CS361

(Software Engineering Program)

Artificial Intelligence II - Applied Machine Learning

Lecture 3

Evolutionary Computation: Differential Evolution as an Optimization Method

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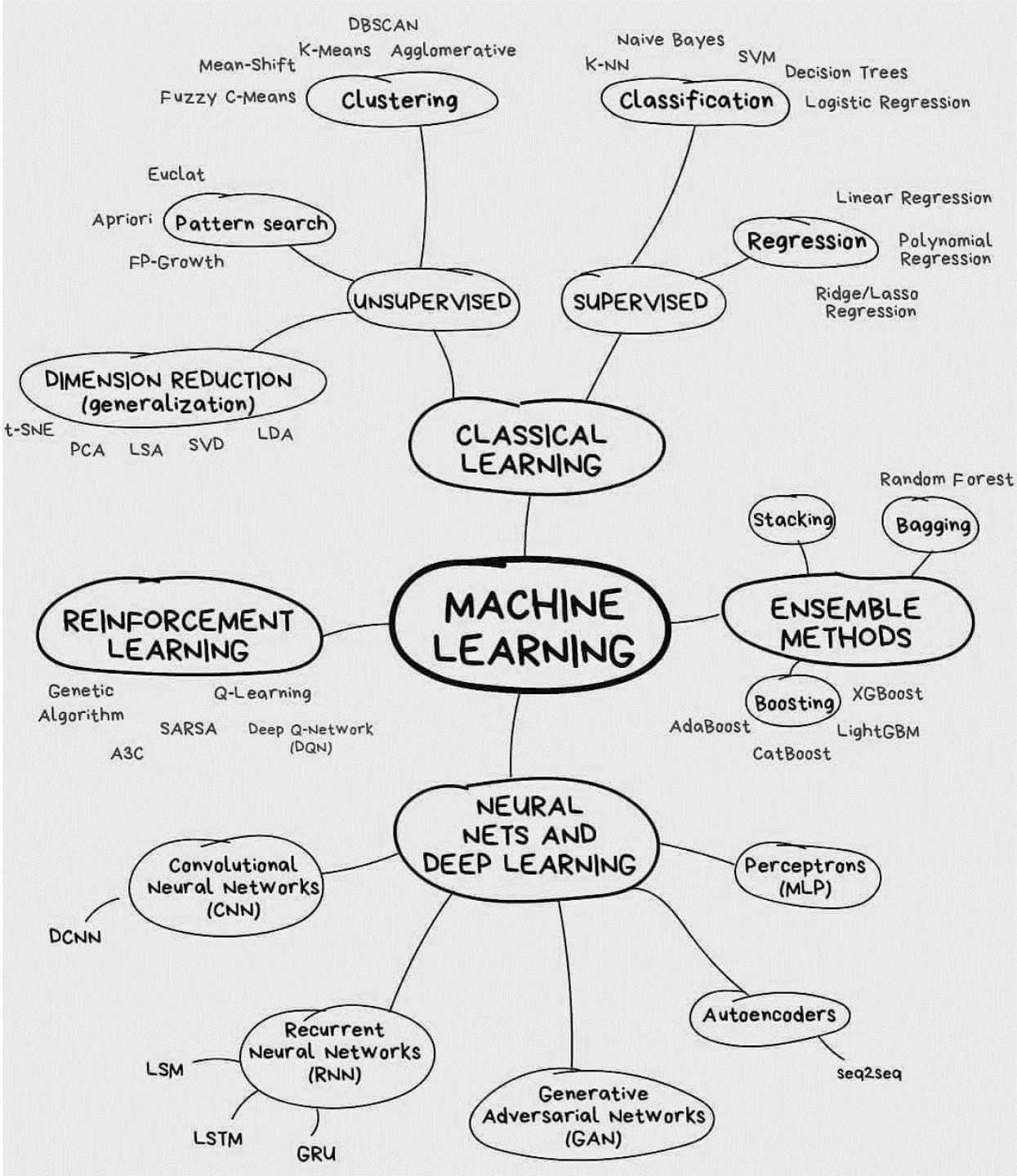
Lecture is based on its counterparts in the following courses (and the following resources):

- *Evolutionary Computing*, **University of Vaasa (Finland),** Electrical & Energy Eng.
- *Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces*, **Journal of Global Optimization** 11: 341–359, 1997.
- *Data Clustering Method based on Mixed Similarity Measures*, **(ICORES 2017),** 192-199
- *Differential Evolution,* **University of Colorado (USA),** Colorado Springs.

Machine Learning?

{*Artificial Intelligence*}
Machine Learning Map

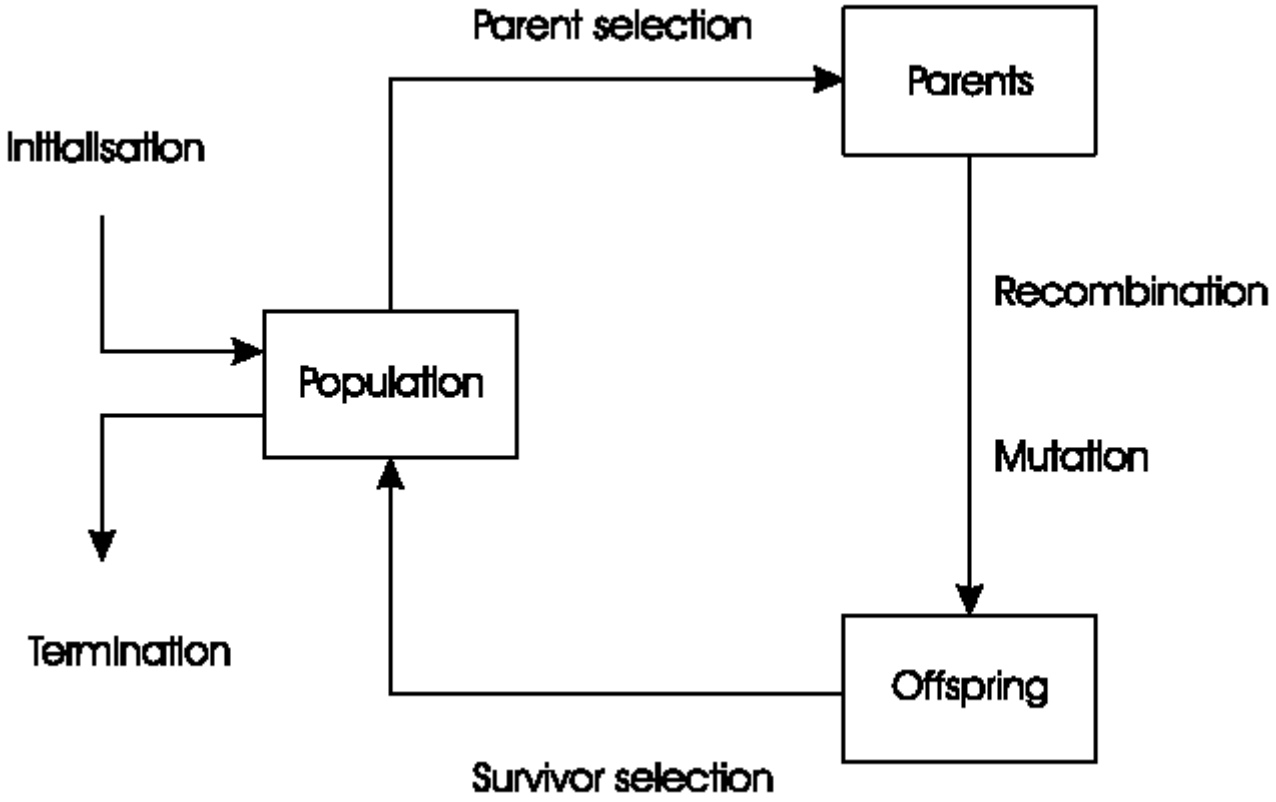


Recap:
Evolutionary Computation
Genetic Algorithms

Evolutionary Computation (EC)

- Evolutionary Computing *(or Evolutionary Algorithms "EA")* is a field of science and engineering that tries to apply some phenomena that appears in the nature to the optimization.
- Most notable is the adaptation of Charles Darwin's evolution theory in order to solve difficult search and optimization tasks.
- The method is universally applicable, since it has been successfully applied to almost all thinkable search & optimization problems in engineering, science & people's everyday life.

