Worked examples of different Cost of Capital Calculations for Finance Exam

On the first three pages are the cost of capital questions – these just contain the information necessary for the cost of capital calculations.

The solutions start on page 4.

Company ungeared beta and WACC calculation

1. The risk free rate of interest is 5.35% and the market risk premium is 6.5%. The project will be funded 25% with debt and 75% with equity. The company has an asset beta of 1.1, and the debt beta is 0.2. The project is modest in proportion to the size of the company and it is in the same line of business and has the same financing proportions as the parent company. The tax rate is 30%.

   What is the WACC?

Company and project ungeared betas and WACC calculation

2. Tax is 30% and is paid in the year of the cash flow. The parent company has an equity beta of 1.1 and has 25% of its total value financed by debt. The debt has a beta of 0.15. The project will be funded 35% with debt. The parent company's revenues fluctuate by 1.05 times that of overall economic activity whereas with the project the volatility is 1.25 times that of economic activity. The project's debt beta is 0.5.

   The risk free rate of interest is 5% and the stock market risk premium is 6%. What is the WACC?

Company and project ungeared betas and WACC

3. XYZ has an equity beta of 1.4 and is financed 80% by equity and 20% by debt. The beta of debt is 0.1. The risk free rate of interest in the UK is 5% and the stock market risk premium is 6%. The UK company tax rate is 30%.

   The project will be financed with 40% debt and 60% equity. The beta of the debt for the project will be 0.25. The project will have fixed costs of 55% of cash flows, whereas the parent company has fixed costs of only 45% of cash flows.

What is the WACC?

Debt rate using Yield to Maturity, then WACC

4. Olympian plc is a quoted optical instrument company in the UK stock market. The beta of the company's shares is 1.35. The company has a bond outstanding that has exactly seven years to maturity. It has a coupon of 7%, and a current price of £92.32 (the par value is £100). The risk free rate of interest is 5.5% and the stock market risk premium is 6.5%.

    The company has a debt to equity ratio of 40:60, and the company tax rate is 30%. The company is undertaking a project in its normal line of business, which will involve an outlay of £75m today and produce free cash flows in today's money of £20m per year for seven years.

    What is the cost of equity, cost of debt and weighted average cost of capital for Olympian?

Leasing cost of capital

5. The machine costs £60,000 and will have a useful life of 6 years. It will be depreciated down to zero value on a straight line basis. The company faces a tax rate of 30%. If the company leases the machine it will have to make seven lease payments of £10,750 with the first payment being made now. The company can borrow at a rate of 9%.

    The company has a target debt to total capital ratio of 0.4. The company has an equity beta of 1.25, its share price is 360p and the dividends have been growing at a rate of 5% per annum recently. The risk free rate of interest is 5% and the stock market equity risk premium is 6%.

    What is the cost of capital that you will use?

Using a Proxy company to calculate cost of capital

6. The project will be funded as always with JJB Packaging from retained earnings. JJB faces a tax rate of 30%. JJB is a private company.

There is a company very similar to JJB Packaging listed on the stockmarket. It has 36% debt in its total capital structure, and the debt has a beta of 0.25. The risk free rate of interest is 6% and the stock market risk premium is 6.5%. The stockmarket company has an equity beta of 1.3.

International project – cost of capital

7. Suntona plc. is a UK specialist retailer and is considering a project in the US. There is no similar US company to Suntona. Suntona has an asset beta of 1.10, and a debt beta of 0.25. The stock market risk premium is 6% in both the UK and the US. Suntona's target debt-equity ratio is 40:60. The company's pre-tax cost of debt is 6.00%. The UK has a one year risk free rate of 4.50% and it is 2.75% in the US, and the term structure is flat in both countries over the next five years. Suntona's tax rate is 30% (assume US tax rate also 30%). The spot exchange rate is $1.90/£1.

Cost of Capital – Worked Solutions

Company ungeared beta and WACC calculation

1. The risk free rate of interest is 5.35% and the market risk premium is 6.5%. The project will be funded 25% with debt and 75% with equity. The company has an asset beta of 1.1, and the debt beta is 0.2. The project is modest in proportion to the size of the company and it is in the same line of business and has the same financing proportions as the parent company. The tax rate is 30%.

