's average return but is not as high as the company's capital suppliers could earn on comparable-risk investments in the marketplace.

On the other hand, investment B in Figure 7.7 presents the company with the opportunity to make the exact opposite error. Because B's risk is less than the company's average, investment B should be evaluated with a criterion less stringent than the WACC. Were the latter to be used, B, and investments like it, would be erroneously rejected.

The lesson of Figure 7.7 is now clear: a company should **not** generally apply its WACC as an investment criterion. The WACC will give the correct answer only when an investment's risk is the same as the average risk of the entire company.

We should not leave you with the erroneous impression that companies regularly make shocking errors in investment decisions of the sort implied by Figure 7.7 (though it would be as incorrect to think that serious errors of that type are never made). Many companies are aware that projects can differ in risk, and that some adjustment of criterion is advisable depending upon the risk of the investment relative to the company. Usually this takes the form of fixed increments or decrements to the company's average criterion. For example, investments that are low-risk, such as some 'cost-saving' investments (where savings are based upon known engineering as opposed to product market estimates) are automatically granted an x per cent reduction in discount rate (where x is set by the company). Scale increasing investments, which are likely to be much the same as the average risk of the company, are evaluated with the company's WACC, whereas riskier projects, including new product lines or research and development, are judged with y or z per cent additions to the company's WACC (again, variables y and z being set by the company).

Such approaches are an improvement over complete ignorance of investment risk differentials, but do not completely solve the problem of evaluating investments whose risk differs from that of the company as a whole. Remember that the **market's** valuation of an investment's desirability will depend upon the investment's expected return and systematic risk. Unless the adjustments to the WACC of the company described above are related to the systematic risks of projects, the criterion will not be identical to the valuation process felt by the capital suppliers of the company. An SML-based criterion is designed to provide such adjustments.

7.4.1 Estimating Systematic Risk of Company Investments

To use the SML for estimating required returns we must first be able to specify the amount of systematic risk (the size of the coefficient) of a project. There are several ways that a company might be able to do this. First, suppose that the project is of the same risk as the existing company. If the company's shares are traded in the stock market, we can merely look up the β coefficient of the company's shares in one of the financial reporting services that supply that sort of data. (There are several such services, usually offered for a subscription fee by financial reporting services or 'free' by brokerage houses to customers, but occasionally marketed – as superior estimates – by independent consulting companies.) These services use various processes, encompassing a wide range of statistical sophistication, to estimate the β coefficient of a company's shares. Nevertheless, they all embody the essential idea of a 'line of best fit' approach much like that shown in Figure 7.5.

If the project is not simply an increase in the scale of the company but is different from the average risk (either above or below), the company's β coefficient will not do. In that situation the investment, though evaluated as different from the company's overall level of risk, may be similar to another (for example, entering a new line of business where there are already other companies present). In such situations, the β coefficients for the other companies, with the same systematic risk as the project, can be used. This approach is also valuable when the shares of the investing company are not traded, but the shares of a similar company are, and the investment is simply a scale change. Here you can use the company's WACC, but estimate it with a similar company's β coefficient.

The above are the easy ways to estimate the systematic risk of a project, and are fine in the situation to which they apply. But often there is no company whose shares are traded and whose systematic risk can be expected to be the same as the proposed investment. What can be done to estimate the β coefficient of such investments in this situation?

When market-generated β coefficients are unavailable, the systematic risk measure must be constructed artificially. In other words, we must find what the β coefficient would be if it were to exist. The best approaches to such estimates begin with a β coefficient for the company or division thinking of undertaking the project, and adjusting that coefficient for the differences between the project and the company or division. However, if the coefficients for the company or division are also unknown, a market-traded company can usually be found with a β coefficient to which economic attributes of a project can legitimately be compared.

Remember that the β coefficient reflects the extent to which the returns from a project are expected to accentuate or dampen the returns on the market as a whole. In constructing β coefficients from the characteristics of the investment itself, it is necessary to concentrate upon the underlying factors affecting the returns on the project. For example, one such factor is the extent to which the project's revenues react to changes in overall market activity. If the project's revenues are expected to be quite volatile (i.e. go up a lot in good times and go down a lot in bad) relative to the company or divisional average, an adjustment to the coefficient must be made. Similarly, on the cost side, if the fixed costs of a project comprise a relatively high proportion of its costs, the β coefficient of the project must be adjusted upwards, because net results will be more variable with high fixed costs (described as '**operational gearing**').

The basic process is conceptually straightforward: begin with a 'benchmark' β coefficient (from a company or division that can be either similar to or different from the project in question) and then successively multiply that coefficient by the ratios of the project's volatilities to those of the benchmark. This description probably does not sound particularly instructive, but the numerical illustration that follows will dispel any ambiguity in the actual methods to be used.

There is one preliminary adjustment which must be performed when constructing β coefficients artificially. If the value you are beginning with (the 'benchmark' β coefficient) is from a company that has borrowed money, the systematic risk of that company's shares is higher than it would be if the company were financed solely with equity capital, and the value must thus be purified of this effect before the other adjustments are made. This is an adjustment for **financial gearing**. (If the project is to be partially debt-financed, you will also subsequently readjust the constructed β coefficient upward to reflect the financial leverage intended. But that amount may be different from what the original benchmark value contained, and so the latter must first be cleansed of financial leverage.)

Let us look at a numerical illustration of how a β coefficient can be artificially constructed. Suppose that European Aerospace Research (EAR), a fictitious manufacturer of airframe parts, is considering entry into the business of offering consulting advice to governments. EAR's shares on the market have a β coefficient of 1.6, and the company has £250 000 of equity and £100 000 of debt outstanding in market values. The company's debt has a β coefficient of 0.44. β coefficients being additive (the β of a *combination* of assets is simply the value-weighted average of the individual asset βs), we can deduce the basic ungeared β coefficient (β_u) of the company with:

$$\beta_u = \beta_e \frac{\text{E}}{\text{V}} + \beta_d \frac{\text{D}}{\text{V}}$$

where β_e and β_d are observed equity and debt β coefficients respectively, E and D are their observed market values, and V is the sum of E and D. (This illustration is a bit simplified because it ignores tax effects, but the essential points are intact.) Solving for the ungeared β of EAR, we get:

$$\beta_u = 1.6 \frac{250}{350} + 0.44 \frac{100}{350} = 1.27$$

This would be EAR's β coefficient if it had no debt outstanding. We now must adjust for revenue and operational gearing differences between airframe part manufacturing and government consulting. EAR's marketing people inform us that the company's revenues have tended to fluctuate about 1.2 times those of overall economic activity, whereas consulting is expected to have a comparable ratio of only 0.95. To adjust EAR's now ungeared β coefficient for revenue risk differentials, we simply multiply it by the ratio of the investment's revenue volatility to that of the company:

$$\begin{aligned}
\text{Revenue-adjusted}\beta &= \beta_u \frac{\text{Project revenue volatility}}{\text{Company revenue volatility}} \\
&= 1.27 \frac{0.95}{1.2} \\
&= 1.01
\end{aligned}$$

Next, we are informed by EAR's accountants that the fixed costs of operation in the airframe parts business comprise about 25 per cent of net cash flow, whereas in the consulting business they are expected to be only 15 per cent. Once we adjust for this consideration we shall have arrived at the ungeared β for the consulting business:

$$\begin{aligned}
\text{Project } \beta_u &= \text{Revenue-adjusted}\beta \frac{(1 + \text{Project fixed cost \%})}{(1 + \text{Company fixed cost \%})} \\
&= 1.01 \frac{1.15}{1.25} \\
&= 0.93
\end{aligned}$$

The final step that remains is to readjust the reconstructed and ungeared β coefficient for any financial gearing planned for the project. In order to do so, we must know the β coefficient for the debt that will be issued by EAR for the project. Suppose that the financial experts at EAR have decided that the consulting operation can sustain a 20 per cent gearing ratio (this is a market value of debt to total debt-plus-equity ratio, D/V). Rather than going through an intricate and similar adjustment illustration for the β coefficient of the debt of EAR, let us assume that investment bankers have advised EAR that debt issued solely to finance this project, and in the 20 per cent gearing ratio, would have a β_d of 0.22. (We shall show you where this figure comes from in the next section.)

We now know the ungeared β coefficient of the project (0.93), the β coefficient of the debt of the project (0.22), and the gearing ratio of the project (20 per cent). With this information we can calculate the equity β coefficient of the project:

$$
\begin{aligned}
\beta_u &= \beta_e \frac{\mathrm{E}}{\mathrm{V}} + \beta_d \frac{\mathrm{D}}{\mathrm{V}} \\
0.93 &= \beta_e(0.80) + 0.22(0.20) \\
\beta_e &= [0.93 - 0.22(0.20)]/0.80 \\
&= 1.11
\end{aligned}
$$

So the β coefficient of the shares of the consulting project of EAR, were such shares to be sold independently in the market, would be 1.11, in contrast to EAR's β_e of 1.6. Government consulting is evidently less risky than building airframe parts, including the effect of financing.

The foregoing has shown how a β coefficient for a project can be constructed in situations where a market β coefficient is unavailable. Of course it would have been much easier for EAR to arrive at the investment's β coefficient if there were a company exclusively engaged in government consulting and whose shares were traded on an exchange. (Remember that even in that situation there would be an adjustment necessary for financial gearing if EAR intended to finance the project with a different debt proportion.) It is also true that the financial gearing adjustment could have been done in just one step, by using a similar ratio calculation as that used for operational gearing and revenue volatility. We presented the ungeared β coefficient as an interim step because the notion of an ungeared return is familiar and will appear again.

7.4.2 Estimating the WACC of an Investment

We are now aware that β coefficients for individual projects can be estimated, and we already knew that such coefficients can be used with the SML to generate required rates to be used as discount rates for investment analyses. This section examines the procedure for arriving at such a discount rate, based upon β coefficients of the type described in the previous section. You can tell from the title of this section that the idea we shall develop is in part familiar: the notion of using a WACC for investment decisions. What will be new is how to estimate the WACC using β coefficients and the SML.

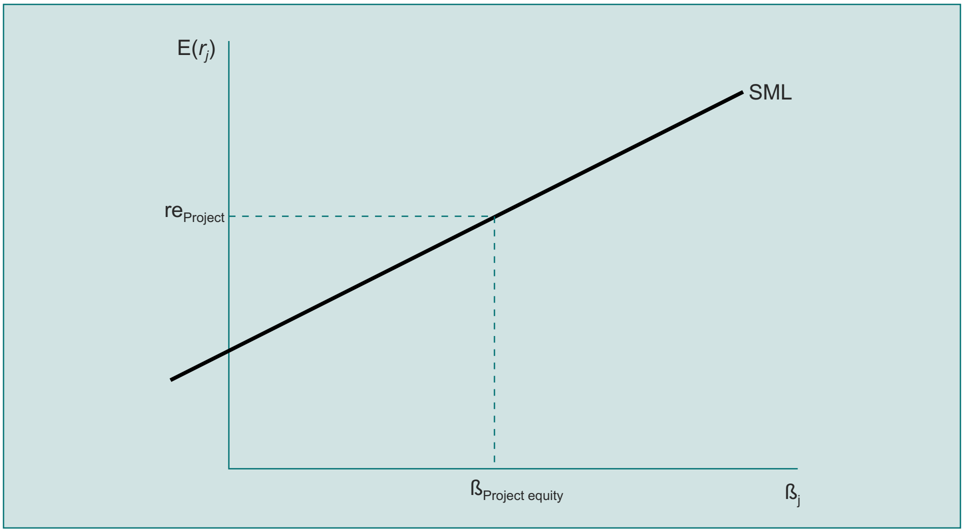

Figure 7.8 **Estimating required returns with the SML**

Figure 7.8 illustrates what we must do. In it is depicted the SML and the β coefficient of a project's equity, such as we found for EAR above. Invoking the SML relationship will allow us to find the return required on the equity of the project. Once that is known, we simply calculate the WACC of the project in the same way that we would for any company. We have seen such calculations often in previous modules.

The quantitative estimation of a required return using the SML requires that we refer to its Equation 7.1:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$

There are three pieces of information necessary to get an answer from Equation 7.1. We need rf, the risk-free return; $E(r_m)$, the expected return on the market portfolio; and β_j, the systematic risk of asset j. We have dealt with estimating the β, but what about the other information?

First, the risk-free return. This is easy to find. You would simply look up the government bond interest rates (YTMs) for comparable-maturity investments in such bonds. (There are some quite sophisticated methods of dealing with such estimates, adjusting for cash-flow patterns and interest rate term structures, but unless the interest rate structure is steep, these adjustments will not make much difference.)

More intricate is the estimation of $E(r_m)$. In terms of historical statistics, market portfolio return data have been collected in several countries for long periods of time. Using such numbers for $E(r_m)$ would, however, not be advisable. The problem is that $E(r_m)$ itself, as you can see by substituting appropriately into Equation 7.1, is a function of the risk-free rate, which is quite variable across time. To avoid this problem, common practice is not to estimate $E(r_m)$, but instead to estimate [$E(r_m)$-rf], on the (correct) theory that the latter (the **difference** between the market return and the risk-free rate) is more stable over time than is the market return itself. For example, the London Stock Exchange's historical average annual market return has been 9.1 per cent **above** the risk-free rate, whereas in the US a somewhat broader index indicates an 8.8 per cent market portfolio 'excess' average return, both for periods of time of more than fifty years. Such averages are by no means constant over time, but they are more so than $E(r_m)$ itself and are thus used until something better presents itself.[6]

We now have enough information to illustrate the use of the SML to find a required return. The EAR example from the previous section will serve this purpose well. We have discovered that, adjusted for all important operating and financial characteristics, a reasonable estimate of the β coefficient of the equity of the EAR consulting project is 1.11. Using 10 per cent as an estimate of the existing risk-free rate rf and the 9.1 per cent excess market return of the UK (where EAR is assumed to be based), the SML tells us that:

$$\begin{aligned} \mathrm{E}(r_j) &= rf + [E(r_m) - rf]\beta_j \\ \mathrm{E}(r_e) &= 0.10 + (0.091)1.11 \\ &= 0.201 \; or \; 20.1\% \end{aligned}$$

To capture the opportunity costs of its shareholders, EAR must use an equity required return of 20.1 per cent in evaluating its government consulting project. This rate will compensate shareholders for the undiversifiable risk that the project will add to their asset portfolios.

Should EAR then use 20.1 per cent as the discount rate for the project's free cash flows? Of course not. Remember that the appropriate discount rate for a project's FCF* must reflect the necessary returns (or capital costs) to the *full* financing of the project. The EAR consulting project does have equity financing, but is also partially to be financed with debt. We can recall from Module 4 that the project will require that EAR use a WACC (or rv*) to evaluate the project. And this WACC is not the WACC for the entire company but that which would uniquely apply to the consulting project were it to exist as a separate company. This is because the capital suppliers of EAR are effectively being asked to **add** the consulting project to their portfolio of holdings, and thus it must stand or fall on that basis.

Let us find the WACC that EAR should apply to the cash flows of the consulting project. You recall from Module 4 that the WACC is defined as:

$$rv^* = \frac{\mathrm{D}}{\mathrm{V}}(rd^*) + \frac{\mathrm{E}}{\mathrm{V}}(re)$$

We have all of the necessary information to calculate rv^* except for rd^*, the after-tax cost of debt capital. Fortunately, we also have enough information to calculate it, too. Remember that we have been informed that the β_d for the project is 0.22. Thus:[7]

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6 There are reporting services which sell such estimates, many of which are numerically lower than those quoted above.

7 Recall that the β_d of 0.22 was assumed to have been supplied to us by investment bankers. In practice, the investment bankers would have probably supplied us with the 12 per cent debt rate itself, and we could have thence deduced from Equation 7.1 the β_d.

$$\begin{aligned} \mathrm{E}(r_j) &= rf + [E(r_m) - rf]\beta_j \\ \mathrm{E}(r_d) &= 0.10 + (0.091)0.22 \\ &= 0.12 \ or \ 12\% \end{aligned}$$

If we assume a 50 per cent company income tax rate, the WACC of the project is:

$$\begin{aligned} rv^* &= 0.20[0.12(1 - 0.50)] + 0.80(0.201) \\ &= 0.173 \ or \ 17.3\% \end{aligned}$$

EAR should use a discount rate rv^* of 17.3 per cent to evaluate the consulting project's free cash flows, FCF*.

We have already seen in Figure 7.7 the kinds of errors that a company can make by evaluating projects with the company's WACC. It is instructive to see whether EAR would have likely made an error in its evaluation of the consulting project by using the company's WACC. Of course, in order to shed light on that issue, we must calculate the WACC for EAR. Fortunately, we have sufficient information to do that quite easily.

EAR's equity β coefficient, we were earlier informed, is 1.6. Therefore we can find the company's re by again invoking the SML:

$$\begin{aligned} \mathrm{E}(r_j) &= rf + [E(r_m) - rf]\beta_j \\ \mathrm{E}(r_e) &= 0.10 + (0.091)1.6 \\ &= 0.2456 \ or \ 24.56\% \end{aligned}$$

EAR's equityholders are requiring a 24.56 per cent return. If we assume that the company's debt requires a 14 per cent return (higher than the project because of the company's higher operating risks and financial gearing relative to the project), and using the known debt and equity proportions of 100/350 and 250/350 respectively, we find EAR's WACC to be:

$$\begin{aligned} rv^* &= \frac{\mathrm{D}}{\mathrm{V}}(rd^*) + \frac{\mathrm{E}}{\mathrm{V}}(re) \\ &= 0.29[0.14(1 - 0.50)] + 0.71(0.2456) \\ &= 0.1947 \ or \ 19.47\% \end{aligned}$$

EAR's WACC is 19.47 per cent, whereas the correct WACC for the project is 17.3 per cent.

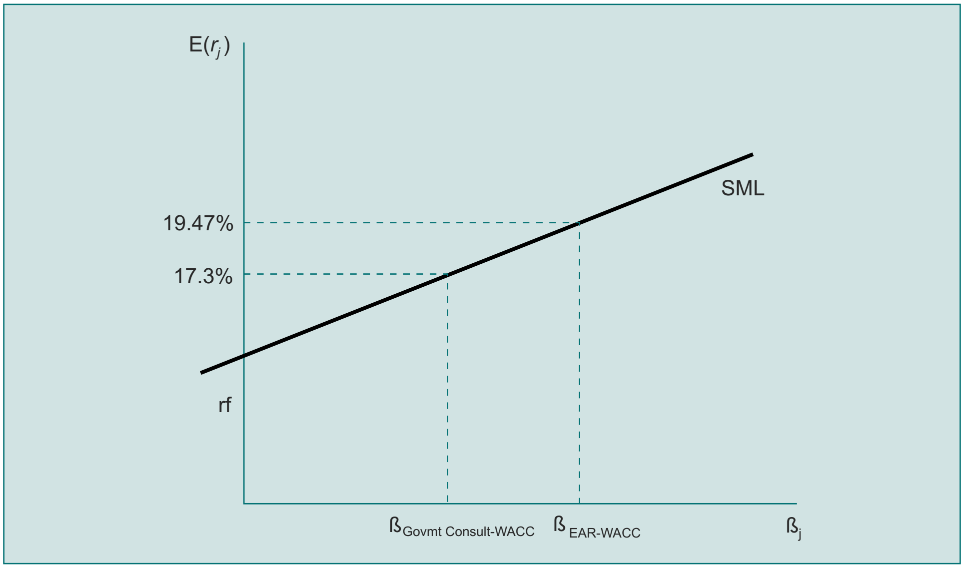

Figure 7.9 EAR's WACC and the SML

Figure 7.9 applies the arguments in Figure 7.7 to the EAR government consulting project case. Note that, were EAR to use its own WACC to evaluate the consulting project, it would be applying too stringent a standard and (depending upon the pattern of cash flows expected for the project) could well reject a desirable choice. (If the consulting project's IRR is greater than 17.3 per cent but less than 19.47 per cent, just such an error would be made.) As before, government consulting seems to be less risky than airframe parts manufacture, and the investment analysis should reflect this.

7.5 Other Considerations in Risk and Company Investments

We have invested significant time and effort in discussing how companies can use the CAPM and SML as aids in making decisions about risky investments. Finance analysts continue to study the correct application of these ideas in various situations, and this section mentions two such applications and extensions of the model into real investment situations.

7.5.1 Certainty Equivalents

The first application is the use of **certainty equivalents** in investment decision making. To this point we have illustrated a method of recognising risk by adjusting upward the discount rates used in present-value calculations. Specifically, the SML Equation 7.1 is designed to add a 'risk premium' to the risk-free rate so as to produce a risk-adjusted discount rate that recognises the opportunity costs of portfolio-holding capital suppliers. Conceptually, that is not the only mechanism that is available to adjust present values for risk. It is alternatively possible to adjust downward the expected future cash flow itself for its risk characteristics, creating a

certainty-equivalent cash flow, and to discount that (now 'certain') cash flow at the risk-free rate.

It is possible to create such certainty equivalents in a manner consistent with the SML. Essentially, the question asked is: 'How much money would I now agree to accept **in the future for certain** in exchange for the risky cash flow of the investment?'

To use the SML to answer the question we must change the Equation 7.1 a little. Instead of stating it in terms of discount rates, if we restate it in terms of cash flows, the SML adjustment becomes:

$$CF_{ce} = CF - \frac{\mathrm{E}(r_m) - rf}{\mathrm{Variance}(r_m)} \mathrm{Covariance}\,(\mathrm{CF}, r_m) \tag{7.2}$$

Equation 7.2 is thus the SML, but stated in terms of cash flows instead of discount rates. The certainty-equivalent cash flow *(CF_{ce})* is found by subtracting from the expected risky cash flow (CF) an adjustment for its systematic risk. That adjustment uses a variant of the 'market price of risk', $[\mathrm{E}(r_m)\text{-rf}]/\mathrm{Var.}\ (r_m)$, multiplied by a measure of the systematic risk of the cash flow, Covar. (CF, r_m). The only new terms in the Equation are Var. (r_m) and Covar. (CF, r_m); they are the **variance** (the squared standard deviation) of the market return, and the **covariance** (the cash flow's β coefficient multiplied by the variance of the market return) of the cash flow with the overall market.

Equation 7.2 is no better nor worse than Equation 7.1. The same information is required for each of them to function, and the answers obtained by using them are the same. The only difference is that instead of learning from Equation 7.1 what risk-adjusted discount rate to use in a valuation of cash flows, you learn from Equation 7.2 what risk-adjusted cash flow to discount with the risk-free rate. The present-value answers are identical if the equations are used correctly.

Let us briefly examine an example to test this incongruity. Suppose your company is evaluating an investment wherein the following information is known:

$$\begin{aligned} \mathrm{E}(r_m) &= 0.18 \\ \mathrm{Var.}\,(r_m) &= 0.01 \\ CF_1 &= £150\,000 \\ \mathrm{Covar.}\,(CF_1, r_m) &= £2400 \\ rf &= 0.10 \end{aligned}$$

Substituting into Equation 7.2, we have:

$$\begin{aligned} CF_{ce} &= £150\,000 - \frac{(0.18 - 0.10)}{0.01} £2400 \\ &= £130\,800 \end{aligned}$$

This result tells us that the market would be willing to exchange an absolutely certain £130 800 for the uncertain £150 000 cash flow of the investment, adjusting for its systematic risk. This exchange, however, would be made at the future time point t_1, so the present value of the cash flow is the discounted value of the certain £130 800, which using the risk-free rate is:

$$\begin{aligned} \text{PV(CF)} &= \frac{£130\,800}{1+0.10} \\ &= £118\,909 \end{aligned}$$

So the present value of the risky future cash flow is £118 909, found by the certainty-equivalent method. Were we to have used the risk-adjusted discount rate Equation 7.1, we would, instead of the covariance of the cash flow, have needed to know the β coefficient of the investment's return. Without going into a lot of algebra and statistical theory, take our word for it that the relationship is given by:

$$\beta(r_j) = \frac{\text{Covar.}\,(CF_j, r_m)}{\text{Var.}\,(r_m)PV(CF_j)} = \frac{£2400}{0.01(£118\,909)} = 2.01835$$

Substituting the above into Equation 7.1 for this example, we obtain:

$$\text{E}(r_j) = 0.10 + (0.18 - 0.10)2.01835 = 0.261468$$

So the cash flow would require slightly over a 26 per cent return (we are using a lot of decimal places to make the answers come out the same). Applying this discount rate to the cash-flow expectation:

$$PV(CF_1) = \frac{£150\,000}{(1.261468)} = £118\,909$$

This, of course, is exactly the same value we found using the certainty equivalent method.

Is there anything to choose between the two techniques? Not really. Some may feel more comfortable using one as opposed to the other for psychological reasons, but the answers we get are the same. Nevertheless, your education as a finance person would be incomplete if you had never heard of certainty equivalents.

7.5.2 Risk Resolution across Time

One commonly encountered complexity in investment analyses is that the risk of an investment can be foreseen to change as time passes, yet a decision as to whether the investment is to be undertaken must be made now, before the effects of such changes in risk have been resolved. It is easiest to understand this phenomenon with an example.

Suppose that your company is considering the introduction of a new product in a line of business about which you have little information. Specifically, your plan is first to test the market with a small investment, so as to get some idea of the extent and character of demand, the costs of production and marketing, and so forth. Only then, if the information indicates it is worthwhile, will you enter the market in a big way.

The initial tests are expected to cost £500 000. If the tests indicate good prospects, there is to be an additional £5 000 000 investment, which is expected to generate an NPV of £2 500 000. This NPV is the result of annual £750 000 FCF*s discounted at 10 per cent rV^* in perpetuity. Your best guess is, however, that the odds are only one-in-two that the initial test will indicate that the larger investment will be pursued.

A common error made in such analytical situations is to treat the investment's entire cash flow set as having the same risk (which in this case would be very high given the odds of finding a good market in the initial tests). But that is not the correct view of the riskiness of the investment. Think of the sequence of events over time: first, the relatively small test marketing is undertaken, for which there is a fairly riskless outlay of £500 000. Then one of two things will happen: either you will reject the project because it is no good – another low-risk outcome – or you accept the project because the study has indicated it to be worthwhile, again a relatively low-risk outcome. The high risk (the 50 per cent chance of a bad outcome) is felt **before** the very expensive investment commitment would be made.

How can such a decision situation be structured? The basic question is whether to undertake the £500 000 initial test-market outlay, where the desirability of that outlay depends upon the outcome of the test. Obviously if the tests indicate rejection of market entry, the £500 000 will have produced no good result. On the other hand, if the tests are positive, you will gain £2 500 000. (That gain will be one period in the future, so would need to be discounted at the risk-free rate to be comparable to the £500 000 outlay.) Essentially, the investment is asking you to give up a certain £500 000 now in exchange for a lottery that has a 50 per cent chance of a £0 payoff and a 50 per cent chance of a £2 500 000 payoff one period hence. Expected payoff to the lottery is thus:

$$0(0.5 + £2\ 500\ 000(0.5) = +£1\ 250\ 000$$

The £1 250 000 is of course an uncertain amount, but will occur only one period in the future. In order to be unacceptable, the discounted certainty equivalent of the £1 250 000 would necessarily be less than £500 000, not itself a very likely condition (requiring a 150 per cent discount rate.) The decision to invest the initial £500 000 is thus a good one.

Suppose we had instead felt that the risk of the negative outcome of the test market should be allowed to affect the discount rate of the future larger investment. It would have been necessary only to increase the discount rate on the investment's £750 000 annual cash flow from 10 per cent to 15 per cent (with its £5 000 000 outlay) to render it (erroneously) worthless, a quite likely result if the riskiness of the test market result is included in the evaluation of the larger investment's cash flows.

What is the lesson of this illustration? Simply that the passage of time, to the extent that it will alter the risk of an investment's cash flows, should be considered by investment analysts. Such concerns will be important particularly when time will be witness to the significant resolution of uncertainty with respect to the returns to future outlays of investments.

Learning Summary

This module has introduced us to a quantitative method of adjusting discount rates for the risk inherent in the cash flows of a project. The basis for such adjustment is a 'market model' of how such rates are determined. We call that model the security market line (SML) or capital asset pricing model (CAPM). The essential characteristic of the model is that it recognises that capital suppliers to a company themselves hold portfolios of assets that are well diversified (or where the market prices their securities as if they do). With this characteristic, markets will only compensate (with higher rates of return) investors for the amount of risk that cannot be diversified away. We call this risk 'systematic risk' and it is measured by the β coefficient that appears in the risk-adjusted return Equation 7.1.

This idea of the market rewarding only a particular type of risk bearing is a very important one. Not only has it produced a theory of risk-adjusted discount rates superior to anything yet available – and with impressive empirical validity – but it allows for certain nice intuitions about company investments and risk in general, which would be impossible without it.

For example, one of the lessons of the SML is that company diversification for the purposes of reducing risk is not destined to be a desirable activity from the shareholders' perspective. A company may indeed reduce the risk of its cash flows by undertaking projects whose cash flows are less than perfectly correlated with those of the company. Similarly, two firms with less than perfectly correlated cash flows can merge, thus reducing the risk of their combined cash flows. But think whether that activity is of benefit to shareholders. If the shareholders already hold well diversified portfolios, there is nothing to be gained in their portfolios by the company reducing only its **diversifiable** risk, which would be the sole result of such investments and mergers. On the other hand, if the shareholders were not well diversified, they could have simply themselves bought the shares of the two unmerged firms and thereby produced the risk-reduction benefits of the merger in their own portfolios, and at doubtless lower costs than those entailed in a company merger. The lesson is that if shareholders are already well diversified, diversification at the company level is irrelevant (and probably costly) to them.

One counter-argument offered to the above statements is that company risk reduction through diversification is beneficial not because shareholder risk is reduced but because, with debt in its capital structure, the likelihood of a company defaulting on its debt and the attendant unpleasantries is reduced. This plausible-sounding argument is also incorrect.

Take, for example, the situation of two companies merging and thereby reducing the risk of their debt. It is true that the debt of the combined company will likely be of lower risk than the average risk of the debt of the two companies uncombined. But think for a minute how that risk reduction came to pass. The imperfect correlation of the companies' cash flows is the cause. And the fact that debt is now less risky implies that there was at least some chance before the merger that a default on debt would have taken place in one or the other (or both) of the companies. Now, however, such risk has been reduced, if the basic cash-flow expectations of the companies themselves have not been affected by the merger but merely combined, the only place the debt risk reduction could have arisen is from equityholders. In essence, the equityholders of one firm are now providing insurance to the debt suppliers of the other firm, and vice versa. (Thus the term '**coinsurance**' is used to describe this phenomenon.)

What does this mean? Simply that the result of such a merger will be to increase the value of the debt of the companies, but reduce the value of the shares of the companies by the same amount. What about any benefits of diversification? The debt-holders of the company are better off, but the shareholders are worse off, and by the same amounts; thus the only 'benefits' of risk reduction through diversification have been to shift some shareholder wealth to debt-holders. Financial managers who do that sort of thing are not destined to have long, prosperous careers.

What is the lesson of this? Simply that companies should not try to provide something to shareholders which the shareholders can do perfectly well on their own. Since shareholders can perfectly well provide to themselves the benefits of diversification (by holding the shares and bonds of many companies, by buying diversified mutual funds, by belonging to retirement or pension arrangements that hold diversified portfolios, and many other low-cost arrangements) companies should spend their time trying to find high NPV investments as opposed to diversifying so as to reduce risk.

We have presented the CAPM and SML as if these ideas are completely settled and agreed upon by all concerned. You should be aware that there still exists much debate in the finance world, both academic and practitioner, as to the validity of the ideas and measures we have herein presented. But one fact stands out: the ideas we have studied in this chapter are at this point the only serious contenders for estimating risk-adjusted discount rates based upon some consistent economic model of compensation for risk bearing.

There are at least one or two other promising approaches,[8] not inconsistent with this, that embody less stringent assumptions as to investor behaviour, where risk-adjusted rates are deduced from groups of securities, including options of various types, which promise exactly the same risks as the opportunity in question. But such approaches have not yet been made as convincingly applicable to company investments as has been the SML.

To review what we have learned in this module:

1. Diversification reduces risk. The total risk inherent in an investment can be thought of as comprising diversifiable and undiversifiable risk, the former of which disappears when the investment is included in a well diversified portfolio.
2. Capital suppliers require higher rates of return for bearing higher risk, but the risk that they bear is only that which is undiversifiable in the portfolios they hold.
3. The market, in setting the required returns on assets and securities, depends only upon their undiversifiable systematic risks. It 'prices securities as if they were held in well diversified portfolios'.
4. This pricing process of financial markets can be captured in an equation that specifies required return as a function of systematic risk, or β coefficient. The equation is Equation 7.1, and that for the SML is depicted in Figure 7.6.

[8] One promising alternative to the CAPM is the **APT** (or **arbitrage pricing theory**), which posits a somewhat more complex set of factors – inflation, the shape of the yield a curve, economic activity, etc. – each of which has a β coefficient for each company.

5. This SML relationship can be used to estimate the appropriate risk-adjusted discount rates for company investment projects.
6. If an investment project is subject to risk that is very similar to that for the company undertaking it, and will be similarly financed, the company's WACC can be used as the discount rate for the investment's free cash flows. The WACC of a company can be estimated by finding its *re* value with the SML and using the standard weighting process for the *re* and *rd* (the debt rate or rates).
7. If an investment differs in risk profile from the company considering it, the investment must be evaluated with a WACC consistent with the investment's risk and financing. Such rates may be estimated as was the WACC for the company but using the β coefficient of another company whose risk is similar to the investment in question.
8. If there is no β coefficient available to be observed for the investment (for example if the comparable-risk company's shares are not traded in the market) the β coefficient for the investment must be artificially constructed. The process for accomplishing this requires that a 'benchmark' β coefficient (one from a company or investment about which relative volatility statements can be made) be adjusted for the extent to which the investment's characteristics differ from the benchmark. Section 7.4.2 details the procedure to accomplish this adjustment.
9. It is possible to construct an SML relationship that generates systematic risk-adjusted certainty-equivalent cash flows which are discountable at the risk-free rate, and arrive at the same answers available with the risk-adjusted discount rate SML approach.
10. To the extent that an investment's cash flows do not have constant risk across time, analyses become more complicated. When investment risk is expected to be resolved as time passes, this must be taken into account in the analysis, or risk may be overestimated for future cash flows beyond the resolution point.

Review Questions

7.1 Even though it was generally accepted in financial circles that the standard deviation of rates of return is an appropriate measure of risk, few empirical studies in finance could find a good relationship between that risk measure and the actual rates of return being earned by holders of securities across time. Which of the following gives the most important reason for that?

A. The level of sophistication available in statistical analysis was not high enough to detect the subtleties of such a relationship.
B. Investors in securities are not really risk-averse in the sense that they require higher rates of return for bearing risk; they are only interested in becoming wealthy, and are willing to take risks to do that.
C. Actually, any such tests are destined to produce questionable results because they are attempting to measure the unmeasurable: the feelings and expectations held by investors as they buy and sell securities.
D. The risk measure was incorrect. People only regard as risk that part of return standard deviation that cannot be diversified away when they include the security in a well diversified portfolio. So an undiversifiable risk measure was required.

7.2 Suppose that you already hold many securities and are considering investing in the following four additional ones. You are convinced that the data provided are correct:

	Expected return	**Standard deviation of return**	**β coefficient**
Security W	15%	10%	0.8
Security X	18%	15%	1.0
Security Y	19%	20%	1.1
Security Z	21%	21%	1.5

The risk-free rate is 10% per cent, and the excess of the market return over the risk-free rate, $(E(r_m)\text{-rf})$, is 8 per cent. You would:

I. Invest in either X or Y or some combination of the two because they are the only securities that offer enough return for the risk they contain.
II. Invest in any or all four because the returns and standard deviations of return are ordered the same; that is, the higher-return securities have the higher risk, and vice versa.
III. Invest in only the highest return security if not risk-averse, only in the lowest risk security if very risk-averse, or in one of the others if somewhere in between in your preference for risk bearing.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both I and III.

7.3 Anglo-Auto plc, whose shares are traded on the stock exchange and are widely held, is considering an investment in a gold mining operation. The gold mined will have no particular use in Anglo-Auto's operations, and will simply be sold on the market. Anglo-Auto is expecting to pay a fair market price for the mine. The stated purpose behind the venture, according to the managing director of Anglo-Auto, is diversification: 'We at Anglo have found that the company's fortunes are too much dictated by the state of the overall economy, which has been quite volatile in the last decade. By investing in the gold mine we are providing a nice offset to the risks of operating in the economy as a car manufacturer, because we have discovered that gold prices tend to do well when our company does poorly, and vice versa.'

Which of the following is correct?

A. Anglo-Auto is making a mistake by thinking that an investment in a gold mining operation is beneficial in reducing the risks of the company. Gold mining is itself a very risky operation, and adding two high-risk operations together is simply bad strategy.

B. The plan is a good one, if a bit extreme. Since the cash flows of car manufacturing and gold mining are less than perfectly correlated, combining them into a single stream will likely offer to the market a less risky stream than either would provide individually. This should result in an increase in the price of Anglo-Auto's shares.

C. It is true that Anglo's risk will likely be reduced, but the risk reduction is in total, not systematic, risk. So the benefits of such diversification will appear only at the corporate level, not for shareholders.

D. Either B or C could be correct, depending upon whether the shareholders of Anglo-Auto are well diversified.

7.4 Consider the risks and returns available from combining into a portfolio the shares of the companies listed below.

I. General Automobile Manufacturers

II. General Steel Company

III. General Pawnbrokers plc

IV. General Bakeries.

Which pair of shares is likely to produce the greatest reduction in risk due to diversification effects?

A. General Automobile Manufacturers and General Pawnbrokers plc because the two would be expected to have opposite relationships with overall economic activity.

B. General Automobile and General Steel, because these two companies' shares would produce a beneficial source of raw materials for the car manufacturing component, and a ready market for the steel produced.

C. General Bakeries and General Steel, because the former will not be affected much by overall economic activity, whereas the latter would.

D. Either General Auto and General Pawnbrokers or General Steel and General Pawnbrokers for the basic reason stated in A for General Auto and General Pawnbrokers.

7.5 Examine the graphs in Figure 7.10. Each shows a scatter diagram of the historical relationship between the returns of a particular security and an overall market portfolio or ‘market index’.

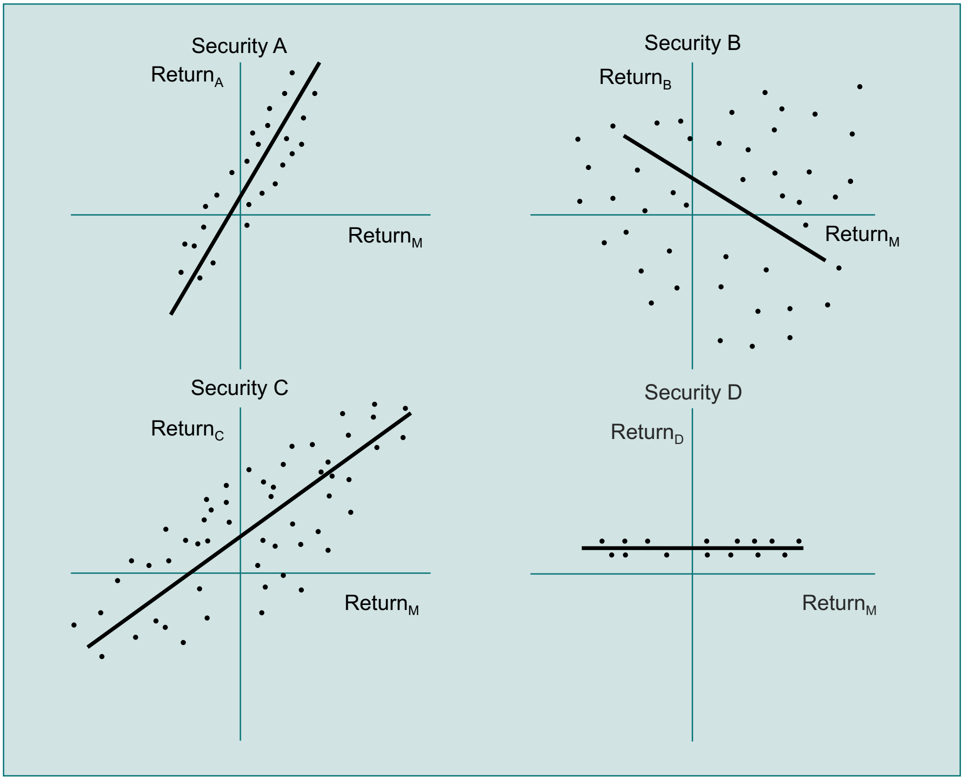

Figure 7.10 Scatter diagrams of security and market returns

Drawn on each diagram is a regression line formed by the ‘least squares’ method, or ‘line of best fit’. Rank the securities in order of declining:

I. β coefficient

II. total risk.

7.6 Within the context of the scatter diagrams in Figure 7.10, describe the manner in which the total risk of securities is transformed into systematic risk when these securities are held in well diversified portfolios.

7.7 Engineered Genetics (EG) is considering a foray into the development and manufacture of an organism that, when combined with red *vin ordinaire*, would transform cheap wine to taste exactly like 1945 Chateau Mouton Rothschild. The odds of such a development being successful in an engineering sense are about one in four, would take three years to resolve, and cost £3 000 000. EG figures that such project risks would require a 25 per cent discount rate per period. On the other hand, were the research to be successful, EG feels that the product would generate free cash flow of £3 000 000 for ever, after an initial additional outlay of £5 000 000 for a production facility. Alternatively, EG could at that point sell the patent on the process to a certain French company (to remain unnamed) for £25 000 000. Given the popularity of such a product, success is virtually assured, and thus EG is comfortable with a 10 per cent discount in that phase. The risk-free rate is 8 per cent. Should EG begin developing the product or not?

Case Study 7.1: NOSE plc

National Olfactory Speciality Enterprises plc (hereafter NOSE) is in the business of manufacturing and selling appliances to improve the function and appearance of the nose. Up to now the company's products have been of reasonably standard technology, utilising accepted techniques of design and manufacture, running the gamut from olfactometers to prosthetic noses (the latter to replace those lost through unfortunate illnesses and accidents or unskilled pugilistic adventures).

NOSE has recently been approached by an inventor who seems to have been successful in engineering the first 'smelling aid', which can loosely be considered the counterpart of the well known 'hearing aid'. NOSE's production, accounting and marketing experts have been busy formulating projections for the cash flows that would be associated with the company offering the device (which they have dubbed 'Cyrano' because it has yet to prove amenable to much miniaturisation). These analyses having been performed, NOSE is now ready to decide whether to undertake the project.

NOSE's shares are traded on the Stock Exchange, and have a β coefficient of 1.2. The company also has debt outstanding, comprising 35 per cent of NOSE's total value and having a β coefficient of 0.20. The company is in a 50 per cent income tax bracket, interest being deductible.

The project is expected to be about 50 per cent more revenue-volatile than the rest of the company's products, and will have fixed costs equal to 47 per cent of revenues compared with fixed costs of only 5 per cent of revenues in the rest of the company. Investment bankers have advised NOSE that Cyrano can be financed with a 15 per cent interest debt claim, as long as NOSE will supply 70 per cent of the (market value) equity for the project, and that the project's cash flows alone will service the debt.

The appropriate risk-free return for the period in question is 10 per cent per annum, and the difference between the market's expected return and the risk-free rate is 9.1 per cent.

1 Advise NOSE as to the correct WACC for it to use in evaluating 'Cyrano', and contrast this with NOSE's WACC.

Module 8

Company Dividend Policy

Contents

Learning Objectives

A company's shareholders are the most important capital suppliers to the firm, and financial decision making is aimed toward maximising their wealth. One of the important financial decisions made within a company is its dividend policy; that is, the strategy the company uses to transfer cash to its shareholders. The module begins with some basic economics of dividends, showing how dividends can simultaneously be both the basis for share value and an irrelevancy. Actually, the argument is that the dividend decision, under certain idealised conditions, makes no difference to shareholder wealth, as long as other financial decisions (such as real asset investments and capital structure) are given. When those idealised conditions are removed, however, dividend policy may become important. When taxes, brokerage fees, consumption patterns of investors, and information frictions are considered, dividend policy has the potential to be quite important. The module goes to some lengths to display what is known and not known about the impact of the choice of dividend policies on shareholders' wealth.

8.1 Introduction

This module addresses company dividend policy. Most simply, the question of dividend policy is how much money the company should pay to shareholders across time. **Dividends** are the amounts of cash that a company distributes to its shareholders as the servicing of that type of capital. Shareholders have invested in the company by purchasing shares, and dividends are the company's direct compensation to shareholders for their investment.

Dividends, unlike interest and principal payments to debtholders, are not a contractual right of shareholders. There is no requirement that a company pay its shareholders any particular amount of dividend at any particular time. However, since shareholders are the residual claimants of the company's cash flows, they do in a sense 'own' the amounts of cash that the company produces net of all other contractual requirements (other claims being the costs of operations, taxes, interest, and so forth). Financial managers of a company must decide how much of its residual cash should be paid as dividends to shareholders. Since any residual cash not paid as dividends is still 'owned' by shareholders, this retained cash is reinvested in the company on behalf of shareholders. The dividend decision is thus also, in mirror image, a **cash retention** or **reinvestment decision**. Any reasonable discussion of dividend policy questions must treat both aspects of this decision.

As we promised for all of our discussions of company financial decisions, our analysis of company dividend policy will assume that managers of companies try to make shareholders as wealthy as possible. So the real question of dividend policy boils down to this: with amounts of cash that could be distributed to shareholders across time, and given that any amounts not so distributed would be reinvested by the company in the name of the shareholders, is there a particular strategy of dividend distribution that would produce more wealth for shareholders than any other? The task of this module is to investigate possible answers to that question.

8.2 Dividend Irrelevancy I

As the title of this section implies, there is a set of arguments that says that the dividend decision of a company is unimportant; that nothing the company does in the way of paying or not paying a dividend has any effect upon the wealth of its shareholders. Actually, there are two trains of thought that would lead us to this conclusion, both of which we shall investigate in the module. The first, which is the subject of this section, is not really taken as a serious real-market consideration by financial thinkers because it is 'unrealistic' in the sense that a number of important phenomena are ignored. Nevertheless, the valuable aspect of this view of dividend policy is the light it can shed upon the more complex arguments that follow. This set of ideas contains the basic economics of all real-market dividend decisions, but in an admittedly simplistic form. Studying the simple setting first will make our later, more sophisticated discussions much easier.

Fortunately we have already performed much of the work of developing this simple concept of dividend policy. Recall the discussions we undertook in Module 2 about the effects upon shareholders of a company undertaking an investment. We used the Simple Corporation to illustrate that an investment's NPV accrues to the wealth of its shareholders. In that illustration the company was facing a decision about an investment with an initial cash outlay of £100 000 and a positive NPV of £400 000. The final result of the discussion was that the shareholders of Simple got the entire £400 000 NPV regardless of whether the investment was financed by reducing the company's dividend to retain cash, or by continuing to pay the dividend and raising the money for the investment from new shareholders. That argument is exactly the set of ideas embodied in this first view of 'dividend irrelevancy', so we should review it here.

The Simple Corporation was to make a £100 000 cash outlay so as to undertake the investment. Upon the acceptance of the project, Simple's market value (it had borrowed no money so it had only equity shares outstanding) would increase by £500 000. Thus the investment would cost £100 000 and produce £500 000 in value, netting the aforementioned £400 000 NPV. The question was: from where should the cash be obtained to finance the investment – from reducing the planned dividend, or by issuing new shares? Note that this is really a question of **company dividend policy**. Another way is to ask the equivalent question: does it make any difference to shareholders if Simple reduces its dividend instead of floating new shares so as to undertake the investment?

In this example the answer to the question was in the negative. If the company retained the cash to undertake the investment, shareholders were less wealthy by the £100 000 they did not receive as a current dividend, but were better off by the £500 000 increase in their market value, netting the £400 000 NPV.

On the other hand, were the company to pay the planned dividend, shareholders would not forego the £100 000 cash investment outlay, but the company would be forced to seek outside capital to finance the investment. Raising the £100 000 from new shareholders would doubtless require that the company give the new shareholders in return shareholdings worth £100 000. Thus the original shareholders get to keep their £100 000 cash in the form of the company dividend, but lose £100 000 of the £500 000 increase in market value to the new shareholders, again netting the £400 000 NPV. This means the original shareholders' wealth is the same regardless of the decision the company makes about the dividend.

There are two important lessons to be learned from this example. First, the net result of changing the company's dividend is the substitutability of **capital gains** (i.e. share value increases) as the dividend is reduced for **cash** when the dividend is paid. In other words, higher cash dividends mean lower market value to existing shareholders, and lower cash dividends mean higher market value to existing shareholders. This is common to all dividend policy changes when other financial decisions are to be held the same (e.g. here the £100 000 investment decision has already been made, and will not be affected by the decision about dividends). If existing shareholder dividends are reduced, their claim upon future dividends, and thus current market value, is **higher** because less new equity is raised. If existing shareholder dividends are increased, their claim upon future dividends, and thus current market value, is **lower** because more new shares must be sold.

In effect, by accepting their current dividend, and given the investment plans of the company, current shareholders are selling part of their future dividends to new shareholders. But that sale is at a market price equal to the dividend received. So the gains and losses offset each other, and there is no change in existing shareholder wealth, only in its composition. If they accept higher dividends, current shareholders have more cash and less share value; if they accept lower dividends, they have more market value and less cash. You can see this easily in Table 8.1, which reproduces Table 2.1 for the Simple Corporation's investment (and dividend) decision. (Note that reducing the dividend produces a loss of cash and increase in value, while the payment of the dividend produces no loss of cash but a smaller value.)

Table 8.1 The Simple Corporation: changes in shareholder wealth due to investment

	Cash	Value	Net
If dividend is not paid:			
Old shareholders	−£100 000	+£500 000	+£400 000
New shareholders	0	0	0
If dividend is paid:			
Old shareholders	0	+£400 000	+£400 000
New shareholders	−£100 000	+£100 000	0

The second important point about this example is the connection it illustrates about the company's dividend and investment decision. This is not unique to the Simple Corporation's dividend decision, but is common to all dividend policy questions. What is the connection between dividends and investment? Figure 8.1 may help you to visualise the relationship. The figure shows the company's residual cash available to be allocated between dividends and retentions, along with the company's plans as regards investments. The investments can be financed with retentions of residual cash, or by new share issuances (for simplicity we again assume an all-equity company).

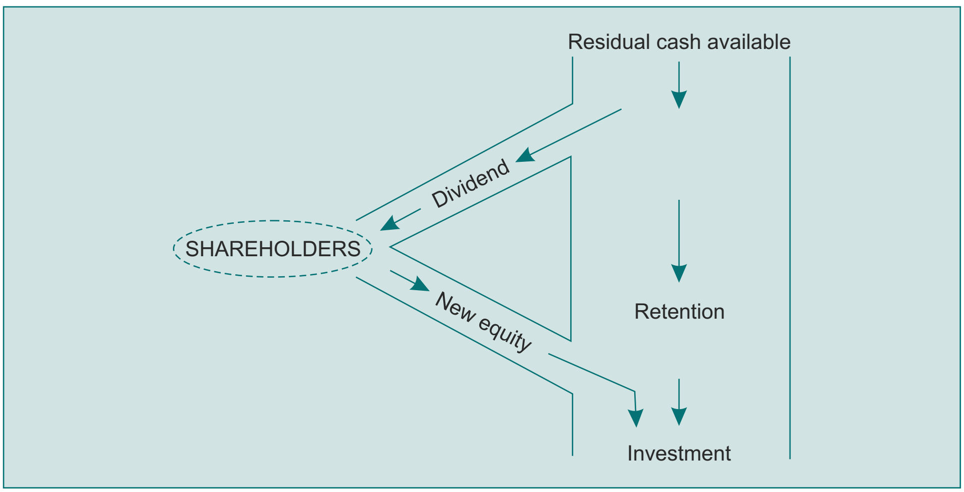

Figure 8.1 The dividend decision

Think of the situation this way: any dividend paid by the company will come from a pot of money that is necessarily split into only two parts, dividends to shareholders and cash retention by the company. So any increase in dividends will reduce cash retention, and any reduction in dividends will increase cash retention. Further, all retentions are by definition part of the company's investments for that period (even increases in bank deposits are investments subject to analytical choice). So the connection between dividends and investments can be regarded as a truly organic one for the company: if dividends are changed, and no other action undertaken, the company's investments will also change. In Figure 8.1 an increase in dividends would be shown as a widening of the 'Dividend' pipe and a narrowing of the 'Retention' pipe, resulting in a diminution of the 'Investment' amount, or vice versa for a reduction in the amount of dividends. Either would change the investment that the company made.

The question at hand, however, is company dividend policy, not investment decisions. So to isolate the effect of its dividend choices, we must keep the company's investment plans intact as dividends are changed. To hold the investment decision unchanged, any increase in dividends must be accompanied by cash being raised from outside the firm to replace the reduced cash retention. Of course a reduction in dividends would require less cash to be raised from outside. In Figure 8.1 this would mean that keeping the size of the 'Investment' pipe the same regardless of changes in the 'Dividend' pipe would require that the 'New equity' pipe be increased or decreased in the same amounts as the changes in dividends. In other words, if the 'Dividend' pipe is increased, the 'New equity' pipe must also be increased by the same amount, or the 'Investment' pipe would change. A reduction in dividends would also require a reduction in outside share issuances in the same amount, to keep investment plans intact.

Recall the Simple Corporation's dividend decision. One important reason that dividends are irrelevant in this example is that we have isolated the decision from all other financial decisions that the company is making. Here the sole function of a dividend alteration is to shift the source of cash for investments from old to new shareholders, and vice versa. In the Simple Corporation, as long as the new shareholders pay exactly as much cash into the company as they receive in value of new shares, and as long as the company's investment decision is maintained, the wealth of old shareholders will not be affected by how much dividends they do or do not receive.

Thus the Simple Corporation example is not in any sense a contrived example. The dividend–investment interaction shown therein is common to all dividend decisions by companies. Simple is 'simple' in a number of other important ways that we shall now begin to discuss, but the basic economics of dividends is well displayed in that exercise.

To review, the notion of dividend irrelevancy proceeds as follows: leaving other financial decisions intact, higher dividends require more new shares to be sold, lower dividends require fewer. As long as the new shareholders insist upon receiving full value for the cash they contribute, and company share value in total is unchanged (because other decisions are intact), existing shareholder wealth is unaffected by the dividend decision. Any change in the cash portion of current shareholder wealth (due to changes in cash dividends) will be exactly offset by changes in the value of their shareholdings (due to changes in the amounts of new shares issued by the company).

There is one final point to be made about the discussion of dividend irrelevancy in this context. Many experienced finance people, when confronted with the irrelevancy assertion, have significant trouble believing it. As we shall see below, there are good reasons for their scepticism, but one of the arguments against irrelevancy is definitely incorrect, and it is one of the most common. This is the 'bird in the hand' view of dividend policy. It argues that dividends are preferable to capital gains because the former is actual cash in hand, and the latter is based upon future dividends not yet received. Thus a policy that substitutes future uncertain dividends for current certain dividends is by its very nature designed to increase the riskiness of the company's shares. This sounds plausible, but is wrong.

Remember that one of the primary functions of financial markets is to place current (certain) values upon future (uncertain) cash flows. That is exactly what has been done in the setting of the prices of companies' shares. So the current market share price includes an adjustment for the riskiness of future dividends. In no sense is that an incorrect price, regardless of the dividend policy of the company.

Now consider the situation of a shareholder whose company adopts a policy of reducing dividends (and also, of course, of reducing the sale of new shares). That shareholder will have the same total wealth, but the wealth will have a different composition: it will comprise more wealth in share value and less in cash from dividends. That is indeed a riskier situation for the shareholder. But if the shareholder were unhappy with this portfolio composition, there is nothing to prevent the shareholder from selling enough shares to duplicate the cash dividend that was not received. Were that to be done, the shareholder's wealth would have the identical composition as it would if the company had not adopted the lower dividend payout.

For example, suppose that the company's dividend policy had produced £1000 less cash because of a dividend reduction, but this had caused the company to issue £1000 less new equity. The shareholder's wealth is exactly the same total, but differently comprised. There is now £1000 more in value and £1000 less in cash. If the shareholder was happier with the £1000 in cash rather than share value, shares in that amount can be sold, and the shareholder will end up with the same cash and value holding that would have resulted from the company paying the dividend and issuing the new shares itself. The shareholder has effectively issued the shares himself, by selling some of his shares to new shareholders. Thus up to this point there is no reason to think that dividend policy makes any difference to shareholders.

There is actually a deeper lesson in this illustration, which will serve us well in discussions of all company financial decisions. In the above illustration the shareholder has in effect 'undone' the company-level dividend decision in the shareholder's own portfolio. When the effect of company financial decisions upon shareholders' portfolios can be undone by offsetting actions of shareholders, **the company financial decision is irrelevant**. This is a very powerful insight, and will eventually allow us to understand the importance of various financial decisions much more deeply.

8.3 Dividends and Market Frictions

The Simple Corporation example in the above section is instructive of the basic cash flow and value economics of dividends, but ignores some real-market phenomena that can have important effects upon company dividend policy. What types of phenomena are we talking about here? Essentially three: taxes, transaction costs and flotation costs. Let us examine each of these in turn.

8.3.1 Taxation of Dividends

When a company pays a dividend, the cash thus distributed must make its way through whatever tax system exists in the economy before the dividend is useful to the shareholder. From the shareholder's perspective, it is after-tax (both company and personal) dividends that are of interest. Similarly, the substitute for dividends, capital gains, are also potentially liable for taxation. As we have argued many times before, tax systems are essentially arbitrary and can be quite different between countries. It is generally the case, however, that dividends are more heavily taxed than capital gains.

In countries where the amounts of cash available to pay dividends are net of company taxes, and the dividends paid are taxed again at the shareholder level, dividend payment to taxable shareholders is expensive. Those shareholders would likely be better off receiving their wealth in the form of share price increases that are either not taxed or taxed at lower rates than the dividends.

Suppose, for example, that companies pay income tax at the rate of 50 per cent and shareholders in addition pay 30 per cent tax on dividends received. Furthermore, suppose that a company has £1000 profit before tax (assume this is a cash amount) and is planning to invest £500 in assets during this period, so that £500 of cash must either be retained or raised from shareholders. The results of choosing to pay dividends or not are given in Table 8.2.

Table 8.2 The Simple Corporation: changes in shareholder wealth due to investment

	Pay dividends	Do not pay dividends
Profit before tax	£1000	£1000
Company income tax	−500	−500
Profit after company income tax	500	500
Dividends	500	0
Shareholder income tax	150	0
Net after-tax cash to shareholders	350	0
Increase in share value to existing shareholders	0	500

In either case, the shares of the company will increase by £500, but should current shareholders receive the dividend, they cannot also receive the share price increase. It will go to the new shareholders who contribute the £500 necessary for the investment outlay. If the shareholders **do not** receive the dividend, they **do** receive the £500 share value increase because the company was not forced to raise cash from new shareholders, and because the money for the investment was retained instead of being used to pay the dividend.

The important thing to understand about this illustration is that total taxes paid by the company **and** its shareholders are higher when dividends are paid than when they are not. Shareholders receiving the dividend have only £350 cash in hand after tax whereas they would have had £500 in share value had dividends not been paid. The company needs £500 for investment, and can raise that money from new or old shareholders. But the damage has already been done: the £150 of taxes is lost to government.

When dividends are not paid, shareholders receive no cash, but neither do they pay taxes. Such shareholders paying no cash into the company, nor having others contribute cash, retain their full claim, and have a value increase of £500. The difference between the final wealth of shareholders who receive dividends and those who do not is thus £500-£350=£150, the difference in the taxes paid. (If and when shareholders realise 'capital gains' by selling shares, there may be some tax paid, but it is usually later and sometimes less than that paid upon the receipt of dividends.)

In such a tax system where 'double taxation' of dividends is unavoidable, there is a strong tax incentive against the payment of dividends for companies seeking to please their shareholders. (Interestingly, the empirical evidence as to whether high dividend-paying companies' shares are adjusted in price for the taxability of the dividends is mixed, or at least not agreed by consensus of researchers. Some think that in these economies there are enough dividend tax-avoidance transactions available so that this tax detriment of dividends is really unimportant.)

In some countries (such as the UK), dividends are not taxed as heavily. These 'imputation' systems make some attempt to alleviate the double taxation of dividends by imputing an amount of company taxes to shareholders based upon the dividends that companies pay, and then giving shareholders a credit on their taxes for that amount. The effect of such systems is to cause less of a bias against dividends than systems that tax both company profits and shareholder dividends.

Using the above example, assume that the tax system is such that dividends received by shareholders are imputed by tax authorities as having had 30 per cent tax already paid on their behalf by the company. Further, companies paying dividends pay the same total tax as companies not paying dividends, even though shareholders are given tax credits for the imputed tax on their dividends. Our example now becomes as in Table 8.3.

Table 8.3 **Results of the dividend payment decision under the imputation tax system**

	Pay dividends	Do not pay dividends
Profit before tax	£1000	£1000
Company income tax	−500	−500
Profit after company income tax	500	500
Dividends	500	0
Shareholder income tax	214.29	0
Tax credit	214.29	0
Net after-tax cash to shareholders	500	0
Increase in share value to existing shareholders	0	500

Note two things about this example. First, shareholders receiving dividends pay no net tax on them, so that there is no wealth difference between the position of those who receive dividends and those who do not. The tax credit exactly matches the tax liability. (The reason that the liability and the credit are more than 30 per cent of the dividend paid is the nature of the 'imputation' calculation itself, wherein the dividends received are assumed to have been **after** the 30 per cent tax is levied, so that the dividend received is actually 70 per cent of the imputed dividend. The latter is taxed at the 30 per cent rate, meaning that the actual dividend generates a tax liability and credit of 30/70 of the cash amount paid.)

Secondly, note that there actually is a tax liability on the part of the shareholder, against which there is a **fixed** credit. In this example, the tax and the credit are the same amount, so there is no net tax paid on the dividend and shareholders would be indifferent between receiving them or not. But suppose that there is a personal tax liability that is higher than the tax credit received (because of higher personal tax brackets by shareholders). Here the credit would not cover the tax liability, and the shareholders would be worse off receiving dividends than not. Suppose also that shareholders' personal income tax rates were 40 per cent. The situation now becomes as set out in Table 8.4.

Table 8.4 **Results of the dividend payment decision for taxpayers in higher tax bracket**

	Pay dividends	Do not pay dividends
Profit before tax	£1000	£1000
Company income tax	−500	−500
Profit after company income tax	500	500
Dividends	500	0
Shareholder income tax	285.71	0
Tax credit	214.29	0

	Pay dividends	Do not pay dividends
Net after-tax cash to shareholders	428.58	0
Increase in share value to existing share-holders	0	500

Here again there would be a bias against the payment of dividends, even in an imputation system. (In the UK there are further aspects of the company-versus-personal dividend imputation system that produce a bias against dividend payments, having to do with the fact that the imputed shareholder dividend tax is paid in advance. **Advance corporation tax (ACT)** must be paid by the company even if there is no company income tax. The company can set this off against its income tax – **mainstream corporation tax (MCT)** – if and when this is eventually paid. You are familiar enough with the time value of money to see the penalty involved when a company with no income tax liability pays dividends.)

8.3.2 Transaction Costs of Dividend Payments

The second friction with which dividends interact is **transaction costs**. Recall the example of the Simple Corporation. There, company dividend policy affected not the total of shareholder wealth but the composition of that wealth. Higher dividends meant more cash and less value, and vice versa. From the perspective of shareholders, if they can costlessly shift their portfolios from shares to cash and back, there is no reason to prefer one payment to the other.

For example, suppose Simple had chosen to reduce its dividend so as to make the investment outlay. Shareholders would, relative to the opposite decision, have found themselves with £100 000 less cash, and £100 000 more share value. If shareholder preferences were to consume some of that £100 000, they would merely sell some shares for cash. (If they sold exactly £100 000 worth of shares, they would put themselves in the same situation that they would have been in had the company instead paid them the dividend and sold shares.) The opposite would have been true had the shareholders received a dividend but wished to consume only part (or none) of it. They would then have costlessly converted the cash into shares. So in the Simple Corporation, dividends did not matter.

But in real markets, shareholders cannot shift costlessly between shares and cash. There are usually brokerage fees to be paid when such transactions take place. (And there may be the forced 'realisation' of capital gains and the taxes thereby due.) So shareholders may prefer one dividend policy to another depending upon their preferences for consuming wealth across time and the costs they would pay to achieve the desired consumption pattern, given a particular dividend policy by the company.

8.3.3 Flotation Costs

Finally, companies themselves incur costs in raising money from capital markets when they pay dividends so high as to require new shares to be sold. These are called **flotation costs**, and they can be significant for the issuance of new shares, depending upon the mechanism of sale. If intermediaries such as investment bankers are used, the costs can be as high as 5 to 25 per cent of the total value of issued shares.

8.3.4 Combined Frictions

So real-market considerations of taxes, transaction costs and flotation costs are potentially significant considerations for companies in their dividend policy decisions. Though the sizes and impact of these frictions are essentially an empirical question, and can differ in different countries, it is probably true that the net bias in most cases is against the payment of cash dividends and toward the retention of cash by the company. Given the average tax brackets of shareholders and other empirical elements of these frictions, the result of substituting capital gains for dividends is diminution of transaction and flotation costs, and delay or reduction of taxes.

The resulting optimal dividend policy of companies would be as follows: find all of the investments that have positive NPVs, and retain as much cash as is necessary to undertake these investments; if there is cash left over, only then might a dividend be paid; only raise new equity capital when internally generated funds are insufficient to provide the cash necessary to undertake all good investments. This is sometimes called a '**passive residual dividend policy**'.

If our discussion of dividends were complete, we would expect to see evidence that companies actually pursued such a policy. What we see instead is that companies' dividends across time are much more stable than a passive residual policy would require. We also would expect to see the shares of companies that pay relatively high cash dividends valued less highly than otherwise identical companies who instead tend to retain cash. The evidence here is mixed, and not overwhelmingly in favour of retention as we would expect.

Why do companies not follow a passive residual strategy of dividend payment? Probably because passive residual dividends are not the best option for companies. (Either that or companies are not aware of the frictions discussed above, which is unlikely.) If passive residual dividends are not optimal, our discussion of dividend decisions cannot be complete. There are evidently additional factors to be considered in the dividend decision.

8.4 Dividend Clienteles: Irrelevancy II

In our discussion of the frictions existing in real markets for dividends, we pointed out the potential importance of considerations such as taxes and transaction costs, but only in a general way. In other words, we admitted that taxes exist and can affect how much post-tax wealth is available to shareholders from a given dividend policy, but did not talk about the fact that shareholders are not all taxed the same. Nor do shareholders all have the same preferences for consuming income across time, and thus likelihoods of incurring transaction costs in rebalancing portfolios.

The recognition is important that shareholders are not all alike in the exposure they have to dividend and capital gains taxation and that preferences for consumption of wealth across time differ. Intuitively it should be easy for you to see that one type of shareholder, say those in high personal tax brackets, would prefer one kind of dividend policy (low cash payout), whereas another kind of shareholder in a low tax bracket might well prefer high cash payout. Such different kinds of shareholders have come to be called **clienteles** in finance. The interpretation is that they comprise groups that would be willing to pay extra to get the type of dividend policy that is best suited to their own tax and consumption proclivity. Said another way, they have probably been attracted to the shares of a company that pursues a policy that to them is attractive.

What does this mean for the dividend decisions of companies? Consider, for example, the Complex Corporation. Its shares are traded in a financial market wherein the companies and shareholders are both subject to the full range of 'frictions' we discussed in the previous section. Complex Corporation could undertake to study the average tax situation of all shareholders, brokerage fees, proclivities to consume wealth, and so forth, so as to reach an opinion as to the dividend policy that would be of most appeal to the average of all shareholders, or even to its own average shareholder. In a market having only one firm or just a few firms providing wealth to shareholders, that might make sense. The best policy for shareholders could be designed, and might produce a wealth increase for shareholders.

But there are not just a few companies providing wealth disbursements to shareholders; there are many. And those companies provide a wide range of dividend strategies to the market. Given the number and types of dividend payout patterns available, we can raise questions as to whether anything a particular company can do to change its dividend policy is likely to give its shareholders something that they could not acquire elsewhere. As a matter of fact, this idea is one of the underpinnings of current thinking about company dividend policy, and it brings us back to the original 'irrelevancy' conclusion, even in realistic financial markets.

Think of the situation this way. As we have argued exhaustively above, there are many potential different shareholder 'clienteles' for various dividend policies by companies. Some shareholders (commonly called 'fat cats') want low payouts because of high personal income tax brackets. Others ('widows and orphans') want high payouts because of low personal taxes and preferences to consume their wealth in the form of cash payments. And there is a continuum in between of possible variations on these themes. For example, managers of institutional share portfolios (pension fund and mutual fund managers) might well prefer low dividend payout if their salaries depend upon the total value of the shares they manage, because cash dividends in some instances must be paid out to those who are serviced by the institution. Some other institutions may be legally constrained to hold only shares that maintain a relatively high payout. These clienteles will each tend to be attracted to the shares that provide the dividend policy they desire. Companies are thus in a competitive market for dividend policies as well as for their other goods and services.

If a clientele is under-served, a company could likely increase the value of its shares by adopting a dividend policy that appeals to the under-served clientele (the under-served clientele would likely be willing to pay a premium for shares providing the policy it desires). But that is true of all companies. And in a competitive market companies seeking to maximise their share values will cause them to serve each clientele until the share price benefit of switching from one policy to another disappears. So there would be no benefit from a company changing its dividend policy, even in real, complex financial markets.

The implication for company dividend policies of these observations about clienteles is straightforward. Clienteles are important, and definitely have preferences about optimum company dividend policies. However, it is unlikely that a company choosing one policy over another will be of benefit to its shareholders because there are likely to be no relatively under-served clienteles willing to pay a premium for the company to change its policy.

In a sense we have come full circle from our original dividend irrelevancy argument under frictionless, taxless markets. We again have a situation where the choice of dividend by a company will not produce a benefit for its shareholders, but from a much more complex, realistic perspective.

Actually, 'irrelevancy' is perhaps too strong a term to describe the prescription for dividends that this argument produces. A better term than irrelevancy is probably 'inertia'. Remember that the financial market causes transaction costs to be incurred when switching shareholdings from one company to another. Suppose that your company switches from its present policy to another, and there is no effect upon the prices of your shares, as predicted above. But consider now the situation of your shareholders. They, having likely chosen your shares expecting your prior policy, will now be forced to seek another company's shares (perhaps identical in all respects to your shares prior to the dividend policy change). This realignment of shareholdings will cause your shareholders to incur transaction costs in the form of brokerage fees and perhaps capital gains taxes. Therefore 'irrelevancy' is not strictly true from the full perspective of shareholders, even though there is no share price effect of dividend policy alteration.

The above boils down to the following. Dividend policies are important to various clienteles, but competitive markets have likely equalised the desirability of serving one clientele as opposed to another. And a company's switching from one policy to another is costly to its existing share clientele, so **whatever** a company's current dividend policy is, it is likely to be optimal. Dr Pangloss would be delighted with this instance of his 'best of all possible worlds'.

8.5 Other Considerations in Dividend Policy

8.5.1 Dividends and Signalling

One of the empirical findings about company dividends is that these cash payouts seem to be more stable in monetary terms across time than any particular residual or clientele hypothesis for dividend policy can explain. Company financial managers seem to be loath to pay a dividend unless they think it can be sustained for some period of time by the expected cash flows of the firm. This seems to be true even when the company would be retaining cash beyond its current need for investment funds. And in other instances, companies simultaneously pay a cash dividend and sell new shares, an obviously expensive combination.

How can this be explained? One obvious explanation is that company financial managers do not understand the arguments about company dividend policy that we have made up to this point in the text. In a competitive market for financial managers, however, that would not be a very convincing argument. There must be something else afoot.

One explanation for the 'smoothing' across time of company dividends that seems more reasonable is the **signalling** value of dividends. The argument for 'signalling' goes as follows. Real financial markets have frictions not only in the form of transaction costs, brokerage fees and taxes, but also in the free flow of information. Companies find themselves operating under various constraints in informing shareholders about the future prospects of the company. And it is in the interest of shareholders that shares may be bought and sold at prices that fully reflect this information. These constraints take the form of restrictions on public forecasts of cash flows by managers – by the accounting profession through its 'generally accepted accounting principles', through government regulatory provisions in company security issuances, through the fear of litigation being brought by disappointed share purchasers should the forecasts prove incorrect, or through the fear that competitors will receive valuable information about the company's plans, thus reducing the value of those plans.

Yet it is obviously in the interests of managers and shareholders to have share prices reflect new information as quickly as possible. (This is true even when it is bad news, because you are likely to fool the market only once or twice by hiding or delaying bad news, and thereafter would have a difficult time getting good news believed.)

How then might shareholders be informed of events that cannot be explicitly broadcast? This can take place through subtle signals that the company gives – by alterations, for example, in its dividend payment. There is evidence, though by no means conclusive, that companies do just that when changing dividends.

It is now easier to see why dividends must be reasonably smooth over time. If, for example, a company pursued a policy of paying as much cash dividend as possible (a 'widows and orphans' policy), the time pattern of dividends would be driven by the occurrence of 'residual' cash across time. For most companies this is likely to be a rather random process. And it is difficult to the point of impossibility to signal information by changing dividends if the base pattern from which the signal is to be interpreted is random. The same would hold true for any other clientele-based policy.

So one reason for the smoothing of dividends across time is that the dividend announcement can be made to be a 'surprise' (either good or bad) to the market. Based upon the past pattern of dividends (either with respect to time or relative to other measures such as earnings), the market will have developed an expectation for the dividend to be announced. If the announcement is higher than expected, the news is good; if lower, bad.

The phenomenon of signalling can also explain certain financial market actions that have no good reason to exist otherwise. Consider, for example, the transaction called a 'share dividend'. Here a company declares a dividend, but does not pay cash to its shareholders. Instead, each outstanding share is paid an additional partial share of the company. For example, if you hold one share of Complex Corporation, and the company declares a 10 per cent share dividend, you will now own 1.1 shares of the company's equity. That sounds fine, until you remember that all shares of Complex received the same dividend. Therefore the transaction is completely neutral with respect to shareholdings. The company has more shares outstanding, but each shareholder has exactly the same proportional claim on the company's cash flows as they did before. A moment's thought will produce the inescapable conclusion that there is no obvious reason for companies to do this (especially since there are transaction costs involved).

Now, however, we can offer a reason for share dividends to exist. They can be used as a signal, just like cash dividends. There is convincing evidence that companies use these transactions (including 'share splitting' wherein a 'two-for-one' split is the same as a 100 per cent share dividend) to signal to shareholders, and that shareholders pay attention to – and act upon – these signals.

8.5.2 Dividends and Share Repurchase

On occasion a company may repurchase its shares on the open market. When that is done, the company will also usually announce that the transaction is being undertaken so as to have shares for various uses (merger and acquisition purposes, employee stock option exercise, and so forth); very often the company will announce that it considers its shares 'underpriced' and thus a good investment, and is therefore investing in itself.

The first set of reasons is a bit suspicious, because there is seldom a reason why the company could not simply issue new shares to employees and merger candidates instead of going to the trouble to repurchase. The latter reason (a company 'investing in itself') is rubbish, and the company knows it. No one is really fooled by these pronouncements accompanying share repurchases by companies. Why then are such transactions undertaken? And why are these announcements made?

The truth is that share repurchases are nothing more than a cash dividend to shareholders. If all shareholders sell back to the company the same proportion of their holdings, it is easy to see that the net effect is to shift cash from the company to shareholders, leaving undisturbed the proportional claim of each shareholder.

Even if all shareholders do not sell their shares to the company in the same proportions, you can see by our earlier discussion about opposite personal portfolio transactions that shareholders can end up having the same proportional claims after the repurchase if they so choose. This would be accomplished simply by buying or selling shares among themselves on the open market, using the money that the company distributed.

What of the claim of 'investing in itself'? A company can no more invest in itself by repurchasing its shares than a snake can successfully nourish itself by chewing upon its tail. Eventually truth will out. There will always be a 100 per cent equity claim outstanding as long as there is one share not repurchased, and since that single share would claim the entire firm, it would take the entire equity value of the company to repurchase it. This exposes share repurchase for what it truly is: a payment of cash dividend (or, in the extreme, a liquidation of the company).

Does this mean that company share repurchases are to be scorned and avoided as duplicitous? Not at all. Remember, the market is never fooled by such transparent statements. Why then are such actions taken? One reason is that in some countries the money received by shareholders in share repurchase transactions is taxed more lightly (or even not at all) in comparison with cash dividends declared by the company. Obviously it is bad public relations for a company to announce that by share repurchase it is seeking to help its shareholders avoid paying their income taxes on dividends. So the announcement of 'investing in itself' is made. Share repurchases on the open market also show some signs of being signalling attempts that receive a positive response from shareholders.

There is one type of share repurchase that is not so positive for shareholders, however. In some countries a company can undertake a 'targeted' share repurchase. This is a transaction wherein a company offers to repurchase only particular shares (usually held by an individual or group which the company's managers are frightened will take over the company and make things less pleasant for existing management). The repurchase price is usually at a significant premium above the existing market price of the company's shares. And the shares of the company on the open market usually decline in price even more than would have been expected by the loss of the cash premium paid.

Evidently the market thinks that targeted share repurchases are bad news for shareholders, in that existing managers will now be left to make decisions about the company without the implied oversight of the external 'market for managerial talent' evidenced by the now bought-out shareholdings.

Learning Summary

The discussion of dividend policy in this module has run the full gamut from a clear, simply analysed decision in perfect financial markets to a very complex process with many different alternatives and tactics available to the company in its wish to maximise shareholder wealth. We have seen that in 'frictionless' financial markets (without taxes and transaction costs, and with the free flow of information) dividends make no difference. We have also seen that no markets are really so frictionless, and that there is at least the possibility that shareholders may prefer one dividend policy to another because of real frictions that are experienced.

Companies, however, in seeking to maximise shareholder wealth can be expected in aggregate to have offered the market the mix of dividend policies that 'clears the market' of potential share premiums for any particular dividend policy, even in the presence of dividend clienteles. That leaves us with the notion that dividends are not really irrelevant, but that marginal gains from switching dividend policies are unlikely, and may even be costly to shareholders.

Finally, we saw that dividends may be used as signals by managers to shareholders for information that would be expensive to disseminate otherwise. That may cause companies to 'smooth' their dividends across time more than a basic wish to serve a particular clientele would suggest. Such signals could also be accomplished by many other techniques, such as share dividends instead of cash dividends.

Though these are the main ideas about company dividend policy, they are not exhaustive of all the implications of company dividends for shareholder wealth. For example, one effect on the dividend a company declares might be that the debt the company has issued has a legal restriction on the amount of dividend that a company is allowed to pay. (And companies may pay less than the maximum allowable dividend in some years so as to build up a reservoir of allowable dividends should they face a time of plentiful residual cash and few positive NPV investments.)

There are those who think that dividends also play a role in the problem of conflicts of interest between shareholders, managers and creditors. These are called **agency considerations**. The payment of cash dividends can be regarded as a shifting of control of these assets from managers to shareholders, the latter then having the option of whether or not to allow managers to regain operating control over such assets. This may constrain managers to behave more in the interests of shareholders.

Dividends can be used to shift assets out of the company and therefore from the potential claim of creditors. Dividend payments can also change the overall riskiness of the company's asset base. Both of these can be detrimental to creditor wealth, and creditors will doubtless take pricing or contractual actions to offset these potential uses of dividends.

Company dividend decisions are thus not the simple process of deciding how much cash is left after all commitments and plans have been executed, and paying that amount to shareholders. The considerations of signalling, agency and the effects of market imperfections upon optimal dividends are important dimensions about which financial managers must be aware.

Review Questions

8.1 Suppose that ABT plc is a company in a financially frictionless, taxless, information-efficient world. ABT's next dividend is a propsed cash dividend of £10 per share, and the market expects this dividend to continue for ever, increasing at an annual rate of 5 per cent. ABT's equity discount rate is 15 per cent per annum. Using the perpetuity-growth valuation technique:

$$\text{Price/share} = \frac{\text{Dividend}_1}{re - g} = \frac{£10}{0.15 - 0.05} = £100$$

ABT's shares thus now sell for £100 apiece. ABT is considering an alteration in its dividend payout from the existing 30 per cent of available cash to 60 per cent of that amount, a doubling of the cash payment, to begin at the next dividend payment date. Any shortfall of cash retention will be made up with new share issuances. Which of the following will be the effect upon ABT's share prices of the dividend change?

A. The shares will double in value, because the share price is a function of the expected future dividends, which are now expected to be twice as much.
B. The shares will have the same price, because the increase in annual dividend for the current shareholders will be exactly offset by a decrease in the rate of growth of their dividend.
C. The shares will decline in value because the existing shareholders are effectively liquidating the company by taking such a large dividend.
D. The shares will have the same price, because shareholders will find half of the dividend now taxed away.

8.2 You are a fledgling financial manager in a company operating in a taxless, frictionless, information-efficient world. Your boss comes into your office and says, 'What's all this I hear about dividends being irrelevant? I remember very clearly from my courses in finance that dividends are the very basis for share value! Does that imply that a company's share value is irrelevant? Explain things to me.'

Which of the following should be your response?

A. Explain that dividends themselves are not at all irrelevant, and are in fact the basis for share prices, which are themselves representative of shareholder wealth, but dividend policy irrelevancy is a useful pedagogical device that finance texts employ to illustrate how real companies begin thinking about the dividend decision.
B. Explain that dividends themselves are not at all irrelevant, and are in fact the basis for share prices, which are themselves representative of shareholder wealth, but dividend policy irrelevancy can happen even with complex taxation and preferences for consuming wealth by shareholders as long as there are many companies competing in offering dividend policies to the market. This is because the forces of demand and supply for particular dividend policies would cause their 'prices' (or benefits for switching from one policy to another) to be equal.

C. Explain that dividends themselves are not at all irrelevant, and are in fact the basis for share prices, which are themselves representative of shareholder wealth, but dividend policy is irrelevant because shareholders do not care how they receive their wealth, either in cash or in capital gains, and the dividend decision merely allocates a constant amount of wealth differentially between those two forms.
D. Explain that dividends themselves are not at all irrelevant, and are in fact the basis for share prices, which are themselves representative of shareholder wealth. Further explain that modern finance theory is not that dividend policy is irrelevant. Irrelevancy only occurs when all real-market frictions such as taxes and transactions costs are assumed away.

8.3 Cellular Telesystems plc pays, on average, a high percentage of its income as dividends to its shareholders. This dividend is not a constant percentage of earnings but tends to be reasonably stable in value, increasing in absolute cash amount over time. Cellular, being a high-technology growth company, has large needs for investment outlays over time, and thus often goes to the capital market to raise money for investment purposes.

Which of the following best describes Cellular's dividend policy?

A. A very poor one, because it is engaging in unnecessary transaction costs of raising new equity capital across time. If the dividend payout were lowered, less outside cash would be necessary from new shareholders and the attendant costs would be reduced, to the benefit of existing shareholders.
B. Excellent, because the company is maximising its share value by paying as high a cash dividend as it can, because dividends are the basis for share value.
C. Good, because the shareholders of the company are 'fat cats' in high tax brackets, and thus prefer the policy that the company has chosen.
D. Reasonable, because the stable pattern of dividends allows the company to 'signal' events to the market, and the split between capital gains and cash dividends has likely attracted a shareholder group that prefers it.

8.4 A company finds itself with cash from operations in excess of its profitable investment needs, and well in excess of its dividend payment in the prior period. It must decide what to do with the money. Which of the following would be a reasonable choice?

A. Declare an 'extra' dividend to shareholders, with the understanding that the cash amount is not to be taken as a 'good news' signal about future earnings prospects.
B. Declare a cash dividend for the excess amount of cash along with the usual dividend.
C. Retain the cash and invest it in government securities.
D. Retire (i.e. pay off) some debt.

Module 9

Company Capital Structure

Contents

Learning Objectives

Companies must decide how they are to be financed, and in particular what mix of debt and equity claims are appropriate for the firm to issue. Module 9 develops the set of ideas and techniques by which companies can decide which mix offers the greatest promise of enhancing shareholder wealth. First the module shows how debt in a company's capital renders its equity claims more risky. The 'EBIT-EPS' chart illustrates this risk dimension of gearing or leverage (the existence of debt). As with dividend policy, the module then shows an idealised set of conditions under which shareholders would be indifferent about capital structure, followed by an explanation of the effects of real market phenomena such as taxes, competing tax benefits and bankruptcy costs on optimal capital structures of firms. The module concludes with some specific advice to companies as to how capital structure decisions can be made. You will learn from the module not only valuable techniques for deciding how much debt and equity a firm should issue but also some important financial economics of such decisions, which can readily be extended to making decisions in less familiar situations.

9.1 Introduction

Suppose that you are the financial manager of a company. While about your tasks one day, a proposal to undertake a new investment project comes across your desk. The proposal has been thoroughly evaluated by your junior staff and clearly has a positive NPV (or IRR exceeding its opportunity cost), and so it should be accepted by the company. After examining the company's financial resources, however, it is also clear to you that the firm does not have enough money in the bank (or in other liquid assets) to pay the costs of the project, nor are operating revenues large enough to cover the investment outlay.

Obviously, the company will be forced to raise new capital so as to be able to undertake the project. At this point the major question to be addressed in this module makes its appearance: should the money be borrowed, or should it be raised from shareholders?

This situation and the question it presents is one way of thinking about the **capital structure decision** companies face. A company's capital structure is the extent to which it is financed with each of its capital sources (debt and equity). Actually, a company's capital structure is the entire mix of financing it uses, which may include types other than pure debt and equity (such as preference shares, warrants and so forth), but for the purposes of this discussion the above definition will serve well.

In this module we shall explore the company capital structure decision. As with the financial decisions we have examined in prior modules, our concern will primarily be the decision's potential effects upon shareholder wealth, here due to the choice of financing type: debt or equity.

9.2 Capital Structure, Risk and Capital Costs

How can a company's capital structure decision affect shareholder wealth? There are several correct answers to that question; some of the answers are easy to understand, and others are more subtle. There are also seemingly plausible – but incorrect – answers to that question, which are particularly insidious. Dispensing first with these can actually teach us some valuable lessons about capital structure.

Consider the following argument: 'Debt is more expensive than equity in a company's capital structure because debt carries required interest payments whereas equity does not.' This argument is quite easy to dismiss, because it ignores the fact that dividend payments (and capital gains, the allocation between which depends upon dividend policy) to shareholders are legitimate capital costs of the company. The thought that a company could raise money from shareholders without compensating them in terms of their required returns is ridiculous. So equity carries the equivalent of interest costs in the form of expected dividends and capital gains. Debt is in no sense more expensive simply because interest payments are contractual while dividends are not.