General Scheme of Genetic Algorithms

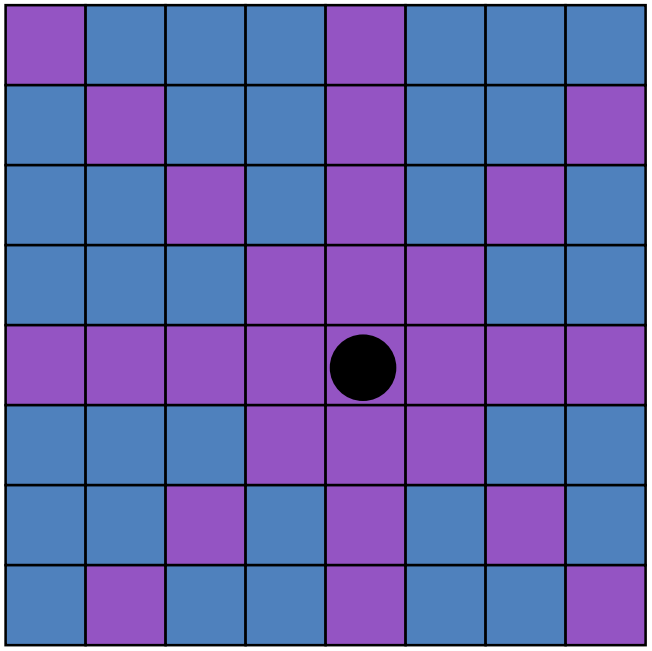


Pseudocode for a Typical Genetic Algorithm (GA)

```
BEGIN
  INITIALISE population with random candidate solutions;
  EVALUATE each candidate;
  REPEAT UNTIL ( TERMINATION CONDITION is satisfied ) DO
    1 SELECT parents;
    2 RECOMBINE pairs of parents;
    3 MUTATE the resulting offspring;
    4 EVALUATE new candidates;
    5 SELECT individuals for the next generation;
  OD
END
```

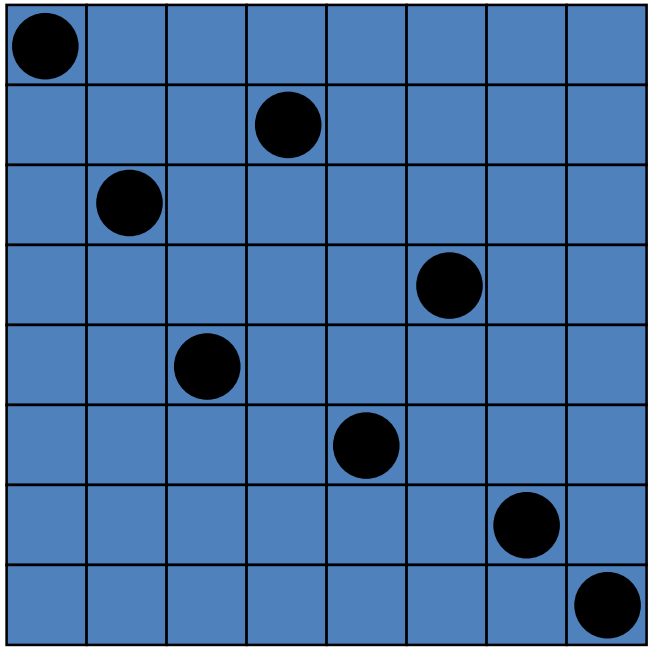
Example: The 8 Queens Problem ..

- Place 8 queens on an 8x8 chessboard in such a way that they cannot check each other.



The 8 Queens Problem: *Representation ..*

Phenotype:
A board's configuration ..



Genotype:
A permutation of
the numbers 1 : 8

Obvious mapping

1	3	5	2	6	4	7	8

The 8 Queens Problem: *Fitness Function (Evaluation) ..*

- Penalty of one queen: the number of queens she can check.
- Penalty of a configuration: the sum of the penalties of all queens.
- Note: penalty is to be minimized
- Fitness of a configuration: inverse penalty to be maximized

The 8 Queens Problem: *Parents' Selection for Recombination (Crossover)..*

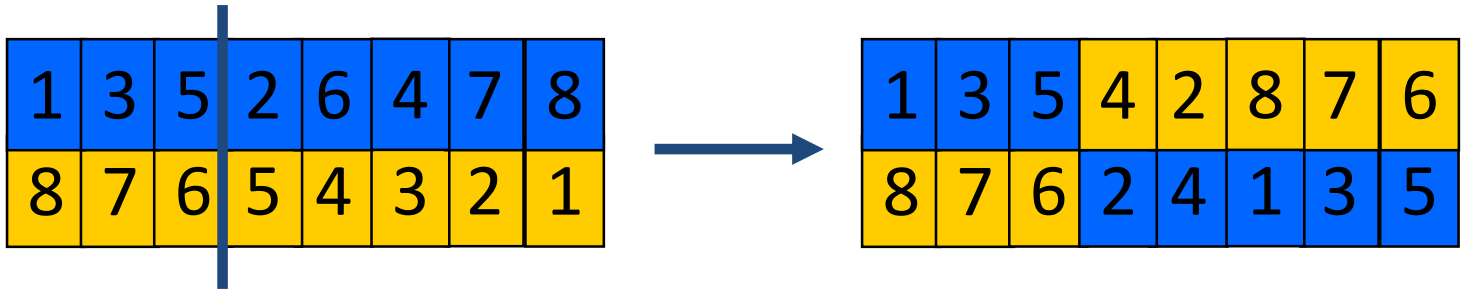
Selecting Parents for Crossover/Recombination:

- For e.g., by picking randomly 5 parents, and then select the best two to undergo crossover.

The 8 Queens Problem: *Recombination (Crossover) ..*

Combining two chromosomes *(in this case, permutations)* into two new permutations:

- Choose a random crossover point ..
- Copy first parts into both children ..
- Create the second part by inserting values from the other parent:
  - In the order they appear there (at that parent) ..
  - Beginning after the selected crossover point .. and ..
  - Skipping values already in the child.



The 8 Queens Problem: *Mutation ..*

- Small variation in one permutation:
- *For e.g.*, swapping values of two randomly chosen positions.

The 8 Queens Problem: *Parents' Selection for Replacement ..*

Survivors' Selection *(Replacement; "Creating a new generation")*

.. For Example: when inserting a new child into the population, choose an existing member to replace by:

- Sorting the whole population by decreasing fitness.
- Enumerating this list from high to low.
- Replacing the first with a fitness lower than the given child.

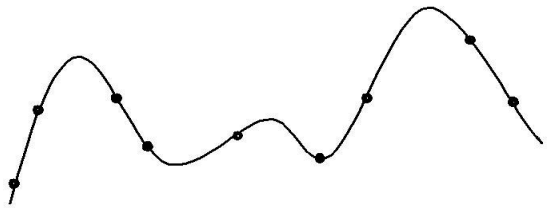
The 8 Queens Problem: *Summary ..*

Representation	Permutations
Recombination	"Cut-and-crossfill" crossover
Recombination probability	100%
Mutation	Swap
Mutation probability	80%
Parent selection	Best 2 out of random 5
Survival selection	Replace worst
Population size	100
Number of Offspring	2
Initialisation	Random
Termination condition	Solution or 10,000 fitness evaluation

Note that this is ***only one possible***
set of choices of operators and parameters.

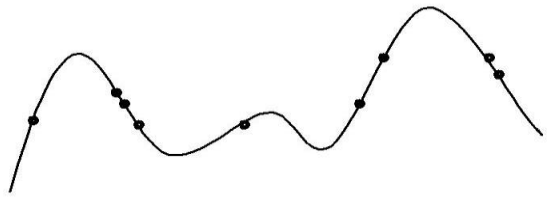
Typical Behaviour of an EA ..

Phases in optimising on a 1-dimensional fitness landscape:



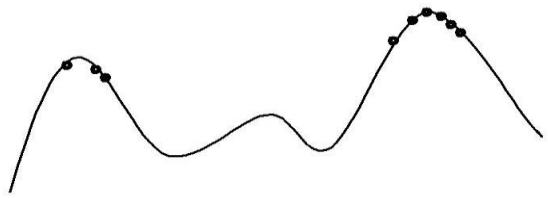
An Early Phase:

Quasi-random population distribution.



A Middle Phase:

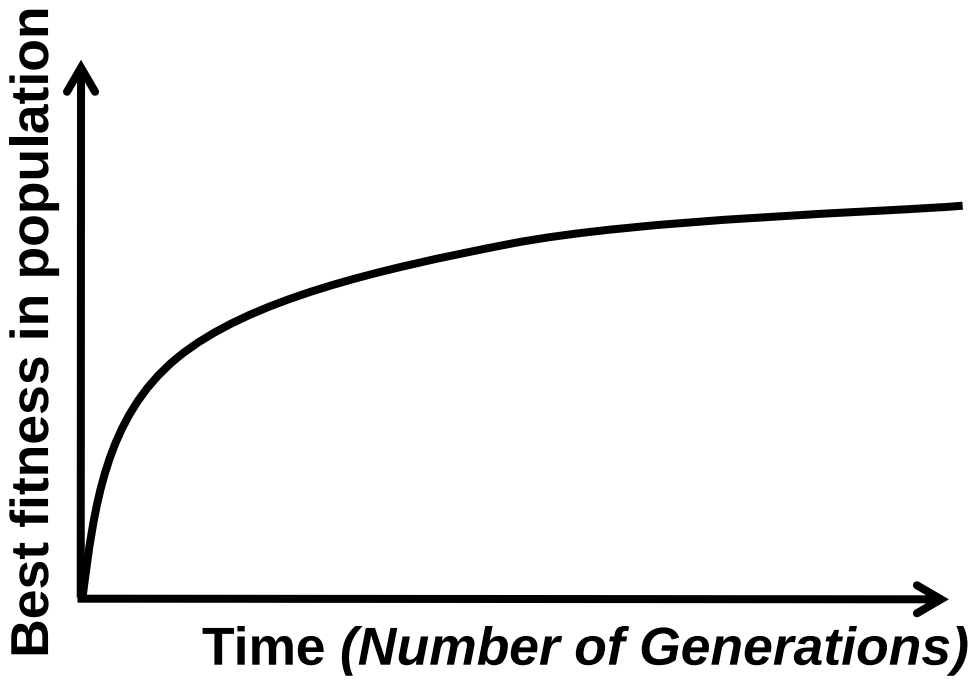
Population arranged around/on hills.