   What is the WACC?

   Solution

   You are given; rf, (rm-rf), project (D/(E+D)) & (E/(E+D), company asset beta (β_u) & debt beta (β_d). With (rm – rf), if the question says 'equity risk premium' or stock market risk premium' or 'market risk premium', it means this calculation has already been done and you are being given (rm – rf).

   You are told that the project is in the same line of business as parent, and has the same financing.

   What is missing? The equity beta (β_e) is missing and is needed to calculate the cost of equity (re).

   How do we obtain β_e?

   You need to ungear the company beta. Use the formula;

   $\beta_u = (\beta_e*(E/(E+D)) + (\beta_d*(D/(E+D))$ [this formula is given in the exam formula sheet]

   So, using the numbers provided in the question;

   *1.1 = 0.75 β_e + (0.25 * 0.2)*

   0.75 β_e = 1.1 – 0.05

   β_e = 1.05/0.75

   β_e = 1.4

   Now we have the equity beta and the debt beta, so the CAPM can be used to calculate the cost of equity and the debt rate.

   re = rf + β_e(rm – rf)

   re = 5.35 + 1.4(6.5) = 14.45%

   rd = rf + β_d(rm – rf)

rd = 5.35 + 0.2(6.5) = 6.65%

And then the WACC;

WACC = (re * (E/(E+D)) + (rd * (D/(E+D)) * (1 – tax)

WACC = (14.45 * 0.75) + (6.65 * 0.25)*(0.70)

WACC = 10.8375 + 1.16375

WACC = 12.00125

WACC = 12%

Company and project ungeared betas and WACC calculation

2. Tax is 30% and is paid in the year of the cash flow. The parent company has an equity beta of 1.1 and has 25% of its total value financed by debt. The debt has a beta of 0.15. The project will be funded 35% with debt. The parent company's revenues fluctuate by 1.05 times that of overall economic activity whereas with the project the volatility is 1.25 times that of economic activity. The project's debt beta is 0.5.

   The risk free rate of interest is 5% and the stock market risk premium is 6%. What is the WACC?

   Solution

   You are given Parent company equity and debt betas, the debt weight in the capital structure. You are given the debt weight in the project financing and the project's debt beta. You are also given information on the differences in risk between the parent and the project with the revenue volatility data.

   What do you need to do?

   You need to work out the parent ungeared beta and then adjust for the differences in risk.

   Use the ungeared beta formula;

   $\beta_u = (\beta_e *(E/(E+D)) + (\beta_d*(D/(E+D))$

   and using the numbers provided in the question;

   $\beta_u = (1.1 * 0.75) + (0.15 * 0.25)$

   $\beta_u = 0.825 + 0.0375$

   $\beta_u = 0.8625$

This ungeared beta represents the basic operating risk of the parent company, it excludes the impact of debt. We can now adjust this to obtain the ungeared beta of the project and then use that to calculate the equity beta.

The formula for adjusting the company ungeared beta is;

Company ungeared beta * Project revenue volatility/Company revenue volatility

$\beta_{u(project)} = 0.8625 * 1.25/1.05$

$\beta_{u(project)} = 1.02678$

This is the ungeared beta for the project and represents the business risk or operating risk of the project. If the project were just funded with equity (100% equity), you could use this beta directly in the CAPM calculation for the cost of equity (re).

In this project, there is debt funding, so we need an equity beta and debt beta to calculate the costs of equity and debt. The question gives the debt beta (0.5) and the debt weight (35%). We can now use the ungeared beta formula to calculate the missing equity beta;

$\beta_u = (\beta_e*(E/(E+D))) + (\beta_d*(D/(E+D)))$

and if we insert the known figures;

$1.02678 = (0.65*\beta_e) + (0.35*0.5)$

$0.65\beta_e = 1.02678 – 0.175$

$\beta_e = 0.85178/0.65$

$\beta_e = 1.31$

Now we have the equity and debt betas and the weights of debt and equity, so the cost of equity and the debt rate can be calculated;

$re = rf + \beta_e(rm – rf)$

$re = 5 + 1.31(6) = 12.86\%$

$rd = rf + \beta_d(rm – rf)$

$rd = 5 + 0.5(6) = 8\%$

Then calculate the WACC;

$WACC = (re * (E/(E+D))) + (rd * (D/(E+D))) * (1 – tax)$

$WACC = (12.86 * 0.65) + (8*0.35)*(1-0.30)$

$WACC = 8.359 + 1.96$

$WACC = 10.319\%$

Company and project ungeared betas and WACC

3. XYZ has an equity beta of 1.4 and is financed 80% by equity and 20% by debt. The beta of debt is 0.1. The risk free rate of interest in the UK is 5% and the stock market risk premium is 6%. The UK company tax rate is 30%.