Consider the next argument: 'So equity's required returns cannot be ignored in the company's capital structure decision. Since equity's residual claim on the company means that equity's risk is higher than debt's, equity's returns (via the SML) must also be higher. Therefore equity must always be a **more** costly capital source than debt. So the capital structure decision is easy: it will always be cheaper to borrow than to raise equity capital because the required returns to debt suppliers are always lower than the required returns to shareholders.' This argument, though incorrect, is difficult to dismiss as easily as the first, because it does seem to be plausible at initial hearing. The error is that the argument ignores important interactions within the set of a company's required capital returns. An illustration of these interactions will be instructive of the fallacy of the argument, and will also advance our understanding of company capital structure decisions.

9.2.1 Capital Structure and Risk – an Illustration

It is wrong to think that debt is cheaper than equity because interest rates are lower than equity required returns. To understand why that is, we must first study why the required rates to these two types of capital are ordered the way they are.

Recall our discussion in Module 2 about the nature of the equity claim on companies. We said there that shareholders' claims are **residual** to all other claims upon the company. They are 'last in the queue'. That being the case, it must always be true that for any company, its debt claims require lower returns than its equity, because equity, being residual, must bear greater risk than debt. Since debt has a claim prior to equity, whatever risk is inherent in the operational cash flows of a company will be shared unequally by debt and equity. Debt will have less risk (and thus a lower required rate), and equity will have more (and thus a higher required rate).

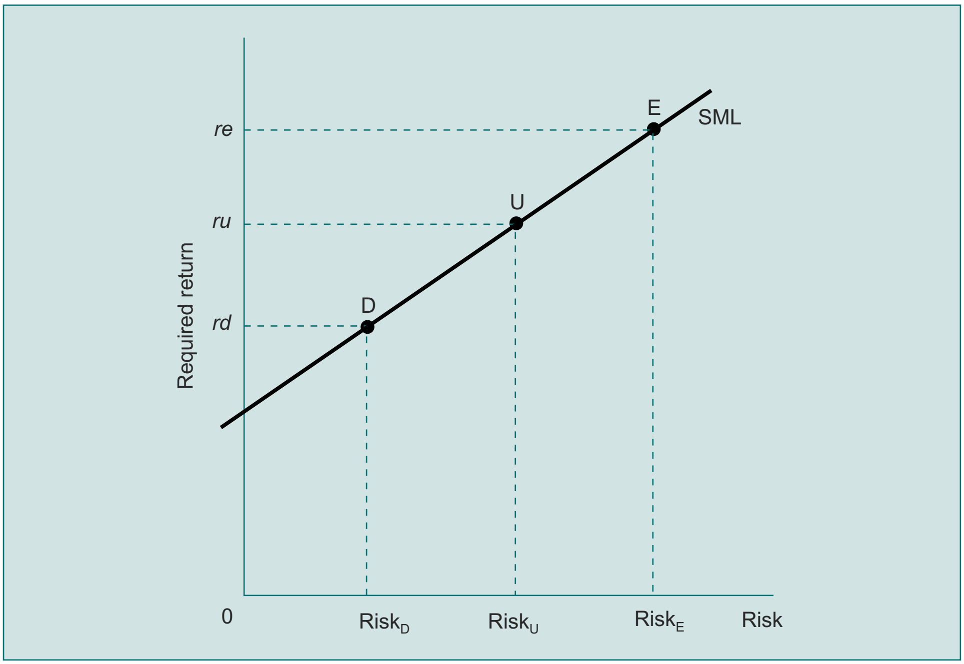

Figure 9.1 **The security market line and required returns to capital claims**

Figure 9.1 illustrates this point with the help of the SML. Here you see three points plotted on the SML. Point U is the amount of risk and required return inherent in the ongoing operations of the firm itself, independent of its capital structure. (The letter 'U' appears because it signifies the amount of risk that would exist for the shareholders of the company if the firm were 'ungeared', or had no debt in its capital structure. Here the only risk borne by shareholders is from the operations of the company.) Points D and E in Figure 9.1 indicate the amounts of risk and return that exist for the company's capital claims if the firm is partially financed with debt (D) as well as equity (E). Note that, because of debt's higher position in the hierarchy of capital claims, it gets 'first chance' at the company's operating cash flows. And because the company is only partially financed with debt, debt does not bear the full operating risk β_u of the company. Equity, on the other hand, now has a residual claim, with debt taking precedence. So the equity of a company that uses debt as part of its financing must be riskier than ungeared equity. Thus point E, the geared equity risk–return location, must be higher up the SML than point U and require a higher return.

The rationale for the questioner's assertion that debt's return is always lower than equity now becomes clear. What is not yet clear is why we can counter-assert that it is not correct to think that debt financing is therefore 'cheaper' than equity.

Perhaps the simplest way to understand why debt is not cheaper is to return to Figure 9.1 with a slightly different perspective. Suppose that there is an ungeared company with the risk–return location U. The corresponding value of *ru* will be its required return, an ungeared equity return. Suppose further that there is another company, identical in operations, but with some debt in its capital structure, as represented by points D and E in Figure 9.1. If the questioner above were correct, we would see the average capital costs of the geared firm being **lower** than those of the ungeared firm. Do we see that in Figure 9.1? It is not clear that we do.

Examine the relative risk and return locations of U, D and E. It is true that D, debt's risk and return, is less than U. This is consistent with the questioner's point about debt being less costly than equity. But note the relative locations of U and E. The existence of the debt's prior claim has **increased** the risk (and thus the required return) to equity. Geared equity (E) is **riskier** than ungeared equity (U). Thus debt is 'cheaper' in the sense of carrying an interest rate or required rate of return less than equity, but the very existence of debt's prior claim serves to increase the risk of equity to be higher than it would be without debt. This means that debt is not in the fullest sense necessarily a 'cheap' capital source when the interactions between debt and equity financing are considered.

A more careful examination of the economics of company capital structure as evidenced by Figure 9.1 shows that it is entirely possible that whatever gains are had from issuing low-interest debt might well be offset by increases in the risk and attendant returns required by shareholders. (Another perspective on this same question is to ask yourself which company, the geared or the ungeared, has the lower WACC. You should be able to tell from Figure 9.1 that it is not at all clear which is lower, if the 'average' required rate is used to judge the WACC.)

9.2.2 Capital Structure and Risk – an 'EBIT-EPS' Example

A numerical example of the relationship between gearing and risk should be useful at this point. Suppose Ungeared plc is expected to make £120 000 (perpetual) per annum earnings before interest and taxes (for obvious reasons this is often called EBIT, or 'operating income'). We shall assume for simplicity that this is also a cash number so as not to be troubled by the tricks that accountants do with numbers.

The important thing to remember about EBIT is that it is a measure of the total amount of money available to service both debt and equity (of course taxes must be subtracted, but again, for simplicity, let us assume taxes away).

Ungeared plc is financed totally by an equity claim that has a market value of £1 000 000. So the company's shareholders expect a perpetual £120 000 per annum and value that at £1 000 000.

Geared plc is a firm identical to Ungeared except that Geared uses some borrowing in its capital structure. Geared has borrowed £500 000 and has agreed to pay interest of £40 000 per annum on the loan. Thus Geared bondholders expect to get £40 000 per annum and its shareholders look forward to a perpetual £80 000 per annum, based upon the operating EBIT of £120 000 with a £40 000 prior claim due to the lenders.

Geared shareholders obviously have a smaller investment in their company than do Ungeared shareholders. On a value basis, Geared is using 50 per cent debt, so let us assume that it has only half the number of shares outstanding that Ungeared has. (That would make the value **per share** the same for each company.) Specifically, let us assume that Ungeared has 10 000 shares outstanding worth £1 000 000 (or £100 per share), whereas Geared has 5000 shares worth £500 000 (thus also worth £100 per share).

To see how the existence of debt in a company's capital structure causes its equity to be riskier, we shall now illustrate that Geared's equity is riskier than Ungeared's. This will be sufficient to prove our point because the companies are identical in all ways save capital structure.

In Module 7 we showed that risk is best measured by the β coefficient, or the extent to which a capital claim's returns vary with the market. The β coefficient, you recall, can be thought of as the standard deviation of a security's returns multiplied by its usual relationship with the market. Other things held the same (as they are in this example), we have that the higher the variability of a security's returns, the higher its risk. So we can test whether Ungeared's or Geared's equity is the riskier by seeing which has the higher variability of returns.

But the only information we have about the returns of the two companies' shares is the expected returns. How about the potential variability of their returns? To answer this, remember that companies' operating cash flows (or EBIT in this case) are not known for certain. We have said that each here expects EBIT of £120 000, but that is merely an expectation. Let us further specify the probability distribution of EBITs by assuming that for both companies it is as in Table 9.1.

Table 9.1 Spread of EBITs for Ungeared and Geared

Outcome	Probability
£15 000	0.10
£120 000	0.55
£150 000	0.35

Here we have been more specific about the EBITs that might actually occur each year for the companies, and the likelihoods of those occurrences. Note that the expectation or mean of the probability distribution is in fact the £120 000 that we have earlier portrayed. (Multiplying each outcome by its probability of occurrence and adding yields £120 000.)

With this new information about their operating or EBIT risks, let us now turn our attention to the risks of the two companies' equity claims. Ungeared plc has only equity outstanding, so the entire EBIT goes to shareholders. (Again, for simplicity we assume away complexities of depreciation, retention, investments, and so forth.) Therefore the risk of Ungeared plc equity is exactly the same as its EBIT, given above.

To be able to make reasonable comparisons between the two companies, however, we must somehow recognise that the shareholders of Ungeared plc have invested a different amount (£1 000 000) than those of Geared plc (£500 000). The easiest way to adjust for this difference is to compare the returns and risks on a 'per share' basis (remembering that the shares are selling for the same £100 price per share).

Thus for Ungeared, we can find the distribution of returns to shareholders on a per-share basis by dividing the outcomes by the number of shares outstanding (10 000). Table 9.2 does that, and shows the probability distribution of returns to shareholders of Ungeared on a per-share basis.

Table 9.2 **Ungeared plc earning per share**

EBIT outcome	EPS outcome	Probability
£15 000	£1.50	0.10
£120 000	£12.00	0.55
£150 000	£15.00	0.35

Geared plc in contrast has debt in its capital structure so as to require only 5000 equity shares of financing. On a per-share basis, the risk to Geared's shareholders comprises both the company's operating (EBIT) risk and also the risk inherent in the existence of higher-priority (often called 'senior') interest payments to the debt capital. Table 9.3 shows the probability distribution of returns for Geared's equity and debt.

Table 9.3 **Geared plc earnings per share**

EBIT outcome	Interest outcome	Equity outcome	EPS outcome	Probability
£15 000	£40 000	−£25 000	−£5.00	0.10
£120 000	£40 000	£80 000	£16.00	0.55
£150 000	£40 000	£110 000	£22.00	0.35

The lesson about the risk of equity in the presence of company borrowing is easily seen by comparing the EPS outcomes and probabilities of Table 9.2 and Table 9.3. Table 9.2 shows what happens to the per-share income of equityholders in Ungeared plc when operating income (EBIT) varies from £15 000 to £150 000. Note that the range of this variability is from £1.50 to £15.00 per share. On the other hand, the shareholders of Geared plc experience a range of per-share incomes of −£5.00 to £22.00 when the company's operating income has the same variability as Ungeared's. The reason that Geared shareholder returns are more risky is obvious from Table 9.3: the £40 000 interest payments stand 'first in the queue' to receive any cash forthcoming from the operation so geared. The variability of Geared's operating income is thus amplified (or 'leveraged') upwards to shareholders by the existence of company borrowing.

Comparison of Table 9.2 and Table 9.3 indicate that the range of possible returns to shareholders is much greater (with the same probabilities) for Geared than for Ungeared. Geared's equity must therefore be more risky. Geared shareholders are obviously in the more risky situation, yet the only difference between the two companies is that Geared plc has to some extent used debt in its capital structure, whereas Ungeared plc is ungeared.[1]

Figure 9.2 uses the operating income and shareholder return information in Table 9.2 and Table 9.3 and plots these in a graph. Note that operating income (EBIT) and shareholder return exhibit a much more volatile relationship for Geared than for Ungeared. A given change in operating income for Geared produces a much greater change in shareholder result than for Ungeared. (Incidentally, this graph has a name in finance. It, predictably, is called an **EBIT-EPS chart**. Such displays are used to illustrate graphically the risk to shareholders inherent in using particular amounts of borrowing in a company's capital structure. And the terms '**gearing**' or '**leverage**' are verbal descriptions of the differences in steepness of the Geared and Ungeared lines in EBIT-EPS graphs.)

[1] A standard deviation calculation would show this, but is superfluous.

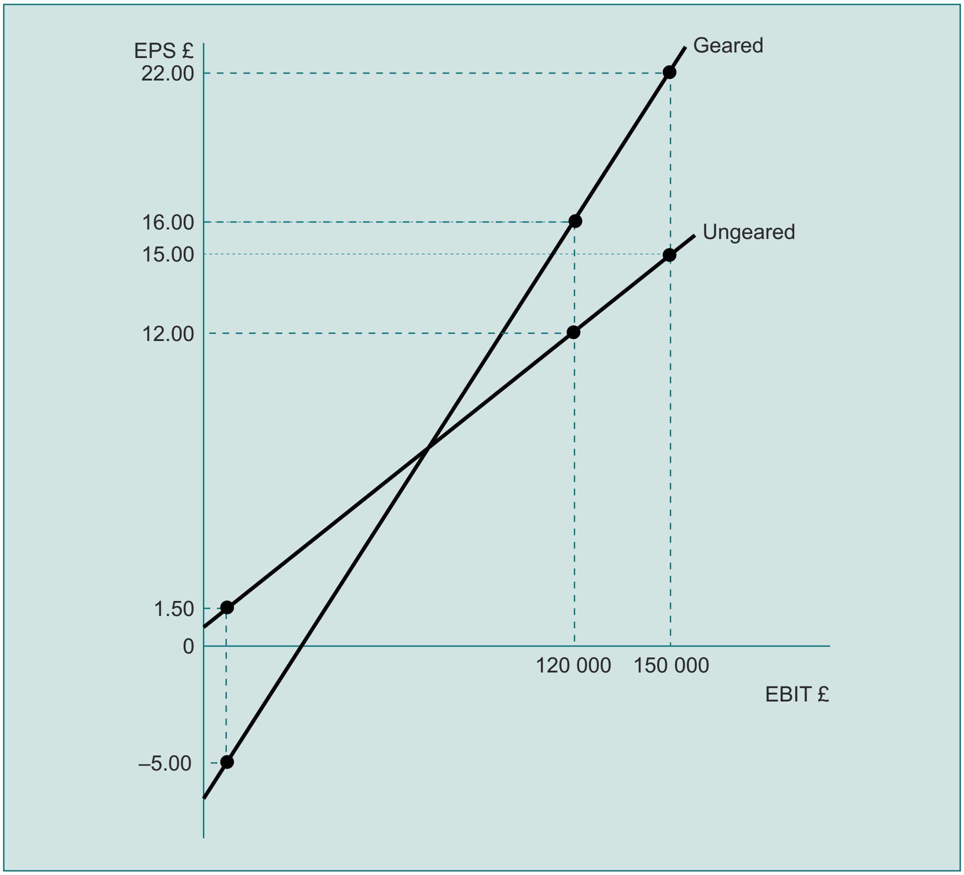

Figure 9.2 **An EBIT-EPS chart for Geared plc and Ungeared plc**

It should also be clear that the more borrowing a company does, the steeper will be the line depicting the EBIT-EPS relationship, and, also, the higher the equity risk. This type of risk is often called **financial risk** to distinguish it from the underlying 'line of business' or operating risk (also known as 'operating leverage') that resides in EBIT.

The above discussion should be sufficient to convince you that company borrowing causes the returns to shareholders to be more risky than it would otherwise be. One or two other issues may, however, have caught your attention in that discussion, and we will now move on to deal with them.

First, you may have been curious how Geared could generate a negative EPS (−£5.00) when the £15 000 operating income outcome results. The arithmetic is, of course, easy: £15 000 of income minus £40 000 of interest is a negative £25 000, which divided by the company's 5000 shares yields −£5.00 per share. The bothersome aspect of the result is the seeming contradiction with shareholders' limited liability. In other words, one way in which the negative EPS would have been produced would be for the company to raise the £25 000 from shareholders so as to pay interest to debt. And shareholders under limited liability are under no legal obligation to make good on company debts. (Another way to raise the money would be for Geared to sell some assets.)

Only if such cash is paid to creditors will the £40 000 interest be legitimately included in the company results, and shareholders need not pay it. The question is: would they? The answer is that they probably would, given that unless they do, creditors can foreclose on the debt and take over the assets (and future expectations) of the company. A glance at the probability distribution of operating results and EPS for Geared plc would indicate that the odds are that shareholders will do much better on the next period's 'roll of the dice', and therefore would be wise to pay the interest even though they need not.

Of course a company can default on an interest payment whenever it chooses, not just when operating income is less than the interest due. One can thus in general regard the payment of interest by companies as essentially a transaction wherein shareholders repurchase the assets of the firm from creditors, the price being the interest payment. The counterpart to this view of the transaction is that when companies originally borrow, they sell their assets to creditors but withhold an option to repurchase the assets by making interest payments. Keep this thought somewhere in the back of your mind as we continue to discuss company capital structure decisions. It will serve us well in discussions to come.

Finally, notice that there is a nice consistency in the required returns of the various capital claims of Ungeared and Geared. Turn your attention to Figure 9.3, remembering that the two companies are operationally identical. Ungeared, with only equity claims outstanding, is worth £1 000 000. That £1 000 000 is generated by a perpetuity expectation of £120 000 per annum, implying a required return re of 12 per cent. This is plotted as point U in Figure 9.3.

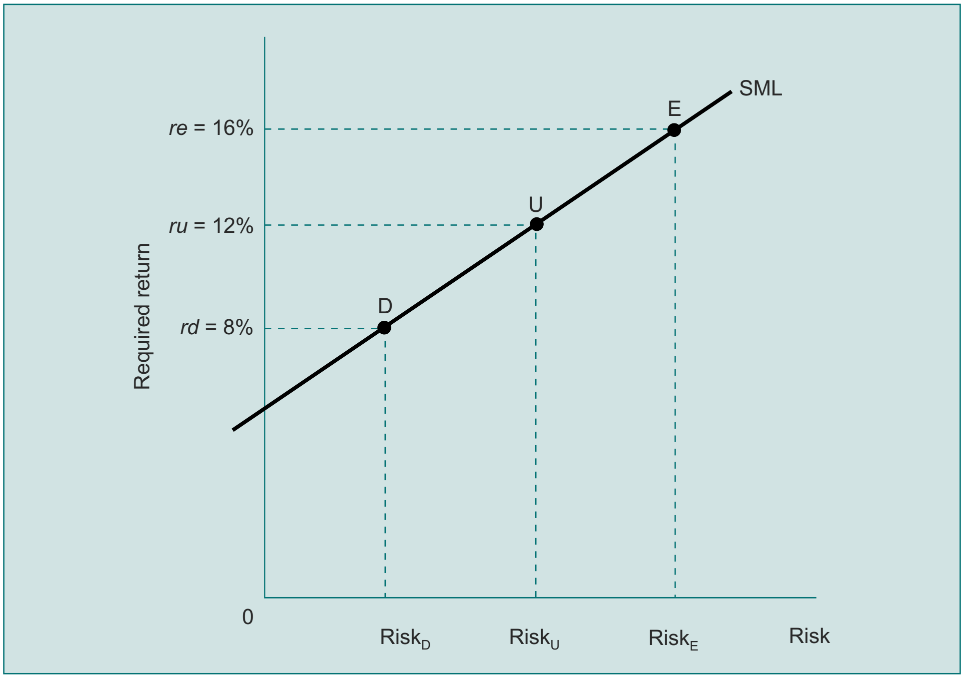

Figure 9.3 **The security market line and required returns**

Geared has two claims, debt and equity outstanding. Its debt requires an 8 per cent return (£40 000 of interest on £500 000), while equity requires a 16 per cent return (£80 000 of dividend on £500 000). These plot as points D and E, respectively, in Figure 9.3. The reason why Geared plc's equity needs a higher return is the company debt, which has first claim on the operating cash flow. (And the reason why the return on Geared's debt is lower than that on Ungeared's equity return is that the debt not only is the first claim but also requires only £40 000 of the £120 000 expected operating income, a lower-risk prospect than even Ungeared's equity.)

Our original reason for beginning this discussion of capital structure and risk, you should recall, was to show why debt is not necessarily a cheaper capital source than equity even though debt's required return is lower than ungeared equity return. That reason, we are now doubly aware, is that the taking-on of debt causes equity's risk to increase, and therefore increases geared equity's required return. This increase in general equity risk and return is often called the '**implicit cost**' of borrowing. With increased equity required returns, the lower cost of debt is in danger of being offset by the higher cost of equity, leaving no benefit. (As a matter of fact, in the example we have just studied, there was indeed no benefit.)

To see the truth of this last statement, recall that Ungeared has an average capital cost of 12 per cent, based upon its ungeared equity claim. Geared, on the other hand, has an 8 per cent debt rate and a 16 per cent geared equity rate. Since Geared's claims each comprise half of the value of the company, their average is $(0.5 \times 8\%) + (0.5 \times 16\%) = 12\%$. Thus Geared has the same average capital cost as Ungeared, and there is no benefit (nor detriment) to company borrowing. The increase in the (generalised) geared shareholder required return exactly offsets the lower cost of borrowing.

As you would expect, this numerical result was designed into the arithmetic of the example. But there **is** an important set of economic arguments that produces the same result, not by artificial construction but by financial market actions. There are still other important capital structure arguments that yield quite different predictions as to what happens to company capital costs when debt is issued. We now have enough of an introduction to such a company financial decision as to look at these debates and learn what is known about the best way to decide how much a company should borrow.

9.3 Capital Structure Irrelevance I: M&M

Capital structure as a concern of finance asks the question: 'Does borrowing as opposed to issuing equity capital affect the wealth of shareholders?' We have seen in the previous section that another way to ask the same question is to examine what happens to the company's average capital cost when debt is substituted for equity in a company's capital structure. We found that debt is not in itself cheaper just because it carries an interest rate lower than the required return of the equity it replaces (because it raises the risk and return of the remaining equity).

But we have not resolved the initial question for, although the example of Geared plc and Ungeared plc was designed to produce no gain to shareholders from company gearing, there is no reason at this point to regard that effect as a general phenomenon. Actually, little light was shed on this issue until some early work by John Burr Williams[2] was expanded by Franco Modigliani and Merton Miller.[3] These finance theorists offered an economic argument about company capital structure decisions that has proved very useful. Like the early dividend irrelevancy arguments we studied in Module 8, this one is most valuable for the general principles it elucidates rather than the managerial instruction it gives. But these general principles underlie much of subsequent practical capital structure ideas, so they must be of primary concern to us now.

Let us return to the world of Geared plc and Ungeared plc in the previous section. The companies in that example will serve us well in investigating the arguments of 'M&M', as Miller and Modigliani have come to be known. First, we shall tell you the final result of this analysis, so you know where we are heading. The M&M economics of company capital structure predict that for companies like Geared and Ungeared, it would make no difference to shareholder wealth whether the companies borrowed money or not. Financial markets would ensure that this is the final result. Let us now see how M&M arrive at this conclusion.

Remember that for purposes of simplicity we have assumed away taxes in this example, and let us also specify that there is a well-functioning capital market that

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2 Williams, J.B. (1938) *The Theory of Investment Value*, Harvard University Press, Cambridge, Mass.

3 Modigliani, F. and Miller, M. (1958) 'The Cost of Capital, Corporation Finance and the Theory of Investment', *American Economic Review*, 48 (June), 261–97.

does not discriminate among the various types of claims upon which it sets prices. This condition means simply that two claims promising the same probability distribution of future cash flow will sell for the same price.

Now suppose that there are many firms like Ungeared plc in the market and there is no company like Geared plc. Ungeared is thinking of becoming geared by borrowing some money and is interested in the effect of that capital structure alteration upon its shareholders. To become geared, Ungeared would borrow £500 000 at an interest rate of 8 per cent, and the market is prepared to lend money to the company at that rate. The question is thus: 'What will happen to the wealth of Ungeared shareholders if the company alters its capital structure in that fashion?'

The answer to this question will require that we discover what happens to Ungeared's equity value when it becomes geared. To see this, recall that prior to the borrowing, Ungeared equity is worth £1 000 000 (based upon claiming a risky £120 000 perpetuity at a 12 per cent required return). The £500 000 cash that the company borrows as it becomes geared must somehow be given to its shareholders (either through a dividend, share repurchase, or cancellation of a new equity flotation), or else there would also be a company asset alteration.

So the shareholders of Ungeared will immediately get £500 000 more cash, but have given up something in return. They have effectively sold to lenders a £40 000 prior claim on the future operating cash flows of Ungeared, receiving the £500 000 in compensation. Of course, the loss of first priority claim on £40 000 of future operating income in perpetuity must cause the equity value of Ungeared to fall as it becomes geared. The question is how much it will fall.

Table 9.4 illustrates the situation of shareholders. If equity value decreases less than £500 000, shareholders of Ungeared will gain wealth from becoming geared (at the same time renaming the company Geared plc). If equity value declines more than £500 000 shareholders will be worse off, and if it drops exactly £500 000 they will experience no wealth change as a function of the company's alteration of its capital structure.

Table 9.4 Ungeared plc to Geared plc: Shareholder wealth and capital structure

	Equity value	+	Cash	=	Wealth
Ungeared	£1 000 000	+	0	=	£1 000 000
Geared	?	+	£500 000	=	?+£500 000

What will be the equity value of the renamed company? Here is where M&M make their important contribution. They argue that the equity value of Ungeared must change to equal exactly £500 000 as Geared equity, and leave unchanged the wealth of shareholders. Their argument is as follows. Suppose the equity value of Ungeared changes to £600 000. That equity now will be claiming a future cash flow of the company's operating EBIT of £120 000 minus the £40 000 company interest payment, namely £80 000 per year in perpetuity. Ungeared shareholders will have experienced a wealth increase of £100 000 because they get £500 000 in cash from the debt issuance and retain £600 000 of equity value, totalling £1 100 000. Prior to the capital structure alteration their wealth was £1 000 000 of ungeared equity.

But is this likely to happen? Can Geared's equity value actually be £600 000? M&M would say no. Again, think of the situation of the Geared shareholders. They are holding shares worth £600 000 which entitle them to £80 000 in perpetuity coming from the company's operating cash flow of £120 000 with a £40 000 prior claim by lenders. That £600 000 of equity is not, however, a viable value in the capital market. Why not? Because the **identical** future cash-flow expectation can be achieved **in a different manner, and less expensively**. Let us now see how.

Suppose that Mr Wrench wishes to acquire an expectation of £80 000 in perpetuity, with the same risk characteristics as Geared's equity cash flow. This could be accomplished by simply buying the shares of Geared for £600 000, or by buying the shares of Ungeared but borrowing personally so as to arrive at the same £80 000 expectation (£120 000 from the company, with a £40 000 personal interest payment).

Suppose that Mr Wrench personally borrows £500 000, the same amount as Geared did, and puts up enough cash to purchase the shares of Ungeared. At what interest rate could he borrow? If the shares of Ungeared plc are offered as collateral, Mr Wrench would be able to borrow at the same 8 per cent interest rate that the renamed Geared plc does. This is because the company and personal loans would be identical. Geared borrows by promising first claim on its future operating cash flows, and Mr Wrench will be borrowing to purchase Ungeared by offering identical (Ungeared operating cash flow) collateral.

After this transaction, Mr Wrench now finds himself owning all of the shares of Ungeared, which entitles him to an operating cash-flow expectation of £120 000 per annum. He has personally borrowed £500 000, offering only the shares of Ungeared as collateral, and has acquired an 8 per cent interest rate in the non-discriminatory capital market. That creates a £40 000 prior claim upon his personal cash-flow expectation from the Ungeared shares. So Mr Wrench now has an £80 000 net personal cash-flow expectation arising from a £120 000 risky Ungeared operating cash flow with a £40 000 prior claim to lenders. Table 9.5 illustrates this comparison.

Table 9.5 Shareholder cash-flow expectations: Personal and company borrowing

	Company borrowing (Geared shareholders)	Personal borrowing (Mr Wrench borrowing and buying Ungeared)
Company operating cash flow	£120 000	£120 000
Company interest payment	£40 000	0
Net cash from company	£80 000	£120 000
Personal interest	0	£40 000
Net personal cash flow	£80 000	£80 000

Note that in terms of future cash flows, having borrowed personally, Mr Wrench has the same expectations and risks as do the shareholders of Geared plc. So in terms of their personal portfolio of holdings, they have identical future cash-flow expectations. Note also, however, that the portfolio of personal borrowing and Ungeared shares has cost Mr Wrench only £500 000 out of his pocket (the £1 000 000 Ungeared share value minus the £500 000 personal borrowing), whereas the Geared shares are valued in the market at £600 000.

This is an untenable situation. The Geared shareholders would quickly sell their shares and borrow personally to purchase Ungeared shares (getting the same future cash flows and netting themselves a nice £100 000 in the bargain). In fact, everyone would avoid the overpriced Geared shares. (Said another way, who would be silly enough to buy the shares of Geared for £600 000 when the identical future cash flow is available through Ungeared and personal borrowing for only £500 000? The answer is, of course, no one.) In an efficient market the shares of Geared **must** sell for just what it would otherwise cost to acquire the same future cash-flow expectations. The Geared shares must sell for £500 000 – no more and no less. (We could illustrate a similar mispricing if Geared shares sold for less than £500 000, based upon others willing to pay £500 000 for Geared's future cash flows, but the intuition that such an argument can be made is sufficient.)

What does this example of company and personal gearing tell us about company capital structure decisions? It says something that seems surprising at first: company capital structure decisions do not matter. When Ungeared became Geared, with Geared shares necessarily worth £500 000, shareholder wealth is unchanged. (Substitute this result in Table 9.4 to see the truth of the statement.) Because shareholder wealth will be the same regardless of company capital structure, the latter is irrelevant.

This statement must, however, be carefully interpreted. Company capital structure is irrelevant because it has no effect upon shareholder wealth in the market we have herein designed. Because shareholders can borrow and lend on the same basis as companies, any benefit (or detriment) residing in company borrowing can be duplicated (or cancelled) by shareholder borrowing or lending transactions in their own personal portfolios. So company capital structure does not matter.

Company capital structure in this illustration does affect the risk and return of equity. Recall that Ungeared shareholders' required returns increase when the company becomes Geared, recognising the increased risk of their holdings in the company. But the shareholders' wealth did not change (the Geared shareholders have riskier shares and riskless cash rather than the lower-risk shares of Ungeared). And if the now-Geared shareholders are not pleased with their £500 000 of cash and £500 000 of riskier shares, they need merely sell their Geared shares and use the resulting total £1 000 000 of cash to purchase all of the shares of a company such as Ungeared plc. Then the shareholders would be right back where they were before Ungeared became Geared. So company capital structure is completely irrelevant in the world of Geared and Ungeared.

Parenthetically, we also note that Geared's cost of capital is the same as Ungeared's (you can find the numerical demonstration at the end of the previous section), which makes the same point about irrelevancy but more succinctly.

One thing that often bothers those who hear this M&M argument for the first time is that it seems to depend upon very artificial assumptions, to the extent that the real market applicability of it is not clear. As we shall see very soon, there are indeed complications that must be addressed in real markets. But the basic economics of the M&M argument will remain intact and are very important.

There is one criticism of M&M that we should address directly at this point, however. Many, upon hearing this discussion, are disturbed that it seems to require several operating companies, all identical to our Ungeared plc, so as to work correctly. This is not at all the case. The assumption that there were many identical operating companies available to shareholders for their personal portfolio reallocations made the illustration simpler to understand, but is not necessary for its correct functioning. All that is necessary is that shareholders have other opportunities to acquire the same risks, **from whatever source**. And such risk-altering opportunities are widely available in real markets. For example, there are non-identical companies that have the same operating risks; or there are non-identical companies that can be bought with personal borrowing or lending to duplicate the ungeared operating risks of any other company. Or there are any of a number of other non-share investments (such as real estate, precious metals, and so forth) that shareholders could use so as to realign the risks of their portfolios and accomplish the M&M result. M&M were perfectly aware of this, but doubtless felt that pedagogical clarity required them to keep the illustration as simple as possible.

9.3.1 Arbitrage and Prices – a Digression

The illustration above is a very famous one in finance. It is famous not so much for what it teaches us about company capital structure decisions (though that is by no means trivial) but more for the market pricing mechanism it utilises. This mechanism is called '**arbitrage**.'

'Arbitrage' is a transaction wherein an instantaneous risk-free profit is made, as the shareholders of Geared plc would have achieved by selling at the higher (£600 000) price, borrowing, and buying the shares of Ungeared plc. Efficient financial markets abhor arbitrage as much as Nature abhors a vacuum. Arbitrage opportunities cannot be expected to exist for any significant time in a market with well informed investors. Market prices **must** adjust to cause all equivalent future cash flows, **sold in whatever form as single securities or as portfolios of holdings**, to sell for the same price.

Thus there is a very important kind of oversight that is brought to bear in financial markets. The seeking of greater wealth by participants causes an elegant consistency among security prices. Not only must identical securities sell for the same prices, but also all portfolios of securities having the same cash-flow expectations must have the same total value.

Think of what this means. It tells us that the market is ever vigilant for mispricings of single securities and all of the **combinations** that can be formed from securities. Even if two portfolios have wildly different securities in them, if the portfolios' aggregate future expectations are identical, the portfolios must be equally valuable, **and the individual security prices within them will adjust to accomplish that**. If the prices do not, there will be an arbitrage opportunity, which would cause investors to leap at the chance to increase their wealth at no risk. The forces of demand and supply will quickly cause prices to adjust so as to destroy the arbitrage opportunity. Arbitrage opportunities thus disappear when such portfolios have the same values.

From the perspective of company capital structure decisions, remembering that company and personal borrowing and lending are types of securities, we now know that the market is capable of pricing these as well as any other security or portfolio. This will ensure that the same consistency among future cash-flow expectations and market prices reigns with respect to capital structure decisions, as it does with any other portfolio-altering action.

9.3.2 Summation of Capital Structure Irrelevancy I: Connections with Capital Costs and Values

The discussion of the contributions of M&M to our understanding of the essential economics of company capital structure decisions cannot be overstated. Because this is such an important set of ideas, it is worth our effort here to review what we have learned, and tie these ideas to useful concepts from other parts of the course.

We have found that under the set of conditions specified for capital markets to this point (no taxes, or other 'frictions' that would cause any difference between the cash flows a company produces from its operations and those that are actually claimed by capital suppliers), how much or how little debt a company has in its capital structure is irrelevant. This idea interacts with familiar concepts of capital costs and company values in a way that might not have been obvious before this point.

This view of a company's capital structure decision implies that its bond, share and total values behave in a very specific manner as the company changes its capital structure to have a higher or lower proportion of debt financing. The M&M ideas to this point make clear that that total value of the company must be unaffected by a change in its capital structure. This means that as the company substitutes debt for equity in its financing, the total value of the company must remain the same. This in turn implies that each £ increase in debt value must be accompanied by a £ decrease in equity value, or else the company's value (defined simply as the sum of the two) must change (which according to M&M, it cannot). Using the numerical data from our Geared and Ungeared companies, the following set of relationships must hold, as shown in Figure 9.4.

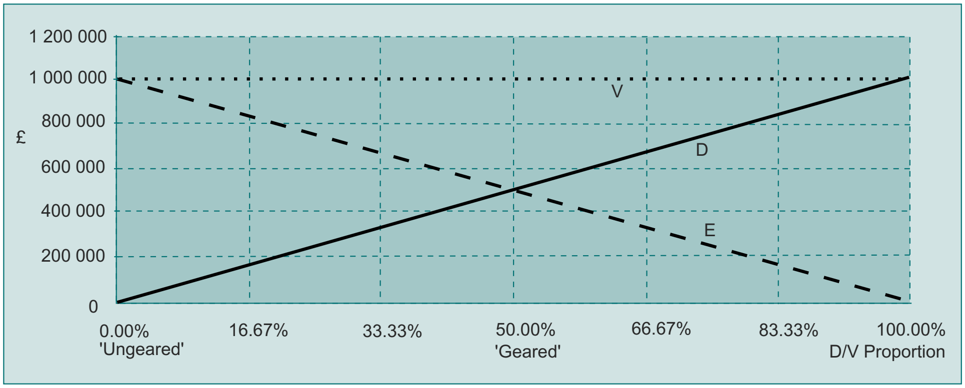

Figure 9.4 Capital structure and company values (without taxes)

Figure 9.4 generalises the relationships between overall company value (V) and the values of debt (D) and equity (E) as the company's capital structure changes. Notice that the overall value of the company as it changes its capital structure from ungeared with no debt, or D/V = 0, to geared with D/V = 50% (and beyond), does not change, and is constant at £1 000 000. As the value of the debt claims on the firm increases, so the value of the equity claims decreases £ for £. The company cannot change its total value by changing its capital structure; all it can do is repackage its capital claims to comprise differing proportional claims of debt and equity and, as we have seen, this does not change shareholder wealth.

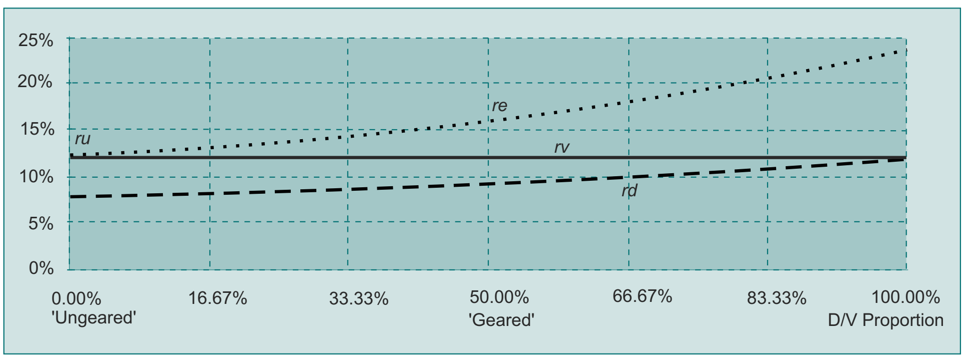

Figure 9.5 Capital structure and capital cost (without taxes)

Figure 9.5 illustrates the behaviour of the required rates of return and overall capital cost of the company: it alters its capital structure. Notice that as the company changes from ungeared D/V = 0 to geared D/V = 50% (and beyond), its overall required return (*rv*) remains constant (at 12%). Recall from Module 4 that *rv* is simply the value-weighted average of the company's debt required return *(rd)* and its equity required return *(re)*. We see that the company's required debt rate rd=8% at its geared location of D/V = 50%, and that the *rd* rate would be higher with more debt and lower with less debt, which makes sense. Further, Figure 9.5 shows the equity required rate *(re)* increasing from its lowest level *(ru)* when unleveraged to higher levels with higher D/Vs, consistent with equity's higher riskiness as debt is added to the capital structure.

Notice, however, that with both the equity and debt rates increasing with D/V, there is no increase in the company's overall capital cost or required return, *rv*, their value-weighed average, which remains steady at 12%. How can this happen? How can it be that *rv* does not increase even though its components *re* and *rd* do? The answer is best understood by returning to the value relationships in Figure 9.4. There we see that V, total company value, does not change as D/V increases. Since company value V must be simply its cash flows discounted by its overall required returns, or

$$\mathrm{V} = \frac{\mathrm{FCF}}{rv}$$

we must have that FCF as well as V is left unchanged as D/V increases. Therefore from the above simple value relationship, the company's overall capital cost (*rv*) must also be left unaffected by alterations in the company's capital structure.[4]

With respect to the specific weighted average relationships determining *rv*, from Module 4, we have

$$rv = \frac{D}{V}(rd) + \frac{E}{V}(re)$$

which implies

$$rv = \frac{D}{V}(rd) + \left(1 - \frac{D}{V}\right)(re)$$

Even though both the equity *(re)* rate and the debt *(rd)* rate increase with increases in D/V, notice that the value weight [1 − (D/V)] on the (higher) equity rate *(re)* will steadily decline, and the value weight D/V on the (lower) debt rate *(rd)* will steadily increase as D/V increases. **The higher proportion of lower-cost debt exactly offsets the lower proportion of higher-cost equity, such that their weighted average is unchanged**. This is not magic. It is dictated by the constant-value relationship above and, more directly, reflects simply the forces of the capital market at work. By the arbitrage examples we have studied, it would be impossible for any other result to occur in the financial markets as we have constructed them up to this point.

9.4 Capital Structure Decisions and Taxes

The M&M arbitrage argument produces the result of company capital structure irrelevancy in the type of market they illustrate. This market is unrealistic in the sense that there are no taxes. When we look at the same kind of analysis with taxes, the results can be different.

9.4.1 Capital Structure Relevance with Taxes

A company is taxed by the government on the amount of 'income' or 'profit' it makes. As we saw in Module 4, when companies can report the payment of interest

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4 That a company's value is the present value of its future free cash flow should be clear from our work in Module 4.

for income tax purposes, the companies' taxes are lower by an amount called the **interest tax shield**.

For example, let us assume that there is a company income tax of 50 per cent and that this applies to the world of Ungeared and Geared from the previous section. In the absence of any other kinds of taxes (such as taxes on personal income), the cash flows to the shareholders would change from the taxless world of Table 9.5 to those of Table 9.6.

Table 9.6 **Shareholder cash-flow expectations: Company and non-company borrowing with company income taxes (50%)**

	Company borrowing	Non-company borrowing
Company operating cash flow	£120 000	£120 000
Company interest payment	£40 000	0
Company profit	£80 000	£120 000
Company taxes	£40 000	£60 000
Net cash from company	£40 000	£60 000
Personal interest	0	£40 000
Net personal cash flow	£40 000	£20 000

Note that the geared company pays £20 000 less in taxes than the ungeared. From the **company** cash-flow perspective of Module 4, the interest tax shield in this case is £20 000, the company interest payment of £40 000 multiplied by its income tax rate of 50 per cent. This cash benefit accrues directly to shareholders of Geared plc, and is unavailable to the personally geared shareholders of Ungeared plc. (Because of different company tax liabilities, the risk attributes of the two shareholder portfolios are no longer the same, but the basic cash-flow effect is sufficient to make this point about tax benefits of company borrowing.)

The deductibility of interest payments by companies should cause there to exist a bias in company capital structures toward the use of borrowing instead of equity capital. The net result of company borrowing in an economy where there is a company income tax with interest deductibility is to cause there to be more after-tax cash available to the capital suppliers of the company than if the company did not borrow. (Note in Table 9.6 that the bondholders and shareholders of Geared plc each get £40 000, netting £80 000; on the other hand, the lenders to the personally geared shareholders of Ungeared plc get £40 000, but the shareholders only net £20 000, the difference going to government in higher taxes on Ungeared.) This is an obvious incentive for companies to borrow, and we can expect that the aggregate value (of both bonds and shares) of borrowing companies will be higher than the shares of companies that do not.

Interestingly, this general phenomenon can still hold true even when there is a **personal** tax on shareholders, and when shareholder personal interest payments are deductible on their personal taxes. Table 9.7 illustrates how this occurs, using a personal tax rate the same as the company rate. (That it would also be true with a lower personal tax rate should be clear, since Table 9.6 in effect uses one equal to zero.)

Table 9.7 **Shareholder cash-flow expectations: Personal and company and borrowing with company income taxes (50%)**

	Company borrowing	Personal borrowing
Company operating cash flow	£120 000	£120 000
Company interest payment	£40 000	0
Company profit	£80 000	£120 000
Company taxes	£40 000	£60 000
Net cash from company	£40 000	£60 000
Personal interest	0	£40 000
Net personal pre-tax cash flow	£40 000	£20 000
Personal taxes	£20 000	£10 000
Net personal after-tax cash flow	£20 000	£10 000

Table 9.7 shows that even when both companies and shareholders are taxed on their income, and with both being able to deduct interest payments on borrowings, shareholders are still better off with the gearing on the company level as opposed to the personal level. The reason is that the company tax benefit occurs with respect to a tax that, once paid, cannot be recouped by shareholders through deductible personal borrowing. Under the system of both company and personal income taxation, in the absence of any other tax provisions there will thus remain an incentive for companies to use borrowing instead of equity, though the relative benefit is smaller, as the comparison of Table 9.6 and Table 9.7 indicates.

The sentence above contains what has been appropriately called a 'weasel phrase' in finance. The phrase is: '… in the absence of any other tax provisions …'. As we are all aware, governments are endlessly inventive in their capacities to generate complex tax rules. One of the more interesting of these 'other tax provisions' appears in the UK tax system, wherein there is an 'imputation' of shareholder personal income tax payment to companies as the companies pay their own income taxes. We have discussed this system in Module 8; if you have forgotten that discussion, you should review it (*see* Section 8.3.1).

The UK 'imputation' system does partially eliminate the tax benefit of company borrowing. Because the imputed shareholder tax rate is less than the company income tax rate, there is still some tax benefit to the issuance of debt by companies, even in the UK. Only if the imputed personal rate is equal to the company rate is the advantage of debt neutralised by an imputation system.

What does this discussion of the interaction of company capital structure decisions and taxes mean? From what we have said so far, it would seem the answer is straightforward: when taxes exist, and when interest is deductible by companies,

companies will tend to use debt as their primary source of capital. The reason is simply that debt is ‘cheaper’ in the sense that the total of taxes paid by companies and their shareholders will be lower than if the companies were to issue equity.

So if we have told the complete story about capital structure, companies would tend to be financed primarily by debt. But that is not what we see. There is in fact a wide range of actual capital structures that exists in real companies. Either the financial managers of those companies are not aware of the tax benefits of borrowing, or there is more to the story than what we have told so far. The latter is of course the case.

9.4.2 Summation of Capital Structure Relevance with Taxes: Connections with Capital Costs and Values

Section 9.3.2 summarised the relationships between values and capital costs as a company changed its capital structure in a world without taxes or other frictions. This section will do the same, but for company capital structure changes when companies are subject to income taxation – definitely a ‘friction’. We recently discovered that when company taxation appears, company capital structure is no longer necessarily irrelevant. This is because the operating cash flows that the company produces are transformed by the taxation system before they can be claimed by capital suppliers. **And the transformation of cash flows is different, depending on the capital structure of the company.** Since interest is deductible for corporate income tax purposes and dividends are not (or are not perfectly deductible), the company can provide more of its operating cash after taxes to capital suppliers if the company uses debt in its capital structure than if it does not.

Because of the tax advantage of company borrowing, a company with debt in its capital structure (and ignoring some important caveats to come) will be more valuable than an otherwise identical company that does not borrow money. The reason is simple: the company that borrows will pay less taxes, and that extra cash flow is worth something.

How valuable is this tax benefit? From Module 4 we are aware that the effect of being able to deduct taxes produces a cash-flow benefit called the interest tax shield (ITS) and equal to the interest payment (I) multiplied by the company income tax rate (T_c). So in each period in which a company enjoys that tax benefit, its post-tax cash flows will be higher by ITS = I(T_c). That amount is a cash flow, however, and not a value. To find the value of this tax benefit, we must discount it appropriately. The appropriate discount rate depends, of course, on how risky this cash flow is. Since the tax benefit cash flow will appear if interest is paid, its risk must be the same as the company's interest payments.[5] And the interest payments require a rate of return equal to rd. So the value of the tax benefit must be equal to:[6]

$$\text{VITS} = \frac{\text{ITS}}{rd}$$

This formula should be somewhat familiar to you. Look in Section 4.5. There you see a similar expression in the APV discussion. Actually, this way of thinking of the effect of tax deductibility on a company's value is very similar to the idea of the APV technique. As a matter of fact, we can profitably think of the entire firm as an APV-type valuation where:

$$\text{V} = VU + \text{VITS}$$

In other words, instead of thinking of the company's value as the sum of its debt and its equity values, we can just as well think of the company's value as the sum of its value if it were unleveraged plus the value of its leveraging-based tax benefits. We would get the same total value using either our familiar 'debt plus equity' or this APV-type valuation, but this latter view helps us understand the workings of capital structure much more easily.

How does this value behave as a company increases the proportion of debt in its capital structure? Figure 9.6 shows the relationships between changes in capital structure and changes in capital claim and company values, with the rules of the capital market as we have developed them to this point.

5 This assumes that the tax benefit will be available only contingent on the payment of interest. Since it is possible that a company could pay interest and not get a tax benefit (for example, if it had no income to report for tax purposes), we are implicitly assuming the existence of ways to obtain the tax benefit regardless of the existence of taxable profits. In most countries' tax systems there are ways to obtain these tax benefits in such situations, with tactics such as (perfectly legal) tax 'carry backs', 'carry forwards' and mergers.

6 The illustration that follows is consistent with our exploration of the WACC in Module 4, and specifically makes the same implicit assumptions about the riskiness of Interest Tax Shields as that module. As we mentioned earlier, Miles and Ezzel (op. cit.) argue that these conditions are too restrictive in many contexts, and suggest that adjustments be made to the ITS valuation formula. Specifically, they suggest that because of uncertain future operating outcomes, even if D/V is held unchanged, the variation in V will cause more uncertainty in the *amount* of interest payments across time. This in turn implies that the ITS should be discounted using rv rather than rd, (because of the indirect influence of operating risk on the amount of interest paid), and the resulting ITS value multiplied by $[(1+\text{rv})/(1+\text{rd})]$ to recognize that the amount of expected interest is knowable one period in advance.

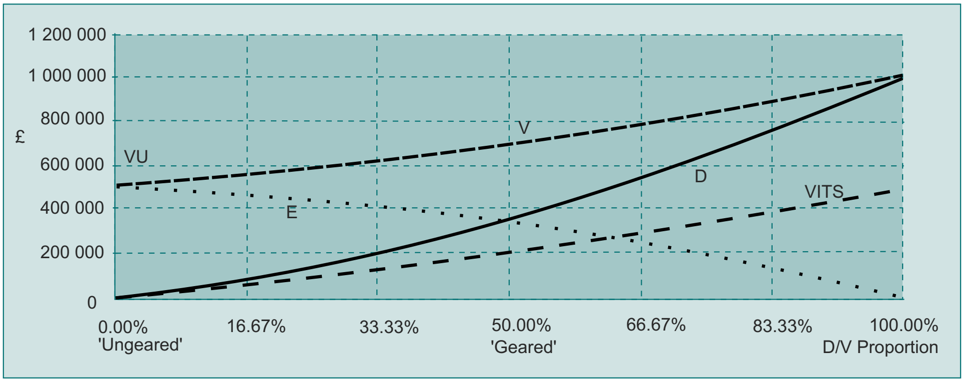

Figure 9.6 Capital structure and company values (with taxes)

Notice from Figure 9.6 that the company's total value (V) increases from an unleveraged value (VU) with increasing D/V, and that this increase is exactly equal to the increase in the value of the interest tax shield (VITS).[7] The interest tax shield becomes more valuable because the deductible interest payments increase (and therefore taxes decrease) as D/V increases, and these tax benefits are only as risky as the interest payments themselves (less risky than the firm's operating cash flows until the point where the interest payments comprise the entire company's operating cash flow). Equity value, of course, declines as D/V increases, but shareholders would be better off because they would be compensated with the cash raised by issuing new debt, based on the higher company value. Notice that the company's value is maximised at the point where it is 100% debt financed (and is worth the £1 000 000 it would be worth in a tax-free world). The reason for this is simply that a company that financed itself this way would pay no taxes, since all of its distributions to capital claimants would be in the form of deductible interest payments.[8]

In addition to effects on values, there are effects on required returns and capital costs. Figure 9.7 looks similar to its 'without tax' version in Figure 9.5. Again, the equity *(re)* and debt *(rd)* rates of required return increase with increases in D/V. The most important aspect of Figure 9.7, however, is its depiction of the behaviour of the company's cost of capital *(rv*)*, or WACC. Notice that the company's cost of capital steadily declines as the company substitutes debt for equity in its capital structure. Why is that?

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7 Notice that the values of the Geared and Ungeared firms are lower than the values we found when the companies were not subject to income taxes. This seems logical, particularly that Ungeared is worth exactly half of its 'tax free' value, since the company income tax rate is 50%. Where did this value go? Into government coffers, of course. Actually a corporate income tax is something like a forced equity participation in the company on the part of the government.

8 Of course, governments do not typically allow companies to do this. There are usually taxation rules that effectively require some minimum amount of equity (and therefore taxes).

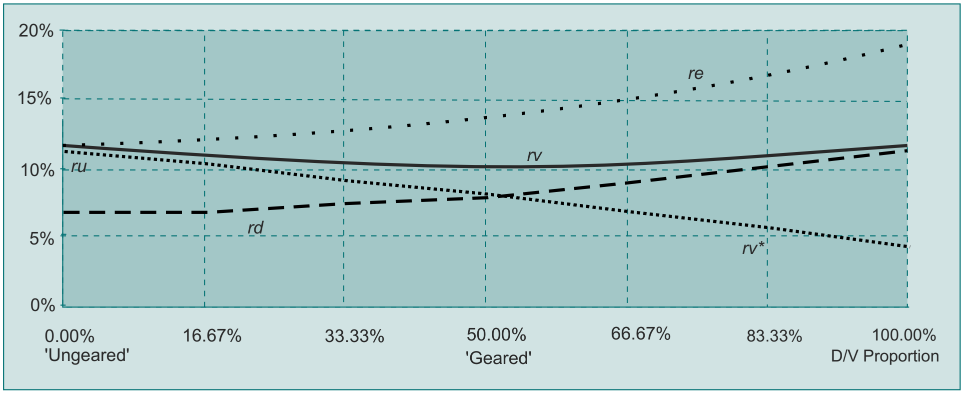

Figure 9.7 Capital structure and capital cost (with taxes)

The reason is similar to the reason that the company's cost of capital did not change in the 'without tax' situation. Recall from Module 4 that the WACC (or rv^*) is the discount rate that gives the correct valuation when applied to a company's ungeared free cash flow ((FCF^*). We see here that the company's value is increasing, but its ungeared free cash flow must (by definition) be unchanged as D/V increases. So by the relationship

$$V = \frac{\mathrm{FCF}^*}{rv^*}$$

from Module 4, if V is increasing and FCF^* is constant, rv^* must be declining as D/V increases. This is exactly as depicted in Figure 9.7.

What is the importance of this? The essential importance is the illustration that a company's cost of capital is affected by the amount of debt in its capital structure, and if interest is deductible for company income tax purposes (again with some caveats to come), the company's cost of capital will be lower the more debt it uses. The reason for this is simply that interest deductibility is a kind of tax subsidy for corporate borrowing that is not available for equity financings.

9.4.3 Capital Structure Irrelevancy II: Taxes

One reason why companies do not use debt exclusively is that it may not be as cheap as it seems from the discussions in the previous section, even with taxation of the type therein discussed. The reasoning for this was offered by Merton Miller,[9] when he asked us to remember that tax benefits of borrowing can only be finally calculated by finding the amount of cash that companies can get into all capital suppliers' pockets after **all** taxes at the company and personal level have been deducted. For example, we must remember that **bondholders** also pay personal taxes on the income that they get from company interest payments. He further cautioned us not to forget that most tax systems have personal tax rates that are 'progressive' (i.e. are higher with higher income).

This latter observation caused Miller to argue that as companies issued more and more debt, they would be forced to pay higher and higher interest rates in order to

[9] Miller, M. (1977) 'Debt and Taxes', *Journal of Finance*, 32 (May), 261–76.

induce those in higher tax brackets to purchase their bonds. This is because bondholders would be paying higher and higher personal income taxes on interest income from companies. Eventually, the increase in interest rates on company borrowing would offset the benefit of deducting interest at the company level, and there would be no particular reason to issue debt instead of equity. In other words, the tax benefits of company deductibility would still exist, but would be erased at the personal level by the high personal tax rates that bondholders would be forced to pay on their interest income.

Competition would cause companies to continue issuing bonds until interest rates had risen enough to cause all **net** (company combined with personal) tax benefits of company borrowing to disappear. Then there would be no reason for borrowing as a source of capital to be preferred to the issuance of equity.

There is still much controversy surrounding Miller's assertion that company capital structure is irrelevant even under taxation and interest deductibility. Various reservations and counter-arguments have been offered, some with apparent merit, but it is probably fair to say that the tax benefits of borrowing are not as high as the simple arithmetic of interest tax shields would imply.

There are yet other considerations within the question of tax benefits of company borrowing. For example, interest is not the only thing that companies can deduct from income so as to reduce income taxes. As we are aware from our cash-flow discussions in Module 4, various depreciation, tax credits and other tax-reducing mechanisms are available to companies. Now consider the situation where a company, because of all of these other tax write-offs, has no 'income' for tax purposes. There is obviously no reason here to choose borrowing for tax reasons, because all tax benefits thereby potentially available have already been exhausted by the company through other means.

The existence of ways other than borrowing to reduce taxes would tend to make borrowing less attractive, but still, for companies without enough of these other write-offs, debt would remain attractive. When we also recall that the amount of 'income' a company will earn in any given year is uncertain, whereas the commitment to pay interest is not, there is a further lessening of potential tax benefits in a probabilistic sense. This implies that borrowing to obtain tax benefits is less desirable in an uncertain world than when future 'income' streams are known with certainty.

9.4.4 Summary of Company Borrowing under Taxation

This section has been a long one, and because it deals with taxes, has been reasonably complex. It would probably be worth the effort to summarise what we have discovered about the company capital structure decision when taxes are considered.

The basic idea we carried into this discussion was M&M's, that company capital structure is unimportant in a taxless world. However, when companies pay income taxes and can deduct interest payments (and not dividends) for tax purposes, debt becomes a cheaper capital source than equity. For example, in that situation it would be in the interests of shareholders of a company for the company to borrow and thereby reduce its taxes rather than use shareholders' capital, the servicing of which does not generate tax benefits.

This tax benefit of borrowing is common to almost all developed economies, and persists even when shareholders themselves pay personal income taxes on their receipts of cash from companies, and can deduct personal interest from their own borrowing.

There are several considerations in real financial markets that tend to diminish this tax benefit of company borrowing, however. In the UK and certain other economies, there is something like deductibility of equity dividends. Such 'imputation' systems, making equity more attractive, tend to diminish debt's relative tax benefits.

In addition, Miller has argued that as more and more borrowing is undertaken by companies in economies with progressive personal taxes, the interest rates necessary to sell bonds to high personal-tax investors will cause the benefits of company borrowing to disappear.

Further, the tax benefits of company borrowing compete with other mechanisms used to reduce taxes (depreciation, credits, etc.). This tends to reduce debt's advantages, particularly when the amounts of 'income' that require shelter from taxes is uncertain.

At this time we have only a quite rudimentary understanding of how these complex relationships between capital structure and taxes interact to produce a final result. Most who think seriously about this question seem of the opinion that there is still some net tax advantage to company borrowing relative to equity issuance, but that gross generalities are dangerous.

9.5 Capital Structure and Agency Problems

Our investigation of company capital structure has led us thus far to the (admittedly tentative) conclusion that taxes cause the borrowing-versus-equity decision to be potentially important. We reached this point by beginning with the elegant economics of the original M&M irrelevancy proposition, and attaching gradually greater reality to the tax situations of companies and their capital suppliers.

This was a valuable step in our understanding of company capital structure decisions, but a bit of thought will show us that there are probably more than tax considerations operating in these decisions. Two pieces of casual empiricism will serve to prove this point.

First, if taxes cause a bias toward company borrowing, we would expect companies to use mostly (as much as the Inland Revenue will allow them to deduct) borrowing as their source of capital. But that is not what we see. Most companies seem to borrow less than what would produce the maximum tax benefits, even considering the other ways that exist to reduce taxable income.

Secondly, though it may strain credulity, there was a time when companies and individuals did not pay income taxes. If taxes are the only reason that capital structure is important, we would have expected to see some indication of irrelevancy of that decision in the fondly recalled days of no income taxes. But that too was not the case. Companies' capital structure actions were no more random then than they are tax-shield-maximising now. Either company financial managers are not behaving rationally, or there are important influences other than taxes on the amount of borrowing that a company undertakes. In competitive markets for financial managers, we would bet upon the latter.

What is the nature of these other influences? **Agency** considerations seem to hold the most promise of being the other important factors in a company's capital structure decision.

The study of 'agency problems' has a long and distinguished history in economics. 'Agency' deals with situations where the decision-making authority of a 'principal' (such as a shareholder or bondholder) is delegated to an 'agent' (such as the managers of a company). Agency considerations concern themselves with the instances where **conflicts of interest** may arise among principals and agents, and how those conflicts of interest are resolved. This may sound esoteric, but it has great practical moment in the study of finance, particularly in the capital structure decisions of companies.

Consider the following situation. A company has two mutually exclusive single-period projects, both with positive NPVs of which it can only choose one:

Mutually exclusive projects

Project A		Project B	
End of period outcome	**Probability**	**End of period outcome**	**Probability**
£8 000	50%	£2 000	50%
£14 000	50%	£20 000	50%

Suppose that Project A is the higher-NPV project, and the company would finance its (assumed) outlay with borrowing, promising an end-of-period £8000 payment collateralised with the project itself.

The lender asked to provide such financing has an agency problem. The lender will recognise that the company has an incentive to switch from Project A to Project B (with the same outlay) once the money has been lent. Why would the company do that? Think of its shareholders as the residual claimants of either Project A or B. Each project has an £8000 debt claim against it, so to find the returns to shareholders we merely subtract the £8000 outlay from each outcome, (remembering that equity has limited liability):

Returns to shareholders

Project A		Project B	
End of period outcome	**Probability**	**End of period outcome**	**Probability**
£0	50%	£0	50%
£6000	50%	£12 000	50%

If the company has the interests of its shareholders foremost in its decisions, there will be an incentive to accept Project B even if it has the lower NPV as calculated using the project's WACC.

What difference does this make to lenders? Let us examine their payoffs if Project B is selected instead of A:

Returns to lenders

Project A		Project B	
End of period outcome	Probability	End of period outcome	Probability
£8000	50%	£2 000	50%
£8000	50%	£8 000	50%

Clearly, lenders would be unhappy if the company indicated that it wished to borrow to undertake Project A, and ended up doing B. (The shift would take **lenders** from a riskless expectation of £8000 from Project A to the risky distribution of B.) The company, however, would predictably tend to make just that shift. This is the 'agency' problem that such a situation presents: there is a conflict of interest between the shareholders and the bondholders of the company.

How would it be resolved? There are several ways, none of them necessarily costless. For example, the lender may refuse to lend without a 'covenant' in the borrowing contract that prohibits the company from switching projects without the lender's permission. This is costly because the lender must monitor the company to see that it is living up to the contract, and you can depend upon such monitoring costs being included in the interest rate that the lender will charge. Or the company may unexpectedly come upon yet a third project that would be better for both it and the lender, but the now-complex conventional contract would make it impossible (or costly) to switch from Project A to the new one. Or the company might succeed in fooling the lender and doing the switch, but then would have accepted a project with a lower NPV, implying less social efficiency and wealth to be distributed to all involved.

The significance of this illustration is that the attractiveness of borrowing by companies may, because of agency costs, be less than would be implied by looking at tax benefits alone. And the illustration given is one of many types of interest conflicts with associated costs of resolution that could be cited.

One of the more important mechanisms that financial markets use to resolve conflicts of interest is by the issuance of complex debt contracts. We have already seen an illustration of one such complexity in the form of constraints on the way a company can operate its assets (there are others of the same kind, including the maintenance of various financial ratios, limitations upon dividend payments, and so forth). All complexities are not of the negative variety, however. There are others, which serve to merge the interests of shareholders and bondholders such that agency conflicts are less likely to arise and where therefore the costs of dealing with them are avoided.

An interesting example of such a 'positive' covenant is the **convertibility** provision that some debt claims carry. 'Convertibility' means simply that under certain conditions (for a specified period and at a given price or exchange ratio) and at the option of the lender, a bond can be exchanged for shares. Not long ago it was often argued that the reason companies issued convertible securities was because such securities carried lower interest rates. It is true that a convertible bond will require a lower return than a non-convertible issue. But that is no reason for a company to use such a security. Why not? Because, if securities markets are efficient, anything the company gains in the way of lower interest rates is almost certain to be lost elsewhere. This loss will be realised by shareholders when bondholders exercise their option to exchange the bonds for shares (at rates beneficial to the bondholders and detrimental to existing shareholders, or bondholders would not convert). What incentive has a company to issue convertible bonds? The incentive is to avoid certain agency costs in issuing debt.

Recall the original example of an agency problem wherein the lenders were justifiably concerned that the company would undertake the riskier project and thereby shift wealth from the bondholders to shareholders. Suppose that company were to issue convertible bonds to raise the money for the chosen project. Here bondholders would be less concerned that the company would undertake to shift to the high-risk investment.

The reason is that, should the company switch to the risky project, the equity component of the convertible security would increase in value so as to offset in part or whole the reduction in bond value. Shareholders would thus not gain, and bondholders not lose, in that transaction. The company's value **in total** would be higher because the bondholders would not require as stringent a set of restrictive or 'negative' covenants and would not be forced to charge for agency monitoring costs. Most importantly, the company itself would no longer have an incentive to switch from the high-NPV project to the low-NPV project (because the shareholders would not benefit at the expense of bondholders), and the securities of the company would be priced with the market expecting the company to undertake the highest NPV projects rather than play games in switching wealth from one group of claimants to another.

Another instance of agency conflict occurs when a company in financial distress (where its bond values have declined due to poor expectations) is unwilling to undertake a profitable investment because the resulting effect would be to help bondholders and not shareholders.

When a company is at the point of being in conflict with the contract it has with lenders, there are additional agency costs that we have not yet mentioned (although one that we *have* mentioned is the foregoing, of profitable investments). One important condition occurs when the company is actually in default of a debt claim.

9.5.1 Default and Agency Costs

It is reasonable that most students of finance are initially under the impression that one of debt's undesirable features is that a company can go bankrupt if it borrows, but cannot if it has only equity claims outstanding. It is true that bankruptcy is only possible if there are creditors. But it is not true that the possibility of bankruptcy is a drawback of borrowing. As a matter of fact, a logical argument can be made that the chance of going bankrupt is a **positive**, not a negative attribute of borrowing.

This surprises many who hear it for the first time, but the logic is inescapable. One key to understanding this important point about borrowing is to remember that the unfortunate economic circumstances that precipitate bankruptcy would exist even if the company had not borrowed. The other key is that bankruptcy's essential effect is merely to change the legal ownership of the company's assets from shareholders to bondholders. It will be easiest for us to understand the issues of default and bankruptcy with an example.

Suppose that Crewboats Limited, a petroleum-exploration company, having borrowed money to purchase vessels, goes bankrupt because oil prices decline and there is no demand for the company's services. The value of the vessels of the company is less than the face value of the debt issued by the company to purchase them. This condition is most unfortunate for the company, but the question of bankruptcy's deleterious effects must be judged from the perspective of whether, **given the decline in demand and the resulting effect on asset values** the company (and its shareholders) are worse off having borrowed than they would have been had they instead issued share capital. It is obvious that shareholders have experienced an unpleasant, wealth-reducing episode. What is not clear is whether or not the type of company financing undertaken (debt) has exacerbated the untoward economic event (the decline in oil prices).

The real problem here is that the boats owned by Crewboats Limited have declined in value due to the decline in demand for what they do. If the company has borrowed such that the amounts of cash flow available from running the boats is insufficient to service the debt, there is little choice but to inform creditors that the debt payments will be defaulted. And if the boats cannot be sold for enough to pay the debt's promised amounts, bankruptcy will probably ensue. But suppose that the company had instead financed itself with equity capital. There is no reason to think that the decline in oil prices would have affected the boats' operating cash flows any differently if the vessels had been financed with equity instead of debt. So the decline in value due to oil price reductions would be felt regardless of financing.

What will happen to the assets of the bankrupt company? The odds are that they will either be taken by creditors and sold, or operated by the company under the supervision of the courts. Either way, the assets will be operated if it is profitable to do so, **given their current market values**, or sold instead if more wealth is thereby generated. It should be easy to see that the same thing would have happened had the company financed itself with equity capital, given the decline in demand.

What then is the importance of bankruptcy? It is **not** that the assets have declined in value due to economic conditions independent of financing; that event would have occurred anyway. The importance is that there may be **extra** costs incurred when a company goes bankrupt (or is otherwise close to financial distress), which would not appear in the absence of borrowing.

The most obvious of these costs are those of litigation, which can be expensive and consuming of management time (better spent in operating assets, developing markets, and so forth). In addition, there are implicit costs such as the delay in using or selling assets caused by being tied up in the legal system. And there are almost surely operational constraints (e.g. suppliers unwilling to extend credit, customers concerned about warranties on products and services to be purchased) that are costly. As important are the conflict-of-interest situations illustrated in the prior section, which are now more than ever likely to be burdensome to the company, and keep it from making optimal least-cost, highest-profit decisions.

So the importance of bankruptcy is perhaps different from what a superficial analysis would indicate. Bankruptcy costs are not the declines in value that precipitate bankruptcy; those are independent of capital structure. The true costs of bankruptcy or financial distress are:

1. the costs involved in pursuing the legal process of realigning the claims on the assets of the company from those specified in the original borrowing contract; and
2. the implicit and opportunity costs incurred in this effort relative to what would have happened had the company been financed instead by equity capital.

We can now explain the doubtless cryptic comment at the beginning of this section, indicating that bankruptcy can be a desirable outcome.

Suppose that there is another company, Crewboats Unlimited, in the same economic position as Crewboats Limited, except that the shareholders of the company do not carry limited liability. Here the decline in asset values below promised debt payments must be made up by shareholders **out of their own pockets**. The company cannot then go bankrupt. Clearly, the shareholders of Crewboats Limited (being able to walk away from the company with no personal liability) are in a better position than those of Crewboats Unlimited. The option to declare company bankruptcy and thereby take advantage of limited shareholder liability is a valuable one.

But as with any event that may be anticipated, we can depend on it that the financial markets have priced the shares and bonds of the two companies appropriately. When Crewboats Limited sold its bonds and shares on the market it would have received more for its shares and less for its bonds (i.e. paid lower returns to shareholders and higher interest rates to bondholders) than would Crewboats Unlimited. There is no 'free lunch' in limited liability. The shareholders' right to walk away from company debt will be priced originally in the company's shares and bonds.

This illustration can be used to remind ourselves of another important aspect of capital structure's financial economics. Suppose that between Crewboats Limited and Crewboats Unlimited there were no differences in the costs of bankruptcy, or in the capacities for shareholders and bondholders to make judgments about the risks and returns from holding the companies' capital claims. In that situation, the **total values** of the two companies' capital claims would be the same, with Crewboats Limited having the higher share (and lower bond) values, and Crewboats Unlimited having the higher bond (and lower share) values. In the same sense as there is capital structure irrelevancy between debt and equity in the original M&M illustration, here, as long as the market sets the prices of securities efficiently, there is nothing to be gained by having limited-liability shares versus unlimited-liability shares. There is a shifting of risk, but it can be anticipated and the market is in the business of valuing just such contingencies. Unless there is some **unique** benefit to the issuance of a particular type of claim (such as is the interest deductibility of debt), there is no reason to think that one type of claim will be any better than another. (Since equity claims exhibit limited liability in almost all economies, there is probably some such unique benefit to that attribute that has not yet been convincingly demonstrated.[10])

9.5.2 Conclusion to Agency Considerations in Borrowing

We have seen that the effect of borrowing by companies is likely to cause agency costs to be incurred, which costs would not be present if there were no borrowing. These agency costs are primarily those of the inefficiencies in operating and investing in assets caused by actual or potential default upon a debt contract, by the restrictive covenants which appear in those contracts to protect the interests of creditors, and by the reasonable assumption made by creditors that, given the chance, managers will take decisions in the interests of shareholders (and adverse to those of bondholders if necessary). All such costs are priced into the interest rates or other terms in bond contracts.

There are some ingenious devices in existence in financial markets to reduce the costs of agency conflicts. We have mentioned convertible securities in the previous section, and could have also mentioned call provisions (the capacity on the part of the company to repurchase its bonds at a fixed price for a given period of time).[11]

Though financial markets are ingenious, it is unlikely that all agency problems of borrowing can be costlessly solved by the issuance of complex securities or by any other mechanism. At some point, as the company continues to add debt to its capital structure, the effect will almost certainly be to increase its cost of capital due to agency costs. Think of a company financed almost totally by borrowing. A relatively small percentage shift in wealth from its creditors will produce a very large percentage increase in wealth to its shareholders. The incentive for the company to undertake such an action is higher the greater the proportion of borrowing. So bondholders, if for no other reason than assuming financial managers are clever enough to invent new wealth-shifting transactions given enough incentive, are likely to cause debt costs to increase as borrowing becomes a larger and larger part of a company's financing.

We would be remiss in neglecting to mention at this point that there are agency costs of issuing **equity** as well as debt. These conflicts of interest are not between shareholders and bondholders, but between shareholders and managers.

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10 One such consideration is probably the high costs of estimating and collecting the personal resources of shareholders lacking limited liability.

11 This one actually lets the bondholder initially assume that the company will make a decision adverse to the interests of bondholders, and thus set low prices on the bonds at initial issuance. But then, when the company does not take such a decision, the bond value increase occurring can be captured by the company for its shareholders through exercising the call provision on the bonds at a price lower than the upwardly revised expectations would require.

Being basically economists at heart, we in finance assume that all parties to a contract will act in their own best interests. To this point we have assumed that the interests of managers and shareholders are congruent. That is closer to being true than it would be to assume that the interests of managers and bondholders are congruent.

For example, shareholders elect the board of directors of the company, who in turn hire and fire management. Further, there is some convincing evidence that the merger and takeover market is really the functioning of the market for managers, wherein a takeover bid is effectively asking shareholders to replace one management group with another. This is likely to be a more significant constraint upon managers to perform in the interests of shareholders than is the vote of a well dispersed shareholder group for the board. Bondholders, of course, have no such oversight capacity on the actions of managers. So managers must be to some extent concerned about how shareholders view their performance.

Nevertheless, there are actions that managers can take that augment their wealth to the detriment of shareholders. One of these is 'perk' consumption beyond the point where management productivity is efficiently enhanced (e.g. a slightly larger and more plush company jet, a more beautiful – or handsome – secretary). Another is asset operations that benefit managers instead of shareholders, such as **conglomeration** (the merging of unrelated firms) to increase the size and reduce the cash-flow risk of the company, thus making management remuneration higher and more stable, with no accompanying benefits to shareholders who own already well diversified portfolios.

Like bondholders, shareholders can anticipate managers undertaking such actions, and price such expectations when they purchase shares of the company. Thus there is an agency cost of simply attempting to raise money outside the firm (when the owners of the company are not also the managers). This 'outside capital' agency cost is doubtless also present in a company's borrowing, but probably less so at moderate debt levels since the debt claim is of higher priority than that of shareholders and would thus be less likely to suffer the pro rata losses from managerial perk consumption than would equity.

What do all of these agency concerns mean to a company's optimal capital structure? They probably mean that there is some level of borrowing that is best for a company from this perspective, and that the level is greater than nothing but less than 100% gearing. This of course is not very useful advice to the fledgling financial manager wondering exactly how much to borrow. That, however, is unavoidable given what we know (and many think it also applies to what we are ever likely to know) about the detailed influences of agency conflicts.

Does that mean that agency concerns are destined to be simply something that financial managers keep somewhere in the backs of their minds, and for which there will never be practical application? No, that is not at all the case. The practical application of agency concerns in the borrowing decision will probably be in the specification of more efficient contracts, which serve to diminish the agency concerns of both debtors and creditors. The convertibility and callability examples are only the simplest of the possible set of such complex provisions that can be designed to avoid the inefficiencies of agency relationships.

9.6 Making the Company Borrowing Decision

At the very beginning of this module, we introduced our consideration of company borrowing decisions by asking that you put yourself in the position of a financial manager faced with a decision as to whether the source of financing for a project would be debt or equity. Since then we have discussed a great number of concerns that might go through the mind of a well educated manager in that situation. It is now time to consider how such a manager would finally decide whether or not to borrow, and if so, how much.

If you are expecting some type of rigorous, quantitative formulation of this decision, we are sorry to disappoint you. There is no such technique available. This should not really be surprising if you have studied carefully what has gone before in the module. Many of the important dimensions of the borrowing decision (especially agency considerations) are at this time simply not well enough understood to be credibly quantified. This is not to say, however, that there is no useful instruction for real-market capital structure decisions that can be garnered from our work to this point. In fact, there are a number of very valuable suggestions we can make.

One thing that stands out as likely to be of particular importance to the optimal amount of borrowing is tax considerations. In very simple tax structures, debt tends to have an advantage over equity, because of the greater capacity of company interest payments to reduce company taxes. With the sole (and anything but trivial) exception of Miller's argument, most agree that there is a tax-induced bias toward using borrowing for company capital. But the practitioner must be very careful to judge the **net** tax benefits of borrowing. In other words, to argue for borrowing instead of equity issuance through tax reasons, one must be willing to argue that the tax benefits are indeed likely to occur and are not grossly uncertain. Companies with quite variable or low accounting (taxable) income must be (and are) aware that tax benefits are more difficult to come by without taxable income (though mergers, carry backs, carry forwards and other schemes can be low-cost mechanisms of getting tax benefits without taxable income). Also, companies with other means of reducing their taxes, such as credits, depreciation and depletion allowances, and so forth, must remember that debt-based tax reductions must not simply replace the deductibilities from other sources. If such replacements are the net effects of deducting interest, there is really no company-wide benefit from borrowing.

There is a common notion in practitioner finance that risky businesses should borrow less (or somehow be lent less) than companies that are not so risky. A moment's thought will indicate that there is very little we have discussed in the module thus far that would seem to validate this notion. Risk can be priced, just as any other attribute of a debt contract, and as long as financial markets are competitive, there would be no reason to think that risky borrowing would be any more costly than risky equity. One must be careful, however, of dismissing common notions and practitioner 'rules of thumb', even when they seem to be arbitrary and uninformed. Very often, not necessarily by any deep analytical process, such rules have evolved valuable content through the Darwinian nature of competitive markets. Said more simply, some 'rules of thumb' work **not** because one knows why they do but because, if they are not used, one goes out of business. Lending and business risk is probably a rule like this. Out of a complete sense of responsibility, however, we should let you know why we think the rule works.

Agency costs are probably the reason why lenders and borrowers are less attracted to risky situations. If you think back to our discussions of these costs, you will see that the presence of uncertainty in the operations of a company would cause these to be more heavily felt. (The difficulties in monitoring the actions of the firm, and the odds of incurring distress costs are obvious examples.) So there seem in fact to be good reasons why borrowing is less attractive for risky companies: risk is likely to make agency costs higher.

9.6.1 Book Values and Borrowing

Another of the interesting comparisons between practitioner and academic views of company capital structure decisions has to do with the simple measurement of how much is borrowed. Both sides, of course, agree on the actual cash amounts transacted. The differences of opinion arise in answering the question of the **extent** to which borrowing (as opposed to equity finance) has been relied upon. Academics schooled in company capital structure theory argue strongly that the correct comparison is debt's proportion of total **market value** of the company, whereas practitioners are just as convinced that **book values** of company capital claims are the correct measures of the extent of borrowing undertaken. From the discussions of value and cash flow in earlier modules, it should be obvious to you that these two approaches to measuring the extent of borrowing will often produce widely differing estimates. Further, the nature of the various biases in book values dictate that for most (non-pathological) situations, book values will show the company as relying more heavily upon borrowing than will market values.

This is not simply a disagreement about the correct rule of thumb. Academics can produce closely argued demonstrations that the correct way to measure the extent of borrowing for a company is to examine market values not book values. These arguments, however, are invariably made within the context of a 'frictionless' world not burdened with costly information searches, agency problems and the assorted other dimensions of real markets with which the practitioners of finance must contend. Make no mistake; academics are also aware of these things but, as we have displayed, have not been able to offer much in the way of rigorous solution. Practitioners, however, must make decisions regardless of the state of theory, and thus do. To help them do so, they evolve arbitrary rules, one of which is this book-value measurement of capital structure.

Why would such a thing work? Why might book values be better than market values as the indicator of a company's reliance on debt? It is true that market values better measure the extent to which future cash flows are expected to be shared between the types of capital claims. However, as we have seen in the previous section, there are times when lenders must think about their capacity to get their hands on the assets of the company itself (in times of bankruptcy or severe distress). In such times, the assets are unlikely to be generating their usual operating cash flows, and may well be offered for sale in liquidation. Book (historic, depreciated) values are here probably better indicators of lender expectations of return than are market values (based upon operating cash flows).

The very nature of tangible assets as collateral for loans may also play a part in the reliance of finance practitioners upon book values. Consider, for example, the differences, from a lender's perspective, between the market values of a computer software company and a gold mine. The software company is likely to have most of its value being generated by the future cash flows from its new products across time, whereas the gold mine's value is based upon the worth of the gold in the ground. Now think of the decisions faced by the companies and their creditors as to the best amount of borrowing for each.

As the software company and its creditors consider the potential for agency problems in the debt of the company, they recognise that in times of financial distress, it is not at all unlikely that there would be adverse operational effects upon the capacity of the company to produce positive cash flows (e.g. concern on the part of skilled staff as to their future, unwillingness of suppliers or government to provide services on credit, management time spent on dealing with financial problems instead of software development, and hence forgone new products). When such difficulties appear, it is highly probable that the existence of debt itself will cause a decline in the value of the company on top of the deleterious event that precipitated the financial distress. The company and its potential creditors must judge the importance of these value declines in deciding upon optimal company capital structure. Because these effects are likely to be significant, the decision is to issue little or no debt.

The gold mine, on the other hand, is unlikely to suffer such distress costs, because its value depends upon an immutable, tangible asset. When the company finds itself upon economic hard times, the existence of borrowing in its capital structure presents few of the same problems experienced by the software company. In times of low gold prices, the value of the gold mine is about the same regardless of whether it is equity-financed or (perhaps in bankruptcy) debt-financed.

For the purposes of this discussion, the primary difference between the gold mine and the software company is in the degree of tangibility of their assets (both are quite 'risky' in the sense of producing rather variable cash flows). Since tangible assets tend to be those that appear on the balance sheets of companies, we begin to see why book value ratios seem to be so important in a company's capital structure decision. Book values are a proxy for the extent to which a company's market value is based upon tangible assets. And tangible assets present fewer problems than do intangible ones in the presence of borrowing. So it is after all not so surprising that book value targets and comparisons play such a big role in company borrowing decisions. This is not because anyone really thinks that book values correctly reflect the true economic value of the enterprise, but simply that book values are a pretty good measure of the extent to which values will not be upset by financial distress when the company is engaged in borrowing ('collateralisation' is a one-word term for the same idea).

9.6.2 Techniques of Deciding upon Company Capital Structure

By what mechanism, then, should companies decide whether or not to borrow and, if so, how much borrowing to undertake? There are essentially two techniques available to companies (or to their consultants and investment bankers giving advice about optimal capital sources). The first has been around for a long time and is very 'low-tech'. It is simply to examine what companies in similar lines of business have decided about the amounts they will borrow. Usually the best way to do this is to look at company averages for borrowing ratios in the industry of interest. (Such industry averages for financial ratios are widely reported and available in most libraries, and from data reporting services.) If the amount of borrowing in question produces a ratio (probably a book value ratio for the reasons mentioned in the prior section) not far from the industry average, the odds are that the decision to borrow is acceptable. If the ratio produced for the company is reasonably well above the industry average, borrowing that amount may be ill advised.

One can see that this approach to deciding about company borrowing depends heavily upon the 'Darwinian rule of thumb' argument. If others have done it and survived in a competitive marketplace, it probably works, regardless of the reasons why. This approach undeniably has its appeal, but we have not spent our arduous efforts in this module to be left only with the advice to examine what others are doing. Evolution is a continuous process and speaks more to the question of survival than to optimality, which is our decision-making concern.

The second general approach to real company capital structure decisions is more consistent with the ideas developed in this module. It is called **financial planning**. Financial planning is also simple in concept, comprising nothing more than a detailed examination of the company's future cash-flow expectations (including those associated with the borrowing alternatives under consideration) so as to decide upon the best choice of financing method. In other words, the company financial planner, in possession of a (usually computer-based) capacity to simulate the cash flow and financial statements of the company across the future, asks a series of 'What if?' questions of the planning model (computer spreadsheet programs are currently much in vogue for this, and are increasingly easy to set up and manipulate). These questions would allow for all financing alternatives under consideration, and for a range of reasonable economic conditions (of product demand, costs, and so forth).

The result of any such analysis is a set of possible future outcomes for the company under the sets of conditions and financing alternatives that the planner chooses to examine. The hope is that one such financing alternative will show itself to be superior to the others in most reasonably expectable future states of the world, and thus be the optimum choice for the company.

On what bases do some financing alternatives prove superior to others? This question is most easily answered by illustration. Consider the situations of the following four companies undertaking financial planning simulations so as to decide upon optimal financing sources:

1. Crewboats Limited, a provider of offshore transport service to the petroleum industry;
2. Eastern Fabricating, a manufacturer of castings and frame parts for aerospace vehicles;
3. Midlands Mineral Resources, with large holdings of minerals in the Midlands;
4. Octopus Enterprises, a 'conglomerate' firm with many subsidiaries engaging in a wide variety of endeavours.

Crewboats Limited has committed itself to purchase some new vessels. The company will require outside financing to have enough cash to buy the boats. The financing alternatives it faces are the issuance of share (equity) capital, or long-term borrowing with the boats as primary collateral. A simulation of the cash flows from operating the boats indicates that only in extremely bad economic conditions would there be any significant chance of defaulting on the terms of the bond contracts. Though the investment in the vessels will produce significant tax benefits to the company, Crewboats is very profitable, and can use the tax write-offs fully. What should the company do?

Crewboats should borrow instead of issuing equity because of all the above, and the additional consideration that boats are a tangible, liquid asset, and thus carry less in the way of agency penalties from lenders looking to them as collateral, and from the company's perspective of operating constraints unique to borrowing.

Eastern Fabricating plans to expand and modernise its plant so as to be more efficient and competitive in the markets it serves. Eastern's profitability is good, but the company has shown significant variability of cash flow in the past, due largely to its heavy reliance upon government contracts and the budgetary vagaries of that process. Eastern's alternatives to finance the expansion are to issue shares or to borrow with straight senior **debentures** (debt collateralised by the overall assets and cash flow of the company). In addition to interest and principal payments, the debentures will require that the company maintain certain operating and financial ratios on its income statements and balance sheets. Simulations of the company's cash flows with the expanded and modernised plant indicate that the interest and principal payments are likely to be affordable, but that, given the likely continued variability in operating results over time, there are significant chances that the company will eventually feel the other contractual constraints. These could require that the company cut its dividend to shareholders, avoid issuing new debt for projects in the future, or actually run foul of working capital requirements in the bond indentures. What should the company do?

Eastern is probably well advised to avoid borrowing, or to choose some mix of equity and debt that would be less constraining.

Midlands Mineral Resources would like to acquire Lowlands Mineral Resources, Midlands being convinced that Lowlands' in-ground reserves are seriously undervalued. Midlands could get the cash to purchase Lowlands by issuing either more debt or more equity. Lowlands has not been doing well, and has large tax losses that would be carried into Midlands. Midlands and Lowlands together have large tax write-offs, which can be foreseen due to depletion of their holdings over time. Simulation indicates that operating cash flows may at times not cover debt service, but this does not trouble the financial managers of Midlands due to the liquidity and tangibility of the companies' resource holdings. What should the company do?