A Late phase:

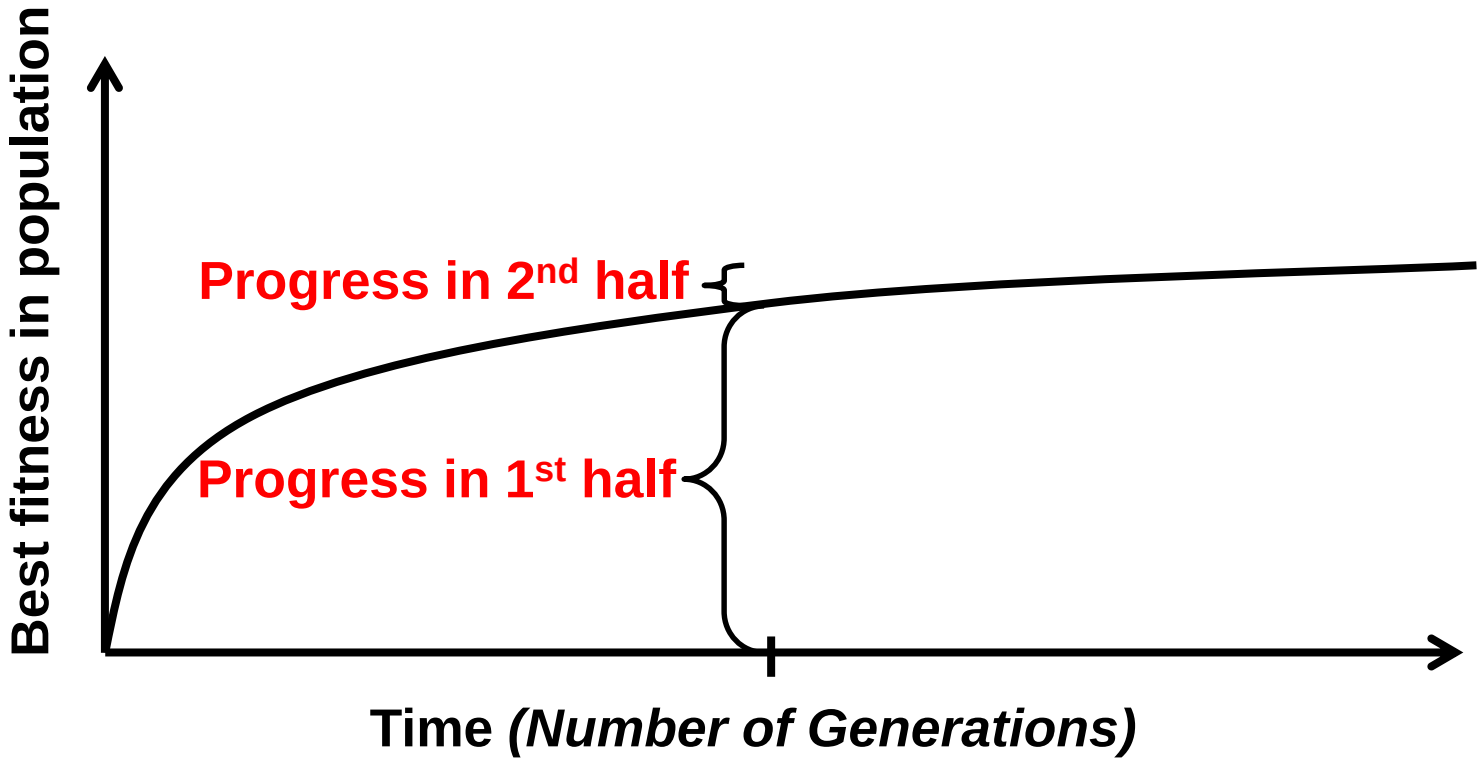
Population concentrated on high hills.

Typical Run: Progression of Fitness



Typical run of an EA shows so-called “anytime behavior” ..

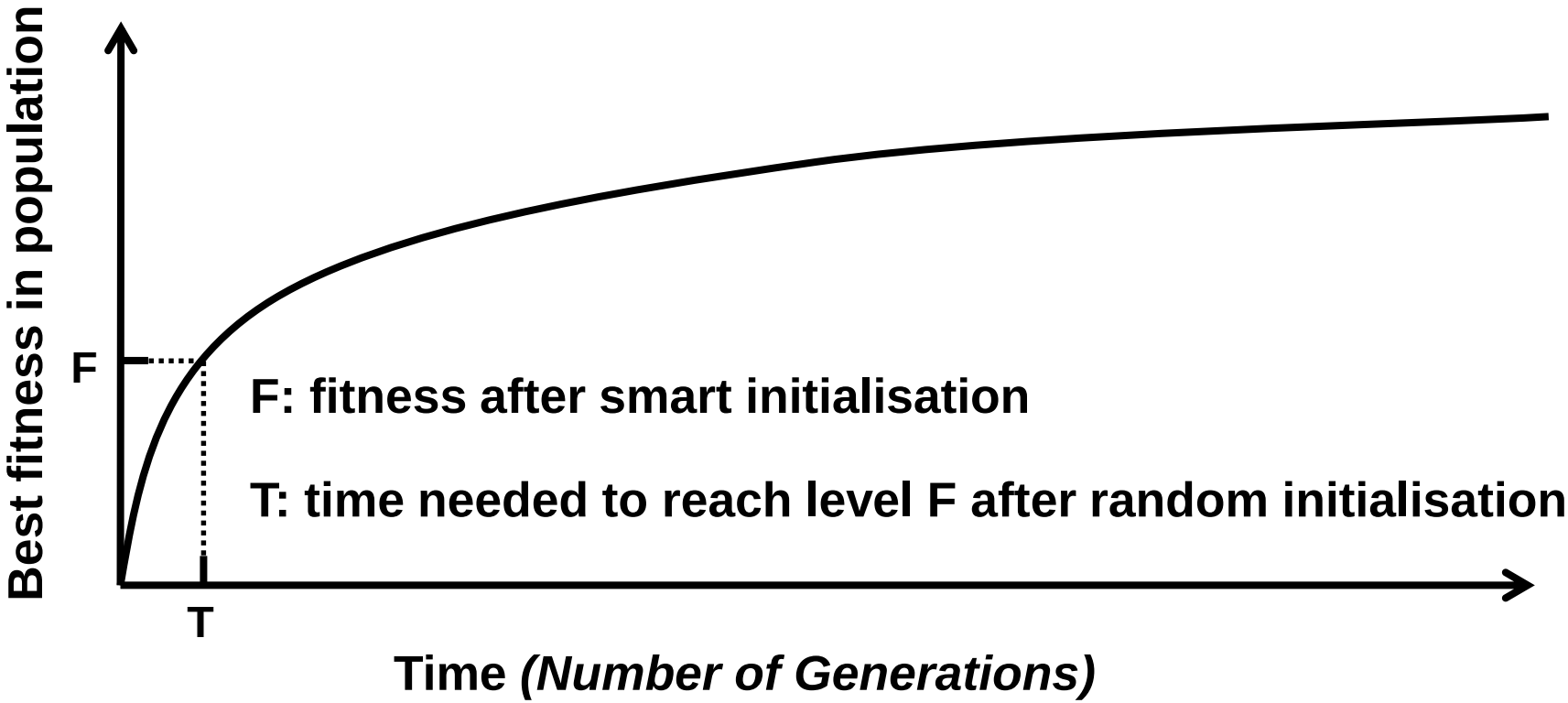
Are long runs beneficial?



Answer:

- *it depends on how much you want the last bit of progress ..*

Is it worth spending effort on smart initialisation?



Answer:

- it depends. Possibly, if good solutions/methods exist ..

Evolutionary Algorithms

Differential Evolution

A generic Nature-inspired population-based global-search metaheuristic optimization algorithm.

Based on vector differences and is therefore primarily suited for numerical optimization problems.

Requirements on a Practical Minimization Technique

- Ability to **handle** *non-differentiable*, *nonlinear*, and *multimodal* cost functions.
- **Parallelizability** to cope with computation intensive cost functions.
- **Ease of use**, in particular, there should be few control variables or parameters.
- **Good convergence** properties, i.e., consistent convergence to the global minimum (optimum) in consecutive independent trials.

Motivation for Introducing Differential Evolution (DE)

- DE was designed to be a stochastic direct search method. In other words, it satisfies the first requirement since it can be used to minimize any function for which a global minimum exists.
- DE uses a population of vectors where the stochastic perturbation of the vectors can be done independently. In other words, parallelization is easy.

