THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4210 (2016/17 Term 1)
Financial Mathematics
Assignment 1 solution

Note: If you have any questions about the solution or assignment score, please let me know by sending an Email to kckchan@math.cuhk.edu.hk

In all of the questions we ignore the effect of interest rate, for simplicity.

1. An investor enters into a short cotton futures contract when the futures price is 50 cents per pound. The contract is for the delivery of $50,000$ pounds. How much does the investor gain or lose if the cotton price at the end of the contract is (a) 48.20 cents per pound; and (b) 51.30 cents per pound?

Solution: When you short a future contracts, you have the obligation to sell the underlying asset at a particular time and particular price. Let P, S be the profit and the cotton price at the day of deiivery respectively. We then have

$$P = 50000(0.5 - S)$$

(a) $P = 50000(0.5 - 0.482) = \$900$
(b) $P = 50000(0.5 - 0.513) = -\$650$

2. You would like to speculate on a rise in the price of a certain stock. The current stock price is \$29 and a 3-month call with a strike of \$30 costs \$2.90. You have $\$5,800$ to invest. Identify two alternative strategies, one involving an investment in the stock and the other involving investment in the option. What are the potential gains and losses from each?

Solution: Let P_t^s, P_t^o S_t be the profit of investing stocks, profit of investing options and the stock price at any time t respectively.

- **Method 1:** Investing stocks

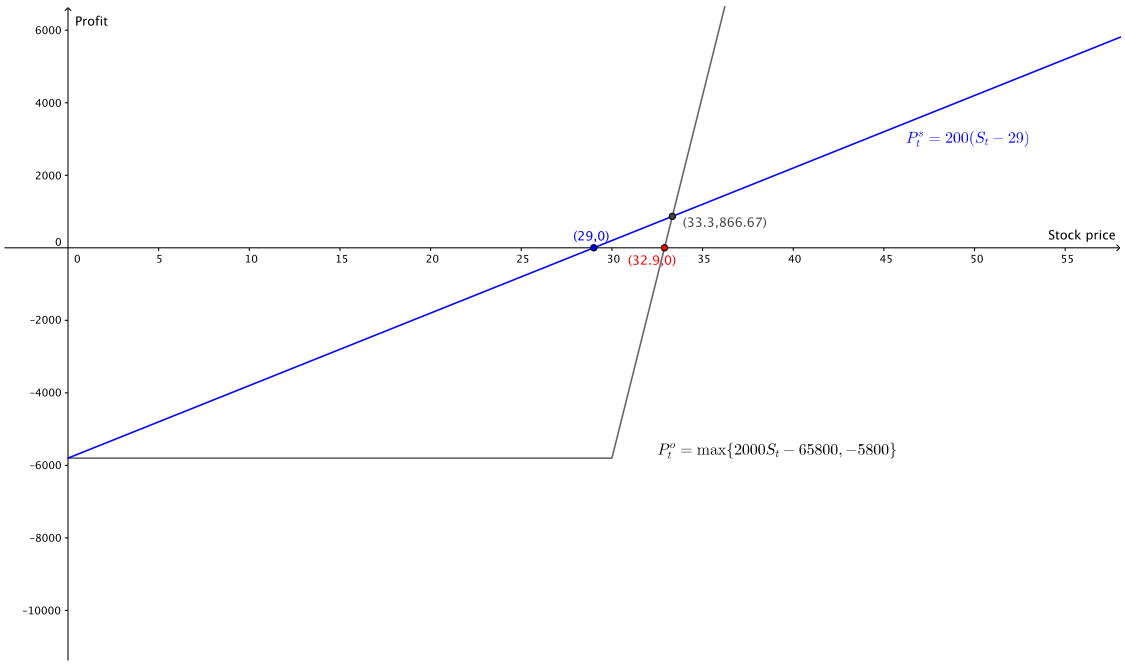
$$P_t^s = \frac{5800}{29}(S_t - 29) = 200(S_t - 29)$$

- **Methond 2:** Investing call options

$$P_t^o = \max\{\frac{5800}{2.9}(S_t - 30),\, 0\} - 5800 = \max\{2000S_t - 65800,\, -5800\}$$

There are some points we can derive from here:

1. $S_t \leq 30 \Rightarrow P_t^o = -5800$
2. $S_t \in (30, \frac{65800}{2000}] = (30, 32.9] \Rightarrow P_t^o = 2000S_t - 65800 \leq 0$
3. $S_t > 32.9 \Rightarrow P_t^o = 2000S_t - 65800 > 0$
4. $P_t^o \geq P_t^s$ iff $S_t \geq 33.33$



3. The price of gold is currently \$500 per ounce. The futures price for the delivery in one year is \$700. An arbitrageur can borrow money at 10% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero.