Midlands should probably not borrow. The reason is not so much that there will be problems with covering the payments of interest and principal, or large distress or asset liquidation costs unique to borrowing. The underlying assets of the companies are such that they are excellent candidates for debt collateral. The problem is that there are few if any benefits that debt can be seen to provide. Since almost all of borrowing's attraction is from the perspective of minimising taxes, and since it is unlikely that any taxes will be paid even if no debt is issued, there is little attraction to debt.

After some years of rough times, Octopus Enterprises has become profitable by streamlining management and 'getting control' of its far-flung and varied pursuits. The company now wishes to change its capital structure so as to take advantage of the tax benefits of borrowing. (Octopus foresees that if it does not do something, its tax bill will increase dramatically in the near future.) The company's expectations are that it will continue to prosper, and there is little reason to think that it will not. It intends to continue acquiring or starting businesses in which it thinks a good return is available. It is willing to take risks to do so. Octopus' simulations indicate that there should be no serious problem in covering the interest and principal payments on the amounts of borrowing it intends. In approaching potential lenders (either through investment banking advice or insurance company 'private placements'), Octopus has discovered that its straight debentures will be asked to pay very high interest rates – much higher than companies of comparable profitability and risk, but whose lines of business are fewer. What should the company do?

Octopus should consider issuing a form of debt more complex than straightforward debentures. The relatively high interest rates that Octopus is being asked to pay are probably caused by lenders' natural concern that Octopus may alter itself to be riskier during the life of the loan. If the total value of the company remained unchanged, its debt values would decline and share values would increase. Creditors have priced such contingencies into the interest they are charging. Octopus may be able to reduce its total capital costs by issuing convertible debt securities that carry the tax benefits of borrowing, but at the same time give lenders a guarantee that any increases in equity values will be shared. By avoiding this 'agency cost', convertibles should produce borrowing terms more favourable for Octopus.

9.6.3 Suggestions for Deciding about Capital Structure

From the descriptions of company capital structure decisions offered immediately above, it should be clear to you that it would be useless to seek a simple all-encompassing process or template that would lead financial managers to make optimal capital structure decisions. Our understanding of the important dimensions of this conclusion lead us rather to suggest the following:

1. Using an appropriate simulation model, the company should attempt to forecast its cash flows and financial statements across the foreseeable future under the

various alternative proposals for financing. These simulations should concentrate upon the reasonable 'worst case' situation, wherein the existence of borrowing might lead to financial distress.

2. If the simulations indicate that there are significant chances for coming into conflict with covenants of the borrowing contracts in ways that would damage optimal operational aspects of the firm (e.g. its wishes to undertake other investments; to operate its assets, including working capital, as it deems best; to incur bankruptcy costs), borrowing should probably be avoided.
3. If the simulations indicate that there are significant chances of the tax benefits of borrowing simply replacing other tax benefits and not adding any new to the company, there is little reason to borrow.
4. If the company's value is largely in tangible assets, more borrowing is sustainable than if the same value is dependent upon intangibles (like the future profitability of investments not yet undertaken, or the skill of managers in inventing or uncovering new opportunities). Industry gearing ratios are useful to see the amounts of borrowing that other similar companies have been able to sustain.
5. If potential lenders may fear that, subsequent to the issuance of debt, the company will take actions that will be to the detriment of bondholders, the company should consider attaching covenants to the bonds so as to alleviate to some degree that concern (e.g. convertibility provisions) or recapture some of the penalty for shareholders (e.g. call provisions on the bonds).
6. Remember that there are also reasons why companies would choose to avoid new equity issuances (e.g. loss of ownership control) and such considerations may outweigh negative aspects of borrowing.
7. When a tentative conclusion about the decision has been made, see if the result would be inconsistent with the capital structures of other companies in the same line of business. If so, some additional attempt should be made to see whether this is in fact an improvement over the usual practice or simply a signal that something has been left out of the analysis.

Learning Summary

The characteristic of company capital structure decision making that creates the most lasting impression upon finance students is the evolution from what seems at its earliest introduction to be a very rigorous, quantitative process to what at its practical application stage is almost exactly the opposite. Witness the capital structure decision descriptions immediately above, which contain not a single number! (There are, of course, numbers that would have appeared in the cash flow and financial statement simulations, but these are simply the grist to be ground by the decision mill.)

The reasons for the contrast between capital structure's theoretical rigour and practical application are a combination of the number and complexity of the relationships involved (the economics of capital claim hierarchies, the influence of company and personal taxes, the costs of resolving anticipatable conflicts of interest, and so forth), and the relatively short period of time that financial thinkers have devoted to formalising these considerations into real-market decision processes.

Today we know tremendously much more about company capital structure decisions than we did when M&M wrote their groundbreaking work in 1958. But there is probably just as much yet to be discovered about this most challenging of financial decisions, particularly in the application of theory to actual decisions taken by financial managers.

Review Questions

9.1 Companies are concerned about the capital structure decision because of which one of the following?

A. It is often necessary to borrow money so as to be able to undertake investments that the company deems desirable.
B. The source of financing may affect the wealth of the capital suppliers of the company.
C. Borrowing is usually cheaper than issuing equity.
D. Equity tends to produce lower-risk capital structures.

9.2 For any given company, the cash flows promised to debt suppliers are bound to be less risky than the cash that shareholders expect. This implies that the required returns to bondholders will always be lower than those of shareholders. However, the more debt issued, the higher will be the riskiness of the company's shares, and hence the higher the required returns to shareholders. This situation implies which one of the following as the best capital structure for a company?

A. The mix of borrowing and shares that produces the lowest risk to the shareholders.
B. The mix of borrowing and shares that causes the value of equity to be maximised.
C. The mix of borrowing and shares that produces a reasonable exposure to risk.
D. None of the above.

9.3 Assume a taxless, frictionless, information-costless world. Leverage Tools plc is a company operating therein with debt in its capital structure. The market value of that debt is £34 000 000. The common shares of Leverage are worth £62 000 000. Leverageless Tools plc is identical in all ways to Leverage, save that Leverageless has not borrowed any money. Leverageless's shares are worth £100 000 000. Suppose that you were interested in bearing the full business risk of such tool companies (so as to get the commensurate return) and were trying to decide how best to do that. The best transaction would be for you to do which of the following?

A. Purchase all of the shares and bonds of Leverage plc.
B. Purchase all of the shares of Leverageless plc.
C. Do either A or B, because both give you what you want.
D. Do neither A nor B, because neither gives you what you want.

9.4 Suppose that in Question 9.3 above there were many Leverage-type companies and only one Leverageless. Do you think that the value situation described in Question 9.3 is likely to persist? Why?

A. Yes, because there are probably people out there who would prefer to hold shares from companies that have not borrowed, and that therefore have lower risk.
B. No, because the rarity of ungeared firms would cause the value of Leverageless to increase.
C. Yes, because the value situation currently exists, and there is no reason to think that anything will happen to change it.
D. No, because the financial market will soon perceive that the shares of Leverageless are too expensive, and their price will fall.

9.5 Your company is considering an investment whose characteristics indicate that it will provide a 10 per cent rate of return (IRR). Your investment banker advises you that the company can raise enough money to undertake the entire investment with a straight debenture debt issue, the after-tax cost of which is 8 per cent. Does this sound like a good deal?

I. Yes, because it provides a rate of return in excess of the cost of funds necessary to finance it.
II. No, because nothing has been said about the riskiness of the project.
III. No, because 8 per cent is not the WACC of the investment.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Both II and III.

9.6 Negative considerations important to the company capital structure decision include:

I. The risk of bankruptcy of the company.
II. The chances that some tax benefits may be lost when the company deducts interest from taxable profits.
III. The chance that the company will not be able to use its assets in the ways it would have, had it not borrowed.
IV. The chance that shareholders will be forced to make good on the company's promised interest and principal payments.
V. Lenders requiring returns that include compensation for the chances that the company will take actions in conflict with the lenders' best interests.

Which of the following is correct:

A. I, II and III.
B. II, III and IV.
C. II, III and V.
D. III, IV and V.

9.7 Conglomerate companies (those having several different types of businesses under one ownership) often issue complex securities in their borrowing. In particular, convertible bonds seem to be popular financing vehicles for these companies. The reason why conglomerates sometimes use convertibles is which of the following?

A. Convertible bonds carry lower interest rates than non-convertible bonds, other considerations held aside.
B. Conglomerates would prefer to issue equity, but because of the high cost they undertake to issue 'delayed equity' in the form of convertibles, which are subsequently exchanged for shares.
C. Convertible bonds are generally of shorter maturity than non-convertibles.
D. Because convertible bonds diminish the fears of bondholders that the company will take decisions to increase share value and reduce bond value.

9.8 Intercontinental Shipping Company tends to use borrowing heavily in its capital structure, while Clever Advertising plc relies mostly upon equity capital. Assuming that each is representative of the industry in which it operates, which of the following is the primary reason for the difference in their choices of financing?

A. Much of Intercontinental Shipping's value is in tangible assets, the operating characteristics of which would not be much affected by financial distress. The opposite is true for Clever.
B. Advertising companies are simply riskier than shipping companies, and thus are capable of sustaining lower amounts of borrowing in their capital structures so as to avoid bankruptcy in bad times.
C. Since capital structure does not matter to companies, the likely reason for the difference is that random choices were made by managers, and there has been no reason to change them.
D. The tax laws favour borrowing for companies such as shippers, which have large investments in depreciable assets, whilst advertising companies have no such tax incentive to use debt.

Case Study 9.1: R-D Star Productions plc

R-D Star Productions plc is in the entertainment business, producing films and other permanent media. The company has been involved in the industry for quite some time, and is generally regarded as one of the best at what it does. The industry itself, however, is of course a rather risky one, subject to not only the usual vagaries of general economic conditions, but also to the strange and unpredictable swings of consumer taste.

The company is now about to undertake a programme of modernisation and expansion (the 'facilities' programme, it is called) of its production facilities. The total cost of this programme is expected to be £150 000 000, of which £130 000 000 is allocated for plant and equipment and £20 000 000 for working capital. Management of R-D Star is convinced that this programme is absolutely necessary for the long-term success of the company.

The company is faced with the decision as to how the facilities programme is to be financed. There are two alternatives being considered. Both alternatives will use £75 000 000 of cash from the ongoing operations of R-D Star, of which £12 000 000 is from depreciation (the company's total depreciation expenses amount to £35 000 000, the residue of which is needed for maintaining other assets), and £63 000 000 is from retained earnings. R-D Star expects that the facilities project will add £22 500 000 to its 2009 earnings before interest and taxes, which project to £160 000 000 without the facilities project. The company pays taxes at a 52 per cent income tax rate.

The first alternative for raising the additional £75 000 000 not supplied by internal funds is to issue shares to the public. R-D Star's merchant bankers are of the opinion that the company could sell shares to net £21 per share (there are currently 20 000 000 shares outstanding, selling at £21.77 per share).

The second alternative is to borrow the necessary cash. R-D Star could borrow £75 000 000 at 14 per cent interest on a ten-year 'sinking fund' arrangement. (A sinking fund means that the company would pay 1/10 of the principal amount each year, in addition to interest on the balance outstanding during the year.) Lenders would also require that R-D Star pay no more than 16 per cent of each year's after-tax profit as dividends, maintain net working capital at a minimum of £80 000 000, and issue no other debt claims of equal or higher priority without the permission of the lenders. Without actually specifying a number, the lenders have also indicated that a lower interest rate is available if R-D Star is willing to put forward as collateral for the loan its extensive library of vintage films, for which it holds international copyrights.

R-D Star's finance department has provided the following historical data (in £m):

	2003	**2004**	**2005**	**2006**	**2007**	**2008**
Current assets	183.9	252.9	311.6	190.3	116.8	137.7
Total assets	735.5	843.0	890.4	761.3	584.2	550.7
Current liabilities	89.2	139.1	170.4	100.6	50.4	65.0
Long-term debt	281.6	345.4	371.5	350.6	261.7	230.0
EBIT	183.9	252.8	222.6	197.9	134.4	154.2
Dividends	5.9	7.2	10.3	10.3	10.3	10.3

The department has also collected certain information about the industry in which R-D Star operates:

Current assets/Current liabilities = 2.0

EBIT/Total assets = 0.26

Long-term debt/Total assets = 0.35

Dividends/Profit after taxes = 0.08

1 Examine this information in the light of what you have learned in Module 9, and advise R-D Star as to the best choice for financing its facilities project.

Hint: You should be aware that there is no fixed method or template that can be used to solve a capital structure case of this type. Very often an analysis of a situation such as presented here is a matter of weighing the various pros and cons of the alternative financing plans in an unstructured format. Essentially the most important question to be addressed is whether the company can foresee there being problems arising in its plans if it chooses the debt financing alternative (assuming that there are probably tax benefits to be obtained from that type of financing compared with equity financing). Though there are important quantitative dimensions to these issues, many of them can be solved almost by inspection (for example, you should not hesitate to use your own approximations, estimates and forecasts to the extent that they allow the analysis to be efficient without misstating essential relationships). Reference back to the summary of capital structure decision making at the end of the module can help to keep you on track to a decent conclusion of the case. Remember that to think there is a given ‘solution’ to a case such as this is misleading. A good outcome would be for you to uncover the most important factors that would influence the decision, and to make an intelligent and defensible recommendation based upon your appreciation of them.

Module 10

Working Capital Management

Contents

Learning Objectives

A company's 'working capital' is its short-term investments in cash, marketable securities, inventories, receivables (debtors) and short-term financings, bank loans and payables (creditors). Because of the short-term nature of these assets and financings, an entirely different set of financial management techniques is used in dealing with them compared with the techniques introduced in the course to this point. The module begins by discussing why short- and long-term assets and financings present different risk profiles, and makes the case for 'maturity matching' in such decisions. Next, the management of specific asset accounts, such as investments in cash balances, and receivables or collection policies, are discussed, with examples of the techniques that companies use to make such decisions. Finally, the module finishes with an application of working capital management bridging into cash budgeting, or the array of the company's detailed and specific expectations as to what is going to happen to the company's cash flows in the near term. This allows the firm to make plans for short-term borrowing or adjustments in other short-term asset and financing accounts.

10.1 Introduction

The course to this point has been somewhat misleading about the day-to-day activities of financial managers. We have investigated major episodic financial decisions in the form of capital budgeting, capital structure and financing decisions. Making these decisions is a truly important function within the company, but there is another set of actions regularly undertaken by financial managers that is as important and often claims more of a typical financial manager's time. This is the management of the company's working capital.

If working capital management is so important, why have we waited until now to introduce the concepts and techniques of such management? A good question. The answer has to do with the nature of the assets and financing involved, and the way in which the company manages this set of concerns.

First we must define specifically what '**working capital**' is. The most common definition, and that which we shall adopt, is the set of balance sheet items that would be included under **Current assets and Current liabilities**. These include the assets of cash, marketable securities, accounts receivable (debtors), and inventories (stocks); and the liabilities of accounts payable (creditors), short-term borrowings, and other liabilities coming due in less than one year.

This definition makes clear that the distinguishing feature of working capital is that it is concerned with the short term. In contrast, essentially all of the decisions we have dealt with in the course up to this point have been, at least potentially, 'long term' in nature. Capital budgeting, capital structure and financing-dividend decisions deal with assets, product lines and capital sources that can be expected to persist for at least some years. This is not the case with working capital items, whose very nature is that they are much shorter-term assets and financings.

This is the first time we have introduced the notion that financial management may be different for assets and financings that have different terms, or 'lifetimes', and so the issue deserves some elucidation. We shall spend some effort in this module discussing differences between short-term and long-term financial management. As a matter of fact, for these introductory comments, the lifetime of a decision is a very important attribute, and we shall use the word 'term' generally to indicate the lifetime or duration of the asset or financing in question.

We must here be clear about another aspect of working capital. We have implied above that managing these assets and financings is different from the finance you have seen to this point. This is quite true, as we shall soon demonstrate. But keep in mind that the basic economic principles of financial management will apply as well to working capital management. That is, management decisions undertaken that change the amount, timing or riskiness of cash flows to capital suppliers can have effects upon the wealths of shareholders. Because working capital is short-term in nature, we are less concerned with discounting far-in-the-future cash flows to present value, but we are nevertheless as ever concerned with discovering the wealth effects of decisions taken.

Before continuing with this module we urge you to turn to the appendix to the module, which deals with financial and ratio analysis of companies, if you have not encountered those procedures before. To continue with this module it is very important that you have a good familiarity with that material. We have relegated it to an appendix for the reasons that (1) many students have studied this or similar material in other contexts and (2) it is not primarily financial decision oriented, but is more in the nature of background material for such decisions.

10.2 Risk, Return and Term

The distinction between short- and long-term asset and financing decisions now having been raised, it is worthwhile to be more specific about how these differ, and why the question of working capital management is important. This section is not about the management of working capital *per se*, but is designed to help you think about the differences in investing in short- versus long-term assets, and in using short- versus long-term finance.

Suppose we were to ask you to categorise a company's term-specified assets and financings with respect to their risk and return characteristics. For example, are investments in short-term assets riskier than long-term assets? How about their respective return attributes? A complete mapping of the relative risk–return characteristics of investments and financings of a typical company can be instructive as to the appropriate mix of these. (If you are impatient to see the answer, you can look ahead to Figure 10.1, which displays the final result of the discussion immediately below.)

10.2.1 Risk and Rates of Return on Assets by Term

Let us consider each of these in turn. First, are short-term (current) assets of higher or lower risk than long-term assets? The answer to this is reasonably intuitive. Think of the types of assets contained in the two categories. Short-term assets include cash, marketable securities, receivables and inventories. Long-term assets are in the form of plant, equipment, real estate, and certain valuable intangibles.[1] Which is less risky? **Short-term assets are less risky** because they possess a much greater amount of **liquidity**, which means the capacity to be turned into cash both quickly and without much loss in value relative to that of their best use.

The reason short-term assets are more liquid than long-term ones is that short-term assets of any firm are more likely to be easily used by a wide range of other companies for about the same things for which these assets were originally used. (Cash is the same for all companies, as are marketable securities. Receivables can be sold for cash – 'factored' – in short order, and inventories can often be quickly liquidated.)

Long-term assets, on the other hand, tend to be much more specific to the line of business in which the company is involved, and thus less easily converted to cash and much more subject to the uncertainties of a particular industry or market. The greater liquidity of short-term assets means they are of relatively low risk. Thus, within a given firm, its short-term assets are less risky than its long-term assets.

How about expected rates of return from investing in short-term versus long-term assets? Consider the things that determine rates of return. From our discussions in Module 7, your answer should be obvious. Since short-term assets are lower in risk, if asset markets are competitive, short-term assets should also exhibit lower rates of return, and long-term assets higher rates of return. There is nothing wrong with that answer, but there is a more intuitive reason for the ordering of asset returns and terms.

Consider the argument that high rates of return can be generated by undertaking high-risk activities, but can also be obtained by having monopoly power in a market. In other words (as an economist would say), 'monopolistic access' to a market

1 Since we in finance are not limited by accounting conventions as to what can be included in a 'balance sheet', we can mention that long-term assets of the latter type could include managerial expertise, brand names, market power of a monopolistic nature, and other 'assets', all of which definitely have value but may not appear in the formal accounting records of the firm.

would allow the monopolist to earn higher returns than a firm in competition. If we extend that argument down to the level of individual assets, the kind of assets in working capital (cash, marketable securities, accounts receivable and inventory) are unlikely to be the ones in which a company has monopoly power. These assets are common (in fact substitutable to a great extent) to all companies, and are thus 'competitive' and unlikely to carry high 'monopolistic' returns.

On the other hand, long-term assets (plant, equipment, managerial ability, technological expertise, brand names and so forth) are exactly the assets within which the firm's specific and unique 'line of business' attributes will lie. These 'line of business' assets are those that can be unique to the firm and that carry its ability to earn high rates of return. If there are higher rates of return to be earned, we would thus expect these to be coming from long-term as opposed to short-term assets.

Thus the risk and return characteristic of a company's asset investment is that short-term assets exhibit relatively **lower** expected risk and return, and long-term assets relatively **higher** risk and return for a firm.

10.2.2 Risk and Rates of Return on Financings by Term

As with asset investments, a company's financing has risk and return implications based upon term. Consider the short-term financing in which a company engages. This comprises payables, short-term borrowings, and other commitments requiring payment within a year (taxes and pension contributions are examples).

Are short-term financings of high or low risk relative to long-term financings (such as long-term borrowing and equity finance)? The answer depends upon what would happen to the firm if it had a particular type of financing and the company's fortunes unexpectedly took a turn for the worse. With short-term financing the commitments require that the principal amount of the financing be validated by the capital supplier at least once per year or even more often. In other words, the bank must agree to allow the company to renew the loan. The short-term extender of credit to the company thus regularly has the option of saying 'no' to renewing the firm's credit.

This is not the case with long-term finance. Even in bad times, the company is required only to abide by its contractual obligations (for example, paying interest on the loan and maintaining certain operating and financial ratios on the company's financial statements), which would prohibit the creditor from demanding the principal balance of the financing until the term of the financing is finished. Equity claims, of course, require even less in the way of contractual performance by the firm.

The nature of short-term finance is that it tends to be risky in the sense of requiring the firm frequently to renew the principal amounts of financing outstanding; this could (and often does) become a problem in hard times. The withdrawal or absence of short-term finance could cause the company to operate in a suboptimal fashion (foregoing good investment opportunities or being forced to liquidate illiquid assets, for example) when it would be least able to sustain such activities. Long-term finance is less likely to cause such situations, and thus is relatively less risky.

The **rates of return** in financing either short- or long-term activities are best understood by considering the costs of the financing types. (High costs produce low rates of return, and vice versa.)

First we should understand that the costs of finance from this perspective have nothing to do with the level of interest rates. This may be surprising, but the explanation is straightforward. We must first remember that financing markets are very competitive, so that if short- or long-term financing were to carry higher or lower interest rates, that would likely reflect simply the costs to lenders of carrying that type of loan. If a firm got lower interest rates on one type of borrowing, that rate reflects simply the lower costs that the lender incurs from that kind of finance, which is commensurate with rates generally available for that risk in financial markets. Since this risk is something that the firm is unable to offer uniquely, there would be no gain to the firm from the lower rates. In other words, the firm is getting paid for exactly what it provides to the market. The fact is, however, that there is no monolithic tendency for short-term rates to be higher or lower than long-term rates.[2] So interest rates are not the reason for return or cost differences between short- and long-term finance.

What then causes the costs of short- and long-term finance to differ? Basically the costs depend upon **reversibility** differences between the types of finance. When a company undertakes a particular type of financing, it commits itself to service that financing for its agreed term. Naturally, short-term sources of capital are of shorter duration than longer-term ones. In the situations where the company finds itself with unforeseen reductions in the need for financing, short-term finance is dispensed with ('reversed') quickly at the end of its term. Long-term finance is not as easily cancelled.[3] So short-term finance is less costly than long-term finance, because short-term sources are less costly to discontinue when unnecessary. Because lower costs mean higher returns, short-term finance exhibits higher returns than long-term.

Thus the risk and return of the characteristic of a company's financing is that short-term financings exhibit relatively **higher** expected risk and return, and long-term financings are characterised by **lower** risk and return. This is exactly the **opposite** of the risk return characteristic of its assets.

10.2.3 Combining Risk and Rates of Return on Assets and Financings

The above discussion of risk and return in assets and financing is summarised in Figure 10.1.

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2 Actually, long-term rates of interest **on the average** have been higher than short-term rates. Economists have argued why this is so, without any absolute resolution of the issue. Regardless of this fact, competition in the financing industry would cause the level of interest rates on short- and long-term financings to be irrelevant to the choice between them.

3 Long-term debt can be retired prior to its maturity, depending upon 'call provisions' in the loans, and with likely early-retirement penalties. Equity can be repurchased by the firm, but this too carries costs if forced in an unforeseen manner. Short-term finance has no such extra costs. This **reversibility** characteristic is **financing's** counterpart to **asset liquidity**.

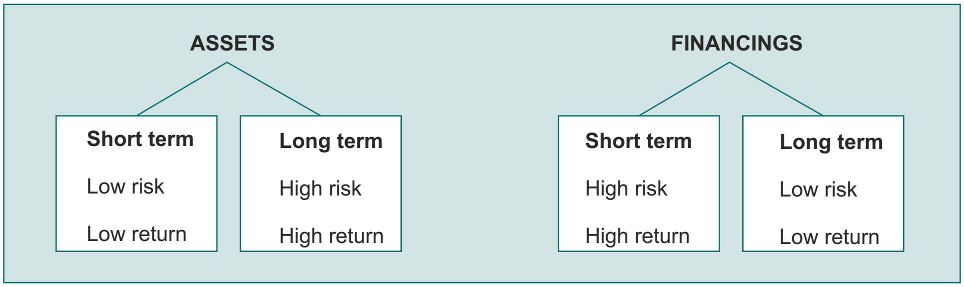

Figure 10.1 Risk and return relationships in assets and financings

A company is continuously faced with the decision as to the best term structure of assets and financings. In other words, given the combinations of short- and long-term choices implied by Figure 10.1, what mix of assets and financings seems most reasonable?

There is an old rule of thumb in finance, which goes: 'Finance short-term assets with short-term liabilities, and long-term assets with long-term liabilities.' This is called **maturity matching**. Figure 10.1 should help to understand why this prescription is a good one. Suppose that a company tends to finance its short-term assets with long-term sources of finance. You can see that this implies a combination of low risk and return on both assets and financings. This tactic produces a very dull company, something like issuing equity capital and using the proceeds to finance government bonds. Though indubitably safe, companies financing short-term assets in this fashion will probably have long-term financing in place when unneeded across time, a costly (low-return) situation.

On the other hand, suppose that the firm tends to finance long-term assets with short-term liabilities. This would be an exciting company indeed. The high-risk, high-return combinations almost certainly imply periods during which the company will find itself hard pressed to validate its quickly-turning-over financing with its illiquid and risky asset base. This situation is the classic and pathological 'cash-flow trap' into which new firms with unseasoned (or no) financial managers regularly fall. A cyclical decline in business causes long-term assets to produce less cash while short-term creditors' principal validation demands persist – an unhealthy situation.

Maturity matching associates low risk-and-return assets with high risk-and-return financing (both short-term), and matches high risk-and-return assets with low risk-and-return financing (both long-term). The result is a mixture of risks and returns that is both potentially profitable and survivable.

Figure 10.2 illustrates the same principles in a slightly different manner. The horizontal axis of the graph depicts the passage of time, while the vertical axis shows the levels of the firm's assets and financings. Note that the required total asset investment varies across time, with most of the variation of asset use being in short-term assets (which have both variable and permanent components). Various choices of the amounts of long-term finance are also shown.

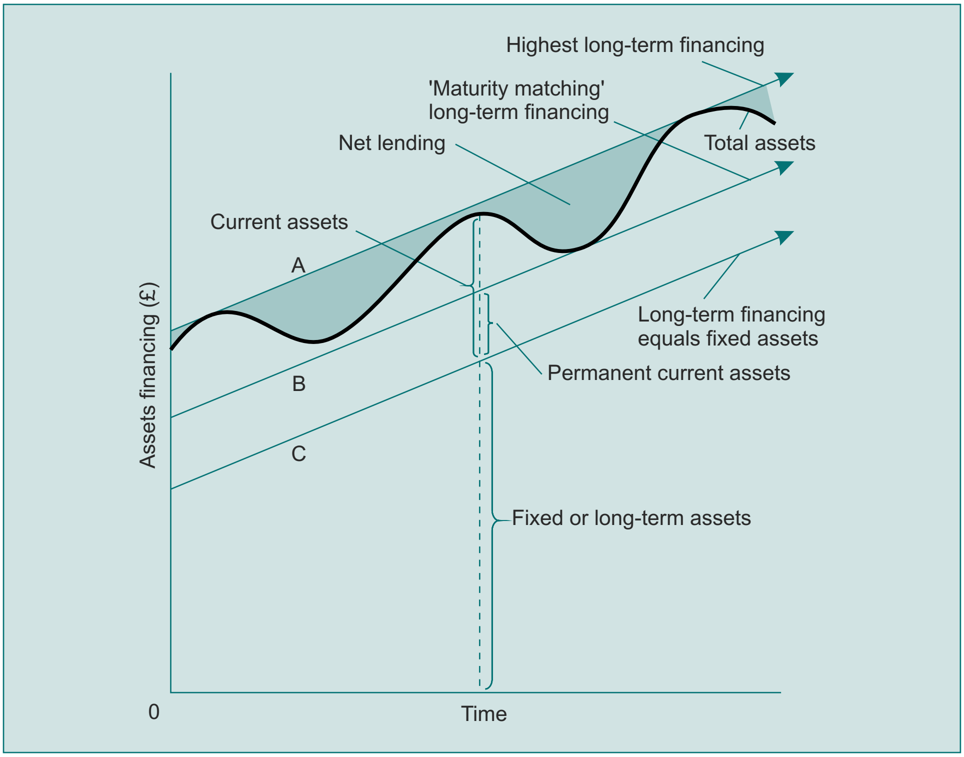

Figure 10.2 **Mix of long- and short-term assets**

The wavy line in Figure 10.2 represents the company's total asset investment, which, although increasing, varies across time. Lines A, B and C depict levels of long-term versus short-term finance. For example, line C shows the company financing its 'accounting definition' assets with the same term of finance. With C as long-term finance, accounting-defined 'fixed assets' are matched with long-term finance, and 'current assets' with short-term. Since some current assets, as the figure shows, have a long-term characteristic (i.e. the company always requires some levels of cash, accounts receivable (debtors) and inventory (stocks), regardless of its immediate market position) there are 'long-term' (permanent current) assets being financed with short-term assets on line C. Line B is true maturity matching, since true long-term investments are matched with long-term financings, as are true short-term assets with short-term financings. Line A uses more long-term finance than matching of maturities would require, with occasional periods when the company would actually lend money to the market awaiting a time when long-term finance could again be used for operational purposes.

Figure 10.2 shows that the greater the amount of low-risk long-term financing, the less is the need for dependence on risky short-term liabilities, but the greater are also the incidences in which the company has unneeded financing. During these periods the firm would be a lender to the capital markets (investing long-term funds in liquid assets), a low-profit activity. The lesson of Figure 10.2 is the same as Figure 10.1: match maturities of assets and financings.

10.3 Management of Short-Term Assets and Financings

Financial managers must make decisions about the firm's investments in short-term assets (cash, marketable securities, accounts receivable and inventories). The analyses that are properly undertaken are more correctly described as 'managing' these assets, as contrasted with the long-term asset decisions that we have seen in prior modules.

We portrayed long-term asset decisions as 'one time' events, wherein a firm decides whether or not to invest in a particular asset based upon that asset's NPV and, aside from potential divestment analyses, the asset then is allowed to function as best it can within the capacities of operating managers to turn a profit.

Financial managers approach the short-term asset decision differently. Rather than considering the desirability of each specific short-term asset, managers adopt **policies** governing the firm's investment in each type. For example, managers might adopt a policy that would replenish the company's cash account by selling marketable securities when the cash account falls below some threshold amount. Here the company does not decide each time how much investment is optimal in cash and securities, but decides initially that the assets should be **managed** in this manner and so manages the accounts thereafter (or until another analysis indicates that the policy should be changed).

In this section we shall examine some of the techniques that managers use to deal with setting policies for short-term asset investment.

10.3.1 Optimisation and Short-Term Investment

There are many specific techniques available for managers to use in dealing with the firm's short-term assets. Some of these techniques are quite simple in concept and application, while others are very sophisticated and complex. But all good techniques share certain common attributes and approaches to the problems at hand. We shall now look briefly at these common attributes of good short-term asset-management techniques.

As with any other investment, the use of short-term assets by a company carries with it certain costs and benefits. A good short-term asset-management process must deal explicitly with these costs and benefits. In general, these techniques attempt to balance costs and benefits in such a manner as to produce the **highest net benefit** or (equivalently) the **lowest net cost** of investing in short-term assets.

What types of benefit and cost exist for short-term asset investment? Consider each of the types of short-term assets appearing in Table 10.1, where the costs and benefits of each asset type are essentially self-explanatory.[4] Cash is necessary to pay

[4] The exception to this may be the costs of a company holding marketable securities. We are assuming here that the market sets prices of these efficiently enough so that the returns provided are commensurate with the risks felt, and thus such investments must be expected to have no positive or negative NPV. Why a 'zero NPV' characteristic of an investment is a 'cost' (given that our previous discussions of NPV would indicate these investments to be at the point of indifference) may not be clear. Such investments are probably costly in the sense that such funds could be paid to shareholders and may thereby avoid some aspects of coming under taxation at both the company and shareholder levels.

anticipated and unanticipated outflows, but does not earn interest.[5] It thus has a 'capital cost'. Accounts receivable (debtors) allow the firm to offer credit to customers, thus increasing sales, but cause cash receipts to be delayed until the customer pays, and sometimes create bad debts. Inventories are necessary so as to avoid stoppages and inefficiencies in the production and selling process, but require that funds be raised or retained to finance these stocks of materials. Further, each time inventory is reordered there are record-keeping and other costs of transacting.

Table 10.1 **Short-term asset benefits and costs**

Asset type	Benefit	Cost
Cash	Highest liquidity	Forgone interest
Marketable securities	Liquidity	Zero NPV
Accounts receivable (debtors)	Increased revenues	Delayed, uncertain cash receipts
Inventories (stocks)	More efficient production schedule, sales flexibility	Capital costs, transaction costs

Managing these asset investments efficiently is a process of balancing their costs and benefits so as to produce the highest company value. All such assets have the common characteristic of benefits increasing over some range of increased usage, and then declining, whilst costs steadily increase with increased usage. Figure 10.3 shows this graphically, and illustrates the notion of optimising by pointing out the usage level consistent with maximum net benefit.

10.3.2 Management of Cash Balances

Though all short-term asset management is similar in concept, the institutional detail and vocabulary of each management process is different. We shall begin the detailed perusal of these processes by examining a company's management of its cash balances. Actually, cash and 'near cash' assets (such as interest-earning bank deposits and short-term marketable securities) are so closely related that their management is usually considered as the same process. As Table 10.1 indicates, these assets confer the 'liquidity' benefit to companies investing in them.

In company operational language, liquidity means that such assets are used for transactions balances, precautionary reserves, anticipatory investment reserves and bank compensating balances. **Transactions uses** of cash are simply the reality that debts must eventually be paid in cash (or equivalently through the transfer of a bank balance in a checking account). **Precautionary** and **anticipatory reserves** recognise that there may be unanticipatable events that require cash (unforeseen price increases in supplies, damaged equipment, and so forth), as well as anticipatable future cash needs of major dimensions (the acquisition of another firm). **Compensating balances** are cash amounts contractually left on deposit with banks (which deposits serve in part to secure the loans made to companies by banks and to help

[5] Some bank accounts may be interest-bearing and be essentially the same as cash.

banks meet reserve requirements). The costs of cash balances are the **transactions costs** of switching between higher and lower interest-bearing securities and accounts, and the **differential interest rates earned.**

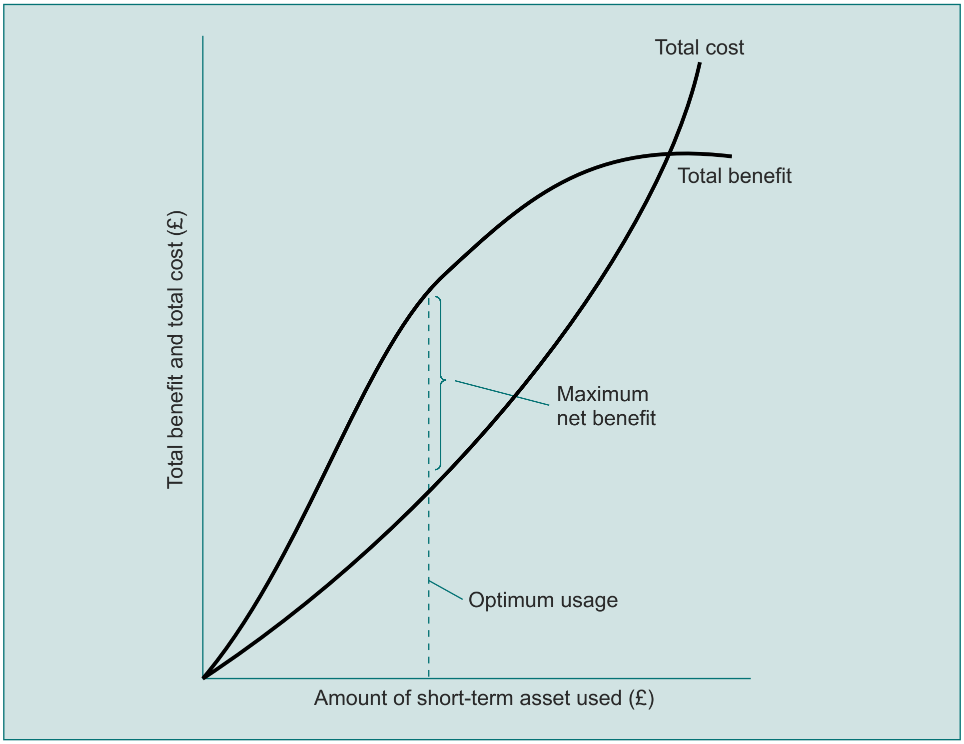

Figure 10.3 **Efficient management of asset investments**

Management of this process requires that there is enough cash on hand to meet the transactions, precautionary, anticipatory and contractual requirements of the company, while at the same time minimising transactions costs and forgone interest. How can such balancing be accomplished?

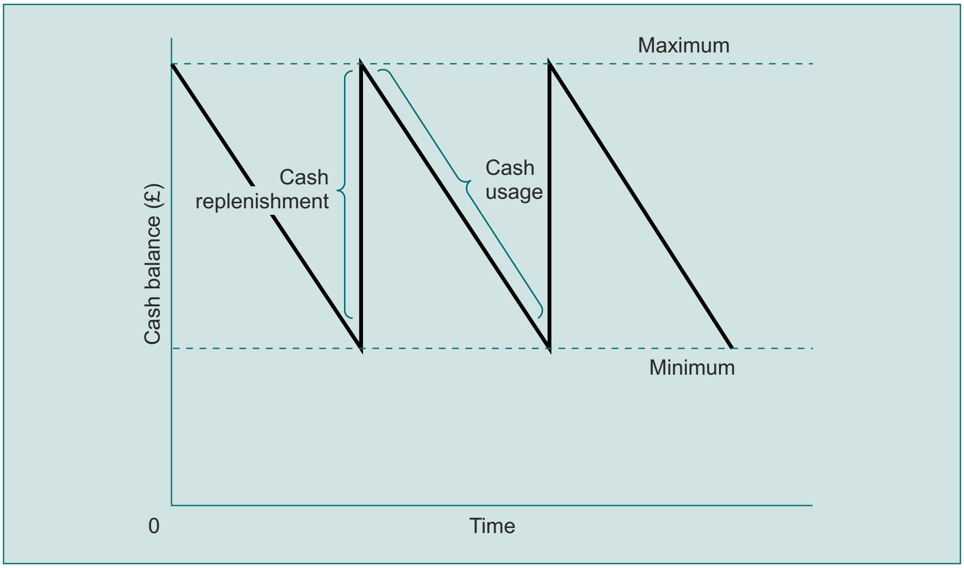

Figure 10.4 **Simple model of cash usage**

To answer that question we must have a picture or model of how cash is used across time in a company. Let us first look at a very simple model of cash usage, wherein a company uses cash quite steadily over time, and requires a fixed minimum balance. Figure 10.4 illustrates the behaviour of cash balances in such a model, where the balances steadily decline as cash is used, down to a minimum acceptable level. Cash is then replenished to some higher balance and again reduced by use, replenished, and so forth. The process is easy to visualise, but it is not clear how the company decides how much cash to add each time it is necessary to replenish. In other words, if a larger amount of cash is added, there will be a longer period when the necessity to replenish again (the 'teeth' in the sawtooth pattern would be larger, with fewer 'teeth' during the year), whilst a smaller replenishment of cash would produce a shorter period between replenishments of cash (smaller but more numerous 'teeth'). How does the firm decide an optimum replenishment amount?

As you would expect, the optimum is a balancing of the costs and benefits of holding cash balances. Let us assume specifically that there is some interest penalty i per cent (interest forgone) in holding cash balances in this very liquid form. Further, assume that each time cash is replenished there will be a (fixed) transaction (reorder) cost of £T. The optimal amount to 'reorder' will be the amount of cash that minimises the net costs of interest forgone and transactions costs during the year. The higher the reorder amount, the fewer the reorder transactions and the lower that cost, but the higher will be the interest forgone because of the higher average cash balances held.

The optimising of cash replenishment amounts in this situation is solved by the following:

$$£r = [(2 \times £D \times £T)/i]^{½} \qquad (10.1)$$

where £r is the optimal amount of cash replenishment, £D is the total annual amount of cash spent by the firm (easily discoverable due to the steady **rate** of cash disbursements) and i and £T are as defined above.[6]

To illustrate, assume that CBA plc expects to disburse £500 000 cash during the year, that each transaction of switching from interest-bearing securities to cash (regardless of amount) costs £100, and that the interest rate differential between the cash and interest-bearing security holding is 10 per cent. Using Equation 10.1, the necessary replenishment amount is:

$$\begin{aligned} £r &= [(2 \times £500\,000 \times £100)/0.10]^{½} \\ &= £31\,622.78 \end{aligned}$$

Thus CBA plc, when it reaches its minimum acceptable cash balance for precautionary purposes, switches £31 662.78 from interest-bearing securities or deposits into cash. Given the interest-rate differential, the rate of cash usage, and the transaction cost, the total of forgone interest and transaction costs are thereby minimised (the solution requires that CBA plans to do this 15.81 times per year – $£500\,000/£31662.78$ – or about every 23 days). For any other replenishment amount the increase in one of the costs would more than offset the reduction in the other, thus increasing the total costs of cash holdings. From the perspective of Figure 10.3, the solution above is the point of maximum net benefit.

The cash-management problem outlined above is a good introduction to the kinds of techniques available to deal with such decision situations, but tends to oversimplify the actual behaviour of a company's cash usage over time. Companies do not usually exhibit steady rates of net cash expenditure across time. Because receipts of cash tend to be seasonal or cyclical, and expenditures tend to be 'lumpy' (occasional large outlays interspersed with smaller outlays), the untended pattern of cash balances of a company would tend to look something like Figure 10.5. Here, the large increases and decreases in cash balances reflect the more common experience of firms in ongoing businesses. Obviously this is a more complex situation than we have observed above. How does optimal cash management take place in this context?

The situation we have described immediately above is actually one in which the manager cannot know in advance either the directions or amounts of cash balance changes in the future.[7] In such a situation, the best the manager can do is specify a probability distribution of **potential** cash balance changes. As it turns out, this allows us to solve a much more realistic cash-management problem. Here the cash balance across time behaves as a 'random walk' in the sense that the change actually occurring is randomly drawn from the (assumed normal) probability distribution of all possible changes that could occur in the cash balance. You can think of the change in cash balance as being determined by a blind monkey choosing a particular coloured ball from a barrel of balls, each colour representing a particular change in

6 The formula, called the 'economic order quantity', was well known in inventory management and first applied to cash balances by William Baumol, an economist at Princeton University. The derivation of formula 10.1 is unnecessary for our purposes, as long as the trade-offs within it are appreciated.

7 This illustration is taken from Miller, M.H. and Orr, D. (1966) 'A Model of the Demand of Money by Firms', *Quarterly Journal of Economics*, 80 (August), 413–35.

cash balance, and the colours represented in the barrel proportionally to the probabilities in the distribution (the monkey is trained to replace each ball after it is examined).

Look at Figure 10.6 – the solution to this cash management problem. As with the original example, there is a minimum amount of cash (we shall call it £*M*) below which the balance is not allowed to fall (due to precautionary or compensating balance reasons), and this is a 'lower bound'. You can see that there are two other levels also indicated in Figure 10.6, namely £*U* (the 'upper bound') and £*R* (a 'return point').

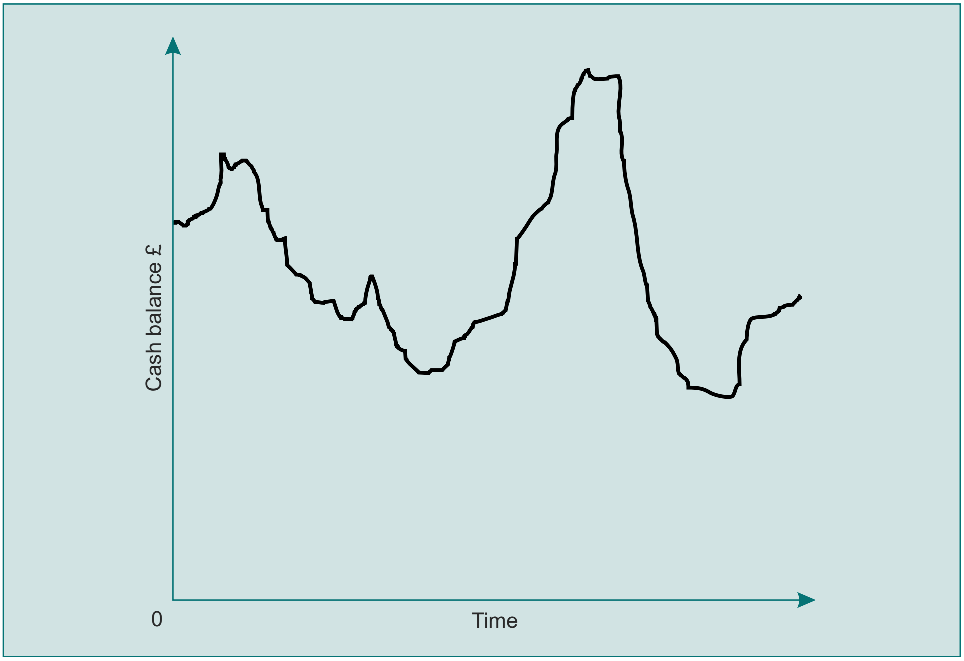

Figure 10.5 **More usual pattern of company cash balances**

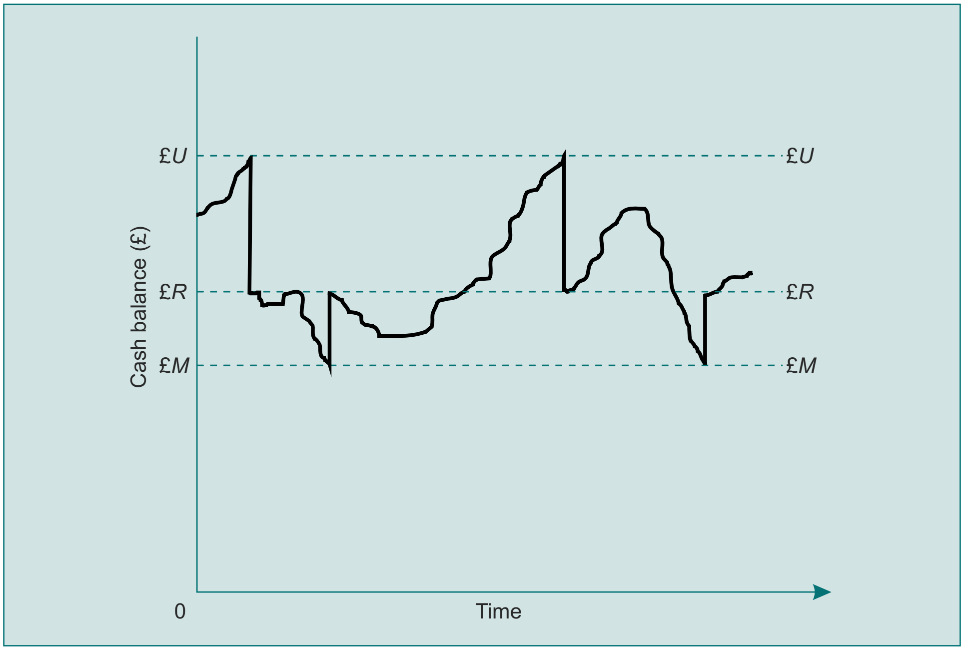

Figure 10.6 **General solution to the cash management problem**

Here is how the process works. The company's cash balance wanders up and down across time. When, due to whatever reason (high cash expenditures, low receipts, etc.) the cash balance falls to £*M*, enough interest-bearing securities are cashed to return the balance to £*R*. When cash balances increase to £*U*, securities are bought with excess cash to bring the balance again to £*R*. If the upper limit £*U* and return point £*R* are well chosen, the costs of maintaining the cash balance are minimised.

What do we mean by a 'well chosen' upper limit and return point? Specifically the formula:

$$£R = [(3 \times £T \times s^2)/4i]^{\frac{1}{3}} + £M \tag{10.2}$$

chooses such a return point £*R*, where £*T* is again the cost per transaction and *i* the interest differential, and here s^2 is the variance of the changes in the cash balances, from the probability distribution mentioned earlier. (As with the prior formula, it is unnecessary for our purposes to derive the proof herein.) Specifically, if the odds are as great for an increase as for a decrease in cash balance, and if the amount of increase or decrease in cash balance is expected, by the probability distribution, to be £*c* for each of the number of times *t* cash balance can change per day, $s^2 = £c^2 \times t$.

A numerical example will help to clarify the point. Suppose that ZYX plc incurs a transaction cost of £*T* = £100 per instance of shifting between cash and interest-bearing securities, has an expected cash-balance change of £*c* = £4000 per hour, experiences an interest differential of 10 per cent (per year, which is equivalent to

0.10/365 per day), and has a minimum cash balance of £*M* of £250 000. Assuming an eight-hour working day, $s^2 = (£4000)^2 \times 8 = £128000000$, and:

$$
\begin{aligned}
£R &= [(3 \times £T \times s^2)/4i]^{\frac{1}{3}} + £M \\
&= [(3 \times £100 \times £128\,000\,000)/(4 \times 0.10/365)]^{\frac{1}{3}} + £250000 \\
&= £282723.12
\end{aligned}
$$

Thus if a £250 000 minimum balance is required, when the lower limit is reached, £32 723.12 of encashing of interest-bearing securities is cost-minimising.

What about reaching the upper limit? As it turns out, the upper limit is part of the solution itself, and is

$$£U = £M + 3(£R - £M) \qquad (10.3)$$

In this example, £$\text{U} = £250000 + 3(£32723.12)$, or £348 169.36. ZYX should, therefore, when its cash balance reaches £348 169.36, purchase 2(£R-£M) or £65 446.24 of interest-bearing securities (which returns the company to £ *R*), or, when its cash balance reaches £250 000, sell £32 723.12 of such securities (also returning the company's cash balance to £*R*). Following the above process will allow the company to minimise its costs of carrying cash (or deposits with lower interest rates than other liquid securities) while maintaining the cash balances at the levels required for running its business lines.

One note of caution should be voiced about the cash-balance decisions described above. Of all the financial decisions that companies make, the cash-management decision is the one that has been most heavily influenced by technological change – and that continues to be so. Eventually, electronic transfer mechanisms for transactions could relegate much of the type of analysis above of little practical significance (when transfers into and out of interest-earning securities of high liquidity can be made instantaneously, automatically and at little or no cost).

The two techniques of managing cash balances mentioned above are simply a sampling of the types of optimisation processes that managers can bring to bear. There are many others, some more and some less sophisticated than these. As was mentioned in the introduction to these methods, however, all good techniques will make an explicit balancing of the benefits and detriments of investing in the asset in question.

10.3.3 Management of Receivables

In addition to cash and marketable securities, companies also usually find it necessary to hold **accounts receivable (debtors)** and **inventory (stocks)**. The management of inventory is essentially similar to cash and securities management (but with costs of shortages considered explicitly), so will not be further discussed.

Table 10.1 shows the essential trade-offs that managers face when deciding what level of accounts receivable to hold. A higher level of receivables promises more credit sales and thus more customers willing to purchase, but at the same time it portends a longer wait (average collection period) until the actual receipt of cash from the sale, and a higher likelihood of never being paid (bad debts). Any good analysis must attempt to trade off between these effects so as to maximise the NPV of the receivables investment.

Unlike the cash management situation illustrated above, there exist no received comprehensive techniques in accounts receivable management that promise simultaneously to combine all relevant costs and benefits. Why this is so is a function of both the complexity of the decision and the lack of sustained attention that finance researchers have focused upon this problem. There are, however, various approaches to solving the problem partially.

One issue that is constantly raised about the investment in receivables is the deterioration in the quality of customer credit accompanying an increase in the amounts owed to the company. This has led to methods of discerning customer creditworthiness. There are many credit-reporting agencies that supply such information to companies (at some cost), and a company's own records of customer payment histories can yield useful information about the likelihood of a customer's paying. Further, the credit-granting decision has been raised to a fine art by banks and other institutions in that line of business. Some companies have found it beneficial to perform sophisticated statistical analyses, using methods such as **discriminant analysis** to examine various customer attributes and thereby make predictions as to creditworthiness.

Such studies always produce an improvement in the capability of a company to filter out bad customers. However, the astute financial manager recognises that at some point rejecting the marginal customer ceases to be worthwhile. That is, the point at which such efforts should be stopped is when the incremental expenditure for search and evaluation exceeds the expected gain from discriminating among good and bad customers. This point will eventually be reached in all such analyses.

Suppose, for example, that a credit-reporting company is willing to offer credit analyses on prospective customers for £25 apiece. Further, suppose that the odds of getting a bad customer (figured, perhaps, from historical records) is 5 per cent, that the average sale generates revenues of £500 and costs of £400, netting £100 profit, and that the company expects to have 100 potential customers during the period in question. If the company does not review customers' creditworthiness and simply accepts everyone, the expected profit from each sale is given as follows:

$$\begin{aligned}\text{Expected profit} &= (\text{Number of good customers} \times \text{Profit per customer}) \\ &\quad +(\text{Number of bad customers} \times \text{Loss per customer}) \\ &= (95 \times £100) + (5 \times -£400) \\ &= £7500\end{aligned}$$

The loss as a result of a bad customer is the company incurring £400 of costs without any revenues. With 100 customers, the company's profits are expected to be £7500 during the period.

If, however, a credit analysis is undertaken on customers at a cost of £25 apiece (and assuming it is always correct in discerning good from bad), the expected profit per customer is:

$$\begin{aligned}\text{Expected profit} &= (\text{Number of good customers} \times \text{Profit per customer}) \\ &\quad +(\text{Number of bad customers} \times \text{Loss per customer}) \\ &\quad - \text{Costs of credit analyses} \\ &= (95 \times £100) + (0 \times -£400) - £2500 \\ &= £7000\end{aligned}$$

Here the company would be better-off not attempting to discriminate between good and bad customers. (*Caution*: The company would be making a bad mistake by letting this information become public. As soon as it was generally known that no analysis of creditworthiness was done, you can be certain that the ratio of bad to good customers would rise.)

There is one approach to the management of receivables that has some generality, and is not terribly difficult to implement.[8] It seeks to calculate the NPV associated with a proposed change in credit terms for a company.

Suppose that the Cashflow Software Company (CFS) was considering an alteration in its credit terms to customers. Currently CFS sells 100 packages per day of software at a price of £250 and variable costs of £175 per package. Customers on the average pay for these 35 days from sale (this is the 'average collection period' mentioned above, and defined in the appendix to this module, which deals with financial analysis). This is generated by a credit policy of 'net 30' which means payment is due 30 days after sale; obviously some customers pay late. There is a bad-debt loss of 3 per cent of sales, and the daily interest rate is 0.04 per cent (or about 15.7 per cent per annum). Working capital other than receivables (the net of cash, inventories and payables) is about 20 per cent of sales.

CFS is considering altering its credit policy to be 'net 35' which will have the predictable effect of increasing sales, but generating a longer collection period and somewhat greater bad debts. Specifically, CFS expects that sales will increase to 125 per day, the collection period will lengthen to 40 days, and the bad debt proportion rise to 4 per cent of sales. Should CFS undertake this change or not?

The answer can be calculated by finding the difference in present values of the company's cash inflows and outflows under the two policies.[9] As the outcome below yields a positive NPV, the change in approach by CFS is worth doing:

8 *See* Hill, N.C., Emery, G.W. and Satoris, W.L. (1985) *Essentials of Cash Management: A Study Guide*, National Cash Management Association.

9 The entries into this calculation are essentially self-explanatory; you can see the appropriate influences of bad debts, the number of days of collectibles lengthening, and unit sales effects simply by comparing the numbers in the calculation with those in the text's explanation of CFS's situation. The only part that might be confusing is the working capital adjustment which appears in the last two lines of the calculation. In it, the change in sales affects the working capital instantaneously (second to last line), while the eventual recouping of the working capital is recognised in the last line of the calculation.

$$\begin{aligned}
\text{NPV} &= \text{Change in PV of sales receipts} - \text{Change in variable costs} \\
&\quad -\text{Change in working capital investment} \\
&= \frac{(125 \times £250)(1-0.04)}{(1.0004)^{40}} - \frac{(100 \times £250)(1-0.03)}{(1.0004)^{35}} \\
&\quad -[(125 \times £175) - (100 \times £175)] \\
&\quad -0.20 \times [(125 \times £250) - (100 \times £250)] \\
&\quad +0.20 \times [\{(125 \times £250)/(1.0004)^{40}\} - \{(100 \times £250)/(1.0004)^{35}\}] \\
&= £1206.30
\end{aligned}$$

There are many possible variants of this situation. Companies may, for example have repeat customers whose probabilities of paying or expected times of paying differ from new customers, for example. Other variants of analysis might attempt to balance discounts given to customers for early payment against the interest (capital) costs of extending credit. Or managers might institute a process of deciding when to undertake collection of bad debts and when to write off a bad receivable. We shall not attempt to give you explicit equations to deal with each situation because in doing so the complexity would outweigh their pedagogical value; the construction of the example and solution above is sufficient to show a good basic approach to dealing with such situations, and it can be expanded or contracted as necessary. Your mastery of earlier modules (such as Module 1 to Module 4) will stand you in good stead here.[10]

10.3.4 Management of Short-Term Financings

Just as a company should decide upon its optimal management process for short-term assets, it is important that short-term financings be planned as well. In aggregate, short-term financing is best considered a function of two phenomena, namely the company's **line of business** and **maturity matching** with short-term assets, as described earlier in this module. This means that the firm should plan to use short-term financings as required by the business lines it pursues, and with the condition that such financings are best done in association with short-term investments, for balancing of risk and return.

The resulting prescription would seem to be a process that, once programmed, could operate almost automatically. Given a desired increase or reduction in short-term assets due to business conditions, the matching of maturities would call for an approximately equal adjustment in short-term financing. Actually, with the exception of the choice among short-term financing sources (bank loans, payables extensions and other deferrals) that is not a bad description of the payables management process.

There are, however, policies that must be set for such a system to be run with any hope of optimality. One consideration is the extent to which payables (creditors) are managed efficiently as a separate unit, and the other is the type of information that would allow a financial manager to set up a well functioning short-term asset/financing system. We shall deal first with the former.

---

[10] The use of 'profits' rather than cash-flow measures in this example is not necessarily inconsistent with earlier arguments against accounting numbers, given the short-term nature of the investments.

One of the important sources of short-term financing that companies use regularly is credit extended by vendors. This is essentially the mirror-image of a firm's own accounts receivable: one company's receivable is another's payable.

When a firm takes advantage of credit extended by a vendor, there is usually a set of payment conditions associated with this 'trade credit' as it is called. These payment conditions almost always include a time when final payment is due, but also a (shorter) time during which payment would produce a discount from the market price of what has been bought. In other words, the price quoted implicitly assumes that payment will not be made until the final due date. (One could argue that it thereby includes a capital cost of the vendor's financing of the purchase price until receipt of the cash from the purchaser.) If payment is made by a particular (earlier) date, the purchaser gets a 'discount' (though an economist might argue that 'discount' to be simply a recognition that the vendor has not incurred the capital cost mentioned above).

Usually these payment terms are described by a phrase such as '2/10 net 30' which signifies that there is a 2 per cent discount which can be taken for payment within ten days of invoicing, that payment beyond that date is at full market price (no discount), and that full payment is expected within 30 days. Why is it important that the process of managing short-term financings deal with this condition of terms of sale?

Consider the decision as to whether the discount in the above example should be taken by the firm, that is whether payment should be made on the tenth day or the thirtieth day.[11] If payment is made, the company obtains a 2 per cent price reduction. Is this good or bad? By what standard should this be judged?

The proper standard is the cost of financing the money that would be used to pay early, or the interest rate on such short-term borrowing. In essence, by not taking the discount the company is 'financing' an extra twenty days of credit from its vendor, and by taking the discount the company is financing the twenty days of credit by borrowing elsewhere for that period. Suppose that the company could borrow at a short-term rate of 12 per cent so as to take the discount. Should they do so in this example?

The answer is clearly 'yes'. Though to the uninitiated a 12 per cent cost may seem higher than a 2 per cent discount, we must remember that the 2 per cent discount would be obtained by paying twenty days early whereas the 12 per cent interest cost is stated on an annual (365 day) basis. The annual equivalent interest cost of a 2 per cent interest rate for 20 days can be found by the intimidating:

$$i = \left[1 + \left(\frac{0.02}{(1-0.02)}\right)\right]^{(365/20)} - 1 = 44.6\%$$

This formula probably looks a bit strange. Some of it is intuitive; Module 1's logic would seem to imply that annualising a 2% interest rate for twenty days gives us a

[11] Naturally, payment would not be made prior to the tenth if the discount is to be taken, nor prior to the thirtieth if not, for the obvious reason of money's time value. Remember also that one transaction may not be very important in absolute value, but since it is a policy being developed, the cumulative effects of choosing the wrong policy can be quite large.

number of compounding periods (365/20), so the exponentiation is familiar. But why not simply then raise (1 + 0.02) to $(365/20)^{th}$ power, if the interest rate is 2%? The reason that would be wrong is the nature of the discount being offered on the payable. To give an example, suppose that the amount owed at 30 days is £100, so the 2% discounted amount is £98.00. The interest rate implied by that relationship is clearly $(100/98)\text{-}1 = 0.02041\ldots$, which is the same as $[0.02/(1\text{-}0.02)]$. Therefore, when a particular percent 'discount' is given for early payment, the annualisation formula is:

$$i = \left[1+\left(\frac{\text{'discount \%'}}{(1-\text{'discount \%'})}\right)\right]^{(365/\text{'discount days'})} - 1 \qquad (10.4)$$

Note that the implied twenty-day interest percentage exceeds the stated discount (2.041% versus 2%). This will be generally true in all such 'discount' transactions, including those 'discounted' loans where the 'interest rate' is stated as a percentage of final principal due rather than on the amount borrowed.

Returning to the example, borrowing at the bank at 12 per cent interest so as to take the discount is superior to incurring an implicit 44.6 per cent interest rate from not taking a 2 per cent discount.

Financial managers must ensure that clear guidelines for the comparison of implicit short-term financing costs are available for these decisions. This is an example of the 'policy' nature of short-term financial management.

To see the potential importance of this, suppose that, in the above example, the firm averaged outstanding payables of £250 000, and the company's financial manager had not set correct guidelines for taking discounts on payables. Over the course of a year the extra twenty days (per thirty) of financing at 44.6 per cent instead of 12 per cent produces a differential of 20/30 × £250 000 × (0.446 − 0.12) = £54 300 per annum. This is quite enough to pay a skilled financial manager for a whole year (or reason enough to dismiss an unskilled one).

In addition to payables, there are other sources of short-term finance. For example, as the above implies, banks regularly lend to companies, and these loans often have contractual provisions (compensating balances, line of credit agreements and take-up requirements) which affect the costs of such loans. The true cost of most such agreements can be deduced by invoking the financial economics which you learned in Module 1, along with a careful reading of the contract. Some more complex borrowing contracts have option characteristics which require more sophisticated valuation processes than this course can offer.

10.4 Cash Budgeting and Short-Term Financial Management

We have argued that management of short-term assets and financings is an ongoing process, and have illustrated this with certain general propositions about the economics of such activities, and examples of their application. Even with this, and the recognition (in the module's appendix) that application of ratio analysis to a company's financial statements can serve to portray its history and condition relative to industry or other standards for such investments and financings, the story of short-term financial management is not complete. Short-term financial management is best pursued within the context of a company's **cash budgeting**.

What is cash budgeting? The name itself is perfectly descriptive: cash budgeting is the setting-forth of the company's expectations for its inflows and outflows of cash over some future time period, usually near-term. It is easy to see why cash budgeting is important to the short-term financial management process. Consider the examples of cash management and receivables management that we examined above. Each of those required a cash projection of some type based upon future expectations of the company's operations. (For cash management it was the net changes in cash balances across time, and for receivables management it was the extent of cash collections of sales.) Without such a projection of a company's cash position, there is no raw data upon which to base management decisions.

Table 10.2 is an example of a simple quarterly cash budget of CSH plc. Though there are many ways to construct an acceptable cash budget, all must have the essential portrayal of cash coming into and going out of the company. CSH's budget begins with the mechanism of generating cash from sales. Rows 1 to 7 indicate the manner in which prior periods' receivables are paid in cash, and cash is generated from this period's sales (our assumption is that 80 per cent of this quarter's sales are paid in cash, and the rest – 20 per cent – is paid next quarter, with no bad receivables). Sales here are estimated by, say, the marketing department.

Table 10.2 Cash budget CSH plc (£000s)

		Qtr 1	Qtr 2	Qtr 3	Qtr 4	Explanation
1.	Receivables at start of period	40.00	26.00	19.00	22.00	7 for prior period
2.	Sales during period	130.00	95.00	110.00	250.00	Estimated
3.	Collections from:					
4.	Current period sales	104.00	76.00	88.00	200.00	80 per cent of 2
5.	Prior period sales	40.00	26.00	19.00	22.00	20 per cent of prior-period sales
6.	Total collections	144.00	102.00	107.00	222.00	4 + 5
7.	Receivables at end of period	26.00	19.00	22.00	50.00	1 + 2 − 6

		Qtr 1	Qtr 2	Qtr 3	Qtr 4	Explanation
8.	Cash inflows:					
9.	Total collections	144.00	102.00	107.00	222.00	6
10.	Other sources	16.00	5.00	5.00	4.00	Estimated
11.	Operating cash inflow	160.00	107.00	112.00	226.00	9 + 10
12.	Cash outflows:					
13.	Payments of payables	100.00	65.00	45.00	55.00	Estimated
14.	Other operating costs	26.00	19.00	22.00	50.00	Estimated
15.	Capital expenditures	0.00	100.00	0.00	0.00	Estimated
16.	Taxes	6.50	4.75	5.50	12.50	Estimated
17.	Interest	3.00	3.00	3.00	3.00	Estimated
18.	Dividends	0.30	0.30	0.30	0.30	Estimated
19.	Total cash outflow	135.80	192.05	75.80	120.80	Sum of 13 to 18
20.	Net operating cash flow	24.20	−85.05	36.20	105.20	11 − 19
21.	Cash balance at beginning of period	10.00	34.20	8.00	44.20	See explanation in text
22.	Net operating cash flow	24.20	−85.05	36.20	105.20	20
23.	Cash balance at end of period	34.20	−50.85	44.20	149.40	21 + 22
24.	Minimum cash balance required	8.00	8.00	8.00	8.00	Estimated
25.	Necessary financing required	−26.20	58.85	−36.20	−141.40	Negative of 23 − 24

Note that row 6 indicates the amounts of cash that CSH expects to take in each quarter from selling its product. These amounts, along with 'other' sources of cash inflow (not including borrowing or capital infusions, but perhaps tax refunds or asset sales), are combined to produce the total operating cash inflow on line 11.

Next, CSH estimates its outflows (note that payment of payables is in cash terms; you can assume that an analysis such as lines 1 to 7 has been performed to generate these cash-flow estimates). Each of the entries in lines 13 to 18 must be the actual cash payment expectation and not an accrual number from accounting statements.[12] Total cash outflows appear in row 19, with the net of operating inflows and outflows appearing in row 20.

Finally, rows 21 to 25 deal with the company's cash situation, and potential shortfalls or surpluses of cash. Row 21 shows the beginning cash balance, which is combined with row 22, the quarter's net operating cash flow, to generate row 23, the end-of-period estimate that would appear if no other action was taken. Row 24

12 For example, companies often report taxes on income statements as accruals of tax liabilities rather than actual cash taxes that are expected to be paid.

is the minimum cash balance with which CSH is comfortable, and subtracting that from row 23 indicates in row 25 any need for additional financing. If row 25 is negative there is a potential cash surplus, which could be invested, or (as in Quarter 2) if row 25 is positive the company must plan to cover the shortfall.

There are several methods by which a shortfall of cash can be covered. It may be best to plan on some additional short-term borrowing in the second quarter. (You can see that it could be paid down by the fourth quarter, given the excess cash amounts shown in the third and fourth quarters.) Or the company might choose to lengthen its payables, or sell ('factor') some receivables, or sell other assets (perhaps marketable securities). Given that the source of cash shortfall is a large capital-asset outlay, the company may wish to consider a long-term borrowing or even an equity issuance.

Which is the best tactic to cover the cash shortfall? This can be decided with the aid of the techniques introduced in this module, and perhaps with material in the capital structure module and ratio analysis that you have studied already. But the importance of a good cash budget is clear: without some detailed knowledge of when and how much cash the company needs or has in excess, there could be at best an embarrassing last-minute plea to the bank (not a confidence-producing exercise), or (worse) other detrimental effects to the business. Excess cash, while not as immediately pathological, is destined to be a low-earning asset.

Most companies have found it useful to have an on-line computer-based cash-budgeting process, which is updated daily or even hourly. The advent of relatively inexpensive computer hardware and 'off-the-shelf' software for this purpose removes any excuse for ignoring such an important dimension of company financial operations.

Learning Summary

In this module we have surveyed the financial management of companies' working capital, its short-term assets and its financings.

Short-term assets and financings are in a sense no different from the longer-term phenomena that we have studied in other modules. Companies can consider the optimal amounts and types of such assets and financings from the perspective of their effects upon shareholders. These short-term decisions are in the abstract no different from any others. However, the best methods of dealing with these decisions are unique. Working capital is best considered as being 'managed' in an ongoing process rather than being decided upon in discrete terms.

We discovered that the rule of thumb urging maturity matching of assets and financings was a good way to see some of the important differences between short- and long-term commitments that a company makes in its asset portfolios and financings, and in particular the risk and return differences among them.

Most importantly, we found that working capital management involves two levels of activity:

1. the 'hands-on' application of management techniques to specific asset and financing decisions (e.g. how much cash to have on hand at any one time, what

credit conditions to set for customers to buy on credit, or whether or not a discount should be taken by paying a supplier before a bill is finally due); and
2. the optimal setting of policies for such decisions, so that each of these small decisions is almost automatically determined by the company's well considered policy.

Finally, working capital management cannot be considered independent of a company's cash budget. Without a plan as to the generation and usage of cash across time, the best management techniques have no data upon which to operate. An up-to-date or 'real time' cash budget is a necessity for effective working capital management.

10.5 Appendix to Module 10: Financial and Ratio Analysis

The purpose of this appendix is to deal briefly with the financial and ratio analysis of companies for those students who may not have encountered such material before. For others it will provide a useful reminder of the principles involved. The contents of this appendix are essential to help you to:
1. understand the need for ratio analysis of external financial statements;
2. appreciate the strengths and weaknesses of ratio analysis;
3. learn the major ratios commonly used, namely liquidity ratios, profitability ratios, capital structure ratios and efficiency ratios; and
4. expand the external analysis to an integrated chart analysis of external and internal accounting information.

10.5.1 Introduction

The emphasis in this appendix is laid on the meaning and basic construction of the principal financial statements prepared by a company: its profit and loss account, its balance sheet and its statement of its sources and uses of funds, all seen from the perspective of a manager who must understand what the firm's accounting numbers tell him. The manager is here instructed in how to dismantle the financial statements in such a way that when one piece of information is matched with another, the manager is in a position to (1) compare the performance of the company this period with last, (2) compare the performance of the company with that of competitors, and (3) detect areas of weakness to which managerial efforts can be directed. Interpreting financial statements is the logical extension to constructing and reading them.

Some preliminary points should help to clarify the picture:
1. Absolute figures as reported in financial statements do not tell the reader very much. For example, to be told that Trossachs Metals made £28 240 000 profit before taxation is not a useful piece of information by itself. This fact must be related to, say, the sales of £628 582 000, which produced the profit, or to the capital employed of £284 506 000 (the net assets in the balance sheet). This relationship between two separate items such as profits before tax and sales is normally expressed as a percentage, in Trossachs' case 4.49 per cent.
2. Percentages, or ratios, permit easy comparison between different corporate entities: divisions within the company, companies within the same group, or

companies within the same industrial sector. A common benchmark used for the purpose of comparison is the data produced by the various 'clearing houses' for industrial statistics. Some of these organisations are operated for profit in the private sector, while others are run by government departments. Typically, a company like Trossachs Metals would contribute its own financial and production statistics to the organisation of inter-firm comparisons, on a strictly confidential basis, and receive in return the computed average percentages and ratios for key performance indicators taken from the information supplied by many of Trossachs' competitors in the metals sector. These percentages and ratios allow Trossachs to gauge its own performance against the industrial sector as a whole. Unless Trossachs' operations by themselves dominate the industrial sector, the average figures (which include Trossachs' statistics) will be a reasonable guide to Trossachs' management as to how their company has performed over the period covered by the industrial averages (normally half-yearly). Note, too, that most organisations for inter-firm comparisons supply upper- and lower-quartile figures in each key area; this allows individual companies to position themselves more accurately than is possible with the overall average percentage or ratio.

3. A word of warning about ratios: they can be used to suppress poor absolute figures. For example, a company could report, say, a 300 per cent growth in turnover, which gives the impression of success and progress. However, the basis for such a calculation could be a sales figure of £100 when all similarly positioned competitors are enjoying sales of £1000. In this situation ratios disguise the real picture.
4. Just as no two sets of company financial statements are the same in style and layout, so too one finds wide diversity in approaches to financial analysis. Analysts do not argue over their individual definitions of, say, 'profit' or 'capital employed' or 'debt'; they recognise that agreement would be impossible because no two companies define these items in the same way. They do agree, however, that they must apply their definitions **consistently** for the same company for many accounting periods; otherwise an inter-period comparison would be invalidated. The organisations that publish inter-firm comparisons usually define in detail the key headings to allow subscribing companies to adjust, where necessary, their own figures and percentages.

A **financial ratio** is a relationship between two quantities, on a company's set of financial statements, that is derived by dividing one quantity by the other. The purpose of using ratios is to reduce the amount of data to workable form and to make it more meaningful. This goal is defeated if too many ratios are calculated – for there are hundreds of ratios that **could** be calculated. The manager must learn which combination of ratios is most appropriate in a specific situation, and must remember that ratios seldom provide conclusive answers but rather lead the manager to ask the right questions and (sometimes) give clues as to possible areas of strength or weakness.

For the purposes of this appendix the following groups of ratios will be explained and discussed using the financial statements of Trossachs Metals provided near the end of this appendix (*see* Section 10.5.8).

1. **Liquidity ratios**. These are designed to measure a company's ability to meet its maturing short-term obligations.
2. **Profitability ratios**. These are designed to measure management's overall effectiveness: does the company control expenses and earn a reasonable return on funds committed?
3. **Capital structure ratios**. These ratios are divided into two groups:
   (a) those that examine the asset structure of the company, and
   (b) those that analyse the financing arrangements of the company's total assets, in particular the extent to which the company relies on debt. This group of ratios is generally known as the **gearing** ratios.
4. **Efficiency ratios**. These give an indication of how effectively a company has been managing its assets.

10.5.2 Liquidity Ratios

The first concern of managers is ensuring the short-term survival of the company. Their attention is thus inevitably drawn to liquidity: is the company able to meet its short-term maturing obligations?

Ratio No. 1: Current Ratio

The current ratio is computed by dividing current liabilities into current assets. Since current assets are cash or assets expected to turn into cash within the current year, and current liabilities are those that must be paid within the current year, a company is in a good position to meet its current obligations if current assets exceed current liabilities by a comfortable margin.

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Trossachs' current ratio} = \frac{281266}{148325} = 1.89 \text{ times}$$

$$\text{Metals industrial sector} = 1.60 \text{ times}$$

Trossachs is thus in a marginally healthier liquidity position than the average of all the other companies in the same industrial sector.

It is often stated as a rule of thumb that a current ratio of 2 indicates a sound financial situation. Unfortunately, this rule can be misleading: one should not conclude that a company can pay its bills because it has a current ratio greater than 2 times. Nor should one conclude that a ratio of less than 2 times indicates that a company cannot pay its bills. For example, a company may have current assets of £4000 (cash £2000 + stock £2000) and current liabilities of £2500 (all creditors), giving a current ratio of 1.6 times. In the next accounting period, by paying off many of its creditors in cash, the company could improve this ratio to 4 times (stock £2000; creditors £500). Since nothing is known about the marketability of stock, it could be claimed that the liquidity position of 1.6 times is healthier than 4 times in this example.

Note: In ratio analysis, it is important to study the **trend** of the ratios calculated rather than attempting to arrive at sound conclusions based on one accounting period's ratios. Readers should calculate Trossachs' 19X1 ratios for themselves on the basis of the information given in Section 10.5.8 and check them against the answers given in the text below.

Ratio No. 2: Quick Ratio (sometimes called the Acid Test)

Analysts recognise that current assets include inventory that is sometimes slow-moving and not so readily realisable into cash as is implied by the current ratio. The quick ratio therefore backs inventory out of the calculation, thus providing a more rigorous test of the company's ability to pay its maturing obligations.

$$\text{Quick ratio} = \frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}}$$

$$\text{Trossachs' quick ratio} = \frac{281266 - 149710}{148325} = 0.87 \text{ times}$$

$$\text{Metals industrial sector} = 1.02 \text{ times}$$

The industrial average is slightly greater than the widely accepted rule of thumb of 1 times (meaning that no matter how great the losses incurred on the sale of inventories, the company would still have enough left to pay its obligations) but the same caution should be heeded about this rule of thumb as was noted with the current ratio.

The position of Trossachs itself gives some cause for further examination. The ratio reveals that the group of companies has not enough cash to meet its obligations if called upon quickly to do so – an unlikely event, one would imagine. It might appear, therefore, that Trossachs is carrying too much inventory compared with its competitors, and management should give immediate attention to the reasons for this and to the likelihood of reducing the high level as quickly as possible. Further analysis of the order book and production plans may shed more light on this potential problem area.

10.5.3 Profitability Ratios

If liquidity is management's first concern for short-term survival, then profitability gives them an insight into long-term survival. Profitability is the result of many managerial policies and decisions. Profitability ratios give some answers as to how effective this managerial action has been.

Ratio No. 3: Profit Margin

Typically, analysts use net profit **after** taxes divided by sales to determine profit margin. However, the tax charge in company accounts is so distorted by a wide variety of fiscal provisions that it often bears little relation to the profit that has actually been earned in the accounting period. Some analysts therefore are beginning to move towards net profit **before** taxes as the numerator of the fraction, although this appendix will use the more conventional after-tax figure. Note that extraordinary items are omitted from the calculation because, by definition, they do not represent normal profit.

$$\text{Profit margin} = \frac{\text{Net profit after taxes}}{\text{Sales}}$$

$$\text{Trossachs' profit margin} = \frac{22547}{630906} = 3.58\%$$

$$\text{Metals industrial sector} = 3.67\%$$

Although Trossachs' profit margin does not seem very different from the other companies' in the metals sector, it should be borne in mind that many industries turn in profit margins each accounting period that are almost constant. It may be that Trossachs' figure is indeed low and should lead management to examine in the first instance their pricing structure (too low?). If their prices are found to be in line with their competitors', then a detailed analysis of costs and overheads should be embarked upon.

Ratio No. 4: Return on Total Assets

Profit is not only a function of sales but also closely related to the assets employed by the company to produce the profit.

$$\text{Return on total assets} = \frac{\text{Net profit after taxes}}{\text{Total assets}}$$

$$\text{Trossachs' return on total assets} = \frac{22547}{432831} = 5.21\%$$

$$\text{Metals industrial sector} = 4.72\%$$

Note that the whole balance sheet figure for total assets has been selected for the fraction: a case can be made for using just the productive assets, omitting such items as goodwill and investments, but in this industrial sector the average is calculated on **total** assets. It can be seen that Trossachs is apparently 'working' its assets harder than its competitors – which is, of course, beneficial for the company.

Ratio No. 5: Return on Specific Assets

It is possible to relate profit to any particular asset if management considers that that asset is crucial to the results of the company. In Ratio No. 2, the quick ratio, it was seen that Trossachs was probably carrying too much stock compared with its competitors. The ratio, return on inventory, may confirm that fear:

$$\text{Return on inventory} = \frac{\text{Net profit after taxes}}{\text{Inventory}}$$

$$\text{Trossachs' return on inventory} = \frac{22547}{149710} = 15.06\%$$

$$\text{Metals industrial sector} = 16.11\%$$

Thus it would appear that the initial conclusion about overstocking was correct: Trossachs' competitors earn a higher return on their inventories.

An interesting additional point emerges: Trossachs earned a higher return on total assets than the rest of the metals industry but a lower return on the specific asset of stock. Therefore it follows that Trossachs is earning a significantly higher return on all its other assets, taken together, than its competitors. Management should consider the possible reasons for this. If Trossachs is 'out of step', is it because the land and buildings figure in the balance sheet is recorded at an unrealistically low figure? Or is it because they have less investment in fixed assets and more in inventory to give them a trading advantage?

Ratio No. 6: Return on Owners' Equity

Perhaps the most important profitability ratio is the one that relates the profit earned to the capital (contributed and accumulated) of the ordinary shareholders of the company. If the company fails to earn a decent return for its shareholders, the share price will fall, thereby prejudicing the company's chances of securing additional capital or loans on beneficial terms.

$$\text{Return on owners' equity} = \frac{\text{Net profit after taxes}}{\text{Owners' equity}}$$

$$\text{Trossachs' return on owners' equity} = \frac{22547}{231433} = 9.74\%$$

$$\text{Metals industrial sector} = 9.39\%$$

Readers should remember, however, that investors are more concerned with relating their returns with the current market price of their shares rather than with the 'book value' of their investment. Another point to note: one normally considers an increase in this ratio to be a good sign. It could in fact signal danger if the firm relies too heavily on debt. But, again, further analysis could indicate whether or not a disproportionate increase in debt caused the improvement in this ratio.

A concluding note on profitability: management possesses more detailed figures than are published in annual reports, and can therefore analyse detailed costs and revenues better than an outsider is capable of doing. A favourite technique to use is the **one hundred per cent statement**. A model is set out below for Company EFG.

	Year ended 31 December 19X1		**Year ended 31 December 19X2**	
	£	%	£	%
Sales	20 000	100	25 000	100
Cost of goods sold	16 480	82.4	20 375	81.5
Gross profit	3 520	17.6	4 625	18.5
Selling expenses	610	3.0	875	3.5
General and administrative expenses	315	1.5	500	2.0
Profits before taxes	2 595	13.1	3 250	13.0
Taxes	1 295	6.5	1 625	6.5
Net profit after taxes	1 300	6.6	1 625	6.5

Sales are set at 100 per cent and each item is calculated as a percentage of sales. It can be seen that Company EFG's percentage savings in cost of sales in 19X2 was lost in the increases in selling expenses and general and administrative expenses. If EFG's management can tighten the controls in this area of spending, then they will enjoy a percentage increase in net profits, something that has escaped them this year.

At the conclusion of the sections setting out the liquidity and profitability ratios it would be helpful to summarise the findings on Trossachs Metals. The liquidity ratios indicate a possible liquidity problem while the profitability ratios indicate a healthy profit situation. Thus it can be concluded that Trossachs' questionable cash position, as revealed in this ratio analysis, is not due to lack of profits. Further analysis may reveal the problem, if indeed Trossachs has a real problem of liquidity at all.

10.5.4 Capital Structure Ratios

The first group of capital structure ratios examines the asset structure of the company: for example, has the company the correct proportion of fixed assets to total assets? As in all ratio analysis, variations can be developed around the principal relationships. The main ratio is given first.

Ratio No. 7: Fixed to Current Asset Ratio

$$\text{Fixed to current asset ratio} = \frac{\text{Fixed assets}}{\text{Current assets}}$$

$$\text{Trossachs' fixed to current asset ratio} = \frac{99043}{281266} = 35.21\%$$

$$\text{Metals industrial sector} = 39.22\%$$

Again the picture is received of Trossachs having more current assets (stock?) than its competitors. If management can pinpoint a satisfactory reason for this – for example a stocking-up in advance of a major sales drive, or a rise in raw material price, or a risk of irregular and short supplies – no action need be taken to trim this investment.

The second group of ratios analyse how the company's assets have been financed. **Gearing** ratios (sometimes known as **leverage**) measure the contributions of shareholders with the financing provided by the company's creditors and other providers of loan capital. Creditors look to the owners' equity, or owner-supplied funds, to provide a margin of safety. If the owners have provided only a small proportion of total financing, the risks of the enterprise are borne mainly by the creditors. On the other hand, by raising funds through debt, the owners gain the benefits of maintaining control of the firm with a limited investment of their own. In such a situation, if the company earns more on the borrowed funds than it pays in interest, the return to the shareholders is magnified. For example, if assets earn 14 per cent and debt costs 11 per cent, the differential of 3 per cent accrues directly to the shareholders. However, if the return on the assets falls to 9 per cent, the differential of 2 per cent between earnings and interest charges must be paid out of profits attributable to shareholders.

Companies with low gearing ratios have less risk of loss when the economy goes into a recession, but they also have lower returns when the economy takes off again. Conversely, highly geared companies are very exposed in hard times (due to a high fixed-interest charge on reducing profits) but enjoy high profits in boom times.

For example, a company has a choice of two capital structures:

	Structure A	**Structure B**
Owners' equity	800	500
10% debt	200	500
	1000	1000
Gearing	20% (low)	50% (high)

The impact of these structures on the return on owners' equity can be seen if three profit-before-interest figures are compared.

Profit before interest	100	50	150
Structure A:			
Interest	20	20	20
Equity earnings	80	30	130
Return on equity	10%	3.75%	16.25%
Structure B:			
Interest	50	50	50
Equity earnings	50	–	100
Return on equity	10%	–	20%

The fall in equity earnings is less dramatic under the low-geared structure (from 10 per cent to 3.75 per cent) than it is under the high geared structure (10 per cent to nil). Conversely, when earnings rise, the equityholders in the highly geared structure enjoy a higher return (20 per cent compared with 16.25 per cent).