Motivation for Introducing Differential Evolution (DE)

- **Ease of use:** To satisfy the third requirement, it is good if the minimization method is self-organizing so that it requires very little input from the user. DE alters the search space using information from within the vector population *(like Genetic Algorithms)*. DE's self-organizing scheme takes the difference of two randomly chosen population vectors to perturb an existing vector. The perturbation is done to every population vector.
- **Convergence:** DE has been found to converge very well in a large number of practical problems although theoretical discussions are not extensive.

Differential Evolution: *Basic Components ..*

- DE is a **nature-based** algorithm that **belongs to the class of genetic algorithms** since it uses selection, crossover, and mutation operators to optimize (**using a global-search metaheuristic**) an objective function over the course of successive generations (Suresh et al., 2009).
- DE is a **parallel population-based** direct search method where the population is comprised of ***NP* vectors** each of **dimension *D***, and is **primarily suited for numerical optimization** problems.
  - *NP* does not change during the algorithm's life time.
- The **initial population is chosen randomly** and should cover the entire parameter space.

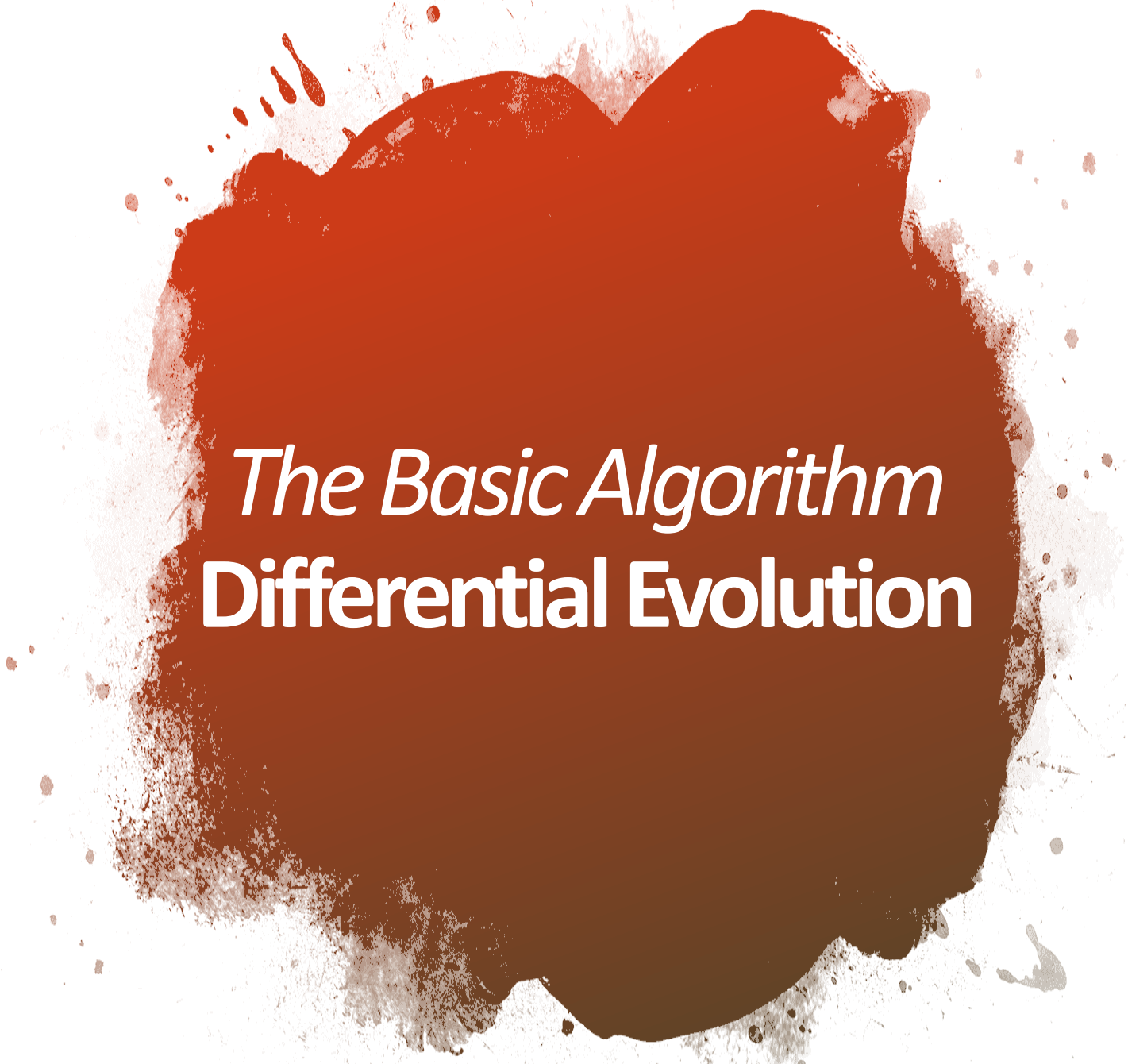
Differential Evolution: *Basic Components ..*

- **Mutation:** DE generates new individuals in the population by adding weighted difference between two population vectors to a third vector (the mutated vector).

- **Crossover:** The mutated vector's parameters are then mixed with the parameters of another pre-determined vector, the target vector, to yield a trial vector.

- **Selection:** If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector in the next generation.

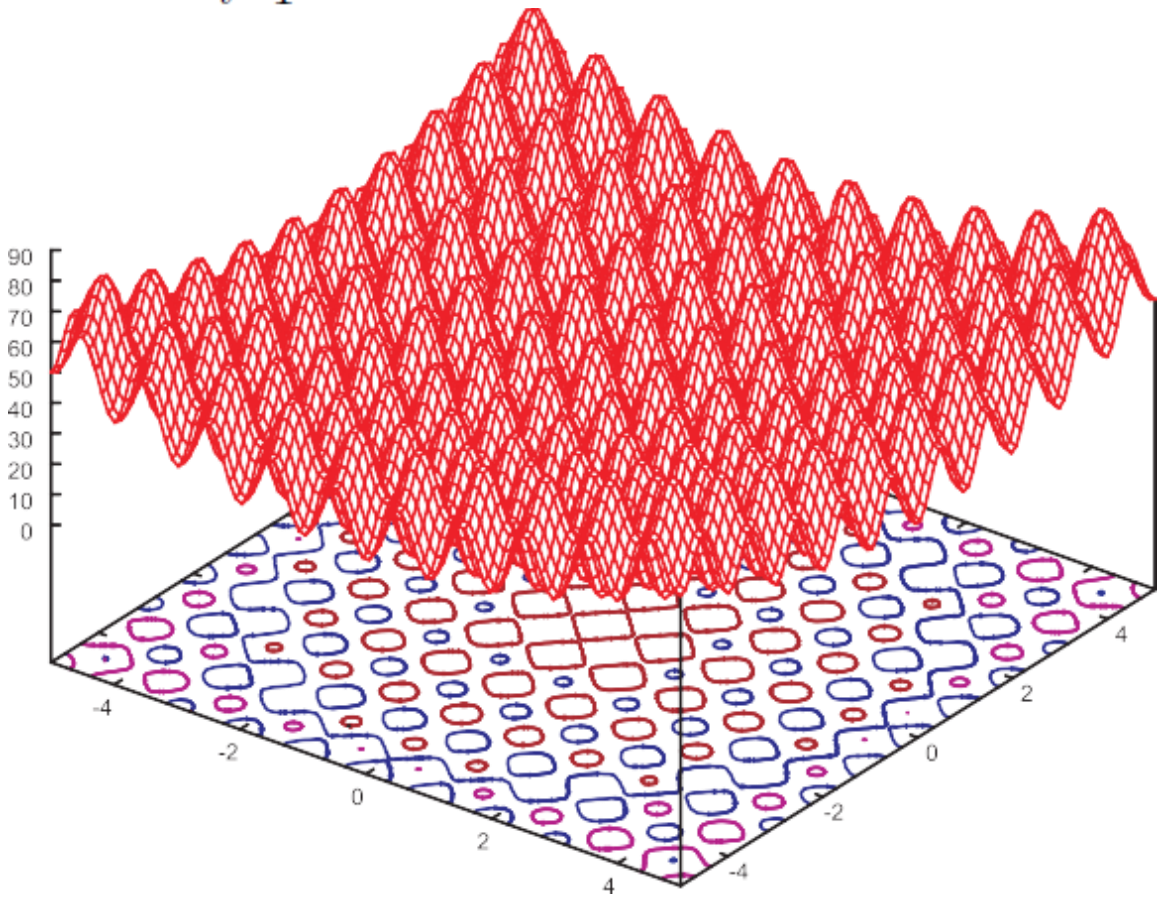
- Each population vector must serve once as the target vector so that NP competitions take place in one generation.


