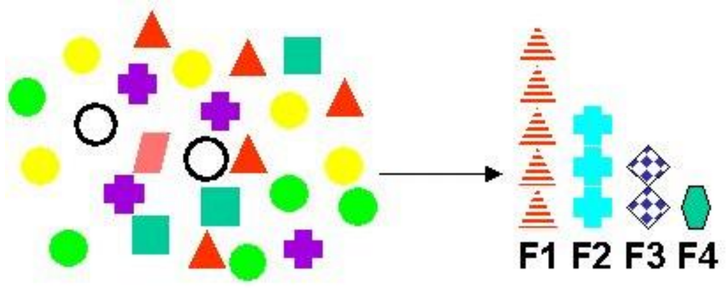
Frequent Item Sets & Association Rules



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Some History

- Bar code technology allowed retailers to collect massive volumes of sales data
  - Basket data: transaction date, set of items bought
  - Data is stored in tertiary storage

- Leverage information for marketing
  - How to design coupons?
  - How to organize shelves?
- The birth of data mining!
  - Agrawal et al. (SIGMOD 1993) introduced the problem of mining a large collection of basket data to discover association rules
  - Many papers followed…

Example: Supermarket Shelf-Management

- Goal: Identify items that are bought together by sufficiently many customers
- Approach: Process the sales data collected with barcode scanners to find dependencies among items
  - ◆Given a set of transactions (market-basket model), find rules that will predict the occurrence of an item based on the occurrences of other items in the transactions
- A classic rule:
  - ◆If one buys diaper and milk, then he is likely to buy beer
  - ◆Don't be surprised if you find six-packs next to diapers!

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Rules Discovered:
{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

Application Examples of Association Rules

- Items = products; baskets = sets of products someone bought in one visit to the store
  - ◆Reveals typical buying behaviour of customers
    - Marketing and sales promotion (suggests tie-in "tricks")
      - a product p appearing as rules' consequent can be used to determine what should be done to boost p sales
      - a product p' appearing as rules' antecedent can be used to see which other products would be affected if the store discontinues selling p'
      - a rule p' -> p an be used to see what products p' should be sold to promote sale of p, e.g., run sale on diapers and raise beer' price
    - Shelf management: position certain items strategically
    - Recommendation, e.g., Amazon's people who bought *X* also bought *Y*
  - ◆High support needed, or no €€'s
    - Only useful if many buy diapers & beer

The Market-Basket Model

- A large set of items, e.g., things sold in a supermarket
  - ◆$I = \{i_1, i_2, \ldots, i_m\}$
- A large set of baskets/transactions, e.g., the things one customer buys in one visit to the store
  - ◆B_i a set of items, and $B_i \subseteq I$

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

- Transaction Database T: a set of transactions $B = \{B_1, B_2, \ldots, B_n\}$
- Our interests: Identify *associations among "items"*, not "baskets"
  - ◆E.g., People who bought Diaper tend to buy Beer

Market-Baskets and Associations

- A many-many mapping (association) between two kinds of things
  - ◆E.g., 90% of transactions that purchase diaper&milk also purchase beer
- Given a set of baskets, discover association rules
  - ◆The technology focuses on common events, not rare events ("long tail")
- 2-step approach
  - ◆Find frequent *itemsets*
  - ◆Generate *association rules*

Rules Discovered:
{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

Causation vs. Association

$$X \longrightarrow Y$$

$$\begin{tikzcd} Z \arrow[d] \arrow[dr] & \\ X \arrow[r, dotted] & Y \end{tikzcd}$$

- In machine learning, $X \rightarrow Y$ usually implies a causal relationship
  - "a change in X (seen as cause) forces a change in Y *(seen as effect)*"
  - *causation* is complex and difficult to prove relationship
- In rule mining, $X \rightarrow Y$ is an association relationship
  - "*X* is associated with *Y* "
  - Much easier to calculate and prove
    - of less interest for medical research than for market research
- Association rules indicate only the *existence* of a statistical relationship between *X* and *Y*
  - They do not specify the *nature* of the relationship

Frequent Itemsets

- Simplest question: find sets of items, called itemsets, that appear "frequently" in the baskets
  - E.g., `{milk, diaper, beer}` is an itemset
- *Support* for itemset *A* = the number of baskets containing all items in *A*
  - Often expressed as a fraction of the total number of baskets
- Given a *support threshold s*, sets of items that appear in at least *s* baskets are called *frequent itemsets*

Support of `{Milk, Diaper}` = 3

Support of `{Milk, Diaper, Beer}` = 2

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
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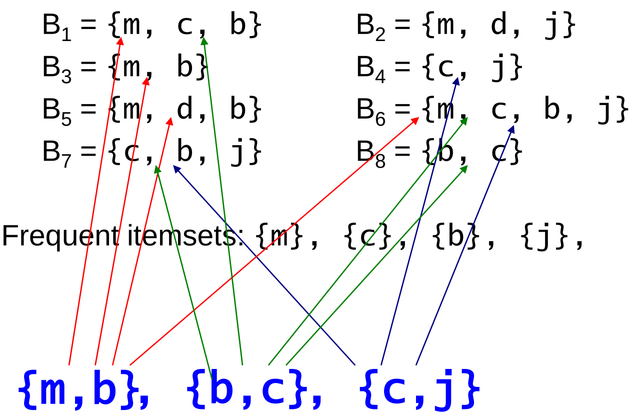
5	Coke, Diaper, Milk

Example: Frequent Itemsets

- Items = {milk, cereal, diaper, beer, juice}
- Support = 3 baskets



B_1 = {m, c, b} B_2 = {m, d, j}
B_3 = {m, b} B_4 = {c, j}
B_5 = {m, d, b} B_6 = {m, c, b, j}
B_7 = {c, b, j} B_8 = {b, c}

- Frequent itemsets: {m}, {c}, {b}, {j},

{m,b}, {b,c}, {c,j}

The Market-Basket Model

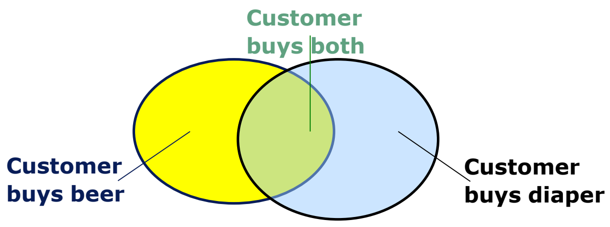
- A *k*-itemset is an itemset with *k* items
  - ◆E.g., A = {milk, diaper} is a 2-itemset
  - ◆E.g., A' = {milk, bear, diaper} is a 3-itemset
- A transaction B_i contains an itemset $A = \{i_1, i_2, \ldots, i_k\}$, if $A \subseteq B_i$
  - ◆E.g., basket B_6= {milk, cereal, bear, diaper} contains the 3-itemset A'
- An association rule is an implication of the form:

$\{i_1, i_2, \ldots, i_k\} \rightarrow \{j_1, j_2, \ldots, j_l\}$, where $\{i_1, i_2, \ldots, i_k\}$, $\{j_1, j_2, \ldots, j_l\} \subset I$, and $\{i_1, i_2, \ldots, i_k\} \cap \{j_1, j_2, \ldots, j_l\} = \emptyset$