Ratio No. 8: Debt Ratio

This ratio measures the proportion of assets that are financed by debt. Debt includes current liabilities and all loans and bonds.

$$\text{Debt ratio} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Trossachs' debt ratio} = \frac{148325 + 44250 + 2428 + 1745}{432831} = 45.45\%$$

$$\text{Metals industrial sector} = 40.00\%$$

The composition of the total debt figure is Current liabilities + Loans + Investment grants + Deferred taxation. The ratio for Trossachs is significantly higher than the industrial average. Trossachs may find it difficult to borrow additional funds without first asking shareholders to provide more capital themselves: there is a danger of Trossachs' management engaging in highly speculative activity with this high gearing ratio because if it is successful, the percentage return to the owners will be substantial.

Ratio No. 9: Times Interest Earned

This ratio switches to the profit and loss account in order to measure the gearing position and margin of safety in relation to earnings. It measures the extent to which earnings can decline without the company finding itself unable to meet the annual interest costs. Should the company default on these payments, then creditors can bring legal action against the company.

$$\text{Times interest earned} = \frac{\text{Profit before tax} + \text{Interest charges}}{\text{Interest charges}}$$

$$\text{Trossachs' times interest earned} = \frac{28240 + 14924}{14924} = 2.89 \text{ times}$$

$$\text{Metals industrial sector} = 3.47 \text{ times}$$

Again, the pattern of potential overgearing is continued: Trossachs' competitors have greater cover for their interest payments. The profitability ratios indicate that Trossachs' earnings are marginally superior; therefore the adverse position in the times interest earned ratio is caused solely by the overgearing observed in the debt ratio. Note that because interest charges (set out in note 1 to the profit and loss account) are paid before tax is computed, the 'profit before tax' figure is selected for this ratio. If Trossachs attempted to borrow more funds, it would lower even further the cover for interest; and so it is unlikely that lenders and providers of funds would put in more money to Trossachs before its shareholders contributed further capital.

10.5.5 Efficiency Ratios

These ratios (sometimes referred to as **activity** or **turnover ratios**) are designed to assist managers and outsiders in judging how effectively a company manages its assets. All the ratios involve comparisons between the figure of sales and the investment in various assets; they assume that management strives to keep a sensible balance between sales and such items as inventory, debtors and fixed assets.

Ratio No. 10: Inventory Turnover

$$\text{Inventory turnover} = \frac{\text{Sales}}{\text{Inventory}}$$

$$\text{Trossachs' inventory turnover} = \frac{630906}{149710} = 4.20 \text{ times}$$

$$\text{Metals industrial sector} = 5.32 \text{ times}$$

Once again, the overstocking position of Trossachs is highlighted; if the company was holding less stock, the turnover of stock in any year would be quicker. It can be seen that Trossachs is holding almost three months' stock while competitors hold only 2.3 months' worth. By doing so, Trossachs is locking up cash in inventory (remember the relatively poor cash position as revealed by the quick ratio) – cash that could be used more profitably. Also, the higher the level of inventory the greater the risk of some part of it becoming obsolete. Trossachs' management should give urgent attention to this matter.

Two additional points should be noted:

1. Sales occur over the entire year, whereas the inventory figure is taken from the closing balance sheet. It could be that Trossachs' pattern of trade dictates a stocking-up at the end of the year; some adjustment should be made to this inventory figure if sufficient information in that regard is available. However, if this seasonal pattern is universal in the metals industry, no adjustment is required in order to compare Trossachs' stocking level with its competitors.
2. Different industries have significantly different inventory turnovers. Metals and engineering are slow because of the length of the production process; retail shops are very rapid, indicating that they need low inventories to service their sales levels.

Ratio No. 11: Average Collection Period

Sometimes called days' sales outstanding, the ratio is calculated by dividing debtors by the daily average of sales (assuming, say, a 360 or 350 working-day year). The ratio represents the average length of time that a company must wait after making a sale before receiving cash.

$$\text{Average collection period} = \frac{\text{Debtors}}{\text{Sales per day}}$$

$$\text{Trossachs' average collection period} = \frac{111948}{630906/350} = 62 \text{ days}$$

$$\text{Metals industrial sector} = 47 \text{ days}$$

Trossachs' sales outstanding are very much greater than its competitors and represent slack credit management. The effect of debtors in any business, necessary though they may be, is that the company who is owed the money is financing the debtors' working capital. Sometimes, if the credit management gets out of control, the selling company has to borrow money in the short term to finance its customers' working capital! This poor ratio in Trossachs' analysis must be tightened if Trossachs wants to restore the reasonable level of liquidity enjoyed by its competitors.

Ratio No. 12: Fixed Assets Turnover

Fixed assets are acquired by a company to produce sellable products: it is therefore not unreasonable to relate the investment in fixed assets to the level of sales generated from them. Again, as in all ratio analysis, the resultant figure is only meaningful when compared with both the previously calculated ratios for the same company and the average ratio for the rest of the industrial sector.

$$\text{Fixed asset turnover} = \frac{\text{Sales}}{\text{Fixed assets}}$$

$$\text{Trossachs' fixed asset turnover} = \frac{630906}{99043} = 6.35 \text{ times}$$

$$\text{Metals industrial sector} = 5.88 \text{ times}$$

As in Ratio No. 4, return on total assets, Trossachs is seen as having a superior ratio than its competitors. One reason may be that Trossachs' management is more efficient at 'working' its assets than other companies, that is Trossachs requires less assets to produce the same level of sales as others. Another reason, however, may be due to a less than realistic valuation of some of its fixed assets, notably land and buildings. If Trossachs' competitors have more up-to-date land valuations in their ratio, then they may report lower turnover rates.

10.5.6 Other Possible Ratios

Companies use many other ratios, not described here. Provided they are used consistently so as to reveal long-term trends, managers should feel free to develop any ratio that they consider gives them insight into their businesses. For example, a popular series of ratios relates sales, profits, capital employed and remuneration with the number of employees in the company. Given the worldwide basis of Trossachs' activities, however, such calculations would yield fairly meaningless results.

The twelve ratios calculated have revealed some possible strengths and weaknesses about Trossachs' financial position. Despite good profitability, the company might experience some difficulty meeting its obligations during 19X3. Also, the high level of inventory gives cause for concern, as does the relatively heavily geared status of the financial structure.

It should be repeated that while the above ratio analysis does not provide conclusive answers, it does indicate to management and outside analysts which areas require further investigation and provide some clues as to what the problems might be. The weaknesses of ratio analysis are not so much the fault of the ratios but of the managers and analysts who misuse them.

10.5.7 Financial Analysis and Internal Accounting: an Integrated Approach

The foregoing sections on ratio analysis were based on the published financial statements of our imaginary Trossachs Metals. Thus the analysis developed therein is equally applicable to managers and external users of the financial statements. Managers, however, also have access to internal accounting information that permits them to engage in a more penetrating analysis. The **chart approach** to financial analysis combines the activity and profitability ratios that have already been described with the detailed costs and revenues obtainable from the internal accounting system.

The **chart approach** (*see* Figure 10.7 below for a typical chart layout) starts with the premise that the percentage return on investment is the proper measure of effectiveness of the employment of capital and that this end result should be presented against a background that emphasises turnover and gross profit margin. The right side of the chart develops the turnover ratio. This section shows how net current assets (cash, debtors, stocks and creditors) added to fixed assets gives total investment. Total investment divided into sales gives the turnover of investment.

The left side of the chart develops the profit margin on sales. When the asset turnover ratio is multiplied by the profit margin on sales, the product is the return on total investment in the firm. This can be seen from the following formula:

$$\frac{\text{Profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Total investment}} = \frac{\text{Profit}}{\text{Total investment}}$$

If management considers that the return on total investment is not satisfactory, it can trace back through the chart in an effort to locate the contributory factors to the poor return. But, again, it must be stressed that the chart will only point managers in the direction of potential problem areas – charts do not solve the problems themselves.

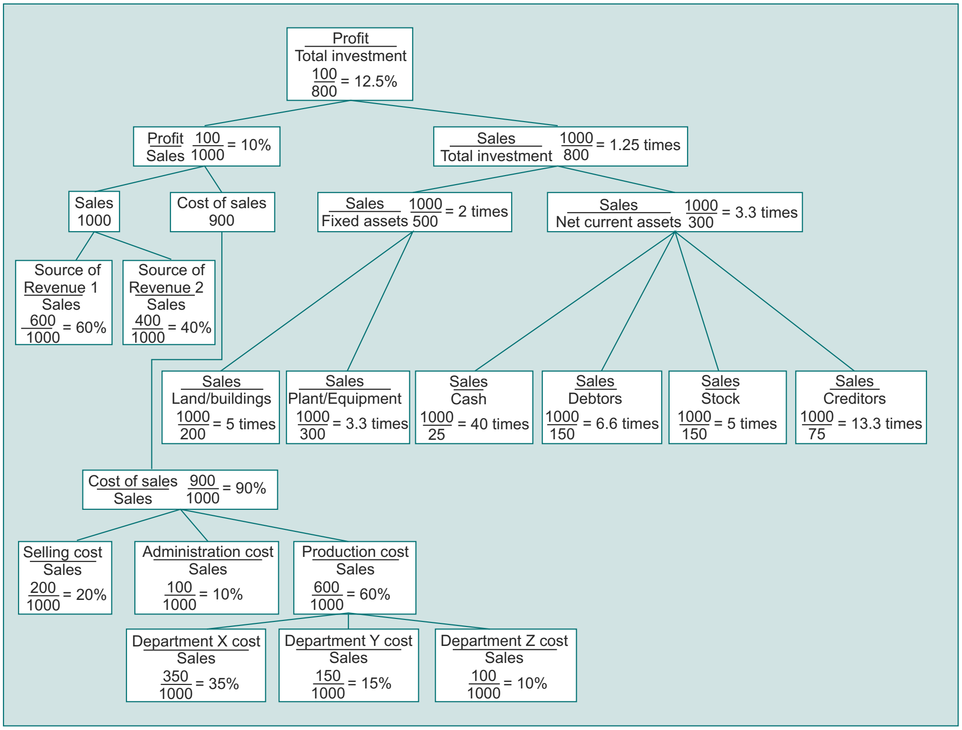

Figure 10.7 Chart analysis of the Cairngorm Company

As in the earlier sections of this module, the real value from a ratio analysis comes from comparing the calculated ratios with:

1. the budgeted ratios for the same period, or
2. the actual ratios for previous periods, or
3. the ratios of other companies or divisions in the same group.

It is unlikely, given the detailed and confidential nature of the costs and revenues required to complete the chart analysis, that companies will be able to compare their own performance with those of competitors, except perhaps through the medium of industry-based average ratios.

Example of Chart Analysis

Because it is not possible to gain the detailed information required for this analysis from the published accounts of Trossachs Metals plc, the following analysis will be made from a simple set of figures for the Cairngorm Company:

The Cairngorm Company

Sales:	£	
Product A	600	
Product B	400	
	£1000	
Cost of sales:		
Production	600	Department X £350
Selling	200	Department Y £150
Administration	100	Department Z £100
	£900	
Profit	100	
Fixed assets:		
Land and buildings	200	
Plant and equipment	300	
	£500	
Net current assets		
Cash	25	
Debtors	150	
Stock	200	
Creditors	(75)	
	£300	
Total net assets (investment)	£800	

The chart analysis of the Cairngorm Company is given in Figure 10.7.

10.5.8 Trossachs Metals Financial Statements

Trossachs Metals plc
Abstract of Annual Report for year ended 31 December 19X2

Accounting Policies

1. *Basis of Consolidation*
   The consolidated accounts include the audited accounts of the Holding Company and its subsidiaries made up to the end of the financial year, together with the Group's share of the audited results of its associated companies. A list of major subsidiary and associated companies appears on page X [not reproduced here].
   Where subsidiary and associated companies are acquired or sold during a year, the Group profit and loss account includes their results from the date of acquisition or to the date of their disposal.
2. *Basis of Profit-Take on Contracting Activities*
   Profit is taken as work progresses. Unless a more conservative approach is necessary, the percentage margin on each individual contract is the lower of the margin earned to date and that forecast at completion, taking account of agreed claims. Full provision is made for all known or expected losses at completion immediately such losses are forecast on each contract. In addition, no profit is taken on contracts in certain overseas territories if it is considered that there may be repatriation problems.
3. *Sales*
   Sales represent amounts invoiced to outside customers, except in respect of contracting activities where they are the value of work carried out during the year for outside customers including estimates of amounts not invoiced. They include the Group's share of sales by associated companies and exclude value added tax.
4. *Depreciation*
   Depreciation on property, plant and equipment is calculated so as to charge to operating profit the cost of assets after deducting government grants over their expected working lives on the straight-line basis, adjusting where necessary for usage and obsolescence.
   The annual rates of depreciation are as follows: Freehold land, nil; Freehold buildings, 2.5%; Leasehold land and buildings: more than 40 years unexpired, 2.5%, less than 40 years unexpired, equally over life of lease; Plant and equipment, at appropriate rates varying from 4% to 25%.
5. *Leased Plant and Equipment*
   An amount equivalent to the cost of leased assets as if they had been purchased is included under property, plant and equipment and depreciated in accordance with the Group's normal depreciation policy. The deemed capital element of future lease payments is included under loans. Deemed interest calculated on the reducing-balance basis is included under finance charges in the profit and loss account.

6. *Stock and Work-in-Progress*
   In general, stocks are valued on the following bases:

Raw Materials:	at average cost
Work-in-progress and finished goods:	at cost of production including related works overheads

   For this purpose cost of production excludes selling, distribution and administrative costs. If the cost of any category of stock, calculated in this way, exceeds its net realistic value, it is written down to the lower figure. The major exception to these principles occurs in the case of one subsidiary, which carries a base stock of 1600 tonnes of copper that, at 31 December 19X2, was stocked at £950 000 (19X1 £590 000) below the lower of cost and net realisable value.
   The main categories of stock are as follows:

	19X2	19X1
Refined and wrought copper and other metals	£72 192 000	£65 445 000
Fabricated products and components	83 974 000	91 444 000

   Work-in-progress in manufacturing activities and all stocks are valued at the lower of cost and net realisable value except for the UK base stock of copper which is valued at a fixed price below net realisable value. Cost incorporates conversion costs incurred, including depreciation and other related fixed and variable production overheads.
   In contracting activities, work-in-progress is valued at estimated sales value. Provision for all known or expected losses to completion and applications for progress payments are deducted from the valuation of work-in-progress to the extent of the valuation on each contract.
7. *Research and Development*
   Research and development expenditure is charged against profits, except for expenditure on fixed assets, which is depreciated in the normal manner.

Consolidated Profit and Loss Account for the year ended 31 December 19X2

	Notes		Thousands of £s 19X2	Thousands of £s 19X1
Turnover			630 906	610 609
Cost of sales			496 247	476 044
Gross Profit			134 659	134 565
Distribution expenses		59 198		56 912

			Thousands of £s	Thousands of £s
Administrative expenses		47 221		43 117
			106 419	100 029
Profit on ordinary activities before taxation	1		28 240	34 536
Tax on profit on ordinary activities			5 693	3 262
Profit on ordinary activities after taxation			22 547	31 274
Minority interests			765	554
			21 782	30 720
Profit on metal stocks after taxation	1		647	1 935
Profit before extraordinary items			22 429	32 655
Extraordinary items after taxation	2		1 063	(3 289)
Profit attributable to members of Trossachs Metals plc			23 492	29 366
Dividends			9 387	9 169
Retained profit for year	3		14 105	20 197
Earnings per share				
excluding extraordinary items			10.8p	15.7p
including extraordinary items			11.3p	14.1p
Movement on Reserves				
At 31 December 19X1 (19X0)			168 213	148 016
Retained earnings for year			14 105	20 197
Amounts taken direct to Reserves			(3 034)	–
At 31 December 19X2 (19X1)			179 284	168 213

Abridged Notes on Consolidated Profit and Loss Account

1. *Profit before Taxation*

   Profit before taxation excludes the effect of changes in the book value of unsold refined and wrought metal stocks due to fluctuations in the price of copper: in 19X2 there was a metal profit, after adjustment for taxation, of £647 000 (19X1 £1 935 000). These items are separately brought into the profit and loss account in arriving at the earnings applicable to shareholders in Trossachs Metals plc.

   Profit before taxation is after charging £2 672 000 for redundancy and reorganisation costs. In addition, the following have been (charged)/credited in arriving at profit before taxation.

		19X2 £000s	19X1 £000s
Share of profits, less losses of major associated companies		725	1 925
Other investment income		4 992	3 185
Interest payable		(14 924)	(11 158)
Depreciation		(12 450)	(11 100)
Audit fees and expenses	– UK	(459)	(406)
	– Overseas	(301)	(259)
Directors' emoluments		(360)	(290)
Hire of plant and machinery		(2 743)	(2 844)

Distributions received from investments in major associated companies amounted to £246 000 (£642 000).

Interest payable	19X2 £000s	19X1 £000s
Bank borrowings	12 364	9 123
Other loans:		
Not wholly repayable within 5 years	1 959	1 795
Wholly repayable within 5 years	601	240
	14 924	11 158

2. *Extraordinary Items*

	19X2 £000s	19X1 £000s
Decrease in value of net fixed assets of overseas subsidiaries due to changes in exchange rates	–	(1 842)
Profit on sale of investments	1 052	–
Other	11	(1 447)
	1 063	(3 289)

3. *Retained Earnings*

	19X2 £000s	19X1 £000s
By company	4 500	18 900
By subsidiaries	8 750	2 703
By major associates	855	(1 406)
	14 105	20 197

(A published set of accounts would include further notes on investment income, directors' emoluments, composition of taxation charge, and the dividends per share and dates for payment of interim and final dividends.)

Balance Sheets as at 31 December 19X2

Thousands of £s 19X1	19X2		Notes		Thousands of £s 19X2	19X1
		Fixed assets				
–	–	Intangible assets	1		31 324	31 120
–	–	Tangible assets	2		99 043	90 894
184 592	206 058	Investment in subsidiaries	3		–	–
2 324	649	Investments			21 198	26 682
186 916	206 707				151 565	148 696
		Current assets				
–	–	Stocks		149 710		150 866
516	450	Debtors		111 948		133 883
13 834	9 758	Cash and short-term deposits		19 608		15 065
27 848	23 156	Dividends receivable from subsidiaries		–		–
42 198	33 364			281 266		299 814
		Creditors: amounts falling				
13 200	20 463	due within one year	4	148 325		167 011
28 988	12 901	Net Current Assets			132 941	132 803
215 914	219 608	*Total Assets less Current Liabilities*			284 506	281 499
		Creditors: amounts falling due				
40 237	38 748	more than one year	4	46 678		43 848
		Provision for Liabilities and Charges				
–	–	Deferred Taxation		1 745		8 455
					48 423	52 303
–	–	Minority Interests			4 650	8 885
175 677	180 860				231 433	220 311

Thousands of £s				Thousands of £s	
19X1	19X2		Notes	19X2	19X1
		Capital and Reserves			
52 098	52 149	Called up share capital		52 149	52 098
123 579	128 711	Reserves	5	179 284	168 213
175 677	180 860			231 433	220 311

Abridged Notes on Balance Sheets

1. *Intangible assets – goodwill*

	£000s
At 31 December 19X1	31 120
Arising on acquisition of minority interests in subsidiaries	204
At 31 December 19X2	31 324

Goodwill, which is shown at cost less amounts written off, is almost wholly the excess of the purchase consideration paid for the acquisition of subsidiaries over their net tangible assets at the date of acquisition.

2. *Fixed Assets*

	Land and Buildings			Plant			Total
	Gross book value £000s	Depreciation £000s	Net book value £000s	Gross book value £000s	Depreciation £000s	Net book value £000s	Net book value £000s
At 31 Dec 19X1	43 723	14 261	29 462	150 794	89 362	61 432	90 894
Capital expenditure	3 012	–	3 012	21 932	–	21 932	24 944
Sales, demolitions and adjustments	(986)	(74)	(912)	(7 944)	(5 692)	(2 252)	(3 164)
Exchange adjustments	(962)	(353)	(609)	(2 492)	(1 920)	(572)	(1 181)
Depreciation for year	–	1 316	(1 316)	–	11 134	(11 134)	(12 450)
At 31 Dec 19X2	44 787	15 150	29 637	162 290	92 884	69 406	99 043

Notes:

i. The gross book value of land and buildings and plant is made up as follows:

	19X2		19X1	
	Land and buildings	**Plant**	**Land and buildings**	**Plant**
	£000s	**£000s**	**£000s**	**£000s**
At cost	31 803	149 561	30 016	136 273
Revaluations by certain subsidiaries principally in 19X0	12 984	12 729	13 707	14 521
	44 787	162 290	43 723	150 794

ii. The gross book value of land and buildings comprises:

	19X2	**19X1**
	£000s	**£000s**
Freehold	38 177	37 475
Long leasehold	5 713	5 344
Short leasehold	897	904
	44 787	43 723

iii. The gross book value of depreciable assets is £203 898 000 (19X1: £191 209 000).

3. *Investments in Subsidiaries*

	Company	
	19X2	**19X1**
	£000s	**£000s**
Shares at cost	102 292	102 795
Amounts owed by subsidiaries	174 091	145 882
Amounts owed to subsidiaries	(70 325)	(64 085)
	206 058	184 592

4. *Current Liabilities*

Amounts falling due within one year

Group creditors include provisions for:

	19X2 £000s	19X1 £000s
Taxation		
UK	6 303	5 093
Overseas	2 769	2 473
Bank overdrafts	28 786	32 847
Funded maintenance of buildings	2 869	3 208
Rationalisation of production facilities	2 260	6 816
Proposed Dividend	5 215	5 210

In addition to the provision for rationalisation of production facilities, Group creditors include £3 900 000 (19X1: £500 000) for redundancy and reorganisation expenditure carried forward at the end of 19X2.

In respect of the Company, creditors include an amount of £4 022 000 for UK taxation (19X1: £3 625 000).

Amounts falling due more than one year

Loans	44 250 2428	43 848

5. *Reserves*

	Company	Group		
		Trossachs and Subsidiaries	Major Associated Companies	Total Group
	£000s	£000s	£000s	£000s
At 31 December 19X1	123 579	151 326	16 887	168 213
Retained profit for year	4 451	13 847	258	14 105
Amounts taken direct to reserves:				
Share premium	67	67	–	67
Adjustment for exchange movements	614	(1 653)	(2 310)	(3 963)
Surplus of net assets over cost of acquiring minorities	–	862	–	862
Transfers between subsidiaries and associates	–	1 276	(1 176)	–

	Company	Group		
		Trossachs and Subsidiaries	Major Associated Companies	Total Group
	£000s	£000s	£000s	£000s
At 31 December 19X2	128 711	165 725	13 559	179 284

Included in reserves is the Company's share premium account of £47 676 000 (19X1: £47 609 000).

Exchange differences arising from the retranslation of the opening net investment in the overseas companies and their retained earnings for the year into sterling at the rates of exchange ruling at 30 September 19X2 have been taken directly to reserves.

Exchange differences in the Company arising from the retranslation at the balance sheet date of currency loans used for overseas investment have been taken directly to reserves.

6. *Commitments and Contingencies*

Group contracts in respect of future capital and investment expenditure which had been placed at the dates of the respective balance sheets amounted to approximately £4 330 000 (19X1: £6 736 000).

The amount of capital and investment expenditure authorised by the Directors but not yet contracted for at the dates of the respective balance sheets was £28 568 000 (19X1: £8 675 000).

The Company's investment expenditure authorised by the Directors but not yet contracted for at the dates of the respective balance sheets was £1 750 000 (19X1: nil).

Group contingent liabilities relating to guarantees in the normal course of business and other items amounted to approximately £9 567 000.

Review Questions

10.1 'Maturity matching' of asset and financing is an accepted rule of thumb in financial management. Which of the following best explains the rationale for maturity matching?

A. Maturity matching ensures that the risks and returns of assets and financings are the same; that is, high risk and return assets are matched with high risk and return financings and vice versa.
B. Maturity matching generates the highest possible rate return for the amount of risk undertaken.
C. Maturity matching is a rather crude, but often effective, rule for avoiding mismatching of risks and returns in assets and financings.
D. Maturity matching associates low risk/return assets with high risk/return financings and high risk/return assets with low risk/return financings so as to generate reasonable risk and return levels.

10.2 The Holiday Toys Company is a very seasonal business. It expects to require levels of fixed and current assets consistent with its business line. The company wishes to understand the implications of various financing plans available, particularly with respect to financing as to which is long-term or short-term. Referring to Figure 10.2 and the related text of Module 10, Holiday Toys can choose to:

I. match assets and financings on the basis of accounting definitions of asset maturity;
II. match assets and financings on the basis of **actual** asset maturity (the notion of permanent current assets); or
III. finance as much as possible with long-term finance.

In terms of the risk exposure caused by Holiday Toys' choices of maturity financing, which of the following is correct?

A. Strategy I will cause a medium risk exposure, III a low risk and II the highest risk exposure.
B. Strategy I will cause the highest risk exposure, II the next highest, and III the least.
C. Strategy I will cause the lowest risk exposure, with II the next lowest, and III the highest.
D. All strategies will produce the same risk.

10.3 Journeys plc is a travel agent with a need to borrow funds from its bank. The bank proposes that the one-year loan of £60 000 to be granted should carry a 14 per cent nominal interest rate and a 10 per cent compensating balance (i.e. Journeys plc must keep £6000 on deposit at no interest earned for the term of the loan), and that interest be paid in advance. Which of the following is the effective interest rate of the loan?

A. 18.4%.
B. 15.6%.
C. 14.0%.
D. 16.6%.

10.4 Tookover plc is in the acquisitions business. It expects to use £4 000 000 cash per annum, evenly spread across the year. To get the cash, the company wishes to use marketable securities in its possession, upon which it earns 13 per cent per annum. The cost per transaction is fixed at £42. The approximate combinations of average cash balance (in excess of the minimum), optimal transaction size and number of transfers per year are which of the following?

A. £25 420; £50 840; 79.
B. £2540; £5080; 787.
C. £141 000; £141 000; 28.
D. None of the above.

10.5 Up ’n Down plc, a trampoline company, has cash inflows and outflows across time that generate changes in its cash balances that are as likely to be up as down. The expected change in either direction is £4000 on any day. Up ’n Down can earn 12 per cent per annum on marketable securities but nothing on cash, the company incurs a £15 charge whenever securities are cashed or whenever cash is put back into securities, and it would be uncomfortable with less than £50 000 cash at any time. Up ’n Down should of course sell securities when its cash balance falls to £50 000. Which of the following state the correct approximate amount of securities that it should sell, the upper limit of its cash balance, and the ‘return point’ or balance resulting immediately after a sale or purchase of securities?

A. £72; £50 216; £50 072.
B. £25 000; £125 000; £75 000.
C. £8180; £74 540; £58 180.
D. None of the above.

10.6 Henry James, the new financial manager of Daisy Publications, has discovered that Daisy’s credit terms are ‘net 60’ while its competitors’ are ‘net 30’. James wishes to consider the effect of coming into line with the industry’s credit terms. Daisy’s marketing department has informed James that the new terms would result in a decline in sales of £75 000 per annum from a base of £650 000, but a reduction in the average collection period from 65 to 40 days, along with a reduction in bad debts from 4 per cent to 3 per cent of sales. Daisy’s variable costs are 80 per cent of sales, working capital stays at about 20 per cent of sales, and Daisy’s interest rate for borrowing is 0.03 per cent per day. Should Daisy undertake the alteration in credit terms, the NPV of the change would be which of the following?

A. £0.78
B. £410.18
C. −£0.78
D. None of the above.

10.7 A recent wet summer has caused a decline in Henley's plc due to the drop-off in sales of racing sculls. The company, in order to have enough cash to preserve its operations, is considering two alternatives:

I. give up the discounts on its payables, which are now 2/10 net 30 and which are all taken on the tenth day, and instead pay on the thirtieth; or

II. borrow the necessary funds from the bank with a 'discounted' monthly note – which means interest in advance – at 16 per cent annual interest.

Henley's should undertake which of the following?

A. Take the loan because it is cheaper than the new payables policy.

B. Pursue the new payables policy because it is cheaper than the loan.

C. Take the loan even if it is more expensive because of the decline in the company's reputation if payables are changed as indicated.

D. Pursue the new payables policy even if the loan is cheaper because a company in this kind of trouble should not be borrowing money.

10.8 As an exercise, program the data and relationships from the CSH plc cash budget into a spreadsheet program on a microcomputer to which you have access. Any decent spreadsheet program will suffice since the problem is straightforward. We suggest that it would be useful to:

i. Examine the effect upon CSH's cash budget of its customers changing the way they pay – say lengthening their payment so that only 75 per cent of this period's receivables are paid in cash.

ii. Examine the effect upon CSH's cash budget of there being a 5 per cent bad-debt proportion of sales.

iii. Construct within the CSH spreadsheet cash budget a payable cash estimation process similar to the one for receivables, and test the effects of CSH's changing its payable's policy (whichever one you have invented so as to generate the process).

iv. Include in your spreadsheet (if the program is sophisticated enough) a 'macro' which calculates the equivalent annual interest rate associated with various discounts allowed to customers or taken by CSH.

Once you have constructed the spreadsheet, you will probably wish to do more than the above, but they are a good start.

Review Questions to Appendix

Questions 10.9–10.18 use the following information:

KY Ltd Profit and Loss Account for year ended 31 December 20XX

	£	£
Sales		5 781 250
Cost of sales		
Opening stock	250 000	
Purchases	5 975 000	
	6 225 000	
Closing stock	1 600 000	4 625 000
GROSS PROFIT		1 156 250

Expenses	596 250
NET PROFIT before tax	560 000
Tax	200 000
NET PROFIT after tax	£360 000

KY Ltd Balance Sheet as at 31 December 20XX

	£	£
Fixed assets		2 000 000
Current assets		
Stock	1 600 000	
Debtors	1 445 313	
Other	1 754 687	
	4 800 000	
Less: Current liabilities	3 200 000	
		1 600 000
NET ASSETS		£3 600 000
Represented by:		
Ordinary shares of £1		1 000 000
Retained profits		
Opening	2 240 000	
20XX	360 000	2 600 000
		£3 600 000

10.9 Which of the following is correct?

The current ratio of KY is:

A. 0.5

B. 1.125

C. 1.5

D. 2.0

10.10 Which of the following is correct?

The quick ratio of KY Ltd is:

A. 0.45

B. 0.55

C. 1.0

D. 1.5

10.11 Which of the following is correct?

The profit margin of KY Ltd is:

A. 6.2%.
B. 13.8%.
C. 20.0%.
D. 38.7%.

10.12 Which of the following is correct?

The return on total assets for KY is:

A. 18.0%.
B. 8.2%.
C. 7.5%.
D. 5.3%.

10.13 Which of the following is correct?

The return on owner's equity for KY is:

A. 9.7%.
B. 10.0%.
C. 20.0%.
D. 80.0%.

10.14 Which of the following is correct?

The debt ratio for KY is:

A. 88.9%.
B. 66.7%.
C. 55.3%.
D. 47.1%.

10.15 Which of the following is correct?

The fixed to current asset ratio for KY is:

A. 41.7%.
B. 55.5%.
C. 80.0%.
D. 88.9%.

10.16 Which of the following is correct?

The inventory turnover for KY is:

A. 28.9 times.
B. 23.1 times.
C. 9.7 times.
D. 3.6 times.

10.17 Which of the following is correct?

Assuming a 350 working-day year the average collection period of KY is:

A. 0.25 days.
B. 87 days.
C. 97 days.
D. 114 days.

10.18 Which of the following is correct?

The fixed asset turnover for KY is:

A. 1.2 times.
B. 1.8 times.
C. 2.9 times.
D. 16.1 times.

10.19 Using figures relating to the year ended 31 December 19X3 (not given here), the current ratio of KY is calculated to be 1.3. The average ratio for the same period for companies in the same industrial sector as KY was 1.6. Three views of this information can be postulated:

I. KY is in a marginally healthier liquid position than the average of all the other companies in the same industrial sector.
II. KY is in a marginally worse liquid position than the average of all the other companies in the same industrial sector.
III. KY is unable to pay its bills since current ratio is smaller than 2.

Which of these statements, if any, are true?

A. I only.
B. II only.
C. III only.
D. Not I or II or III.

10.20 The profit margin for 19X3 for KY was 6.8%. The average figure for other companies in the same industrial sector for the same period of time was 7.0%. The management of KY has the following options as courses of action:

I. Examine pricing structure and compare with competitors.
II. Examine costs and overheads.
III. Look for a new product.

Which courses of action could KY's management be *reasonably* expected to pursue?

A. I and II only.
B. II and III only.
C. I and III only.
D. Not I or II or III.

10.21 The inventory turnover for KY for 19X3 was 4.8 times. The industrial average was 5.6 times. Three views of these facts can be postulated:

I. KY is possibly over-geared and may find it difficult to borrow additional funds.
II. KY has too much money tied up in stock.
III. KY should improve their credit control procedures.

What conclusions could the management of KY be *reasonably* expected to draw from this?

A. I only.
B. II only.
C. III only.
D. Not I or II or III.

Module 11

International Financial Management

Contents

Learning Objectives

More and more companies are expanding their operating environments to be international in scope. These international involvements range from simply exporting and importing goods and services, to engaging in operations in foreign countries, even to raising capital in other countries or in international markets that do not recognise national borders. Financial managers must be educated in at least the basics of such markets and transactions. Module 11 begins with a discussion of foreign exchange, the financial economics of exchange rates, purchasing power parity, interest rate parity and the influence of inflation upon exchange rates. Next it illustrates various techniques that are available to companies to hedge the risk of exchange fluctuations in international transactions. Finally, the module discusses real-asset investment in foreign countries, and the extent to which hedging exchange risk (and other international risks such as confiscation) is possible or desirable. Several exercises in dealing with foreign exchange transactions will help the student understand this area, which has been the least well appreciated of company financial decisions.

11.1 Introduction

Companies today are steadily increasing the amounts of business they do with customers and suppliers in other countries. Sometimes this activity is in the import and export market, that is with a company operating from a domestic base and selling to or buying from customers or suppliers in other countries. Ever more often, however, companies are also choosing to have operations (production facilities, marketing units, etc.) actually residing in other countries (these are the ubiquitous 'multinational firms'). Companies participating internationally in either of these forms face unique financial decisions not encountered by purely domestic firms. This module will introduce you to the most important of these situations.

First a disclaimer. The study of international business is a very complex and challenging subject, only one part of which is financially oriented. International business as a subject area can include not only financial questions, but also issues of government regulations, human capital concerns, trade barriers, political analyses, international accounting differences, and many other topics. We shall deal herein only in the financial dimensions of international business; the rest of international business concerns must be left to other sources.

What do we mean by financial dimensions of international business? Consider the situation of Lookout plc, a UK company selling fabric to customers in Japan. Lookout has found that it sells more fabric to Japanese customers if it agrees to prices stated in yen (¥). Those are good customers who pay in 30 days. Lookout, however, pays its bills and services its capital claims in sterling (£). When Lookout quotes a yen price to its Japanese customers, it can figure its equivalent sterling price by examining the **exchange rate** between yen and sterling. An exchange rate is of course simply the ratio of how many units of one currency are exchangeable into another. For example, if Lookout discovered that £1 = ¥243.06, fabric that Lookout wished to price at £10 per metre would be quoted at ¥2430.60 per metre to Japanese customers.

But Lookout faces a problem here. When it receives the yen in 30 days, there is no guarantee that the exchange rate will still be 243.06 yen per pound sterling. If Lookout does nothing about this uncertainty, the company will be adding a risk of exchange rate fluctuations into the basic business and financial risks it would undertake as a domestic company.[1]

The solution to problems such as Lookout's is one of the important dimensions of international financial management. We shall indicate how companies can deal with the risk of exchange-rate fluctuations in this and other types of decisions they make.

Another instance of a decision that is affected by exchange-rate fluctuations is the **undertaking of a foreign operation**. Aside from the uncertainties residing in the foreign commercial operation itself, since the shareholders of the company will eventually wish to be rewarded in their own currency, and since the basic returns will be generated in the foreign currency, there is potentially an exchange risk problem here also.

In addition to exchange-rate risks there are other international financial concerns with which managers must contend. These others are primarily involved in financing decisions associated with foreign operations, specifically in deciding the optimal **location, security** and **currency** in which to borrow. International capital markets are more varied and complex than purely domestic markets, and financial managers must be aware of the opportunities and challenges of dealing in those markets.

Finally, there are governmental controls and subsidies existing in international financial dealings, and these can importantly influence the profitability of transac-

1 If the exchange rate is greater than the rate upon which Lookout based its pricing decision (which is not necessarily the rate existing on the day an order is taken), Lookout will end up getting less sterling than it anticipated with the contractual number of yen; this is a loss on exchange. On the other hand, if the exchange rate is lower than anticipated by the product pricing decision, Lookout would have an exchange gain. Unless Lookout is a better predictor of exchange rate movements than the market, it will inevitably face exchange risk.

tions and projects that companies undertake. Financial managers must be aware of these also.

11.2 The Foreign Exchange Markets

Foreign exchange markets are one of the least well understood financial markets in existence. One reason is that many companies did not until recently deal much with foreign customers on a regular basis, and thus felt no great need to understand foreign exchange. Another probable reason is the intimidating complexity that this market seems to exhibit. For example, not only are there exchange rates between each pair of currencies, but there are 'spot' and forward exchange rates, options on exchange rates, securities issued in one currency and payable in another or even some combination of currencies, 'Eurocurrency' transactions and securities – and the list goes on.

How can a financial manager hope to deal with this complexity? Historically, financial managers did not. They would delegate this responsibility to a specialist, often a bank or other financial institution, which would provide foreign transaction facilitation for a fee. Today, however, with so many companies depending so importantly upon foreign customers, suppliers and operations, it is no longer acceptable for a financial manager to be uneducated in foreign exchange markets. This section introduces you to the basic financial economics of foreign exchange.

11.2.1 Exchange Rates and the Law of One Price

How do exchange rates come to bear the relationship to each other that they do? The answer is something called '**the law of one price**', or its generalisation, '**purchasing power parity**'. These related ideas are quite straightforward. They say simply that the same thing cannot sell for different prices at the same time. This is nothing more than our old ally the assumption that in free markets arbitrage opportunities cannot long exist. An example will be useful.

Suppose that in London you are fortunate enough to own a fine diamond, and are interested in selling it. The price in London that you can get for it is £10 000 per carat, whereas in New York diamonds of that quality bring $17 000 per carat. Furthermore suppose that the exchange rate between sterling and dollars is $1.60/£1.00. You would naturally send your diamond to New York, collect $17 000 per carat, and exchange that for $17 000/1.60 = £10 625, ending up with £625 more per carat than was available in London (less extra expenses involved in the transaction in New York rather than London).

Obviously such a condition cannot persist. Potential sellers wishing to transact in New York (and buyers in London) will force the $/carat price down to meet the rising £/carat price in London. You can thus see how the term 'one price' derives from that situation. Diamonds are diamonds, and whether they are bought in one currency or another, the same amount of wealth will be expended.

What does this have to do with exchange rates? Consider now expanding the above example to the full set of commodities and services that can be purchased with foreign exchange transactions. Further, suppose – as in our diamond example – that the dollar price of such sales for many commodities in New York is more valuable than the sterling price in London. What will happen? Naturally, sellers of these commodities, like you, will prefer to transact in New York rather than London. Recall that in order to finish your diamond transaction, you sold your dollars ($) and purchased pounds (£). Consider the result if many such transactions took place quickly. There would be selling pressure on dollars ($) as people tried to sell them, and buying pressure on pounds (£) as people tried to acquire them; and with the exchange rate stated as ($)/(£), you can see that the tendency would be for that ratio to increase (increasing the dollar price of pounds), and vice versa for the exchange rate stated as (£)/($). This movement in exchange rates would continue until the incentive to sell dollars and purchase pounds disappears. This latter, of course, will happen when dollar-denominated sales of commodities are neither more nor less valuable than sales in pounds.[2]

Though the above is not the complete story of the economics of exchange rate determination, it is enough to indicate something about how exchange rates get to be what they are. Exchange rates portray relationships in wealth exchanges across national borders in much the same manner as interest rates portray wealth exchanges across time. Interest rates tell us how many £s must be given up today in order to receive £s in the **future**, whilst exchange rates tell us how many £s must be given up now to receive, say, ¥s or $s **now**. And the economics of exchange rate determination is much the same as interest rate determination: interest rates generate the expectation that wealth is preserved across time, while exchange rates do the same thing for purchasing power across countries. If exchange rates are perfectly adjusted, it would not matter in which country you sold your diamond. The amount of sterling you obtained in the final analysis would be the same.[3] Exchange rates adjust to equate the amounts of wealth exchanged across national borders, or tend toward 'purchasing power parity' in the sense of the diamond example.

It would, however, be incorrect to think that exchange rates always and exactly equate purchasing power across all countries. The problem of transactions and information costs is obvious in even the diamond example, and diamonds are one of a very few commodities that travel cheaply for such exchange purposes.

There are significant frictions for other commodities (due to perishability and cumbersomeness for example) and even greater problems with some services (such as managerial talent). As important are the impediments to free exchange that are

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[2] A similar but opposite set of activities on the part of buyers in London, who find things relatively cheaper to purchase there, would produce the same tendency for the ($)/(£) exchange rate to increase, or the (£)/($) exchange rate to fall.

[3] If, for example, the dollar and sterling prices of diamonds remained the same, the exchange rate would be forced to $1.70/£ or £0.58824/$. Since one minor commodity alone is unlikely to force exchange rates to change much, the likelier effect is to lower the $ price or raise the £ price of diamonds. Which would happen depends upon how the ¥ price of diamonds, the € (euro) price of diamonds, and so forth compare with the $ and £ prices of diamonds. If the $ price of diamonds is more inconsistent with all others, it will feel the greatest effect.

If diamonds are only one of many things that are mispriced in dollars relative to the £, the exchange rate will change and domestic prices may not be as much affected.

imposed by governments. These impediments can be negative (from import and export restrictions to currency movement constraints) or positive (subsidisations of various types). Regardless, the effect of these frictions is to cause the purchasing power parity characteristic of exchange rates to be imperfect.

Even with its imperfections there is an impressive elegance and consistency in exchange rates across currencies based upon the general economics of international transactions illustrated above. As we shall see, this consistency is actually even more pervasive than we have described to this point, and extends to many financial transactions and even across time.

11.2.2 Spot and Forward Exchange Rates

Lookout plc (*see* Section 11.1) has a problem with its international transactions in that its future receipts of yen have an uncertain sterling value. Let us see how this type of uncertainty can be eradicated.

Hotlady Mustard plc sells its condiments in the United States. It books and takes orders, delivers, invoices in dollars, collects dollar receivables (debtors) and converts into sterling. The entire process takes about six months from original order-taking until final conversion into sterling. Hotlady obviously faces the same kind of exchange rate risk we uncovered for Lookout plc. What can Hotlady do about this?

Table 11.1 **Foreign exchange quotations for 22 July 2003**

	\$/£	£/\$
	1.5960	0.6266
180 day forward	1.5780	0.6337

Suppose the set of price quotations given in Table 11.1 were those published nationally for 22 July 2003. The first set of exchange rates is called the **spot rate**, which is the basis upon which sterling is exchanged into dollars on that date (or actually for transactions done the day before). If Hotlady receives some dollars on 22 July 2003, they can be exchanged for 0.6266 as many pounds sterling. The second set of exchange rates is called a **forward rate**. A forward exchange rate is the going price for exchanging between currencies at some future time. In this case the reported forward exchange rate is between sterling and dollars six months into the future. By entering into a **forward exchange contract**, a trader commits to purchase or sell an amount of currency at a fixed price at a fixed time in the future.

For example, Hotlady may on 22 July 2003 be negotiating a sale of mustard to a customer in the United States, and will not receive payment in dollars until January 2004. To avoid any uncertainty as to the amount of sterling that those dollars will buy, Hotlady can 'buy sterling forward' at the rate of £0.6337 per dollar. Thus, regardless of the **spot** rate of exchange existing between sterling and dollars in January 2004, Hotlady can guarantee itself an amount of sterling by transacting in the foreign exchange market.

Obviously such transactions are desirable from the perspective of companies like Hotlady (as they would be for other UK companies that might have future commitments to **pay** in dollars, or for US companies with sterling commitments, or any entity with expectations of future cash inflows or outflows in a foreign currency). And that is why the foreign exchange market exists. The purchasers and sellers of forward exchange contracts importantly are companies seeking to avoid the risk of exchange rate fluctuations, or to **hedge** such transactions.[4]

11.2.3 Exchange Rates and Interest Rates

Look again at Table 11.1. The spot and forward exchange rates shown therein can be taken as the actual rates that existed on the day in question. Why did the particular relationship exist? (The relationship on that day was that sterling was selling at a 'forward discount' to dollars and dollars at a 'forward premium' to sterling).[5] The answer will allow us to investigate a most interesting type of international connection between capital markets.

Consider again the situation of Hotlady plc, expecting to receive dollars six months hence. We have illustrated in the previous section how Hotlady can hedge its exchange risk by purchasing sterling with dollars in the forward market. We shall now show you an alternative that Hotlady can use to accomplish the same thing, and in the process we shall illustrate some important economics of international capital markets.

Suppose that Hotlady is in fact expecting to receive \$100 000 on 22 January 2004 and 'today' is 22 July 2003. The forward exchange market tells us that Hotlady can today contract in that market to receive $0.6337 \times 100\,000 = £63370$ six months hence.

Hotlady could instead accomplish the same expectation through another transaction. Suppose that the company were to borrow dollars today at the existing six-month interest rate in the US, and exchange those dollars for sterling at the existing exchange rate. If Hotlady does its maths carefully, it can calculate exactly how many dollars it can borrow so as to pay off the dollar loan with the dollars expected from collecting receivables six months hence:

$$\text{Amount borrowed} = \$\text{receivables}/(1 + \text{Six month \$ interest rate})$$

Let us assume that Hotlady could borrow dollars for six months at an annual rate of 6.5 per cent, so that:

$$\text{Amount borrowed} = \$100\,000/(1.065)^{½} = \$96\,900$$

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4 Of course, one can use the forward exchange market to **speculate** in exchange rates also. If one commits to purchase sterling with dollars but has no future dollar inflow expectation, one is speculating that the actual dollar price of sterling will be greater than that forecasted by the present future price in the market. If the foreign exchange market is efficient, such speculation has a zero expected value, but a positive variance (and thus risk). For companies like Lookout and Hotlady (which have expectations of net future cash inflows in foreign currencies) **not** participating in the foreign exchange market (or otherwise hedging the foreign cash flow) is foreign exchange speculation.

5 Forward discounts and premiums are defined by the relationship between spot and forward rates in a pair of currencies. Since the forward dollar price of sterling (\$1.5780) is lower than the spot price (\$1.5960) there is a **forward discount** on sterling (often quoted as an annual percentage rate, here approximately $2\times(1.5780\text{-}1.5960)/1.5960$ or 2.26 per cent) and a **forward premium** on dollars.

The loan proceeds are then switched to sterling at spot:

$$\text{Sterling proceeds} = 0.6266 \times \$96900 = £60717$$

The sterling proceeds of the loan can then be invested for six months at the UK six-month sterling interest rate, which is 8.94 per cent on an annual basis, yielding at the end of the six months:

$$\text{Sterling (22 January 2004)} = £60717 \times (1.0894)^{½} = £63373$$

Let us review what Hotlady has accomplished. On 22 January 2004 the $100 000 of receivables will be used to repay the dollar loan in the US, so there is no net dollar cash flow to the company at that date. But in the UK, the sterling that Hotlady lent will produce interest and principal of £63 373. Notice that the cash proceeds to the company on 22 January 2004 by this process are **exactly the same** as what would have been produced by purchasing sterling in the forward exchange market.[6]

This is not an accident. It is in fact a **necessity** of foreign exchange and interest rate markets, due to **interest rate parity**. As it sounds, interest rate parity in **forward** exchange markets is a concept comparable to purchasing power parity in **spot** exchange markets. Interest rate parity ensures that borrowing or lending in one currency at the interest rates applying to that currency will produce the same final wealth as borrowing or lending in any other currency at the interest rates applying to the other currencies. Stated that way, the concept is almost self-evidently necessary: interest rates must adjust to ensure such parity or there will be arbitrage opportunities.[7]

How does interest rate parity interact with the forward exchange market? In order for no arbitrage opportunities to exist, the following relationship must hold:

$$\text{Relative interest rates} = \text{Relative forward exchange discount or premium}$$

In the Hotlady example this translates to (rounding apart):

$$\frac{(1.0894)^{½}}{(1.0650)^{½}} = \frac{0.6337}{0.6266} = \frac{1.5960}{1.5780}$$

Given the set of interest rates and a spot exchange rate, only one forward exchange rate is consistent with parity; given the set of spot and forward exchange rates and one country's interest rate, only one interest rate in the other country is consistent with parity.

11.2.4 Forward Exchange, Interest Rates and Inflation Expectations

Our investigation of exchange rates and interest rates has so far argued that there are nice consistencies in purchasing power and interest rates across national borders implied by the two parity principles. If there were not such consistency, a skilled financial trader could turn the international exchange markets into a 'money pump',

6 The £3 difference is caused by rounding of the price and interest quotations.

7 We have calculated the sterling six-month interest rate of 8.94 per cent so as to produce no such opportunities, given the spot and forward exchange rates and the assumed six-month dollar interest rate of 6.5 per cent.

or generator of riskless wealth increases.[8] That is the most important concept to retain from the material in this module.

There is, however, another level of international financial relationships that can help your economic intuition about international financial management, and that ties together forward and spot exchange and interest rates. This is the influence of **inflation** (or more precisely **differential inflation**) upon exchange and interest rate markets.

With a 'simplifying' assumption or two, we can connect inflation differences in currencies to exchange and interest rates in those currencies.[9] The key is to remember that inflation in a given currency affects the future purchasing power of that currency, and that if it is reasonable to think purchasing power parity holds in the **spot** exchange market, it would be unreasonable to think it did not also hold in the **forward** exchange market. Said more simply, an important determinant of the forward exchange rate between any two currencies is the expectation of differential inflation rates in the two currencies.

To understand this, we should first deal with inflation in purely domestic financial markets. When you lend money, the return you contract to receive is usually stated in **nominal** terms; that is, the money you expect to receive in the future is a particular amount of sterling. What is not guaranteed is what that future sterling will buy.[10] If inflation is expected to exist during the period of the loan, the **real** (that is, stated in terms of purchasing power) return is expected to be less than the nominal return. We can formalise this with:[11]

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8 Suppose that the forward rate in the Hotlady example was \$1.60/£ or £0.625/\$. In that situation you could borrow \$1 000 000 at 6.5 per cent (dollar rate) for six months (producing a liability of \$1 031 988 six months hence) and today exchange the proceeds of the loan into £626 000 (see the spot rate in Table 11.1). If you invest £617 961 at 8.94 per cent (sterling rate) for six months it will produce £644 992, which today can be sold forward at \$1.60/£ to cover the \$1 031 988 liability exactly. Undertaking this transaction will be absolutely riskless because all future liabilities are exactly covered by cash inflows in the same currency, and you can place £626 000 − £617 961 = £8 639 (the difference between the sterling proceeds of your dollar loan and the cost of your covering sterling lending) into your pocket and walk away. And you can continue doing this until the market learns its mistake.

What a nice way to make a living! But you should not count on finding such opportunities very often. Remember that your £8 639 did not appear out of thin air. In order for you to have gained that amount, someone else has lost it. Financial markets tend not to be populated by people or institutions who regularly lose money on this type of transaction. Market prices will adjust instantaneously to such a condition.

9 The simplifying assumption is essentially that inflation in each country is either known for certain or that market participants are unconcerned about its uncertainty. Neither of these is true, but the complexities caused by dealing with this rigorously would destroy the pedagogical benefits of any discussion. More advanced texts will help the curious.

10 There are a few governments, the UK and US among them, that have issued bonds with guaranteed purchasing power, but these are rare.

11 This is such a famous relationship that you should know who first stated it formally, namely an economist by the name of Irving Fisher; and it has come to be known as the 'Fisher Effect'. You may recall that Module 6 discussed these concepts in another context.

Nominal interest rate	=	Real interest rate	+	Effect of inflation
$(1 + \text{Nominal rate})^n$	=	$(1 + \text{Real rate})^n$	×	$(1 + \text{Inflation rate})^n$

where n is the number of periods in question.

Suppose in our Hotlady example that the real rates of interest in both the US and UK are 4 per cent annum.[12] The above relationship tells us that expected inflation in the countries is:

$$\text{UK: } (1.0894)^{½}/(1.04)^{½} = (1 + \text{UK expected inflation})^{½}$$
$$\text{UK expected inflation} = 4.75\%$$

$$\text{US: } (1.065)^{½}/(1.04)^{½} = (1 + \text{US expected inflation})^{½}$$
$$\text{US expected inflation} = 2.40\%$$

Under our assumptions, market interest rates in the UK and US indicate expected inflation for the next six months at annual rates of 4.75 per cent and 2.40 per cent respectively. This implies that money residing in sterling over that period of time will experience a reduction in purchasing power of $(1.0475)^{½}$ whereas dollar holdings will experience a $(1.0240)^{½}$ purchasing power reduction. Since money can be kept in either currency, it is only reasonable to expect that purchasing power parity **across time** as well as across currencies will be maintained by the foreign exchange market.

Maintaining purchasing power across time requires that the forward exchange rates for any two currencies be consistent with the inflation expected in those currencies. It should not therefore be surprising to find that the ratio of the forward exchange rate to the spot exchange rate mirrors the ratio of expected inflation rates in the two currencies.[13]

$$\text{Relative inflation} = \text{Relative forward exchange discount or premium}$$
$$\frac{(1.0475)^{½}}{(1.0240)^{½}} = \frac{0.6337}{0.6266}$$

We have now almost completed the story of exchange rates. It remains only to point out that, having already illustrated the relationship between interest rates and exchange rates, and now between inflation and exchange rates, the connection is that interest rate differentials are caused by inflation differentials, which are the root cause of the observed discount or premium on forward exchange.

If we assume that the forward rate of exchange quoted between any two currencies is the market's prediction of the spot rate expected for that future date, you can easily see why the differential inflation expectation is the determinant of the

[12] Determinants of the levels and differences of real rates of interest across countries is well beyond our scope and – to be honest – seems to be beyond the understanding of economists themselves! But if you want to pursue what is known, see any recent international economics text.

[13] Remember our assumption of equal real rates in the two countries. If real rates differ, this ratio relationship will not hold.

relationship between spot and forward exchange in the currencies. This is the international connection between capital markets, of which we spoke earlier.[14]

11.3 International Financial Management

Though our discussions above should have provided significant insight into the basic environment of international financial management and some examples of techniques of operating in that environment, this section will deal more specifically with actual international financial decisions that managers must make.

11.3.1 Hedging International Cash Flows

We have already seen that an expectation of future cash flow (either inflow or outflow) can be hedged in the forward exchange market with appropriate forward purchases or sales, ensuring that the resulting domestic future cash flow has a certain exchange rate. We have also shown that an equivalent result can be obtained by 'covered' borrowing or lending with immediate exchange (see the Hotlady example in Section 11.2). The choice of technique depends upon a comparison of transaction costs and any restrictions on either of the transactions that may exist in either country. Assuming that a company wishes to hedge against exchange rate risk in its future cash flows, either of the two techniques can produce a good hedge.[15]

It remains to be mentioned, however, that companies can have exposure in a currency at several future times, in several different currencies, and with expectations of both inflows and outflows in the same currency. This recognition signals that management of a company's foreign exchange exposure must comprehend the **net** exposure in each currency based upon a detailed comprehension of all monetary accounts.

For a company with extensive transactions in foreign currencies, complete hedging of exchange exposure can be a huge task. There is some feeling that the importance of hedging foreign exchange risk is overstated. As an offset to the risks that a company's domestic shareholders feel from foreign currency exposure is the fact that real assets held in other countries will experience nominal increases in value as inflation increases and exchange rates move down in that currency, so that domestic shareholders may wish some diversification across currencies; and that

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[14] Those readers who are international economists may feel uneasy with the simplicity of this conclusion. Recall, however, that we have, for the very purpose of simplicity, assumed away the complexities of future spot exchange uncertainty, future inflation uncertainty and the behavioural complexities they introduce.

One applicable caveat to our discussion is that forward exchange premiums may be influenced by expectations of exchange restrictions (or, more rigorously, **changes** in exchange restrictions). Fortunately, as this is written, such restrictions are not common; and so we would expect that their expectation is not an important influence upon the forward exchange structure.

[15] If a company does not hedge, it is by definition speculating upon future exchange rate movements. Companies must always remember that to speculate successfully in foreign exchange, it is necessary to predict correctly not only that rates will increase or decrease, but to predict the amounts and directions of change **better** than the set of expected rates implied by the forward exchange structure.

there are significant transactions costs to hedging. Nevertheless, even if a firm is convinced of the above counter-arguments to exchange hedging, close monitoring of that exposure is still necessary since these counter-arguments to hedging depend upon empirical relationships that can and do change over time.

Finally, we should also note that foreign exchange markets and instruments are very much subject to the technological innovations accompanying the computer age. Much of the complexity of deciding upon hedging and tracking the results of such transactions is amenable to automation. The improvements in efficiency and cost of international communications is also important to the new concerns that companies operating internationally have with exchange hedging.

One of the newer securities that have appeared in the recent past is the **foreign exchange option**. An option on forward exchange allows the holder to sell (or buy) foreign currency in a manner similar to a forward exchange contract, but also allows the holder to choose **not** to exercise the option. This is a valuable characteristic in the sense that it offers more than a simple hedging opportunity.

For example, suppose that Hotlady had purchased an **option** to sell $100 000 in January 2004 at £0.6250/$ and the actual exchange rate turned out to be £0.6337/$. Here Hotlady would be better off if the exchange was undertaken at the spot rate of 0.6337 as opposed to the option's contract rate of 0.6250. With an option, Hotlady can do that; the company can simply choose not to exercise the option. Had Hotlady instead chosen to hedge with a forward exchange contract, the company would have been forced to sell its $100 000 at £0.6250/$ with no option.

It would seem from the above that Hotlady (and similarly situated companies) should buy options on forward exchange rather than forward contracts, since with an option they are shielded against adverse movements in exchange rates, while profiting from beneficial movements. Think about this statement for a moment. Do you agree?

If you are suspicious of such a benefit, you are well along the road to becoming a good intuitive financial manager. Certainly holding an option on forward exchange endows Hotlady with a better set of expectations of eventual sterling cash flow than would a forward contract. But this begs the question of the **cost** to Hotlady of buying an option as opposed to a simple forward contract. You can guarantee that the **seller** of the option has weighed very carefully the odds of losing money to Hotlady upon the option's exercise, and has charged accordingly. So whatever Hotlady gains in expectation of future sterling return by purchasing the option instead of the forward contract has likely been offset by the extra cost of the option.[16]

Another dimension of the choice between forward contract and option is the notion of the **purpose** of the transaction. Recall that Hotlady expects to receive $100 000 six months hence. Buying sterling ahead with a forward contract to sell those dollars hedges exchange risk exactly. An option, however, does not hedge; it

[16] Options are in fact much more costly than futures contracts on foreign exchange, and though there is no single 'seller' in the sense of the textual explanation, the 'market' in effect performs the same weighing of odds.

actually creates a position that insures against a bad turn in exchange rates but retains the advantages of a beneficial movement in exchange. This is not a 'hedge' in the sense that it is defined in financial markets. The option is a sort of 'super hedge', but it is one whose financial benefits are likely to be exactly equal to their cost. Unless there is some reason for a company to think it can predict future exchange rates better than the foreign exchange options market, there is no reason for exchange options to be used in transactions where simple hedging of known future currency cash flows is desired. Forward exchange contracts (or similar borrowing and spot exchange transactions) can perform that function quite well.

Module 12 will deal with the theory of options in more detail.

11.3.2 Investing in Foreign Real Assets

The basic process of deciding upon the financial viability of a foreign investment is the same as a domestic one: estimate the expected cash flows from the investment and discount at the investment's cost of capital (or use the cost of capital as a hurdle rate for IRR analysis). There are doubtless complexities in estimating foreign cash flows that do not reside in domestic ones, but to the extent that the expertise to do this is non-financial it must be relegated to the experts in international business mentioned earlier in this module. There is, nevertheless, one aspect of appraising foreign asset investment that does require our attention at this point. The question is most simply viewed as **when** currency translation should be done in the evaluation of an investment.

To see this issue, consider the following. Hotlady Mustard is considering building a production facility in Germany. The company has projected the euro cash flows associated with operating the plant and is now ready to decide whether or not it is a desirable investment. Since the final answer will depend upon the **sterling** NPV or IRR, the question of currency translation arises. The company has two choices for doing the exchange:

1. It can predict future exchange rates between sterling and €, apply those to the € future cash-flow estimates, and discount future sterling cash flows to the present at the sterling discount rate.
2. Or it can discount future € cash-flow estimates at the € discount rate and translate these € values to sterling values using the spot exchange rate.

Having read the above, and if you have understood the discussion of exchange relationships to this point, a logical question which would come to mind is: 'Why the choice? If spot exchange, forward exchange and interest rate markets work as you have explained, there is no difference in the answers you get by using either 1 or 2.'

A smart observation. As a matter of fact you are correct in principle, but the practice is limited by the lack of longer-term quotes for forward exchange (six months to a year is the usual maximum contract offered). Since the forward exchange market is the best source of predictions about expected future spot rates, this source is limited. So choice 1 for an investment with cash flows beyond a year requires that the financial manager seeks projections of future exchange rates from a source other than the forward exchange market. Although there are those willing to make such projections, they are in no sense as reliable as those impounded in the forward exchange structure (which does not exist for long-term investments like this). This procedure, by requiring such information, introduces another level of uncertainty into the analysis.

The best alternative is the second; to use the interest rate structure of the foreign (€) currency to estimate a risk-adjusted foreign currency discount rate, to find the foreign currency (€) NPV, and to translate the resulting foreign currency (€) value at the spot exchange rate to find the domestic (£) value for shareholders.[17] This procedure has the virtue of freeing financial managers from making estimates of long-term future exchange rates, which is very difficult to do well, and which might be influenced by extraneous factors (such as a subconscious wish to have a project look good – or bad).

Another issue in foreign asset investment which should be addressed briefly is that of adjusting for the extra risks entailed in foreign investments *per se*. This is a problematic issue for financial managers, and probably moves to the edge of what is truly international financial management and what is better left to the larger area of international business. Two tenets a financial manager should keep in mind about international investment risks are: first, remember the benefits of diversification; and, secondly, remember the **relative** uncertainties of the investment.

What do these tenets imply for foreign investment? First, if a company's shareholders are not well diversified across international borders, then a foreign investment may well deserve a **lower** risk profile than a purely domestic one if the foreign investment's cash flows **are not** well correlated with a comparable domestic one. If shareholders **are** well diversified across countries, this consideration is neutral.

Second, the kinds of risks inherent in experiencing alterations in foreign tax laws, exchange restrictions, asset confiscations and frictions in repatriation (of foreign wealth) can increase the risk of a foreign investment, but there are also comparable risks in domestic investment. A good analysis of these issues would wish to consider these relative to comparable domestic risks and add a foreign risk premium only if truly deserved (there are doubtless instances in which the latter risks are smaller than domestic ones).

11.3.3 Financial Sources for Foreign Investment

Part of the process of undertaking a foreign investment is the issue of the investment's financing. The primary question in financing foreign investments is the location of the financing source. In other words, should funds to finance the investment be raised domestically, or in the foreign location of the investment, or in

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17 One can get a good indication of the market consensus of expected inflation in the foreign currency by examining the difference between the domestic (£) and foreign (€) long-term interest rates. In projecting future investment cash flows, management's expectation of future domestic inflation, incremented by the difference between long-term foreign and domestic interest rates, can be used as the inflation influence upon foreign investment cash flows. This is consistent with the interest rate parity principle discussed earlier.

a third location? Our discussions of interest rate parity should lead you easily to the conclusion that there would be no reason to choose one financing location over another based upon interest rate differences (of course subsidies offered by governments are the exception).

It may also seem that since the value of the investment will first be generated in the foreign currency, fluctuations in the exchange rate against the shareholder currency is a risk to be faced. Actually, however, this risk is easy to overestimate if economic principles underlying exchange rate movements are ignored.

To understand why there may not be much risk of exchange difficulties in foreign operations, we must first make certain that you understand the difference between **monetary** and **real** assets. Monetary assets are those whose returns are expected to be fixed in nominal or money terms in the future, regardless of the inflation rate in the economy. An example is a receivable (debtor account) held domestically, but denominated in a foreign currency. With this kind of monetary asset, a change in the exchange rate (as we have seen in the examples above) affects the domestic currency value of the asset because the foreign currency value is fixed whilst the exchange rate varies.

Real assets held in foreign countries are not, however, fixed in foreign currency value; these real assets increase in value with increases in foreign inflation. Examples of such assets are primarily plant and equipment and real estate, but managerial talent, distribution channels, technological expertise and other longer-term productive assets also qualify as 'real'.

If an exchange rate changes due to inflation in the foreign country, the **domestic currency value** of a real asset held in that foreign currency will not necessarily change, because the exchange rate effect upon value will be offset by the inflationary effect upon value. Let us illustrate with an example.

Suppose that Hotlady plc builds its German plant and subsequently experiences a depreciation of the € relative to sterling. It would seem on the surface that the sterling value of Hotlady's German plant would thereby be reduced. (This risk could be lessened by borrowing in €, so that any decline in asset values due to exchange would be matched with a decline in liability value.)

However, for the € to fall relative to sterling, it would be very likely that Germany has experienced an increase in inflation relative to Britain (*see* Section 11.2.4). And a German price increase will in turn increase the nominal € costs (and values) of all €-denominated real assets, including Hotlady's German plant. So Hotlady's British shareholders will experience on the one hand a reduction in the value of their German plant due to the increase in the €/£ exchange rate, but will also experience an (offsetting) increase in the €-stated value of the German plant. Both of these effects are due to the same increase in inflation. Although Hotlady shareholders' sterling value per € of German plant has declined, the plant's increase in € value compensates, and the plant's **sterling** value is unchanged.

This implies that multinational firms need not necessarily finance their real-asset investments with liabilities in the same currency as the real asset. To generalise a little, **real** assets (e.g. plant and equipment) held in other countries tend to experience increases in value as inflation increases, whereas **monetary** assets (e.g. accounts receivable) do not. Only the latter are serious candidates for the hedging of exchange risk.

In addition to country locations of financing for foreign investments, we must also mention that there is a truly **international capital market** from which companies can borrow. In this market, long-term funds are usually called 'Eurobonds' whereas short-term borrowings are described as 'Eurocurrency' transactions. These loans are denominated in a particular country's currency, but the funds lent are 'offshore' funds in that they have been paid out from the country but not yet redeposited in a domestic bank in that country. For example, if a US purchaser of British consulting advice pays in dollars, which are then deposited in a UK bank, the UK bank can make a 'Eurodollar' loan to a third party instead of exchanging those dollars for sterling.

Since there are several large, efficient national capital markets willing to make loans to foreigners, why would these 'Euromarkets' exist? The answer is the lack of regulatory oversight in this international capital market relative to domestic markets, because governments make rules in the latter about bank lending, levy taxes and occasionally limit the free flow of money across borders. Though comparable-risk loans will carry very nearly the same interest rates in domestic as in international markets, there is often a slightly lower borrowing rate in Eurocurrency and Eurobond markets than in comparable domestic loans, due to lower regulatory costs.

Finally, companies with international operations should not ignore the potential for financing to interact with the fund repatriation restrictions that countries' governments sometimes place upon foreign investment. For example, it is often easier for a foreign company to export interest payments than it is to pay dividends to shareholders. Other things being equal, this would indicate that borrowing instead of equity investment is the better source in such circumstances.

Repatriation of funds in general is often a problem (if nothing more, a public relations problem) for companies whose shareholders are 'foreigners'. Companies must cope with significant measures in many countries that seek to keep within their borders profits earned by foreign firms. Payments of various types between a parent firm and a foreign subsidiary are commonly used to 'facilitate' the actual flow of cash from subsidiary to parent. Such things as management fees, royalties, loan interest and principal repayments, and transfer payments for other services and goods, are all used (within the legal limitation of the countries involved) to repatriate funds to the domestic parent company.

11.3.4 Financial Solutions to Other International Investment Risks

We mentioned earlier that exchange risks are one of a number of risks that companies face when they operate abroad. Most such risks are not suitable to analysis in a financial context, though of course they do affect the amount of risk adjustment required in valuation of foreign enterprises.

There are, however, some international investment risks other than exchange risk that can be managed in financial transactions. These generally fall under the category of 'moral hazard' risks engendered by dealing with sovereign authorities. In other words, a foreign country is a law unto itself, and can break promises with impunity since its only judge is itself. Examples of such risks range from minor non-compliance with contractual terms to outright confiscation of assets. Any company that holds assets abroad faces this type of risk to some degree or other.

Management of this risk is possible through invoking certain lessons taught in a branch of economics called **agency theory**.[18] The trick is to anticipate potential situations where the host country would be likely to take such action, and build automatic and irreversible counter-incentives into the original agreement that set up the foreign investment. Usually this will involve a third party, whom the host country must appease. For example, participation by the World Bank in a loan that sets up a manufacturing facility will often ensure that the terms of the original contract will not be arbitrarily broached by the host country (due of course to the need for future subsidisations by that institution). Or a contract could specify automatic ownership transferrals in a third country of host country assets (perhaps those of a domestic firm in the host country) in the case of a confiscatory action.

It is usually best to assume that all parties to a transaction will act on the basis of their own best self-interest. It is incumbent upon the good international financial manager to ensure that the best interests of the sovereign decision maker in the host country are in accord with those of the foreign investing firm.

Learning Summary

A company operating and transacting across national borders creates additional 'challenges' (a business euphemism for 'problems') for its financial managers. Most of these complexities involve dealing with exchange risks and restrictions on profit repatriation. In this module we have investigated the basic underlying economics of foreign exchange markets and why exchange rates behave the way they do.

We have learned about hedging against exchange risk on monetary assets, and why real assets are less likely to require such hedging.

We have outlined the preferred technique of analysing foreign real-asset investments, and discussed the primary arguments about choosing a foreign as opposed to a domestic source of international real-asset investment. In the process, we introduced Eurobond and Eurocurrency markets, and their reason for existing.

There are a couple of additional points about international financial management that are important but do not fit easily into one of the structured sections of the module, and so we shall discuss them here.

First, one large problem faced by all multinational firms is the artificiality that can be introduced into financial performance results of foreign subsidiaries by exchange rate fluctuations. As is clear from our investigations in the module to this point,

[18] Agency theory is a very important set of concepts that deal with situations where parties to a transaction or contract have conflicts of interest, and with the economics of how those conflicts are best and most efficiently resolved. Module 9 on capital structure introduced a few such concepts and Module 12 will discuss them in more detail.

such fluctuations can be hedged if they threaten to affect monetary assets and are probably irrelevant for real assets. Nevertheless many companies either do not hedge all of their net monetary exposures and/or do not account for inflationary value increases in real assets in their financial reporting. Both are dangerous biases (the former a 'real' value effect, the latter a reporting distortion) that can and do affect the way managers and foreign operations are judged. Companies should ensure that exchange biases do not cloud the real performance (both good and bad) of foreign subsidiaries.

Secondly, in firms that transact internationally there is often a tendency to be self-congratulatory when exchange rate changes cause monetary returns to be greater than they would otherwise have been, and to blame an unpredictable exchange market when the opposite occurs. We must always keep in mind that there is no need for a company to expose itself to such risks, and little evidence that any company, country, individual or institution is likely to be a long-term winner in betting against the predictions existing in the forward exchange market or in interest rate differentials among countries.

Review Questions

11.1 Dental Specialities plc, with headquarters in the UK, also operates in Incaland, a country in South America. The Incaland subsidiary requires an injection of funds in the amount of Æ60 000 for a year (Æ, or 'eyes' as they are called, are the Incaland currency units). (Yes, the company could not resist an Incaland advertising campaign using 'An eye for a tooth!') At present the exchange rate between sterling and eyes is Æ5 = £1 (i.e. it takes five eyes to a pound sterling). To acquire the funds, Dental Specialities can either borrow sterling for a year at 10.24 per cent or eyes for the same period at 13.36 per cent. If the foreign exchange market is functioning efficiently, which of the following should Dental Specialities do?

A. Not worry which currency it chooses to borrow, because shareholder (sterling) wealth will be the same in either case.
B. Borrow sterling and exchange £ for Æ because the interest rate is lower in the UK.
C. Borrow eyes because the implied inflation rate in Incaland will cause the eyes necessary to repay the loan to be less expensive than the implied lower-inflation sterling.
D. Borrow eyes because the Æ liabilities thereby generated will be repaid from future eyes, thus removing exchange risk from the transaction.

11.2 The one-year forward exchange rate between eyes and sterling in Question 11.1 must be which of the following?

A. Æ5/£.
B. Æ5.5120/£.
C. Æ5.6680/£.
D. Æ5.1415/£.

11.3 Using the data from Question 11.1, suppose that the real rates of interest in the UK and Incaland are identical at 4 per cent per annum. The implied inflation rates in the two countries are thus which of the following?

A. UK inflation = 6.24 per cent, Incaland inflation = 9.36 per cent.
B. UK inflation = 6.00 per cent, Incaland inflation = 9.00 per cent.
C. UK inflation = 10.24 per cent, Incaland inflation = 13.36 per cent.
D. UK inflation = 9.00 per cent, Incaland inflation = 6.00 per cent.

11.4 With the data for the UK and Incaland that you generated in the above questions, assume that you are a UK resident and have £1000 to lend for a year, at which point you wish to spend the resulting cash for consumption. You would be well advised to:

I. Lend sterling because of the exchange risk of coming back into sterling from eyes a year hence.

II. Lend eyes because of the higher eye interest rate, and the fact that since you will be consuming in sterling, the higher eye inflation will not affect your sterling purchasing power.

III. Lend eyes as long as you hedge the future exchange of eyes for sterling.

Which of the following is correct?

A. Both I and III.
B. III only.
C. II only.
D. None of the above.

11.5 Suppose that Dental Specialities foresees an inflow of Æ100 000 one year hence from its Incaland operation. Further, suppose that the company is concerned that the spot rate of exchange of eyes for sterling one year hence will be worse than now. Dental Specialities could:

I. Hedge by borrowing eyes now in an amount requiring a Æ100 000 payment one year hence, and exchange the eye loan proceeds for sterling now.

II. Hedge by selling one-year Æ100 000 in the forward exchange market.

III. Do nothing in the exchange market because the company's financial manager is convinced that in a year the eye/sterling exchange rate will be lower than it is now.

Which of the following is correct?

A. I only.
B. II only.
C. Either I or II.
D. III only.

11.6 If Dental Specialities wishes to pursue strategy (I) in the preceding question, it should do which of the following?

A. Borrow Æ90 711 and exchange to £18 142.
B. Borrow Æ100 000 and exchange to £20 000.
C. Borrow £17 643.
D. Borrow Æ88 215 and exchange for £17 643.

11.7 Dental Specialities should expect to get present value of sterling from the correct transaction(s) in Question 11.5 of how much?

A. £18 142.
B. £17 643.
C. £20 000.
D. £17 157.

11.8 Dental Specialities' marketing programme has been so successful that it is thinking of building a denture manufacturing plant in Incaland. The company's project analysts have generated cash-flow estimates in eyes for the Incaland plant for each year it is expected to operate and have estimated the riskiness of those cash flows. The question now is the most appropriate method to evaluate the project's cash flows. Dental Specialities uses NPV analysis for such decisions. Which of the following should it do?

A. Discount the Æ-stated cash flows with risk-adjusted Æ-based discount rate(s), and convert the resulting NPV to sterling at the current spot rate.
B. Discount the Æ-stated cash flows with risk-adjusted sterling-based discount rate(s), which accomplishes both discounting and exchange in one step.
C. Convert the Æ-stated cash flows in each future period to sterling using forward exchange rates observed or estimated for those periods, and find NPV by discounting with sterling-based risk-adjusted discount rate(s).
D. Convert the Æ-stated cash flows in each future period to sterling using forward exchange rates observed or estimated for those periods, and find NPV by discounting with Æ-based risk adjusted discount rate(s).

11.9 You have been hired as a consultant by Dental Specialities to advise it about hedging the exchange risks of their Incaland operations. The company is particularly concerned about the extreme volatility of eyes, the Incaland currency, and its effect upon the wealth of the company's sterling shareholders. Dental Specialities' operations in Incaland comprise some minor importing and exporting of raw materials and finished product, but now are primarily devoted to manufacturing and selling dentures in the country. Your advice to the company should be to do which of the following?

A. Hedge all Incaland assets by undertaking either forward exchange transactions or foreign borrowing, because changes in exchange rates will affect all asset values.
B. Hedge only the company's monetary assets held in Incaland, because only these are likely to feel a net effect of inflation and exchange rate changes.
C. Hedge the important real assets in Incaland, but not the less important monetary assets, because the import/export side of the business is trivial.
D. Do not hedge.

11.10 Dental Specialities is writing a contract with a joint venturer in Incaland to start up a quite profitable hygienist training school in that country. Dental Specialities is concerned that if the school is a success, the joint venturer will begin a competing school and gradually take away the business. Dental Specialities intends, of course, to include wording in the contract that aims to prohibit such an action, but it is concerned that the Incaland courts would be biased toward a domestic company in any resultant litigation. Which of the following would you suggest to Dental Specialities that it also write into the contract?

A. That the joint venturer's adherence to the contract be collateralised with assets held in either Dental Specialities' home country or an uninterested third country.

B. That the joint venturer's adherence to the contract be collateralised with assets held in the joint venturer's home country.

C. That the son of the joint venturer's managing director be sent to live with the family of Dental Specialities' managing director.

D. That the adjudication of disputes between Dental Specialities and its joint venturer be settled in Dental Specialities' home country's courts.

Module 12

Options, Agency, Derivatives and Financial Engineering

Contents

Learning Objectives

This final module of the course explores topics that are at the cutting edge of financial management practice. To say that these are 'new' ideas or procedures would not be purely correct, because some of the concepts have been studied academically for decades and have taken that long to begin appearing in financial management practice. Other ideas and financial instruments we examine in this module have actually been known to practising financial managers for quite a while, but only recently have their conceptual bases and importance been recognised.

Options are properly termed 'contingent claims' because their value depends upon the value outcome of some other underlying asset. For example, a call option allows its owner to buy an underlying asset at a fixed price for a fixed time. Aside from the importance that these financial assets have taken in their own right (on options exchanges) in the recent past, option characteristics are contained in other not-so-obviously option-oriented securities. The module describes how the common shares of a geared company is such an option. From a simple model of option valuation (the binomial model), it portrays the development of very powerful option valuation mechanisms that are used every day in actual options markets.

Even more important, these option models allow us to understand certain security market phenomena that would be hard to explain without such insights. To investigate these, a more detailed description of a subject briefly mentioned in several earlier modules, that of **agency relationships** is presented. Agency problems arise when a decision maker (an agent, e.g. the managers of a company) acting on behalf of a principal (e.g. the company's shareholders) encounters a conflict of interest with the principal. When this occurs, the agent will tend to act in the agent's own best interest rather than that of the principal. This can produce a net loss to all concerned if poor economic decisions are made due to that conflict. This type of problem occurs in many relationships, most seriously between shareholders and bondholders. When these potential conflicts arise, solutions may be found by a company issuing complex financial instruments (such as convertible bonds), which tend to merge the interests of bondholders and shareholders. The module applies options and agency concepts to illustrate how such solutions can be found.

The set of materials on **derivatives** expands on certain applications of forward and futures markets and international transactions introduced earlier in the course. Financial derivatives are growing quickly and are important markets that are potentially valuable for companies to use in risk-hedging activities, but because they can be made so complex, they are also quite dangerous to participants (both buyers and sellers) who are not well prepared to manage the process of participating in these markets. The student will profit from this exposure to some of the evolving advanced ideas in company finance – in terms of both potentially useful techniques and the example that they represent – of the depth and direction of advances in the field.

12.1 Introduction

As does any active subject area, the field of finance possesses researchers who continually push forward the boundaries of understanding about the field. This module will introduce you to **options valuation, agency relationships, derivatives** and **financial engineering,** ideas that have relatively recently made their way from the realm of pure financial research theory to important applications in financial decision making.

This is a good way to finish your finance course, for two reasons. First, these topics, having recently attracted the attention of sophisticated financial practitioners, allow students to offer excellent signals to the 'marketplace for managers' as to the depth and freshness of the financial education they have received. Secondly, though the topics are different in their essential nature (options, derivatives and financial engineering are primarily a set of quantitative techniques with important current applications, whereas agency concerns are more conceptual), they are interrelated and indicate much about the way finance as a subject has evolved and is evolving. This should help you to evaluate critically new ideas in finance that will appear during your professional career.

12.2 Options

This section will introduce the notion of options, how they are valued, and the extent to which options are much more common in financial markets and decision making than you might have expected. As a matter of fact, viewing some very familiar decisions and securities as options (even though you do not usually think of them as such) will prove to be a very important way to increase the depth of your understanding about finance.

But first we must learn about the basics of options.[1]

12.2.1 Option Characteristics

Options, like other securities, have value that is based upon expectations of cash flows to occur in the future. The characteristic that distinguishes options from other securities is that options are **contingent claims**. This means that the payoffs to an option depend upon what happens to another value or cash flow, often of another security.

We shall begin our investigation of options by examining the simplest of these securities, a '**call option**'. If you own a call option, it allows you to purchase ('call') another security or asset at a fixed price for a fixed period of time.

To see how this works, consider the following situation. You own a call option to purchase 100 shares of 3rd Rate Productions plc at a price of £1.25 per share at any time between now and the end of the year. (The shares of 3rd Rate would be described as the '**underlying assets**' for the option, the price at which the option allows you to purchase the shares is called a '**striking**' or '**exercise**' price, and the last time you can exercise an option is called its '**expiration date**'.)[2] Suppose that 3rd Rate is now selling for £1.50 per share. This means that if you are able to exercise the option to purchase 100 shares of 3rd Rate at £1.25, your net will be:

Value obtained: 100 shares at £1.50 per share = £150
Cost: 100 shares at £1.25 = £125plus call option

So for £125 plus the call option you obtain shares worth £150. The option here is obviously valuable. It is worth at least £25, because you could exercise it and instantaneously gain that much. (When an option can be exercised profitably, it is said to be **in the money**. When it cannot, e.g. where 3rd Rate's shares would be selling for less than £1.25, the option is said to be **out of the money**.)

One of the most important characteristics of an option is captured in its very name. An option owner need not exercise the option if the owner chooses **not to do so**: exercise is 'optional'. Reconsider the call option on 3rd Rate now assuming that 3rd Rate's shares are selling for £1.00. Here exercise would produce a loss to the option owner (by exercising the option, the owner would purchase 100 shares of 3rd Rate at a cost of £1.25 per share plus the option, a cash cost of £125, and obtain a value of 100 shares at £1.00 per share, or £100 – not a good deal). The

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1 Option markets have been around a long time, and have until fairly recently been on the fringe of the more formal markets such as the large stock exchanges. Options markets, probably because of their comparative institutional insularity over the years (and also due to the peculiar individuals who populated these markets), have developed a colourful terminology all their own. To the extent that these terms are acceptably printable in a publicly circulated text, we shall indicate them in bold face below.

2 With respect to when options can be exercised, there are two types: **European** (which can be exercised only at expiration) and **American** (which can be exercised at any time before or at expiration). As you would expect, whether an option is European or American has nothing to do with its geographic location. How those names arose is, we suppose, another story.

ability to avoid exercise is thus a valuable characteristic of an option. If an option's expiration date is at hand and the option is 'out of the money', the owner (usually called the **holder**) always 'has the option' of simply walking away from the deal, just like the unsuccessful gambler at the race track.[3]

Another important point to remember as we investigate options is that for every holder of an option there must be an **issuer**. That is, the option must have originated somewhere. The issuer of an option is often termed the option **writer**.

Suppose that you had 'written' (issued) the call option on 3rd Rate examined above, and that 3rd Rate's shares were selling for £1.50. Your potential losses upon exercise by the option holder would be the mirror-image of the holder's gain, since you would be forced to give up shares worth £150 in exchange for £125 plus cancellation of the option you had written.[4] On the other hand, if 3rd Rate shares were selling for £1.00 (were out of the money), exercise would not happen, and if the expiration date were imminent you could probably count on being allowed to keep any money you got when you originally wrote (sold) the option.

Figure 12.1 illustrates some of the above characteristics of a call option. Note that on the vertical axis of the graph we are describing exercise value, not the value that this call option would carry in the market (we save that discussion for the next section). The vertical axis of Figure 12.1 plots the **exercise** value of the option to the holder, that is, the amount of money that would be netted by the holder if exercise were undertaken immediately, given the value of the underlying security (the shares of 3rd Rate in the above discussion). Note that if underlying security value is less than or equal to striking price (£1.25), the **exercise** value of the option is zero, and that exercise value thereafter increases on a £-for-£ basis with increases in the value of the underlying asset (e.g. at a price of £1.50 for 3rd Rate's shares, the exercise value of the option is £0.25).[5,6]

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3 Contrast this with the **forward** or **futures contracts** that we discussed in Module 1. In those contracts the purchaser of a forward contract to buy a security is forced to complete the transaction even if completion is to the detriment of the purchaser (e.g. the security could be purchased more cheaply in the open market). An option holder would not be required to complete the transaction.

4 Remember a few other things about the transaction. First, as we mentioned earlier, the option writer has no option to avoid the transaction. If you write an option you are allowing **someone else** the choice to transact or not. Secondly, few options on formal options markets are ever really exercised in the sense of shares changing hands; transaction costs would be minimised by having the option writer in this example simply give the option holder the £25 differential. In well-developed options markets, the writer and holder never even meet, and all transactions are cleared in a central computer, with the option exchange standing by the validity of the contracts as written. Finally, if you have written a call option and actually own the underlying securities (e.g. here you own 100 shares of 3rd Rate), you are described as having written a **covered option** or a **covered call**. If you have written the option but do not own the underlying securities, you have written a **naked option** or **naked call**.

5 Henceforth we shall discuss option values on a 'per share' basis (e.g. £0.25 rather than £25) for simplicity.

6 You may sometimes encounter the term 'theoretical value' as a synonym for exercise value. This is a misnomer because, as we shall see very soon, exercise value is not market value, and thus 'theoretical' would also be synonymous with 'wrong', which makes the alternative term of no use.