The Basic Algorithm
Differential Evolution

A Numerical Optimization Example:

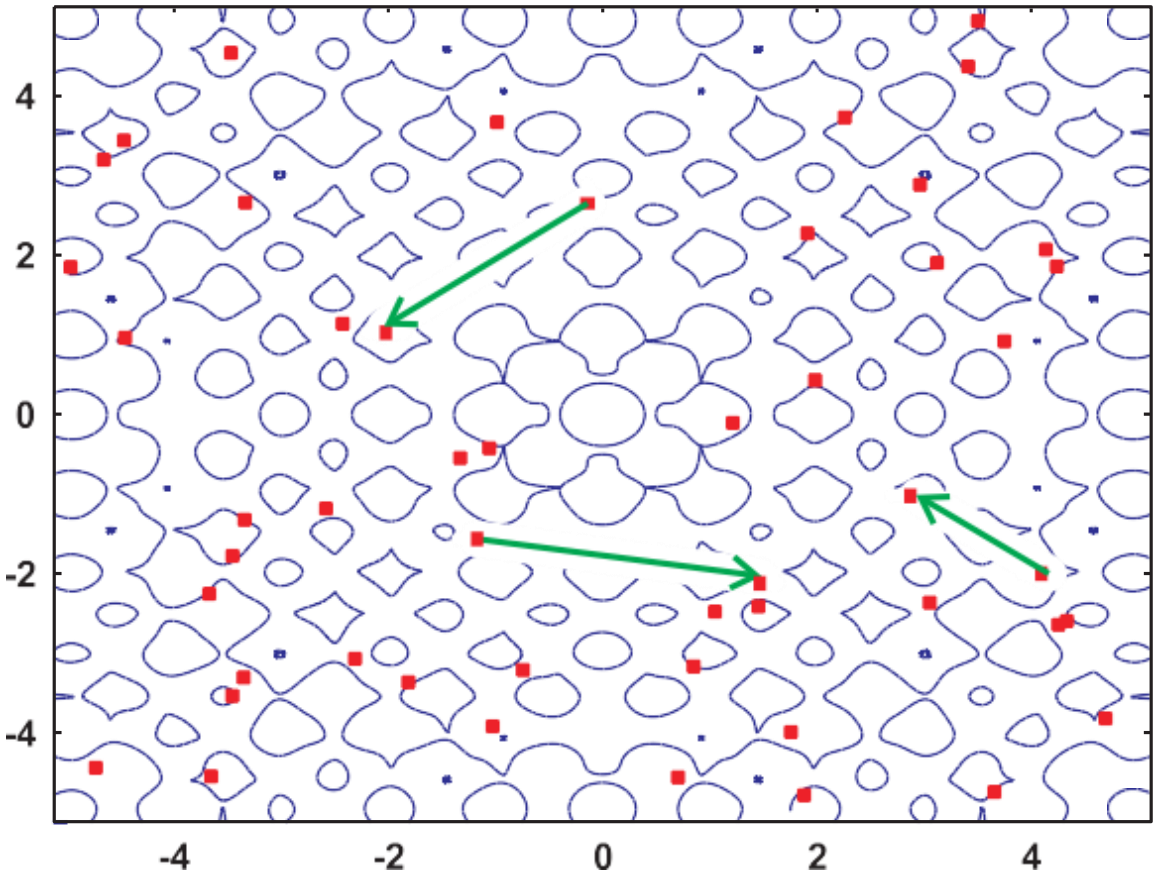
Minimizing a Function ..

$$\text{Minimize } f(\vec{x}) = 20 + \sum_{i=1}^{2} \left(x_i^2 - 10\cos\left(2 * \pi * x_i\right)\right) \text{ for } -5.12 \leq x \leq 5.12$$



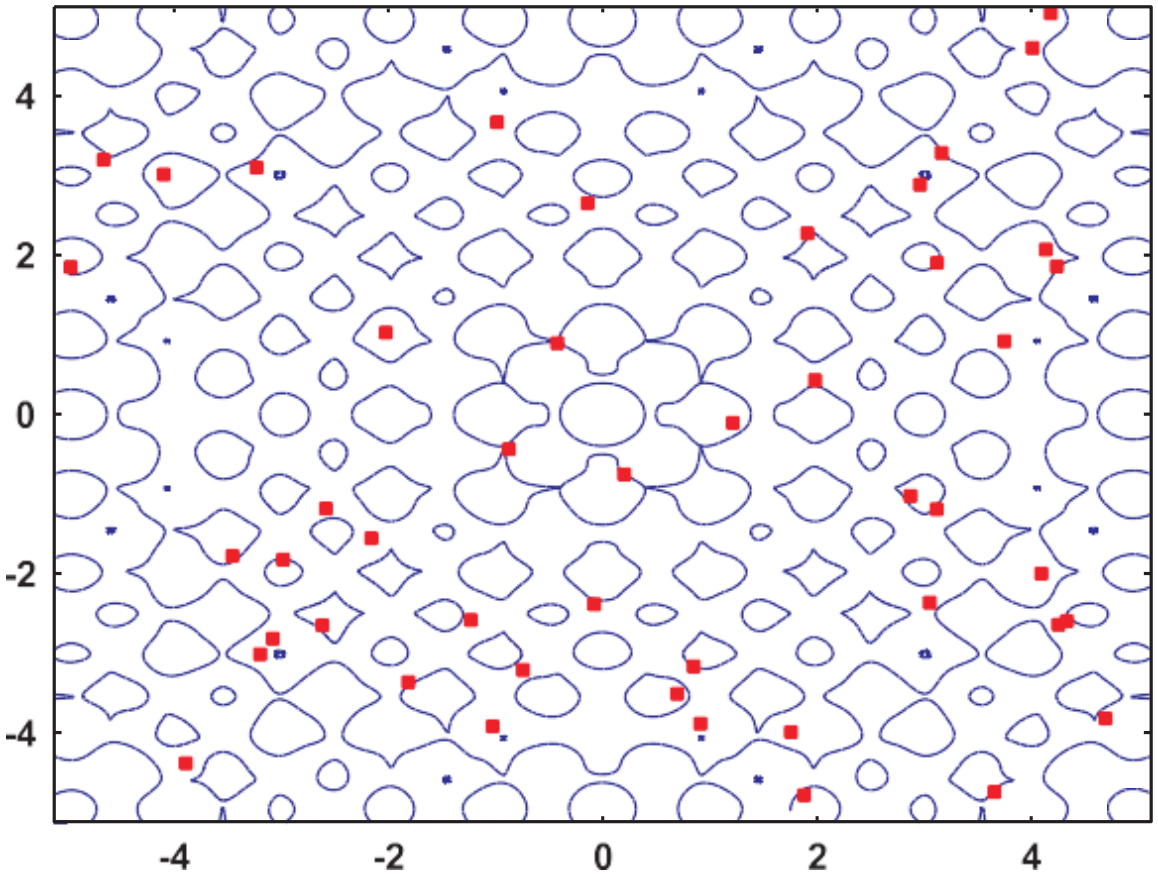
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 0: uniform spread, large differences**



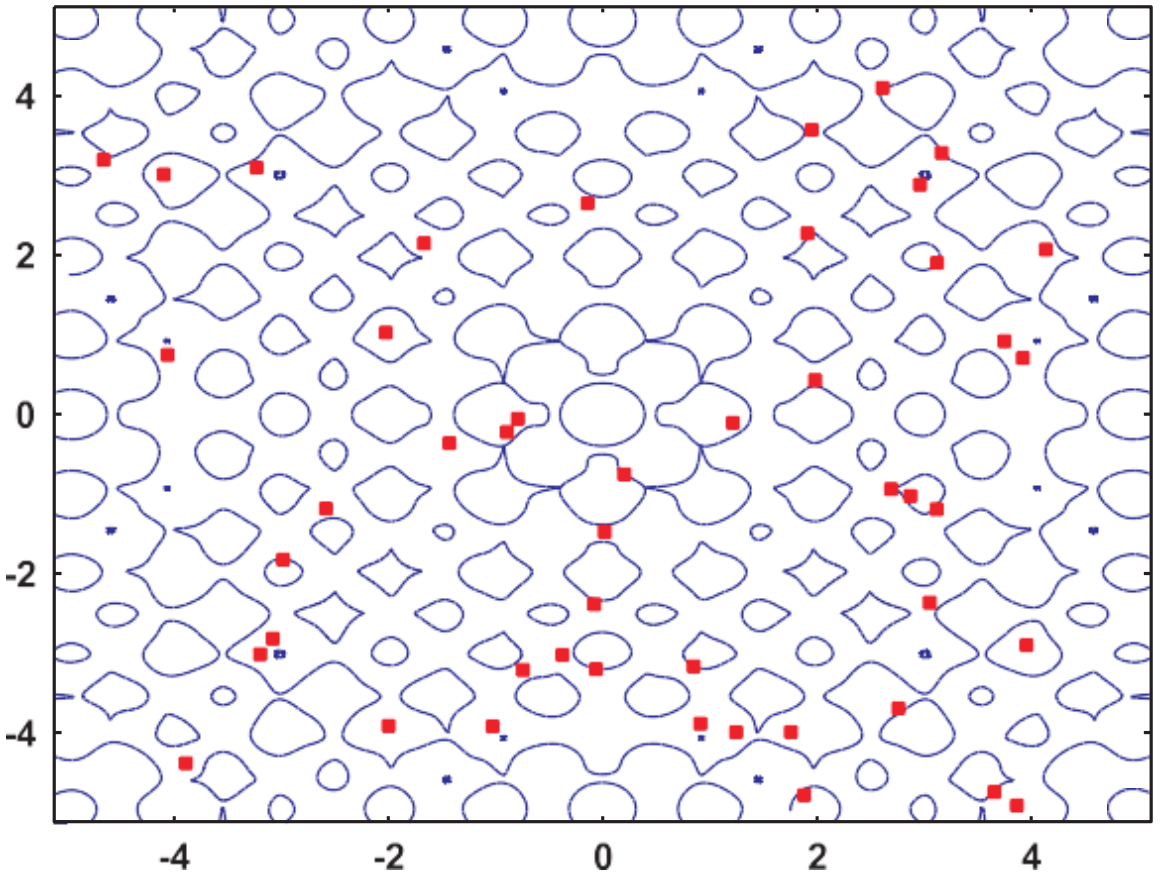
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 1 ..**