   The project will be financed with 40% debt and 60% equity. The beta of the debt for the project will be 0.25. The project will have fixed costs of 55% of cash flows, whereas the parent company has fixed costs of only 45% of cash flows.

   What is the WACC?

   Solution

   You are given parent company equity and debt betas and the weights of debt and equity. You are given the project debt and equity weights and the debt beta of the project. You are also given information on the adjustment needed to the parent company ungeared beta to adjust for the differences in risk between the parent and project, with the fixed cost adjustments.

   What do we need to do? Firstly, ungear the parent company beta;

   $$\beta_u = (\beta_e*(E/(E+D))) + (\beta_d*(D/(E+D)))$$

   and using the numbers provided in the question;

   $$\beta_u = (1.4*0.80) + (0.1*0.2)$$

   $$\beta_u = 1.12 + 0.02$$

   $$\beta_u = 1.14$$

   Now we need to adjust for the differences in the fixed costs of the parent and project;

   *Project β_u = Company β_u * (1+project fixed costs)/(1+company fixed costs)*

   *Project β_u = 1.14 * 1.55/1.45*

   Project β_u = 1.2186

   This is the ungeared beta for the project, representing the operating or business risk of the project. If the project is part funded with debt, then we need the equity and debt betas of the project to calculate the costs of equity and debt.

   The question gives the weights of equity and debt and gives the debt beta, so it is just the equity beta that is missing. Use the ungeared beta formula to find the equity beta;

   $$\beta_u = (\beta_e*(E/(E+D))) + (\beta_d*(D/(E+D)))$$

   $$1.2186 = (\beta_e * 0.6) + (0.25 * 0.4)$$

0.6 β_e = 1.2186 – 0.1

β_e = 1.1186/0.6

β_e = 1.8643

Having the equity and debt betas for the project will allow the cost of equity and the debt rate for the project to be calculated;

re = rf + β_e(rm – rf)

re = 5 + 1.8643(6) = 16.19%

rd = rf + β_d(rm – rf)

rd = 5 + 0.25(6) = 6.5%

Then calculate the WACC;

*WACC = (re * (E/(E+D)) + (rd * (D/(E+D)) * (1 – tax)*

*WACC = (16.19 * 0.6) + (6.5 * 0.4) * (1- 0.30)*

WACC = 9.71 + 1.82

WACC = 11.53%

Debt rate using Yield to Maturity, then WACC

4. Olympian plc is a quoted optical instrument company in the UK stock market. The beta of the company's shares is 1.35. The company has a bond outstanding that has exactly seven years to maturity. It has a coupon of 7%, and a current price of £92.32 (the par value is £100). The risk free rate of interest is 5.5% and the stock market risk premium is 6.5%.

   The company has a debt to equity ratio of 40:60, and the company tax rate is 30%. The company is undertaking a project in its normal line of business, which will involve an outlay of £75m today and produce free cash flows in today's money of £20m per year for seven years.

   What is the cost of equity, cost of debt and weighted average cost of capital for Olympian?

   Solution

   In the question you are given the equity beta, risk free rate and market risk premium, so you can use the CAPM for the cost of equity. For debt you are given the bond price, the bond coupon and the term to maturity of the bond. What this means is that you have to calculate the yield to maturity to get the debt rate. The YTM is the debt rate, because if the company wanted to issue another seven year bond, it couldn't do it with a coupon of

7%. It could only do that if the bond was priced at 100. The bond is priced below 100, so it means that the overall return on the bond if bought today and held to maturity will be greater than 7%, because you will make a capital gain when the bond is bought back off you at 100.