Association Rules

- If-then rules about the contents of baskets
  - $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is *likely* to contain j"
- A general form of an association rule is `Body→Head[Support,Confidence]`
  - *Antecedent*, left-hand side (LHS), body
  - *Consequent*, right-hand side (RHS), head
  - *Support*, frequency
  - *Confidence*, strength
- Example: `diapers → beer [50%, 60%]`
  - *"IF buys diapers, THEN buys beer in 60% of the cases in 50% of the transactions"*

Support and Confidence



- *Support* for the rule A → B: denotes the frequency of the rule within all transactions in the database *T*, i.e., the probability that a transaction contains the union of A and B
  - `support(A → B [s,c]) = p(A B) = support({A,B})`
- *Confidence* of the rule A → B: denotes the percentage of transactions in the database *T*, containing A which also contain B, i.e., the conditional probability that a transaction containing A also contains B
  - `confidence(A → B [s,c]) = p(B|A) = p(A B) / p(A) = support({A,B}) / support({A})`

Example: Confidence

B_1 = {m, c, b}　　B_2 = {m, d, j}
B_3 = {m, b}　　B_4 = {c, j}
B_5 = {m, d, b}　　B_6 = {m, c, b, j}
B_7 = {c, b, j}　　B_8 = {b, c}

- An association rule: {m, b} → c
  - Support ({m, b}) = 4, Support ({m, b, c}) = 2
  - Confidence ({m, b} → c) = 2/4 = 50%

$$\text{conf}(I \to j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Finding Association Rules

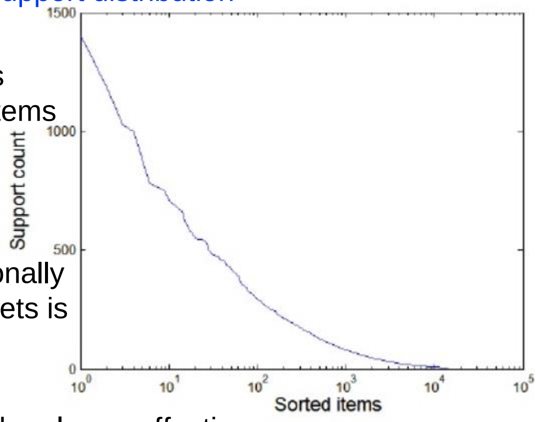
- Goal: Find all rules that satisfy the user-specified *minimum support* (**minsup**) and *minimum confidence* (**minconf**)
  - *support >= s and confidence >=c*

- Key Features
  - Completeness: find all rules
  - Mining with data on disk (not in memory)

- Hard part: Finding the frequent itemsets
  - If A → B has high support and confidence, then both A and B will be frequent

How to Set the Appropriate MinSup?

- Many real data sets have skewed support distribution

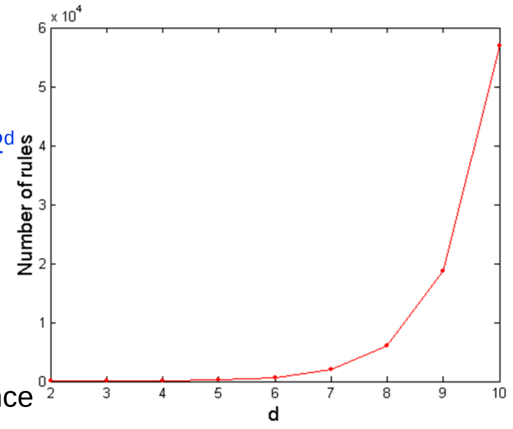


- If minsup is too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
- If minsup is too low, it is computationally expensive and the number of itemsets is very large
- A single minsup threshold may not be always effective

Association Rule Mining Task

- Brute-force approach:
- List all possible association rules
  - ◆Given d unique items:
    - Total number of itemsets = 2^d
    - Total number of ARs = R

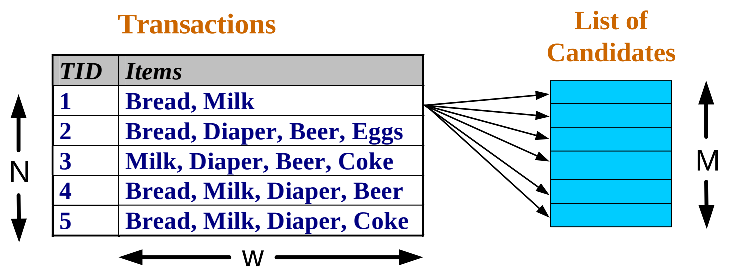
$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$
$$= 3^d - 2^{d+1} + 1$$



- Compute the support and confidence for each rule
  - ◆Prune rules that fail the minsup and minconf thresholds
- Computationally prohibitive!

Counting Frequent Itemsets in One pass

- Each itemset is a candidate frequent itemset
- Count the support of each candidate by scanning the database



TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Match each basket against every candidate
- Complexity ~ O(NMw) => Expensive since M = 2^d !!
  - Need a lot of memory space else swapping counts in/out is very "expensive"

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: M = 2^d
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce N as the size of itemset increases
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing the Number of Candidates: The Apriori algorithm

- Rules originating from the same itemset have identical support but can have different confidence
  - Thus, we may decouple the support and confidence requirements
- Two steps:
  - ❶*Frequent Itemsets:* Find all itemsets `I` that have minimum support
    - usually a computationally expensive phase!
  - Key idea: (anti-)monotonicity property of the support measure
    - If an itemset is frequent, then all of its subsets must also be frequent
    - If an itemset is not frequent, then all of its supersets cannot be frequent

$$\forall A,B: (A \subseteq B) \Rightarrow s(A) \geq s(B)$$

    - The support of an itemset never exceeds the support of its subsets
      - This is known as the anti-monotone property of support

The Apriori algorithm

- ❷*Rule generation:* Use frequent itemsets `I` to generate rules
  - For every subset `A` of `I`, generate rule $A \rightarrow I \setminus A$
    - Since `I` is frequent, `A` is also frequent
  - Variant 1: Perform a single pass to compute the rule confidence
    - $conf(A,B \rightarrow C,D) = supp(A,B,C,D)/supp(A,B)$
  - Variant 2: Filter out bigger rules from smaller ones
    - Observation: If $A,B,C \rightarrow D$ is below confidence, so is $A,B \rightarrow C,D$
  - Output rules above confidence threshold
- In general, confidence does not have an anti-monotone property
  - $conf(ABC \rightarrow D)$ can be larger or smaller than $conf(AB \rightarrow D)$
- But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., I = $\{A,B,C,D\}$: $conf(ABC \rightarrow D) \geq conf(AB \rightarrow CD) \geq conf(A \rightarrow BCD)$
- Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

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Example

B_1 = {m, c, b} B_2 = {m, d, j}

B_3 = {m, c, b, n} B_4= {c, j}

B_5 = {m, d, b} B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}

- Support threshold *s = 3*, confidence *c = 0.75*
- 1) Frequent itemsets:
  - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- 2) Generate rules:
  - ~~b→m: *c*=4/6~~ b→c: *c*=5/6 ~~b,c→m: *c*=3/5~~
  - m→b: *c*=4/5 … b,m→c: *c*=3/4
  - ~~b→c,m: *c*=3/6~~

conf(A→B) = supp(A,B)/supp(A)