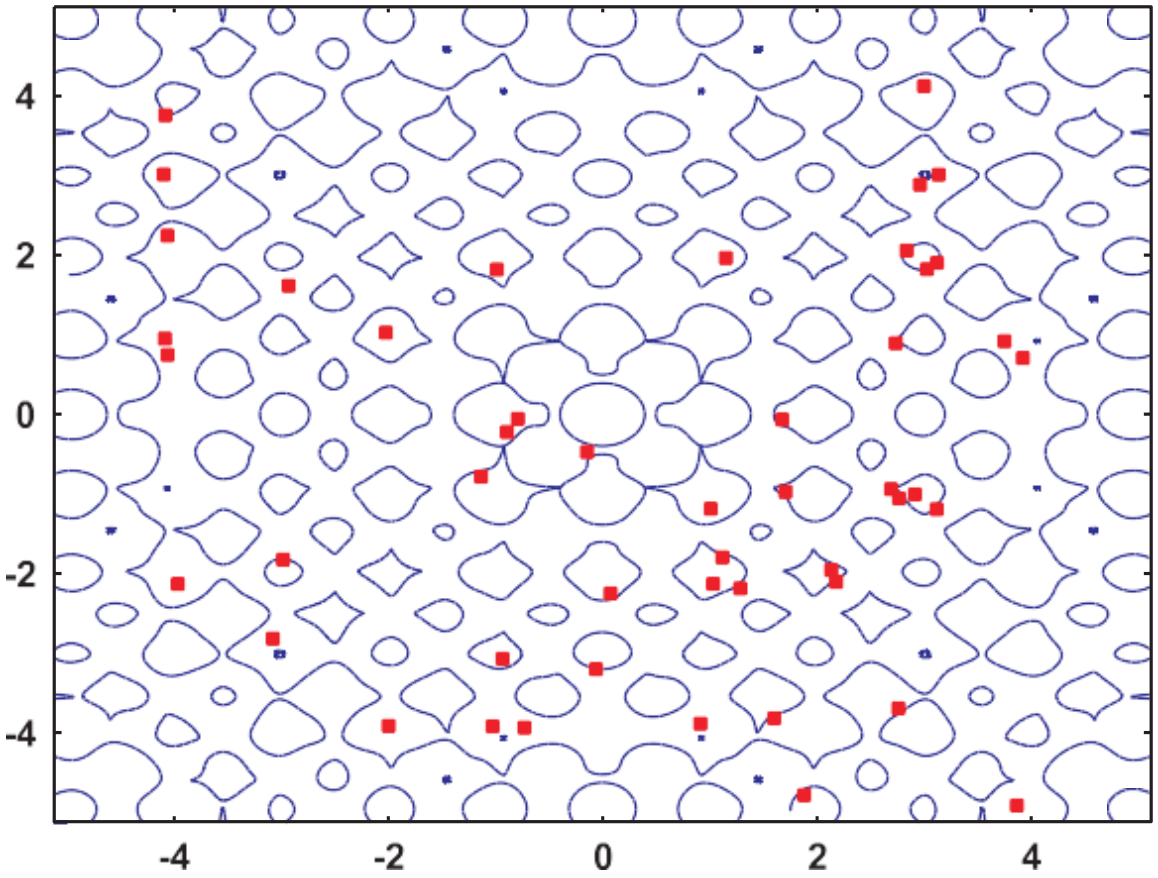
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 2 ..**



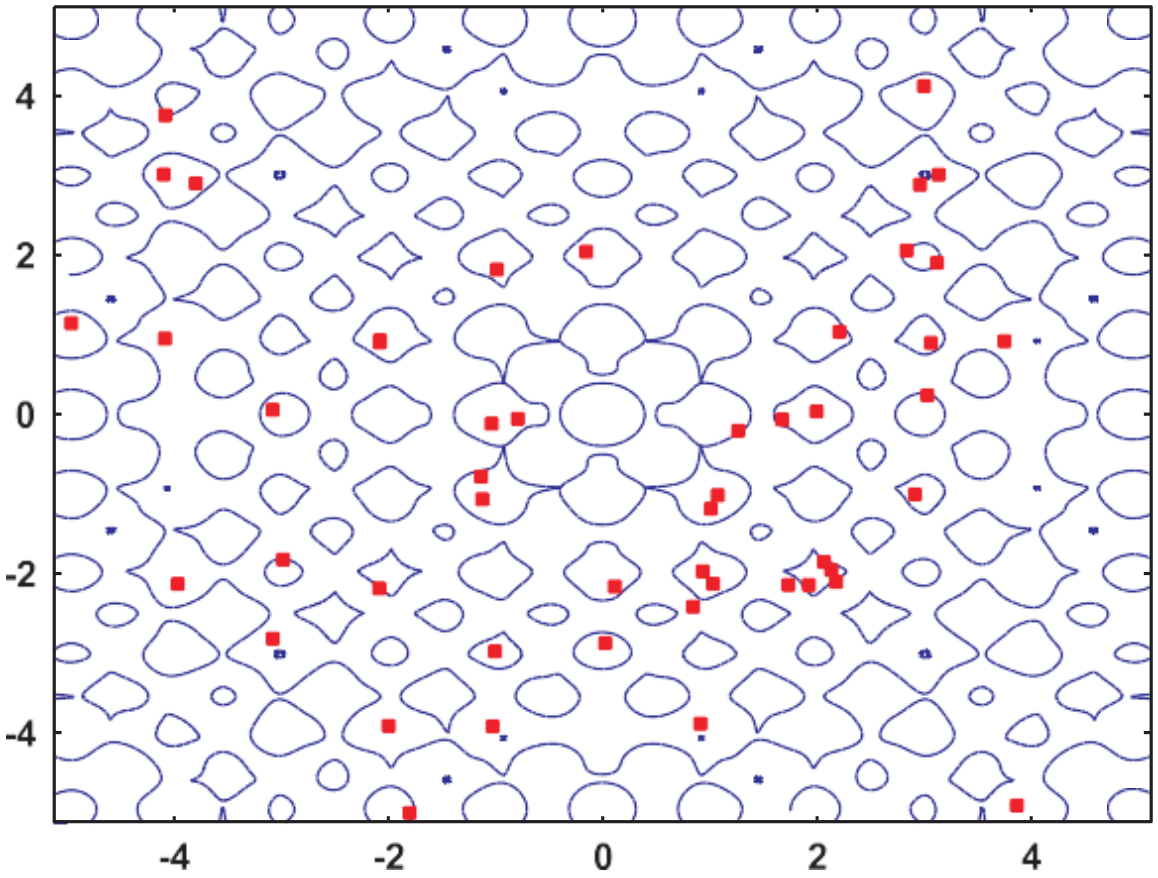
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 5 ..**



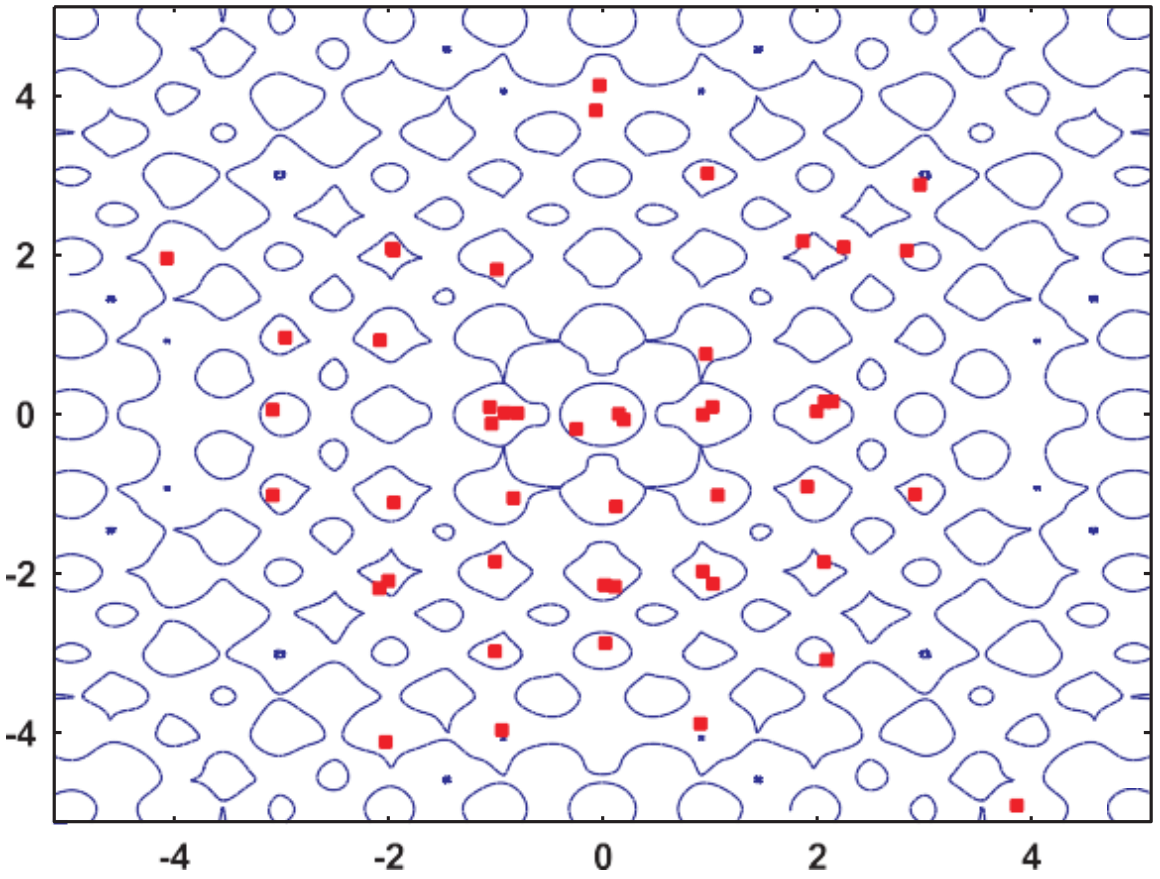
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 10 ..**