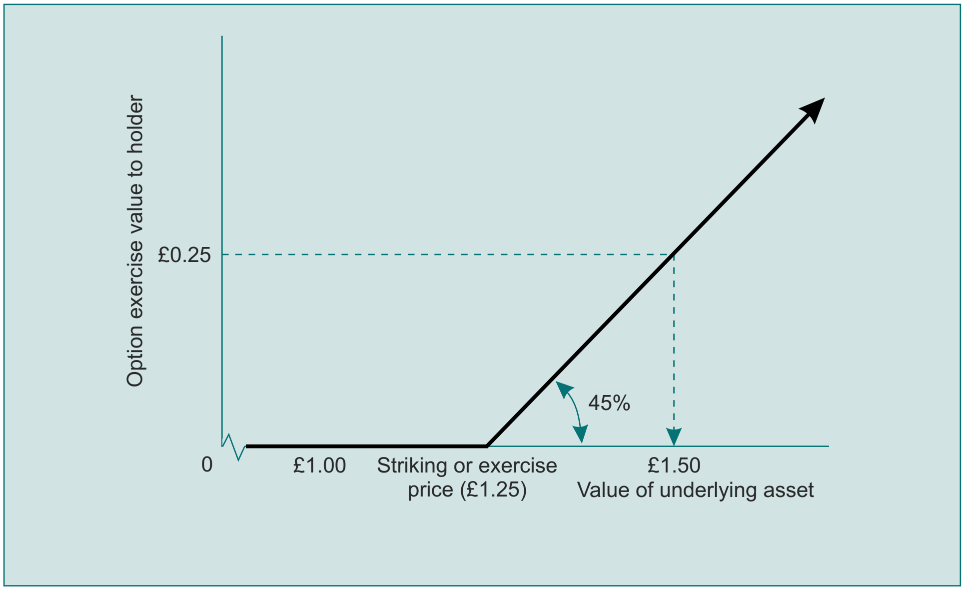

Figure 12.1 Characteristics of a call option

Call options are not the only type of options. One of the more commonly encountered options is a **put**. A put option allows the holder to sell something (usually shares) at a fixed price for a fixed period of time. Puts have positive exercise value to the holder when the underlying asset's value is **less** than the striking or exercise price. In the above example, if you held a put on 3rd Rate at a striking price of £1.25 when 3rd Rate's shares were selling for £1.00, there would be a £0.25 exercise value of the put. If 3rd Rate was selling for £1.25 or greater, the exercise value of the put would be zero, because it would not be exercised. The put **writer** would of course have the mirror-image payoffs upon exercise.

Though puts and calls are the two basic types of options, there are many combinations of these two that can be formed in complex transactions so as to generate a particular risk–return exposure to the holder or writer. Should you ever participate in options markets, you will encounter terms such as '**spreads**', '**strips**', '**straddles**', '**hedges**', '**butterflies**' and others. It is not our purpose here to introduce you to the intricacies of such options market transactions; there are specialised texts written for just that purpose.

Finally, this section would be incomplete without some mention of the extent to which options have recently made their way into other markets. In addition to those written on common shares, there are now market opportunities to buy and sell options on government securities, gold, other commodities, interest rates, stock market indexes, futures contracts and many others. Such markets have grown prodigiously due both to increased appreciation of the economic benefits that options provide to certain transactions, and the relatively recent advances in understanding how markets set prices for these strange securities. The next section takes up the latter topic.

12.2.2 An Introduction to Options Valuation

Consider the situation that we introduced in the previous section. You hold a call option on 3rd Rate shares at an exercise price of £1.25.[7] We have discussed how you would calculate the **exercise** value of that option (e.g. if 3rd Rate were selling for £1.50, the exercise value of the option would be £0.25). We now wish to discuss how to estimate the true or **market value** of that option.

The first question that may come to mind is, 'Why not just use exercise value? What is wrong with assuming that the market price of the option is simply its exercise value?' If you hold the option at an exercise price of £1.25 and 3rd Rate is selling at £1.50, and given what we know about options so far, at first glance there is really nothing obviously wrong with exercise price as market price. But it **is** wrong. Let us see why.

The most obvious illustration of why exercise price is not market price can be shown by considering the market value of an 'at the money' option. Suppose that 3rd Rate shares are selling for £1.25, such that exercise value is zero (this is the point in Figure 12.1 where the exercise value line departs from zero, or where striking price equals underlying security value, such that exercise would cost exactly what it would return).[8] If you owned that option, however, would you be willing to give it away for free? Of course not. As long as there were some chance that the option could have some exercise value prior to expiration, the 'at the money' option would sell for a price greater than zero.

A moment's thought indicates the rationality of this concept. Remember that the option allows you to purchase shares of 3rd Rate at a fixed price for a fixed period of time. Even though **immediate** exercise would net you nothing, that is not to say that **eventual** exercise might not be profitable. As long as 3rd Rate's shares might go above £1.25 between now and expiration date, the 'at the money' option would have some positive value. To say the same thing, you would not be willing to give it away for free, and a rational purchaser would be willing to pay some positive price to buy the option from you.[9]

Thus the market value of the 'at the money' option in 3rd Rate's shares would be greater than zero. But what about 'out of the money' and 'in the money' options? Would they be worth more than exercise value? Yes, they would. Consider an 'out of the money' option on 3rd Rate (suppose that 3rd Rate's shares were selling for £1.00 in association with your £1.25 striking price option – *see* Figure 12.1). Would you be willing to give this away for free? Not as long as 3rd Rate could increase in price to a level above £1.25 between now and expiration. Thus the arguments for 'out of the money' options are the same as for 'at the money' options, though the market prices are likely to be lower for 'out of the money' options identical to 'at the

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7 Since the rest of this module will concentrate on the valuation of **call options**, you should assume that – unless otherwise indicated – 'option' means 'call option'.

8 All of the illustrations in this module are simplistic in that transaction costs (brokerage fees, etc.) are ignored, as they are in this example.

9 If no 'at the money' option existed, you could write one and sell it for some positive amount of money.

money' options since the underlying securities must increase more in value before the option has exercise value.

'In the money' options also sell for more than their exercise values, for essentially the same but somewhat more complex reasons than those above. Though the full reasons will later become more clear, the simplest intuitive reasoning for market value to exceed exercise value for 'in the money' options is that the option holder stands to gain at exercise the benefit of possible interim **increases** in underlying asset or security value, but is not at risk for all possible **reductions** in underlying asset value.

Look again at Figure 12.1, and consider an only slightly 'in the money' option (say, a price of £1.30 for 3rd Rate's shares). Here, if 3rd Rate goes up in price, you can see that exercise value goes up with it, whereas reductions in value of 3rd Rate could not cause the option to decline in value below zero. So there is a kind of positive asymmetry to the possible returns from holding the 'in the money' option that does not exist in holding the underlying security itself. This is why 'in the money' options sell for more than their exercise values.[10]

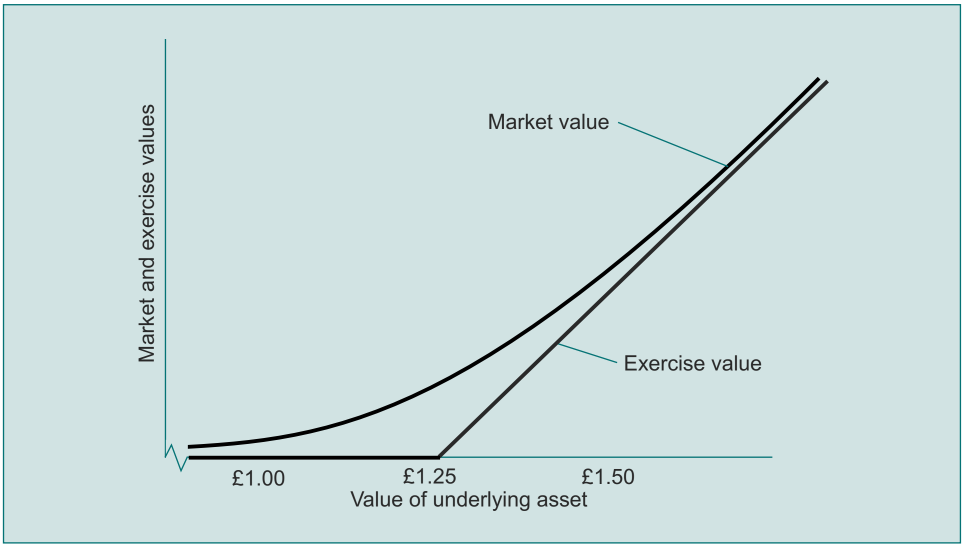

Figure 12.2 **Relationship between exercise and market values of options**

Figure 12.2 summarises the intuitions of the relationship between the exercise and market values of options. Note that option market value is always above exercise value (but less so for well 'in the money' and 'out of the money' options.) The amount by which an option's market value exceeds its exercise value is called the option's **premium**. For example, if 3rd Rate's share price is below £1.25, your call option at a £1.25 striking price will sell for some price above zero, and more above zero the closer 3rd Rate's price comes up toward £1.25. When 3rd Rate sells

[10] Exercise of an 'in the money' option would net the holder shares of the underlying security worth the exercise value. These shares would not have the same 'lower boundary' benefits as the option did; this is another way of seeing why the option is worth more than its exercise value.

for a price above £1.25, your option to purchase will sell for its exercise value **plus** a premium.[11] For example, with 3rd Rate selling for £1.50, your option would sell for a premium **above** its £0.25 exercise value. How much above? The next section will begin developing the ideas that can answer that question.

12.2.3 Calculating the Value of a Simple Option

We now know that options can have value (a 'premium') over and above their exercise value, and we also know some of the reasons why that is so. We do not yet know how to estimate this in a quantitative sense. This section will show us how to do that for a very simple call option. The world we now construct will not at first seem very realistic, but you will soon perceive that not to be a severe limitation to understanding very real options markets.[12]

This option valuation model is simple in that we shall constrain the possible price changes that can occur in the shares of the underlying security. To make the illustration as straightforward as possible, we shall require that the underlying security's price changes during the option's life are '**binomial**'. This means that only one of two values for the underlying security can happen at the time of the option's expiration. Figure 12.3 illustrates such a situation for 3rd Rate, under the assumption that the company's shares are currently selling for £1.50 (so that the option is 'in the money'), and can either increase to £3.00 or decline to £0.75.[13]

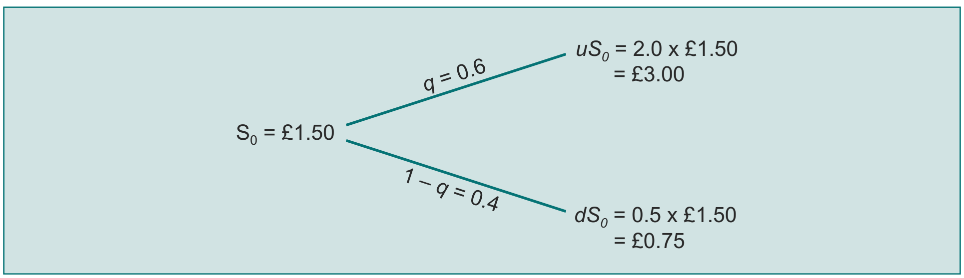

Figure 12.3 **Diagram of common stock outcomes**

[11] One other thing about 'in the money' option values: the further 'in the money' an option is, the less will be the excess of market over exercise value. The reason is that zero minimum exercise value is less and less comfortable, the higher is exercise value. And the further 'in the money' is an option, the higher is exercise value. For example, if 3rd Rate were selling for £20.00, the £1.25 exercise price option with an exercise value of £18.75 would have little downward value protection from reductions in 3rd Rate's share values. Of course, the mirror-image holds for well 'out of the money' options, for the reason that eventual increases in underlying security value to generate positive exercise values are less likely the further 'out of the money' is the option. It should not surprise you to hear that the 3rd Rate option to purchase at £1.25 would not sell for much more than zero if 3rd Rate's shares were selling for only £0.10.

[12] This idea was first formalised in 1979 by Cox, Ross and Rubenstine in the *Journal of Financial Economics* (September 1979), 229-63 as 'Option Pricing: A Simplified Approach', and by Rendleman and Bartter in the December 1979 *Journal of Finance* as 'Two-State Option Pricing'.

[13] This option pricing model will work as well with 'at the money' and 'out of the money' options, as you shall see soon. We are also assuming a 'European' option (exercise only at expiration), though the discrete time nature of this market (in other words prices quoted only 'now' and 'later') makes such an assumption implicit.

Of course these potential changes are uncertain; we do not know which of the two prices will actually happen. Let us assume that there is a 60 per cent chance of the increase and a 40 per cent chance of the decline in the share price of 3rd Rate. Thus we have a simple (it is called a 'binomial') probability distribution of potential price changes in the underlying security.

Figure 12.3 also uses some notation that will eventually serve us well in describing option value:

S_0 = the current price of the underlying security = £1.50
q = the likelihood of the underlying security price increase = 0.6
uS_0 = the underlying security increased price result = £3.00
dS_0 = the underlying security reduced price result = £0.75

We can actually regard the u and d notations as 'multipliers' for original share price, such that:

u = upward multiplier for underlying security price = 2.0
d = downward multiplier for underlying security price = 0.5

You can check to see that the multipliers applied to £1.50 will produce either £3.00 or £0.75 as the price of 3rd Rate's underlying shares at option expiration.[14]

Now let us consider the value of your 'in the money' call option on 3rd Rate. First, we shall specify the final payoffs to you as the holder of the option, depending (**contingent**) upon what happens to the shares of 3rd Rate. But first, some more notation:

C_0 = current market value of the option
C_u = option payoff at expiration if underlying security price is up
C_d = option payoff at expiration if underlying security price is down
X = exercise or striking price of the option

[14] There are some additional technical assumptions we must make to be rigorous. Markets must be 'efficient' in impounding information, and there can be no taxes nor brokerage fees; interest rates must not change during the period in question; 3rd Rate can pay no dividends before option expiration, you must be able to 'sell short' securities without restriction, and there must be a 'stable stochastic process' generating the price changes in 3rd Rate's shares. In addition, this 'binomial' model of option valuation requires that the option extends for 'one period' of time, measured discretely from 'now' until 'expiration'.

None of these assumptions affects very much the real application of this pricing model, so we shall not do more than mention them here. If you wish to pursue actual transactions in options markets, you should be aware of these considerations because they can affect the net profitability of such transactions.

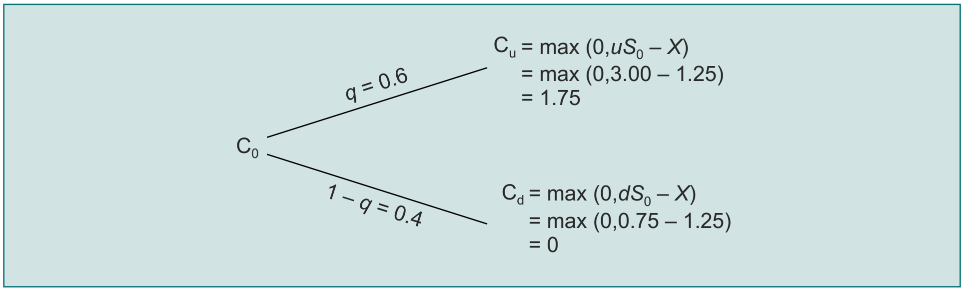

Figure 12.4 Call option payoff diagram

Figure 12.4 indicates that if the underlying security increases in price to $uS_0 = £3.00$, your £1.25 call option will have a C_u = exercise value of £1.75. Note that this potential payoff is $C_u = max(0,uS_0\text{-X})$ which translates as 'the payoff to the option is the greater of (1) zero or (2) the result of exercising the option.' Though it is obvious which you would choose, this is an important perspective on option valuation because it correctly includes the notion that with an option you are not **forced** to exercise, but can choose whether you would be better-off exercising or not.[15]

The importance of this perspective becomes more obvious when we consider the option payoff under the condition of a **decrease** in price for 3rd Rate's shares. Here $C_d = max(0,dS_0\text{-X})$ indicates that your choice is between payoffs of zero and (£0.75-£1.25), or −£0.50. Naturally, you would choose the greater: zero (do not exercise). If 3rd Rate decreases in price, you would be paying £1.25 per share for shares worth £0.75; since you have the option to do so, you decline to exercise the option, and get no payoff: C_d = £0.00.

We have thus discovered (*see* Figure 12.4) that your option has a 60 per cent chance of paying £1.75 and a 40 per cent chance of paying nothing. This is doubtless important information, but the real question still remains: what is C_0? What is your option currently worth?

This is the point where financial theory can help. In earlier modules we have solved thorny issues of valuation by invoking **the law of one price** in financial market economics. We unravelled theories of company capital structure, of dividend payments and of others by demonstrating that two investments offering identical future cash-flow expectations cannot sell for **different** prices in an efficient market. This same law will now again serve us well.

We shall discover the value of your call option on 3rd Rate's shares by calculating the value of another investment that offers the same future cash-flow expectations.[16] This requires that we discover an investment that has the same payoffs as the option but does not include the option, and contains securities whose prices we know. This is not as difficult as it sounds. We can actually find such an investment

15 Not exercising is the 'zero' alternative in the specification of potential payoffs to **holding** a call option. When you have **written** a call option, 'zero' means the holder would not choose to exercise. The same is true for put options.

16 This investment is sometimes termed a '**call-equivalent portfolio**'.

by forming a portfolio of the shares of 3rd Rate itself, in combination with borrowing or lending money.

The portfolio must have payoffs equal to C_u and C_d, with the same likelihoods as those of your option. Let us define three more pieces of notation:

rf = the risk-free interest rate (for either borrowing or lending) for the period of the option; assume this is 10 per cent

Y = the number of underlying shares to purchase (or if negative, to sell short)

Z = the amount lending (if positive) or borrowing (if negative) at the risk-free rate

For the portfolio to duplicate the option's payoffs, it must be true that the final payoffs from the shares in the portfolio (Y times either uS_0 or dS_0) and lending/borrowing (Z times $[1+\text{rf}]^t$) be the same payoffs that the option would produce, given either the increase or decline in 3rd Rate's shares. This can be written formally as:

$$\text{The payoff if 3rd Rate increases: } \mathrm{YuS}_0 + Z(1+rf)^t = C_u$$
$$\text{The payoff if 3rd Rate decreases: } \mathrm{YdS}_0 + Z(1+rf)^t = C_d$$

You can recognise from elementary algebra that the above is a set of simultaneous equations. Leaving out the tedious intermediary steps, we can solve the above for Y and Z, to get:

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£1.75 - £0.00}{£1.50(2.0-0.5)} = 0.7778\ldots \quad (12.1)$$

and:

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)^t} = \frac{2.0(£0.00) - 0.5(£1.75)}{(2.0-0.5)(1+0.10)} = -£0.5303\ldots \quad (12.2)$$

This tells us that if you purchase 0.7778… of a share of 3rd Rate, and simultaneously borrow £0.5303…, you will have the same future cash-flow expectations that would be produced by holding the call option.

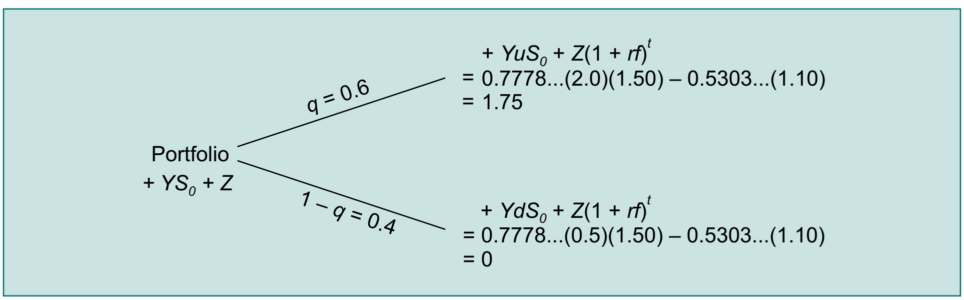

Figure 12.5 Call option payoff diagram from call equivalent portfolio

Let us see whether that is true. Figure 12.5 will help us to visualise the workings of this investment. Note that if the shares of 3rd Rate increase, the 0.7778… shares in this portfolio will be worth $0.7778\ldots \times £3.00 = £2.33$, but the interest and principal on the borrowing will produce a liability of $£0.5303\ldots \times (1.10) = £0.58$, so the net value is £2.33-£0.58 = £1.75, the same as the option. Note that this has the **same** (60 per cent) likelihood as the option's payoff, because both this and the option payoff are contingent upon the same event, namely the increase in 3rd Rate's share price.

If 3rd Rate's shares go down, this portfolio returns $0.7778\ldots \times £0.75 = £0.58$ minus the £0.58 interest and principal, or £0.00 – also the same as your option, and with the same likelihood. So this call-equivalent portfolio duplicates exactly the call option's expectations of the cash flow's amounts, timing and riskiness.

'Very clever', you say. 'But what good does that do us? The above implies only that I can get the same payoffs as the call option from another source. That is nice, but is not the purpose for which we began the exercise. You were supposed to show me what my call option is currently worth in the market.' Believe it or not, we have shown you.

Take another look at Figure 12.5. It shows that the call-equivalent portfolio duplicates the payoffs of the option, and it also shows how to calculate the cost of the call-equivalent portfolio. What good is that information? Remember our 'law of one price': two investments offering identical expectations of future cash flows cannot sell for different prices in an efficient market. This clearly tells us that the cost of the call-equivalent portfolio is in fact the market value of your call option. They must have the same value if they have the same cash-flow expectations.[17] What is the value of your option? Its value is the cost of the call-equivalent portfolio:

$$YS_0 + Z = 0.7778(£1.50) - £0.5303 = £0.6364\ldots \tag{12.3}$$

Market value of call option:

$C_0 = \text{Cost of call-equivalent portfolio} = £0.6364\ldots$

Your call option is currently worth a bit less than £0.64. Notice that the option's exercise value is £1.50-£1.25 = £0.25, and so the option is selling at a 'premium' of about £0.64-£0.25 = £0.39 over its exercise value.

This technique of calculating the market value of an option is perfectly general for 'binomial' cases like this one. All you need do is use Equation 12.1, Equation 12.2 and Equation 12.3. For example, suppose that the price of 3rd Rate's shares were £1.25 so that your option was selling 'at the money', and all other characteristics of the above were the same. The option's value would be:[18]

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17 If they did not, arbitrage opportunities would exist. For example, if the call-equivalent portfolio were cheaper than the option, you could simultaneously write the option and purchase the call-equivalent portfolio, be perfectly hedged at option expiration (because the gains or losses on the option would exactly offset the losses or gains on the call-equivalent portfolio), and pocket the difference in price between the two investments right now.

Such arbitrage opportunities cannot exist for any appreciable time in an efficient market. (Who would be silly enough to **purchase** the higher-priced option which you write, if they could do the same arithmetic as you?) This is the driving force behind the 'law of one price'.

18 Remember to calculate: $C_u = max(0, uS_0\text{-X}) = max(0,(2.0\times £1.25)\text{-}£1.25) = £1.25$

and $C_d = max(0, dS_0\text{-X}) = max(0,(0.5\times 1.25)\text{-}£1.25) = £0.00$

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£1.25 - £0.00}{£1.25(2.0 - 0.5)} = 0.6666 \ldots \tag{12.4}$$

and:

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)^t} = \frac{2.0(£0.00) - 0.5(£1.25)}{(2.0-0.5)(1+0.10)} = -£0.37878 \ldots \tag{12.5}$$

therefore:

$$C_0 = YS_0 + Z = 0.6666 \ldots (£1.25) - £0.37878 \ldots = £0.4545 \ldots \tag{12.6}$$

When the option is 'at the money', its total market value is equal to the premium over exercise value (because exercise value is zero). And as Figure 12.2 would have predicted, the **premium** (£0.45) is higher for this 'at the money' option than that (£0.39) of the 'in the money' option, though the latter has the higher **total** market value (£0.64).

We have illustrated the valuation of this call option by forming a 'call-equivalent' portfolio, and demonstrating that the value of this portfolio determines the value of the call option in an efficient market. This is perhaps the most obviously intuitive approach to option valuation. There is, however, another way of describing the market's valuation of an option. This method (the '**perfect hedge**' approach), though in many ways similar to the 'call-equivalent' approach, is different and important enough to deserve its own explanation, so we present it in an appendix to the module.

12.2.4 Observations about the Simple Option Pricing Model

As interesting as it (probably) has been for you to see in detail how a simple option can be valued, it is now time to consider more generally the things this valuation process is telling us about options. First, notice a few aspects of this model that may have been hidden in the detail of our calculations:

1. Look again at Equation 12.1, Equation 12.2 and Equation 12.3 being used to calculate option value, along with the information necessary to solve the formulas. One important observation about the process is that nowhere in the formulas or data necessary for valuing the option is there any mention of the **probabilities** of the underlying security increasing or decreasing in price. This raises the question of how this valuation mechanism is dealing with risk (our discussions of risk up to this point have always been within the context of probabilities). Is risk being ignored in option valuation? That does not seem reasonable. Actually, we shall see below that risk is *not* being ignored. But for options, valuation is a different process from what we have seen before.
2. Note also that this valuation process is much simpler than those we saw for typical securities in earlier modules. Valuing 3rd Rate's shares by our classic techniques, for example, would have required that we estimate the company's free cash flow to shareholders for the foreseeable future, consult the capital asset pricing model or other market equilibrium theory in order to produce a risk-

adjusted discount rate, and then adjust it for the gearing of the company so as to come to an independent estimate of share value.

To value this call option, however, we dealt only with characteristics of the underlying security, the option's contractual provisions, and the risk-free rate. This is substantially less data – and easier-to-estimate data – than any valuation model we have seen. The reason for this is primarily because the option is a **contingent claim**. In a sense, the market has already done a good part of our valuation task for us by setting the price of the underlying security. By understanding that we are dealing with an option security whose value is purely based upon another value outcome, we have an unprecedentedly efficient valuation mechanism.

3. Finally, it is also important to remember that the basic valuation process we have here relied upon is a 'no arbitrage' one. Recall that we discovered the value of the option by finding the cost of obtaining identical future cash flows with another investment. This is a very simple yet powerful approach to financial valuation, in that it requires that we make very few assumptions about how people and markets behave, other than that mispriced securities cannot long exist (or that arbitrage profits will quickly disappear, which is the same thing).

   This straightforwardness is a very desirable characteristic of option valuation, and we shall see that applications of option valuation are broader than this example has implied. But we must also remember that the elegant simplicity of option valuation does require that an underlying security or asset **already** be valued. Thus there must be instances where option models will not work, and where our original valuation mechanisms will be required.
4. We assumed that the option was a 'European' option in that it could not be exercised before maturity. Actually, most options on active markets are 'American' options, which can be exercised at any time up to or at maturity. But notice something else: the market value of an option is always greater than its exercise value (*see* Figure 12.2). This means that options are not in practice exercised prior to expiration, and that European and American options will therefore have the same value.[19]
5. The illustration of option valuation we have studied above is for a **call option**. Exactly the same valuation process is applicable to finding the value of a **put option**, with respect to specifying the contingent payoffs and their 'put equivalent' portfolio value. Generally, any type of contingent claim is subject to the type of valuation process we have illustrated here; the concepts are applicable as well to more complex contingent claims such as convertible and callable bonds, as we shall soon see.

12.2.5 Valuing More Realistic Options

The binomial model of the previous section is conceptually very rich, but seems to be limited in real-market applicability. How many underlying securities do you know that have only two possible end-of-period prices? Probably none. Does this mean

[19] For some options based upon underlying securities that are expected to pay significant cash dividends prior to option expiration, this may not be the case. Options may also be exercised before maturity if there is any risk that the terms of the option may change.

that we must begin anew in constructing a realistic option-valuation model, one that works when underlying securities can experience the full panoply of real market prices?

Fortunately we shall not be forced to reinvent option valuation to achieve realism within the model as we have developed it. Consider the following:

1. One obvious problem with the simple binomial model of option valuation is that only two underlying security prices can exist at option expiration. Any realistic option model must allow for many possible prices, given what could actually happen to the price of the underlying security.
2. Suppose that, instead of worrying about how many **prices** the model can handle, we instead allow it to cover **two periods** of time. In other words, we maintain the binomial nature of the price changes occurring in the underlying security, but simply allow **two** such changes to take place.

   Figure 12.6 illustrates that situation. Notice that there is still only one of two prices occurring at each time for the underlying security, but that by the end of the second period there are three possible underlying security prices and thus three possible option outcomes. (The two possibilities at the end of the first period generate two apiece at the end of the second, but two of these are identical.) Since valuation of the option (C_0) is essentially the same process here as in our original one-period case, we shall not trouble you with it.

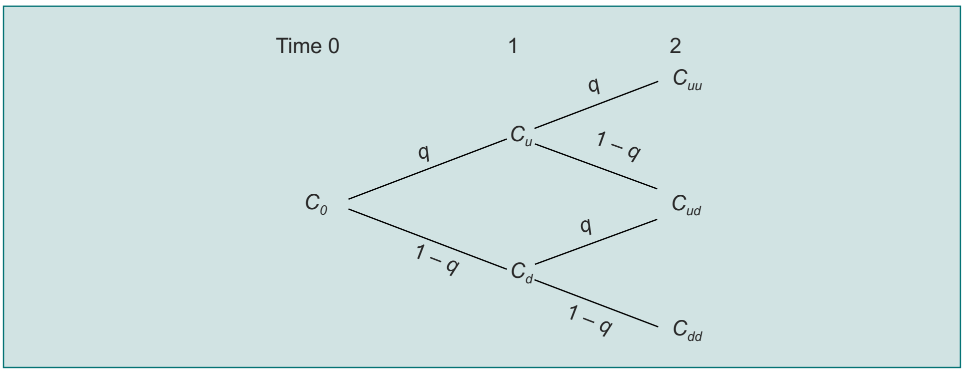

Figure 12.6 **Multiple-period call option diagram**

The binomial model is now a bit more 'realistic' in that three, as opposed to only two, outcomes are possible. 'Terrific,' you say. 'You have added one more possible price to the underlying security. That is a small step in the right direction, but it certainly seems like we have a long way to go to be "realistic" in terms of actual underlying security price movements.'

Actually, we do not have very much more to do to achieve realism. Imagine Figure 12.6 with a third period. It would have four potential outcomes. And adding more periods would add more potential outcomes. Nothing constrains us to limit the number of periods, and thus the number of outcomes to the option payoff. So from the perspective of the number of potential option payoffs, the model is now 'realistic'.

'Wait just a minute,' you say. 'Not so fast. We admit that you have cleverly got the model to have many potential outcomes. But in order to get them, you used so many periods that we would be great-grandparents before the option expired. How "realistic" is that?'

A good, but not unanticipated question. Recall that we never told you anything about the **calendar length** of the periods in the model. As there is nothing to stop us from having a lot of periods, there is also nothing to stop us from making those periods **very** short. Suppose that we told you that each period was 30 seconds long – or even less than a second. Now the model can have lots of outcomes in a very short calendar period.

Envisage Figure 12.6 with hundreds or thousands of periods, each of which was almost infinitesimally short. That kind of binomial market is not a bad representation of what we experience in actual options markets. We could get such a model to produce results. It would still be **conceptually** simple, but its **numerical** complexity would probably render it inefficient for calculations. Fortunately there is an alternative.

Two researchers in 1973 published a groundbreaking option valuation model (this was actually invented before the simple binomial version we just investigated).[20] To make a very complex story as short as possible, you can regard Black and Scholes's option pricing model as essentially the same as the 'many short period' binomial model, except that the time periods are so short that they are 'continuous'.[21] Instead of discrete time periods, time in 'Black–Scholes' (as this model has come to be known) is a continuous process, and security prices can therefore change continuously and thus have no unusual restrictions.[22]

We shall not attempt here to derive the Black–Scholes option model (our work in the binomial method and the appendix to this module is enough of that), but rather portray and interpret its manifestation:

$$C_0 = S_0N(d_1) - Xe^{-\mathrm{rfT}}N(d_2) \tag{12.7}$$

where:

$$d_1 = [\ln(S_0/X) + \mathrm{rfT}]/\sigma(T^{½}) + 0.5\sigma(T^{½}) \tag{12.8}$$

and:

$$d_2 = d_1 - \sigma(T^{½}) \tag{12.9}$$

At first blush, these equations seem overwhelmingly complex. But take heart, they are actually more familiar than they appear. Consider Equation 12.7, which produces the C_0 option value. It is actually doing the same thing that our binomial option valuation process did: calculate the value of a 'call equivalent' portfolio. The

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20 *See* Fisher Black and Myron Scholes's 'The Pricing of Options and Corporate Liabilities' in the May–June 1973 issue of the *Journal of Political Economy*.

21 Recall that Module 1 introduced the concept of continuous time in discounting.

22 Technically, underlying security prices must be generated by a 'stable' process with a 'constant variance'. But you need not concern yourself with that unless you are attracted to such concepts.

first part of the formula, $S_0N(d_1)$ is the number of shares of the underlying security to be purchased, where $N(d_1)$ is the equivalent of our Y in the binomial process, and the second part of the formula $-Xe^{rfT}N(d_2)$ is the amount of borrowing, our Z in the binomial process.[23] The $N(d_1)$ and $N(d_2)$ are actually instructions to take the d_1 and d_2 numbers calculated in Equation 12.8 and Equation 12.9 and find the value of the 'cumulative normal unit distribution.'

What does this mean? It is an instruction to take the value of the 'cumulative normal unit distribution' at the point d_1, where d_1 is calculated in Equation 12.8. The cumulative normal unit distribution is a standard statistical measure, tables for which appear in all statistical reference books.[24] Next is the exercise price (X) multiplied by e^{rfT}. This latter is a continuously compounded interest or discount rate with e being (recall Module 1) the base of the natural system of logarithms, rf being our familiar risk-free rate of interest and T a new variable representing the time remaining until expiration of the option.[25] $N(d_2)$ is another cumulative normal distribution value, this time for the variable d_2 calculated in Equation 12.9.

Equation 12.8 describes the calculation of d_1, with 'ln' being an instruction to find the natural logarithm of the bracketed expression, and the variables within the expression being already familiar, namely the ratio of underlying stock price (S_0) to exercise price (X). The remaining parts of Equation 12.8 are also familiar, with the exception of σ, which is the instantaneous standard deviation of the price of the underlying security. Equation 12.9 has no variables that have not already been mentioned.

Though admittedly intimidating, the above system of formulas is performing a calculation very similar to that which we did in the binomial model of option valuation. The only important difference is that time is now continuous instead of discrete, and thus there are effectively an infinite number of price changes that could occur in the underlying security.

Performing calculations with the Black–Scholes formula is not difficult, particularly if you have access to a typical spreadsheet computer program. Let us go through a Black–Scholes calculation using the 3rd Rate company from the binomial option example. Using the data for the 'in the money' call option:

$S_0 = £1.50,$
$X = £1.25,$
$rf = 10\%,$
$T = 1,$

23 The $N(d_1)$ component is typically called an 'option delta' by professionals who deal in such markets, and is closely related to the option's 'hedge ratio' as explained in the appendix to this module.

24 We do not include such a table here because it is extraneous. These formulas have been widely programmed into computers and even personal calculators, which are readily available to those interested.

25 The time until expiration of the option must of course be calculated in the same units as the interest rate is quoted. For example if T is stated in months, the interest rate must be a monthly rate.

and we shall assume that the standard deviation of 3rd Rate's shares, σ =.69.[26]

From Equation 12.8 we get:

$$d_1 = [\ln(S_0/X) + \text{rfT}]/\sigma(T^{½}) + 0.5\sigma(T^{½})$$

$$d_1 = [\ln(1.50/1.25) + 0.10 \times 1]/0.69 \times 1^{½} + (0.5 \times 0.69 \times 1^{½}) = 0.7542$$

and from Equation 12.9:

$$d_2 = d_1 - \sigma(T^{½})$$

$$d_2 = .7542 - 0.69 \times 1^{½} = 0.0642$$

We now go to our spreadsheet program and have it look up the cumulative normal unit distribution figures for d_1 and d_2:[27]

$N(d_1) = 0.7746$

and

$N(d_2) = 0.5256$

to which we then apply Equation 12.7 to get:

$$C_0 = S_0 N(d_1) - Xe^{-\text{rfT}} N(d_2)$$

$$C_0 = 1.50 \times 0.7746 - 1.25 \times 2.71828^{-0.10\times 1} \times 0.5256 = 0.567$$

The Black–Scholes option value of 3rd Rate's call option is approximately £0.57, given our assumptions as to standard deviation of 3rd Rate's share value.

The above system of equations is important to the realm of options for two reasons. The first is historical. This Black–Scholes option valuation model is the original idea that has fathered a whole industry of practical application and academic theorising (which we shall review briefly below). The model, which began as a theoretical exercise, has proved itself (with some slight modification) to be very robust in predicting the actual price behaviour of options.

The second reason the Black–Scholes model is important to students of options is that the model makes obvious the importance of specific option characteristics in determining option value. Look again at the system of Equation 12.7, Equation 12.8 and Equation 12.9. There are five variables for determining option value:

S_0:	Underlying security price
X:	Exercise (or striking) price
rf:	Risk-free rate of interest
σ:	Standard deviation of underlying security price
T:	Time until expiration

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26 This standard deviation figure is actually consistent with the u and d assumptions we made for 3rd Rate in the binomial valuation method, through the relationship that $u=e^{\sigma/}$ where I is the time interval as a proportion of a year. As we shall see, this actually produces a lower value for the call option than the binomial method. The reason this happens has to do with the contrast between the binomial implication of one 'up or down' per period, as opposed to the Black–Scholes' continuous application of the 'up or down' through the standard deviation.

27 The Excel© spreadsheet uses the function NORMSDIST for this process; other spreadsheets have similar functions.

Understanding the role played by each of these variables in the Black–Scholes model is quite intuitive, given the work we have already done on the binomial model. **Underlying stock price** and **option exercise price** identify the exercise value of the option, and how far 'in' or 'out of' the money the option is located. Recall that we discussed how 'at the money' options would tend to carry the highest premiums.

The risk-free interest rate determines how much money must be set aside so as to exercise the option at expiration. Note that option value **increases** with increases in the interest rate. This may seem counter-intuitive if we think of interest rates as discount rates, but it makes good sense from the perspective of reducing the amount of money that must be saved so as to produce a fixed future sum to exercise the option. In other words, as the interest rate increases, the present value of the future outlay to exercise **declines**, and therefore option value **increases**.

The **standard deviation of underlying security price** is, perhaps at first surprisingly, also positively related to option value. As the riskiness of the underlying security increases, so does the value of the option. Remember we saw that an option will tend to follow upward movements in underlying security price more so than downward movements. This implies that **any** random movement underlying security price is more likely to help than harm option value. Since standard deviation of underlying security price is a measure of the asset's likelihood and amount of a price change (the counterpart of u and d in the binomial model), the higher is σ, the higher is option value, for the same reason that the binomial model indicates that option value increases as (u-d) increases.

As the **time until expiration** increases, option value increases. This reflects simply the higher odds, given a particular σ, of the underlying security experiencing some increase in price (which helps option value) or decrease in price (which does not hurt option value as much), the longer the time until maturity of the option.

12.2.6 Relationships Among Options

To this point we have used the call option to illustrate concepts in option valuation. From the introduction to this module, however, you are aware that there are other types of options. And though valuing them is conceptually a process similar to that you have seen for call options, the exercise of valuing another type of option (a 'put' option) will be very useful in deepening your understanding of option valuation.[28]

Recall that a 'put' option allows the holder to *sell* something for a fixed price during or at a specific time. We shall shortly illustrate how to calculate the value of a put option, but before we do that, it is important to point out that there is an important value relationship between put and call options. That relationship is known in financial practice as *put–call parity*.[29]

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28 Those who choose to pursue more advanced study of options will discover that the more complex and esoteric types of options are actually just combinations of various call and put options.

29 The illustration that follows assumes throughout that put and call options will have the same exercise prices and will be held to expiration. More advanced treatments of option valuation can relax those assumptions, but they do not cause the essential relationships to be much different from those shown.

We can best understand put–call parity by returning to the diagrams of exercise payoffs. Figure 12.7 shows the result of exercising a put option.

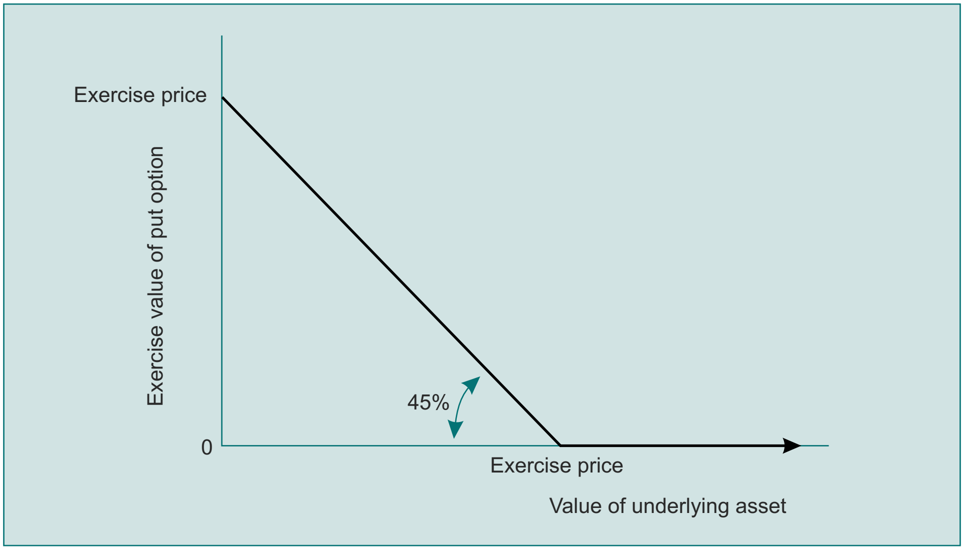

Figure 12.7 Exercise value of a put option

Figure 12.7 illustrates that the exercise payoffs to a put option differ dramatically from those of a call option. Because a put allows the holder to sell at a fixed price, the option is valuable at exercise when the underlying asset is worth less than the put's exercise price. An 'at the money' put option has an exercise value of zero, and as the underlying asset increases in value above the exercise price, the 'out of the money' put option (which allows the holder to sell the asset at the exercise price) remains at zero.[30] Of course, as the underlying asset declines in value below exercise price, the exercise value of the put option increases, because the put holder is allowed to sell the underlying asset at the exercise price even though the underlying asset has declined below that value. As the underyling asset decreases in value toward zero, the put's exercise value increases to a maximum equaling the exercise price.[31]

We are now ready to place a market value on the put option. However, instead of using the same process that we used for a call option, we shall first pursue a different route to finding market value, one based on put–call parity. To do this, we need to illustrate the payoffs of two more securities: a simple holding of the underlying asset and investing in the risk-free asset. Figure 12.8 illustrates the payoffs from investing in the underlying asset itself.

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30 Keep in mind, however, that the put option's *market value* would not be zero 'at the money' or 'out of the money' unless there was no chance that the underlying asset would decline in price to be below the put's exercise price before expiration.

31 This is in contrast to the call's exercise value maximum, which is conceptually unlimited (see Figure 12.1).

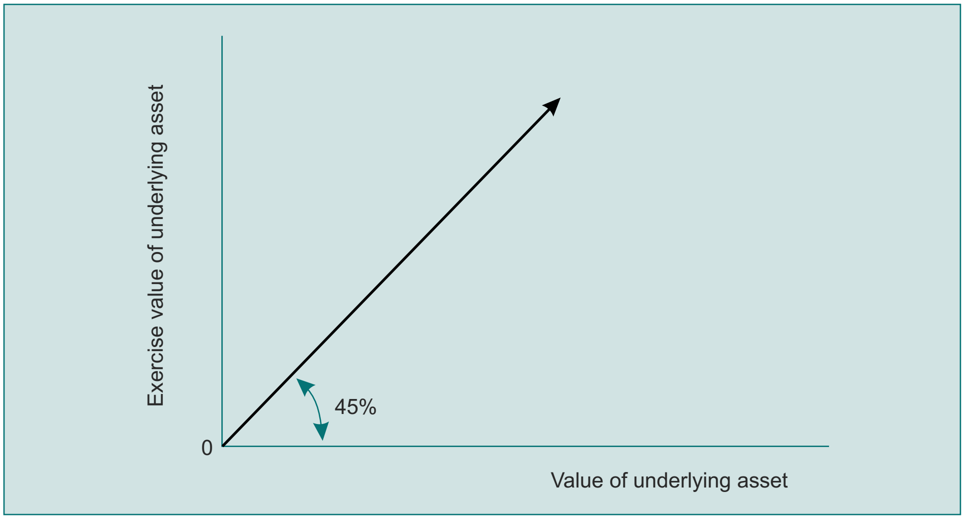

Figure 12.8 **Exercise value of an underlying asset**

Figure 12.8 illustrates, naturally, that holding the underlying asset causes its exercise value to increase or decrease exactly with its market value, with no limit on maximum value and a minimum value of zero (assuming limited liability such as that with corporate equities).

Figure 12.9 illustrates the rather obvious payoffs to investing in the riskless asset.

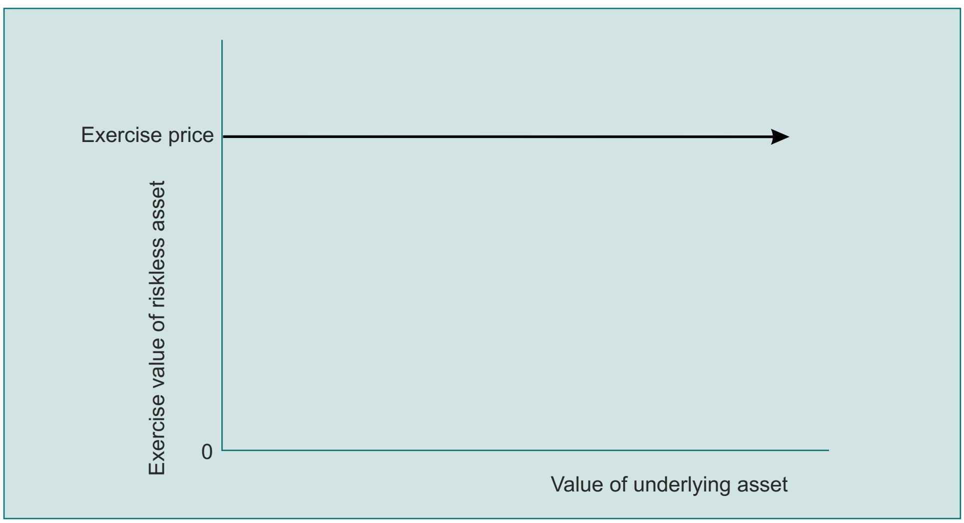

Figure 12.9 **Exercise value of a riskless asset**

Note that in Figure 12.9 we have specified the riskless asset investment to be such that it will be worth the exercise price of the put (or call) option at expiration date.

Now let us consider the payoff to the following: Suppose that you held both a put option and the underlying asset. The potential payoffs from that holding can be found by combining Figure 12.7 and Figure 12.8, which produces the payoffs shown in Figure 12.10.

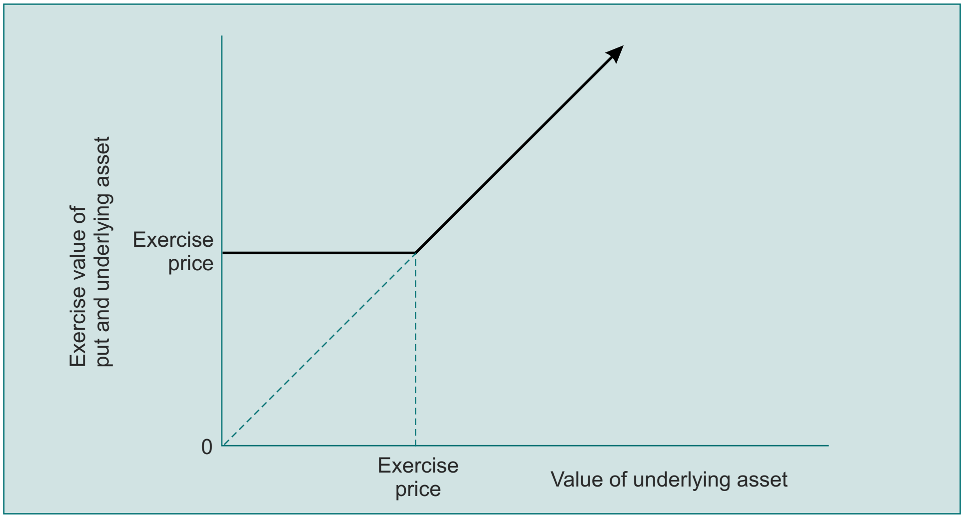

Figure 12.10 Exercise value of put and underlying asset

Figure 12.10 shows that holding both a put and the underlying asset will payoff the same as the underlying asset if the put is 'out of the money' (the underlying asset's market value is greater than the put's exercise price). But if the put finishes 'in the money' (the underlying asset's value is less than the put's exercise price) the combined put and underlying asset will be worth exactly the exercise price. For example if there were a put on 3rd Rank's shares at £1.25 and the shares had a market value of £0.75, the put would be exercised to produce a payoff of £0.50 (by selling shares worth £0.75 for a price of £1.25), which combined with the underlying asset's value of £0.75 produces a total combined value of £1.25, the exercise price.

Now consider a different combination: Suppose that you were to hold both a call option and a riskless asset that will be worth the call's exercise value at expiration. Figure 12.11 shows the payoffs that you would get.

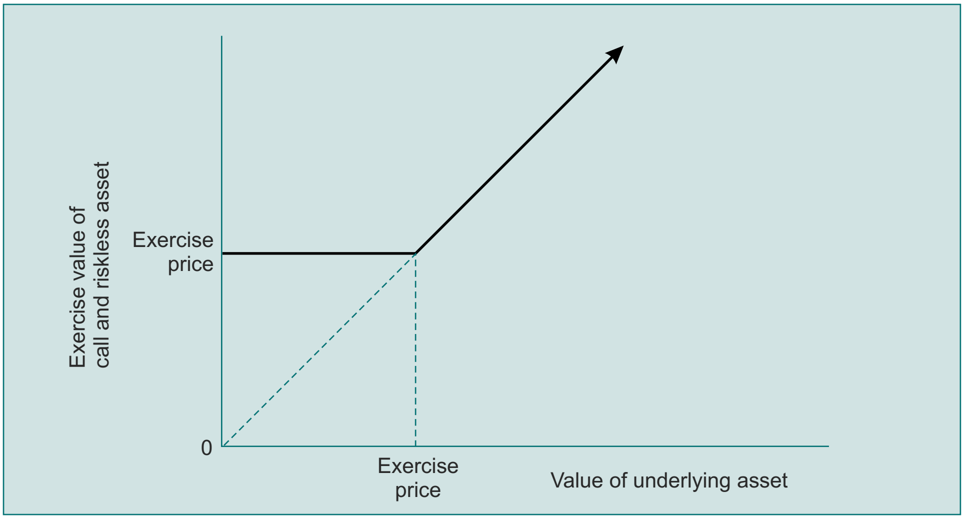

Figure 12.11 **Exercise value of call and riskless asset**

The first thing one notices about Figure 12.11 is that it looks exactly like Figure 12.10: the payoffs from holding a call and the present value of its exercise price appear to be the same as those of holding a put and its underlying asset. That is not an accident. The payoffs are in fact identical. Consider the payoffs in Figure 12.11 if the underlying asset's value is less than the call's exercise price at expiration. Here the call would not be exercised, and your holding would be worth the riskless asset's value of the exercise price. If on the other hand the underlying asset market value at call expiration was greater than exercise price you would exercise the call by using the riskless asset's value to purchase the underlying asset's market value.

Figure 12.10 and Figure 12.11 demonstrate the famous notion of *put–call parity*. They imply that the following relationship must hold:

$$\text{Put} + \text{Underlying Asset} = \text{Call} + \text{Riskless PV of Exercise Price} \tag{12.10}$$

Let us test the put–call parity relationship with our 3rd Rank example. Our earlier valuation of the 'in the money' call option for 3rd Rank showed it to be worth £0.6364. At the same time, 3rd Rank's share value was £1.50, the call's exercise price was £1.25 and the risk-free rate was 10%. Rewriting Equation 12.10 to solve for put value gives us:

$$\begin{aligned}\text{Put} &= \text{Call} + \text{Riskless PV of Exercise Price} - \text{Underlying Asset}\\ &= £0.6364 + £1.25/(1.10) - £1.50\\ &= £0.2727\end{aligned}$$

Put–call parity tells us that a put option on 3rd Rank's shares with the same exercise price as the call option, would have a market value of £0.2727.

We can check that value by using the familiar binomial valuation method, and applying it to this put option. The payoffs to the put option are:

$$P_u = \max(0, X - uS_0) = \max(0, £1.25 - £3.00) = £0.00$$
$$P_d = \max(0, X - dS_0) = \max(0, £1.25 - £0.75) = £0.50$$

Substituting P_u for C_u and P_d for C_d in Equation 12.1 and Equation 12.2:

$$Y = (£0.00 - £0.50)/[£1.50 \times (2.00 - 0.50)] = -.2222$$

$$Z = [(2.00 \times £0.50) - (£0.50 \times £0.00)]/[(2.00 - 0.50) \times (1 + 10\%)] = £0.6061$$

And from Equation 12.3:

$$P_0 = (-.2222 \times £1.50) + £0.6061 = £0.2727$$

The binomial valuation method validates that put–call parity does hold in our binomial example. Though our scope does not extend to proving it, put–call parity also holds in the Black–Scholes world of continuous time. Recall that we valued the 3rd Rank call at £0.5661 using the Black–Scholes method. You can check your understanding of put–call parity by validating our assertion that the Black–Scholes put value should be £0.2020.[32]

Put–call parity not only gives us an easy way to value a put option (if we know the call value). It also specifies a general relationship among puts, calls and their underlying asset. This can be a potentially useful piece of information in that this relationship can be manipulated just as any algebraic one, solving for whatever variable is of interest. We above solved put–call parity for the value of a put, but could just as easily have specified the value of a call as a function of put, underlying asset, exercise price and risk-free rate. This also implies that the put–call relationship can be used to illustrate the creation of securities that might not yet exist. For example if options markets have not yet chosen to write puts on 3rd Rank's shares, one can 'construct' a put by buying a call, investing the present value of the exercise price in the riskless asset, and selling short 3rd Rank's shares.[33]

12.2.7 Applications of Option Valuation

Pure options themselves as securities are increasingly important components of securities markets. The rise of active, efficient options markets now allows investors to use options on debt and equity securities, on various indexes of market activity and on futures to shape portfolio risks in ways that were inaccessible only a few years ago. But this is just scratching the surface of important uses for the options concept. As the Introduction to the module (Section 12.1) implied, there are many financial questions that can profitably be analysed as option pricing issues.

Consider the equity of a firm with debt in its capital structure. This equity is actually a **call option**: if interest and principal are paid to creditors, shareholders own the underlying assets of the firm; if interest and principal are defaulted, creditors will end up with the assets. So the equity is in this very real sense a call option on the assets of the firm. These assets were sold provisionally to the creditors when debt was issued, but can be repurchased by shareholders at the option of the shareholders through the payment of interest and principal to creditors.

[32] For the lazy, this value is £0.561 − £1.50 + £1.25/(1.10) = £0.2020.

[33] Illustrating actual undertaking of these transactions in real markets is beyond the scope of this text, but this type of activity, and much more complex transactions of similar types, is common in sophisticated applications. The section on derivatives in this module mentions some examples as 'exotics.'

'Very clever,' you say. 'It is interesting to see that the equity of a geared firm is really a call option. But what good is that? How does that recognition push forward our understanding of sophisticated financial ideas?'

The answer to that question lies in adapting the discussion of the variables in the Black–Scholes option model in the previous section to the equity of a geared firm. Recall that there are five variables of interest in the option equations, and we investigated how each affects option value. Those variables have familiar counterparts in the leveraged firm. And if we 'experiment' with changes in those variables, we can predict effects upon equity and debt value, given the option formula. These 'changes in variables' in the option formula have quite straightforward translations to financial decisions which we have discussed in other modules. In the process of this experiment we shall gain additional insight into the effects of various financial decisions upon shareholder and bondholder wealth, and the reasons for certain complex contractual provisions of bond issues.

Consider the importance of the ratio of underlying asset value to exercise price (S_0/X). For the option held as equity by shareholders of the company, underlying asset value is the total value of the company's assets, and exercise price is the interest and principal promises to creditors. If this ratio is high, the option is well 'in the money', which implies a low proportion of debt in the company's capital structure. If the ratio is low, the company has lots of debt. A ratio of 1.0 implies that all of the current value of the company is promised to creditors. We learned that the premium above exercise value for an option tends to be greatest when an option is 'at the money' (*see* Figure 12.2). In this context, though the absolute (exercise) value of the equity of the firm is highest when there is no debt (a fully 'in the money' option), equity has its most advantageous option characteristics when the company is very highly geared ('at the money').[34]

One very important influence upon equity value as an option is given by the standard deviation of underlying asset value (σ). Recall that option, and therefore equity, value increases as σ increases. The interpretation of an increase in σ is straightforward: it means that the operating assets of the firm are more risky. For example, by shifting its operations into a new, more risky line of business, the firm can increase the value of its shares.

Other option value variables have similar interpretations. For example, the longer the maturity of the debt, the higher is equity value, other things being held the same.

'That is terrific!' you say. 'We have discovered a new way to increase shareholder wealth. All that is necessary is to examine the company decisions that would increase equity value from the option perspective, take those actions, and our shareholders will be better-off. Their appreciation will certainly be reflected in high salaries for us as financial managers.'

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34 Remember that equity value falls when debt is issued, but the cash proceeds of the debt issue are available to shareholders, so a move from a well 'in the money' to an 'at the money' position would entail the shareholders being compensated for the decline in share price at least to the extent of the exercise value of the option. A 'well out of the money' option in this sense could be a firm in bankruptcy, where the debt claims far exceed the value of the company's assets.

Not so fast. We have not yet repealed the 'law of conservation of investment value', which has served us well to this point. To paraphrase that law: increases in value have to come from somewhere. From where do the equity value increases outlined above come? The answer is that they come from the pockets of debt holders.

Think of the situation this way. Suppose that a leveraged company wishes to increase its riskiness so as to increase equity value. It does this by shifting assets from less risky to more risky activities.[35] Equity value increases. But unless the value of the company's assets increases, and if asset value is the sum of equity and debt value, the only possible source of increases in equity value is reductions in debt value.

This is not difficult to understand. Recall that debt has a fixed claim upon the future cash flows (value) of the firm. Even if the value of the underlying assets remains the same, the riskier operations create greater likelihoods of debt not receiving its promised payments (in bad times). This causes debt value to decline. There will also now, with higher variance company activities, be greater likelihoods of equity receiving very high returns (in good times). Thus equity value increases and debt value decreases, in like amounts.

'Well, so what?' you ask. 'We may feel sorry for debt suppliers, but their wealth loss is their problem. If we can take such an action, we are bound to do so in the interest of our shareholders.'

Exactly. But creditors are not stupid. They can anticipate your feeling exactly as you do. So when they lend money to the company, they insist upon certain provisions in the bond contract that prohibit such actions on behalf of shareholders. Examples of such 'covenants' are prohibitions or restrictions upon asset sales, mergers, dividend payments, seniority rules and a wealth of other actions by firms. In other words, though you may wish to increase equity wealth by undertaking such activities, you will probably be contractually prohibited from doing so. Or if your bond contract allows such activities, you were probably charged 'in advance' by agreeing to a much higher interest rate than you would otherwise have paid.

The above paragraph is an important perspective on option theory applied to business finance, and it is also an introduction to the next topic we shall discuss in this module: the theory of **agency**. Agency relationships are important when there are potential conflicts of interest in decision situations. It is easy to see that complex company capital structures can cause conflicts of interest in financial decision situations.

Our lesson from this application of option valuation theory is that specific actions taken by geared companies, as these appear in the option-value models, can shift wealth from debtholders to shareholders (and of course vice versa, though financial managers would not last long in their jobs if they shifted wealth from

[35] To make the illustrations as simple as possible, assume that the risk increase is unsystematic only, and does not change the value of the assets.

shareholders to creditors). This opportunity for wealth-shifting is anticipated by bond contract provisions that restrict such behaviour by firms.[36]

There are other applications of option theory in financial decisions of firms. For example, we have discussed the conceptual advantages of regarding corporate equity claims as call options. It is also useful to think of these same securities as put options in the sense that the shareholders of a firm can sell the assets of the company to the bondholders for the lesser of the amount owed on the debt (the interest and principal) or the assets themselves (defaulting on the loan, and declaring corporate bankruptcy).

Among the most interesting and valuable of the applications of option valuation in finance is the field of 'real options'. As the next section illustrates, your introduction to options would be incomplete without an exposure to real options.

12.2.8 Real Options and 'Strategic Finance'

In earlier Modules we have devoted significant effort to developing a set of ideas and techniques for companies to use in making *real asset* decisions. Worldwide, modern firms use these techniques, emphasizing the NPV criterion, in making their major asset 'accept or reject' decisions. With near unanimity practitioners and academics in finance agree that there is no superior alternative for making such decisions. But that is deceptive. There are many important real asset and line-of-business decisions that appear to be excellent candidates for the application of classic the NPV technique, but are not. These decisions are ones with *embedded options*. **When an investment proposal carries with it an option to alter, curtail or extend a project's cash flows at some future time, classic NPV is an inadequate evaluation technique**.

Astute financial managers have long recognized that certain investment decisions have option attributes and are therefore impossible to portray in straightforward NPV format. Typically those decisions were categorized as 'strategic' financial decisions, and were made without the quantitative inputs of NPV, relying more on a senior manager's intuition and experience as to which such proposals to accept.

Recent advances in our understanding real asset option attributes have led to a better ability to quantify these 'strategic' decisions. This section of the module introduces the ideas behind options imbedded in real asset decisions or *real options* as they are called, and offers a few examples.

12.2.9 An Example of a Real Option Decision

Consider the typical situation where a decision maker would apply the NPV criterion: suppose that a proposal to undertake a particular type of project, say the WalkPhone example of Module 5, is on the table with its attendant set of cash flow projections and capital cost. The financial analyst applies the NPV technique and

[36] One of the most difficult aspects of such provisions is that, although they constrain shareholders from capturing creditors wealth, they may also constrain the firm from undertaking activities that could increase underlying asset value to the benefit of **all** capital suppliers. This trade-off is central to the capital structure decision of the firm.

recommends either accepting or rejecting the project based on a positive or negative NPV (because of the NPV's clear relationship to shareholder wealth). This is a completely proper process, *as long as the project's cash flow projections fully reflect all of the potential effects on company cash flows implied by the acceptance of the project*. You may be surprised to hear that this situation is more likely the exception than the rule.

Recall the WalkPhone example from Module 5. The essential parameters of the proposal were FCF* cash flows of:

Time	t_0	t_1	t_2	t_3	t_4
*FCF**	−£2 400	+£45	+£1 470	+£1 725	−£30

With a required 18% rv^* discount rate, and an NPV of −£272, the WalkPhone project was (with a negative NPV) not acceptable. But suppose that the company's accepting the WalkPhone project will position it to enter the market for cellular Internet devices in a few years, whereas without the experience in the WalkPhone market the cellular Internet devices (hereafter cId) market will be unavailable to the company.

'Well fine,' you say. 'If the WalkPhone project is itself a loser, but will let the company enter a subsequently very profitable market, all we must do is attach the cId cash flow projections to those of the WalkPhone project and evaluate the NPV of both together.'

Sounds reasonable.

Now suppose that the cash flows that will result from the cId project are quite uncertain, and given what is now known, could be very good or very bad. Much of this uncertainty has to do with the project's not actually beginning for several periods into the future. Further suppose, as is often the case, that as time actually passes the nature of the project's cash flows will become more evident, but the apparent desirability of the project will fluctuate up and down as evidence and opinions evolve. Nevertheless, the best guess currently available as to the cash flows associated with the cId project's cash flows is:

Time	t_4	t_5	t_6	t_7	t_8
*FCF**	−£9 075	+£170	+£5 558	+£6 523	−£113

Further suppose that the 18% rv^* discount rate is also applicable to this project. Applying this discount, the NPV of the cId project is −£1 027 at t_4, and therefore −£530 at t_0. So combining the cash flows of this project with those of WalkPhone would produce a total NPV of −£530 (cId) and −£272 (WalkPhone), or −£802.

'Terrific,' you say. 'What a nice example. Not only should the company avoid the WalkPhone, it should also forget about entering the market for cId. I cannot really say that I am very impressed by your ability to illustrate the importance of real options.'

Careful here; we have not yet dealt with all of the information we were provided about this project. The negative NPV of the cId project is based on the set of cash flow expectations as they exist right now (t_0). But the information we have is that these cash flow expectations could change dramatically and be much better (or much worse) by the time the project actually would begin (at t_4). How can we properly evaluate this more complicated situation? We can do so by regarding the situation as an option valuation problem.

Consider again the implications of accepting the WalkPhone project. Its own cash flows have an undisputed NPV of −£272. But by accepting the WalkPhone project the company will also be purchasing an option of pursuing the cId project four periods hence. Today the cId project does not look so good, but that could change between now and the time the project would be undertaken. Specifically, undertaking the WalkPhone project carries with it a −£272 NPV, **plus** the value (if any) of the *real option* to undertake the cId project.

Now, let us recall one important thing about options: *an option cannot have a negative value*. In other words, the currently negative NPV of the cId project would only be applicable if the company were committing itself to undertaking that project with its currently expected poor cash flows. And accepting the WalkPhone project does not necessarily imply accepting the cId project. Accepting WalkPhone gives the company the ability to decide (four periods hence) whether or not to pursue the cId project. If the cId project still looks bad at that point, the company can simply choose not to accept it, thereby avoiding its poor results. So the currently negative NPV of the cId project, though not completely irrelevant is certainly not a proper basis for thinking of the value of this option. The absolute **worst** result of having this option must be simply to leave the NPV of WalkPhone as it is without the option.

But how then can we think about valuing this option inherent in accepting WalkPhone? To be honest, rigorous valuation of such *real options* is still to some extent in its intellectual infancy.[37] Nevertheless, even our introductory exploration of option valuation to this point can help us conceive how we might estimate the value of this option.

For example, this option sounds like a *call option* (the right to buy something in the future). Call options have (amongst other attributes) an *exercise price*, a *time until expiration*, and an *underlying asset value* that are important parameters in option valuation. Here the exercise price is the outlay to undertake the cId project, or £9 075 at t_4. And the time until expiration is the four periods into the future when the above outlay for the project would be made. So far, so good. But what about the underlying asset value of the option? That is simply the value of the project's cash flows from beyond the date of exercise, discounted to the present *(t_0)*. With a PV of £8 048 at t4, the underlying asset value at t_0 (using the 18% rv^**)* is £4 151.

So we can describe the *real option* imbedded in the WalkPhone project as being a four period option with an exercise price of £9 075 and an underlying asset value of £4 151. Clearly (and consistent with the NPV analysis we have done) the option is 'out of the money' (its exercise price is higher than the underlying asset value). But

[37] This would doubtless be hotly disputed by the several corporate financial consulting firms that are actively marketing advice as to how this is done.

that does not mean the option is worthless. Even 'well-out-of-the-money' options can be valuable. What would cause this option to have value? Of course it would be that there is some chance the underlying asset value will increase to be above the exercise price between now and the exercise date (i.e. of the option going *into the money*, or equivalently, the cId project's expectations producing a positive NPV). And what creates such a chance? The underlying asset value will increase if the currently existing cash flow expectations for the cId project improve (due perhaps to upward revisions in revenues or downward revisions in costs or operating risk, etc.).

It may have occurred to you already that the option valuation parameter that captures this likelihood of the option going 'into the money' is the *standard deviation of underlying asset value*.[38] Recall from those discussions of call option valuation that, other things held the same, an option is **more** valuable the **higher** is the standard deviation of underlying asset value. That should make some sense here. Remember that the company buys a call option on the cId project if it undertakes WalkPhone. And that option is one that can be left unexercised (i.e. reject the cId project) if at the decision point four periods hence the cId project is undesirable. But if the cId project 'turns good' between now and then (and stays that way until t_4) the company would undertake it. *And the higher is the standard deviation of the project's underlying asset value (the more it 'bounces up and down' across time), the higher are the odds that the cId project will 'turn good' between now and the expiration date.* In other words, risky prospective projects like the cId project (with high standard deviations of underlying asset value prior to their being undertaken) are the kind that could produce high option values.

How high must the standard deviation of underlying asset value be in order for the option to have a positive value? Not very high. As a matter of fact, as long as there is any chance at all, no matter how tiny, that the option could go 'into the money' it will have a value greater than zero. **This implies that the −£272 NPV of the WalkPhone project understates the true value of the project to the company because it ignores the almost certainly positive value of the *real option* to undertake the cId project.**

'O.K,' you say. That makes some sense. But should the company undertake the WalkPhone project or not? What you have shown so far is that the WalkPhone's NPV is not as bad as −£272, but you have not shown what it is. At this point we still seem to be in the non-quantitative 'strategy' mode where we began.'

True, but we are not quite finished with the example. We have earlier illustrated specific techniques of valuing options, one of which is the binomial method. That method can be extended to deal with multiple period examples. With information as to the market rate of interest and the u and d parameters of cId underlying asset value for each year we could apply the process and value the cId option implicit in the WalkPhone project. We have also investigated the Black–Scholes call option valuation formula based on the continuous passage of time (a more realistic

38 This is the set of u and d parameters in the binomial illustrations, or the σ parameter in the Black–Scholes option valuation formula.

portrayal of actual option experience). Let us try to apply the Black–Scholes model to this cId real option.

The text's earlier explanation Black–Scholes option valuation indicates that we currently have three of its five necessary parameters (exercise price is £9 075, underlying asset value is £4 151, and time to expiration is 4 periods). If we assume that the observable riskless interest rate is 10%, we have four of the five. What remains to be specified is the standard deviation of the underlying asset value. But here, rather than simply assume a value, it is more informative to return to the decision facing the company: Should WalkPhone (with the cId option) be undertaken? To answer 'Yes' requires that the NPV of the combination must be greater than zero. Because we know that WalkPhone by itself has an NPV of −£272, another way of asking the same question is: does the option have a value at least as high as +£272? If so, WalkPhone should be undertaken, if not, both WalkPhone and its cId option should be rejected. Since the only parameter now missing from the option valuation is the underlying asset standard deviation, a final way of asking the question is: How risky must the underlying asset be, in order for the option to have a value of at least +£272?

Here one can simply begin working the Black–Scholes process until the critical standard deviation is found (the one that causes option value to equal £272), or more efficiently if you have access to a typical spreadsheet computer program, invoke a 'goal seeking' instruction to find that standard deviation after having set the Black–Scholes process into the spreadsheet as described earlier in the text. Either way, if done correctly, you will see that an annual standard deviation of 24.1% is the critical value for the option to be worth at least £272.

With the above parameters, the Black–Scholes process from Equation 12.8 and Equation 12.9 is:

$$d_1 = -0.5924$$
$$d_2 = -1.0737$$

and therefore by taking the cumulative standard normal of each:

$$N(d_1) = 0.2768$$
$$N(d_2) = 0.1415$$

and by the final Black–Scholes Equation (Equation 12.7):

$$C_0 = £272.00$$

So it is possible to estimate the value of the cId option. If the variability of the underlying asset value of the cId project is approximately 25% or more for the next four periods, the company should undertake the WalkPhone project, because the option to pursue the cId project would at least offset the negative NPV of WalkPhone. (Parenthetically, a 24.1% standard deviation is not a terribly high rate, with many average securities on stock exchanges having annual rates of about 20%.)

12.2.10 Other Real Options

The WalkPhone example we have investigated above is one type of real asset decision that has an embedded option. The WalkPhone's option is to allow the company to pursue a subsequent investment. There are other types of real asset decisions with embedded options. Consider the situation where a project can either be operated or liquidated at various points during its projected lifetime. At each of the potential liquidation points (which would be chosen if the project has turned out badly) there is effectively a *put option* that allows the company to sell the project instead of operating it for its remaining lifetime (or at least until the next liquidation option point). The company would choose to liquidate the project if the exercise price (the liquidation value) is greater than the present value of the remaining project cash flows plus the value of the additional options to liquidate further in the future. And therefore the NPV's associated with operating the project through its lifetime must be augmented with the *put option* values inherent in the ability to abandon or liquidate the project. Ignoring abandonment or liquidation options can produce misleading investment NPVs.

More advanced treatments of financial decision-making (such the advanced financial courses in this program) are necessary to examine in any detail the other specific types of options embedded in real asset decisions, but there are many, and more are being discovered with great regularity. As a matter of fact, some financial thinkers have expressed the sentiment that just about all real asset decisions have some kind of embedded option. Think of a run-of-the-mill investment project. How many of these actually lack abandonment alternatives at future times? For what proportion is it impossible to react to altered information across time by changing some characteristic of the investment? Precious few. And only those precious few are purely good candidates for straightforward NPV analysis; the rest ought to be judged by criteria that include their option values.

12.3 Agency

Finance is a fairly complex subject. But the complexity of finance as a set of ideas and techniques is as nothing compared with the complexity of actual financial markets and securities. In fact, students (and skilled practitioners) of finance have often and correctly observed that actual financial markets and securities seem to have a complexity that is unexpected given the more straightforward nature of pure financial economics (e.g. discounting and valuation processes, taxes, risk and claim hierarchies). What is the cause of this complexity? The odds are that it has something to do with the topic of this section: **agency relationships**.

What do we mean by 'agency' as it applies to financial phenomena? The answer itself is a fairly complex one, but is well worthwhile pursuing, and we shall provide that answer soon. But first, a few words of encouragement.

Learning something about agency relationships is worthwhile because of the impressive ability this concept has shown for explaining why certain financial situations **require** complexity, and also why that complexity is intellectually penetrable. In other words, we shall see that agency concepts can explain real market actions which at first seem either irrationally complex or outside the scope of traditional financial economics.

In the discussion below we shall not attempt to describe the area of agency in finance in all of its elaborate detail. Rather, our intent is to familiarise you with the basic concerns that can be profitably addressed with this approach, and show a few examples of its application.

The primary engines driving agency situations are **conflicts of interest**. Recall the situation of shareholders and bondholders of a firm, which we addressed in Section 12.2.6. We were at that point interested in the option characteristics of geared equity, and pointed out that financial managers can shift wealth from bondholders to shareholders by causing the firm to undertake unanticipatedly risky asset investments. Obviously this is a situation where shareholder and bondholder interests diverge. It is a strong candidate for analysis as an **agency problem**.

Now let us define an agency situation.[39] First, to be a problem in agency, there must be an 'agent'. An **agent** is an individual, group or organisation to whom a **principal** has designated decision-making authority. In the context of the shareholder–bondholder conflict, shareholders (or their proxies, the financial managers of the firm) are the agents, and the bondholders are the 'principals'. **Principals** are those who feel the ultimate effects of the decisions taken by agents.

In order for an agency situation to be interesting (to be a 'problem'), it must contain the potential for a conflict of interest between principal and agent. The shareholder–bondholder conflict of interest is an important one, but by no means the only agency problem experienced in company finance.

Other important agency problems arise in the relationship between a firms's managers and its shareholders.[40] Consider that **managers** can be assumed to maximise their personal wealth as a function of the decisions that they undertake for the firm. Depending upon the type of remuneration paid to managers, the efficiencies of the 'market for managerial talent' and the corporate 'takeover' market, managers may find instances of decision making where their personal best interests conflict with those of shareholders. Managers may consume company resources by spending on 'perks' such as thick carpets, private jets and attractive secretaries, all of which return less in improved company performance than they provide as consumption to managers. Managers can also 'consume' company-paid time by shirking duties. The resulting lower value of the firm is an 'agency cost'.

For managers to discontinue such activities, their personal interests must be melded with those of shareholders. To accomplish this, it is not generally enough to give some shares to managers. As long as managers own less than 100 per cent of the firm's shares, managers will gain £1 in personal benefit from consuming £1 of company resources, while losing only £1 multiplied by their percentage holding of all company shares due to the resulting lower value of shares.

If managers are discovered undertaking such company consumption, an efficient market in managerial talent could drive down their salaries. But it is not clear, in a firm with widely dispersed shareholdings, that this kind of compensation pressure

39 These definitions were introduced in Module 9.

40 The first rigorous treatment of agency problems in finance was in this context, when Jensen and Meckling (*Journal of Financial Economics*, 3, 1976) published their groundbreaking work.

can be brought to bear upon managers (who probably control the board of directors). Thus many economists think that an efficient market for company **takeovers** is an important solution to the agency problem of manager–shareholder conflict.

In a takeover, existing shareholders are offered a premium above existing market price to sell their shares, usually by an outside group of managers. The outsiders are effectively telling the shareholders that existing management is inefficient, and thus the increase in company value that will appear upon more efficient new managers taking over (and somehow avoiding the prior agency difficulties) can be allocated between old and new shareholders.[41]

There are some important general lessons about agency problems in the above discussion. First, note that there is an **incentive to solve** the agency problem. The **overall** incentive is that the increase in the value of the company if managers perform without unproductive personal consumption exceeds the value to the managers of their personal consumption. So, in general, managers would lose less than the company would gain if such activities were curtailed. But unless managers are provided **personal** incentives to change their behaviour, this value is permanently lost (an agency cost). The 'solutions' illustrated above include buyouts by outside management groups.[42]

There are other possible solutions to this agency problem, from complex management employment contracts, to managerial stock options, to auditing by independent directors, and many more. Sometimes good solutions to conflicts of interest can be derived from such devices. At other times a takeover or buyout is the ultimate solution.

But any solution to an agency problem requires that (1) there be an **overall** gain to solving the problem, which is allocated in such a way that (2) the agent has an incentive to participate in the solution. This holds, as we shall see, for bondholder–shareholder conflicts as well as shareholder–manager conflicts.

Agency problems can also arise when '**information asymmetries**' exist. An information asymmetry occurs when a buyer and seller in a transaction may have different amounts or quality of information about the decision at hand.

Consider the situation of a company attempting to sell bonds so as to undertake a positive NPV project. Suppose that the managers inside the firm have specific information as to the NPV of the project, but that the potential bondholders have only a general notion of its NPV (as is often the case, due to legal and competitive constraints upon information release).

If there is a range of possible NPVs, bondholders likely assume that NPVs lower than those known by the firm are possible. Therefore the firm will not be able to

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41 When a company 'goes private', by the **managers** buying the firm from shareholders, similar premiums above prior market values are also paid. Managers evidently themselves recognise that holding 100 per cent of the shares will cause different behaviour on their **own** parts.

42 These outside management groups are often described as 'hostile'. Though they are usually hostile to existing managers, they are not usually so to existing shareholders. As a matter of fact, entrenched management groups often go to great lengths (with publicity, lawsuits, delaying actions, and so forth) to discourage potential takeovers. Their incentives for doing so are obvious.

sell bonds at prices commensurate with the actual higher NPVs of the project. The lower bond prices are the agency cost of this situation.

Is there a 'solution' to this agency problem? It would seem that the basic conditions are there (the incentive to increase the value of the bonds by revealing the true NPV of the project). A simple solution would require that the bondholders must somehow be assured of the project's NPV before the bonds are sold, and this is usually impossible. Eventually, of course, if the project is undertaken, its NPV will be revealed by its outcome, and bond prices will increase. The problem with this situation is that bondholders instead of shareholders will receive at least part of the NPV of the project, as bond prices increase. With that foreseeable, the shareholders may not allow the project to be undertaken in the first place.[43]

One solution to this agency problem may be to issue a security more complex than a simple bond. Suppose that the company issued a bond with a **call provision**. A call provision, you remember, allows the company to repurchase the bond from the bondholders at a set price for a given period of time.[44] Call provisions in effect allow the shareholders to buy the bond at a fixed price no matter what it would be worth without the call provision. How does this solve the agency problem caused by information asymmetry about the investment?

The agency problem is that the bond is not as valuable as it should be (thus the firm does not take in enough cash from selling it originally). The call provision, being a call option on bond value, must also therefore be undervalued. The bondholders, by undervaluing the NPV of the project and thus the bond value, are also undervaluing the call provision that shareholders have upon the bond. So when the project finally proves itself, the company can call the bonds and gain their increased value.[45] Thus the agency problem caused by information asymmetry is solved by this complex security, the call-provisioned bond.

There are innumerable such agency situations that arise in finance. (One of the other important ones we have already discussed in Module 9 on capital structure: the bankruptcy costs of borrowing.) Interestingly, the 'solutions' to these problems were invented by financial markets well before researchers appreciated the agency problem itself. Actually, agency theories in finance have been most impressive in explaining financial conditions and securities that already existed but that seemed to be irrational.[46]

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43 In a more rigorous sense, the NPV of the project to shareholders will be lowered by the present value of the expected eventual amount of bond value increase.

44 Yes, the call provision is in fact a call option on the bond.

45 The actual transaction would be for the firm to set a call price for the bond such that the bondholders, because of their relative pessimism about the project, would not assign much likelihood to the bond's price increasing so as to make the call provision attractive. The firm, of course, is aware that because of the higher NPV project, the call provision is more likely to be exercised than the bondholders assume.

Actually, all of this will be accomplished in present-value terms as soon as the bond with call provision is originally sold.

46 The call provision is a good example of this. Simplistically, it is possible to think of a bond's call provision as a bet between the bondholder and shareholder about their respective capacities to predict changes in future interest rates. (Of course, if interest rates go up, bond prices go down, and vice versa.

12.4 Derivatives

12.4.1 What They Are and Are Not, and the Reasons for their Reputation

During the past few years, the popular press has produced a number of dramatic headlines describing spectacular financial market losses incurred by companies and other institutions that have participated in investments called **derivatives**. Well known names such as Procter & Gamble, Barings Bank, Orange County in California and Sumitomo Bank are but several examples of the many organisations that have lost spectacularly, in the hundreds of millions of dollars, in very brief periods of time.

But what exactly are derivatives? Are they the frighteningly complex securities that are used only by those unfortunate enough to be led (or misled) into these markets by their own ignorance or avarice? Are they designed to accomplish nothing more than quickly emptying the pockets of participants (and perhaps their shareholders or citizens)?

Actually, derivatives can be very dangerous financial instruments, so complex that those who buy them and sell them (and even at times those who invent them) do not understand enough to make responsible decisions about them. But that is not the inherent nature of derivatives. A 'derivative' is simply any financial security whose return or outcome set is derived from some other asset's value or return outcome. By this definition we have already studied a number of derivatives in the course.

Recall, for example, our review of forward and futures contracts in Module 1. The prices and returns on such securities are purely derived from the prices of underlying (sometimes called 'primitive') assets such as grain, metals, foreign exchange or even other financial securities such as government bonds. Thus these contracts properly fit the definition of 'derivatives', but in no sense are they terribly difficult to understand or necessarily financially dangerous. The same would hold true for other examples of derivatives we have seen, such as options and other contingent claim securities. Our study of these securities has more often than not indicated uses that tend to reduce risk (through hedging activity) rather than expose buyers and sellers to large risks of loss.

Why then have derivatives acquired this popular reputation as being very complicated and dangerous? Well, one reason is that they can be. There has evidently been

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A call provision, which makes a bond cheaper for the bondholder to buy, will produce a gain to shareholders if interest rates turn out to be lower than the price of the original call provision implied, and vice versa.)

But why in the world would shareholders and bondholders be interested in betting against each other on the behaviour of interest rates in the future by attaching a call provision onto a bond? All of the best evidence is that neither group would be expected to be better than the other in predicting changes in interest rates. The answer is that shareholders and bondholders are not in the least interested in participating in such a lottery (it is a 'fair' lottery in that the expected value is equal to its cost). What they are doing is avoiding the agency cost of asymmetric information. Only agency theory could have offered that explanation (though the market discovered the call provision solution well before we academics were able to explain why it existed).

a tendency on the part of some financial security issuers to offer some frighteningly complicated derivative securities, the risks of which even they do not understand. And even simple derivatives can be very dangerous when misapplied.

Consider for example a simple futures contract in copper. Our study of financial securities such as those in Module 1 implies that the participants who would logically buy and sell these securities would be producers or users of copper seeking to hedge the uncertainties of future price movements in that metal. But there is nothing (save the size of one's initial wealth) that would stop anyone from taking positions, either long or short, in copper futures. If you took a large position in copper futures, and the price unexpectedly changed in a direction adverse to your position, you could lose a lot of money very quickly, particularly if you bought on margin (leveraging), as is typically done. But engaging in such a transaction would be simply taking a bet that you have better anticipated changes better in the price of copper than has the futures market in the metal, an obviously naive assumption on your part.

This latter scenario has been an unfortunate characteristic of many of the large losses sustained by participants in derivatives markets, including the Barings, Sumitomo and Orange County fiascos. In each of these, there were either irresponsible or unsophisticated participants that led their institutions into holding derivative positions that had immense – if not bizarre – exposures to changes in metals prices, interest rates or foreign stock market indexes. Each of these stories is great fun to read, to review the embarrassing outcomes and to puzzle over how they could have happened. But the causes seem to be mostly organisational shortcomings at the institutions in question rather than any real problem in the markets for the derivatives that were the proximate cause of the losses.

There is absolutely no reason for Orange County, California, to be taking large leveraged bets on whether interest rates in the United States are going to increase or decrease. US Treasury securities markets are populated by tremendously well informed traders who spend all their time seeking to anticipate and arbitrage very small mispricings of such securities. The notion that a political subdivision in California would have any consistent ability to beat such traders at their own game is ludicrous; yet it tried.

The other typical scenario of large losses in derivatives markets is more complicated than the Orange County or Barings examples. In this alternative scenario (which we can call the Procter & Gamble type), a company with a legitimate reason for participating in derivatives markets (usually to hedge interest-rate risks stemming from other financings or working capital) bought into (or was led into, depending upon whom you believe) positions in very complex, even one-of-a-kind securities. Rather than producing hedges, the positions had exposures to changes in interest rates likely not to be comprehended by either the holder or the issuer. And interest rates did change in ways that caused huge unanticipated losses to the holders. What were thought to be hedged positions turned out to be very risky bets on particular types of changes in interest rates.

12.4.2 Participation in Derivative Markets

What is the significance of this rather lengthy discussion of the dangers of derivatives? Clearly, there are lessons to be learned from the unfortunate experience of the above organisations. For example, there should be very careful control and oversight of those responsible for committing firms to positions in derivative markets. And there clearly should be someone in the organisation who understands enough about these markets to appreciate the risks inherent in a proposed commitment (even if the understanding is simply that the proposed investment is too complicated to be understood).

But these cautions about participation in derivative markets should not be taken as instruction that these markets should always be avoided. In fact, if the great publicity accorded these examples causes companies to avoid derivative markets, it would be a most unfortunate outcome. The potential benefits of participating in derivative markets are very large for many organisations, if done correctly. We have seen some of those already. For example, in Module 11 we studied foreign exchange futures and options, and in Module 1 we discovered important uses for interest rate futures transactions. All of these are derivatives. And there are many more, from shipping companies transacting forward in bunkers (fuel for ships), to complicated (but well understood) transactions in financial markets where underlying securities such as packages of home mortgages are split into separate interest and principal payment securities and resold to financial institutions to offset mirror-image risks.

These and other derivative transactions have produced huge markets, with positions estimated to be approximately £20 000 billion at any given time (rivalling the magnitude of traditional bond and stock markets around the world). Clearly, there is something going on in these markets quite different from the alarming stories mentioned above.

What is the purpose of these derivative transactions? By far the most common type of transaction in derivative markets is the hedging of risks.[47] Properly used, derivatives serve to *reduce*, not increase, risk.

12.4.3 The Types of Derivatives

Though we have encountered many of the available derivative types and their uses elsewhere in the text, it will be worthwhile here to introduce a list of their types.