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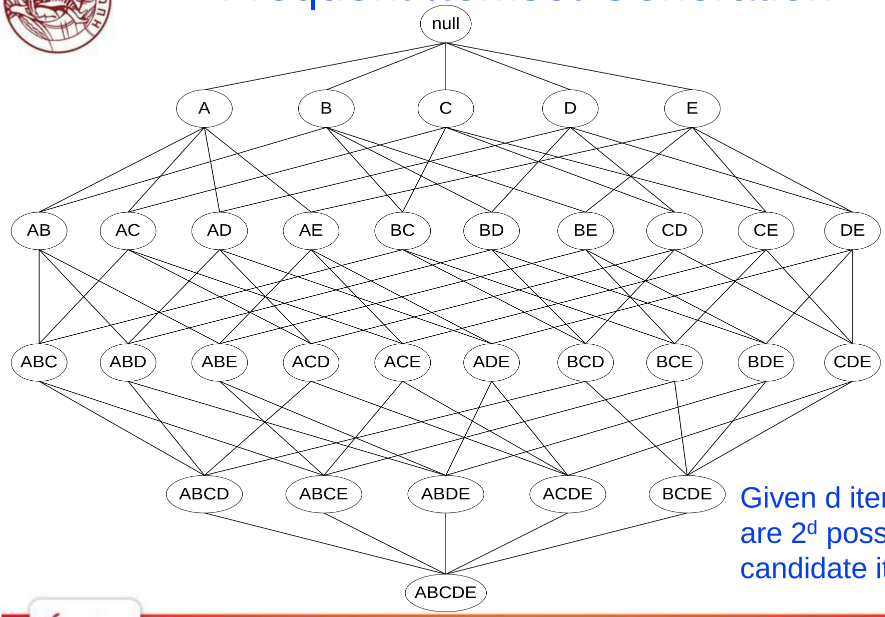
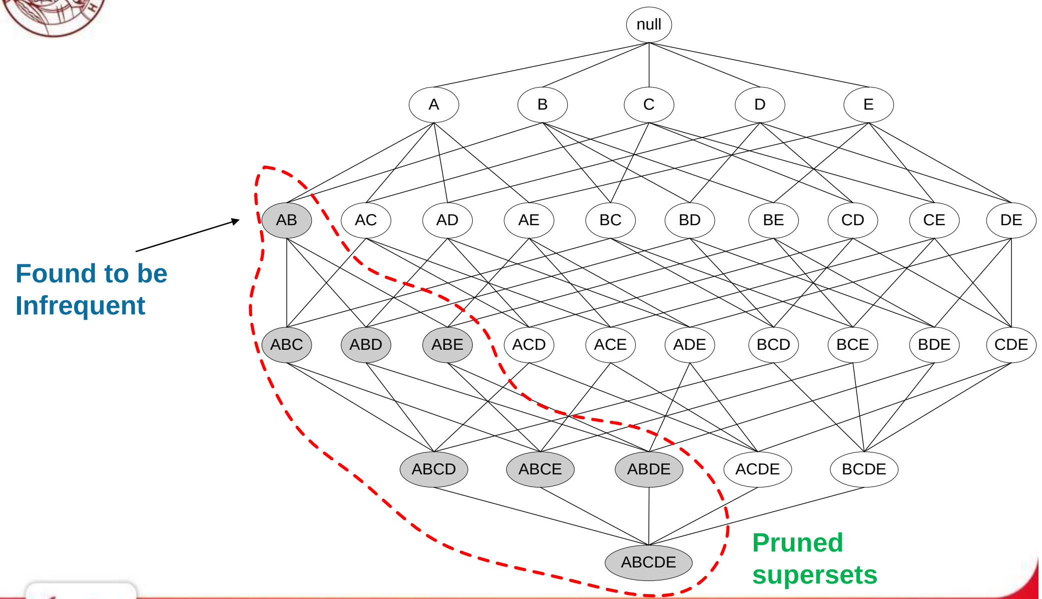
Frequent Itemset Generation

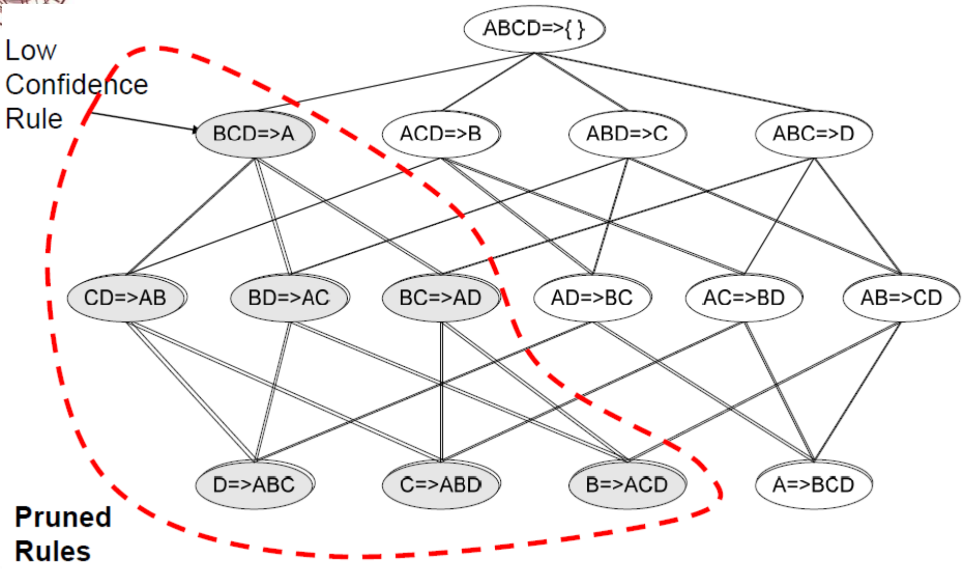


Given d items, there are 2^d possible candidate itemsets

Illustrating the A-Priori Principle



Rule Generation Example

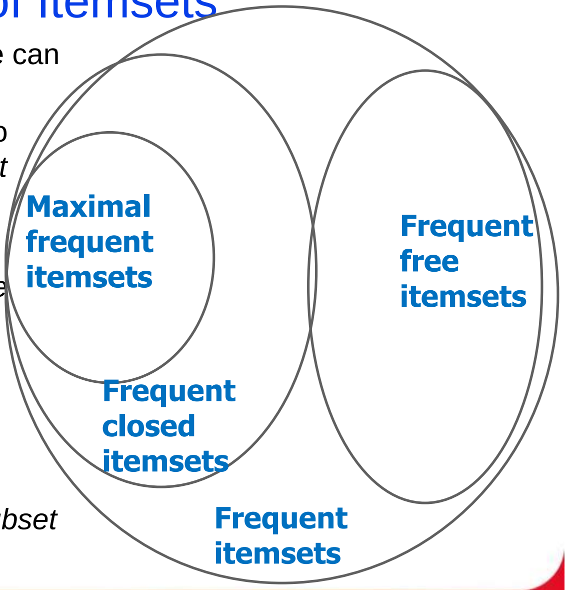


How to Improve A-priori Efficiency?