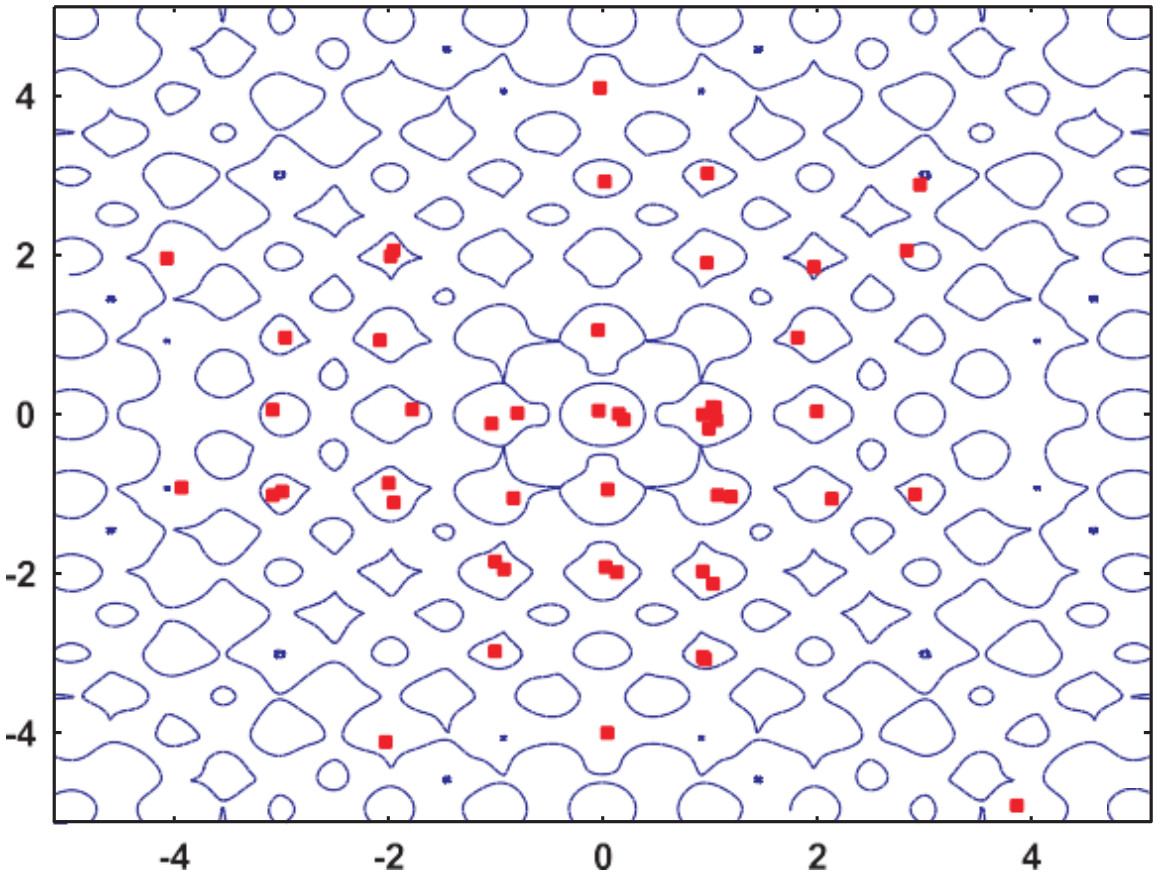
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 20: concentration around the center**



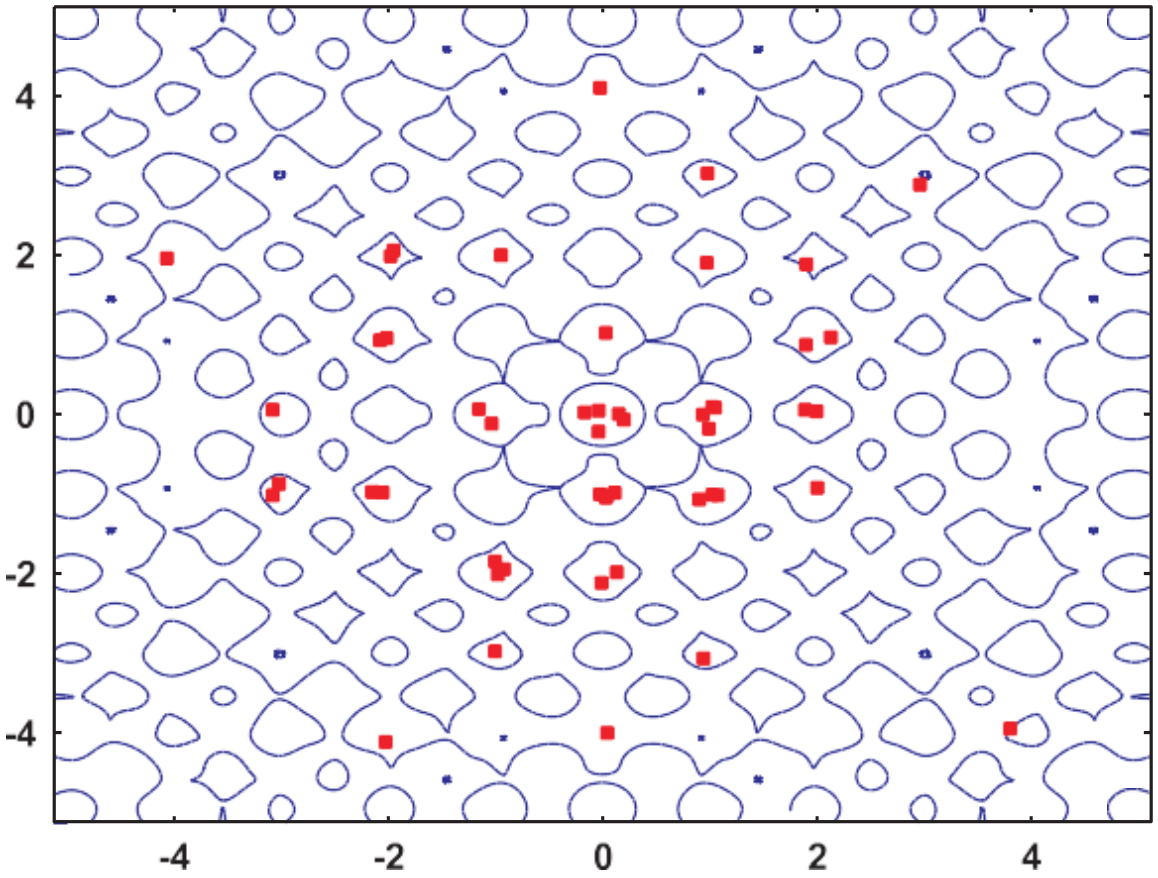
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 30 ..**