- Interest Rate
  - Forward Contracts
  - Futures Contracts
  - Options
  - Swaps

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47 This not to say that one can only hedge in these markets. Obviously the positions of Barings and Orange County were not hedges. There are potentially non-hedged and non-pathological positions that could be taken in derivative markets, but such transactions essentially require that the participant has monopolistic information about future movement in prices of the assets that underlie derivatives. Consistently obtaining such information is most unlikely.

- Stock Market
  - Futures Contracts on Market Indexes
  - Options on Market Indexes
  - Options on Individual Securities
- Foreign Exchange
  - Forward Contracts
  - Futures Contracts
  - Options
  - Swaps
- Mortgage
  - Complex Derivatives[48]
- Real Asset
  - Forward Contracts
  - Futures Contracts
  - Options Contracts
- Hybrids and Exotics

As you peruse the list, you will doubtless notice that many of the securities listed are familiar. Actually, the only ones that you may not be able to identify are likely to be the ones named 'swaps,' 'hybrids and exotics' and perhaps the mortgage-based derivatives. We shall relegate the study of the latter to those of you who are interested in pursuing those specialised securities beyond the depth permitted in this text. **Swaps**, however, are commonly enough encountered so that at least a passing familiarity with them is appropriate for a student of finance at your advancing level.

12.4.4 Swaps

'Swaps' are derivatives designed to allow hedging the risks of interest rate and foreign exchange-rate movements. A swap is a conceptually straightforward transaction where one party to the transaction exchanges one stream of cash flow with a 'counterparty', who provides the other stream of cash flow to be exchanged. There are often 'side payments' of some type to or from one or the other party or a third-party facilitator to the swap.

With that general definition of a swap, an example is in order. To illustrate an interest rate swap, suppose that a firm had issued a long-term bond with a set of promised interest and principal payments extending for some period of time. And suppose that this firm was in a line of business that caused its operating cash flows to be volatile, such that there was the chance of operational constraints due to the need to cover the periodic required interest and principal payments. If the volatility of the firm's operating cash flows was due to a type of risk that appeared in some other type of interest rate-based security (say, due to raw materials costs or credit

[48] There are many different types of mortgage-based derivatives, more than a listing here could credit. Mortgage derivatives differ from interest-based derivatives primarily in that mortgage-based derivatives have risks contingent upon changes in the timing of principal repayment by borrowers as interest rates fluctuate. Texts have been written on this subject alone.

terms that fluctuated in response to the same things that caused short-term interest rates to fluctuate), the firm could undertake an interest rate swap to reduce the risk caused by the fixed-interest payments of its long-term bonds.

Investment bankers or other financial institutions would be happy to 'facilitate' such an 'interest rate swap,' where the firm exchanges its responsibility for making fixed long-term interest payments for a responsibility to make interest payments that fluctuate up and down in concert with the firm's operating fortunes (thus in effect hedging the firm's risk of operational disruption due to financial distress). To do that, the firm transacts with a counterparty to make the firm's fixed interest payments in exchange for the firm making variable interest payments on behalf of the counterparty.[49] Thus the firm is relieved of the risk of its operating cash flows being disrupted by the necessity of making fixed interest payments.

'Wait,' you say. 'We already know how to do that another way. The firm could have bought or sold some kind of interest-rate futures. Why do we need another hedging instrument?' A good question. There really is no risk of interest rate or exchange rate movement that could not be hedged by forward or futures transactions instead of swap transactions, at least in concept. The reason for these kinds of swaps to exist is that they are cheaper to undertake than futures or forward interest-rate transactions (in the sense of avoiding a long stream of commissions to brokers on these transactions), or because those markets are not 'complete' enough (in other words, they may not offer exactly the type of contract that would hedge the risk in question), or both.

Hedging risks of adverse interest or exchange rate movements is probably the most important use of swaps, but there are others. Swaps have been used in complex combinations in exchange and interest-rate markets – to avoid certain kinds of costly national securities regulations or adverse taxation consequences, for example.

That swap transactions are important is without question. Within the last few years there has been a huge increase in the number and size of swaps, both in terms of interest-based and exchange-based risks and opportunities.

12.4.5 Exotics

'Exotics' are true derivatives, but ones formulated from combinations or mixtures of other types of derivatives. These are conceptually important derivatives. They are tailored to the very specific risk exposures of a single firm, and can be very complicated – so complicated, in fact, that there have been instances where there is at least an allegation that not even the inventor of the security understood the potential

[49] This transaction is typically done through a third-party investment banking house, who may or may not be the counterparty. And the firm is not at all released from its legal liability to make its long-term interest payments, but rather will use the payments it receives from the counterparty to make those payments. Obviously, the quality of the counterparty's promise is important, and should be no less good than the firm's, or the firm will be less well off. On occasion, the investment banking house will offer some type of guarantee, though there may well be a counterparty who would have exactly the opposite type of need.

range of outcomes that could be experienced by the firm entering into the transaction (the Procter & Gamble scenario is an example).

A detailed example of this type of derivative is difficult for this text, not only because of its complexity but also because of its length. Think, nevertheless, of a security that paid off on the contingency that some type of interest rate increased above a particular 'cap' rate, or declined below a particular 'floor' rate (the distance between the two rates sometimes being termed, appropriately enough, a 'wedding band'). Pricing such a security and shaping the risks it can hedge is formidable indeed. And further think of a security that paid off on the 'inverse' contingency of 'X% minus' some interest rate standard.

You begin to see the reason for the term 'exotics', and the above are not particularly complicated examples of the types of transactions being offered in these markets.

Are such transactions useful? Do they accomplish any reasonable economic purpose? Apparently so, or we should not expect to see anything like the volume that we do. They are important because of the very sophisticated type of 'financial engineering' they exemplify.

12.4.6 Conclusion to Derivatives

Derivatives are important financial securities, not because they are necessarily complex or even particularly new (though some are indeed both complex and new). But derivatives are significant financial phenomena that are becoming steadily more important. Giving you the capability to transact in the full range of derivatives is beyond the scope of this text, but not necessarily of advanced texts available. Many firms now regularly use derivatives to hedge a wide range of operational and financial risks. And firms that ignore derivatives run the risk of competitive disadvantage to the firms that do not.

12.5 Financial Engineering

12.5.1 The Definition of Financial Engineering and its Uses

'Financial engineering' is a very new terminology in the practice of finance, and has yet to receive a universally accepted definition. But the core of ideas applied in financial engineering is widely recognised by advanced researchers and practitioners in the field. Financial engineering is closely intertwined with the ideas and markets of derivative securities and risk hedging that we have discussed in the text to this point.

One perspective on financial engineering is that it is the 'nuts and bolts' of the process of designing 'hybrid' or 'exotic' financial securities to fit very specific risk-shaping intentions of a firm or other institution.

Suppose, for example, that a firm finds itself with the expectation of undertaking a project in a foreign country at some point in the future, that the project will require substantial purchases of some real asset, that when undertaken the project will exhibit particular kinds of cash-flow patterns of size and volatility, and that the project will have complex interactions with national regulations and taxation regimes. Financial engineering can be employed to design purchases and sales of various financial securities (complex combinations of futures and forward contracts in interest and exchange, swap commitments, options and other participations) that serve to reduce the risks of adverse movement in each of the risk exposures implied by the project (changes in interest rates, exchange rates, real asset prices, etc.), and to reduce the costs of national regulations and taxation.

Of course, designing such complex and specific shapings of risk, contingencies, timings and locations of present and future cash flows is not quantitatively a simple task. The semi-pejorative term 'rocket scientists' is often employed to describe the individuals with training and expertise in producing these financially engineered exposures. And there have been some rather spectacular failures due to poorly designed packages of contracts, as we have discussed above. But the writing on the wall is clear: financial engineering is not a mere passing fad, and will be an increasingly important set of tools for the use of financial managers.

Most true financial engineering activities are 'one-off' analyses requiring great technical expertise and familiarity with not only the financial instruments that can be used but also the company or organisation demanding the service. And because very few operating companies have the expertise on their staff to perform the necessary analyses, most financial engineering services are purchased from second-party advisors or investment bankers. But the technology of financial engineering is becoming more widely available, in both professional training and automated computational contexts.

12.5.2 The Elements of Financial Engineering

The above describes some aspects of financial engineering, but could leave the impression that financial engineering is little more than an elegant name for a set of procedures that are complex but nevertheless common in financial practice. In other words, is 'financial engineering' little more than a term used to enhance the marketability of a set of services performed by financial consultants and investment bankers, or is there some unique substance implied by the term? To be honest, there is probably some salesmanship in the selection of the term 'financial engineering', because of the implied rigour and quantitative reliability. But there is also some substance to the essential ideas of financial engineering over and above the understanding of the individual financial securities it uses and their interactions.

Financial engineering embodies the notion that there are a few elemental 'building blocks' of financial contracts that can be combined in a rigorous fashion to produce an almost unlimited variety of 'non-standard' cash-flow expectations.[50] These building blocks can be classified as having essential characteristics of **credit extension** (such as loans, bonds, etc.) or **price fixing** (such as futures and forward contracts) or **price insurance** (such as call and put options of various types).

[50] *See*, for example Smith and Smithson's *The Handbook of Financial Engineering*, Harper & Row, New York, 1990.

The conceptual contribution of financial engineering is the idea that all familiar financial securities can be regarded as comprising some combination of these three aforementioned (and highlighted) characteristics, in greater or lesser proportion; and that inventing new or 'hybrid' financial securities to fit exactly some requirement of an individual financial market participant is a matter of combining these essential elements into a package of claims that produces a profile of cash-flow expectations meeting this specific need.

Recall the complicated situation of the firm facing the project described in Section 12.5.1. A financial engineer would dissect or decompose the elemental nature of the exposures and opportunities of that firm's position with respect to the project, and design securities that would appropriately hedge the interest, exchange and real asset risks and exploit the regulatory and taxation opportunities. These elemental exposures and opportunities would have the essential characteristics of credit extension (borrowing and lending), price fixing and price insurance.

12.5.3 Conclusion to Financial Engineering

The specific procedures used by financial engineers to form these hybrid or exotic securities is well beyond the scope of this text, and as of this writing remain somewhat intellectually inaccessible to any but highly quantitatively expert practitioners. As we have seen, even these practitioners are not immune from making mistakes in the design and implementation of these complicated contracts.[51]

That having been said, financial engineering is one of the most exciting new applications of financial ideas, and promises upon its maturing to be another very valuable tool for financial practitioners.

Learning Summary

The purpose of this module has been to introduce you to more advanced topics in finance, from two perspectives. First, well-educated finance students should be aware of the general direction of advanced thinking in the field, even if they are not deeply involved at those levels. This is important because finance is a growing, active field of research and new application, with an almost constant flow of innovative concepts. Even practitioners beginning in the field must have some capacity to appreciate and make judgments about these new applications and ideas.

Our second purpose has been to show you two specific advances in financial thinking: options and agency theory. The module's presentations of these contained ideas that were unknown only a short time ago. Yet the applications of options and agency concepts are growing very quickly in financial markets, and have even greater promise. You can rely upon encountering many instances where these concepts are invoked to explain or solve financial problem.

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[51] Over and above generating a very specific and complex set of derivative cash flows, these contracts must of course be priced by the sellers (typically an investment house or commercial bank), who then have the resulting problem of hedging their own exposures on the other sides of the contracts.

12.6 Appendix 1 to Module 12: an Alternative Derivation of Binomial Call Option Value

In the main text of the module, we discovered a method of valuing a binomial call option, the method being based upon the value of a 'call-equivalent' portfolio (a portfolio formed from holdings of the underlying security and risk-free borrowing/lending, and having the same payoffs and probabilities as the call option). There is another technique of arriving at a binomial call option value, which is similar in general concept to a 'call-equivalent' portfolio, in that the technique assumes only 'no arbitrage' assumptions about the financial market, but differs in terms of the investment used.

Instead of constructing a 'call-equivalent' portfolio, this method combines the call option itself with holdings (positive or negative) of the underlying security so as to form a portfolio that has an absolutely risk-free return. Such a portfolio is termed **perfectly hedged**, in that no matter which event (or d) happens to the price of the underlying security, the same money return occurs in the portfolio.

In other words, adverse movements in share prices in the portfolio are exactly offset by positive option payoff results, and vice versa. If we can calculate the money amount of the known riskless payoff to this perfectly hedged portfolio, we can simply discount this money payoff at the known risk-free rate of interest to find the present (market) value of the portfolio.

That portfolio has two value components, namely (1) the shares of the underlying security and (2) the option. Since we know both the value of the underlying security and the portfolio in which the security and the option are held, it is simple arithmetic to deduce the value of the call option as the difference between the two.

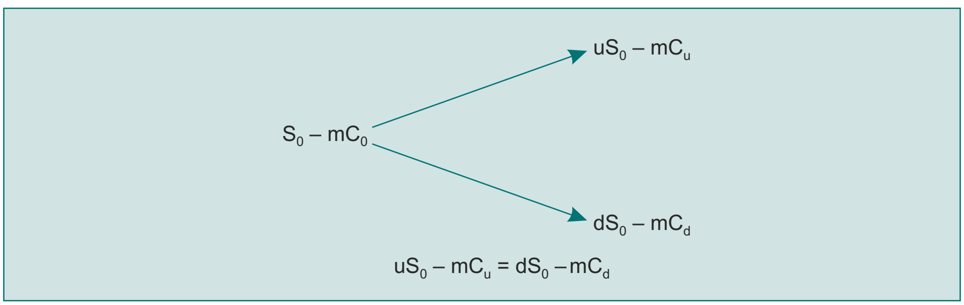

Figure 12.12 Diagram of a perfectly hedged portfolio

Let us return to our binomial example in the text of the module to see how this works. Figure 12.12 displays the essential idea of the perfectly hedged portfolio. Reading that figure from the left, S_0-mC_0 is the value of the hedged portfolio, formed from one share of the underlying security (S_0), and the 'hedging' of that security's price movements by writing call options against the security. The -mC_0 element is properly read as the cash taken in (thus the negative 'cost' to the portfolio) by writing m call options at a price of C_0 per call option. So S_0-mC_0 specifies the hedge portfolio of positive ('long') holdings of the underlying security and the call options written ('short') against the security holdings.

On the right-hand side of Figure 12.12 are the payoffs to the hedge portfolio. The expression $uS_0\text{-}mC_u$ is the amount of money that the portfolio would get if the underlying security increases in price, whereas $dS_0\text{-}mC_d$ is the portfolio's payoff if the share price goes down. Notice that the payoffs comprise both the share price change and the payoff to having written the call option.

Also in Figure 12.12 is the condition necessary for the portfolio to be perfectly hedged:

$$uS_0 - mC_u = dS_0 - mC_d \tag{12.11}$$

In other words, the portfolio payoffs must be the same no matter what happens to underlying security price or option payoff.

We can solve Equation 12.11 for m so as to find out the only thing we do not know: how many call options to write:

$$m = \frac{S_0(u-d)}{C_u - C_d} \tag{12.12}$$

The variable m actually has a special name: it is called the **hedge ratio** because it tells us how many options to write against each share of the underlying security so as to achieve the perfectly hedged portfolio:[52], [53]

Let us try out the above process on 3rd Rate's shares and call options. Recall from the module that:

S_0 = the current price of the underlying security = £1.50
q = the likelihood of the underlying security price increase = 0.6
uS_0 = the underlying security increased price result = £3.00
dS_0 = the underlying security reduced price result = £0.75

Further recall that the exercise price of the call option is £1.25, creating:

$C_u = £1.75$

and:

$C_d = £0.00$

We can now solve for the hedge ratio:

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[52] This is obviously an important concept to those who are interested in using options markets to alter the riskiness of their security holdings. One caution is that hedge ratios in real options markets (i.e. non-binomial markets) are somewhat more complex to calculate (though by no means are they inaccessibly so).

A more serious caution is that – as Equation 12.12 indicates – the hedge ratio itself is a function of the underlying stock price and option payoffs, so that any change in these over time will cause the hedge ratio to change, requiring revisions in the hedge portfolio. Many options holders have learned an expensive lesson that such actions must be taken very fast in actual markets, and sometimes are impossible to execute before the damage has been done (see the major market moves of October 1987, and the financial press's reporting of 'portfolio insurance' – which did not work as well as its promoters had hoped during that period).

[53] The astute reader may have noticed that the hedge ratio m is also the inverse of Y from the 'call equivalent' valuation process. The economic significance of this is better left to more advanced treatments of option issues.

$$m = \frac{S_0(u-d)}{C_u - C_d} = \frac{£1.50(2.0-0.5)}{£1.75-£0.00} = 1.28571 \tag{12.13}$$

This tells us that we can form a perfectly hedged portfolio by buying one share of 3rd Rate's equity and simultaneously writing 1.28571 call options.

Let us see if that works. We can test whether it does by examining the portfolio's payoffs under the two possible conditions:

$$\begin{aligned} uS_0 - mC_u &= £3.00 - (1.28571)\,£1.75 = £0.75 \\ dS_0 - mC_d &= £0.75 - (1.28571)\,£0.00 = £0.75 \end{aligned}$$

The portfolio is perfectly hedged. If 3rd Rate's shares go up and the option has its higher payoff, the portfolio returns £0.75. If 3rd Rate's shares decline and the option is worthless (unexercised), the portfolio returns £0.75, the same amount.

So we have accomplished the difficult part: reconstructing the perfectly hedged portfolio. To calculate the value of the option is child's play. The hedged portfolio will return £0.75 no matter what, so its market value can be found by discounting the £0.75 at the risk-free return:

$$\text{Portfolio value} = \frac{£0.75}{(1.10)} = £0.68181\ldots$$

And we know that the total value of the portfolio comprises a known positive 3rd Rate security value and unknown negative (written) call option amount, and so:

$$S_0 - mC_0 = £1.50 - (1.28571)C_0 = £0.68181\ldots$$

Solving this equation for the value of the call option, we obtain:

$$C_0 = (£1.50 - £0.68181)/1.28571 = £0.6364$$

This is the same result that we obtained using the 'call-equivalent' portfolio method in the main text of the module (*see* Section 12.2.3).

Note: This hedging portfolio process of option valuation is often shortened by writing the direct equations for the calculations of option value we have just accomplished. These equations are:

$$C_0 = [pC_u + (1-p)C_d]/(1+rf) \tag{12.14}$$

where p is defined by:

$$p = (1 + rf - d)/(u - d) \tag{12.15}$$

We shall not trouble with the derivation of these formulas. You can check to see that they do indeed work.

This approach to valuing an option is valuable in that it traces in a discrete time format the same process that Black–Scholes used in developing their groundbreaking and realistic continuous time model described earlier, in Section 12.2.5 of the module.

12.7 Appendix 2 to Module 12: A Numerical Application of Agency Theory

To complete this module about new ideas in finance we shall in this section present a numerical illustration of how issuing a complex security can solve a severe agency problem. In a sense this is a 'capstone' example for this module, in that we shall use elements of both options and agency theory in constructing the example. It is probably one of the better illustrations of the paths down which financial theorists are now exploring.[54]

The basic situation is this. A firm is interested in issuing debt. The proceeds of the debt will be used to undertake an investment project. The agency problem is a classic one between bondholders and shareholders: bondholders are concerned that shareholders will cause the firm to undertake a more risky investment than bondholders have been led to believe (thus shifting wealth from bondholders to shareholders), and therefore bondholders are unwilling to buy the bond at a price consistent with the lower-risk investment.

This agency problem will be solved by issuing **convertible** bonds; that is, bonds that can, at the option of the bondholders, be converted into common shares. This will solve the agency problem by causing the bonds to recapture any wealth shifts gained by shareholders.[55]

Until examples like this were cited, the reason for the existence of convertible bonds were not well understood in financial markets. Many finance texts described convertible bonds as being desirable because of the lower interest rates a firm could get by issuing these claims. Of course it is true that the interest rates are lower on convertible bonds than on non-convertible bonds, because the convertibility option granted to bondholders makes the bonds more valuable. But that extra value has to originate somewhere. If convertible bonds accomplish nothing in the way of net value creation, the higher value of the bonds must be coming from lower share values. This illustration will show that such bonds do create net value, by virtue of erasing an important agency cost.

Let us now begin building the example. Suppose that the assets of the firm undertaking the investment have a present value of £2000, based upon the expectation of cash flows resulting from the investment. Specifically, let us assume that the firm will operate for two periods, with value expectations at the end of the two periods based upon (from the binomial option model) $u = 1.4$ and $d = 0.6$. This implies values at t_1 of either £2000(1.4) = £2800, or £2000(0.6) = £1200. Similarly for the

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[54] This illustration is taken from a working paper by John Hannum, titled 'Convertible Debt and the Incentive to Shift Risks'.

[55] Note that the bonds need to be converted into shares to accomplish this recapture. Just like American call options (which are not exercised before expiration because market value always exceeds exercise value), the conversion attribute of a bond causes the bond's market value to be higher due to the simple **ability** to convert.

following period, the implied asset values at t_2 are (*uu*) £3920, (*ud*) £1680 and (*dd*) £720. This set of value expectations is shown in the top panel of Figure 12.13.[56]

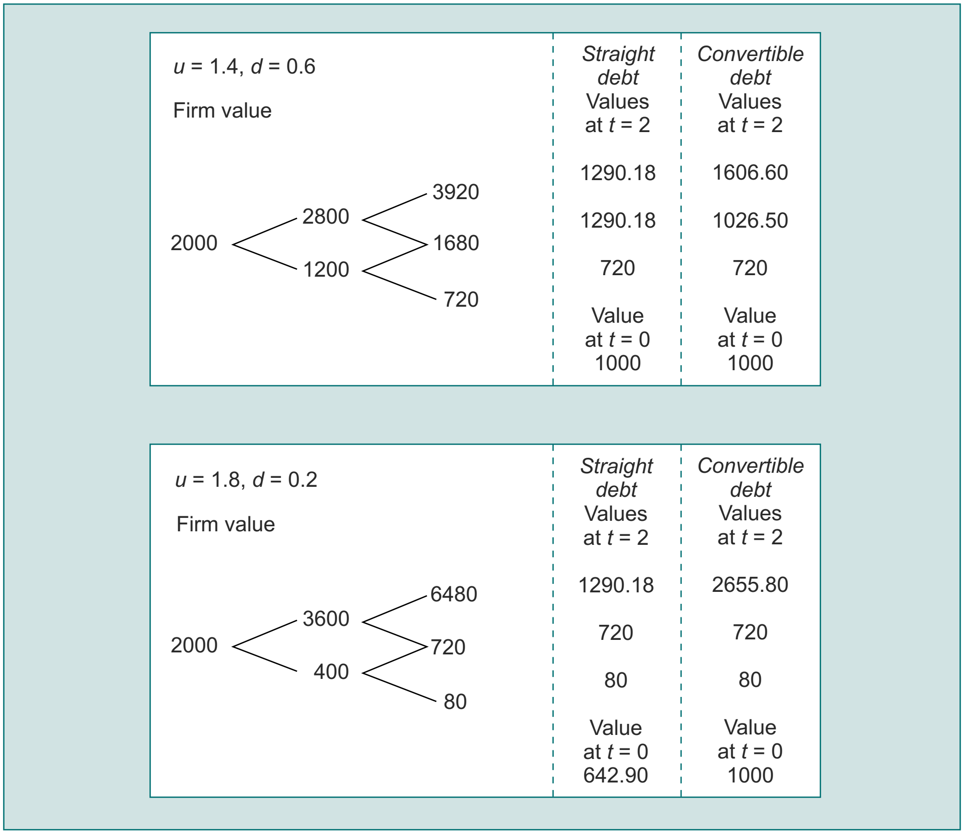

Figure 12.13 A convertible bond as an option

Now suppose that the company wishes to issue debt at t_0 (assume no taxes or bankruptcy costs) that has a promised payment at t_2 of £1290.18. If debt suppliers concur with the value expectations in the top panel of Figure 12.13, we can calculate the value of their bond using the binomial model of option valuation.

Calculating option value in two periods with the binomial model is straightforward. We first calculate the values of the option (bond) at t_1, based upon possible payoffs at t_2, and then with these values in hand we calculate the t_0 value.[57]

First, the bond payoffs. Figure 12.13 indicates that the bond will receive its promised payment of £1290.18 if one of the events *uu*, *ud* or *du* occurs because the asset values of the firm will be enough to pay the bond promise, but the bond will receive only £720 if *dd* occurs (the bond here defaults on its promised payment, and

56 Note its similarity to Figure 12.6. The only difference is that Figure 12.7 leaves out the irrelevant probabilities.

57 We have not before characterised a straight bond as being an option. But a bond is a 'contingent claim' in the same sense that a share is: the final payoffs to a bondholder are **contingent** upon the asset values that the firm actually obtains.

the bondholders end up with the entire asset value of the firm).[58] Thus at t_2, $C_{uu} = C_{ud} = C_{du} = £1290.18$, and $C_{dd} = £720$. Assuming an interest rate of 10 per cent per period, we can calculate the possible values of the bond at t_1. If u occurs during the first period, bond value at t_1 calculated using Equation 12.1 is:

$$Y = \frac{C_{uu} - C_{ud}}{S_1(u-d)} = \frac{£1290.18 - £1290.18}{£2800(1.4 - 0.6)} = 0.00$$

and, applying Equation 12.2:

$$Z = \frac{uC_{ud} - dC_{uu}}{(u-d)(1+rf)} = \frac{1.4(£1290.18) - 0.6(£1290.18)}{(1.4-0.6)(1+0.10)} = £1172.89$$

Therefore (from Equation 12.3):

$$C_1 = YS_1 + Z = 0(£2800) + £1172.89 = £1172.89$$

If d occurs during the first period, using Equation 12.1, the bond value will be (at t_1):

$$Y = \frac{C_{du} - C_{dd}}{S_1(u-d)} = \frac{£1290.18 - £720}{£1200(1.4 - 0.6)} = 0.59394$$

and, applying Equation 12.2:

$$Z = \frac{uC_{dd} - dC_{du}}{(u-d)(1+rf)} = \frac{1.4(£720) - 0.6(£1290.18)}{(1.4-0.6)(1+0.10)} = £265.79$$

Therefore (from Equation 12.3):

$$C_1 = YS_1 + Z = 0.59394(£1200) + £265.79 = £978.52$$

Taking the two possible values of the bond at t_1 as the outcomes for the bond at that time, we can now find the value of the bond at t_0:

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£1172.89 - £978.52}{£2000(1.4 - 0.6)} = 0.12148$$

and:

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)} = \frac{1.4(£978.52) - 0.6(£1172.89)}{(1.4-0.6)(1+0.10)} = £757.04$$

Therefore:

$$C_0 = YS_0 + Z = 0.12148(£2000) + £757.04 = £1000$$

So the bond will be worth £1000 (see the top panel of Figure 12.13) and therefore, with the total value of the firm at £2000, the shares must also be worth £1000.

The above assumes that the bondholders will believe the company's assessment of the risk of the project. Suppose, however, that the bondholders are concerned that the firm will take their money and put it in a higher-risk project than the one implied by the company values above. Specifically, let us assume that the bondholders fear that the firm will choose an investment yielding the asset values in the bottom panel of Figure 12.13. You can see the higher risk evident with $u=1.8$ and

58 The shareholders will not take up their option to purchase the assets of the firm from the bondholders at a price equal to the promised payment, because the promised payment exceeds the value of the assets.

d=0.2. The **company** is still worth £2000, but this value is generated by a greater spread of final values at t_2, as Figure 12.13 indicates (the highest is now £6480 instead of £3920, and the lowest is only £80 instead of £720).

The effect of this fear on the part of bondholders is that they are not willing to pay £1000 for the bond, even though the bond carries the same promised payment of £1290.18 at t_2. The higher risk of final asset values changes the possible final payoffs to bondholders to those shown in the lower panel of Figure 12.13. With this riskier project, the bondholders are looking at payoffs of either £1290.18, £720 or £80 as opposed to the £1290.18, £1290.18 or £720 implied by the lower-risk project.[59]

The lower panel of Figure 12.13 indicates that bond value will fall to £642.90.[60] Bondholders will only be willing to provide the firm with this much cash, since they are aware that, given the chance, the firm will take the higher risk project so as to benefit shareholders.

The problem here is that the firm **must** under this situation take the riskier investment, because if it does not, the bond would then increase in value, and shares decline by the same amount. This is an agency problem. But it has a nice solution: the firm can issue a convertible bond instead of the simple one used in the example thus far.

Suppose that the company, instead of issuing a straight bond, issues a convertible bond with a promised payment of £1026.50 along with an option for the bondholder to convert the bond into 40.985 per cent of the company's common stock at any time.[61]

The payoffs to such a bond are illustrated in the right-hand columns of the top and bottom panels of Figure 12.13. If the firm issues this bond and subsequently takes the lower-risk investment (top panel), the bond's payoffs are either $C_{uu} = £1606.60$ (the bondholders convert their bonds into 40.985 per cent of the shares, $0.40985 \times £3920 = £1606.60$, because this is greater than the bond's promised payment of £1026.50), $C_{ud} = £1026.50$ (the bondholders do not convert because 40.985 per cent of £1680 is obviously less than the promised payment of

[59] The higher variance of asset value increases the value of the option that shareholders have to repurchase the assets of the firm from the bondholders for the promised payment. In simpler terms, the higher variance benefits the shareholders by increasing the size of their potential highest payment without hurting the size of their potential lowest payment. This increases equity value. And since total company value is unchanged, the value shareholders gain comes from the pockets of bondholders.

[60] We shall assume either (1) that you are willing to take our word for the new bond value or (2) you can substitute the new payoffs in the sample calculations from the top panel of Figure 12.7 (which we just presented) to validate the figure.

[61] Two points about this convertible bond:

a. First, the 40.985 per cent is obviously not chosen randomly; we have chosen that specific number for a reason that will become clear.
b. Secondly, the terms by which a convertible bond can be changed into common stock are specified in the bond contract, and are usually quoted in terms of a 'conversion ratio' (number of shares per bond) or 'conversion price' (price of shares per £ of bond face value at conversion) both of which amount to simply a fixed percentage of shares.

£1026.50) or C_{dd} = £720 (the bond is in default of its promised payment, and the bondholders take the entire assets of the firm).

Valuing the convertible bond under the assumption that the firm accepts the lower-risk investment, if u occurs at t_1:

$$Y = \frac{C_{uu} - C_{ud}}{S_1(u-d)} = \frac{£1606.60 - £1026.50}{£2800(1.4-0.6)} = 0.25897$$

and:

$$Z = \frac{uC_{ud} - dC_{uu}}{(u-d)(1+rf)} = \frac{1.4(£1026.50) - 0.6(£1606.60)}{(1.4-0.6)(1+0.10)} = £537.66$$

Therefore:

$$C_1 = YS_1 + Z = 0.25897(£2800) + £537.66 = £1262.78$$

If d occurs during the first period, bond value will be (at t_1):

$$Y = \frac{C_{du} - C_{dd}}{S_1(u-d)} = \frac{£1026.50 - £720}{£1200(1.4-0.6)} = 0.31927$$

and:

$$Z = \frac{uC_{dd} - dC_{du}}{(u-d)(1+rf)} = \frac{1.4(£720) - 0.6(£1026.50)}{(1.4-0.6)(1+0.10)} = £445.57$$

Therefore:

$$C_1 = YS_1 + Z = 0.31927(£1200) + £445.57 = £828.69$$

Taking the two possible values of the convertible bond at t_1 as the outcomes for the bond at that time, we can now find its value at t_0:

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£1262.78 - £828.69}{£2000(1.4-0.6)} = 0.27131$$

and:

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)} = \frac{1.4(£828.69) - 0.6(£1262.78)}{(1.4-0.6)(1+0.10)} = £457.38$$

Therefore:

$$C_0 = YS_0 + Z = 0.27131(£2000) + £457.38 = £1000$$

The convertible bond has a value of £1000 at t_0 if the firm adopts the lower-risk investment.

Recall that the agency problem was that straight bondholders feared that the firm would not take the lower-risk investment, but would instead adopt the higher-risk investment so as to shift wealth from bondholders to shareholders. The lower panel of Figure 12.13 shows (right-hand column) that the convertibility characteristic of the bond changes its payoffs substantially relative to the straight bond when the firm takes the higher-risk investment. Notice particularly that the upper limit of payoff to the convertible bond increases to £2655.80 (bondholders here obviously choose to convert to shareholders); this recognises that the convertibility provision of the bond is like an insurance policy for bondholders. If the firm switches from the lower risk to the riskier investment, the reduction in size of the minimum payment to the convertible bond (from £720 to £80) is compensated for by an increase upon conversion in the highest payment (from £1606.60 to £2655.80).

Most importantly **the value of the convertible bond is the same, regardless of which investment the firm undertakes**. This is the critical part of the example to appreciate. The bond will be worth £1000 regardless of whether the firm accepts the high- or low-risk investment.[62]

Why is this important? Because this complex security has caused the agency problem to disappear. Convertible bondholders would have no reason to fear the firm choosing the high-risk investment, for they are as well off under either investment choice. And the firm would have no **incentive** to choose the higher- (or lower-) risk investment, since there is no wealth shift implied between shareholders and bondholders as a function of the choice of investments.[63]

This example shows the essence of an agency problem – and the solution to it. First, a conflict of interest that causes the decision maker to choose suboptimally, and then a solution that merges the interests of the conflicting parties, producing an optimal decision.

62 Again, to save space we do not show the calculation for the high-risk bond, though it proceeds identically to that of the low-risk.

63 Suppose that the low-risk investment had an NPV slightly higher than the high-risk investment. In that situation, a straight bond issue would force the firm to choose the lower NPV investment, whereas the convertible bond issue would allow the firm to choose the higher NPV investment. We did not design this into the example because the numbers were complicated enough already.

Review Questions

12.1 One of the important variables influencing option value is the variance of return on the underlying security or asset. As the variance of the underlying asset increases, a call option on that asset becomes which of the following?

A. More valuable because in a call option higher underlying asset variance means positive returns are accentuated and negative returns are dampened.
B. Less valuable because the call option is thereby more risky.
C. More valuable because Equation 12.7, Equation 12.8 and Equation 12.9 say so.
D. Less valuable because the chances of the option ever going 'in the money' have decreased.

12.2 Another of the important variables influencing option value is the market interest rate. As interest rates increase, which of the following does the value of the option become?

A. Lower, because the discount rates applied to the option's future cash flows are now higher.
B. Higher, because the present value of the amount of money necessary to exercise the option is lower, and option value varies inversely with exercise price.
C. Indeterminate, because although discount rates are higher, the present value of exercise price is less.
D. None of the above.

12.3 The equity of a firm that has borrowed money is correctly considered a 'call option' because of which of the following?

A. The shareholders can repurchase the bonds if there is a call provision on them.
B. The shareholders can repurchase the assets of the firm by paying the interest and principal on the bonds.
C. The bondholders can keep the assets if they wish.
D. The bondholders hold a put option to resell the assets of the firm to the shareholders.

12.4 Using the binomial option model, calculate the value of a call option due to expire in one period, with an exercise price of £22, underlying share price of £25, u = 1.2, d = 0.7 and rf = 10 per cent.

12.5 If a company has bonds outstanding, the bonds are almost certain to have contractual provisions that restrict the ability of the company to undertake activities such as a merger, dividend payments, entering a new line of business and others. The net effect of these provisions is that they:

I. Probably hurt the value of the firm as a whole because they inhibit the ability of the firm to pursue otherwise optimal operating and financial activities.

II. Are necessary so as to get the bondholders to agree to lend money to the firm, so are a required part of having debt in the company's capital structure.

III. Are good for the firm in that they hold managerial decision making to some kind of an external, easily measurable standard.

Which of the following is correct?

A. Both I and II.

B. II only.

C. I, II and III.

D. I only.

12.6 The reason why convertible bonds exist in financial markets is which of the following?

A. They are a mechanism to merge the conflicting interests of shareholders and bondholders.

B. They will increase shareholder wealth if share prices turn out to be lower than what was expected at the time that the conversion characteristics of the bond were priced by the market.

C. Interest rates are lower on convertible bonds than on non-convertible bonds.

D. When the company issues convertible bonds, with other provisions the same as a non-convertible bond, the company takes in more money.

Appendix 1

Statistical Tables

Table A1.1 Present value of £1 to be received at time point *t*

$$\frac{1}{(1+i)^t}$$

Period *t*	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091	.9009	.8929
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264	.8116	.7972
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513	.7312	.7118
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830	.6587	.6355
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209	.5935	.5674
6	.9420	.8880	.8375	.7903	.7462	.7050	.6663	.6302	.5963	.5645	.5346	.5066
7	.9327	.8706	.8131	.7599	.7107	.6651	.6227	.5835	.5470	.5132	.4817	.4523
8	.9235	.8535	.7894	.7307	.6768	.6274	.5820	.5403	.5019	.4665	.4339	.4039
9	.9143	.8368	.7664	.7026	.6446	.5919	.5439	.5002	.4604	.4241	.3909	.3606
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855	.3522	.3220
11	.8963	.8043	.7224	.6496	.5847	.5268	.4751	.4289	.3875	.3505	.3173	.2875
12	.8874	.7885	.7014	.6246	.5568	.4970	.4440	.3971	.3555	.3186	.2858	.2567
13	.8787	.7730	.6810	.6006	.5303	.4688	.4150	.3677	.3262	.2897	.2575	.2292
14	.8700	.7579	.6611	.5775	.5051	.4423	.3878	.3405	.2992	.2633	.2320	.2046
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394	.2090	.1827
16	.8528	.7284	.6232	.5339	.4581	.3936	.3387	.2919	.2519	.2176	.1883	.1631
17	.8444	.7142	.6050	.5134	.4363	.3714	.3166	.2703	.2311	.1978	.1696	.1456
18	.8360	.7002	.5874	.4936	.4155	.3503	.2959	.2502	.2120	.1799	.1528	.1300
19	.8277	.6864	.5703	.4746	.3957	.3305	.2765	.2317	.1945	.1635	.1377	.1161
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486	.1240	.1037
21	.8114	.6598	.5375	.4388	.3589	.2942	.2415	.1987	.1637	.1351	.1117	.0926
22	.8034	.6468	.5219	.4220	.3418	.2775	.2257	.1839	.1502	.1228	.1007	.0826
23	.7954	.6342	.5067	.4057	.3256	.2618	.2109	.1703	.1378	.1117	.0907	.0738
24	.7876	.6217	.4919	.3901	.3101	.2470	.1971	.1577	.1264	.1015	.0817	.0659
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923	.0736	.0588
26	.7720	.5976	.4637	.3607	.2812	.2198	.1722	.1352	.1064	.0839	.0663	.0525
27	.7644	.5859	.4502	.3468	.2678	.2074	.1609	.1252	.0976	.0763	.0597	.0469
28	.7568	.5744	.4371	.3335	.2551	.1956	.1504	.1159	.0895	.0693	.0538	.0419
29	.7493	.5631	.4243	.3207	.2429	.1846	.1406	.1073	.0822	.0630	.0485	.0374
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573	.0437	.0334
35	.7059	.5000	.3554	.2534	.1813	.1301	.0937	.0676	.0490	.0356	.0259	.0189
40	.6717	.4529	.3066	.2083	.1420	.0972	.0668	.0460	.0318	.0221	.0154	.0107
45	.6391	.4102	.2644	.1712	.1113	.0727	.0476	.0313	.0207	.0137	.0091	.0061
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085	.0054	.0035

13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
.8850	.8772	.8696	.8621	.8547	.8475	.8403	.8333	.8000	.7692	.7407	.7143	.6667
.7831	.7695	.7561	.7432	.7305	.7182	.7062	.6944	.6400	.5917	.5487	.5102	.4444
.6931	.6750	.6575	.6407	.6244	.6086	.5934	.5787	.5120	.4552	.4064	.3644	.2963
.6133	.5921	.5718	.5523	.5337	.5158	.4987	.4823	.4096	.3501	.3011	.2603	.1975
.5428	.5194	.4972	.4761	.4561	.4371	.4190	.4019	.3277	.2693	.2230	.1859	.1317
.4803	.4556	.4323	.4104	.3898	.3704	.3521	.3349	.2621	.2072	.1652	.1328	.0878
.4251	.3996	.3759	.3538	.3332	.3139	.2959	.2791	.2097	.1594	.1224	.0949	.0585
.3762	.3506	.3269	.3050	.2848	.2660	.2487	.2326	.1678	.1226	.0906	.0678	.0390
.3329	.3075	.2843	.2630	.2434	.2255	.2090	.1938	.1342	.0943	.0671	.0484	.0260
.2946	.2697	.2472	.2267	.2080	.1911	.1756	.1615	.1074	.0725	.0497	.0346	.0173
.2607	.2366	.2149	.1954	.1778	.1619	.1476	.1346	.0859	.0558	.0368	.0247	.0116
.2307	.2076	.1869	.1685	.1520	.1372	.1240	.1122	.0687	.0429	.0273	.0176	.0077
.2042	.1821	.1625	.1452	.1299	.1163	.1042	.0935	.0550	.0330	.0202	.0126	.0051
.1807	.1597	.1413	.1252	.1110	.0985	.0876	.0779	.0440	.0254	.0150	.0090	.0034
.1599	.1401	.1229	.1079	.0949	.0835	.0736	.0649	.0352	.0195	.0111	.0064	.0023
.1415	.1229	.1069	.0930	.0811	.0708	.0618	.0541	.0281	.0150	.0082	.0046	.0015
.1252	.1078	.0929	.0802	.0693	.0600	.0520	.0451	.0225	.0116	.0061	.0033	.0010
.1108	.0946	.0808	.0691	.0592	.0508	.0437	.0376	.0180	.0089	.0045	.0023	.0007
.0981	.0829	.0703	.0596	.0506	.0431	.0367	.0313	.0144	.0068	.0033	.0017	.0005
.0868	.0728	.0611	.0514	.0443	.0365	.0308	.0261	.0115	.0053	.0025	.0012	.0003
.0768	.0638	.0531	.0443	.0370	.0309	.0259	.0217	.0092	.0040	.0018	.0009	.0002
.0680	.0560	.0462	.0382	.0316	.0262	.0218	.0181	.0074	.0031	.0014	.0006	.0001
.0601	.0491	.0402	.0329	.0270	.0222	.0183	.0151	.0059	.0024	.0010	.0004	.0001
.0532	.0431	.0349	.0284	.0231	.0188	.0154	.0126	.0047	.0018	.0007	.0003	.0001
.0471	.0378	.0304	.0245	.0197	.0160	.0129	.0105	.0038	.0014	.0006	.0002	.0000
.0417	.0331	.0264	.0211	.0169	.0135	.0109	.0087	.0030	.0011	.0004	.0002	.0000
.0369	.0291	.0230	.0182	.0144	.0115	.0091	.0073	.0024	.0008	.0003	.0001	.0000
.0326	.0255	.0200	.0157	.0123	.0097	.0077	.0061	.0019	.0006	.0002	.0001	.0000
.0289	.0224	.0174	.0135	.0105	.0082	.0064	.0051	.0015	.0005	.0002	.0001	.0000
.0256	.0196	.0151	.0116	.0090	.0070	.0054	.0042	.0012	.0004	.0001	.0000	.0000
.0139	.0102	.0075	.0055	.0041	.0030	.0023	.0017	.0004	.0001	.0000	.0000	.0000
.0075	.0053	.0037	.0026	.0019	.0013	.0010	.0007	.0001	.0000	.0000	.0000	.0000
.0041	.0027	.0019	.0013	.0009	.0006	.0004	.0003	.0000	.0000	.0000	.0000	.0000
.0022	.0014	.0009	.0006	.0004	.0003	.0002	.0001	.0000	.0000	.0000	.0000	.0000

Table A1.2 Present value of £1 per period for t periods

$$\frac{1-\frac{1}{(1+i)^t}}{i}$$

Period *t*	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.9009	0.8929
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.7125	1.6901
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4437	2.4018
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.1024	3.0373
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6959	3.6048
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.2305	4.1114
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.7122	4.5638
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	5.1461	4.9676
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.5370	5.3282
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.8892	5.6502
11	10.368	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	6.2065	5.9377
12	11.255	10.575	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.4924	6.1944
13	12.134	11.348	10.635	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.7499	6.4235
14	13.004	12.106	11.296	10.563	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.9819	6.6282
15	13.865	12.849	11.938	11.118	10.380	9.7122	9.1079	8.5595	8.0607	7.6061	7.1909	6.8109
16	14.718	13.578	12.561	11.652	10.838	10.106	9.4466	8.8514	8.3126	7.8237	7.3792	6.9740
17	15.562	14.292	13.166	12.166	11.274	10.477	9.7632	9.1216	8.5436	8.0216	7.5488	7.1196
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.3719	8.7556	8.2014	7.7016	7.2497
19	17.226	15.678	14.324	13.134	12.085	11.158	10.336	9.6036	8.9501	8.3649	7.8393	7.3658
20	18.046	16.351	14.877	13.590	12.462	11.470	10.594	9.8181	9.1285	8.5136	7.9633	7.4694
21	18.857	17.011	15.415	14.029	12.821	11.764	10.836	10.017	9.2922	8.6487	8.0751	7.5620
22	19.660	17.658	15.937	14.451	13.163	12.042	11.061	10.201	9.4424	8.7715	8.1757	7.6446
23	20.456	18.292	16.444	14.857	13.489	12.303	11.272	10.371	9.5802	8.8832	8.2664	7.7184
24	21.243	18.914	16.936	15.247	13.799	12.550	11.469	10.529	9.7066	8.9847	8.3481	7.7843
25	22.023	19.523	17.413	15.622	14.094	12.783	11.654	10.675	9.8226	9.0770	8.4217	7.8431
26	22.795	20.121	17.887	15.983	14.375	13.003	11.826	10.810	9.9290	9.1609	8.4881	7.8957
27	23.560	20.707	18.327	16.330	14.643	13.211	11.987	10.935	10.027	9.2372	8.5478	7.9426
28	24.316	21.281	18.764	16.663	14.898	13.406	12.137	11.051	10.116	9.3066	8.6016	7.9844
29	25.066	21.844	19.188	16.984	15.141	13.591	12.278	11.158	10.198	9.3696	8.6501	8.0218
30	25.808	22.396	19.600	17.292	15.372	13.765	12.409	11.258	10.274	9.4269	8.6938	8.0552
35	29.409	24.999	21.487	18.665	16.374	14.498	12.948	11.655	10.567	9.6442	8.8552	8.1755
40	32.835	27.355	23.115	19.793	17.159	15.046	13.332	11.925	10.757	9.7791	8.9511	8.2438
45	36.095	29.490	24.519	20.720	17.774	15.456	13.606	12.108	10.881	9.8628	9.0079	8.2825
50	39.196	31.424	25.730	21.482	18.256	15.762	13.801	12.233	10.962	9.9148	9.0417	8.3045

13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
0.8850	0.8772	0.8696	0.8621	0.8547	0.8475	0.8403	0.8333	0.8000	0.7692	0.7407	0.7143	0.6667
1.6681	1.6467	1.6257	1.6052	1.5852	1.5656	1.5465	1.5278	1.4400	1.3609	1.2894	1.2245	1.1111
2.3612	2.3216	2.2832	2.2459	2.2096	2.1743	2.1399	2.1065	1.9520	1.8161	1.6959	1.5889	1.4074
2.9745	2.9137	2.8550	2.7982	2.7432	2.6901	2.6386	2.5887	2.3616	2.1662	1.9969	1.8492	1.6049
3.5172	3.4331	3.3522	3.2743	3.1993	3.1272	3.0576	2.9906	2.6893	2.4356	2.2200	2.0352	1.7366
3.9975	3.8887	3.7845	3.6847	3.5892	3.4976	3.4098	3.3255	2.9514	2.6427	2.3852	2.1680	1.8244
4.4226	4.2883	4.1604	4.0386	3.9224	3.8115	3.7057	3.6046	3.1611	2.8021	2.5075	2.2628	1.8829
4.7988	4.6389	4.4873	4.3436	4.2072	4.0776	3.9544	3.8372	3.3289	2.9247	2.5982	2.3306	1.9220
5.1317	4.9464	4.7716	4.6065	4.4506	4.3030	4.1633	4.0310	3.4631	3.0190	2.6653	2.3790	1.9480
5.4262	5.2161	5.0188	4.8332	4.6586	4.4941	4.3389	4.1925	3.5705	3.0915	2.7150	2.4136	1.9653
5.6869	5.4527	5.2337	5.0286	4.8364	4.6560	4.4865	4.3271	3.6564	3.1473	2.7519	2.4383	1.9769
5.9176	5.6603	5.4206	5.1971	4.9884	4.7932	4.6105	4.4392	3.7251	3.1903	2.7792	2.4559	1.9846
6.1218	5.8424	5.5831	5.3423	5.1183	4.9095	4.7147	4.5327	3.7801	3.2233	2.7994	2.4685	1.9897
6.3025	6.0021	5.7245	5.4675	5.2293	5.0081	4.8023	4.6106	3.8241	3.2487	2.8144	2.4775	1.9931
6.4624	6.1422	5.8474	5.5755	5.3242	5.0916	4.8759	4.6755	3.8593	3.2682	2.8255	2.4839	1.9954
6.6039	6.2651	5.9542	5.6685	5.4053	5.1624	4.9377	4.7296	3.8874	3.2832	2.8337	2.4885	1.9970
6.7291	6.3729	6.0472	5.7487	5.4746	5.2223	4.9897	4.7746	3.9099	3.2948	2.8398	2.4918	1.9980
6.8399	6.4674	6.1280	5.8178	5.5339	5.2732	5.0333	4.8122	3.9279	3.3037	2.8443	2.4941	1.9986
6.9380	6.5504	6.1982	5.8775	5.5845	5.3162	5.0700	4.8435	3.9424	3.3105	2.8476	2.4958	1.9991
7.0248	6.6231	6.2593	5.9288	5.6278	5.3527	5.1009	4.8696	3.9539	3.3158	2.8501	2.4970	1.9994
7.1016	6.6870	6.3125	5.9731	5.6648	5.3837	5.1268	4.8913	3.9631	3.3198	2.8519	2.4979	1.9996
7.1695	6.7429	6.3587	6.0113	5.6964	5.4099	5.1486	4.9094	3.9705	3.3230	2.8533	2.4985	1.9997
7.2297	6.7921	6.3988	6.0442	5.7234	5.4321	5.1668	4.9245	3.9764	3.3254	2.8543	2.4989	1.9998
7.2829	6.8351	6.4338	6.0726	5.7465	5.4509	5.1822	4.9371	3.9811	3.3272	2.8550	2.4992	1.9999
7.3300	6.8729	6.4641	6.0971	5.7662	5.4669	5.1951	4.9476	3.9849	3.3286	2.8556	2.4994	1.9999
7.3717	6.9061	6.4906	6.1182	5.7831	5.4804	5.2060	4.9563	3.9879	3.3297	2.8560	2.4996	1.9999
7.4086	6.9352	6.5135	6.1364	5.7975	5.4919	5.2151	4.9636	3.9903	3.3305	2.8563	2.4997	2.0000
7.4412	6.9607	6.5335	6.1520	5.8099	5.5016	5.2228	4.9697	3.9923	3.3312	2.8565	2.4998	2.0000
7.4701	6.9830	6.5509	6.1656	5.8204	5.5098	5.2292	4.9747	3.9938	3.3317	2.8567	2.4999	2.0000
7.4957	7.0027	6.5660	6.1772	5.8294	5.5168	5.2347	4.9789	3.9950	3.3321	2.8568	2.4999	2.0000
7.5856	7.0700	6.6166	6.2153	5.8582	5.5386	5.2512	4.9915	3.9964	3.3330	2.8571	2.5000	2.0000
7.6344	7.1050	6.6418	6.2335	5.8713	5.5482	5.2582	4.9966	3.9995	3.3332	2.8571	2.5000	2.0000
7.6609	7.1232	6.6543	6.2421	5.8773	5.5523	5.2611	4.9986	3.9998	3.3333	2.8571	2.5000	2.0000
7.6752	7.1327	6.6605	6.2463	5.8801	5.5541	5.2623	4.9995	3.9999	3.3333	2.8571	2.5000	2.0000

Table A1.3 Future value of £1 t periods hence

$(1 + i)t$

Period *t*	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1100	1.1200
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2321	1.2544
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.3676	1.4049
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5181	1.5735
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.6851	1.7623
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.8704	1.9738
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.0762	2.2107
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.3045	2.4760
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.5580	2.7731
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	2.8394	3.1058
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.1518	3.4785
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.4985	3.8960
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	3.8833	4.3635
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.3104	4.8871
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	4.7846	5.4736
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	5.3109	6.1304
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	5.8951	6.8660
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	6.5436	7.6900
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1471	6.1159	7.2633	8.6128
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	8.0623	9.6463
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	8.9492	10.804
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	9.9336	12.100
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	11.026	13.552
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	12.239	15.179
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.835	13.585	17.000
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.918	15.080	19.040
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.245	13.110	16.739	21.325
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.167	14.421	18.580	23.884
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.172	15.863	20.624	26.750
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.063	13.268	17.449	22.892	29.960
35	1.4166	1.9999	2.8139	3.9461	5.5160	7.6861	10.677	14.785	20.414	28.102	38.575	52.800
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.286	14.974	21.725	31.409	45.259	65.001	93.051
45	1.5648	2.4379	3.7816	5.8412	8.9850	13.765	21.002	31.920	48.327	72.890	109.53	163.99
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.902	74.358	117.39	184.56	289.00

13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
1.1300	1.1400	1.1500	1.1600	1.1700	1.1800	1.1900	1.2000	1.2500	1.3000	1.3500	1.4000	1.5000
1.2769	1.2996	1.3225	1.3456	1.3689	1.3924	1.4161	1.4400	1.5625	1.6900	1.8225	1.9600	2.2500
1.4429	1.4815	1.5209	1.5609	1.6016	1.6430	1.6852	1.7280	1.9531	2.1970	2.4604	2.7440	3.3750
1.6305	1.6890	1.7490	1.8106	1.8739	1.9388	2.0053	2.0736	2.4414	2.8561	3.3215	3.8416	5.0625
1.8424	1.9254	2.0114	2.1003	2.1924	2.2878	2.3864	2.4883	3.0518	3.7129	4.4840	5.3782	7.5938
2.0820	2.1950	2.3131	2.4364	2.5652	2.6996	2.8398	2.9860	3.8147	4.8268	6.0534	7.5295	11.391
2.3526	2.5023	2.6600	2.8262	3.0012	3.1855	3.3793	3.5832	4.7684	6.2749	8.1722	10.541	17.086
2.6584	2.8526	3.0590	3.2784	3.5115	3.7589	4.0214	4.2998	5.9605	8.1573	11.032	14.758	25.629
3.0040	3.2519	3.5179	3.8030	4.1084	4.4355	5.7854	5.1598	7.4506	10.604	14.894	20.661	38.443
3.3946	3.7072	4.0456	4.4114	4.8086	5.2338	5.6947	6.1917	9.3132	13.786	20.107	28.925	57.665
3.8359	4.2262	4.6524	5.1173	5.6240	6.1759	6.7767	7.4301	11.642	17.922	27.144	40.496	86.498
4.3345	4.8179	5.3503	5.9360	6.5801	7.2876	8.0642	8.9161	14.552	23.298	36.644	56.694	129.75
4.8980	5.4924	6.1528	6.8858	7.6987	8.5994	9.5964	10.699	18.190	30.288	49.470	79.371	194.62
5.5348	6.2613	7.0757	7.9875	9.0075	10.147	11.420	12.839	22.737	39.374	66.784	111.12	291.93
6.2543	7.1379	8.1371	9.2655	10.539	11.974	13.590	15.407	28.422	51.186	90.158	155.57	437.89
7.0673	8.1372	9.3576	10.748	12.330	14.129	16.172	18.488	35.527	66.542	121.71	217.80	656.84
7.9861	9.2765	10.761	12.468	14.426	16.672	19.244	22.186	44.409	86.504	164.31	304.91	985.26
9.0243	10.575	12.375	14.463	16.879	19.673	22.901	26.623	55.511	112.46	221.82	426.88	1477.9
10.197	12.056	14.232	16.777	19.748	23.214	27.252	31.948	69.389	146.19	299.46	597.63	2216.8
11.523	13.743	16.367	19.461	23.106	27.393	32.429	38.338	86.736	190.05	404.27	836.68	3325.3
13.021	15.668	18.822	22.574	27.034	32.324	38.591	46.005	108.42	247.06	545.77	1171.4	4987.9
14.714	17.861	21.645	26.186	31.629	38.142	45.923	55.206	135.53	321.18	736.79	1639.9	7481.8
16.627	20.362	24.891	30.376	37.006	45.008	54.649	66.247	169.41	417.54	994.66	2295.9	11223.
18.788	23.212	28.625	35.236	43.297	53.109	65.032	79.497	211.76	542.80	1342.8	3214.2	16834.
21.231	26.462	32.919	40.874	50.658	62.669	77.388	95.396	264.70	705.64	1812.8	4499.9	25251.
23.991	30.167	37.857	47.414	59.270	73.949	92.092	114.48	330.87	917.33	2447.2	6299.8	37877.
27.109	34.390	43.535	55.000	69.345	87.260	109.59	137.37	413.59	1192.5	3308.8	8819.8	56815.
30.633	39.204	50.066	63.800	81.134	102.97	130.41	164.84	516.99	1550.3	4460.1	12348.	85223.
34.616	44.693	57.575	74.009	94.927	121.50	155.19	197.81	646.23	2015.4	6021.1	17287.	*
39.116	50.950	66.212	85.850	111.06	143.37	184.68	237.38	807.79	2620.0	8128.5	24201.	*
72.069	98.100	133.18	180.31	243.50	328.00	440.70	590.67	2465.2	9727.9	36449	*	*
132.78	188.88	267.86	378.72	533.87	750.38	1051.7	1469.8	7523.2	36119.	*	*	*
244.64	363.68	538.77	795.44	1170.5	1716.7	2509.7	3657.3	22959.	*	*	*	*
450.74	700.23	1083.7	1670.7	2566.2	3927.4	5988.9	9100.4	70065.	*	*	*	*

* Interest factors exceed 99999.

Table A1.4 **Future value of £1 per period for t periods**

$$\frac{(1+i)^t - 1}{i}$$

Period t	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1100	2.1200
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3421	3.3744
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7097	4.7793
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.2278	6.3528
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	7.9129	8.1152
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	9.7833	10.089
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.260	10.637	11.028	11.436	11.859	12.300
9	9.3685	9.7546	10.159	10.583	11.027	11.491	11.978	12.488	13.021	13.579	14.164	14.776
10	10.462	10.950	11.464	12.006	12.578	13.181	13.816	14.487	15.193	15.937	16.722	17.549
11	11.567	12.169	12.808	13.486	14.207	14.972	15.784	16.645	17.560	18.531	19.561	20.655
12	12.683	13.412	14.192	15.026	15.917	16.870	17.888	18.977	20.141	21.384	22.713	24.133
13	13.809	14.680	15.618	16.627	17.713	18.882	20.141	21.495	22.953	24.523	26.212	28.029
14	14.947	15.974	17.086	18.292	19.599	21.015	22.550	24.215	26.019	27.975	30.095	32.393
15	16.097	17.293	18.599	20.024	21.579	23.276	25.129	27.152	29.361	31.772	34.405	37.280
16	17.258	18.639	20.157	21.825	23.657	25.673	27.888	30.324	33.003	35.950	39.190	42.753
17	18.430	20.012	21.762	23.698	25.840	28.213	30.840	33.750	36.974	40.545	44.501	48.884
18	19.615	21.412	23.414	25.645	28.132	30.906	33.999	37.450	41.301	45.599	50.396	55.750
19	20.811	22.841	25.117	27.671	30.539	33.760	37.379	41.446	46.018	51.159	56.939	63.440
20	22.019	24.297	26.870	29.778	33.066	36.786	40.995	45.762	51.160	57.275	64.203	72.052
21	23.239	25.783	28.676	31.969	35.719	39.993	44.865	50.423	56.765	64.002	72.265	81.699
22	24.472	27.299	30.537	34.248	38.505	43.392	49.006	55.457	62.873	71.403	81.214	92.503
23	25.716	28.845	32.453	36.618	41.430	46.996	53.436	60.893	69.532	79.543	91.148	104.60
24	26.973	30.422	34.426	39.083	44.502	50.816	58.177	66.765	76.790	88.497	102.17	118.16
25	28.243	32.030	36.459	41.646	47.727	54.865	63.249	73.106	84.701	98.347	114.41	133.33
26	29.526	33.671	38.553	44.312	51.113	59.156	68.676	79.954	93.324	109.18	128.00	150.33
27	30.821	35.344	40.710	47.084	54.669	63.706	74.484	87.351	102.72	121.10	143.08	169.37
28	32.129	37.051	42.931	49.968	58.403	68.528	80.698	95.339	112.97	134.21	159.82	190.70
29	33.450	38.792	45.219	52.966	62.323	73.640	87.347	103.97	124.14	148.63	178.40	214.58
30	34.785	40.568	47.575	56.085	66.439	79.058	94.461	113.28	136.31	164.49	199.02	241.33
35	41.660	49.994	60.462	73.652	90.320	111.43	138.24	172.32	215.71	271.02	341.59	431.66
40	48.886	60.402	75.401	95.026	120.80	154.76	199.64	259.06	337.88	442.59	581.83	767.09
45	56.481	71.893	92.720	121.03	159.70	212.74	285.75	386.51	525.86	718.90	986.64	1358.2
50	64.463	84.579	112.80	152.67	209.35	290.34	406.53	573.77	815.08	1163.9	1668.8	2400.0

13%	14%	15%	16%	17%	18%	19%	20%	25%	30%	35%	40%	50%
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.1300	2.1400	2.1500	2.1600	2.1700	2.1800	2.1900	2.2000	2.2500	2.3000	2.3500	2.4000	2.5000
3.4069	3.4396	3.4725	3.5056	3.5389	3.5724	3.6061	3.6400	3.8125	3.9900	4.1725	4.3600	4.7500
4.8498	4.9211	4.9934	5.0665	5.1405	5.2154	5.2913	5.3680	5.7656	6.1870	6.6329	7.1040	8.1250
6.4803	6.6101	6.7424	6.8771	7.0144	7.1542	7.2966	7.4416	8.2070	9.0431	9.9544	10.946	13.188
8.3227	8.5355	8.7537	8.9775	9.2068	9.4420	9.6830	9.9299	11.259	12.756	14.438	16.324	20.781
10.405	10.730	11.067	11.414	11.772	12.142	12.523	12.916	15.073	17.583	20.492	23.853	32.172
12.757	13.233	13.727	14.240	14.773	15.327	15.902	16.499	19.842	23.858	28.664	34.395	49.258
15.416	16.085	16.786	17.519	18.285	19.086	19.923	20.799	25.802	32.015	39.696	49.153	74.887
18.420	19.337	20.304	21.321	22.393	23.521	24.709	25.959	33.253	42.619	54.590	69.814	113.33
21.814	23.045	24.349	25.733	27.200	28.755	30.404	32.150	42.566	56.405	74.697	98.739	171.00
25.650	27.271	29.002	30.850	32.824	34.931	37.180	39.581	54.208	74.327	101.84	139.23	257.49
29.985	32.089	34.352	36.786	39.404	42.219	45.244	48.497	68.760	97.625	138.48	195.93	387.24
34.883	37.581	40.505	43.672	47.103	50.818	54.841	59.196	86.949	127.91	187.95	275.30	581.86
40.417	43.842	47.580	51.660	56.110	60.965	66.261	72.035	109.69	167.29	254.74	386.42	873.79
46.672	50.980	55.717	60.925	66.649	72.939	79.850	87.442	138.11	218.47	344.90	541.99	1311.7
53.739	59.118	65.075	71.673	78.979	87.068	96.022	105.93	173.64	285.01	466.61	759.78	1968.5
61.725	68.394	75.836	84.141	93.406	103.74	115.27	128.12	218.04	371.52	630.92	1064.7	2953.8
70.749	78.969	88.212	98.603	110.28	123.41	138.17	154.74	273.56	483.97	852.75	1491.6	4431.7
80.947	91.025	102.44	115.38	130.03	146.63	165.42	186.69	342.94	630.17	1152.2	2089.2	6648.5
92.470	104.77	118.81	134.84	153.14	174.02	197.85	225.03	429.68	820.22	1556.5	2925.9	9973.8
105.49	120.44	137.63	157.41	180.17	206.34	236.44	271.03	538.10	1067.3	2102.3	4097.2	14962.
120.20	138.30	159.28	183.60	211.80	244.49	282.36	326.24	673.63	1388.5	2839.0	5737.1	22443.
136.83	158.66	184.17	213.98	248.81	289.49	337.01	392.48	843.03	1806.0	3833.7	8033.0	33666.
155.62	181.87	212.79	249.21	292.10	342.60	402.04	471.98	1054.8	2348.8	5176.5	11247.	50500.
176.85	208.33	245.71	290.09	342.76	405.27	479.43	567.38	1319.5	3054.4	6989.3	15757.	75752.
200.84	238.50	283.57	337.50	402.03	479.22	571.52	681.85	1650.4	3971.8	9436.5	22057.	*
227.95	272.89	327.10	392.50	471.38	566.48	681.11	819.22	2064.0	5164.3	12740.	30867.	*
258.58	312.09	377.17	456.30	552.51	669.45	811.52	984.07	2580.9	6714.6	17200.	43214.	*
293.20	356.79	434.75	530.31	647.44	790.95	966.71	1181.9	3227.2	8730.0	23222.	60501.	*
546.68	693.57	881.17	1120.7	1426.5	1816.7	2314.2	2948.3	9856.8	32423.	*	*	*
1013.7	1342.0	1779.1	2360.8	3134.5	4163.2	5529.8	7343.9	30089.	*	*	*	*
1874.2	2590.6	3585.1	4965.3	6879.3	9531.6	13203.	18281.	91831.	*	*	*	*
3459.5	4994.5	7217.7	10436.	15090.	21813.	31515.	45497.	*	*	*	*	*

* Interest factors exceed 99999.

Table A1.5 **Normal distribution table for N(x) when $x \leq 0$**

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

The table should be used with interpolation. For example:

$$\begin{aligned} \mathrm{N}(-0.1234) &= \mathrm{N}(-0.12) - 0.34[(\mathrm{N}(-0.12) - \mathrm{N}(-0.13)] \\ &= 0.4522 - 0.34[0.4522 - 0.4483] \\ &= 0.4509 \end{aligned}$$

Table A1.6 **Normal distribution table for N(x) when $x \geq 0$**

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

The table should be used with interpolation. For example:

$$\begin{aligned} \mathrm{N}(0.6278) &= \mathrm{N}(0.62) + 0.78[(\mathrm{N}(0.63) - \mathrm{N}(0.62)] \\ &= 0.7324 + 0.78[0.7357 - 0.7324] \\ &= 0.7350 \end{aligned}$$

Appendix 2

Examination Formula Sheet

YTM/IRR (Interpolation)

$$\text{YTM} = r1 + \left[\frac{(\text{NPV1})}{(\text{NPV1}+\text{NPV2})} \times (r2 - r1)\right]$$

Dividend growth model

$$P0 = \frac{D1}{(r-g)}$$

Capital asset pricing model

$$\text{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$

Ungeared beta

$$\beta_u = \beta_e \frac{\text{E}}{\text{V}} + \beta_d \frac{\text{D}}{\text{V}}$$

Weighted average cost of capital

$$rv^* = \frac{\text{D}}{\text{V}}(rd^*) + \frac{\text{E}}{\text{V}}(re)$$

WACC–NPV

$$\text{NPV}_0 = \sum_{t=0}^{n} \frac{\text{FCF}_t^*}{(1 + \text{rv}\,*)^t}$$

APV

$$\text{APV}_0 = \sum_{t=0}^{n} \left[\frac{\text{FCF}_t^*}{(1 + ru)^t} + \frac{\text{ITS}_t}{(1 + rd)^t}\right]$$

Profitability index

$$PI = \frac{\sum_{t=1}^{n} \frac{\text{FCF}_t^*}{(1 + rv^*)^t}}{-\text{FCF}_0^*}$$

Cost–benefit ratio

$$\text{CBR} = \frac{\sum_{t=0}^{n} \frac{\text{Inflows}_t}{(1 + rv^*)^t}}{\sum_{t=0}^{n} \frac{\text{Outflows}_t}{(1 + rv^*)^t}}$$

Beta coefficient

$$\text{Beta}_j = \frac{\text{Standard deviation of return}_j \times \text{Correlation of } j \text{ with market}}{\text{Standard deviation of market return}}$$

This can be expressed as:

$$\beta_j = \frac{\sigma_{jm}}{\sigma_m^2}$$

Certainty equivalent

$$CF_{ce} = CF - \frac{\mathrm{E}(r_m) - rf}{\mathrm{Variance}(r_m)} \mathrm{Covariance}\ (\mathrm{CF}, r_m)$$

Optimising cash replenishment

$$£r = [(2 \times £D \times £T)/i]^{½}$$

and:

$$£R = [(3 \times £T \times s^2)/4i]^{\frac{1}{3}} + £M$$

Binomial option

$$Y = \frac{C_u - C_d}{S_0(u-d)}$$

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)^t}$$

$$C_0 = YS_0 + Z$$

Black–Scholes

$$C_0 = S_0 N(d_1) - Xe^{-\mathrm{rfT}} N(d_2)$$

where:

$$d_1 = [\ln(S_0/X) + \mathrm{rfT}]/\sigma(T^{½}) + 0.5\sigma(T^{½})$$

and:

$$d_2 = d_1 - \sigma(T^{½})$$

Appendix 3

Practice Final Examinations

Contents

This section contains two practice final examinations, each of which is indicative of the type and level of material that appears in the Heriot-Watt University final MBA degree examination in Finance.

The duration of the examination is 3 hours. The marks value of each section is shown. Within the total time of 3 hours, students may allocate their time among sections as they see fit. The pass mark is 50%.

There is no choice in the selection of questions to be answered.

In the true/false and multiple choice questions, there are no marks deducted for wrong answers.

For each question, a solution is provided, which will allow students to assess their performance. The examination serves two purposes: to test understanding of the course and to provide information on standards required to pass the university final degree examination.

The rationale for providing two examinations is that students who have worked through the course, have taken the first practice examination and, on the basis of their performance in that examination, are not satisfied that they have attained mastery of the material, will be able to study the course again and have a second opportunity to test themselves. Where the first examination is satisfactory, the second may be used for additional practice.

Practice Final Examination 1

The examination is in 3 sections:

Section A – True/False Questions
20 questions each worth 1.5 marks.
Total marks available in Section A: $20 \times 1.5 = 30$

Section B – Multiple Choice Questions
30 questions each worth 2 marks.
Total marks available in Section B: $30 \times 2 = 60$

Section C – Essay/Problem Questions
2 questions each worth 45 marks.
Total marks available in Section C: $2 \times 45 = 90$

Total marks available for examination: = 180

Pass mark = 50% of 180 = 90

Section A – True/False Questions

1 £1.00 today is worth more than £1.00 expected to be received tomorrow because £1.00 can be invested today. T or F?

2 NPV and IRR are totally different techniques, using different information. T or F?

3 Financial markets are probably the most competitive markets in existence. T or F?

4 Shareholders may not always wish the firm to undertake all investments that have positive NPVs to the firm. T or F?

5 Depreciation is not a cash flow, and therefore it can be ignored in estimating cash flows of an investment project. T or F?

6 One of the most common mistakes made in estimating project cash flows is that analysts forget to subtract interest payments. T or F?

7 With mutually exclusive investments, if the IRR of the incremental cash flow exceeds the hurdle rate, the investment with the higher non-discounted cash flow should be accepted. T or F?

8 It may be in the best interests of a firm's shareholders to accept a project that has a negative NPV if it interacts with the cash flows of another project in such a way as to produce a higher NPV overall. T or F?

9 With well functioning financial markets, there is no reason to think that borrowing is better than lending in times of high inflation. T or F?

10 For the potential to reduce risk by diversifying between two assets, the correlation between the two assets must be negative. T or F?

11 When one investment has twice the systematic risk of another, the required return of the riskier should be twice as high. T or F?

12 The β coefficient of a geared or leveraged company's equity will always be larger than the β coefficient of the firm's total assets. T or F?

13 If cash dividends are more heavily taxed than capital gains, we may expect to see companies that pay high cash dividends being subject to higher required returns. T or F?

14 Since all clienteles for particular dividend policies have probably been adequately served by companies seeking to maximise share price, it really would affect no one's wealth if a company were to change its dividend policy. T or F?

15 In a frictionless 'M&M' world, the only thing that would cause a company to choose one debt level over another is the likelihood of encountering bankruptcy. T or F?

16 The only positive reason for companies to issue debt is because of tax advantages. We know this because prior to the composition of company income taxes, only equity was issued. T or F?

17 Agency and other constraints on the benefits of borrowing must have important effects because we do not see all firms borrowing to the full extent for which the taxation authorities will allow interest to be deducted. T or F?

18 The best strategy for accounts receivable is one that offers the greatest likelihood of being paid. T or F?

19 Options sell for more than exercise value because options tend to increase in value regardless of the direction of price change in underlying assets. T or F?

20 If managers themselves own shares of the company, their incentives to consume company resources disappear because each £1.00 of such consumption reduces share price accordingly. T or F?

Section B – Multiple Choice Questions

For each question choose one option only.

(Quantitative results are rounded to the nearest unit, for example £134.56 would be shown as £135.)

21 The financial market asks and pays interest of 9 per cent per year, and you have currently the expectation of receiving £2000 per year, paid now and at the end of the next two years (three cash flows). What is the maximum amount of money you can spend from this source (including that from transacting with the financial market) **at the end of the first year?**

A. £2180.
B. £5985.
C. £6000.
D. £6015.

22 You have been offered the opportunity to purchase an annuity that promises an initial payment to you of £1000 at the end of this year and each following year for 17 years (including the first payment). If comparable-risk, comparable-timing investments are presently offering 11 per cent interest, what would you be willing to pay now for this annuity?

A. £7000.
B. £7549.
C. £8380.
D. £17 000.

23 The following investments are available. Their free cash flows for each period are shown, and the appropriate discount rate is 12% per period.

		£000s		
Investment	t_0	t_1	t_2	t_3
#1	(1500)	845	670	500
#2	(2600)	105	200	3500

Using the NPV technique, which investment(s) should be undertaken?

A. #1 only.
B. #2 only.
C. Both #1 and #2.
D. Neither #1 nor #2.

24 You expect to receive £1000 per annum for 50 years and wish to value that expectation at a 10 per cent discount rate, but your present-value tables do not have enough periods and your pocket calculator cannot exponentiate. So you are facing a quite tedious calculation to find the exact value. By how much would you err in using a simple perpetuity valuation?

A. $£10\,000/(1.10)^{50}$
B. A huge amount, since an infinite amount of money will be ignored.
C. $£1000/(1.10)^{50}$
D. $50(£1000/1.10)$

25 What is the effect when a company undertakes an investment with a positive NPV?

A. Shareholder wealth goes up by that amount.
B. Managers wealth goes up by that amount.
C. Bondholder wealth goes up by that amount.
D. Wages and salaries go up by that amount.

26 The price/earnings ratio of a company

I. always provides useful information about the company's prospects.
II. provides useful information about the company's prospects when compared with firms in other industries.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

27 What do free cash flows of an investment project include when they are appropriate to be discounted with the project's WACC?

A. All expected payments between the firm and its capital suppliers.
B. All changes in the payments between the firm and its capital suppliers induced by the project.
C. All changes in the payments between the firm and its capital suppliers induced by the project, were the project to be financed only with equity.
D. All changes in the payments between the firm and its capital suppliers induced by the project, were the project to be financed only with equity, but incremented with the expected interest tax shields.

28 Payback period as an investment decision technique

I. is undesirable because it ignores discounting and cuts off consideration of cash flows beyond the payback point.

II. even if improved by adding discounting and explicit consideration of all cash flows, cannot be made to work in all cash-flow conditions.

III. is appealing because of its simplicity.

Which of the following is correct?

A. I only.
B. II only.
C. I and II only.
D. I, II and III.

29 NPV **must** be chosen as the technique for evaluation instead of IRR only when

I. there are multiple sign changes in projected cash flows.

II. projects are mutually exclusive.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

30 The CBR and PI techniques of investment analysis

I. each give the same indication of investment acceptability, which indication agrees with NPV if investments are independent.

II. are at times superior to NPV in that they give an additional idea of the return relative to the investment or cost.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

31 IRR is more likely to give incorrect indications of investments ranking among a group of mutually exclusive investments when the investments

I. are of significantly different magnitude.

II. are of significantly different duration.

III. have very different NPVs.

Which of the following is correct?

A. I only.
B. II only.
C. I and II.
D. I, II and III.

32 Which of the following is correct?

Capital-rationing situations tend to occur when companies

A. are trying to raise more money than capital suppliers can afford to provide.
B. have not planned correctly and run out of funds toward the end of a fiscal year.
C. discover more good projects than is typical.
D. have complicated management structures that cause problems in information flows.

33 Suppose that your firm is considering two alternative roofing materials for a new plant it is building. The plant is expected to last indefinitely, but the roofing material will of course require occasional refurbishment. Fibreglass shingles will last an expected 25 years, whereas slate's expected lifetime is 32 years. Fibreglass costs £130 000 to install, and has annual maintenance costs incurred at year ends that is expected to average £11 000, whereas slate costs £216 000 and has expected annual maintenance costs of only £2000. Because of differing underlayment structures, once the company decides on a roofing material, it must stick with it or else incur additional large costs of renovation. If a 10% discount is appropriate, what should the company do? (*Hint*: You may be forced to calculate a numerical result from the formula at the top of page A1/3, since Table 2 does not have a specific listing for 32 years, or you may be fortunate enough to have a calculator or spreadsheet that has that formula built in.)

A. Choose neither because they both have negative NPVs.
B. Choose slate because its equivalent annual cost is less than £25 000, compared with fibreglass's in excess of £25 000.
C. Choose fibreglass because the present value of its costs is less than £230 000 compared with slate's in excess of £230 000.
D. Choose fibreglass because its equivalent annual cost is less than £25 000 compared with slate's in excess of £25 000.

34 Inflation is best handled in investment decision making by

I. estimating real cash flows and applying real discount rates.
II. estimating nominal cash flows and applying nominal discount rates.
III. estimating cash flows using current prices and using currently observable discount rates.

Which of the following is correct?

A. I only.
B. II only.
C. III only.
D. Neither I, II nor III.

35 The reason why a portfolio's risk cannot be reduced to zero by diversification is that

I. mathematical techniques of combining securities into optimal portfolios are not well enough understood.

II. there is a common influence among most investments that causes their returns to be positively correlated

III. many investments are difficult to discover, especially the ones with negative correlations to others.

Which of the following is correct?

A. II only.
B. II and III only.
C. III only.
D. I, II and III.

36 Investment A has an expected return of 14% and a standard deviation of 5%, while investment B has an expected return of 10% and a standard deviation of 3%. Which of the following is true if you were to put half of your wealth into each investment?

A. The risk of the portfolio will be 4% and the expected return 12%.
B. The risk of the portfolio will be at least 4% and the expected return will be 12%.
C. The risk of the portfolio will be no greater than 4% and the expected return 12%.
D. The question does not provide enough information to make any of the comments about risk in A, B or C.

37 The β coefficient you have decided is appropriate for your firm's equity investment in a new manufacturing facility for fertiliser is 1.16. If the risk-free rate is 9% and the excess of the market's expected return above the risk-free rate is 8%, what return will you require on the equity investment?

A. Approximately 17.0%.
B. Approximately 17.4%.
C. Approximately 18.0%.
D. Approximately 18.3%.

38 Which of the following is correct?

Diversification on the part of an equity-financed company so as to reduce cash-flow variability is generally

A. desirable due to the portfolio effects of risk reduction.
B. undesirable due to the fact that opportunities for risk-reducing investments are rare and expensive.
C. neutral if shareholders are not well diversified.
D. neutral if shareholders are well diversified.

39 Which of the following is correct?

The 'passive residual' dividend policy of a company says dividends should

A. equal the profits of the firm because profits belong to shareholders.
B. equal the retained earnings of the company because this amount is what is 'left over' after all other company transactions.
C. be based upon shareholder needs for cash for consumption purposes.
D. be equal to the amount of cash flow left over after all good investments have been funded.

40 A reasonably stable dividend policy is consistent with

I. the wish to satisfy a shareholder clientele of 'widows and orphans' who depend upon that cash flow for their personal consumption needs.
II. efficient signalling of value-altering news events by the company.
III. legal requirements of portfolio admission by certain financial institutions.

Which of the following is correct?

A. I only.
B. I and III only.
C. II and III only.
D. I, II and III.

41 Debt in a company's capital structure may alter the company's cost of capital and shareholder wealth, relative to what these would have been without debt. Among the sources of such alterations are

I. equity's increased risk due to debt's senior claim on company cash flows.
II. debt's capital cost being less than equity's due to debt's senior claim.
III. interest deductibility.

Which of the following is correct?

A. I only.
B. III only.
C. I and II only.
D. II and III only.

42 Suppose that the only tax in an economy was company income tax, and that there were no other 'frictions' (such as transactions costs, bankruptcy costs, or other agency costs). Which of the following is correct?

Optimal capital structure for companies in this economy would likely be

A. very little debt, since shareholders would not be paying personal income taxes.
B. a reasonable balance of debt and equity, such that in difficult economic times the odds of defaulting on interest and principal promises was acceptable.
C. as much debt as they could issue with deductible interest.
D. zero debt because interest would not be tax deductible.

43 Suppose that Intelligent Micro Designs plc (IMD) is seeking your advice about financing a proposed research and development project. IMD's business is designing computer motherboards, accomplished through a very talented group of design engineers. IMD can raise either equity or debt capital. IMD currently has a very low proportion of debt in its capital structure, and has shown an impressive history of earnings increases. Interest and principal payments on the company's proposed borrowings seem to be modest relative to the company's net cash flows. What would your recommendation be?

A. That IMD should issue equity rather than debt because of very high bankruptcy or financial-distress costs.
B. That IMD should issue debt because it has not exploited sufficiently debt's tax benefits.
C. That IMD can issue either debt or equity without differentially affecting its shareholder wealth.
D. That IMD should issue a complex debt instrument like a convertible bond.

44 What is the primary trade-off that occurs in managing Accounts Receivable (Debtors)?

A. In order to get creditworthy customers, you must give them especially good credit terms.
B. Good customers sometimes take longer to pay than is best for the company.
C. The costs of finding out who will be good customers can be more than the money saved by avoiding poor credit risks.
D. Increasing revenues by reducing credit standards versus having secure receivables with lower sales revenues.

45 Inventories

I. are costly to hold, but allow revenues to be higher.
II. should be kept at the absolute minimum that allows the firm to operate.
III. can be made cheaper to hold if the firm reorders less often.

Which of the following is correct?

A. I only.
B. II only.
C. I and III only.
D. II and III only.

46 Suppose that the current one-year interest rate in the US is 9.4% and the UK rate is 11%. Further, suppose that the spot exchange rate of dollars for pounds is 1.5 $/£. What should your prediction of the one-year forward rate for $/£ be?

A. 1.351
B. 1.50
C. 1.478
D. 1.665

47 Your firm is building a plant and beginning to sell on credit terms in South Aerok, and is concerned with the potential risks arising from the indications of growing inflation in that country. Your suggestion about hedging this risk should be

I. hedge the receivables.

II. hedge the plant.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

48 A call option has a striking price of £4.50 and the shares upon which it is based are selling for £4.65. There are several months until the option expires. What will the premium of the call option be?

A. Zero, because share price is above option value and exercise would simply produce the 'in the money' gain of £0.15.
B. Negative, because 'in the money' options may go 'out of the money' between now and expiration.
C. Positive, because 'in the money' options stand to benefit more than they lose from price volatility.
D. Positive, negative or zero depending upon the values that appear for the parameters of the Black–Scholes option-valuation formula.

49 Which of the following is correct?

The common shares of a geared or leveraged company can legitimately be considered

A. a put option because they can be sold at anytime.
B. a call option because they allow the repurchase of the bonds of the company.
C. a put option because they allow the shareholders to sell the assets of the firm to its bondholders.
D. a call option because they allow the shareholders to buy the assets of the firm from bondholders.

50 For an 'agency problem' to be solvable, necessary conditions of the situation are

I. an overall gain in value from solving the problem.

II. an incentive for the agent to participate in the solution.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

Section C – Essay/Problem Questions

Marks available for each part of the question are given with each question.

Problem 1

You are given the following information about the market for risk-free government bonds (all of which pay interest at the end of the year):

- There is a 9% coupon bond maturing in two years (it has remaining payments at the end of this and the next year), and it is currently selling for £1017.07.
- There is a three-year 10% coupon bond the price of which you do not know, but the coupon from the first interest payment (the end of this year) is selling in the market for £91.74.
- The three-year spot interest rate is 7%.
- There is a four-year 11% coupon bond currently selling for £1164.24.

Please answer the following questions about the bond market (and in answering, you should round to the nearest £0.01 and to the nearest 0.01%):

51 What is the price of a one-year 8% coupon bond now in the market?

(4 marks)

52 What is the current price of the three-year 10% coupon bond?

(6 marks)

53 What is the four-year spot rate currently in the market?

(5 marks)

54 What is the one-period forward rate beginning one period hence?

(4 marks)

55 Suppose that you owned the four-period 11% coupon bond and were approached to sell the third coupon payment (the £110 that is expected for the end of year three). At what price would you agree to sell now?

(3 marks)

56 With the same assumptions as in **5** above, at what price would you now agree to sell the same coupon two years hence?

(7 marks)

57 Suppose that there is another four-year risk-free bond in the market, which promises to pay as follows:

End of Year 1	End of Year 2	End of Year 3	End of Year 4
£973.38	£110	£110	£110

What would be the current price of this bond?

(5 marks)

58 If your calculator is capable of producing Yield to Maturity, calculate it for both four-year bonds. If you do not have access to such a calculator, make an educated guess as to the relative sizes of the YTMs of these two bonds, and **explain** in either case the reason why the bonds' YTMs are such as they are.

(11 marks)

TOTAL = 45 marks

Problem 2

You are considering 'going short' (issuing, or 'writing') a call option on one share of UK Aero plc. The call option has an exercise price of £18, and one share of UK Aero is currently selling for £17. Your opinion is that at the end of the single period until option expiration, only two UK Aero share prices have any probability of occurring – either £13.60 (probability of 0.35) or £22.10 (probability of 0.65). The risk-free market interest rate is 10%, and all other market conditions are as specified in the section in Module 12 on the binomial model of option valuation.

59 How much money would you collect for issuing the option right now? Construct an explanation to an intelligent but uninitiated student as to why the market is willing to give you money for issuing an option that lets the holder buy something for £18 that is currently selling for £17.

(16 marks)

60 Suppose that you were concerned about the risk of issuing a 'naked option', and wished to hedge your issuance of this option. Portray the exact transaction or set of transactions that would make your position risk-free.

(18 marks)

61 Suppose that you were in fact seeking the risk–return characteristic exhibited by the issuance of this call option, but the option was not available. Illustrate the transaction(s) you could undertake so as to produce the same risk and return exposure as you would have derived from issuing the option.

(11 marks)

TOTAL = 45 marks

Practice Final Examination 2

The duration of the examination is 3 hours. The marks value of each section is shown below. You may allocate your time as you think fit. The pass mark is 50%.