- Dynamic itemset counting
  - ◆Add new candidate itemsets only when *all* of the subsets are estimated to be frequent
- Transaction Reduction
  - ◆A transaction that does not contain *any* frequent k-itemset is useless in subsequent scans
- Hash-based itemset counting
  - ◆A k-itemset whose corresponding *hashing bucked count* is below the threshold cannot be frequent
- Partitioning
  - ◆Any itemset that is potentially frequent in DB must be *frequent in at least one of the partitions* of the DB
- Sampling
  - ◆Mining on a subset of given data, *lower support threshold* and consider a method to determine completeness

Compacting Output Rules: Classes of Itemsets

- To reduce the number of rules we can post-process and only output:
  - ◆Maximal Frequent itemsets: no *immediate superset is frequent*
    - Can generate all frequent itemsets (without support)
  - ◆Closed itemsets: no *immediate superset has the same count* (>0)
    - Can generate all frequent itemsets and their support
- Alternately:
  - ◆Free itemset: no *immediate subset has the same count* (>0)



Example: Maximal/Closed

	Count	Maximal (s=3)	Closed
A	4	No	No
B	5	No	Yes
C	3	No	No
AB	4	Yes	Yes
AC	2	No	No
BC	3	Yes	Yes
ABC	2	No	Yes

Frequent, but superset BC also frequent

Frequent, and its only superset, ABC, not freq

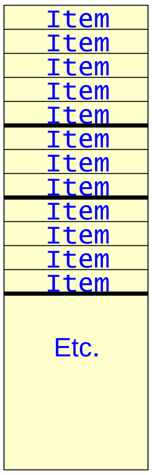
Superset BC has same count

Its only super-set, ABC, has smaller count

Finding Frequent Itemsets

Computing Itemsets

- Back to finding frequent itemsets
- Typically, data is kept in flat files rather than in a database system:
  - ◆Stored on disk, basket-by-basket
  - ◆Baskets are small but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use k nested loops to generate all itemsets of size k
- Note: To find frequent itemsets, we have to count them
  - ◆To count them, we have to generate them



Items are positive integers, and boundaries between baskets are –1

Computation Model

- In practice, association-rule algorithms read data in passes
  - ◆We measure the cost by the *number of passes* over the data
  - => Cost of mining is the *number of disk I/Os*
- The approach:
  - ◆We always need to generate all the itemsets
  - ◆But we would only like to count/keep track of those itemsets that in the end turn out to be frequent
- For many frequent-itemset algorithms *main-memory* is the critical resource
  - ◆The number of different things we can count as we read baskets is limited by main memory

Finding Frequent Pairs

- The hardest turns out to be finding the frequent pairs of items $\{i_1, i_2\}$
  - Often, frequent pairs are common, frequent triples are rare
    - The probability of *being frequent drops exponentially with the itemset size*; number of itemsets grows more slowly with size
- Naïve Algorithm:
  - Read file one, counting in main memory the occurrences of each pair
    - From each basket of n items, generate its $n(n-1)/2$ pairs using two nested loops
- Problem: fails if n^2 exceeds main memory
  - Suppose 10^5 items, counts are 4-byte integers
  - Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$
  - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

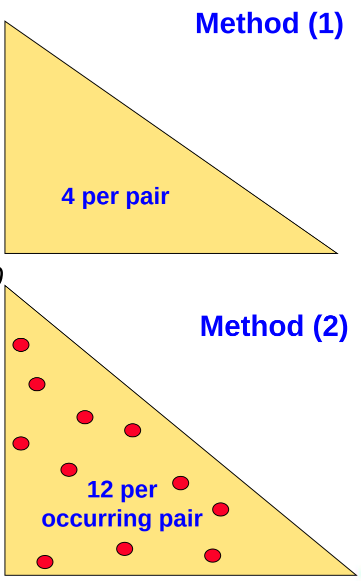
Counting Pairs in Memory

Two approaches:

- Approach 1: Count *all* pairs using a matrix keeping only the counts c
- Approach 2: Keep a table of triples $[i, j, c]$ = "the count of the pair of items $\{i, j\}$ is c"
  - If integers and item ids are 4 bytes, we need approximately12 bytes for pairs with *count>0*
  - Plus some additional overhead to organize the table for efficient search ("hashtable")

Note:

- Approach 1 only requires 4 bytes per pair
- Approach 2 uses 12 bytes per pair (but only for pairs with count > 0)



Triangular Matrix

- Approach 1: Triangular Matrix
  - ◆ n = total number of items
  - ◆ Count pair of items $\{i,j\}$ only if $i<j$
    - So use only half of the two-dimensional array
  - ◆ A more space-efficient way is to use a one-dimensional triangular array
  - ◆ Keep pair counts in lexicographic order:
    - $\{1,2\},\{1,3\},\ldots,\{1,n\},\{2,3\},\{2,4\},\ldots,\{2,n\},\{3,4\},\ldots$
  - ◆ Pair $\{i,j\}$ is at position: $(i-1)(n-i/2)+j-i$
  - ◆ Total number of pairs $n(n-1)/2$; total bytes= $2n^2$
  - ◆ Triangular Matrix requires 4 bytes per pair

a11	a12	a13	a14	a15	…
a21	a22	a23	a24	a25	…
a31	a32	a33	a34	a35	…
a41	a42	a43	a44	a45	…
…	…	…	…	…	…

Every time you see a pair $\{i,j\}$ from a basket, increment the count at the corresponding position in triangular matrix

Comparing the two Approaches

- Approach 2 uses 12 *bytes* per occurring pair *(but only pairs with count > 0)*
  - ◆ Total bytes used is about 12p, where p is the number of pairs that actually occur
  - ◆ Beats Approach 1 if less than 1/3 of possible pairs actually occur
  - ◆ May require extra space for retrieval structure, e.g., a hash table

Problem is if we have too many items so the pairs do not fit into memory.
Can we do better?

A-Priori Algorithm

Example

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pairs (2-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

(no need to generate candidates involving Coke or Eggs)

Triplets (3-itemsets)

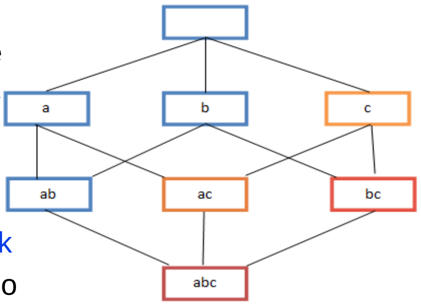
Itemset	Count
{Bread,Milk,Diaper}	2

Minimum Support = 3

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
6 + 6 + 1 = 13

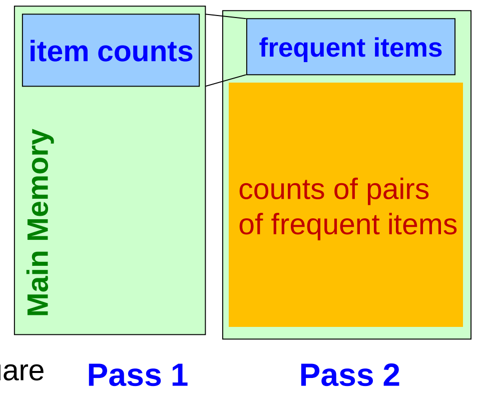
Candidate Generation

- Contrapositive for pairs: if item *i* does not appear in *s* baskets, then no pair including *i* can appear in *s* baskets
- Basic principle (Apriori):
  - An itemset of size k+1 is candidate to be frequent only if all of its subsets of size k are known to be frequent
- Main idea:
  - Construct a candidate of size k+1 by combining two frequent itemsets of size k
  - Prune the generated k+1-itemsets that do not have all k-subsets to be frequent
- So, how does A-Priori find frequent pairs?
  - A two-pass approach limiting the need for main memory counts