To calculate YTM you need to use trial and error (it is the same calculation as internal rate of return, IRR). But you can make a rough guess to begin with so your rates are closer to the actual YTM. If you had bought the bond your return would be a mix of income and capital gain or loss. With this bond your return will be £7 per annum for income and about £1 per annum capital gain ((100 – 92.32)/7). So you will make about £8 per year on the bond, say 8%, so your guesses should be plus 1% and minus 1% from that rate. So use 7% and 9%. What you want is one positive answer and one negative.

The cash flows for the calculation are;

(92.32) + 7 + 7 + 7 + 7 + 7 + 7 + 107

These cash flows have to be discounted first by 7% then 9%. A quick way is to value the coupon cash flows as an annuity and the principal repayment as a single present value calculation.

Using 7%

$= PVAIF_{7\%, 7} * 7 = 5.3893 * 7 = 37.73$ >> using Appendix 1, Table 2

and $PVIF_{7\%,7} * 100 = 0.6227 = 62.27$ >> using Appendix 1, Table 1

Add these two together $= 100$

The 7% is the original coupon on the bond, so discounting the coupon cash flows by 7% will give a value of 100 (discounting any bond cash flows by the original coupon rate will give 100).

The 100 is too high compared to the 92.32, so we need the higher rate to get a lower value.

Using 9%

$= PVAIF_{9\%,7} * 7 = 5.0330 * 7 = 35.23$

And $PVIF_{9\%,7} *100 = 0.5470 = 54.70$

Add together $= 89.93$

So with 7%, the rate is too low and with 9% it is too high. The YTM will be inbetween 7% and 9%. Work out the difference between each rate's answer and the bond price;

7%: $100 – 92.32 = 7.68$

9%: $92.32 – 89.93 = 2.39$

In the YTM calculation you ignore the sign, so both answers above are entered into the calculation as positive values. You are trying to work out the distance the point will be between 7% and 9%.

The YTM calculation is;

*YTM = r1 + (((NPV1/(NPV1 + NPV2)) * (r2 – r1))*

Where r1 = first guess, r2 = second guess, NPV1 = difference for rate 1, NPV2 = difference for rate 2 (this is the difference between the actual bond price and the bond price calculated at that particular rate).

*YTM = 7 + ((7.68/(7.68 + 2.39)) * (9 – 7))*

*YTM = 7 + (0.763 * 2)*

YTM = 7 + 1.526

YTM = 8.526%

The cost of equity is obtained using CAPM;

re = rf + β$_e$(rm – rf)

re = 5.5 + 1.35(6.5) = 14.275%

With the cost of equity and the YTM, the WACC can be calculated;

*WACC = (re * (E/(E+D)) + (rd * (D/(E+D)) * (1 – tax)*

*WACC = (14.275 * 0.6) + (8.526 * 0.4) * (1 – 0.30)*

WACC = 8.565 + 2.387

WACC = 10.95%

Leasing cost of capital

5. The machine costs £60,000 and will have a useful life of 6 years. It will be depreciated down to zero value on a straight line basis. The company faces a tax rate of 30%. If the company leases the machine it will have to make seven lease payments of £10,750 with the first payment being made now. The company can borrow at a rate of 9%.

   The company has a target debt to total capital ratio of 0.4. The company has an equity beta of 1.25, its share price is 360p and the dividends have been growing at a rate of 5% per annum recently. The risk free rate of interest is 5% and the stock market equity risk premium is 6%.

   What is the cost of capital that you will use?

Solution

*The cost of capital for lease cash flows is the after tax cost of debt. Here that is 9% * (1 – 0.30) = 6.3%.*

If there was a salvage value for the equipment, that would be discounted by the WACC, and here you just need to work out the cost of equity (as the debt rate is given);

re = 5 + 1.25(6)   = 12.5% (the dividend information is incomplete and can't be used to calculate re. You would need to know what the next dividend is going to be to use that method).

The WACC is

*WACC = (re * (E/(E+D)) + (rd * (D/(E+D)) * (1 – tax)*

*WACC = (12.5 * 0.6) + (9 * 0.4) * (1 – 0.30)*

WACC = 7.5 + 2.52

WACC = 10.02%

Using a Proxy company to calculate cost of capital

6. The project will be funded as always with JJB Packaging from retained earnings. JJB faces a tax rate of 30%. JJB is a private company.