There is no choice in the selection of questions to be answered.

In the multiple choice questions, no marks are deducted for wrong answers.

The examination is in 2 sections:

Section A – Multiple Choice Questions

30 questions each worth 2 marks.

Total marks available in Section A: 30 × 2 = 60

Section B – Essay/Problem Questions

2 questions each worth 45 marks.

Total marks available in Section B: 2 × 45 = 90

Total marks available for examination: = 150

Pass mark = 50% of 150 = 75

Section A – Multiple Choice Questions

1 A bank pays an interest rate of 15%. How much must be placed in an account at the bank on 1.1.10 if the following amounts are to be withdrawn from the account and the balance after the last withdrawal is to be zero?

Date	Amount to be withdrawn
1.1.12	£2000
1.1.13	£2000
1.1.14	£2000
1.1.15	£2000
1.1.16	£5238

A. £7230.
B. £8314.
C. £11 511.
D. £13 238.

2 You see an advertisement that makes the following offer.

> 'Make seven annual payments of £1000 to a bank with the first payment on 1.1.92 and the last on 1.1.98. Receive from the bank £1000 annually in perpetuity with the first payment on 1.1.99'.

Ignoring taxes, what interest rate does an investor receive if he takes up this offer.

A. 7.05%.
B. 10.41%.
C. 11.51%.
D. 20.51%.

3 Suppose you have a project having a positive NPV. Now, if this project is funded by debt and equity, the resulting increase in company NPV will go to:

A. both bondholders and old shareholders.
B. both bondholders and new shareholders.
C. both old shareholders and new shareholders.
D. old shareholders only.

4 Rite Bite Enterprises sells toothpicks. Gross revenues last year were £350 000 and total costs were £50 000. Rite Bite is an all-equity firm with one million shares outstanding. Gross sales and costs are expected to grow at 5% per year. The appropriate discount rate is 15% and all cash flows are received at a year's end. There is no corporate tax; and all cash flows are paid out in dividends at the end of the year. The dividend for last year has just been paid. What is the current price per share of Rite Bite stock?

A. £0.50
B. £3.15
C. £3.50
D. £4.00

5 If the one-year spot rate is 7% and the two-year spot rate is 12%, what is the one-period forward rate commencing one year from now?

A. 4.67%.
B. 9.56%.
C. 17.23%.
D. 25.44%.

6 You have won the local lottery. Lottery officials have offered you a choice of the following alternative payments:

I. £10 000 one year from now.
II. £20 000 five years from now.

What discount rate would make both the alternatives equally attractive?

A. 10.9%.
B. 18.9%.
C. 11.9%.
D. 16.9%.

7 The book value of the debt of the Burp Gas Company is £10 million. Currently, the market value of the debt is 90% of book value and is priced to yield 12%. The one million outstanding shares of Burp stock are selling for £20 per share. The required return on Burp stock is 20%. The tax rate is 34%. Given the above information, what is the WACC for the company?

A. 12.25%.
B. 13.25%.
C. 16.25%.
D. 17.25%.

8 Missile, Inc is considering a £20 million modernisation project in the power systems division. The project's after-tax cash flow will be £8 million in perpetuity. Missile's cost of debt is 10% and its cost of equity is 20%. The target debt–equity ratio is 2 (i.e. value of debt is twice the value of equity), and the firm is in the 34% tax bracket. What is the WACC and NPV? Should the firm undertake this project?

A. 0.1000; £60 000 000; yes.
B. 0.1107; −£52 267 389; no.
C. 0.1107; £52 267 389; yes.
D. 0.1507; £32 267 389; yes.

9 There are some factors that need to be distinctly identified in determining incremental cash flows, such as

I. sunk costs.
II. opportunity costs.
III. side effects (i.e. cash flows from interactions of the investment with the other activities of the corporation).

Which of the following is correct?

A. I and III only.
B. II only.
C. II and III only.
D. All of the above.

10 Company management believes that inflation is going to fall steeply and stay at a low level. Market prices do not reflect this expectation. If management decides to act on its belief, it will immediately

A. replace long-term debt with short-term debt.
B. replace short-term debt with long-term debt.
C. raise the debt–equity ratio.
D. not worry about inflation and make no changes in its financial plan.

11 Consider the following projects.

	C_0	C_1	C_2
X	−£4000	£2500	£3000
Y	−£2000	£1200	£1500

Which of the following would you select if the discount rate is 10% and there are no constraints in the availability of funds?

A. X.
B. Y.
C. Both X and Y.
D. Neither X nor Y.

12 Consider the following projects being looked at by the Nitrex company. Nitrex has a required rate of return of 15% for this type of project, but temporarily has no access to the capital market. It has a maximum of £500 000 available for investments. The projects cannot be postponed.

Project	Initial outlay	NPV at 15%	IRR
V	500 000	125 000	23%
W	250 000	75 000	17%
X	150 000	25 000	35%
Y	100 000	50 000	25%
Z	150 000	50 000	25%

Which of the following should be chosen?

A. V.
B. W, Y, Z.
C. W, X, Y.
D. X, Y, Z.

13 Mrs T. Potts of Ideal China has a problem. The company has ordered a new kiln for £400 000. Of this sum, £50 000 is estimated to be the installation cost. Mrs Potts does not know whether this will be classified by the tax authorities as current expense or capital investment. Assume a five-year straight-line depreciation schedule, no salvage value, a tax rate of 34%, and the opportunity cost of capital of 5%. What is the present value of the tax shield of the installation cost if it is classified as current expense and there is a one-year delay in making tax payments?

A. £14 190.
B. £14 850.
C. £16 190.
D. £16 850.

14 Using the data from the previous problem, what is the present value of the tax shield if the installation cost is classified as capital investment?

A. £14 718.
B. £15 850.
C. £16 190.
D. £16 850.

15 The probability that the economy will experience moderate growth next year is 0.6, and the probability of a recession is 0.2. The other possibility is rapid expansion. If the economy falls into recession, you can expect to receive a return on your portfolio of 5%; with moderate growth your return would be 8%; and under rapid expansion your portfolio will return 15%. What is the standard deviation of the portfolio?

A. 0.0331
B. 0.0533
C. 0.0880
D. 0.1000

16 Portfolio risk (standard deviation) is equal to the weighted average (linear combination) of the risks of individual assets when the individual asset risks are which of the following?

A. positively correlated.
B. negatively correlated.
C. perfectly correlated.
D. not correlated at all.

17 Bald plc is the most similar market-traded firm to the investment you are intending to make in a hair-care centre. Bald is financed with 25% debt and 75% equity, has a reported equity β coefficient of 1.5 and a debt β coefficient of 0.8. You intend to finance your project with equity. The β coefficient you should use in setting the required rate of return for your project is which of the following?

A. 0.714
B. 1.325
C. 1.565
D. 2.333

18 If the β coefficient for an equity investment is 2.0, the risk-free rate 10%, and the market premium is 10%, your expected return on the investment is how much?

A. 10%.
B. 20%.
C. 30%.
D. 40%.

19 The following data have been developed for Gamma plc.

Variance of Gamma returns:	0.62662
Variance of market returns:	0.052851
Covariance of the returns on Gamma and market:	0.0607172

If the market risk premium is 8.5% and the expected risk-free return is 6.6%, what is the expected return of Gamma plc?

A. 7.5%.
B. 15.1%.
C. 16.0%.
D. 16.4%.

20 Which of the following statements is correct?

I. Legislation that caps dividends would be expected to hold down stock prices, because the stock price is the present value of its expected dividend stream.
II. As dividends are shareholders' wages, they should be regulated under wage control.

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

21 Which of the following statements is correct?

I. A capital investment opportunity that is risky but offers an expectation of a 10% DCF rate of return must be an attractive project to a company that can borrow the whole amount at 8%.
II. One important reason why firms should operate at conservative debt levels is that the more debt the firm issues, the higher the interest rate it must pay.

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

22 Which of the following is correct?

A. Stock prices fall when equity issues are announced and when debt issues are announced.
B. Stock prices rise when equity issues are announced and when debt issues are announced.
C. Stock prices fall when equity issues are announced but not when debt issues are announced.
D. Stock prices rise when equity issues are announced but not when debt issues are announced.

23 Consider the following statements:

I. Stockholders benefit from indentures in bond contracts in times of distress.

II. Bondholders benefit from indentures in bond contracts when the bonds are issued.

Which of the following is correct?

A. I only.
B. II only.
C. Both I and II.
D. Neither I nor II.

24 XYZ plc has traded at £12 for the last three years, whereas ABC plc has moved from £50 to £100. Over this time the market has risen by 100%. Which of the following is correct?

A. XYZ has a low β coefficient, where ABC has a high β coefficient.
B. XYZ has a market β coefficient.
C. ABC has a market β coefficient.
D. Price volatility does not indicate the values of β coefficients.

25 Thrift plc is considering a lock-box proposal which will reduce its collection and processing time by two days. The cost of the lock box is an annual fee of £15 000 plus £0.25 per transaction. The interest rate is 6.0% per year. The average customer payment is £4500. How many customers each day on average must use the system to make it profitable?

A. 30.
B. 34.
C. 40.
D. 44.

26 The Allen Company has monthly credit sales of £600 000. The average collection period is 90 days. The cost of production is 70% of the selling price. What is the Allen Company's average investment in accounts receivable?

A. £42 000.
B. £540 000.
C. £1 260 000.
D. £5 400 000.

27 The spot rate for the dollar per sterling pound is \$1.50/£. If the interest rate is 7% in the US and 4% in the UK, what is the appropriate one-year forward rate if no arbitrage opportunities exist?

A. \$1.458/£
B. \$1.543/£
C. \$1.560/£
D. \$1.605/£

28 Storm Equipment Inc has come across an investment in Germany. The project costs €10 million and is expected to produce a cash flow of €4 million in year one, and flows of €3 million in years two and three. The current spot exchange rate is $1.50/€1 and the current risk-free rate in the US is 7%, while it is 4.5% in Germany. The appropriate discount rate for the project is estimated to be 11%, the US cost of capital for the company. The company could be sold for €2.1 million at the end of three years. Given the above information, the NPV is:

A. +$368,614
B. –$232,453
C. –$348,680
D. –$1,030,000

29 If the nominal interest rate in the UK is 10.24% and that in the US is 13.36%, and the real rate of interest in each country is 4%, then the inflation rate in the UK and the US, respectively, are:

A. 6% and 6%.
B. 6% and 9%.
C. 9% and 9%.
D. 9% and 6%.

30 If the exercise price of a call option you own is £2.45 and the stock price is currently trading at £2.40, then:

A. the call is 'in the money' and its value is positive.
B. the call is 'out of the money' and its value is positive.
C. the call is 'out of the money' and its value is zero.
D. the call is 'out of the money' and its value is negative.

Section B – Essay Questions

Problem 1

Heritage Leisure plc has a business that involves purchasing historic houses, castles etc. from their owners and developing the property so as to attract visitors. This usually involves building restaurants and bars, providing car parking, improving the property itself and often adding subsidiary attractions such as a small zoo or a miniature steam railway.

Heritage has been expanding and aims to continue to expand. Several other companies have noticed this business opportunity, partly as a result of Heritage's early success, and competition to buy suitable properties is now intense. Heritage is aware of the danger that the market will become oversupplied but believes there will be several more years before this happens. Heritage is aware that leisure spending is very income-sensitive. In a year of recession, revenue would fall by 25%, while operating costs could only fall by 5%. Although there has been no recession recently, Heritage knows that, based on past experience, one year in five is a recession year and recessions occur irregularly and unpredictably.

Heritage now has a total of £25 million invested in its seven properties. This amount is made up of both the initial purchase price and the subsequent capital expenditure on improvements. This investment is funded by £10 million of long-term debt (at a fixed interest rate of 11%), £5 million of short-term debt (current interest rate 8%) and equity of £10 million. There are 10 million shares outstanding, of which 5.5 million are owned by Mr Taylor, the founder and Chief Executive of the company. The remainder are held by outside investors, with no single investor holding more than 3% of the shares. The current market price of Heritage's shares is £2.00.

The income statement for the year just ended (2010) is as follows:

	£ (million)
Revenue	20.0
Operating costs	11.0
Depreciation	3.0
EBIT	6.0
Interest	1.5
Earning before tax	4.5
Tax at 33⅓%	1.5
Earnings after tax	3.0
Dividend 15p/share	1.5
Retained earnings	1.5

The depreciation charge matches the reinvestment in the properties needed to maintain their attractiveness.

The recent record of earnings per share and dividends per share is as follows:

Year	**2007**	**2008**	**2009**	**2010**
EPS (pence) (after tax)	20.7	24.8	24.2	30.0
Dividend per share (pence)	10.0	12.0	13.0	15.0

Growth in earnings has taken place because the number of properties operated by Heritage has grown. The revenue from a property once it is operated shows no particular long-term trend. Inflation has been negligible.

Heritage has recently had the opportunity to purchase Manderley Castle. The price would be £2 million and an additional £1 million would be needed to improve the property. The incremental financial figures from Manderley, in a non-recession year, are forecast to be:

	£ (million)
Revenues	2.0
Operating costs	1.1
Depreciation	0.3
EBIT	0.6

The general expectation of financial forecasters is that interest rates, will remain stable. Mr Taylor's private forecast is that interest rates, both long- and short-term, are likely to fall by about 1% over the coming year and will thereafter remain stable.

Heritage has decided to buy Manderley and now has to choose how to finance the purchase. The retained earnings are being used for a marketing programme, and so the full £3 million must be raised. The alternatives are:

A. To sell 1.7 million new shares at a price of 177p per share to outside investors (current shareholders would not be given a prior right to buy the shares). The company believes this is the highest price at which these new shares could be sold.
B. To borrow £3 million on a long-term fixed interest rate basis at 11%.
C. To borrow £3 million for one year at 8%. At the end of the year Heritage would require to renegotiate the interest rate and other conditions of the loan.

If the money is borrowed, the bankers would require Heritage to offer the Manderley property as primary collateral. They would also require that Heritage agree not to pay dividends for any year greater than the amount of after-tax earnings in that year. There is no such condition regarding dividends attached to the present borrowings.

Required

Note: Questions 31 and 32 of this problem are general and do not require reference to the information about Heritage.

31 According to the classic Miller–Modigliani analysis, and ignoring taxes, how does the choice between debt and equity affect the wealth of current shareholders?

(5 marks)

32 How does company tax modify the answer to **31**?

(5 marks)

33 How does Heritage's dividend policy affect the choice of finance?

(5 marks)

34 How does the probability of a recession in Heritage's market affect the choice of finance?

(5 marks)

35 How does the ownership of Heritage's shares affect the decision?

(5 marks)

36 How does the dividend constraint imposed by potential leaders affect the decision?

(5 marks)

37 How should Mr Taylor's expectation of future interest rate changes affect the decision?

(5 marks)

38 What conclusion do you draw on the balance of advantages and disadvantages of using long-term rather than short-term debt in this cases?

(5 marks)

39 What conclusion do you draw on the balance of advantages and disadvantages of using equity rather than debt in this case?

(5 marks)

TOTAL = 45 marks

Problem 2

You are given the following information about three bonds, each with a face value of £100 and interest payable annually. The next interest payment will be in one year's time.

Bond	Maturity (Years)	Price today	Coupon rate
A	1	£99.0566	5.0%
B	2	£104.6741	10.5%
C	3	£104.5604	10.0%

40 What is the current price of a zero coupon bond maturing in one year; two years; three years?

(10 marks)

(*Note*: The following questions are quite independent of B in the table above. The information given in B in the table above is irrelevant.)

The nominal one-, two- and three-year spot rates are 7%, 9% and 8.5% respectively.

41 What are the prices of zero coupon bonds, face value £100 with one, two and three years to maturity?

(10 marks)

42 What are the nominal forward rates one year and two years hence?

(10 marks)

43 What is the price of the three-year bond expected to be in one year's time?

(5 marks)

44 Assume the expected real interest rates are as follows:

1st year	1%
2nd year	2%
3rd year	2.5%

What are the expected inflation rates in each of the next three years?

(10 marks)

Examination Answers

Practice Final Examination 1

Section A – True/False Questions

1 The correct answer is True. This is true because the £1.00 invested today at interest will accrue to more than £1.00 in the future.

2 The correct answer is False. This is false because there are many similarities between NPV and IRR, in that they use the same data, are both discounted cash-flow methods, and usually give the same final indications of investment desirability.

3 The correct answer is True. This is true because financial markets are not only countrywide, but effectively worldwide, and deal in a product that is largely homogeneous: future cash-flow expectations. No single participant is large enough to affect that market with its own sales or purchases (save some large governments); information flows quickly, inexpensively and with little bias; and there are few impediments to the free flow of capital across securities or national borders. No other market is as close to a perfectly competitive ideal.

4 The correct answer is True. This is true because there may be times, especially when the firm is geared and in some financial distress, that bondholders would benefit at the expense of shareholders if a positive NPV investment were undertaken.

5 The correct answer is False. This is false because ignoring depreciation will cause cash-flow estimates to deal incorrectly with the tax effects of depreciation.

6 The correct answer is False. This is false because finance analysts should not subtract interest payments at all. Interest payments are cash flows to capital suppliers and thus are correctly included (not subtracted) in FCF* to which the WACC is applied.

7 The correct answer is False. This is false because of the use of the word 'accepted'. It is true that the higher non-discounted NPV investment is better than the other, but not necessarily true that it is acceptable (its IRR may be less than the hurdle rate).

8 The correct answer is True. This is true because investment projects that have economic (causal) cash-flow interactions must be considered only with such interactions specified. In other words, it is irrelevant if an investment, taken alone, has a negative NPV – as long as that investment adds to the positive NPV of another.

9 The correct answer is True. This is true because markets are very good pricers of existing expectations, and though it is likely that either the net borrower or the net lender will benefit at the expense of the other, in order to know in advance which it will be requires an ability to predict inflation and interest rate changes better than the financial markets. Though markets may not always be terribly good at that, no better alternative has yet been found.

10 The correct answer is False. This is false because risk is reduced by diversification as long as correlation is not perfectly positive. This includes all positive correlations less than 1.0.

11 The correct answer is False. This is false because it would assume that the SML always has a slope equal to 1.0. There is no reason to think that to be the general situation, and in fact there is evidence that the SML changes its slope in reaction to changes in the risk-free rate and expected market return.

12 The correct answer is True. This is true because the equity of an ungeared firm will be the equity of its assets, and since gearing increases the risk of equity, moving from an ungeared to a geared firm will increase equity's risk and β coefficient (its responsiveness to overall market return changes).

13 The correct answer is True. This is true because in order to attract a clientele that pays taxes, such firms would be forced to offer the same after-tax return as other firms with lower dividend payouts and higher capital gains. To duplicate the after-tax return, a higher pretax return may be necessary. (If there was a significant clientele that did not pay taxes, or that had very high transaction costs of turning capital gains into necessarily consumable cash, this general proposition may not hold.)

14 The correct answer is False. This is false because it is costly for clienteles to move from one firm's shares with a particular policy to a different firm's shares with another policy.

15 The correct answer is False. This is false because bankruptcy is costless in such a world, and would present no detriment to borrowing versus equity.

16 The correct answer is False. This is false because we know there are other positive reasons (such as agency costs of equity) for issuing debt, and corporate borrowing did in fact exist before the appearance of company income taxes.

17 The correct answer is True. This is true because mere tax advantage would argue that firms borrow more than we actually see them borrowing.

18 The correct answer is False. This is false because such a strategy is costly to implement in terms of making judgments about customers, and there is probably some balance of bad customers who are worthwhile putting up with so as to get the higher revenues from all customers.

19 The correct answer is False. This is false because options both increase and decrease in value as underlying asset values change (calls move in the same direction, and puts move in the opposite direction). The distinguishing feature of options in this context is that the value movements are positively biased (e.g. calls do not decline as much as underlying assets).

20 The correct answer is False. This is false because each £1.00 of consumption of corporate assets produces £1.00 of benefit to managers, while managers' share value will only decline by £1.00 times the proportions of all shares held by managers.

Section B – Multiple Choice Questions

21 The correct answer is D. There are several techniques of finding the correct answer, but they have in common the goal of finding the value of all three cash flows at the end of the first year. This could be accomplished by discounting the end-of-third-year cash flow to the end of the second year, adding the undiscounted second year

cash flow, and adding the end-of-first year cash flow with one year's accrued interest:

$$£6015 = £2000(1.09) + £2000 + \frac{£2000}{(1.09)}$$

The same result would occur if all cash flows were first discounted to present value and then accrued outward at interest for one period, or any of a number of other alternatives. Using present and future value tables would also produce the same result, save that multiplicative 'factors' (e.g. 0.9174) would be used instead of discounting (e.g. dividing by 1.09) as shown.

22 The correct answer is B. The annuity present-value table indicates that a 17-year annuity at 11% interest has a factor of 7.5488. Multiplying this by £1000 yields the correct answer.

23 The correct answer is C. The two investments each have an NPV of £144:

$$\#1\ \ £144 \quad = \quad -£1500 + \frac{845}{(1.12)} + \frac{670}{(1.12)^2} + \frac{500}{(1.12)^3}$$

$$\#2\ \ £144 \quad = \quad -£2600 + \frac{105}{(1.12)} + \frac{200}{(1.12)^2} + \frac{3500}{(1.12)^3}$$

24 The correct answer is A. If the correct cash-flow expectation is 50 years of £1000 per year, a perpetuity will miss an infinite number of £1000 per annum cash flows beyond year 50. At year 50, the value of those is simply their perpetuity value of £10 000. The present value of that £10 000 is of course found by the process in A. (Incidentally, the present value is £85.19, or 0.852% – less than one percent – of the £10 000 estimated value, a very small error.)

25 The correct answer is A. A correctly calculated project NPV, being based upon residual cash flows and a capital cost that accounts for all required returns to capital suppliers, produces the net increase in wealth of shareholders, the residual claimants of the corporation.

26 The correct answer is D. The price/earnings ratio of a company can, on proper comparison, provide information about the future prospects of a firm, but can yield misleading or incorrect impressions if used carelessly. One such misuse is that mentioned in alternative II, namely cross-industry comparisons.

27 The correct answer is C. Answer A is incorrect because it implies that all of the firm's cash flows are included in a project's NPV. Answer B is incorrect because it would include all financing cash flows including interest tax shields, and answer D is incorrect in part for the same reason.

28 The correct answer is D. Payback period's shortcomings are well described in I; II is correct because the various discounting and other improvements to payback still relegate it to uses that are at best specialised; and III is correct because payback is seductively simple.

29 The correct answer is D. IRR can be made to work in any situation if one is wise enough to recognise the additional analysis required to make it work. In situations of mutual exclusivity, IRR will give good decisions if implemented in an incremental cash-flow context. In situations of multiple sign changes, explicit assumptions of

reinvestment flows will allow IRR to function. And if multiple discount rates occur across time, a market yield from comparable cash-flow securities can provide a hurdle rate. These are, however, formidable conditions, and given that NPV always works and is a good bit easier to calculate – especially in the situations listed above – IRR may not be an optimal choice of technique.

30 The correct answer is A. The CBR and PI techniques work correctly to indicate acceptability of investments when there is no economic relatedness among proposals, and each can be considered alone for a 'yes' or 'no' verdict. The techniques are not in any sense superior to NPV as answer B implies, and though PI may be useful as a crude first-approximation ranking device in certain capital-rationing situations, it certainly is not reliable enough to be used as is implied in answer B. Answer C is contradicted by answer A.

31 The correct answer is C. The problems which arise with IRR are in divergent patterning of mutually exclusive project cash flows. Both different sizes and timings of cash flows fit that definition; thus answers A and B are incorrect. Answer D is incorrect because IRR's confusion is not generally related to the size of NPVs of competing projects.

32 The correct answer is D. Capital rationing is not having enough money to pursue all desirable projects. Answer A is incorrect because capital markets have more money than any single firm could ever find projects (if capital markets will not provide enough money, it is because they disagree as to the desirability of projects, not because of any shortage of funds). Answer B is incorrect because financial markets stand ready at any time to provide funding for good projects. Answer C is incorrect for the same reason. The text indicates that capital rationing is a reaction to an information shortfall in either the market or the company, and thus answer D is consistent with the latter.

33 The correct answer is B. This is a problem in which the investments are 'repeatable' or 'renewable'. The correct technique (other than specifying all cash flows into the future until the two alternatives have a common terminal point, an unrealistic calculation in this case, since the first common terminal point is 800 years in the future!) is to find each alternative's 'equivalent annual cost'. To do this, first you find each alternative's NPV in its first cycle, and divide the result by the annuity factor for that particular duration:

For fibreglass:

$$\begin{aligned}\text{NPV}_f &= (25 - \text{year annuity factor} \times £11\,000) + £130\,000\\ &= (9.077 \times £11\,000) + £130\,000\\ &= £229\,847\end{aligned}$$

For slate:

$$\begin{aligned}\text{NPV}_8 &= (32 - \text{year annuity factor} \times £2000) + £216\,000\\ &= (9.526 \times £2000) + £216\,000\\ &= £235\,052\end{aligned}$$

The equivalent annual costs are respectively:

$$\begin{aligned}
\text{EAC}_f &= \text{NPV}_f/25-\text{year annuity factor} = £229\,847/9.077 \\
\text{EAC}_f &= £25\,322 \\
\text{EAC}_8 &= \text{NPV}_8/32-\text{year annuity factor} = £235\,052/9.526 \\
\text{EAC}_s &= £24\,675
\end{aligned}$$

Thus slate is cheaper as measured by equivalent annual cost, because its higher installation cost is offset by the longer duration between installations and the lower annual maintenance. Answer A is incorrect because the negative NPVs are appropriate due to these being 'cost-minimising' investments.

34 The correct answer is B. Nominal cash flows discounted with nominal discount rates is the superior technique due to the difficulties of estimating real discount rates. Though answer A could be made to work in concept, in reality its application is prohibitively difficult. Answer C is incorrect because it amounts to something like using nominal discount rates on real cash flows, which will bias NPVs downward.

35 The correct answer is A. As the text described, most investments are influenced by a 'common factor', which is usually described as the 'market'. This common factor produces a situation where investments have some lower limit on the extent to which diversification reduces risk. Some newer models of market equilibrium (such as the APT) have seemed to uncover more than one 'common factor'. I is incorrect because we have plenty of mathematical skills to do such analyses. III is simply incorrect.

36 The correct answer is C. A portfolio's risk can never be any higher than the linear combination of the risks of its component assets (*See* Figure 7.3). When assets within a portfolio are perfectly positively correlated, portfolio risk is a weighted average (linear combination) of the risks of the component assets. When correlation is less than perfectly positive, portfolio risk will be less than the linear combination. Correlations cannot be greater than perfectly positive, and answers A and B are incorrect for that reason. Answer D is incorrect because we can confidently ascribe answer C as the correct answer.

37 The correct answer is D. This is simply an application of the SML:

$$\begin{aligned}
\text{Required return} &= rf + \text{E}(rm - rf)\beta \\
\text{Required return} &= 0.09 + (0.08)1.16 \\
\text{Required return} &= 0.1828 \text{ or } 18.3\%
\end{aligned}$$

38 The correct answer is D. Diversification for the purpose of portfolio risk reduction only helps shareholders if shareholders are not themselves already well diversified. To the extent that a firm has debt in its capital structure, it may help through bankruptcy risk reduction, but hurt because of 'coinsurance' of debt claims (increasing their value at the cost of equity).

39 The correct answer is D. A 'passive residual' dividend policy means that dividends are fully driven by other decisions that the company makes. In other words, after all other policies (including how much cash should be held by the company in bank accounts) are carried out, and if any cash is left over, it may be paid as a dividend. Retained earnings of the company do not follow this definition, because retained earnings are in effect channelled into investments. Thus answer B is incorrect, and

answers A and C are clearly not 'passive' dividend policies since they are driven by some other concern.

40 The correct answer is D. Shareholder consumption preferences, signalling, and legal-professional portfolio constraints can all influence a company's wish to maintain a reasonably stable dividend across time.

41 The correct answer is B. The only relevant factor in the list is interest deductibility, which causes debt to be more attractive than equity, other things being held the same. Option I is incorrect because equity's higher risk (and required return) is offset by debt's lower cost. Option II is incorrect for the mirror-image reason of option I.

42 The correct answer is C. With interest-deductible debt, no other taxes and no other frictions or agency costs, the shareholder wealth-maximising capital structure would be to issue as much debt as carries interest deductibility. Though other deductibilities (like depreciation) might cause less than all of the interest tax shield to be utilised, the company will be no worse off than it would have been with equity, and on most occasions would pay less taxes. Answer B is incorrect because we have assumed away all of the problems caused by too much debt in the capital structure, and answer A is nonsense, as is D.

43 The correct answer is A. Most of IMD's value is in intangible assets, made up of its engineering skill. In times of financial distress (generated by declines in the computer industry, which do happen) such assets quickly lose a good bit of their value as employees leave for greener pastures. Debt suppliers, foreseeing this, would require much higher interest rates as compensation for this asset intangibility. Answer B is incorrect because, though IMD could perhaps gain interest tax shields, such tax benefits may be otherwise available (through R&D credits, for example), which do not carry bankruptcy costs. Answers C and D propose tactics that would be applicable to firms in other situations: C is an irrelevancy suggestion, which IMD does not represent; D would be applicable if bondholders felt that IMD could 'shift risks' of investments after borrowing.

44 The correct answer is D. Receivables management is primarily a trade-off between acquiring more sales by easing credit terms and thereby delving more deeply into the pool of questionable customers, versus having lower sales but with faster and more reliable payments. There is no empirical evidence that would support answers A or B, and answer C, though correct, is not the **primary** trade-off of receivable management.

45 The correct answer is A. Physical inventories allow the firm to respond to the requirements of markets more flexibly. However, inventories, including cash, are assets and require financing, and thus cannot be expanded without eventually reaching uneconomic size. Option II is wrong because it offers too stringent a standard by the above description of the role of inventories. Option III is opposite to the truth.

46 The correct answer is C. As Module 11 points out, the ratio between interest factors (1.11/1.094) = 1.01463 must be the same as the ratio between spot and forward exchange rates. Thus (1.5/One-year forward rate) must also equal 1.01463; therefore

the one-year forward rate must equal 1.5 ÷ 1.01463 or 1.478. The other rates shown are inconsistent with this required economic relationship.

47 The correct answer is A. Real assets tend to increase in value along with inflation, so there is no need to hedge the plant. Financial assets (such as receivables), however, have future payoffs stated in nominal currency terms, and if inflation is unexpectedly high, future receipts in both the foreign and domestic economy will be less valuable than anticipated when received. Thus answer A is the only correct one of those listed.

48 The correct answer is C. An option allows the holder to benefit from positive events (in this case increases in price of the underlying security) in greater degree than to lose from negative events. Specifically, increases in stock price above £4.65 will produce gains in current £0.15 exercise value on a £-for-£ basis, while reductions in stock price below £4.64 cannot reduce exercise value below £0.00. Thus the premium (the amount by which option value exceeds exercise value) must be positive; the option must be selling for more than £0.15. Answers A and B are incorrect because they predict non-positive values, and answer D is incorrect because premiums must be either positive or zero (and will only be zero if there is no time until expiration).

49 The correct answer is D. When a company borrows money by issuing bonds, it (in economic effect) is selling the assets of the firm to the bondholders (the bondholders have first claim on those assets in the case of default) in exchange for the money borrowed. Shareholders, however, (in effect) retain an option to repurchase the assets of the firm from bondholders upon the firm paying the bonds' contractual interest and principal (and otherwise abiding by the terms of the bond indenture). Answer A is incorrect because the option characteristic of common stock has nothing to do with the shares' marketability. Answer B is incorrect, because shareholders may not in fact be able to buy back the bonds (and even if they could it would not be the characteristic that allows shares to be considered options). Answer C is closer to being correct than A or B, and in some contexts may be another way of looking at share value; but in the sense explained by Module 12, is not as good an answer as D.

50 The correct answer is C. Both of conditions I and II are required. If there is no overall gain, there is no value increase to be allocated between principal and agent, and thus no prospect of inducing a change in behaviour. Even with such an increase in overall value, however, the agent must somehow be allocated a part of the gain, or there is no incentive for the (decision-making) agent to change behaviour.

Section C – Essay/Problem Questions

Problem 1

51

The problem implies the following price and cash-flow information, along with the 7% three year spot rate:

Now	End of Year 1	End of Year 2	End of Year 3	End of Year 4
£1017.17	£90	£1090		
?	£100	£100	£1100	
£91.74	£100			
£1164.24	£110	£110	£110	£1110

The price of the one-year 8% coupon bond depends upon the receipt of £1080 one year hence, discounted at the one-year spot rate. The one-year spot rate is not given, but can be deduced from the price of the end-of-first-year coupon of the three-year bond. Since the £1080 cash flow from the one-year bond and the £100 cash flow from the three-year bond have the same risk and timing, they must be subject to the same discount rate:

$$\begin{aligned}
£91.74 &= \frac{£100}{(1+i_1)} \\
(1+i_1) &= \frac{£100}{£91.74} = 1.09 \\
i_1 &= 9\%
\end{aligned}$$

With a 9% one-year spot rate, the current price of the one-year 8% coupon bond is £1080/1.09=£990.83.

52 The price of the three-year 10% coupon bond is a function of its known cash flows discounted with the one-, two- and three-year spot rates. From Question **1**'s answer we now have the one-year spot rate, and the three-year spot rate is given in the problem as 7%. So the remaining unknown is the two-year spot rate. It can be deduced by analysing the two-year 9% coupon bond:

$$\begin{aligned}
£1017.07 &= \frac{£90}{(1+i_1)} + \frac{£1090}{(1+i_2)^2} \\
&= \frac{£90}{(1+0.09)} + \frac{£1090}{(1+i_2)^2} \\
&= £82.57 + \frac{£1090}{(1+i_2)^2} \\
£934.50 &= \frac{£1090}{(1+i_2)^2} \\
(1+i_2)^2 &= \frac{£1090}{£934.50} = 1.1664 \\
(1+i_2) &= 1.08 \\
i_2 &= 8\%
\end{aligned}$$

With a two-year spot rate of 8%, the price of the three-year 10% bond is:

$$\begin{aligned}
£? &= \frac{£100}{(1+i_1)} + \frac{£100}{(1+i_2)^2} + \frac{£1100}{(1+i_3)^3} \\
£? &= \frac{£100}{(1+0.09)} + \frac{£100}{(1+0.08)^2} + \frac{£1100}{(1+0.07)^3} = £1075.40
\end{aligned}$$

(This question can also be solved using forward interest rates.)

53 If the current price of the four-year bond is known, and if the first three spot rates are known, it is simple algebra to find the four-year spot rate:

$$\begin{aligned}
£1164.24 &= \frac{£110}{(1.09)} + \frac{£110}{(1.08)^2} + \frac{£110}{(1.07)^3} + \frac{£1110}{(1+i_4)^4} \\
&= £100.92 + £94.31 + £89.79 + \frac{£1110}{(1+i_4)^4} \\
(1+i_4)^4 &= \frac{£1110}{£879.22} = 1.26248 \\
(1+i_4) &= 1.06 \\
i_4 &= 6\%
\end{aligned}$$

54 The one-year forward rate one year hence, ${}_1f_2$, can be found most easily by working with the cash flows and prices of the two-year bond:

$$\begin{aligned}
£1017.07 &= \frac{£90}{(1+i_1)} + \frac{£1090}{(1+i_1)(1+\ {}_1f_2)} \\
&= \frac{£90}{(1.09)} + \frac{£1090}{(1.09)(1+\ {}_1f_2)} \\
(\ {}_1f_2) &= \frac{£1090}{£934.50(1.09)} = 1.07 \\
{}_1f_2 &= 7\%
\end{aligned}$$

55 You have already calculated that answer in **3**. The value is £89.79.

56 This question is asking for a 'forward price' of the year-three £110 coupon, expected to exist as of the end of the second year. To find this value, you must discount the year three cash flow by the forward interest rate for that year, ${}_2f_3$. This latter forward rate has not yet been calculated (unless you solved one of the earlier answers with forward rates instead of spot rates). The simplest way to calculate ${}_2f_3$ is to recall that:

$$\begin{aligned}
(1+i_3)^3 &= (1+i_2)^2(1+\ {}_2f_3) \\
(1+\ {}_2f_3) &= (1.07)^3/(1.08)^2 = 1.0503 \\
{}_2f_3 &= 5.03\%
\end{aligned}$$

Therefore the end-of-year-two forward price of the end-of-year-three £110 coupon of the four-year bond is:

$$£? = £110/(1+\ {}_2f_3) = £110/1.0503 = £104.73$$

57 Pricing this bond requires only that the specified cash flows be discounted at the appropriate spot or forward rates:

$$\begin{aligned}
£? &= \frac{£973.38}{(1.09)} + \frac{£110}{(1.08)^2} + \frac{£110}{(1.07)^3} + \frac{£110}{(1.06)^4} \\
£? &= £1164.24
\end{aligned}$$

Note that this price is the same as that of the four-year 11% coupon bond.

58 The four-year 11% coupon bond's YTM is 6.23%, whereas the bond in **7** has a YTM of 7.97%. (These YTMs are, of course, the constant discount rates that would produce the correct market prices of the bonds.) If you did not have access to a

calculator that could find the YTMs easily, you should have said that the YTM of the 11% coupon bond must be expected to be **less** than the YTM of the other four-year bond. The reason why the yield of the coupon bond is less than the yield of the other bond is that the pattern of cash flows presented by the coupon bond is such that much of that bond's present value is generated by its final cash flow (£879.22 of its £1164.24 value), whereas the other bonds' value is generated almost equally across time. Since the coupon bond's value comes from a cash flow being operated upon by a much lower discount rate than the average (note that, in this market, rates are lower farther into the future), the YTM, which is a funny 'value weighted average' of the spot rates, must be less for the coupon bond because of the importance to its value of the lower-rate-generated portion compared with the other bond. Both bonds are, of course, subject to exactly the same set of spot and forward market interest rates. The difference in YTMs is caused solely by differences in cash-flow patterns.

Problem 2

59 The call option that you issue will bring you (assuming no brokerage fees) cash equal to the value of the option. To value the option, first you must find uS_0 and dS_0. With share price increasing from £17 to £22.10 the former is 1.3, and the latter is 0.8 with a decline from £17 to £13.60.

$$C_u = \max\{0, uS_0 - X\} = \max\{0, 1.3(£17) - £18\} = £4.10$$
$$C_d = \max\{0, dS_0 - X\} = \max\{0, 0.8(£17) - £18\} = £0$$
$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£4.10 - £0}{£17(1.3 - 0.8)} = 0.48235$$

and:

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)} = \frac{1.3(£0) - 0.8(£4.10)}{(1.3-0.8)(1+0.10)} = -£5.96364$$
$$YS_0 + Z = 0.48235(£17) - £5.96364 = £2.24$$

The value of the call option is £2.24. This is the amount of cash that you would receive from writing the option. Note that the exercise value is zero, yet you are paid a positive amount for issuing the option. The reason you are given this money for issuing this option even though it is 'out of the money' is that you have promised to sell something for a fixed price (£18), and it has at least some probability between now and option expiration of being worth £22.10. The buyer of the option is purchasing the chance of gaining the spread between exercise price (£18) and £22.10 (netting £4.10) or, if share price instead goes down, simply walking away and not exercising the option. The market values this 'lottery ticket' at £2.24.

60 The condition for a 'perfect hedge' from the Appendix to Module 12 is:

$$uS_0 - mC_u = dS_0 - mC_d$$

The hedge comprises the information as to how may options to write in connection with the purchase of one share of the underlying stock:

$$m = \frac{S_0(u-d)}{C_u - C_d}$$
$$= \frac{£17(1.3-0.8)}{£4.10 - £0.00} = 2.07317$$

This tells us that we can form a perfectly hedged portfolio by buying one share of equity and simultaneously writing 2.07317 call options. But we have already issued just one call option, so we could either issue another 1.07317 call options or purchase less than one share. Of course the partial share would bear the same ratio of one per 2.07317 options. If we were to write only one option, we would purchase 0.48235 (the inverse of the hedge ratio) shares to hedge perfectly.

To test that assertion, for an increase in share price we have:

$uS_0 - mC_u = £22.10 - (2.07317)£4.10 = £13.60 \text{ [one share]}$

or

$uS_0 - mC_u = £22.10(0.48325) - £4.10 = £6.58 \text{ [one option]}$

and for a decline in share price we have:

$DS_0 - mC_d = £13.60 - (2.07317)£0.00 = £13.60 \text{ [one share]}$

or

$DS_0 - mC_d = £13.60(0.48325) - £0.00 = £6.58 \text{ [one option]}$

The portfolio is perfectly hedged: if UK Aero's shares go up and the option has its higher payoff, the portfolio returns £13.60 if hedged by issuing a total of 2.07317 options with a purchase of one share, or goes up £6.58 if hedged by issuing only one option but purchasing 0.48325 shares. If UK Aero's shares decline and the option is worthless (unexercised), the portfolio returns either £13.60 or £6.58, depending upon the hedge transaction but the same amount as it would if the share price had increased.

61 You have already actually done the calculations for the answer to this part of the problem. Remembering that Y is the number of underlying shares to issue so as to form the 'call equivalent' portfolio, and Z is the amount of lending (if positive), the transaction that would duplicate the issuance of the option would be to sell Y shares and borrow Z (if positive):

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£4.10 - £0}{£17(1.3-0.8)} = 0.48235$$

and

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)} = \frac{1.3(£0) - 0.8(£4.10)}{(1.3-0.8)(1+0.10)} = -£5.96364$$

You can sell 0.48235 shares and lend (since negative) £5.96364 at the risk-free rate. Your payoff is then, if share price increases:

$$-Y(uS_0) + Z(1+rf) = -0.48325(£22.10) + £5.96364(1.10) = -£4.10 \quad \text{(A3.1)}$$

and if share price declines:

$$-Y(dS_0) + Z(1+rf) = -0.48325(£13.60) + £5.96364(1.1) = £0.00 \quad \text{(A3.2)}$$

Note that these payoffs are exactly the same (if you have not rounded the results) as those for the option payoffs.

Practice Final Examination 2

Section A – Multiple Choice Questions

1 The correct answer is A. The amount you are required to place in the account today is the present value of the sum you hope to receive in the future (as measured from 2010). In this case it is the present value of £2000 two years in the future, £2000 three years in the future, £2000 four years in the future, £2000 five years in the future and £5238 six years in the future. To do this easily, use the data in Table A1.1 in Appendix 1, go to the column headed 15% to find the appropriate number in each case to multiply the future value by to get the present value, and then add all these present values together.

2 The correct answer is B. The easiest way to solve this problem is to invoke the formulas that apply to the transactions you are undertaking. The basic proposal asks you to invest a constant amount for seven years, and then exchange that value for a set of perpetuity payments to yourself from the investment. That means you are essentially giving up a future value seven years hence for a perpetuity beginning at that point. The formula for the future value of a seven-year annuity is deducible from the general one given at the top of Table A1.4 in Appendix 1 (stated per £), namely

$$\frac{(1+i)^7 - 1}{i}$$

and the present value of an annuity at that point is, of course, simply:

$$\frac{1}{i}$$

If you are giving up the seven-year payment stream to get the perpetuity at that point, your interest rate on the whole set of cash flows is found by finding the rate that causes the two to be the same, i.e.:

$$\frac{(1+i)^7 - 1}{i} = \frac{1}{i}$$

which reduces to

$$(1+i)^7 = 2$$

You can solve this for the answer, 10.41%, if you have a computer or powerful enough calculator, or you can simply look in Table A1.3 in Appendix 1 for the future value seven years hence of a single payment (the obvious form of the reduced solution), which the Table indicates to have a value of 2 approximately half-way between the interest rates of 10% and 11% for a seven-year period.

3 The correct answer is D. Both new debt and new equity will be sold at market prices; that is, on the basis that the PV of the prospective cash inflows, calculated at the risk-adjusted rate established in the market, just equals the price at which new

debt or new equity is sold. So the purchasers of new debt and new equity get no NPV. All the gain from the positive-NPV project goes to the old shareholders.

4 The correct answer is B. If dividends are expected to grow at a constant rate, the formula $D_1/(r\text{-}g)$ can be used to value the shares of the company, where D_1 is the next dividend, r the required rate of return and g the growth rate of dividends. Next year's dividend will equal next year's gross revenue (£350 000 × 1.05) less next year's total costs (£50 000 × 1.05). This dividend of £315 000 will be distributed amongst one million shares. Therefore the value of each share will be

$$\frac{£0.315}{(0.15-0.05)} = £3.15$$

5 The correct answer is C. Using the formula:

$$(1+i_2)^2 = (1+\ {}_0f_1)(1+\ {}_1f_2)$$

we obtain

$$\begin{aligned}({}_1f_2) &= (1+i_2)^2/(1+\ {}_0f_1)\\ &= (1.12)^2/1.07\\ &= 1.1723\end{aligned}$$

Thus the implied one-year forward rate is 17.23%.

6 The correct answer is B. The choices are equally attractive if they have the same NPV. When the discount rate is 18.9%, $£10\,000/1.189 = £20\,000/(1.189)^5$.

7 The correct answer is C. The total value of Burp Gas Company is the value of its debt (£9 million) plus the market value of its equity (£20 million), totalling £29 million. The Weighted Average Cost of Capital (WACC) is determined by the product of the cost of debt and the proportion of the debt to the value of the firm (adjusted for the tax shield) plus the product of the required rate of return for equity and the proportion of equity to the market value of the firm. Thus:

$$\begin{aligned}\text{WACC} &= [(1-0.34)\times(0.12\times(9/29))] + [0.2\times(20/29)]\\ &= 0.1625\end{aligned}$$

8 The correct answer is C. Using the above formula:

$$\begin{aligned}\text{WACC} &= [(1-0.34)\times(0.1\times(2/3))] + [0.2\times(1/3)]\\ &= 0.1107\end{aligned}$$

Using the WACC as the discount rate, the NPV of an investment of £20 million yielding £8 million in perpetuity is

$$(£8\,000\,000/0.1107) - £20\,000\,000 = £52\,267\,289$$

A positive NPV means the firm should undertake the project.

9 The correct answer is C. Sunk costs represent money that has previously been spent. Investment analysis is concerned with the cash flow resulting from the investment being considered: money spent before the investment has been undertaken is not cash flow resulting from the investment decision. This is in clear contrast to opportunity costs – the value of opportunities forgone by undertaking a particular course of action – and any side effects that will occur after the investment is made.

10 The correct answer is A. If the management's expectation is correct, interest rates will tend to fall with inflation as lenders require less compensation for the loss in

their purchasing power resulting from their decisions to postpone consumption. Choosing option A will allow the firm to reduce its borrowing costs. Option B would commit the firm to higher real and nominal interest rates. Option C might be considered if the management expected high and persistent inflation that was not reflected in market prices.

11 The correct answer is C. Both projects have a positive net present value. Both investments should be chosen as there are no capital restraints constricting the choice of action.

12 The correct answer is B. Projects W, Y, and Z give the highest total NPV for the initial outlay of £500 000.

13 The correct answer is C. If the installation cost is a current expense, it can all be deducted from taxable income this year and there will be a cash saving of $£50\,000 \times 0.34$ in one year's time. The present value of this saving is:

$$\frac{£50\,000 \times 0.34}{1.05} = £16\,190$$

14 The correct answer is A. If the installation cost is treated as a capital investment, the deduction from taxable income is £10 000 per year for 5 years. The tax savings are 34% of these amounts delayed for one year.

$$\frac{£10\,000 \times 0.34}{1.05} + \frac{£10\,000 \times 0.34}{1.05^2} + \frac{£10\,000 \times 0.34}{1.05^3} + \frac{£10\,000 \times 0.34}{1.05^4} + \frac{£10\,000 \times 0.34}{1.05^5} = £14\,718$$

15 The correct answer is A. The average expected return of the portfolio is 8.8% (the weighted average of all possible outcomes). The standard deviation of the portfolio is simply the weighted average of the square root of the weighted average of the difference between the possible outcomes and the average return, that is

$$\sqrt{[0.2 \times (0.05 - 0.088)^2] + [0.6 \times (0.08 - 0.088)^2] + [0.2 \times (0.15 - 0.088)^2]} = 0.0331$$

16 The correct answer is C. Look again at Figure 7.3. It is only when the assets are perfectly correlated (that is, when the correlation coefficient is plus one) that the risk of the portfolio can be a weighted average of its components. The risk of a portfolio with assets that are not perfectly correlated (that is, the correlation coefficient is less than one) is a non-linear function of asset risk and correlation with each other, and this is far more complex than the simple weighted average.

17 The correct answer is B. The β coefficient of the required rate of return is the weighted average of the β coefficient of debt and the β coefficient of equity, namely:

$$(0.25 \times 0.8) + (0.75 \times 1.5) = 1.325$$

18 The correct answer is C. The Capital Asset Pricing Model should be used to find the expected return on the investment,

$$\text{Expected return} = 10\% + [2 \times (20\% - 10\%)] = 30\%$$

19 The correct answer is D. Once again, the Capital Asset Pricing Model should be used to find the expected return for Gamma plc. To do this, we have to calculate Gamma's β coefficient using the data on covariances and variances provided. Thus:

$$\beta_{Gamma} = \frac{\text{Covariance of Gamma's return with the market}}{\text{Variance of the market}}$$
$$= 0.0607172/0.052851$$
$$= 1.1488373$$
$$\text{E(R)}_{\text{Gamma}} = 6.6 + (1.1488373 \times 8.5)$$
$$= 16.365$$

20 The correct answer is D. Statement I is incorrect as dividends are not relevant: shareholders will be compensated through higher capital gains. Statement II is incorrect as dividends are not shareholders' wages: the wage to shareholders is the return they receive through dividends and capital gains.

21 The correct answer is D. Statement I is incorrect as it does not take the required return for equity into account. Similarly, statement II is incorrect as higher gearing does not necessarily imply greater risk.

22 The correct answer is C. Stock prices fall when bad news is announced and rise when good news is announced. The ability to raise debt is generally seen as good news as it indicates lenders' confidence in the company. Conversely, issuing equity is often seen as bad news as it is often a sign that the firm is short of capital and cannot raise funds elsewhere.

23 The correct answer is D. Statement I is incorrect: indentures give bondholders the right to any assets in the case of bankruptcy; shareholders only get what is left after the bondholders get what is owed to them. Statement II is incorrect. Indentures reduce the return required by bondholders by reducing their risk.

24 The correct answer is D. The β coefficient for each company is determined by the covariance and variances of their returns (dividends and capital gains) with the market. Prices alone are not enough to work with.

25 The correct answer is B. Let the average number of daily customers be *N*. The savings are two days of interest on an annual $£4500 \times \text{N} \times 365$ of payments. This equates to:

$$4500 \times N \times 365 \times \frac{(0.06 \times 2)}{365} = 540N$$

The costs are $15\,000 + (0.25 \times 365 \times \text{N}) = 15\,000 + 91.25\text{N}$

Break-even occurs when

$$540N = 15\,000 + 91.25N$$

Therefore:

$$N = \frac{15\,000}{(540 - 91.25)} = 33.43$$
$$= \text{34 customers daily (rounded)}$$

26 The correct answer is C. The cost of each month's sales is

$$£600\,000 \times 0.70 = £420\,000$$

On average, 3 months' sales are outstanding at any time, so the amount invested in accounts receivable is

$£420\,000 \times 3 = £1\,260\,000$

27 The correct answer is B. The appropriate one-year forward rate is determined by $\$1.50{\times}(1.07/1.04)$.

28 The correct answer is A. The risk-free interest rates are used to calculate the implied exchange rates. Thus, in round terms:

					Exchange rate 1.50 $/€ *Discount rate 11%* *US interest rate 7.0%* *German interest rate 4.50%*
	0	**1**	**2**	**3**	
Euro cash flow	–10	4	3	5.1	
Adjust by		1.0239	1.0484	1.0735	*Adjust by (1+US interest rate)^t/(1+German interest rate)^t*
To get exchange rate	1.50	1.53589	1.57263	1.61025	
Convert to US$	–15.00	6.14	4.72	8.21	*Multiply euro cash flow by each year's exchange rate*
Discount factor	1	0.9009	0.8116	0.7312	*Discount US$ cash flows with US discount rate*
Discounted cash flows	–15.00	5.53	3.83	6.00	
				NPV=	0.368614 $m **$368,614**

The NPV is calculated in dollars using the dollar discount rate.

29 The correct answer is B. The real rate of interest is determined using the following formula:

$$(1 + \text{inflation rate}) = \frac{(1 + \text{nominal interest rate})}{(1 + \text{real interest rate})}$$

30 The correct answer is B. An option is 'out of the money' if it is cheaper to buy the stock than exercise the option. An option has positive value if it is out of the money because of the possibility that it will be 'in the money'.

Section B – Essay Questions

Problem 1

31 The debt–equity decision has no effect on the wealth of current shareholders, according to M&M.

32 If interest is tax-deductible but dividends are not, then the use of debt results in a tax saving. The wealth of shareholders is increased by the present value of those future tax savings. If an incremental amount of debt D is taken on permanently, the tax saving is TD (where T is the tax rate).

33 An amount of 45% of Heritage's shares are held by outside investors and the shares are quoted on the market. A failure to maintain the dividend would probably lead to a loss of confidence in the company and a sharp fall in the price of the shares. Keeping the share price up is important for Heritage's long-term growth plans, since it may want to fund future expansion by share issues.

With Manderley, the income statement in a recession year would be as follows:

	Share funding	Long-term debt	Short-term debt
Revenue	16.5	16.5	16.5
Operating costs	11.495	11.495	11.495
Depreciation	3.3	3.3	3.3
EBIT	1.705	1.705	1.705
Interest	1.5	1.83	1.74
Earnings before tax	0.205	(0.125)	(0.035)
Cost of maintaining dividend at 15p/share	1.755	1.5	1.5

With share funding, the dividend can be maintained using the cash flow from depreciation and the small amount of after-tax earnings. With debt funding, the dividend would have to be cut.

34 The possibility of a recession makes Heritage's business more risky. The chance of disaster in the form of bankruptcy depends partly on the risk of the business and partly on the level of debt. Heritage can offset an increased operating risk in its business by reducing the amount of financial risk (debt) in its balance sheet.

35 Mr Taylor has 55% of the shares at present, which gives him unquestioned control. If the Manderley development is funded by a share issue to outside shareholders, his share will go down to 47%. This is still normally enough to control the business, since it is unlikely that all the other shareholders would combine to vote against Mr Taylor. Nevertheless, Mr Taylor may prefer to maintain a majority of votes in his hand and this would suggest the use of debt finance.

36 See **3** above. In a recession year the dividend could be maintained out of current cash flow but not out of after-tax earnings. If debt is used to finance Manderley, Heritage will lose the ability to maintain the dividend in a bad year.

37 If Mr Taylor believes that interest rates will fall, it would be a mistake to make a long-term commitment to borrow at rates that are temporarily high. This factor would argue against the use of long-term debt.

38

- Long-term interest rates are generally higher, and so profits will look better in the short run if short-term debt is used.
- According to the expectations hypothesis, the fact that short-term debt is cheaper now suggests that it is expected to be more expensive in the future, and there is no clear long-term benefit from using one type of debt rather than the other.
- Short-term debt adds an additional element of risk, because Heritage cannot predict what interest rate it will have to pay when it rolls its short-term debt over. Many companies prefer to match the maturity of assets and liabilities for this reason.

39 Debt brings tax advantages and maintains Mr Taylor's majority holding in the company. On the terms proposed, it threatens the company's ability to maintain its dividend. Perhaps the best solution would be for Heritage to borrow less than the full purchase price of Manderley on terms that do not restrict dividend payments. A small share issue to raise the rest of the money would not destroy Mr Taylor's voting majority, although Mr Taylor must accept that, if he wishes Heritage to grow quickly, he will have to relinquish his majority position before very long.

Problem 2

40 Bond A maturing in one year gives a final cash flow of £105. The value of bond A paying £100 is thus:

$$£99.0566 \times \frac{100}{105} = £94.340$$

The one-year interest rate is therefore:

$$\frac{100}{94.340} - 1 = 6.00\%$$

Bond B maturing in two years gives a cash flow of £10.5 (after 1 year)
£110.5 (after 2 years).

The present value of the first coupon payment is:

$$\frac{£10.5}{1.06} = £9.9057$$

So the present value of the final payment of £110.5 is:

$$£104.6741 - £9.9057 = £94.7684$$

The value of 100 to be paid after two years is therefore:

$$£100 \times \frac{£94.7684}{£110.5} = £85.763$$

The two-year interest rate is:

$$100 \times \left[\left(\sqrt{\frac{100}{85.763}}\right) - 1\right] = 7.98\%$$

The three-year bond C pays:

£10 after one year, present value is $\frac{10}{1.06} = 9.4340$

£10 after two years, present value is $\frac{10}{(1.0798)^2} = 8.5766$

Total = 18.0106

The £110 paid after three years therefore has a present value of:

$£104.5604 - £18.0106 = £86.550$

The value of 100 after three years is $100 \times \frac{86.550}{110} = 78.68$

41 One year to maturity = $\frac{£100}{1.07} = £93.458$

Two years to maturity = $\frac{£100}{(1.09)^2} = £84.168$

Three years to maturity = $\frac{£100}{(1.085)^3} = £78.291$

42 One-year nominal forward rate = $\frac{(1.09)^2}{1.07} - 1 = 11.037\%$

Two-year nominal forward rate = $\frac{(1.085)^3}{(1.09)^2} - 1 = 7.507\%$

43 Three-year bond in one year's time = $£78.291 \times 1.07 = 83.771$

44 First-year inflation rate = $\frac{1.07}{1.01} - 1 = 5.9406\%$

Second-year inflation rate = $\frac{1.11037}{1.02} - 1 = 8.8598\%$

Third-year inflation rate = $\frac{1.07507}{1.025} - 1 = 4.8849\%$

Appendix 4

Answers to Review Questions

Contents

Module 1

Review Questions

1.1 The correct answer is B. The 'maximum you can consume' implies that all of your consumption will be at t_0, so in addition to the £3000 of cash flow already at that time, you must shift your future cash flow to the present. Since your future cash flow is at t_1, it must be discounted to t_0 using the 11% rate given. Thus:

$$\begin{aligned}
\text{PV(maximum)} &= CF_0 + \frac{CF_1}{(1+i)} \\
&= £3000 + \frac{£5328}{(1+0.11)} \\
&= £3000 + £4800 \\
&= £7800
\end{aligned}$$

This calculation is the final arithmetic of an actual financial market transaction that would result in that amount of t_0 cash flow being available. For example, a lender in the financial market would be willing to provide the £4800 of t_0 cash to you, if you provided in return a claim upon your t_1 cash flow. The £4800 would require an interest payment of £4800(0.11) = £528, plus the return of the original £4800 lent, to total your £5328 at t_1. The lender of course must concur in your expectation that the £5328 will actually appear at t_1.

Questions 1.1 to 1.11 refer to **Module 1**, Section 1.3. The graphical solution to these questions is given in Figure A4.1.

Figure A4.1 Financial exchange line and investments

1.2 The correct answer is D. This is the process of shifting all of your cash flow to t_1 instead of t_0 as you did in the question immediately above. The £5328 of t_1 cash flow stays where it is, but the £3000 at t_0 must be shifted to the end of the period:

$$\begin{aligned}\text{Maximum } t_1 \text{ consumption} &= CF_0(1+i) + CF_1 \\ &= £3000(1.11) + £5328 \\ &= £3330 + £5328 \\ &= £8658\end{aligned}$$

The actual financial market transaction that would shift all of your cash flow to t_1 is simply the act of lending all of your t_0 cash flow to the market at an 11 per cent interest rate for one period. The £3000 at t_0 increases by $£3000(0.11) = £330$, and that amount of £3330 plus the original t_1 £5328 makes £8658 available.

1.3 The correct answer is A. Since the amount you wish to consume is greater than your initial t_0 cash flow, you must shift enough of your t_1 cash flow to t_0 so as to increase your consumption at that time point from £3000 to £5000. The additional £2000 you need at t_0 requires that £2000(1.11) = £2220 be given up at t_1, the equivalent of the interest and principal on a loan in the amount of £2000 at t_0 for one period at an 11 per cent interest rate. Since your initial t_1 cash flow is £5328, you are left with the expectation of $£5328 - £2220 = £3108$ at t_1.

1.4 The correct answer is B. Here you are shifting some of your t_0 cash flow to t_1. The ratio between the amounts shifted is £1.11 at t_1 for each £1 at t_0. To consume

£7548 at t_1, an additional £2220 in excess of your £5328 initial cash flow at that time point, you would necessarily give up (lend) £2000 at t_0. The £2000 given up serves to reduce your original t_0 £3000 to £1000.

1.5 The correct answer is D. The present value of all of the consumption combinations in the questions above is £7800. This is apparent in Figure A4.1 as all of these are points along the same exchange line emanating from the t_0 intercept of £7800. The **reason** the present values are all the same is because, with £7800 of t_0 resources, all of the combinations are exactly attainable by various borrowing and lending activities at the market interest rate; no other t_0 amount can do that. Since total resources shifted to t_0 is by definition present wealth, and £7800 is that amount for you, the (B) answer is in that sense tautological.

1.6 The correct answer is C. The easiest way to see this is to notice that, in Figure A4.1, any financial market transaction for the purpose of altering consumption patterns simply serves to move the participant up or down the same exchange line, and that exchange line has a single t_0 intercept, present wealth. The economic importance of the observation that wealth is unchanged by financial market transactions is that such transactions are best used by participants to reallocate their resources across time so as to consume a fixed amount of wealth according to their preferences. But financial markets, if they efficiently price the securities in them, do not allow participants to change their wealth by transacting in those markets. The timing of consumption does not affect wealth, nor does the fact that with a positive interest rate, the amounts consumable at various times may not be the same. You may prefer one pattern of consumption to another, but that does not affect your wealth, defined as the present value of all your future cash flows or resources.

1.7 The correct answer is B.

$$\text{NPV}(1) = -£1000 + \frac{£1250}{(1.11)} = +£126.13$$

$$\text{NPV}(2) = -£500 + \frac{£650}{(1.11)} = +£85.59$$

$$\text{NPV}(3) = -£1500 + \frac{£1650}{(1.11)} = -£13.51$$

Investments 1 and 2, in producing positive NPVs, are expected to have future returns that exceed the returns that investments in the financial market would provide. Investment 3, in producing a return of £1650 at t_1 in exchange for a £1500 investment at t_0, is not providing a return as high as the financial market would for a similar investment ($£1500 \times 1.11 = £1665$). Positive NPVs are associated with returns in excess of market rates; negative NPVs come with investments whose returns are less than what the market would provide.

1.8 The correct answer is B.

$$
\begin{aligned}
0 &= -£1000 + \frac{£1250}{[1 + \mathrm{IRR}(1)]} \\
\mathrm{IRR}(1) &= 25\% \\
0 &= -£500 + \frac{£650}{[1 + \mathrm{IRR}(2)]} \\
\mathrm{IRR}(2) &= 30\% \\
0 &= -£1500 + \frac{£1650}{[1 + \mathrm{IRR}(3)]} \\
\mathrm{IRR}(3) &= 10\%
\end{aligned}
$$

Investments 1 and 2 offer average per-period rates of return (IRRs) in excess of those generally available on similar investments in the financial market, so are acceptable. Investment 3's 10 per cent return is less than the 11 per cent return the market would provide, so it is rejected. Note that the two criteria, NPV and IRR, have in this instance indicated the same 'correct' decision in each case.

1.9 The correct answer is A.

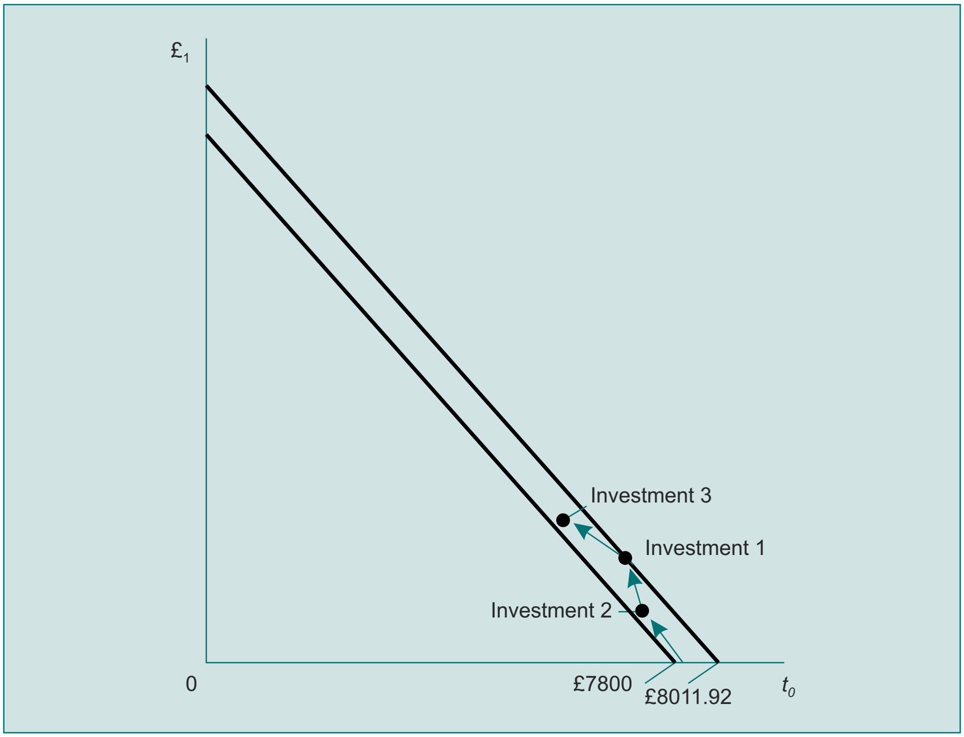

Figure A4.2 Multiple investments and the financial exchange line

There are a number of ways to arrive at this result. The simplest is to remember that NPV is the change in present wealth of the participant accepting the investment, and since you are the sole shareholder of the company undertaking the investments the sum of the NPVs of the investments will accrue directly to you. £211.72 is the sum of the NPVs of the acceptable investments 1 and 2.

Another way to see this is given in Figure A4.2. In this graph we have plotted the investments (we have them emanating from the original present wealth rather than the original cash-flow amounts, which has no effect upon the results but is the more common convention in finance), the new exchange lines that would be associated with them, and the resulting present wealth. You can see that if investments 1 and 2 are undertaken, wealth will be maximised, and the quantitative increase is from £7800 to £8011.72, an increase of £211.72.

1.10 The correct answer is D. In the financial market the way we have designed it to this point, it would make no difference to your wealth (as long as the investments were somehow or other accepted) whether you provided the money or not, whether the money was derived from your borrowing or the company borrowing, or by the company issuing new shares. Since the investments' cash flows and discount rates are known for certain, and since there are no frictions like taxes and brokerage fees in the market, money will be provided at an opportunity cost of 11 per cent, and the resulting values will obtain. You should not reject the company's request for funds simply because your preference is to consume an amount equal to all of your income at t_0, because if you give money to the company you can always borrow what you need from the market. Since the investments for which your money will be used will earn more than it costs you to borrow the money, providing the money the company requests is worthwhile (but no more nor less worthwhile than the company borrowing the money or issuing new shares).

The only caution we would point out is that whenever the financial market is approached for funds, either for borrowing or for issuing new ownership shares, the market must be made aware of and concur in the expectations associated with the investments that will claim the funds. (It is particularly important that share prices increase to reflect the NPVs **before** the new shares are sold, or else the new shareholders will capture part of your NPV.) If that concurrence is not the case, the market may not provide the money for investments at the same opportunity rate that you used as the discount rate to evaluate the investments. The resulting wealth effects of the investments would not then be the same as you expected, and could actually produce wealth decreases. This applies to both borrowing and raising new ownership funds.

1.11 The correct answer is A. You should accept the investment with the highest NPV, because it increases your wealth the most. (Incidentally, though we did not ask it, the investment that does that is investment 1.) You should not accept one investment over another on the basis of the rates of return they earn, because rates of return are not the same as potential consumption amounts, and it is the opportunities to consume wealth that motivate participants in financial markets. To see this, notice that investment 2 has a higher IRR than investment 1, even though investing in 2 would produce a smaller excess cash return than would investing in 1. If you put your money in investment 2, after opportunity costs are subtracted, you end up with $£650 - [£500(1.11)] = £95$ at t_1, whereas investing in 1 gives you $£1250 - [£1000(1.11)] = £140$ at t_1. So, regardless of the fact that investment 2 promises a return of 30 per cent to investment 1's 25 per cent, investment 2 does not produce as much wealth. (Note that the present (t_0) values of the excess amounts calculated above are in fact the investment NPVs.)

Students often here raise the point that investment 2 would be preferred to investment 1 if we were allowed to invest more than the £500 outlay in 2 at the 30 per cent return. That is indisputably true. If you invest £1000, for example, at a 30 per cent return you would receive £1300 at t_1, exceeding the return from investing in 1. But the odds are that you could not do that, because the investments are real assets like machines, manufacturing plants, inventory or even education. Just because the first machine earns 30 per cent is no reason to think that the next one will, and even less so the third! If more money poured into it and yet investment 2 continued to earn 30 per cent per pound invested, we would argue that the investment analyst was at fault for not specifying in the original analysis that investment 2 had that capacity. The truth is that securities in financial markets are the only investments that generally have the characteristic of offering the same IRR regardless of the amounts invested (because the supply of such assets is essentially unlimited relative to our individual capacities to invest in them). And we have already discussed the chances of increasing wealth by investing in such financial assets.

1.12 The correct answer is A. You should choose location 1 because it has the higher NPV:

$$\text{NPV}(1) = -£2500 + \frac{£1250}{(1.10)} + \frac{£1300}{(1.10)^2} + \frac{£1450}{(1.10)^3} = +£754.70$$

$$\text{NPV}(2) = -£2500 + \frac{£1300}{(1.10)} + \frac{£1300}{(1.10)^2} + \frac{£1300}{(1.10)^3} = +£732.91$$

1.13 The correct answer is A. You should choose location 2 if your opportunity costs are 25 per cent per period, as indicated by a recalculation of NPVs:

$$\text{NPV}(1) = -£2500 + \frac{£1200}{(1.25)} + \frac{£1300}{(1.25)^2} + \frac{£1450}{(1.25)^3} = +£34.40$$

$$\text{NPV}(2) = -£2500 + \frac{£1300}{(1.25)} + \frac{£1300}{(1.25)^2} + \frac{£1300}{(1.25)^3} = +£37.60$$

The interesting point about Questions 1.12 and 1.13 is that they show how changing the discount rate alters not only your wealth (note that the NPVs of both alternatives are lower with the 25 per cent opportunity cost than with the 10 per cent), but also the relative desirability of the alternatives (location 1 is preferred at the lower discount rate, and location 2 at the higher). The reason for that result appears when we sketch a graph of the relationship between discount rates and NPVs for these two alternatives, as we have done in Figure A4.3. Notice that as the discount rate increases, the NPV of location 1 declines more quickly than location 2's, such that even though location 1 was preferred at the lower discount rate, eventually – as the discount rate increases – location 2 becomes the preferred alternative. The arithmetic reason for this is not difficult to see. Look at the patterns of cash flows that the two alternatives exhibit: location 1 has its largest cash flows further into the future and is thus more subject to the vagaries of discount rate changes than location 2 whose cash flows are more evenly spread across time. So changes in discount rates will tend to cause greater changes in the NPV of 1 than 2, and that is exactly the characteristic of which we have taken advantage in designing this example.

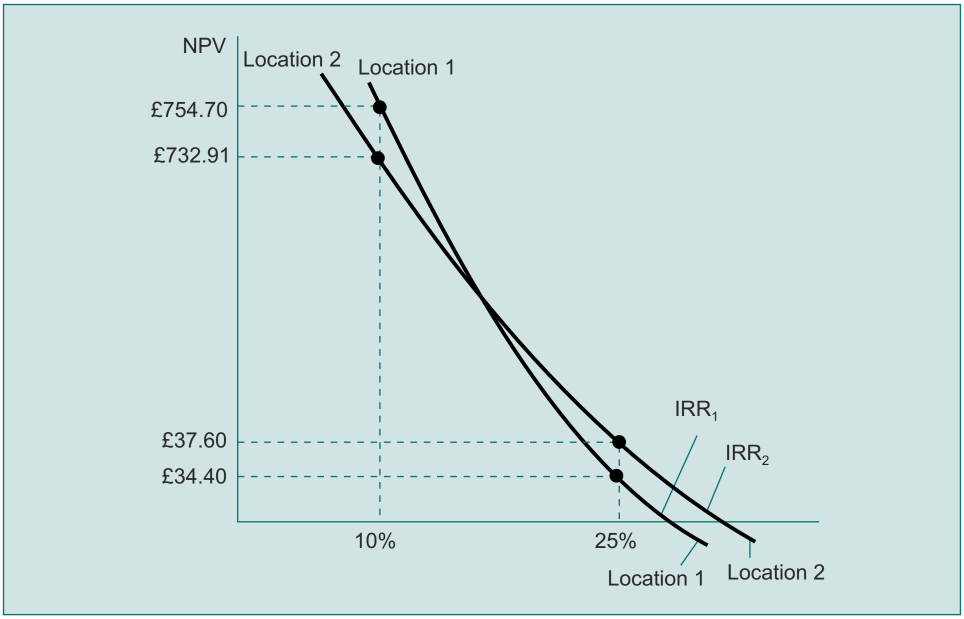

Figure A4.3 **NPV, IRR and the crossover rate**

There is another important lesson in this example that you will appreciate when we discuss the ways that companies actually make their real-asset investment decisions. Suppose that you had decided between these alternatives on the basis of IRR instead of NPV. Without actually calculating IRR, you can see that there is a problem with that technique in this example. Calculated in the usual manner, there is no way that the IRR could have 'changed its mind' about the relative desirability of the two alternatives the way that NPV did. A change in the discount rate would not change the IRR of either alternative, and whichever had the highest IRR with the 10 per cent opportunity cost would also have the highest IRR with the 25 per cent opportunity cost. This means that if you decided on the alternative with the higher IRR, you might be choosing the one with the lower NPV, and thus not maximising your wealth. You can see this problem clearly in Figure A4.3, where investment in location 2 has the higher IRR (the horizontal intercept) but has the higher NPV only at higher discount rates. There is a way to fix this problem with the IRR as a decision technique, and we shall show it to you in a later section.

1.14 The correct answer is B. It would be necessary for the second investment to have a constant per-period rate of increase in cash flow of 2 per cent so as to have the same present value as the first investment:

$$PV(1) = \frac{£1000}{0.10} = £10000$$

$$PV(2) = \frac{£800}{0.10 - 0.02} = \frac{£800}{0.08} = £10000$$

1.15 The correct answer is C. The first investment would necessarily be expected to run for about seventeen years so as to have the same present value as the second investment running for ever. The easiest way to arrive at this answer is to see that the second investment, as a non-growth perpetuity, has a present value of $£800/0.10 = £8000$. To be equally desirable, the first investment must also have a present value of £8000, assuming they cost the same. This means that the first investment, with a per-period cash flow of £1000, must have a ratio between cash flow and value of eight times so as to produce a present value of £8000. Since we know that this is an annuity, and we know the discount rate (10 per cent), we can go to the annuity tables in **Appendix 1** and look in the 10 per cent discount rate column for a number close to 8. The table indicates that an annuity running for 17 years has a present value equal to 8.0214 times its per-period cash flow.

1.16 The correct answer is C. You would necessarily put aside £63.67 per month for a year, with 10 per cent annual interest compounded monthly so as to end up with £800 at the end of the year. The following steps will arrive at that result:

1. First find the present value of the £800. This is $£800/[1 + (0.10/12)]^{12} = £724.17$.
2. Next, find the present value of £1 per month for one year at the interest offered. This is 11.3745, an annuity value that is the sum of the series $1/(1 + 0.10/12)^1 + 1/(1 + 0.10/12)^2 + ... + 1/(1 + 0.10/12)^{12}$.
3. Finally, divide the present value of what you want to end up with (£724.17) by the present value per pound of what you will put in (11.3745), and the result (£63.67) is the number of pounds per compounding period that you must put in so as to end up with the £800 at the end of the year.

There is a somewhat easier way to solve this problem, which requires the use of the formulas for future and present values of annuities that we have not presented. Because of the many variants that these can assume, and the limited usefulness of the full range of these formulas, we recommend that you refer to a good reference book of mathematical tables to get the one that applies to any particular problem you may be facing.

1.17 The correct answer is B. It would be necessary for you to deposit £723.87 in the bank at the beginning of the year at a continuously compounded rate of 10 per cent so as to finish with £800 at the end of the year. This is nothing more than the present value of £800 one year hence at a continuously compounded discount rate of 10 per cent: $PV = CF/e^{it} = £800/e^{0.10} = £723.87$.

1.18 The correct answer is C. You should undertake the investment because it has a positive NPV:

$$\begin{aligned} NPV &= -£15000 + \frac{£7000}{(1.10)} + \frac{£11000}{(1.10)^2} \\ &= +£454.54 \end{aligned}$$

But since there is risk that ${}_1f_2$ will change, you can simultaneously hedge the t_2 investment cash flow's present value by selling an interest rate future for the second period.

If ${}_1f_2$ should increase to 20 per cent, as the question indicates, your NPV of the investment will become:

$$\begin{aligned}\text{NPV} &= -£15000 + \frac{£7000}{(1.10)} + \frac{£11000}{(1.10)(1.20)} \\ &= -£303.03\end{aligned}$$

This would cause you to regret having made the investment. But if you had at t_0 sold an interest rate future based upon an £11 000 t_2 cash flow, the result would differ. That interest rate future, when you sold it under the original term structure (with an implied ${}_1f_2$ of 10 per cent), would have a fixed t_1 price of £11 000/(1.10) = £10 000. So you would own a contract that commits you to sell an £11 000 t_2 cash flow for £10 000 at t_1. When the ${}_1f_2$ rate increases to 20 per cent, the t_1 value of a t_2 £11 000 becomes £11 000/(1.20) = £9166.67. So you own the right to sell something worth £9166.67 for £10 000, a gain to you of £833.33. But this gain is as of t_1. At t_0 (with an unchanged i_1 of 10 per cent), this has a value of £833.33/(1.10) = £757.57. So the interest rate future contract that you own will increase in value £757.57 at t_0. On the other hand, your investment NPV has decreased from +£454.54 to −£303.03, a reduction of £757.57. Thus the change in value of your (sold) interest rate future exactly hedges you against the risk of investment NPV diminution due to interest rate changes.

Case Study 1.1: Bond and Interest Rate Arithmetic

1 This case refers to **Module 1**, Section 1.4.5 and after.

The basic information provided in this case can be arranged in the approximate form that such information was in the text:

Price		**Cash flows**		**YTM**	**Bond**
t_0	t_1	t_2	t_3		
£919.97	£40	£1040		?	(A)
?	£100	£1100		8.5595%	(B)
£1014.59	£80	£80	£1080		(C)

In addition you are provided with the information that the current one-period spot rate is 10 per cent. (This latter information is not literally necessary, as the very astute student may have discovered: the one-period spot rate can be deduced from the simultaneous equation solution to the information about bonds A and B, the two-period bonds. At this stage of the text, however, it would be unfair to think that everyone would see this relationship.)

The current price of bond B can be found by remembering that the YTM is the discount rate that equates the bond's price to its cash-flow promises:

$$\text{PV(B)} = \frac{£100}{(1.085595)} + \frac{£1100}{(1.085595)^2} = £1025.49$$

2 The current two-period spot rate of interest can be found with the information about the price of either of the two-period bonds and the one-period spot rate. For example, using bond A, the present value of its t_1 cash flow is £40/1.10 = £36.36. Since its total present value is £919.97, the present value of its t_2 cash flow is £919.97 – £36.36 = £883.61. That value can be regarded as being produced by the two-period spot rate in the equation £883.61 = £1040/(1 + i_2)^2. Solving for i_2 we get 8.489 per cent as the two-period spot rate.

An interesting check on this result is to see if it works on bond B. We should be able to reproduce B's present value using the spot rates for the first two periods, since they are supposed to apply to all bonds of equal risk. Using the spot rates, the present value of B's t_2 cash flow is £1100/(1.08489)^2 = £934.59, and the present value of its t_1 cash flow is £100/(1.10) = £90.91. These sum to £1025.50, the price of bond B (rounded) that we found initially by using its YTM as the discount rate.

3 The one-period forward rate of interest for the second period is the rate of exchange for shifting resources from t_1 to t_2. The easiest way to calculate that rate is to use the relationship between spot rates and forward rates: 1 plus the spot rate of interest raised to the nth power (where n is the number of periods that the spot rate covers) is equal to the product of {1 plus each of the intervening forward rates}. That rule translates in this situation to:

$$(1+i_2)^2 = (1+ {}_0f_1)(1+ {}_1f_2)$$

Remembering that the one-period spot rate is the same thing as the first period's forward rate (${}_0f_1 = i_1$), we know every rate in the above except the forward rate for the second period, ${}_1f_2$. Solving for it, we have:

$$\begin{aligned}(1+ {}_1f_2) &= (1+i_2)^2/(1+ {}_0f_1)\\(1+ {}_1f_2) &= (1.08489)^2/(1.10)\\(1+ {}_1f_2) &= (1.07)\\{}_1f_2 &= 7\%\end{aligned}$$

4 The one-period forward rate of interest for the third period can be calculated by the same technique as we used for ${}_1f_2$; that is, if we know the spot rate for the entire span of time between now and the end of the period from which we wish to extract the forward rate, we need merely to multiply together {1 plus all of the forward rates but the one we wish to find}, and divide that product into the spot rate raised to the nth power. The problem that we face here in doing that is that we do not known the three-period spot rate.

The three-period spot rate requires that we find the rate of exchange between t_0 and t_3. This information is somehow contained in the cash flows and price of the three-period bond. If that bond were a 'pure discount' bond (a bond that had no interim interest payments) we could find the spot rate by dividing the current price into the t_3 cash flow, then taking the cube root and subtracting one. But that tactic will not work with our three-period bond because its current price also includes the values of its interim t_1 and t_2 coupon payments. Our first step must be to find the present value of the t_3 payment alone. That obviously requires that we subtract the present values of the t_1 and t_2 cash flows from the current market price of bond C. We know that the interim cash flows can be valued with the spot or forward rates for the appropriate periods:

$$\mathrm{PV}(\mathrm{CF}_1) = £80/(1 + 0.10) = £72.73$$
$$\mathrm{PV}(\mathrm{CF}_2) = £80/(1 + 0.08489)^2 = £67.97$$

The sum of the present values of the first two cash flows of C is thus £72.73 + £67.97 = £140.70. These two cash flows comprise that amount of C's present market value. Since the total market value of C is £1014.59, the present value of its t_3 payment is £1014.59 – £140.70 = £873.89. We now have enough information to calculate the three-period spot rate:

$$\begin{aligned}(1 + i_3)^3 &= CF_3/[\mathrm{PV}(\mathrm{CF}_3)] \\ (1 + i_3)^3 &= £1080/£873.89 = 1.23585 \\ (1 + i_3) &= 1.07314 \\ i_3 &= 7.314\%\end{aligned}$$

The three-period spot rate of interest is 7.314 per cent. It is now child's play to find the third period's forward rate:

$$\begin{aligned}(1 + {}_2f_3) &= (1 + i_3)^3/(1 + {}_0f_1)(1 + {}_1f_2) \\ (1 + {}_2f_3) &= (1 + i_3)^3/(1 + i_2)^2 \\ (1 + {}_2f_3) &= (1.07314)^3/(1.08489)^2 \\ (1 + {}_2f_3) &= 1.05 \\ {}_2f_3 &= 5\%\end{aligned}$$

(By looking at the sets of spot and forward rates in this example can you tell which set we initially assumed? Students often correctly think that if they arrive at an answer that is a clean, whole number they have probably found the correct result since those who make up problems often start with the answer and work backwards, and also tend to begin with whole numbers in designing problems. Of course, astute problem designers know you know those tendencies!)

5 In the process of finding the third period's forward rate you have already found the three-period spot rate ($i_3 = 7.314\%$).

6 You would expect to find the YTM of bond A to be less than the YTM of bond B. Remember in the text where we discussed the 'coupon effect' on the YTM? We said that the pattern of a bond's cash flows interacts with the set of spot and forward rates to produce the YTM as a complex weighted average of the rates and cash-flow pattern. The patterns of the two bonds' cash flows in this case are such that A has the lower coupon, and thus its YTM is less heavily influenced by rates close to the present and more heavily influenced by rates further into the future than is bond B with the higher coupon. Since rates are higher closer to the present, the higher-coupon bond (B) will have the higher YTM, and the lower coupon bond (A) will have the lower coupon. Should you be interested, the YTM of A is about 8.52 per cent compared with B's 8.56 per cent. You should now be able to explain to the interested listener over cocktails why two bonds of equal risk and equal maturity but different coupons have different YTMs, and that in no true economic sense are their 'interest rates' different.

7 The current expectation for the t_1 price of bond B can be found by discounting to t_1 the cash flows to be received after t_1 at the set of forward rates between t_1 and the times that the cash flows are to be received. It is more difficult to say that than to do it:

$$\begin{aligned} V_1(\mathrm{B}) &= CF_2/(1 + {}_1f_2) \\ &= £1100/(1.07) \\ &= £1028.04 \end{aligned}$$

8 You have already calculated the current price of this security (based on a claim upon the t_2 cash flow of bond C). In Question **4** of the case you discovered that *PV(CF_2)* for bond C is £67.97. That is, in fact, under the conditions of the question, the t_0 price of a claim on the £80 interest payment of C at t_2.

9 By using futures markets in financial securities you can, if faced with the type of uncertainty posed in this case, ensure that a particular NPV will actually result. Here, though the amounts of cash that are expected to appear across the future are known with certainty, the interest rates that will apply between future time points are not. The existing set of spot and forward rates is known (and was used, we assume, to calculate the NPV).

The problem can be illustrated by supposing that, immediately after the commitment to undertake the project at t_0, interest rates rise. Now, there is nothing we can do to make interest rates go back to where they were. Therefore the NPV of the investment itself is irrevocably lower. But there is something that we should have done to have our **wealth** remain intact. We should, at the same time we commit to the investment, also engage in another transaction that has exactly the opposite exposure to the risk of interest rate changes that the investment has. One such transaction is to sell forward contracts in financial securities at the same time that the investment commitment is made.

A forward contract is a commitment to purchase a financial security at a fixed price at a set future time point. When you sell one of those, you are committing yourself to sell the security on the terms of the contract. As interest rates (discount rates) increase, security prices decline. So as interest rates rise at t_0 and security prices fall, the price of the contract to purchase a security in the future at a fixed price also falls, but your negative holding of the contract (selling it) **increases** in value.

In other words the commitment that you own (to sell the security at a fixed price) becomes more valuable as the security's price itself declines, and the security's price declines as interest rates increase. Finally, since the NPV of the real asset declines with the increase in interest rates, that decline will tend to be offset by the increase in value of the contract to sell a financial security. Naturally, for the hedge (as this tactic is called) to be perfect, great care must be taken in deciding exactly what kind of a contract (or, more realistically, contracts) to sell. But that analysis is well beyond us at this point, and not really necessary to the basic lesson that such activities are possible.