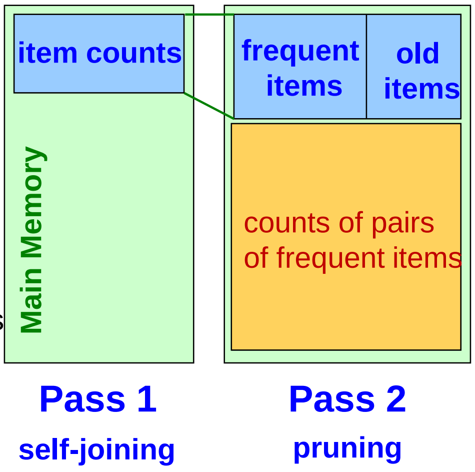
A-Priori Algorithm

- Pass 1: Read baskets and count in main memory the occurrences of each item
  - Requires only memory proportional to #items
  - Items that appear at least *s* times (minsup) are the *frequent items*
- Pass 2: Read baskets again and count in main memory only those pairs where both elements were found in Pass 1 to be frequent
  - Requires memory proportional to square of *frequent* items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)



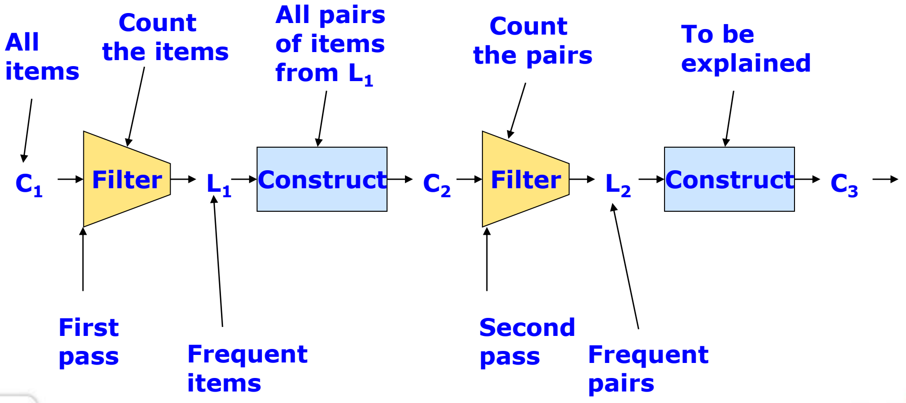
Details for A-Priori

- You can use the triangular matrix method with m = number of frequent items
  - May save space compared with storing triples
- Trick: re-number frequent items `1,2,…,m` and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- For each k, we construct two sets of k *–tuples* (sets of size k):
  - C_k = *candidate* k -sets = those that might be frequent sets (support $\geq s$) based on information from the pass for $k-1$
  - L_k = the set of truly frequent *k-tuples*



The Apriori algorithm

Level-wise approach

C_k = candidate itemsets of size k
L_k = frequent itemsets of size k

1. k = 1, C_1 = all items
2. While C_k not empty

Frequent itemset generation

3. Scan the database to find which itemsets in C_k are frequent and put them into L_k

Candidate generation

4. Use L_k to generate a collection of candidate itemsets C_{k+1} of size k+1
5. k = k+1

Recall: Example from Last time

B_1 = {m, c, b}	B_2 = {m, d, j}
B_3 = {m, c, b, n}	B_4= {c, j}
B_5 = {m, d, b}	B_6 = {m, c, b, j}
B_7 = {c, b, j}	B_8 = {b, c}

- Frequent itemsets (s = 3):
  - {b}, {c}, {j}, {m}
  - {b,m} {b,c} {c,m} {c,j}
  - {m,c,b}
- How we can compute them with A-Priori?

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A-Priori Algorithm Example

Generate C_1 = `{ {b} {c} {j} {m} {n} {p} }`
Count the support of itemsets in C_1
Prune non-frequent: L_1 = `{ b, c, j, m }`

Generate C_2 = `{ {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }`
Count the support of itemsets in C_2
Prune non-frequent: L_2 = `{ {b,m} {b,c}  {c,m}  {c,j} }`

Generate C_3 = `{` $\underline{\texttt{\{b,c,m\} \{b,c,j\} \{b,m,j\} \{c,m,j\}}}$ `}` ******
Count the support of itemsets in C_3
Prune non-frequent: L_3 = `{ {b,c,m} }`

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1. But that one can be more careful with candidate generation. For example, in C_3 we know `{b,m,j}` cannot be frequent since `{m,j}` is not frequent

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A-Priori Algorithm: Memory Details

- The first pass of A-Priori
  - Create two tables
  - Translate items(e.g. strings) to numbers
  - Counters of singletons
- Between the passes of A-priory
  - Many singletons won't be frequent
  - Create new numbering 1..m just for frequent items
  - Create frequent-items table: array of size n
    - i-th element is zero if not frequent or number 1..m
- The second pass of A-Priori
  - Count all the pairs that consist of two frequent items
  - Maintain triangular matrix of $4{*}m^2/2\ bytes$(or triples structure)

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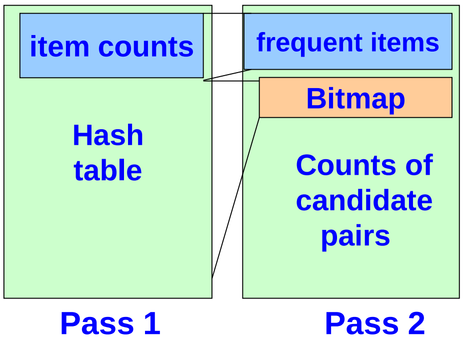
Improvements to A-Priori

Observations

- In pass 1 of the Apriori scheme
  - Only individual item counts are stored
  - Remaining memory is unused
- In pass 2 of the Apriori scheme, it is possible that `(i,j)` is not frequent even though `i` and `j` are frequent
  - But we still must count them (and hence need to store them in memory)
- Can we use the idle memory (in pass 1) to reduce the memory required in pass 2?