   There is a company very similar to JJB Packaging listed on the stockmarket. It has 36% debt in its total capital structure, and the debt has a beta of 0.25. The risk free rate of interest is 6% and the stock market risk premium is 6.5%. The stockmarket company has an equity beta of 1.3.

   Solution

   You are told that JJB is a private company, and that the project will be all equity financed. Because JJB is private, neither the CAPM nor the dividend growth model can be used directly to calculate the cost of equity. But there is a proxy company that does the same activity as JJB and is quoted on the stock market, so its information can be used to construct a cost of capital for JJB.

   The proxy company will have the same operating risk as JJB, so we need to ungear its beta;

   $\beta_u = (\beta_e*(E/(E+D)) + (\beta_d*(D/(E+D))$

   $\beta_u = (1.3 * 0.64) + (0.25 * 0.36)$

   $\beta_u = 0.832 + 0.09$

   $\beta_u = 0.922$

Because JJB is all equity financed we can use this ungeared beta directly to calculate the cost of equity;

re = 6 + 0.922(6.5) = 11.99%

This is the cost of capital for the project.

International project – cost of capital

7. Suntona plc. is a UK specialist retailer and is considering a project in the US. There is no similar US company to Suntona.   Suntona has an asset beta of 1.10, and a debt beta of 0.25. The stock market risk premium is 6% in both the UK and the US. Suntona's target debt-equity ratio is 40:60. The company's pre-tax cost of debt is 6.00%. The UK has a one year risk free rate of 4.50% and it is 2.75% in the US, and the term structure is flat in both countries over the next five years. Suntona's tax rate is 30% (assume US tax rate also 30%). The spot exchange rate is $1.90/£1.

    Solution

    In this question it is a UK company doing a project in the US. First thing to do is to work out the UK WACC.

    You are given the debt and equity weights and the ungeared (asset) beta and the debt beta. What is missing is the equity beta, so use the ungeared beta formula to solve for the missing equity beta;

    $\beta_u = (\beta_e*(E/(E+D))) + (\beta_d*(D/(E+D)))$

    1.10 = (β_e 0.6) + (0.25 * 0.4)*

    0.6β_e = 1.10 = 0.1

    β_e = 1.0/0.6

    β_e = 1.667

    The cost of equity and the debt rate can be calculated;

    re = 4.5 + 1.667(6)   = 14.5%

    rd = 4.5 + 0.25(6)   = 6%

    The WACC can now be calculated as we have the debt and equity return rates and the weights of debt and equity;

    *WACC = (re * (E/(E+D)) + (rd * (D/(E+D)) * (1 – tax)*

    *WACC = (14.5 * 0.6) + (6 * 0.4) * (1 – 0.30)*

WACC = 8.7 + 1.68

WACC = 10.38%

This is the WACC for the company in the UK and would be used to discount cash flows for UK projects. This project is in the USA, so the WACC has to be adjusted for the risk difference between the US & UK.

This is done by multiplying (1 + UK WACC) by the interest rate differential between UK & US;

(1 + US WACC) = (1 + UK WACC) * (1+Foreign Interest rate)/(1+Domestic Interest rate)

The Domestic interest rate is the interest rate of the country with the single (1) unit in the forex quote (here the quote is $1.90/£1, so the £ is the Domestic and the US is the Foreign).

(1 + US WACC) = 1.1038 * (1.0275)/(1.045)

(1 + US WACC) = 1.1038 * 0.98325

(1 + US WACC) = 1.0853

US WACC = 8.53%

(REMEMBER to Multiply (1+UK WACC), not just the (UK WACC). If you multiply the (UK WACC) by the interest rate differential, there will be hardly any difference and you will have the wrong WACC and get the wrong NPV. A quick check to see if you are roughly correct is to take the difference between the two interest rates, (4.50 – 2.75 =) 1.75 and check that the difference between the UK WACC and the US WACC is around that level (here it is 10.38 – 8.53 = 1.85. That is just a quick check – if you had multiplied the UK WACC by the interest rate differential you would have 10.21% as your US WACC – completely wrong (10.38 * (1.0275/1.045) = 10.21).