Solution: Consider the following action: *Borrow money and buy one ounce of gold, at the same time short one gold future delivering one ounce of gold one year later.*

Then the cash flow at $t = 0$ equals 0 assuming no money transaction in trading futures. But at the delivery day, we have

$$\text{payoff} = 700 - 550 = 150 > 0$$

Therefore an arbitrageur should borrow as much as possible and use all the money to buy gold and short gold futures at the same time.

4. Describe the payoff from the following portfolio: a long forward contract on an asset and a long European put option on the asset with the same maturity as the forward contract and a strike price that is equal to the forward price of the asset at the time the portfolio is set up.

Solution: Let S, E be the asset price and strike price at delivery day (delivery day is the same for both the contract and option). Note that we have

$$\text{strike price} = \text{forward price} = E$$

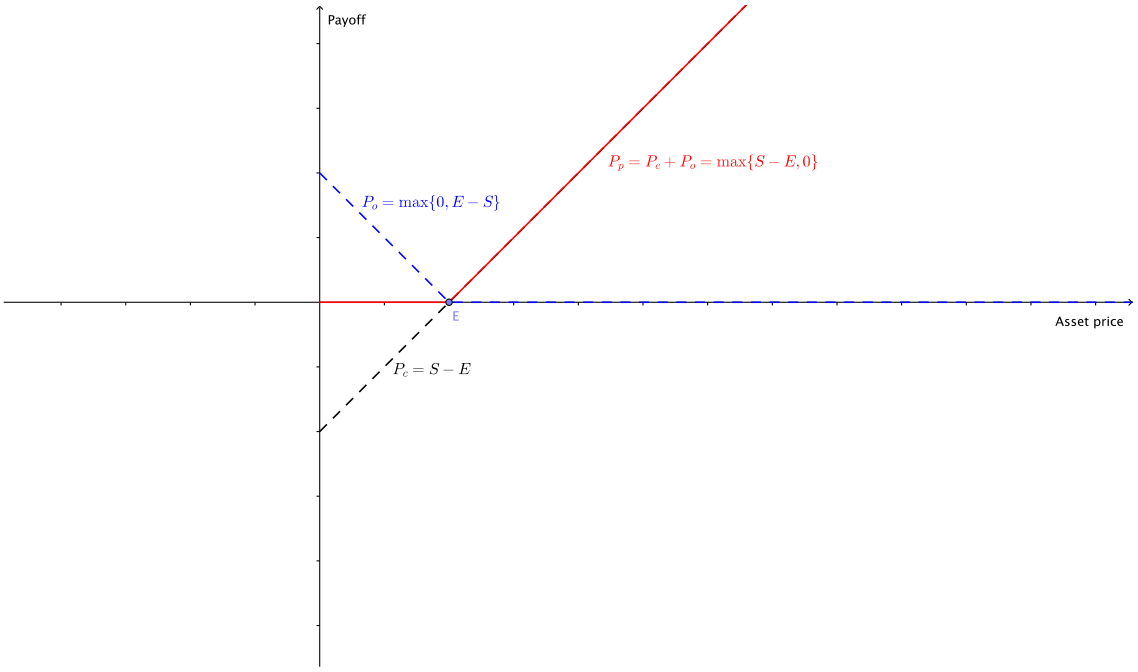
Let P_c, P_o, P_p be the payoff (not profit!) from buying a forward contract, buying an European put option, and the portfolio respectively. Then

$$P_c = S - E$$
$$P_o = \max\{0, E - S\}$$

Thus,

$$P_p = (S - E) + \max\{0, E - S\} = \max\{S - E,\, 0\}$$

An graphical illustration is shown below:



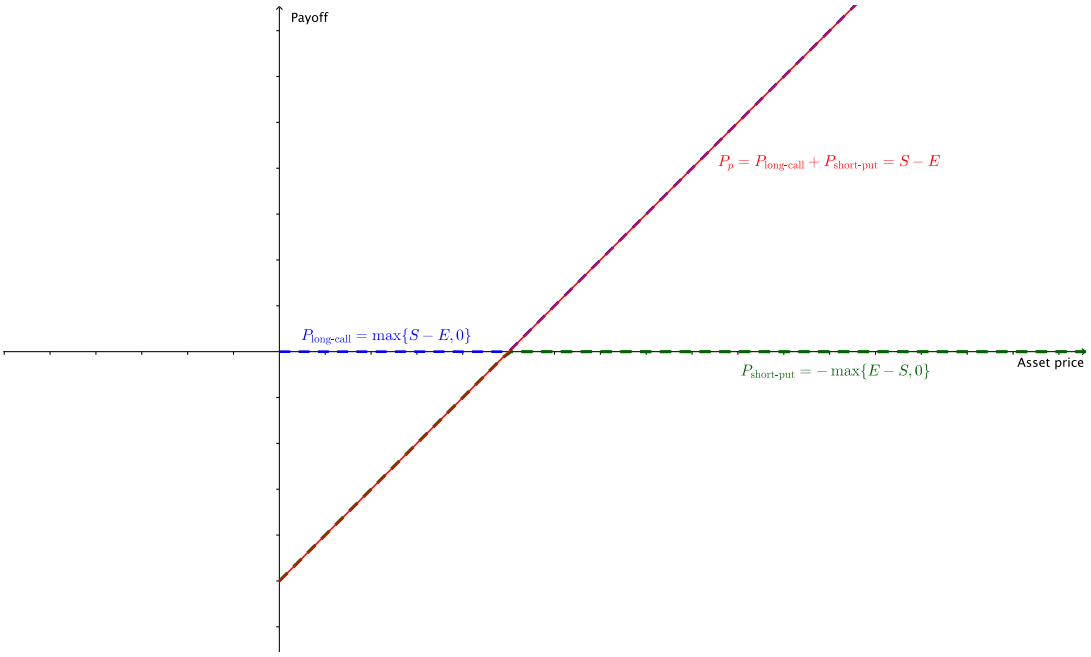
5. A long forward contract is equivalent to a long position in a European call option and a short position in a European put option." Explain this statement.

Solution: In this question we discuss only the payoff.
By using same notation as Q4 and let P_c, P_p be the payoff from buying a forward contract and the portfolio respectively. We then have

$$\begin{aligned} P_p &= \max\{S - E,\, 0\} - \max\{E - S,\, 0\} \\ &= \max\{S - E,\, 0\} + \min\{S - E,\, 0\} \\ &= S - E \\ &= P_c \end{aligned}$$

Here we assume that all of them have **same delivery day** and **forward price equals strike price**.

In terms of payoff, they are indeed equivalent.



Remark:

1. Many of you confused the word ***payoff*** and ***profit***. When we talk about the payoff of an option, we usually don't include the premium. For example, when you long a call option, the payoff equals $\max\{S - E, 0\}$. But the profit equals $\max\{S - E - p, -p\}$ as you have to pay a premium to the seller. Graphically, ***payoff diagram*** and ***profit diagram*** are different.
2. Try to be more precise. This means you should try to write your answer in a more rigorous way. For example, some of you said *"The profit is unlimited"*. Then I will ask: To what extent it is unlimited? You may make use of mathematical symbols and equations to help illustrate your point. For example, you may say

   $$P = S - 100 \text{ ant therefore } P \to \infty \text{ when } S \to \infty$$

   by defining suitable symbols before using them.
3. When you want to show some relations, don't just give examples and then conclude it is true. Usually we don't need examples, what we need is general result. Examples are used to illustrate your claim, but not to prove your claim. In this homework I didn't consider them wrong, but it's a good habit to prove things in a more general way.
4. Try to use equations and diagrams to help you. Many of you wrote a long paragraph to illustrate you idea. This is correct but inefficient. Instead, you can show the mathematical forumula and illustrate by formula. In this way, you ideas can be more clearly evaluated.