PCY (Park-Chen-Yu) Algorithm

- Pass 1 of PCY: In addition to item counts, maintain a hash table with *as many buckets as fit in memory*
  - ◆Keep a count for each bucket into which pairs of items are hashed (not the actual pairs that hash to the bucket!)
  - ◆Number of buckets can be smaller than number of pairs
    - Collision is possible!
- *Multistage* improves PCY (latter)



Observations about Buckets

- We are not just interested in the presence of a pair, but whether its count is at least the support s threshold
- If a bucket contains a frequent pair, then the bucket is surely frequent
- However, even without any frequent pair, a bucket can still be frequent (*false positives*)
  - ◆So, we cannot use the hash table to eliminate any member (pair) of a “frequent” bucket
- If a bucket is not frequent, no pair that hashes to that bucket could possibly be a frequent pair
  - ◆For a bucket with total count < s, none of its pairs can be frequent
  - ◆Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- Pass 2 of PCY: we only count pairs that hash to frequent buckets
  - ◆There are still infrequent pairs that slipped through

PCY Algorithm – Pass 1

- Pairs of items need to be generated from the input file
  - ◆they are not present in the file!
- Before Pass 1 Organize Main Memory
  - ◆Space to count each item: One (typically) 4-byte integer per item
  - ◆Use the rest of the space for as many integers, representing buckets, as we can

```
     FOR (each basket) {
         FOR (each item in the basket)
           add 1 to item's count;
New ⎡    FOR (each pair of items) {
 in ⎨      hash the pair to a bucket;
PCY ⎣      add 1 to the count for that bucket
         }
     }
```

PCY Algorithm – Between Passes

- In pass 2, only need to count pairs that hash to frequent buckets
  - ◆We must count again because we did not keep the information on the pairs, and also because of the collision
  - ◆However, we do not need the count information from pass 1 any more
  - ◆What we need is an indication on whether a pair is possibly frequent or not
- Bit vector serves this purpose well (and consumes less space)
  - ◆1 means bucket count exceeds the support **s** (i.e., is frequent); 0 means it did not
  - ◆The hash value now corresponds to the bit position
- 4-byte integers are replaced by bits, so the bit-vector requires 1/32 of memory
- Also, decide which items are frequent and list them for the second pass

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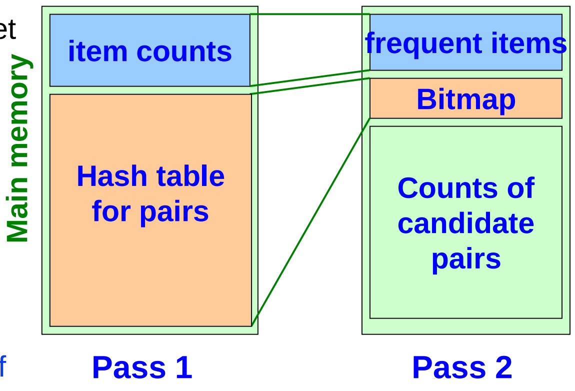
PCY Algorithm – Pass 2

- Count all pairs $\{i, j\}$ that meet the conditions for being a candidate pair:
  - Both i and j are frequent items
  - The pair $\{i, j\}$, hashes to a bucket number whose bit in the bit vector is 1
- Both conditions are necessary for the pair to have a chance of being frequent



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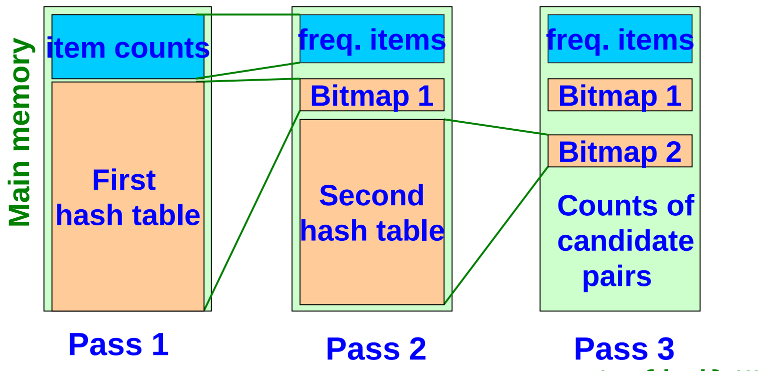
PCY Scheme (Pass 2): Memory Details

- Buckets require a few bytes each
  - Note: we don't have to count over s
  - # buckets is `O(main-memory size)`
- On second pass, a table of `(item, item, count)` triples is essential
- Cannot use triangular matrix scheme. Why?
  - Pairs of frequent items that PCY avoid counting are placed randomly within the triangular matrix
  - No known way of compacting the matrix to avoid leaving space for the uncounted pairs
- Thus, the hash table must eliminate 2/3 of the candidate pairs for PCY to beat A-priori

Refinement: A *Multistage* Algorithm

- Limit the number of candidates to be counted
  - Remember: memory is the bottleneck
  - Still need to generate all itemsets but we only want to count/keep track of the ones that are frequent
- Key idea: After Pass 1 of PCY, rehash only those pairs that qualify for Pass 2 of PCY
  - *i* and *j* are frequent, and
  - `{i,j}` hashes to a frequent bucket from Pass 1
- On *middle* pass, fewer pairs contribute to buckets, so fewer *false positives* –frequent buckets with no frequent pair
- Uses several successive hash tables---requires more than two passes

Multistage Picture



Count items
Hash pairs `{i,j}`

Hash pairs `{i,j}` into Hash2 iff:
i,j are frequent,
`{i,j}` hashes to freq. bucket in B1

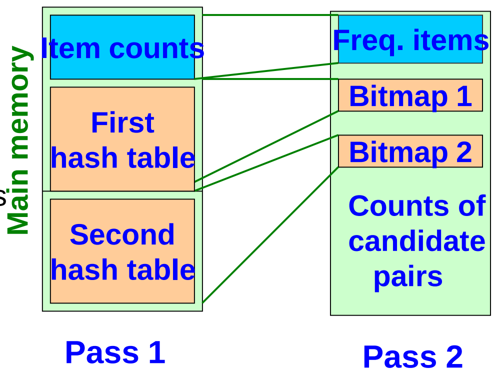
Count pairs `{i,j}` iff:
`i,j` are frequent,
`{i,j}` hashes to freq. bucket in B1
`{i,j}` hashes to freq. bucket in B2

Multistage – Pass 3

- Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:
  - Both i and j are frequent items
  - Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1
  - Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1
- Important Points
  - The two hash functions have to be independent
  - We need to check both hashes on the third pass
    - If not, we would wind up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: The Multihash Algorithm

- Key idea: use several independent hash tables on the first pass
- Risk: halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes!