There is one other point that should be made in this discussion of hedging against discount rate uncertainty in real-asset investments. The astute student may have already asked the question, 'Suppose that you had borrowed money to undertake the investment. If you had made a commitment to pay a fixed interest rate for the duration of the real-asset investment, how does all this business of changing interest rates apply? It would seem that since you are certain of the cash you are going to take in and the cash you are going to pay out, interest rate changes are irrelevant.'

A good point. The essential intuition of this view is satisfactory, but needs a bit of refinement. It is true that the original example, in order to be an accurate reflection of the risk of interest rate changes, would necessarily assume that however the investment was financed, its interest rates (or other required returns to capital suppliers) must be able to vary with changes in market rates. If they do not, as in the question just posed, the prices of the securities you have issued to finance the investment will fluctuate with changes in market interest rates. In our example, when rates go up, the amount you would have to pay to creditors would not increase if the interest payments were fixed, and therefore the values of those claims would decline. This is economically the same effect as hedging in futures markets, but is done here by making a fixed commitment to pay interest amounts rather than to sell a security. It is possible to hedge your wealth against changes in future interest rates by financing a project with commitments that would vary in value exactly opposite to your investment's cash flows as discount rates change.

10 The actual hedging transaction in this marketplace would be to sign a futures contract to sell at t_2 a security that claims a t_3 cash flow of £1000. The contract price for t_2 will be $£1000/(1+ {}_2f_3) = £1000/(1.05) = £952.38$. Should ${}_2f_3$ change at any time, increasing or decreasing the NPV of your investment by changing the present value of its £1000 cash inflow at t_3, your futures contract's present value will also change in the opposite direction, in exactly the same amount. You are thus hedged against the risk of a change in ${}_2f_3$ upon your investment NPV.

Case Study 1.2: A Multiple-Period Resource Reallocation

1 **This case refers to all the material in Module 1, Section 1.4.4.**

The basic information in this case is that initially you are to deal with a time-defined resource endowment of £12 000 now, £13 000 at the next time point, £14 000 at the following, and £15 000 at the final. The market rate for borrowing and lending is a constant 8 per cent per period.

Present wealth is:

$$PW = £12000 + \frac{£13000}{(1.08)} + \frac{£14000}{(1.08)^2} + \frac{£15000}{(1.08)^3} = £47947.26$$

2 The current prices at which you would be able to sell each of your expected future cash flows are:

$$PV(\mathrm{CF}_1) = \frac{£13000}{(1.08)} = £12037.04$$

$$PV(\mathrm{CF}_2) = \frac{£14000}{(1.08)^2} = £12002.74$$

$$PV(\mathrm{CF}_3) = \frac{£15\,000}{(1.08)^3} = £11\,907.48$$

Naturally these, along with the £12 000 present cash flow, sum to equal the total present value calculated in Question **1**.

3 Given the expectations that exist now, you would expect to be able to sell your t_2 cash flow at t_1 for:

$$\mathrm{V1}(\mathrm{CF}_2) = \frac{£14\,000}{(1.08)} = £12\,962.96$$

The t_2 cash flow must be discounted one period to t_1.

4 Similarly, the value currently expected for the t_3 cash flow at t_1 is:

$$\mathrm{V1}(\mathrm{CF}_3) = \frac{£15\,000}{(1.08)^2} = £12\,860.08$$

The t_3 cash flow must be discounted two periods to t_1.

5 The solution to this question, of how much you can consume as a maximum constant amount at each time point, requires an annuity calculation. But instead of finding the present value of the annuity, we must find the annuity cash-flow amounts that produce an already known present value. The reason that the solution is structured in this way is that we know the present wealth to be allocated across the future but we do not know the constant consumption amounts that would discount to equal that present value.

There will be four equal consumption amounts, one now and one at each of the next three time points. Taking 'CC' to stand for the constant consumption amount at each time point, the amount of present wealth to be allocated across the **future** is equal to total present wealth minus present-time-point consumption (PW − CC). If we divide that amount by the annuity present-value factor for the correct number of periods (three) and the correct discount rate (8 per cent), the result will be CC, the cash flow per period that will produce the known present value of future consumption. That amount is given thus:

$$\begin{aligned} CC &= \frac{PW - CC}{\text{Annuity factor}} \\ &= \frac{PW}{1 + \text{Annuity factor}} \\ &= \frac{£47\,947.26}{1 + 2.5771} \\ &= £13\,403.95 \end{aligned}$$

With an initial set of cash-flow expectations as given, and a constant discount rate of 8 per cent per period, you can consume £13 403.95 per period.

There are an infinite number of ways that you can arrive at that consumption pattern by borrowing and lending in the financial market. We present one such below, but any of the others would also be correct. One can be reasonably certain of a correct solution if the result is the constant consumption amount at each time point with no residue at the end.

	t_0	t_1	t_2	t_3
Initial resources	£12 000.00	£13 000.00	£14 000.00	£15 000.00
Borrowing	£1 403.95	£516.27		
Interest paid		£112.32	£153.62	£118.22
Repayment of loan			£442.43	£1 477.83
Lending				
Interest received				
Net (consumption)	£13 403.95	£13 403.95	£13 403.95	£13 403.95

The amounts of borrowing shown are the **new** borrowings at each time point, whereas the interest amounts are calculated on the basis of the **total** borrowings outstanding for each period. For example, at t_2 the interest is calculated on a borrowed amount of £1920.22, which is the sum of the two borrowings at t_0 and t_1 (£1403.95 + £516.27). The slight discrepancy at t_3 is due to rounding at earlier time points.

Note that this solution has no lending. There is nothing that would have prevented you from borrowing more than was necessary at any time point (except t_3) and lending the extra. There would have been no wealth effect of that, since the borrowing and lending rates are the same and there are no costs incurred on a 'per transaction' basis.

6 In this question we have changed the market interest rates to a one-period spot rate of 6 per cent, a two-period spot rate of 8 per cent, and a three-period spot rate of 9 per cent. Otherwise the situation is the same. Your present wealth is now:

$$PW = £12\,000 + \frac{£13\,000}{(1.06)} + \frac{£14\,000}{(1.08)^2} + \frac{£15\,000}{(1.09)^3} = £47\,849.64$$

Your wealth with the constant 8 per cent market rate was £47 947.26, and so the change in market interest rates has caused your wealth to decrease marginally.

7 The separate cash-flow present values under the two interest rate regimes are:

	8% constant	**6%, 8%, 9%**
$PV(CF_0)$	£12 000.00	£12 000.00
$PV(CF_1)$	£12 037.04	£12 264.15
$PV(CF_2)$	£12 002.74	£12 002.74
$PV(CF_3)$	£11 907.48	£11 582.75

Note that in **present-value terms** the t_1 cash flow has increased, the t_2 cash flow has stayed the same, and the t_3 cash flow has decreased. The straightforward reason is that the opportunity costs of acquiring those amounts (the market interest rates) have changed in exactly the opposite directions as the cash-flow present values. (The amounts necessary to invest at t_0 in order to end up with the initial resource amounts have changed because market interest rates have changed.) One of the present values has changed in the same direction as the overall wealth change, one in the opposite direction, and the third has not changed at all. The net effect of those three changes is the overall wealth decrease.

8 The correct answer to this question depends upon an interpretation of exactly what is being asked. If the question is asking whether you can consume exactly the **amounts** that appeared in the answer to Question 5, the response must be 'No, you cannot.' The easiest way to discover this is to find the present value of the preferred consumption pattern and see whether your present wealth under the new interest rate regime is as large. If present wealth is as large as the present value of the consumption pattern, there will be some set of financial market transactions using your initial resources that will allow you to reach the desired pattern:

$$\begin{aligned} \mathrm{PV(CC)} &= £13\,403.95 + \frac{£13\,403.95}{(1.06)} + \frac{£13\,403.95}{(1.08)^2} + \frac{£13\,403.95}{(1.09)^3} \\ &= £47\,891.22 \end{aligned}$$

The present value of the consumption pattern is £47 891.22, whereas your present wealth is £47 849.64. This means that you cannot consume £13 403.95 now and at each of the next three time points with the altered interest rates. If you do not believe this, try to find a set of borrowing and lending transactions with your resources that would produce that consumption pattern.

On the other hand, if the question is interpreted to be asking whether you can consume a maximum constant amount per period under the new interest rate structure, the answer is of course that you can. Be cautioned, however, that the exact constant amount that can be consumed at each time point is more difficult to calculate than it was when interest rates were constant across the future, because annuity tables are not usable unless rates are constant. The most straightforward means of arriving at an answer would probably be some type of search process.

9 Yes, you would accept the investment (using the NPV criterion) under the constant 8 per cent rate structure:

$$\mathrm{NPV} = -£5060 + \frac{£1500}{(1.08)} + \frac{£2000}{(1.08)^2} + \frac{£2480}{(1.08)^3} = +£12.27$$

10 No, under the non-constant rate structure the NPV is negative:

$$\mathrm{NPV} = -£5060 + \frac{£1500}{(1.06)} + \frac{£2000}{(1.08)^2} + \frac{£2480}{(1.09)^3} = -£15.21$$

11 Reversing the t_1 and t_3 cash flows of the investment causes its **relative** desirability under the two rate structures to change:

$$\begin{aligned} \mathrm{NPV} &= -£5060 + \frac{£2480}{(1.08)} + \frac{£2000}{(1.08)^2} + \frac{£1500}{(1.08)^3} = +£141.72 \\ \mathrm{NPV} &= -£5060 + \frac{£2480}{(1.06)} + \frac{£2000}{(1.08)^2} + \frac{£1500}{(1.09)^3} = +£152.58 \end{aligned}$$

The investment now is acceptable under the non-constant rate structure. As a matter of fact, the investment generates a greater NPV under this structure than it does under the constant-rate structure. The reason for this becomes apparent when we examine the interaction of the cash-flow pattern change with the alternative rate structures. The switch of t_1 and t_3 cash flows has substituted a higher for a lower cash flow nearer to the present, and vice versa. The effect of that change is to cause **both** NPVs to increase (as you can see by comparing the NPVs immediately above to those originally calculated). But notice also that the constant-rate structure has its highest discount rate at the period furthest into the future and its lowest at the period nearest the present. The original cash flows had the same pattern, which resulted in the largest discount on the highest cash flow. The reversal of cash flows caused the opposite association under the non-constant structure, namely the smallest discount on the highest cash flow. This is sufficient not only to cause the investment to be acceptable under this rate regime, but also to make it more desirable than it would be under the constant-rate regime.

Module 2

Review Questions

2.1 The correct answer is A. Financial managers should attempt to maximise the wealth of currently existing shareholders because that capital supplier group is unprotected by specific contractual performance dimensions, whereas all other entities with which the company transacts are protected. If the decision processes of managers were at odds with the wealth of shareholders, equity capital would be much more expensive or even impossible to raise.

In addition, the other interests mentioned in (C) (the poor, customers, the environment) are themselves protected in their transactions with the company, either by government or some other entity. Further, there should be little confidence that company managers could efficiently and fairly adjudicate among the conflicting interests of such groups with respect to the various allocations of company-generated wealth that is under the managers' control. Such decisions are probably best left to the elected officials of the country, who usually have taxation, wealth-transfer and other allocative powers, and are answerable to the groups through the election process.

2.2 The correct answer is C. The method of arriving at this result is to recognise that the market price of Perpetual Payments plc at any time is correctly considered to be either the discounted value of its entire set of future dividends or the present value of {the final share price at which it can be sold at some future time plus the interim dividends}. Thus if you are convinced that you can sell the shares for £210 a year hence, and given the growth perpetuity nature of the company's dividends, you may use the formula:

$$\begin{aligned}\text{Share price}_0 &= \frac{\text{Dividend}_1 + \text{Share price}_1}{(1+re)} \\ &= \frac{£10 + £210}{(1+0.10)} \\ &= £200\end{aligned}$$

So the price you would now be paying for the shares of Perpetual Payments plc is £200. To find the implicit growth rate, we have:

$$
\begin{aligned}
\text{Share price}_0 &= \frac{\text{Dividend}_1}{(re - g)} \\
g &= re - \frac{\text{Dividend}_1}{\text{Share price}_0} \\
&= 0.10 - \frac{£10}{£200} \\
&= 0.05
\end{aligned}
$$

It is also possible to solve this problem by using the price at t_1 (£210) and the dividend at t_2 (which would be £10 multiplied by 1.05, or £10.50). Either way, the implied growth rate is 5 per cent.

The lesson intended for this question is to reiterate that the market prices of capital claims are the discounted values of the amounts of cash that holders of those claims expect to receive in the future. It makes no difference whether these amounts of cash are paid by the company to the holder as a dividend or paid by another outside purchaser of the shares to the holder as share price. Such purchasers must, in the end, be looking to the eventual dividends paid by the company, as would the holder were the shares not sold.

2.3 The correct answer is D. Both II and III are correct. Indeed, II must be correct if III is; the source of financing for the project is irrelevant as long as the value and outlay amounts are as shown.

If the project is financed by a dividend reduction, old shareholders lose the amount of the dividend reduction (which is equal to the outlay for the project), but gain the full value increase, the net of which is NPV = £1 000 000. If the project is financed by new equity, old shareholders lose no dividend, but gain only £1 000 000 of the £3 000 000 value increase because the company would have necessarily given £2 000 000 in value to new shareholders for them to have contributed that amount for the project's outlay.

If the project is financed with debt, the same argument as for new equity applies, as would similar ones for any combination of dividend reduction, new equity or debt.

2.4 The correct answer is D. When attempting to interpret the information implicit in a company's price/earnings ratio, an analyst must be careful to recognise that the P/E ratio is a type of very crude valuation formula, which relates earnings (a proxy for the first period's cash flow) to the market price of the shares. Given that an accurate value formula must deal with size, timings and riskiness (through the discount rate) of all of the expected cash flows of the company, the P/E ratio can only give unambiguous information when all but one of these several factors that can differ among companies are held constant in the comparison. This is not the case when comparing a company's P/E ratio to an average, which would include companies with different risks, cash-flow timings and amounts expected for the future. Valuable comparisons are likely to be made amongst only very similar firms, usually within a single industry.

Module 3

Review Questions

3.1 The correct answer is D. Both I and II are correct, because the numbers recorded in the income statements of companies are in some cases not cash flows at all, or omit cash flows that actually occur, and can be mislocated in time with respect to when the cash flows are actually expected to occur. III is incorrect in that the accounting numbers are not 'wrong' (in the sense of being incorrect) but are merely developed and used for purposes other than the types of decisions comprehended by financial cash flows.

3.2 The correct answer is B. It is true that finance is concerned only with cash flows, but it is not true, as the other answers imply, that finance is unconcerned with keeping track of the amounts that are owed the company by customers. Answers (A) and (C) are obviously incorrect in this dimension. Answer (D) is correct in the implication that the cash flows across time will capture the actual or expected payments of cash by customers, but is incorrect in the assertion that owed amounts can therefore be ignored – amounts owed the company by customers are as much assets as machines and equipment, and must be managed as carefully.

3.3 The correct answer is B. Option II is correct because, consistent with our insistence upon 'cash flows, only cash flows, and all cash flows', the inclusion of the cash amounts taken in when the asset is eventually expected to be sold is the correct perspective from which to compare the information validity of depreciation expenses with financial cash flows. In other words, the cash flow generated upon the eventual disposal of the asset will in effect accomplish the same result as recording the deterioration of its usefulness across time by depreciation expenses, and record this more correctly in terms of the amounts and timings of associated cash flows.

Answer (A) is wrong because it implies that cash flows would not include those from the eventual sale of the asset, and answer (C) is wrong because it implies that the company can be unconcerned with the period-by-period value changes or deterioration in the asset. The company must be concerned about all of these things, but the free cash flows thereby expected to be generated are the things that will cause the capital claims of the company to have value.

3.4 The correct answer is D. Because we are concerned with the changes that will take place in the cash flows of the company were it to accept the given project, and since taxes are cash flows, we must account for the changes in the company's taxes that will be induced by the new product. If the new product will reduce the company's taxes because of negative accounting income, those cash-flow changes are legitimately included for the periods during which they are expected to occur. This, of course, implicitly assumes that the company has taxable income from other sources against which such losses can be written.

The other answers are incorrect because they either ignore the legitimate tax cash flows in question or they make unsupportable assertions about tax cash flows offsetting each other across time. Even if tax increases and decreases were identical and in opposite directions, the fact that they are expected to occur at different times would affect their present values, so as to produce little likelihood of exactly offsetting effects.

3.5 The correct answer is A. 'Free cash flow' is the amounts of cash that the company is expected to be able to pay to (or must raise from) capital suppliers in order to validate its plans for the future. It is neither necessarily the maximum that could be paid at any time (the company may plan to retain cash for investment purposes which could have been paid as a dividend to shareholders), nor is it simply the sum of all dividends, interest and principal amounts expected to be paid to capital suppliers (the company may, for example, both pay dividends and raise new share capital, whence only the net amount of those is 'free' in the financial sense).

'Free cash flow' is not strictly determined by any particular legal encumbrances on a company's cash flow, though such legal constraints may affect the free cash flow in any period. The correct interpretation of free cash flow is the residue after all plans and expectations of the company are considered. Some of these may be legally required, while others may be entirely voluntary.

Module 4

Review Questions

4.1 The WACC–NPV of the robot reel fabricator project is calculated as follows:

$$\mathrm{NPV}_0 = \sum_{t=0}^{n} \frac{\mathrm{FCF}_t^*}{(1+rv^*)^t}$$

where the WACC is:

$$rv^* = \frac{D}{V}(rd^*) + \frac{E}{V}(re)$$

and $rd^* = rd(1 - \text{Tax rate})$

As described in the question the appropriate cash flows are thus:

Weir Fishing Tackle plc

	Robot Reel Fabricator (£s)			
	Now	**Yr 1**	**Yr 2**	**Yr 3**
Customers		+10 000.00	+14 000.00	+9 000.00
Operations		−3 000.00	−6 000.00	−4 000.00
Assets	−7 000.00			+1 000.00
Government		−4 122.47	−5 095.84	−1 065.22
FCF*	−7 000.00	+2 877.53	+2 904.16	+4 934.78

and the WACC is:

$$rv^* = 0.25(0.12)(1 - 0.50) + 0.75(0.16) = 0.135$$

Therefore the project's net present value is:

$$\begin{aligned} \mathrm{NPV}_0 &= \sum_{t=0}^{n} \frac{\mathrm{FCF}_t^*}{(1 + rv^*)^t} \\ &= -£7000 + \frac{£2877.53}{(1.135)} + \frac{£2904.16}{(1.135)^2} + \frac{£4934.78}{(1.135)^3} \\ &= +£1165 (\text{rounded to the nearest } £) \end{aligned}$$

As the NPV of the project is positive, the recommendation must be to proceed.

4.2 The correct answer is B. It is not correct to include the tax reductions caused by interest deductibility induced by the project's financing in the cash flows for a WACC–NPV calculation. The reason is that the debt rate used, rd^*, is **already** reduced to reflect the effect of interest deductibility. To include that effect in the cash flows again would be double counting.

Further, the implication of answer (C) that we can arrive at the same answers as WACC–NPV by including the interest tax shields in the cash flows but not the discount rates is correct (see the text), but the technique then would not be WACC–NPV.

4.3 The correct answer is C. Answers (A) and (B) are not really incorrect, but tell us less about the economic content of the WACC than about its function. Answer (D) is simply wrong; it is a description of the APV technique.

4.4 The correct answer is D. All three descriptions of the NPV are correct, and address the importance of this measure from various perspectives.

4.5 The APV of the project is found by:

$$\mathrm{APV}_0 = \sum_{t=0}^{n} \left[\frac{\mathrm{FCF}_t^*}{(1 + ru)^t} + \frac{\mathrm{ITS}_t}{(1 + rd)} \right]$$

The necessary data are all either given in the question (*ru*) or are available from the answer to Question 4.1 (FCF* and *rd*), excepting the interest tax shields. To find these, we merely multiply the interest expectations by the tax rate (50 per cent):

	(£s)			
	Now	**Yr 1**	**Yr 2**	**Yr 3**
Interest	–	244.94	191.68	130.44
Interest tax shield	–	122.47	95.84	65.22

Thence we solve for the APV as indicated in the formula:

$$\begin{aligned}\text{APV} &= -£7000 + \frac{£2877.53}{(1.1507)} + \frac{£2904.16}{(1.1507)^2} + \frac{£4934.78}{(1.1507)^3} \\ &\quad + \frac{£122.47}{(1.12)} + \frac{£95.84}{(1.12)^2} + \frac{£65.22}{(1.12)^3} \\ &= £933 + £232 \\ &= £1165\end{aligned}$$

Note that the APV in amount is identical to the NPV. The two techniques produce the same result, but by different methods of calculation.

4.6 The correct answer is A. The APV technique can in concept include other financing effects by simply listing the cash flows and discounting them at appropriate rates. The NPV formulation makes it difficult to do so within the WACC. Answer (C) is the virtue of the NPV, not the APV, and answer (D) actually pertains to those who adhere to the IRR.

Module 5

Review Questions

Case Study 5.1: PC Problems plc

Topics for review and action

1 In beginning a cash-flow estimation for an investment proposal, it is valuable to remember the method by which the analysis is to proceed so as to undertake steps in a reasonable order and avoid unnecessary or confusing estimation processes. One such confusion can occur if we are forced to interrupt one procedure to do another in order to find a number necessary to the analysis at that point. For example, at some point taxes will have to be calculated, and that will require depreciation expenses for the project to be estimated. If we had not already calculated the depreciation expenses, our process would be interrupted to do so. To avoid that, it is a good idea to get all such preliminary work out of the way first:

The depreciation expenses for the depreciable assets are (£s):

	t_1	t_2	t_3	t_4	t_5
Vans	15 000	15 000	15 000	0	9 000
Phones	1 000	1 000	1 000	1 000	0
Total depreciation	16 000	16 000	16 000	1 000	9 000

The depreciation schedule includes the 'write-off' of the vans, which is allowed upon their sale at the end of year five.

At this point we are ready to take the next step, which is to calculate the changes in taxes that PC Problems will experience if it accepts the project. To do so we must calculate the changes in taxable income implied by the project's acceptance (in £s):

	t_1	t_2	t_3	t_4	t_5
Sales revenue	75 000	150 000	250 000	275 000	302 500
Other revenue					15 000
Total revenue	75 000	150 000	250 000	275 000	317 500
Direct expense:					
Labour	20 000	40 000	46 000	52 900	60 835
Other	11 250	22 500	37 500	41 250	45 375
Management	10 000	11 000	12 100	13 310	14 641
Marketing	15 000	10 000	10 000	10 000	10 000
Total direct expense	56 250	83 500	105 600	117 460	130 851
Depreciation	16 000	16 000	16 000	1 000	9 000
Total expenses	72 250	99 500	121 600	118 460	139 851
Profit before tax	2 750	50 500	128 400	156 540	177 649
Taxes (52%)	1 430	26 260	66 768	81 401	92 377

There are several interpretations of the original information provided that may cause your answers to differ from those above. For example, note that we have included the revenue from selling the vans at t_5, and that we have included the new manager's compensation as opposed to that of the person who will be working on this project (£10 000 instead of £15 000, at t_1). The reason for the latter is that £10 000 is the **change** that will take place in the company's cash flows were the project to be accepted.

One interpretation of the data not subject to debate is that the expenditure for the market survey is irrelevant to the issue of the cash flows of this project. The market survey is a classic 'sunk cost'. No decision taken by the company at this point can have an effect upon the cash flow associated with the market survey, so the market survey cash flow can be ignored.

Further, note that we have ignored the overhead allocation of managerial compensation in general to the project, because the information did not indicate that any other compensation would change as a result of the project. Note also that we have not charged the project's profits with the interest that PC Problems intends to pay. The reason is that the WACC–NPV technique specifically adjusts for such interest and its tax effects in the discount rate used.

The final calculations necessary before we can deduce the relevant cash flows of the on-site service proposal are those necessary to recognise the investments that the company is making in what is called 'working capital'. You recall from the module that the easiest way to recognise these for tax purposes is to find the increase or decrease each year in the company's net working capital accounts. This can be used as an adjustment to the accounting profit figures to produce a set of cash-flow estimates. When done, all of the cash-flow effects of changing inventory, cash on hand, accounts receivable and accounts payable will be included. Using the information provided, the **levels** of the accounts are (£s):

	t_0	t_1	t_2	t_3	t_4	t_5
Accounts receivable	0	15 000	30 000	50 000	55 000	0
Accounts payable	0	7 500	15 000	25 000	27 500	0
Cash and inventories	7 500	15 000	25 000	27 500	30 250	0

The indications at t_5 that the amounts are zero is consistent with the liquidation potential mentioned in the information given initially. Though the company may not be seriously intending to liquidate the project at that point, its reluctance to deal with estimates beyond year five would require you to estimate present values of remaining net assets at that point, and this process is as good as any. Failure to 'liquidate' in this or similar fashion would bias the analysis, and ignore value that is expected to exist. Note that we are specifically assuming that the company will not 'work down' the inventory during the year, but will sell the intact inventory at the end of the year; thus instantaneously before the liquidation the cash and inventory is £30 250 at t_5. (Receivables and payables are higher than their t_4 amounts at t_5, being £60 500 and £30 250 respectively.)

Thus the **changes** in the accounts from the prior year, and thus the changes in net working capital (remembering that changes in accounts payable are subtracted from the changes in receivables, cash and inventories) are (£s):

	t_0	t_1	t_2	t_3	t_4	t_5
Accounts receivable	0	+15 000	+15 000	+20 000	+5 000	−55 000
Accounts payable	0	+7 500	+7 500	+10 000	+2 500	−27 500
Cash and inventories	+7 500	+7 500	+10 000	+2 500	+2 750	−30 250
Change in net working capital	+7 500	+15 000	+17 500	+12 500	+5 250	−57 750

We are now ready to find the cash flows of the project:

	t_0	t_1	t_2	t_3	t_4	t_5
Sales revenue		+75 000	+150 000	+250 000	+275 000	+302 500
Total direct expense		−56 250	−83 500	−105 600	−117 460	−130 851
Change in net working capital	−7 500	−15 000	−17 500	−12 500	−5 250	+57 750
Assets	−58 000					+15 000
Taxes		−1 430	−26 260	−66 768	−81 401	−92 377
FCF*	−65 500	+2 320	+22 740	+65 132	+70 889	+152 022

2 The application of WACC–NPV to the on-site proposal requires that the appropriate cost of capital for the project is estimated. The information in the question indicates that the following is the calculation necessary:

$$\begin{aligned} rv^* &= (0.25)(0.12)(1-0.52)+(0.75)(0.20) \\ &= 16.44\% \end{aligned}$$

This rate is the market-value weighted average of the after-tax rates for debt and equity (with debt's rate adjusted for the deductibility of interest). Application of this rate to the free cash flows derived in Question **1**'s answer produces:

$$\begin{aligned} \text{NPV} &= -£65500+\frac{£2320}{(1.1644)}+\frac{£22740}{(1.1644)^2}+\frac{£65132}{(1.1644)^3} \\ &\quad +\frac{£70889}{(1.1644)^4}+\frac{£152022}{(1.1644)^5} \\ &= +£104106(\text{rounded}) \end{aligned}$$

The on-site project seems well worthwhile, with a large positive NPV.

3 The intention of PC Problems plc to borrow £58 000 at t_0 in order to buy vans and telephones is not exactly consistent with the company's policy of maintaining a 25 per cent proportion of debt in its capital structure. This can be deduced by seeing that the cash flows above imply that the project would increase the market value of the company's capital as follows.

$$\begin{aligned} \text{Increase in market value of all capital} &= \text{NPV} - \text{FCF}_0^* \\ &= £169606 \end{aligned}$$

Of this amount, 25 per cent is £42 402 (rounded). In other words, the company must borrow only this amount – and not £58 000 – if capital structure policy is to be maintained. If the higher amount is borrowed, not only will the company be borrowing more than its policy permits, but the weights used in the WACC calculation, and the rates used (especially for shareholders) therein, are probably wrong. You should advise the company of this.

The technique suited for the use of such borrowing information is the APV method described in **Module 4**. APV applied to this example would not necessarily produce exactly the same answer as the above NPV because the implied debt amounts are different, as above. For the same answers to be produced, one obvious solution is to abandon the borrowing plan described and use one that implies borrowing £42 402 at t_0, reducing that borrowing as the project runs its course and always maintaining the 25 per cent market-value debt ratio.

4 The appropriate response to this question is that such erosion in related products should indeed be included as a reduction to the effect upon the company's revenues of accepting the proposal, but only if such a reduction would not have been expected to occur because of competitive conditions in the service market regardless of the introduction of the on-site service by PC Problems plc.

Module 6

Review Questions

6.1 The correct answer is D. Payback is not generally a preferred technique because it ignores the discounting process and the cash flows beyond the payback point. But under a few limited instances (constant cash flows and equal-risk projects, for example), a company could design a payback criterion that produced correct answers.

6.2 The correct answer is D. AROI cannot generally or even specifically be made to produce correct answers to investment analyses (i.e. those identical to NPVs). The problem with AROI is that it relies upon the accounting profit numbers developed for purposes entirely different from cash-flow estimation. Further, AROI being an average of profit rates upon depreciated investment across time, does not duplicate the discounting process whereby financial markets set values upon capital claims, and therefore does not give a good measure of the effects of shareholder wealth that are caused by company investment activities.

6.3 The NPVs and IRRs of the investments are thus:

	NPV	IRR
Investment 1	£107	12%
Investment 2	£89	13%

The correct choice is Investment 1 because it would increase shareholders' wealth by £107 whereas Investment 2 would increase their wealth by only £89, even though Investment 2 has a higher per-period average earnings rate on the funds invested.

6.4 If there is a requirement that IRR should be used as the investment-evaluation technique, it is necessary to do an 'incremental cash flow' analysis of the two alternatives in order to arrive at a correct result. First, the incremental cash flows are calculated by choosing Investment 1 as the 'defender' because its undiscounted cash flows are the most attractive:

	t_0	t_1	t_2
Investment 1	−£1500	+£180	+£1680
Investment 2	−£1000	+£130	+£1130
Incremental investment	−£500	+£50	+£550

Next, by trial and error, we discover that the IRR of these incremental cash flows is 10 per cent. We compare this rate to the appropriate discount rate for the projects, which is 8 per cent. The incremental IRR exceeds the rv^* for the projects, so the 'defender' or Investment 1 is the choice, since its IRR also exceeds the hurdle rate of 8 per cent.

The explanation of this technique's capacity to uncover the correct investment alternative is that, by using the incremental cash flows, it finds the IRR of the **extra** £s invested in the defender given an investment in the challenger. If the IRR of the extra investment in the defender is larger than the discount rate, the extra investment must be worthwhile relative to investing only in the challenger. In order to undertake this extra investment, we reject the challenger altogether, and by accepting the defender we get the cash flows of both the challenger and the incremental or extra investment, which together are of course simply the cash flows of the defender.

6.5 The correct answer is B. The cost–benefit ratio does in fact have almost the same drawbacks as the IRR in choosing among investment alternatives, because a ratio suffers the same limitations as a rate of return when comparisons as to wealth implications are required.

6.6 The correct answer is B. Everything said in the answer to Question 6.5 about the cost–benefit ratio can also be said about the profitability index.

6.7 The correct answer is D. The other criteria, as above, do not necessarily order projects correctly (i.e. put them in the order of their wealth-increasing potential), and so they cannot generally be depended upon to produce correct orderings of projects to be considered. Further, even if they did, accepting projects in that order under a budget constraint could result in a total package or portfolio of projects being accepted that was not optimal in the sense of being the one with the greatest **total** NPV within the budget constraint (see the illustrative example within the main text of **Module 6**).

The only entirely correct prescription is to examine all possible combinations of projects that are within the budget constraint and choose the combination with the highest NPV. There are several programming software packages available that can help to make that search more efficient.

6.8 The correct answer is that Happy Pooch can choose either {S and F} or {S, H and F} and expect to get the same NPV. Both of these combinations have NPVs of £165 000 and none other has as high. The display of project combinations and their interactions are (in £000s):

	Outlay	PV of future FCF*	NPV
S	140	210	70
H	25	55	30
F	210	305	95
S + H	180	295	115
H + F	220	360	140
S + F	350	515	165
S + H + F	435	600	165

6.9 The correct answer is C. Icky should purchase the Ugh because the present value of the future costs of the Ugh is less than the Echh, considering future replacements for both. The Echh has a single-cycle NPV of −£84 824 (its three future operating outlays discounted at 12 per cent and added to its cost), whilst the Ugh's single-cycle NPV is −£98 496. But the equivalent annual cost of the two, found by dividing their single-cycle NPVs by the annuity present-value factor for the discount rate and number of periods appropriate to each, is:

$$\text{Echh} = \frac{-£84824}{2.402} = -£35314$$

$$\text{Ugh} = \frac{-£98496}{3.037} = -£32432$$

Thus the Ugh is the preferred aerator. The equivalent annual cost has correctly captured the effect on the present-value costs of the machines of their replacement cycles, their relative initial costs, and their relative annual operating costs.

6.10 The correct answer is D. The cardinal rule for treatment of inflation is to discount inflated cash flows with inflated discount rates, and real (uninflated) cash flows with real discount rates. Further, the approach of dealing with inflation by using inflated cash flows and actual market-based discount rates (which are already inflated) is the preferred method because it does not require that company financial managers estimate what the market is expecting the effect of future inflation to be. The latter would be necessary if one were to attempt to deal with 'real' discount rates and cash flows to arrive at correct NPVs.

The implication of answers (B) and (C) that this approach is either wrong or no better than the real discounting process is wrong. For the detailed discussions of why, see the text of the module.

Note that answer (A) is also not correct, it being an unqualified approval of the existing technique. The cautions voiced in answer (D) are appropriate in that they recognise that the technique as described did not ensure that cash-flow estimates coming from operating parts of the company were non-inflated to begin with. (If they included inflation, and if the finance department again increased them for inflation, they would be double-counting inflation.) The additional suggestion that operating units may well be better inflation estimators in their own cash flows than the finance department also seems valid.

Module 7

Review Questions

7.1 The correct answer is D. Standard deviation is indeed an appropriate risk measure for the **entire portfolio of assets** held by someone. But when an individual asset is added to or removed from that portfolio, the effect upon the portfolio's risk – and thus upon the risk felt by the individual – is only the asset's systematic or undiversifiable risk. All of the remainder of the standard deviation of the security's return disappears through the phenomenon of diversification in the portfolio.

There is also some truth to the answer in (C). Some legitimate reservations about the nature of the testing being done upon the 'market model' have been voiced by responsible investigators. But this is not the main reason for the result mentioned in the question.

7.2 The correct answer is A. You would consider investing only in securities X and Y. The reason can be seen by using the SML as represented by Equation 7.1:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$

where rf=0.10 and [E(r_m)-rf]=0.08, as given, for each of the β coefficients of the securities. The resulting returns indicate what investments of comparable systematic risk would be expected to return in the market. Applying this formula to each security, we have:

	β	Required return	Expected return
W	0.8	16.4%	15%
X	1.0	18.0%	18%
Y	1.1	18.8%	19%
Z	1.5	22.0%	21%

Only in the cases of securities X and Y are the returns that the securities expected to provide as high as those generally available in the market for investments of comparable systematic risk (listed as 'Required return' and calculated with the SML formula).

No reasonable investor would have chosen answers (B) or (D), but someone who did not diversify (for some perhaps irrational reason) could choose answer (C), which implies holding only a single security. This of course is not a correct answer when you are already holding a well diversified portfolio, as implied by the question.

7.3 The correct answer is C. As was argued in the Learning Summary of **Module 7** (*see* Learning Summary, the market will set prices on securities under the assumption that the shareholders have invested in well diversified portfolios. Thus any corporate-level diversification of the type proposed by Anglo-Auto plc is destined to be at best useless and at worst costly in the sense of transactions costs paid by Anglo-Auto to acquire the mine. If that type of investment was desirable for shareholders, they could have simply bought shares in a gold mine (and may well have already done so, in which case Anglo's actions could cause them to incur costly portfolio rebalancing transactions to get rid of their gold stocks).

Answer (D) is incorrect because, even though shareholders may be less than well diversified, the market will price the shares as if they were, and thus there would be no price increase in reaction to the acquisition.

Finally, remember that the investment was expected to be of no operational use to Anglo-Auto. This is a necessity for the above arguments to be correct. Were there to be any real economic interaction between the companies' cash flows (e.g. if Anglo-Auto used gold in the cars, and thus avoided the gold market by buying the mine, or if the gold mine used cars and it was cheaper for the merged firm to get cars to the gold mine than to acquire them on the market, minus any profits for Anglo-Auto), then the merger may or may not be profitable for Anglo-Auto, depending upon the price paid for the mine and the economic gains to be made. An analysis of that question would involve treating the acquisition as any other proposed investment by Anglo-Auto, and evaluating it with a WACC based upon the systematic risks of the implied free cash flows of the project.

7.4 The correct answer is D. This question is designed to test your intuitive understanding of what diversification is, and when it is relevant. To answer the question, you must posit that auto and steel companies are likely to react very positively to overall economic conditions (good when the economy is good and vice versa), pawnbrokers very negatively, and bakeries not much at all. Thus General Automobile Manufacturers and General Steel Company would likely have correlations with overall economic activity that are positive and strong. General Pawnbrokers plc, on the other hand is likely to have a negative correlation with overall economic activity. Thus combinations of these companies would provide large diversification effects.

The argument in answer (B) concerns real economic interactions between merging companies, not those effects that would accrue in portfolio combinations; further, the effects listed are not 'diversification' in the sense of stochastic risk reduction. Answer (C) is wrong because diversification effects, though likely to exist, would be less between General Bakeries and **any** of the other companies, because the bakery has a lower overall volatility with respect to economic conditions.

7.5 The β coefficient is the slope of the line of best fit in Figure 7.10 for each security. Thus the highest-sloped lines are the highest β coefficients, and vice versa. Specifically, the ranking by β coefficient is:

- Security A
- Security C
- Security D
- Security B.

The total risk of a security can be visualised as the spread of dots with respect to the vertical axis (the rate of return for the individual security). Thus the securities with the greatest spread of dots viewed from the vertical axis are those with the highest total risk. Specifically, the ranking by total risk is:

- Security B

- Security C
- Security A
- Security D.

7.6 The transformation of total risk into systematic risk in Figure 7.10 can be visualised as the dots themselves disappearing (being diversified away), with the only remaining uncertainty being that associated with the securities' relationship with the market, as represented by the slope of the line of best fit, or the β coefficient. Thus as the overall market return varies along the horizontal axis, securities' returns are varying along the vertical axis as dictated by the line of best fit. This reduction of risk from total to systematic is contingent upon these securities being held in well diversified portfolios, for otherwise the 'off-line' dots would still be present and the market would not fully explain the risk of the securities.

7.7 The correct manner in which to analyse this situation is to separate the two different-risk components of the project. For example, the successful development would imply market value of the investment as of three years hence of:

$$\text{Value}_3 = \frac{£3000000}{0.10} - £5000000 = £25000000$$

(which is exactly what the unnamed French company would be willing to pay). But the odds are only 25 per cent that such an outcome will occur, creating an expected NPV at that point of 0.25 × $Value_3$ or £6 250 000. This, however, is only the expected NPV three years hence, and must be compared on a correct basis with the £3 000 000 present outlay for research. To be acceptable, the certainty equivalent of the risky £6 250 000 must be at least equal to $£3\,000\,000 \times (1.08)^3$, i.e. about £3 780 000 at that future time point. Alternatively, a risk-adjusted discount rate of about 27.7 per cent (sufficient to discount £6 250 000 three years hence to equal £3 000 000 now) is the maximum that would still generate a non-negative NPV for the project. The calculation of the 27.7 per cent rate is:

$$\text{Maximum discount} = (£6\,250\,000/£3\,000\,000)^{\frac{1}{3}} - 1 = 27.7\%$$

So one correct comparison is to decide whether exchanging a certain £3 780 000 for a 25 per cent chance of £25 000 000 and 75 per cent chance of nothing (an expected value of £6 250 000) three years hence is desirable. Another correct comparison is whether a 27.7 per cent three-period IRR is acceptable for the current outlay of £3 000 000 in this project, whose risk resolution, but not final cash flow, will be three years hence. There is not enough information in the problem to decide, but there is enough to see how the problem should be approached.

Note that we can for no purpose use the 25 per cent discount rate mentioned in the problem, for that is inclusive of all risks, including those that would continue to exist after the technical success of the project had been resolved. It would also obviously imply that the project is undesirable, given that the 25 per cent discount rate would be applied to all of the project's cash flows, even those after the major risk had been resolved.

Case Study 7.1: NOSE plc

1 There are several steps involved in arriving at Cyrano's WACC. The beginning point is the β coefficients for NOSE, which can be used to derive NOSE's ungeared β coefficient, thus:

$$\beta_u = \beta_e \frac{\mathrm{E}}{\mathrm{V}} + \beta_d \frac{\mathrm{D}}{\mathrm{V}}$$
$$= 1.2(0.65) + 0.20(0.35) = 0.85$$

Thus the ungeared β coefficient for NOSE is 0.85. This must be further adjusted for the difference in revenue volatility in Cyrano relative to NOSE:

$$\text{Revenue} - \text{adjusted}\beta = \beta_u \frac{\text{Revenue volatility of Cyrano}}{\text{Revenue volatility of NOSE}}$$
$$= 0.85(1.5) = 1.275$$

The revenue-adjusted β must then be adjusted by the relative fixed costs of Cyrano so as to arrive at Cyrano's ungeared β coefficient, thus:

$$\text{Project}\beta_u = \text{Revenue} - \text{adjusted}\beta \frac{(1 + \text{Project fixed cost\%})}{(1 + \text{Company fixed cost\%})}$$
$$= 1.275(1.4) = 1.785$$

Thus Cyrano's ungeared β coefficient is 1.785. But Cyrano will be in part financed with debt, so the project's ungeared β must be separated into its debt and equity components. With the information that NOSE can issue debt for Cyrano at a 15 per cent discount rate, and that market and risk-free returns are as indicated, we can deduce the beta coefficient for Cyrano's debt from the SML formula:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$
$$0.15 = 0.10 + (0.091)\beta_d$$
$$\beta_d = 0.55$$

With this information we can find the β coefficient for the equity of Cyrano:

$$\beta_u = \beta_e \frac{\mathrm{E}}{\mathrm{V}} + \beta_d \frac{\mathrm{D}}{\mathrm{V}}$$
$$1.785 = \beta_e(0.70) + 0.55(0.30)$$
$$\beta_e = [1.785 - 0.55(0.30)]/0.70$$
$$= 2.31$$

Thus Cyrano's equity β coefficient is 2.31, in contrast to NOSE's 1.2.

The next step in finding Cyrano's WACC is to calculate the re implied by its equity β coefficient and the SML:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$
$$\mathrm{E}(r_c) = 0.10 + (0.091)2.31$$
$$= 0.31 or 31\%$$

This required equity return is higher than that of NOSE plc as a whole, which we can deduce is given via the same Equation 7.1:

$$\mathrm{E}(r_j) = rf + [E(r_m) - rf]\beta_j$$
$$\mathrm{E}(r_c) = 0.10 + (0.091)1.2$$
$$= 0.21 or 21\%$$

Cyrano's equity required return (31 per cent) is substantially higher than NOSE's of 21 per cent. Note that this is caused by the revenue, fixed cost (operational gearing) and financial gearing differences between NOSE and Cyrano. The same SML relationships apply to each, however.

This now allows us finally to calculate Cyrano's WACC:

$$\begin{aligned} rv^* &= \frac{\mathrm{D}}{\mathrm{V}}(rd^*) + \frac{\mathrm{E}}{\mathrm{V}}(re) \\ &= 0.30[0.15(1-0.50)] + 0.70(0.31) \\ &= 0.24 or 24\% \end{aligned}$$

Thus Cyrano's free cash flows (FCF*) should be discounted at a WACC of 24 per cent per period. To contrast this with NOSE's WACC, we require one additional piece of information about NOSE, its rd. We know NOSE's β_d is 0.20, so it is a simple matter to find rd, again via our Equation 7.1:

$$\begin{aligned} \mathrm{E}(r_j) &= rf + [E(r_m) - rf]\beta_j \\ \mathrm{E}(r_d) &= 0.10 + (0.091)0.2 \\ &= 0.12 or 12\% \end{aligned}$$

Now we are ready to find NOSE's WACC:

$$\begin{aligned} rv^* &= \frac{\mathrm{D}}{\mathrm{V}}(rd^*) + \frac{\mathrm{E}}{\mathrm{V}}(re) \\ &= 0.35[0.12(1-0.50)] + 0.65(0.21) \\ &= 0.16 or 16\% \end{aligned}$$

Cyrano needs a 24 per cent WACC while the company undertaking the project requires only a 16 per cent WACC. Were NOSE to have evaluated Cyrano with the company's WACC, the project would appear to be more attractive than it is.

Module 8

Review Questions

8.1 The correct answer is B. The shares will not change in price, because the growth rate in the future dividend to the existing shareholders will decline by the exact amount necessary to offset the cash dividend increase. To see this, remember that ABT must now plan to raise £10 more per share in new equity each period, also growing at the 5 per cent per annum rate. These new shares will claim a steadily increasing portion of the future dividends of the company. The effect of this steadily increasing claim of new shareholders is to depress the growth rate of dividends for existing shareholders. Specifically, the £10 per share dividend increase in the first year is 10 per cent of the value of the equity of the company, and thus that percentage must now be raised from new shareholders. Therefore, 10 per cent of the next year's dividend will go to new shareholders. This is effectively a reduction of 10 per cent in the growth rate of the future dividend. This process continues *ad infinitum*, so that:

$$\begin{aligned}\text{Price per share} &= \frac{\text{Dividend}_1}{re - (g - 10\%)}\\ &= \frac{£20}{0.15 - (0.05 - 0.10)}\\ &= \frac{£20}{0.20}\\ &= £100\end{aligned}$$

Current shareholders have effectively traded lower growth rates in dividends for higher near-term cash dividends. They will find that they are progressively giving up their proportional claim on the company to whomsoever buys the new shares. Answer (C) is thus incorrect, because the company is not liquidating itself (it is still making the same investments that it always has, but now is raising the money from new shareholders instead of paying a lower dividend and retaining the cash); existing shareholders are liquidating their claim.

Answer (A) is incorrect because, although dividends are in fact doubled, the net rate of future dividend increase to the currently existing shares exactly offsets that in a value sense. Answer (D) is wrong because there are no taxes.

8.2 The correct answer is B. The best answer to this question is (B), even though there are elements of truth in all but (D).

Answer (B) is the most 'modern' depiction of the dividend irrelevancy argument because it would hold even in realistic markets, whereas the statements in answers (A) and (C) would be true only in the restrictive cases of frictionless markets, not really the most 'modern' explanation of irrelevancy. Note the distinction between '**dividends**' as cash flows (which are not at all irrelevant, and form the basis for share value), and **dividend policy** (i.e. how the residual cash is split between dividends and retention, which can be irrelevant).

Answer (D) is wrong because it ignores the arguments in answer (B). One should, of course, also mention that the issues raised by dividend irrelevancy arguments in this context also ignore questions of signalling, agency and so forth, which could argue against irrelevancy.

8.3 The correct answer is D. **The best answer is (D).**

One is usually at first attracted to the logic of answer (A), because it is correct that the company would probably be able to reduce its capital-raising costs by lowering its dividend payout and retaining more cash. But this argument ignores the fact that the policy the company has been pursuing has probably attracted a set of shareholders who prefer that policy. Shifting to a lower payout would not be likely to raise the share price, because that would require another group to pay a premium for the new pattern of dividends, and there are plenty of companies offering each possible pattern. The gain would be the transaction costs saved, and the loss would be the costs to shareholders of rearranging their portfolios to find a different company with a dividend policy they prefer. It is not at all clear that the net result would be positive.

The 'signalling' rationale is also reasonable.

8.4 The correct answer is A. By simply paying out the excess cash as a regular dividend, the company is in danger of giving the market a misimpression about future 'good news' – hence the 'extra dividend' (a very common practice in real markets). Retaining the cash to invest in government securities is only worthwhile if there are foreseeable future profitable real investment needs that will require the money, or if there are some unusual tax characteristics of the decision, neither of which are specified in the question. Finally, the retirement of debt is not on the face of it a desirable choice, there being no special reason to think that the company is presently using the wrong amount of debt capital. (There is a natural bias to think that it is always good to retire debt when one can. This is not at all true for companies. We explore this issue in Module 9 on capital structure.)

Module 9

Review Questions

9.1 The correct answer is B. The module has attempted to show that the choice between equity or borrowing can affect the wealth of shareholders (and bondholders) of companies. To the extent that managerial decision makers are concerned about the interests of these groups, the decision is important to the company. Answer (A) is incorrect because a company is never **forced** to borrow; it can instead issue shares, though perhaps at higher cost. Answer (C) is incorrect as a general principle, which is demonstrated in the several arguments in the module about the pros and cons of borrowing. Answer (D) is correct to some extent but, without the recognition that there are costs to low-risk capital structures (such as forgone tax benefits), the argument is incomplete.

9.2 The correct answer is D. None of the other answers is truly correct. Answer (A) is obviously wrong, because a company achieves minimum risk exposure simply by having no borrowing. There are often benefits to borrowing that offset to some extent the increased risk. Answer (B) is incorrect because the highest equity value is probably in an ungeared firm. In such a situation, equity value is high because shareholders have so much invested, not because there has been a good decision about capital structure. Though difficult to argue with as a homily, answer (C) is not destined to be useful as a decision criterion for financial managers.

If there is a positive but general answer to the question, it is as follows. 'The issue of risk from the perspective asked is irrelevant. Of course, increased borrowing causes increased risk. But if shareholders are concerned about such risk, it is easily neutralised (for example, by altering their personal portfolios to include fewer shares and more bonds of the company). Capital structure alterations of the type implied by the question serve simply to reallocate the operating risk of the company among capital claims. Shareholders can undo this at will, and so such concern is unwarranted (unless there would be high transactions costs in doing so).' The foregoing statement is simply another way of saying the same thing that M&M did in their first capital-structure 'irrelevancy' argument.

9.3 The correct answer is A. By purchasing all of the securities of either company, you will experience the full business risk of the tool industry because you will thereby claim all of the operating cash flows of the firm (and the companies have identical operating cash flows). The reason that Leverage is better than Leverageless is simply that the cost of Leverage's shares and bonds, £96 000 000, is less than Leverageless's £100 000 000 shares. Since either purchase would claim the same future cash flows, one naturally chooses the less expensive. With this information, all answers other than (A) are clearly wrong.

9.4 The correct answer is D. As Question 9.3 indicates, anyone interested in the business risk of the tool industry would do better by buying the bonds and shares of Leverage rather than just the shares of Leverageless. The same general aversion to Leverageless will also apply to anyone thinking of purchasing the shares of that company. It simply is offering nothing that cannot be more cheaply purchased by buying Leverage's capital claims. That being the situation, the price of Leverageless's shares must fall to be worth £96 000 000, the value of Leverage's claims.

Answer (A) is incorrect not because there are no such people with proclivities to obtain the risk and return of an ungeared firm, but because Leverageless is simply too expensive. Leverageless's risk can be obtained for £4 000 000 less by buying all of the bonds and shares of Leverage.

Answer (B) is incorrect because rarity is irrelevant in the sense used. The **risk and return** that a correctly priced Leverageless would offer is widely available, not only from Leverage as described above, but also from any number of other investments (other companies in other industries, real estate, artwork, and so forth) if correctly combined with personal borrowing or lending. The only thing rare about Leverageless, as far as the market is concerned, is that its price is wrong. Arbitrageurs will quickly ensure that this mispricing disappears.

Answer (C) is wrong because there is an incentive to cause the price of Leverageless to decline. For example, those who hold the shares of Leverageless could sell those shares for £100 000 000 and purchase the bonds and shares of Leverage for £96 000 000, obtaining the same future cash-flow expectations as with Leverageless but with £4 000 000 in their pockets.

The only interesting point implied by answer (C) is how the price of Leverageless got to be wrong in the first place. The answer to that question is, of course, that we designed it that way so as to make the question interesting. In other words, we arbitrarily caused the financial market to make a mistake, and expected you to spot it.

9.5 The correct answer is D. Option II is correct because we cannot be certain how to evaluate the 10 per cent IRR being predicted for the investment. If the company's assets in general are to be pledged as collateral for the loan (a straight debenture), the 8 per cent rate asked may be reflecting the risks of the company in general rather than the investment in particular. Comparing the investment's return with the interest on the debt used to 'finance' it is thus wrong, since the company itself – not the investment – is being 'financed'.

Option III is also correct, because the 8 per cent cost of borrowing ignores the effect of the borrowing upon the shares of the company. Even though the cash outlay for the investment is fully funded by the borrowing, that borrowing comprises the 'financing' of the investment only if the debt collateral is limited to the investment, **and** if the investment has no NPV. (With a positive NPV, part of the 'financing' is the equity value created by the investment's NPV, in the sense that the future cash flows of the project must cover the expectations of the creditors **and** the shareholders assigning the positive NPV and attendant share price increase.) With a straight debenture, the company's assets in general have collateralised the loan, and thus are at risk. The WACC can capture such considerations; the debt cost cannot. Option I is obviously wrong because it implies that the investment is financed solely by the borrowing.

9.6 The correct answer is C. Option II is correct because one of the important reasons that companies borrow is to reduce their taxes. If a company has weighed the pros and cons of borrowing and decided that there are net benefits to using debt rather than equity, but has ignored the chances that tax benefits through debt are merely replacing other deductions from income, the net benefit of borrowing may be incorrect.

Option III is correct because even when the company is not in violation of borrowing covenants, it may be constrained to forego profitable activities either involuntarily (e.g. because an indenture covenant precludes mergers, or constrains dividend payments) or voluntarily (e.g. because the positive NPV of a project would accrue to bondholders rather than shareholders). Furthermore, the company may be forced to undertake actions it would rather not – for example, maintaining more current liquid assets, or even liquidating operating assets to pay interest and principal.

Option V is correct because lenders will anticipate that, given the chance, the company will take any opportunity to shift wealth from bondholders to shareholders (by undertaking risky investments, by funnelling resources to shareholders and managers instead of bondholders, and so forth). These contingencies will be priced in advance by lenders, and cause borrowing costs to be higher than they would otherwise be. To some extent, such 'agency' costs may be avoided by activities such as writing constraining contracts that do not materially affect the optimal asset operations of the company and have low monitoring costs, or issuing complex securities such as convertible or callable bonds that reduce the costs or chances of such conflicts of interest.

Option I is wrong because bankruptcy in and of itself is merely the shifting of asset ownership from shareholders to bondholders. This is in fact a benefit, not a detriment, because in the absence of the capacity to declare bankruptcy, companies' shareholders, in their role as residual owners, would be forced to make good on the company's interest and principal promise. This also explains why option IV is incorrect. The only problem with bankruptcy is that there are likely to be both direct costs incurred in litigation, and indirect operational costs and inefficiencies associated with this process of shifting asset ownership.

9.7 The correct answer is D. Conglomerates engage in so many unrelated activities that it is difficult for a lender to know with any great certainty the mix of riskinesses that will appear in the company over time. If a company succeeds in convincing the lender that a particular amount of risk will be experienced, and then causes the actual risk to be higher, the lender's bond value will be lower and, given an unchanged total company value, shares will be more valuable. There will have been a wealth shift from bondholders to shareholders. Bondholders judge the likelihoods of this when they originally purchase bonds (thereby setting the required returns). Thus, for a conglomerate the costs of borrowing would be higher due to the lenders charging in advance for the wealth shift likelihood. Convertible securities do away with this charge, because the conversion characteristic causes any increases in share value to be reflected in the bond values. Wealth shifts are no longer a problem, lenders can charge lower interest rates, and the company has avoided the costs of potential conflicts of interest.

Answer (A) is incorrect because, though interest costs are indeed lower for convertible bonds, it is not the lower rate *per se* that causes the bonds to be desirable financing vehicles. Even if there were no potential conflicts of interest, if the conversion terms of the bond are set so as to be valuable, the bonds will carry a lower interest rate than non-convertible bonds. (For example, a company's shares are selling for £50 apiece, and a £1000 face-value bond is convertible into 25 shares; such a bond would obviously be worth more than a straight debenture of the same face value and coupon interest rate, even if the bond were issued by a firm with no chances to shift wealth to shareholders from bondholders.) The lower interest rate on this convertible, however, is 'paid for' in full by the shareholders when the bonds are exercised at advantageous conversion rates, and the new shareholders claim their pro rata portion of company cash flow and value.

Answer (B) is incorrect because convertible bonds are not at all necessarily 'delayed equity issuances'. As a matter of fact, there is no particular reason why bondholders – unless forced to do so – would ever choose to convert their bonds into shares. With convertible bonds, they maintain their higher-priority claim upon interest and principal payments, whilst at the same time enjoying the benefits of equity share-value increases.

Answer (C) is wrong because there is no such general relationship between maturities of convertible and non-convertible bonds.

9.8 The correct answer is A. From the perspective of both lenders and borrowers, it is important that the operational cash flows arising from a firm not be heavily disturbed by the existence of borrowing when the company comes upon hard times. With ships and other tangible assets as the primary basis of value for the company, Intercontinental Shipping and its creditors can feel secure that there will be few **additional** operational problems with the firm's assets should the shipping company or its industry come upon hard times. The ownership and collateralisation of the assets is clear, and there is a ready market in ships.

Most of Clever Advertising's value, on the other hand, likely derives from expectations of future cash flows generated by the skill of its copy writers, and 'brainstorming' of account executives. This is precious little collateral for a lender to look toward in bad times. It is likely that such resources would simply seek other employment rather than be operated or 'sold' by creditors should Clever turn out to be dull. Any adverse economic conditions would be thus exacerbated by the existence of borrowing.

Answer (B) is wrong because risk as a unique consideration (the variability of future cash flows) is not a final determinant of company capital-structure decisions. Shipping companies are, in fact, at least as risky in this sense as are advertising companies. The real test is: **given the risk**, is borrowing a further detriment? For Intercontinental Shipping, it is not; for Clever Advertising, it is.

Answer (C) is wrong because capital structure is probably **not** irrelevant. The world of the original M&M theory is not the world in which real companies operate. And Miller's arguments about real market irrelevancy, though interesting, were probably incomplete.

Answer (D) is wrong because there is nothing in the tax situations of the companies that would indicate that borrowing is better for Intercontinental Shipping than for Clever Advertising. As a matter of fact, with the capacity that tangible assets have for shielding income from taxes, Intercontinental's tax situation in terms of borrowing may be worse than Clever's.

Case Study 9.1: R-D Star Productions plc

1 Because this case concerns itself primarily with the debt versus equity decision, it will be worth our while to review some general precepts about optimal decision making in this context.

First, keeping in mind that we are seeking to make the shareholders of the company as wealthy as possible, we are in a sense seeking the least-cost type of financing for the facilities project. Though the case tells us the pre-tax cost of borrowing (in the form of an interest rate), that is essentially the only capital-cost information with which we are explicitly provided. Does this mean that we cannot estimate a cost of equity capital for comparison purposes? No, it does not. There is enough information in the case to allow someone concerned with the question of equity's cost to estimate this (by knowing the industry, looking up the average β coefficient in a suitable reference, and applying the CAPM, for example). But this begs the question of why we would wish to estimate the costs of debt and equity in the first place.

Admittedly, discovering the implied capital costs seems an obvious first step to offering a solution to the company's financing question. But a moment's reflection about our discussions of company capital structure in the module can lead us to a more efficient decision process. Recall that we found borrowing to be the lower after-tax capital cost alternative, as long as several tests of the company's operating results in the presence of borrowing were satisfactorily met. If these tests can be performed on R-D Star with reasonably unambiguous results, there is no need actually to calculate the costs of debt and equity capital. Remember, the question is not how much they cost (which could be of issue in the question of estimating a WACC for the purposes of making the decision to invest in the facilities project in the first place, which decision has already been made). The question is instead:

which of the two capital types is cheaper? If that can be answered without reference to the particular costs of each, there is no reason to pursue the latter issue.

The first test of debt's desirability is to see whether its tax benefits are likely to be realised. This in turn depends upon whether the company will have income subject to taxation. With acceptance of the facilities project, R-D Star is expected to have EBIT of £182 500 000 in 2009. Interest on the new borrowing is 14 per cent of £75 000 000, or £10 500 000. Thus the best guess is that the company will report taxable income of £139 800 000 in 2009 if it finances itself with debt. (This is the £182 500 000 minus the £10 500 000 of interest on the new debt and minus £32 200 000 interest on the £230 000 000 of old debt, assuming the same interest rate on old and new borrowing). Though there is significant variability in the company's past income stream, the odds that operating results would deteriorate sufficiently during the life of the loan to cause the tax benefits of interest deductability to disappear are low. This information, and the risk attributes of gearing as discussed in the module, are depicted in the EBIT-EPS chart of Figure A4.4. (The EPSs are after tax, assuming deductible interest, and based on either the 20 000 000 shares outstanding if debt is issued, or 23 571 429 shares if equity is issued.)

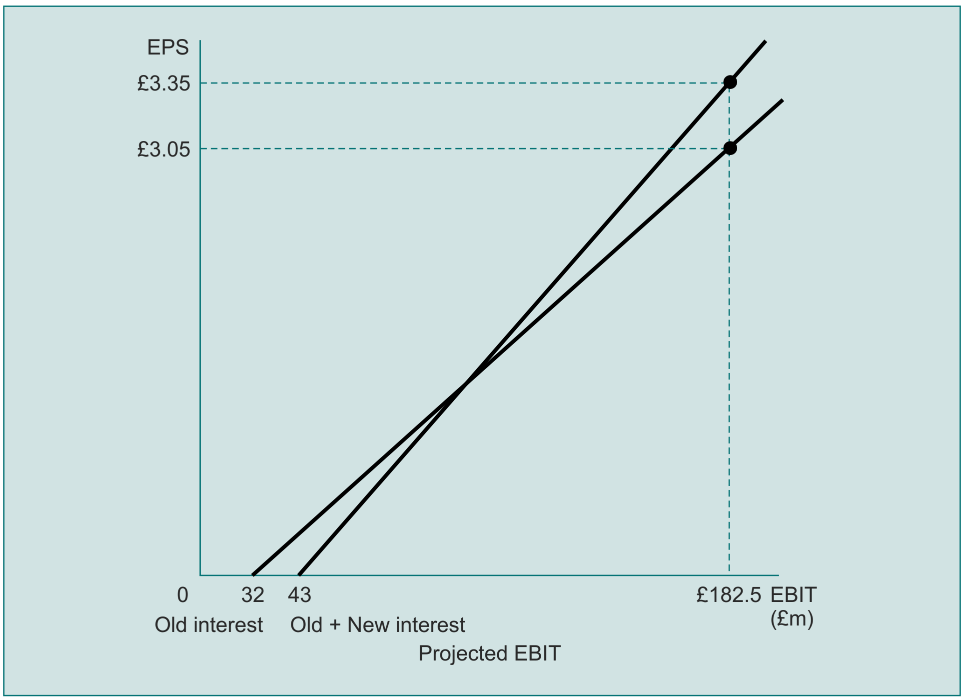

Figure A4.4 EBIT-EPS for R-D Star Productions plc

If debt is to be cheaper than equity on an after-tax basis, what is our hesitancy? Why do we not simply opt for borrowing and have done with it? The reason is that the question of being able to deduct interest is not the only difference between borrowing and issuing equity capital. Interest deductibility does indeed make borrowing more desirable than issuing shares, but borrowing can also have negative implications for the company and its shareholders. Having now decided that borrowing will likely carry tax benefits, we must see if there are any significant negative considerations in R-D Star's borrowing alternative.

What can go wrong when a company borrows? The most obvious danger in borrowing is that interest and principal payments may not be met, and the company may go into default from that breach of the debt contract. A glance at Figure A4.4 should be enough to convince you that there is little likelihood that R-D Star will not be able to meet its contractual payments to creditors. (Note also, however, that interest and principal on total borrowing – not merely on new debt – must be covered, and that the interest alone may be 14 per cent of £305 000 000, or £42 700 000.)

The more significant considerations are probably in the other covenants of the debt contract. Recall that there are constraints upon dividend payments, net working capital, and other borrowing that will apply to R-D Star if it enters into the debt contract.

Consider the first constraint. The company's dividend has been stable in absolute terms for the past few years, even in the face of declining and variable income. In the history presented, there has not been a reduction of dividend to R-D Star's shareholders. This is an indication that a forced reduction in dividends might be costly to the shareholder clientele of the company. With projected EBIT of £182 500 000, and interest at £42 700 000, after-tax profits are 48 per cent of the difference, or £67 104 000. Dividends are constrained to be no more than 16 per cent of this, or approximately £10 700 000. This is slightly above last year's figure, but not by much. It would take only a small shortfall in operating income to force R-D Star to cut its dividend to shareholders if borrowing is used.

Net working capital must be kept to at least £80 000 000. In 2008 the difference between Current assets and Current liabilities is only £72 700 000, but the facilities project will use £20 000 000 of its funding for an increase in net working capital, and so this constraint is not likely to be onerous.

The limited information available in the case makes it more difficult to judge the importance of the bond contract's constraint upon future borrowing. Such constraints, when binding, most often have the effect of forcing the company to use some other type of (more costly) financing for future projects. Whether this is a problem for R-D Star is not clear.

What has this review of the contractual constraints upon the company indicated about the desirability of borrowing? At this point it seems that the major problem is a capacity to maintain the company's dividend payment. Dividends, however, are only one of a number of things intended to be paid from R-D Star's EBIT. Remember that the company plans to use £63 000 000 of retained earnings as part of the internally generated funding for the facilities project. The other uses for EBIT comprise dividends, interest and principal. We can arrive at the expected claims upon projected EBIT by listing the plans of R-D Star, and the amounts of EBIT they would require. Remembering that taxes must be paid upon everything save interest, we have:

Interest:	
New borrowing	£10 500 000
Old borrowing	£32 200 000
Principal:	
New borrowing	£15 600 000
Old borrowing	£47 920 000
Facilities project:	
EBIT required	£131 250 000
Dividends:	£21 450 000
Total EBIT required:	£258 920 000

(These calculations assume that a 52 per cent tax must be paid upon non-deductible EBIT amounts in order to arrive at after-tax cash. For example, the £63 000 000 of facilities investment requires £63 000 000/0.48=£131250000 of EBIT. Principal payments and interest rates on old borrowing are assumed to be on the same terms as the new borrowing.)

The obvious conclusion is that R-D Star cannot hope to do all that it expects to do with a projected EBIT of only £182 500 000. The shortfall is nearly £80 000 000. If the company wishes to pursue the facilities project with borrowing, it will at the same time be forced to alter its expected usage of EBIT. This may require refunding the old debt issue (thereby effectively foregoing its principal payments this year), cutting its dividend, or some combination of the two.

Another valuable exercise in evaluating the financing choice is to see whether R-D Star will deviate significantly from the industry norms if it accepts the debt alternative. Taking all of these intentions of the company, we can construct a set of results for 2009:

R-D Star: 2009 Projected Results (£m)

Current assets	157.7
Total assets	700.7
Current liabilities	65.0
Long-term debt	274.5
EBIT	182.5
Dividends	10.3

(These figures assume that 10 per cent of old borrowing is paid off, and that the net increase in working capital is due to the addition of current assets, as opposed to the addition of both assets and liabilities.)

The projected results can be translated into a set of operating and financial ratios, which can be compared to the industry averages:

	Industry	R-D Star
Current assets/Current liabilities	2.00	2.42
EBIT/Total assets	0.26	0.26
Long-term debt/Total assets	0.35	0.40
Dividends/Profits after taxes	0.08	0.15

The facilities project financed with borrowing would cause R-D Star to be somewhat more liquid, about as profitable, more highly geared and to continue paying a higher rate of dividends than other comparable companies.

Actually, as we have seen, R-D Star cannot finance all it wishes to do in 2003 without changing some of the decisions implied by the above ratios. Re-examining the ratios in this light can be instructive as to the alternatives available to R-D Star. For example, with liquidity high, perhaps some current assets can be reduced, thus reducing financing needs and therefore borrowing necessities. Or R-D Star, with debt as projected being above-average in gearing, perhaps should abandon plans to borrow. On the other hand, dividends are higher than what comparable companies pay to shareholders, and thus these could perhaps be cut without major problems.

There are additional considerations that must also not be neglected. For example, it is not clear how important the restriction upon future borrowing is for R-D Star. If the company has additional expansion plans, it may be better to issue equity now so as to avoid the restrictions upon future borrowing; or perhaps it would be best to bargain with lenders to loosen somewhat the restriction on future borrowing in exchange for a higher interest rate. The proposal to negotiate new terms of the loan based upon the tangible value of R-D Star's inventory of old films may also be important.

What, then, is the 'solution' to this case? In truth, as with most interesting real-market problems, there is both no solution and an infinite number of solutions. Any plan that can be financed with expected EBIT and borrowing or equity is a 'solution' in the limited sense of allowing the company to carry out its expansion and probably survive. But a plan that aspires to optimality must balance the positive and negative effects of its financing choices. All things considered, in the current case many financial managers would choose to issue equity instead of borrowing. The reasons cited would be the high gearing ratio that would be produced by the debt issue, and the likely necessity to reduce dividends.

On the other hand, reasonable managers might also argue that borrowing is best, and the liquidity and dividend cutting is a good tactic to gain the tax benefits of debt. They would further argue that using more of EBIT than is available in 2009 is not significant, because that phenomenon is for one year only, and disappears in the year 2010 with no need for a large expansion claim on available cash. The 2009 shortfall can be easily covered by one year's rolling-over in principal payment on old borrowing, or by a short-term loan.

Finance managers might further argue that the benefits of the tangibility of R-D Star's film inventory is already included in the interest rate on the proposed debt issue, and though a lower rate could be achieved by a specific mortgage, there would be no net gain because other lenders would thereby be forced to charge the company higher rates because these lenders would now have less collateral for their loans.

As with most cases, there is not enough information provided in the case study to come down with great authority on one side or the other without making additional assumptions about the company or its environment. A large, planned, future expansion might argue for equity now and borrowing later. Concern about controlling interest in shareholdings might cause there to be reticence to issue equity.

Module 10

Review Questions

10.1 The correct answer is D. This is exactly the description that occurs in the text (*see* Section 10.2.3). Answer (C) is also a decent answer, but not as detailed as (D). Answer (A) is wrong because it mixes up the matching order of this technique, and answer (B) is wrong because it endows maturity matching with more power than it can justifiably claim.

10.2 The correct answer is B. When a company matches asset and financing terms on the basis of strategy I accounting definitions of term, it will finance 'current assets' with short-term finance and 'fixed assets' with long-term finance. Since current assets overstate the true short-term assets of a company due to the permanent nature of some short-term assets, this strategy will produce less long-term (low-risk) finance and more short-term (high-risk) finance than will strategy II, which uses the real categorisations, or III, which maximises long-term finance. Answers (A), (C) and (D) are inconsistent with (B), and are wrong.

10.3 The correct answer is A. Interest of £8400 per annum will be paid, based upon a £60 000 loan at 14 per cent interest. To find the correct effective interest rate, interest must be divided by the amount borrowed (on a one-year loan with repayment of principal at the end of the year) – the calculation is just that simple. The amount borrowed is net of the £8400 interest paid in advance and the 10 per cent (£6000) compensating balance, making a net borrowing of £60 000-£6000-£8400 = £45 600. Dividing that into the £8400 interest gives a rate of 18.4 per cent.

10.4 The correct answer is A. Substituting correctly in Equation 10.1 from the module (reproduced below) produces the combination shown in answer (A). If your answer was (B), you divided by 13 instead of 0.13 in the formula. If you answered (C) it was likely nothing but a wild guess. Since there is a correct answer, (D) is wrong.

$$\pounds r = [(2 \times \pounds D \times \pounds T)/i]^{1/2} \qquad \text{(A4.1)}$$

10.5 The correct answer is C. This is found by substituting correctly into Equation 10.2 from the module (reproduced below). Incorrect answers may have been generated by failing to square the £4000 change or failing to approximate the daily interest rate as 0.12/365. The other answers are wrong, and could have been arrived at by errors such as the above.

$$£R = [(3 \times £T \times s^2)/4i]^{\frac{1}{3}} + £M \qquad \text{(A4.2)}$$

10.6 The correct answer is A. You can review your understanding of this answer by referring to Section 10.3.3 of the module. If Daisy Publications undertakes the alteration in its credit policy as proposed, it will be better off by £0.78 (per day) in present-value terms. The calculation that produces this result is taken from the Hill *et al.* work mentioned in footnote 8 of the text of the module, and it proceeds as set out hereafter.

The reduction in sales will cost the firm £166.72 as the discounted value of receipts, net of bad debts. This is found by discounting the new daily sales of £1 575.34 × (1 − 0.03) at $(1.0003)^{40}$ and subtracting from that the original £1 780.82 × (1 − 0.04) discounted at $(1.0003)^{65}$. The remainder of the calculation is analogous in this respect with that of the text:

$$\begin{aligned}
\text{NPV} \quad = \quad & \text{Change in PV of sales receipts} \\
& -\text{Change in costs} \\
& -\text{Change in working capital investment} \\
= \quad & \frac{(£1\,575.34)(1-0.03)}{(1+0.0003)^{40}} - \frac{(£1\,780.82)(1-0.04)}{(1+0.0003)^{65}} \\
& -[0.8 \times (£1\,575.34 - £1\,780.82)] \\
& -\left[0.20 \times \left[£1\,575.34 - £1\,780.82 - \left(\frac{(£1\,575.34)}{(1+0.0003)^{40}} - \frac{(£1\,780.82)}{(1+0.0003)^{65}}\right)\right]\right] \\
= \quad & £0.78
\end{aligned}$$

10.7 The correct answer is A. The loan is much cheaper than the giving-up of discounts. As Section 10.3.4 of the module indicates, the annual interest rate equivalent of foregoing the discount is:

$$i_1 = \left(1 + \frac{0.02}{1-0.02}\right)^{\frac{365}{20}} - 1 = 44.58\%$$

The cost of the loan can be found by:

$$i_2 = \frac{0.16}{(1-0.16)} = 19.05\%$$

The reason that this gives us the cost of the loan is that the interest cost is 0.16 times the 'principal amount' of a discount loan, but the actual amount that the bank gives you at the beginning of the loan period is only the 'discounted' amount, or (1-0.16) = 0.84 times the 'principal amount'. Because you do not get the entire 'principal amount', you are paying interest at a rate higher than the stated rate, and the ratio [0.16/(1-0.16)] = 0.1905 displays the effective interest rate on the discount loan.

Thus the loan is clearly less expensive. Answer (C) is not entirely unreasonable but, since the loan is cheaper, it is debatable. Actually, simply giving up discounts is not nearly as bad as being a late (or non-) payer. Answer (D) is wrong because it forgets that payables are simply another form of borrowing, and this is well known to capital markets.

10.8 There is no answer provided for this exercise.

Review Questions to Appendix

10.9 The correct answer is C.

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} = \frac{4800000}{3200000} = 1.5$$

10.10 The correct answer is C.

$$\text{Quick ratio} = \frac{\text{Current assets less inventories}}{\text{Current liabilities}} = \frac{4800000 - 1600000}{3200000} = 1.0$$

10.11 The correct answer is A.

$$\text{Profit margin} = \frac{\text{Net profit after tax}}{\text{Sales}} \times 100\% = \frac{360000}{5781250} \times 100\% = 6.2\%$$

10.12 The correct answer is D.

$$\begin{aligned}\text{Return on total assets} &= \frac{\text{Net profit after taxes}}{\text{Total assets}} \times 100\% \\ &= \frac{360\,000}{2\,000\,000 + 4\,800\,000} \times 100\% = 5.3\%\end{aligned}$$

10.13 The correct answer is B.

$$\text{Return on owner's equity} = \frac{\text{Net profit after taxes}}{\text{Owner's equity}} \times 100\% = \frac{360\,000}{3\,600\,000} \times 100\%$$
$$= 10\%$$

10.14 The correct answer is D.

$$\text{Debt ratio} = \frac{\text{Total debt}}{\text{Total assets}} \times 100\% = \frac{3\,200\,000}{2\,000\,000 + 4\,800\,000} \times 100\% = 47.1\%$$

10.15 The correct answer is A.

$$\text{Fixed to current asset ratio} = \frac{2000000}{4800000} = 41.7\%$$

10.16 The correct answer is D.

$$\text{Inventory turnover} = \frac{\text{Sales}}{\text{Inventory}} = \frac{5781250}{1600000} = 3.6\text{times}$$

10.17 The correct answer is B.

$$\text{Average collection period} = \frac{\text{Debtors}}{\text{Sales per day}} = \frac{1445313}{5781250 \div 350} = 87\text{days}$$

10.18 The correct answer is C.

$$\text{Fixed asset turnover} = \frac{\text{Sales}}{\text{Fixed assets}} = \frac{5781250}{2000000} = 2.9 \text{ times}$$

10.19 The correct answer is B. The higher the current ratio, the greater the liquidity position, and therefore KY Ltd has a lower ratio than its competitors in 19X3. But note that 1.3 still represents a healthy position for current assets in relation to current liabilities.

10.20 The correct answer is A. When a company's profit margin is under pressure it must exercise maximum control on costs and overheads. Then it attempts to increase prices in line with its competitors. A new product, by itself, is not necessarily the answer to its problems.

10.21 The correct answer is B. A relatively slow inventory turnover indicates too much inventory, and therefore B is the correct option. The other two options are not relevant to inventory.

Module 11

Review Questions

11.1 The correct answer is A. If Dental Specialities borrows sterling, it must repay (Æ60 000)/5 × 1.1024 pounds sterling, or £13 228.80 at the end of the year. If the company instead borrows eyes, it must repay (Æ60 000) × 1.1336 eyes, or Æ68 016.00. But the latter is denominated in eyes, not sterling, the shareholder-wealth currency. The question is thus how much sterling must in effect be paid at the end of the year if the loan is now taken out in eyes. The answer depends upon the exchange rate of sterling for eyes at the end of the year. If, as the question specifies, the forward exchange market is working efficiently (and otherwise has the characteristics assumed in the text of the module), the forward exchange rate will be given by:

$$\frac{\text{Forward exchange rate}}{5} = \frac{1.1336}{1.1024}$$

$$\text{Forward exchange rate} = 5.1415$$

Thus the sterling necessary to pay off the loan is Æ68 016/5.1415=£13 228.80, exactly the same amount as if the loan were made in sterling originally.

One implicit assumption made in the above explanation is that the actual future exchange rate that will exist between sterling and eyes is well predicted by the existing forward exchange rate. Though this is empirically valid as an **expectation**, there is uncertainty about the actual rate. This uncertainty can be hedged by engaging in a forward exchange transaction if funds are borrowed in sterling but will be repaid from eye revenues (by selling eyes ahead), or the reverse if funds are borrowed in eyes but will be repaid by sterling revenues (by selling sterling ahead). Answer (D) is a correct answer only if in fact the loan liability will be liquidated with eye revenues.

Answer (B) is wrong because interest rates are not really 'lower' in the UK, given that present and future exchanges of currencies must be made, and the exchange rates imply interest rate differentials, as the above calculations show.

Answer (C) is wrong because, though the implied inflation rate in Incaland is higher than the UK (assuming real interest rates are the same), the higher Incaland interest rates will compensate exactly for the 'cheaper' eyes used to repay the loan.

11.2 The correct answer is D. As the answer to Question 11.1 indicated, the correct response is (D).

This is the only result consistent with 'interest rate parity' between the UK and Incaland.

11.3 The correct answer is B. It is necessary to invoke only the relationship:

Nominal interest rate = Real interest rate + Effect of inflation

For the UK: $1.1024 = (1.04) \times (1 + \text{Expected inflation})$

$\text{Expected inflation} = 6\%$

For Incaland: $1.1336 = (1.04) \times (1 + \text{Expected inflation})$

$\text{Expected inflation} = 9\%$

11.4 The correct answer is A. You would be as well off lending sterling as lending eyes, as long as you hedged your eye transaction and assuming the transaction costs were the same. You can verify that this is true simply by referring to the calculations in the answer to Question 11.1, and regarding the outflows as inflows since you would be lending instead of borrowing (and in the amount of £1000 rather than Æ60 000, so the cash amounts – but not the interest percentage returns – would differ). Option II is wrong because there is no incentive to lend eyes due to the higher interest rates in Incaland, because the eyes in which the loan would be repaid to you would thence require conversion into sterling at an exchange rate that must be expected to compensate for the relative inflation rates in the two currencies.

11.5 The correct answer is C. The company's sterling inflow (either now or at the end of the year) will be the same regardless of whether tactic I or II is used. With tactic I, the eye interest rate is 13.36 per cent, which implies that Æ 100 000/1.1336= Æ88 215 can be borrowed now with the interest and principal covered by the eventual end-of-year eye receipt. At an exchange rate of Æ5 = £1, Dental Specialities can obtain £17 643 by immediately exchanging the loan proceeds, resulting in an eye liability fully covered by an eye asset, and that much sterling in hand. With tactic II, the Æ100 000 end-of-period eye receipt can be sold on the forward exchange market at the rate of Æ5.1415 = £1, or a guaranteed £19 450 at the end of the year. At a 10.24 per cent sterling interest rate, the company can borrow a fully covered £19 450/1.1024 = £17 643, the same as under tactic I. Thus either I or II is correct.

Answer (D) is wrong because there is no reason to think *a priori* that a corporate officer of a dental supply company could be a better predictor of changes in exchange, interest and inflation rates in the UK and Incaland than is the forward exchange market.

11.6 The correct answer is D. See the above discussion of tactic I in the answer to Question 11.5.

11.7 The correct answer is B. See the above discussion in the answer to Question 11.5.

11.8 The correct answer is A. In general, the result should eventually appear as a sterling NPV. Naturally, cash flows in a particular currency must be discounted with discount rates that comprehend the interest (or inflation) in that currency. This implies that either answer (A) or answer (C) can produce the correct answer, but answers (B) and (D) cannot.

The reason why answer (A) is preferred to (C) is that (C) requires that future exchange rates for each year be predicted by Dental Specialities' financial managers, a task that is unlikely to be as successful as the procedure in answer (A), which requires no exchange projections and uses data likely to be more readily available. (Forward exchange markets generally do not extend as far into the future as real-asset investments of companies). Incaland inflation predictions are accessible from the comparison of interest rates in the UK and Incaland, as per the text.

11.9 The correct answer is B. As the text maintains, only monetary assets feel the full brunt of unexpected exchange rate changes, because only their payoffs are denominated in fixed amounts of the foreign currency. Real assets, on the other hand, have values that are likely to fluctuate positively with inflation, and thus adverse movements in exchange rates would tend to be offset by this phenomenon. Thus answer (A) encourages higher transactions costs and may in fact unbalance the 'natural' hedging of real assets. Answer (C) is simply perverse, and answer (D) ignores the fact that monetary assets, though admittedly not great for this operation, may still require hedging.

11.10 The correct answer is A. The correct answer is (A) (though (C) may be effective depending upon the extent to which the joint venturer is family-oriented).

The key to this answer is that sanctions brought to bear upon the joint venturer must be invoked by powers outside Incaland but must also be enforceable on the joint venturer inside of that country. Answer (B) obviously does not fit this description, but answer (D) also falls short since UK courts probably have no jurisdiction inside Incaland. Answer (A) has both attributes necessary to enforce the contract. Whether or not the joint venturer will accept such terms depends upon the profits foreseeable, and other sources of financing and expertise to undertake the venture.

Module 12

Review Questions

12.1 The correct answer is A. As we discussed in the module, the very nature of option-contingent payoffs dictates that underlying security variances are positively related to option values. The reason is that an option holder is allowed to take advantage of upswings in underlying asset prices but does not as fully experience downswings. Answer (C) is technically correct, but is trivial. Answers (B) and (D) go in the wrong direction, and so must be wrong also.

12.2 The correct answer is B. As interest rates increase, in effect the less money must be put aside now so as to take advantage of (exercise) the option at expiration. The reduction in the present value of exercise price is an increase in option value. (This is true even though few options are ever truly 'exercised'.) Answer (A) is wrong because the future cash flows in the question are negative, not positive. And a reduction in the value of negative cash flows increases (positive) option value. The above illustrates why answers (C) and (D) are also wrong.

12.3 The correct answer is B. One proper manner of considering the value of geared equity is to regard it as a call option on the assets of the firm, which can be exercised by paying interest and principal to bondholders who bought the assets by purchasing bonds, but at the same time have issued a call on those assets to shareholders.

Answer (A) is true, but not a correctly general answer to the question. Answers (C) and (D) are incorrect descriptions of the contractual rights of bondholders.

12.4

$$C_u = \max(0, uS_0 - X) = \max(0, 1.2(£25) - £22) = £8$$
$$C_d = \max(0, dS_0 - X) = \max(0, 0.7(£25) - £22) = £0$$

Using Equation 12.1:

$$Y = \frac{C_u - C_d}{S_0(u-d)} = \frac{£8 - £0}{£25(1.2 - 0.7)} = 0.64$$

and (based on Equation 12.2 and Equation 12.3):

$$Z = \frac{uC_d - dC_u}{(u-d)(1+rf)} = \frac{1.2(£0) - 0.7(£8)}{(1.2 - 0.7)(1 + 0.10)} = -£10.1818\ldots$$

$$YS_0 + Z = 0.64(£25) - £10.1818\ldots = £5.82$$

The value of the call option is £5.82. Note that its exercise value is £25-£22=£3, so it is selling at a 'premium' of £2.82.

12.5 The correct answer is C. All three descriptions of the role of constraining indenture provisions are probably correct. The module text discusses the first two in some detail, showing them to be consistent with the agency relationship between shareholders and bondholders, and one mechanism (though not necessarily the most efficient one) of insuring bondholders against actions by the firm to shift wealth from them to shareholders. Option III is a new concept, but consistent with the type of agency problem that the module discussed as existing between managers and all outside capital suppliers. With interest and principal payments (and other specific performance-type bond indenture provisions) existing on debt claims, managers' degrees of freedom to consume corporate resources is inhibited. This is one more reason why companies may borrow.

12.6 The correct answer is A. The last example in the text of the module illustrated that the best interpretation of the reason for a convertible bond to exist is to guarantee bondholders that they will participate with shareholders in any increase in share value, including those that would otherwise be shifts of wealth from bondholders to shareholders.

All of the other 'reasons' listed are correct descriptions of what happens when a company issues convertible bonds instead of non-convertible bonds, but they are not **reasons** for convertible bonds to exist. Reasonably efficient securities markets can set appropriate prices on all of those phenomena at the time of the bonds' issuance, and there is no reason to think that shareholders would either benefit or lose relative to bondholders due to them. In order to have 'survival value' in such a market, a security must be able to do something better (or at least as well) as any other competing security. The 'benefit' of convertibles is their insurance characteristic for bondholders, which (as the module illustrates) allows a firm to make other decisions that are value-maximising.

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