So far, …

- Numerous approaches and refinements have been studied to keep memory consumption low
  - PCY and its refinements (multistage, multihash)
- Either multistage or multihash can use more than two hash functions
  - In multistage, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
  - For multihash, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$

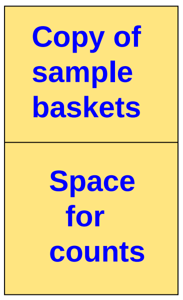
Limited Pass Algorithms

All (Or Most) Frequent Itemsets in $\leq$ 2 Passes

- A-Priori, PCY, etc., take *k* passes to find frequent itemsets of size *k*
- Can we use fewer passes?
- Use 2 or fewer passes for ALL sizes, but may miss some frequent itemsets
  - Approximate solution
    - Simple algorithm: Use random sampling
    - Savasere, Omiecinski, and Navathe (SON) algorithm
    - Toivonen

Random Sampling

- Take a random sample of the market baskets
- Run A-priori or one of its improvements (for itemsets of all sizes, not just pairs),
  - load the sample into the main memory
    - so you don't pay for disk I/O each time you increase the size of itemsets
  - reduce support threshold proportionally to match the sample size
  - be sure you leave enough space for counts
- Use as your support threshold a suitable, scaled-back number
  - E.g., if your sample is 1/100 of the baskets, use *s* /100 as your support threshold instead of *s*



Random Sampling:– Option

- False positives will result
  - ◆Itemset may be frequent in the sample but not in the entire dataset (because of the reduced minsup threshold)
  - ◆Run a second pass through the entire dataset to verify that the candidate pairs are truly frequent
    - Can remove false positives totally
- False negatives will also result
  - ◆Itemset is frequent in the original dataset but not picked out from the sample
  - ◆Scanning the whole dataset a second time does not help
  - ◆Using smaller threshold helps catch more truly frequent itemsets, but requires more space

SON Algorithm

- Repeatedly read small subsets (chunks) of the baskets into main memory and perform the first pass of the previous algorithm on each subset
  - ◆This is not sampling but processing the entire file in memory-sized chunks
- An itemset becomes candidate if it is found to be frequent in *at least one* subset of the baskets using a scaled-back support threshold
- On a second pass, count all the candidate itemsets and determine which are frequent in the entire set
- Key “monotonicity” idea: an itemset cannot be frequent in the entire set of baskets unless it is *frequent in at least one subset*
  - ◆A subset contains a fraction p of whole file (number of subsets is 1/p)
  - ◆If itemset is not frequent in any subset, then the support in each subset is less than p * s
  - ◆Hence, the support in whole file is less than s: not frequent!
    - (1/ p) p * s = s

SON Distributed Version

- SON lends itself to *distributed data mining*
  - Map Reduce
- Baskets distributed among many nodes
  - Subsets of the data may correspond to one or more chunks in distributed file system
  - Compute frequent itemsets at each node
    - Phase 1: Find candidate Itemsets
  - Distribute candidates to all nodes
  - Accumulate the counts of all candidates
    - Phase 2: Find true frequent Itemsets

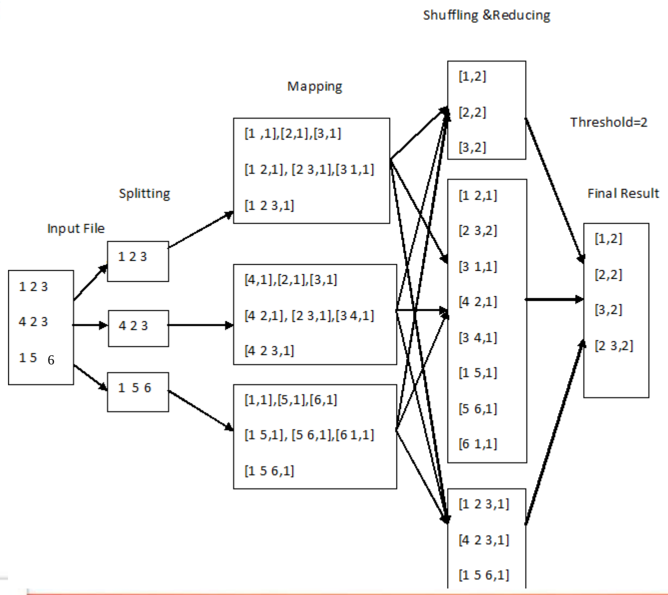
SON MapReduce: Phase 1

- Map
  - Input is a chunk/subset of all baskets; fraction p of total input file
  - Find itemsets frequent in that subset:
    - Use support threshold =s * p
  - Output is set of key-value pairs `(FrequentItemset,1)` where `FrequentItemset` is found from the chunk
- Reduce
  - Each reduce task is assigned a set of keys, which are itemsets
  - Produce keys that appear one or more times
  - Frequent in some subset; these are candidate itemsets

SON MapReduce: Phase 2

- Map
  - ◆Each Map task takes a chunk of the total input data file as well as the output of Reduce tasks from phase 1
    - All candidate itemsets go to every Map task
  - ◆Output is set of key-value pairs `(CandidateItemset,support)` where the `support` of the `CandidateItemset` is computed among the baskets of the input chunk
- Reduce
  - ◆Each Reduce task is assigned a set of keys, which are candidate itemsets
  - ◆Sums associated values for each key: total support for `CandidateItemset`
  - ◆ If total support of itemset >=s, emit itemset and its count

SON Map Reduce (2 in 1)



www.hadooptpoint.com/finding-frequent-itemsets-using-hadoop-mapreduce-model/

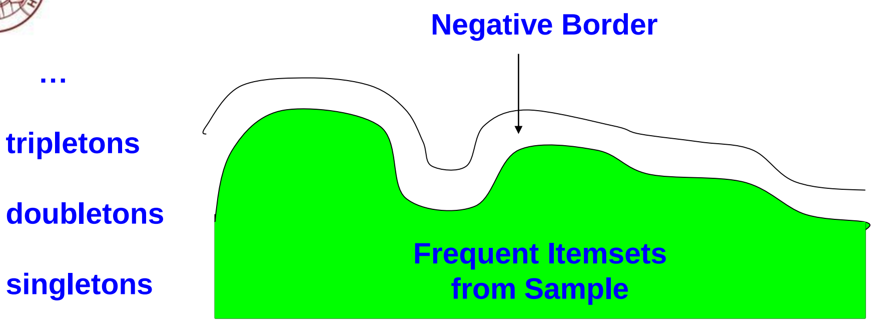
Toivonen's Algorithm

- Toivonen's algorithm is a *heuristic* algorithm for finding frequent itemsets from a given set of data
- Given sufficient main memory, uses one pass over a small sample and one full pass over data
  - Gives no false positives (always check against the whole)
- BUT, there is a small but finite probability it will fail to produce an answer
  - Will not identify frequent itemsets (false negatives)
- Then must be repeated with a different sample until it gives an answer
  - Need only a small number of iterations

Toivonen's Algorithm

- Start as in the random sampling algorithm, but lower the threshold slightly for the sample
  - For fraction p of baskets in sample, use 0.8ps (0.9ps) as support threshold
  - Example: if the sample is 1% of the baskets, use *s* /125 as the support threshold rather than *s* /100
- Goal: avoid missing any itemset that is frequent in the full set of baskets
  - The smaller the threshold the more memory is needed to count all candidate itemsets and the less likely the algorithm will not find an answer
- Add to the itemsets that are frequent in the sample their *negative border*
  - An itemset is in the negative border if it is not deemed frequent in the sample, but *all* its immediate subsets are (subset by deleting a single item)

Example: Negative Border



- *ABCD* is in the negative border if and only if:
  1. It is not frequent in the sample, but
  2. All of *ABC*, *BCD*, *ACD*, and *ABD* are
- *A* is in the negative border if and only if it is not frequent in the sample
  - Because the empty set is always frequent
  - Unless there are fewer baskets than the support threshold (silly case)

Toivonen's Algorithm

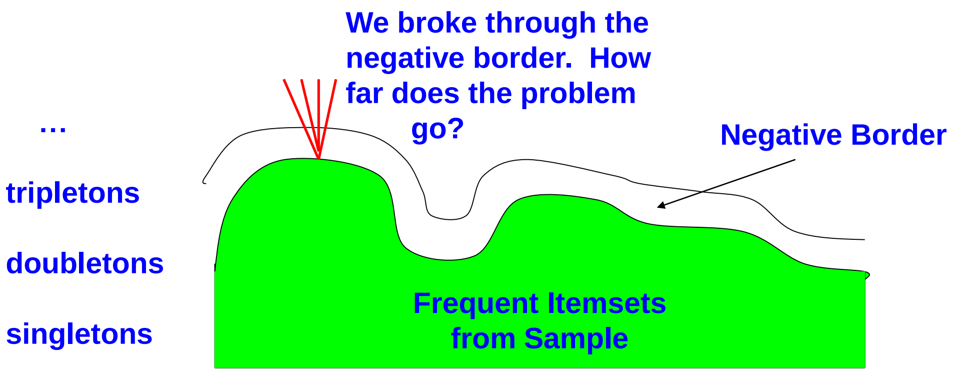
- In a second pass, count all candidate frequent itemsets from the first pass, and also count their negative border

- If no itemset from the negative border turns out to be frequent, then the candidates found to be frequent in the whole data are *exactly* the frequent itemsets

- What if we find that something in the negative border is actually frequent?
  - We must start over again!

- Try to choose the support threshold so the probability of failure is low, while the number of itemsets checked on the second pass fits in main-memory

If Something in the Negative Border is Frequent . . .


We broke through the negative border. How far does the problem go?
…
tripletons
doubletons
singletons
Negative Border
Frequent Itemsets from Sample

Theorem 1

- Given a data set D and a sample set S | S ≤ D, if there is an itemset T that is frequent in D but not frequent in S, then there is an itemset T' that is frequent in D and is in the negative border of S
  - False negatives appear in the negative border
- Proof: Suppose not; i.e.;
  1. There is an itemset *T* frequent in D but not frequent in S, and
  2. Nothing in the negative border is frequent in the whole
- Let *T* ' be a smallest subset of *T* that is not frequent in S
- All subsets of T are also frequent in the whole (*T* is frequent + monotonicity)
  - *T'* is frequent in the whole
- *Thus, T is in the negative border* (else not "smallest")

Theorem 2

- Given a data set D and a sample set S | S ≤ D, if there is an itemset T that is frequent in D and the negative border of S, then there is an itemset that is frequent in D and but not frequent in S
    - By definition, any itemset in the negative border of S is not frequent in S. Hence T is frequent in D but not frequent in S
- During the second pass of the algorithm, if we found an itemset T of the negative border to be frequent in D, then we can assume by this theorem that there is an itemset that is frequent in D but not frequent in S;
    - in such a case, we are forced to *restart the algorithm* as we have already failed to discover at least one itemset that is frequent in D
- If we found no itemset of the negative border to be frequent in D, then by the previous theorem we are permitted to terminate the algorithm as we have discovered all the frequent itemsets of D

Toivonen’s Algorithm

- Toivonen's algorithm is a powerful and flexible algorithm that provides a simplistic framework for discovering frequent itemsets in large data sets while also providing enough flexibility to enable performance optimizations directed towards particular data sets.
- Its deceptive simplicity allows us to discover all frequent itemsets through a sampling process
- Numerous optimizations and approximations can be made to improve the algorithm's performance on data sets with particular properties
    - For instance, using a slightly lowered threshold will minimize the omission of itemsets that are frequent in the entire dataset as such omissions result in additional passes through the algorithm
    - However, the support threshold should also be kept reasonably high so that the counts for the itemsets in the second pass in main memory

Summary

- Market-Basket Data and Frequent Itemsets
  - ◆Many-to-Many relationship
- Associating rules
  - ◆Confidence and Suport
- The A-Priori Algorithm
  - ◆Combine only frequent subsets
- The PCY algorithm
  - ◆Hash pairs to reduce candidates
- Multistage and Multihash algorithm
  - ◆Multiple hashes
- Randomized and SON algorithm
  - ◆Sample, Divide into Chunks and treat as samples by MapReduce
- Toivonen's Algorithm
  - ◆Negative Border

References

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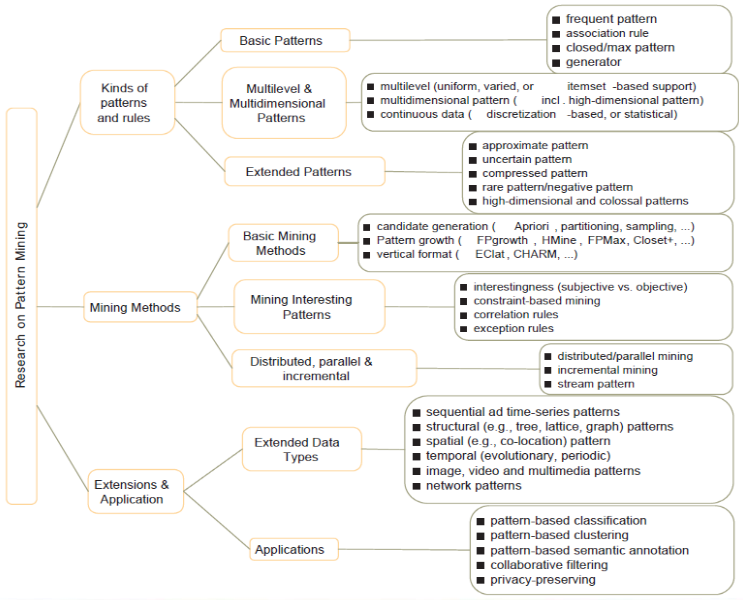
Research on Pattern Mining: A Road Map


Research on Pattern Mining
Kinds of patterns and rules
Basic Patterns
frequent pattern
association rule
closed/max pattern
generator
Multilevel & Multidimensional Patterns
multilevel (uniform, varied, or itemset -based support)
multidimensional pattern (incl . high-dimensional pattern)
continuous data (discretization -based, or statistical)
Extended Patterns
approximate pattern
uncertain pattern
compressed pattern
rare pattern/negative pattern
high-dimensional and colossal patterns
Mining Methods
Basic Mining Methods
candidate generation (Apriori , partitioning, sampling, ...)
Pattern growth (FPgrowth , HMine , FPMax, Closet+, ...)
vertical format (EClat , CHARM, ...)
Mining Interesting Patterns
interestingness (subjective vs. objective)
constraint-based mining
correlation rules
exception rules
Distributed, parallel & incremental
distributed/parallel mining
incremental mining
stream pattern
Extensions & Application
Extended Data Types
sequential ad time-series patterns
structural (e.g., tree, lattice, graph) patterns
spatial (e.g., co-location) pattern
temporal (evolutionary, periodic)
image, video and multimedia patterns
network patterns
Applications
pattern-based classification
pattern-based clustering
pattern-based semantic annotation
collaborative filtering
privacy-preserving