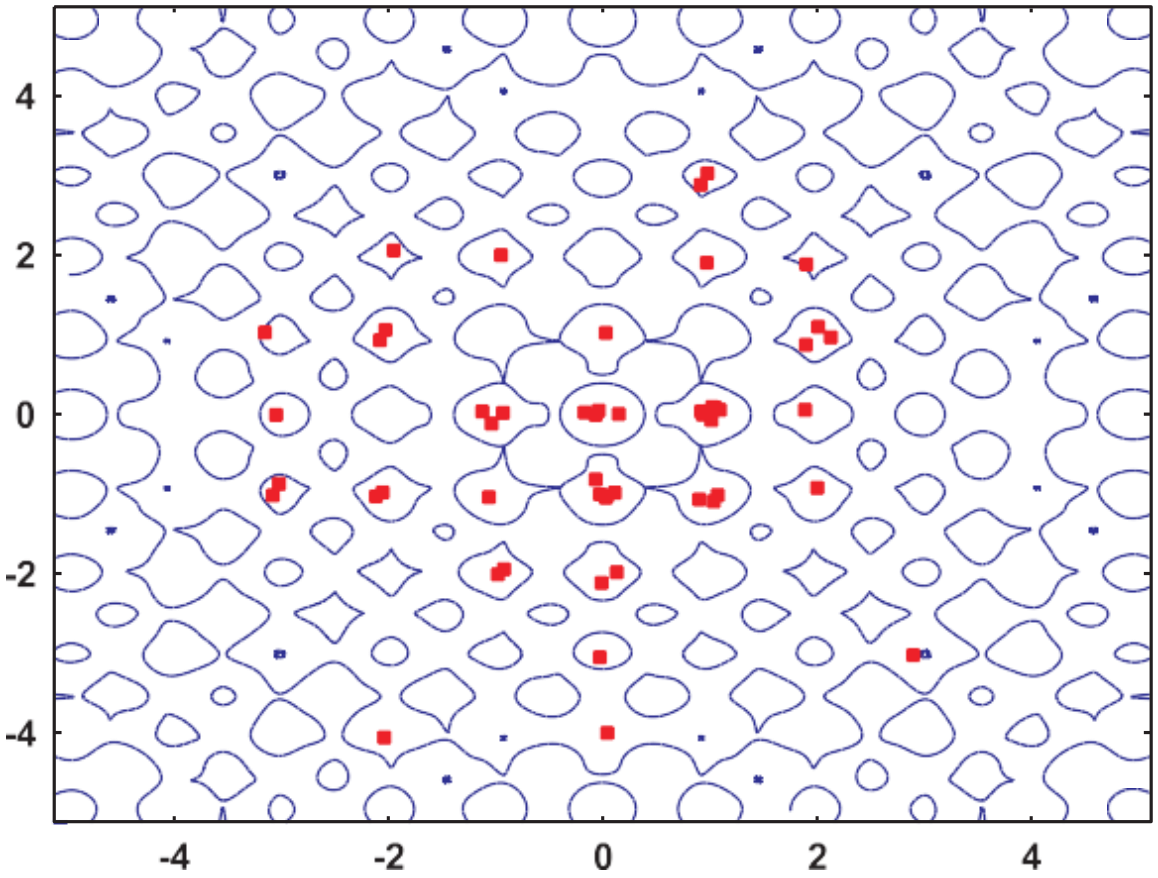
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 40 ..**



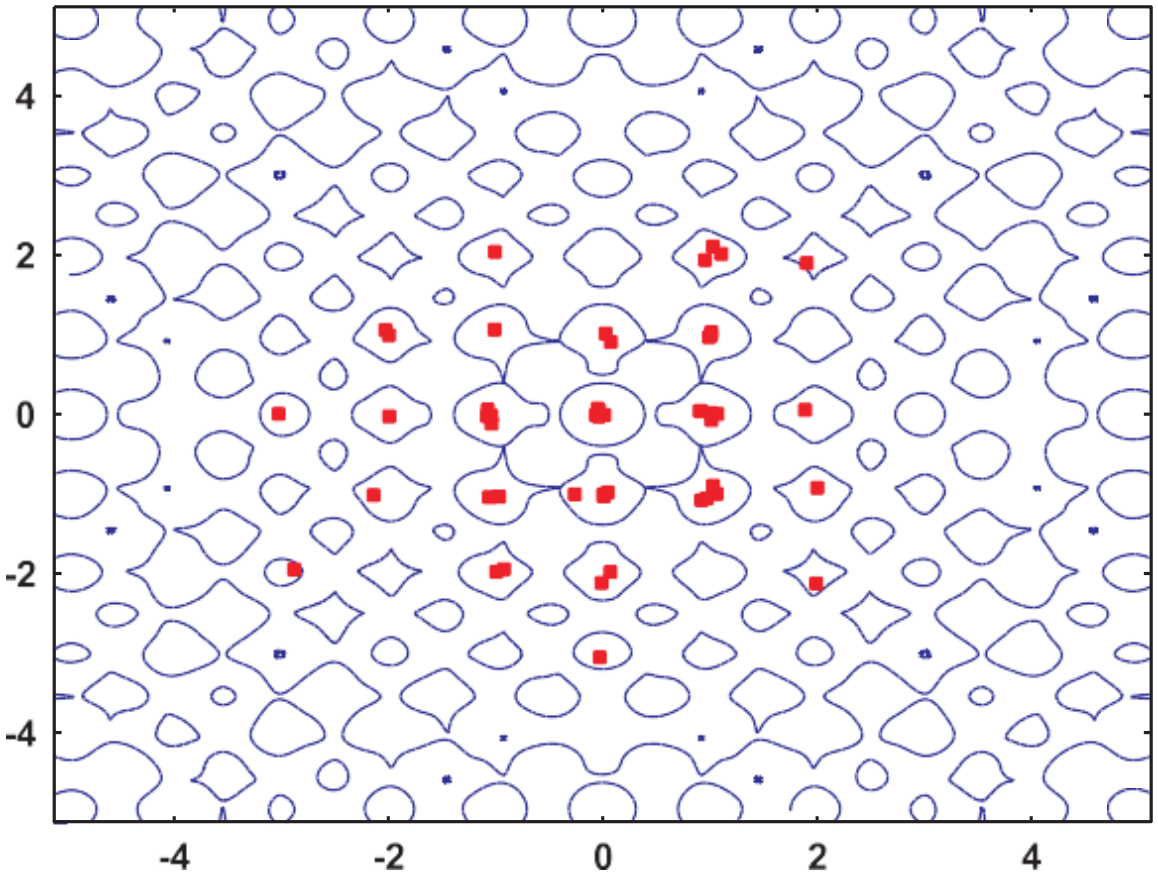
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 50 ..**



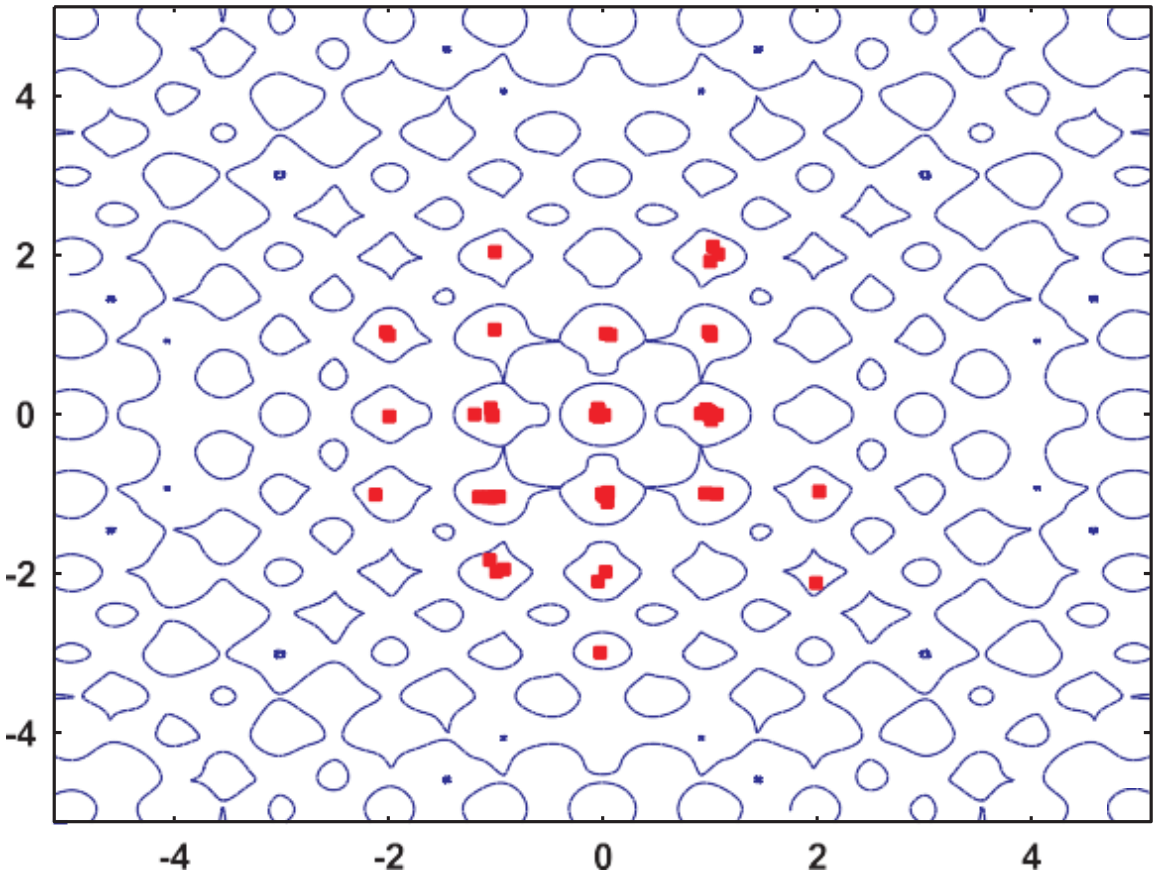
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 100 ..**



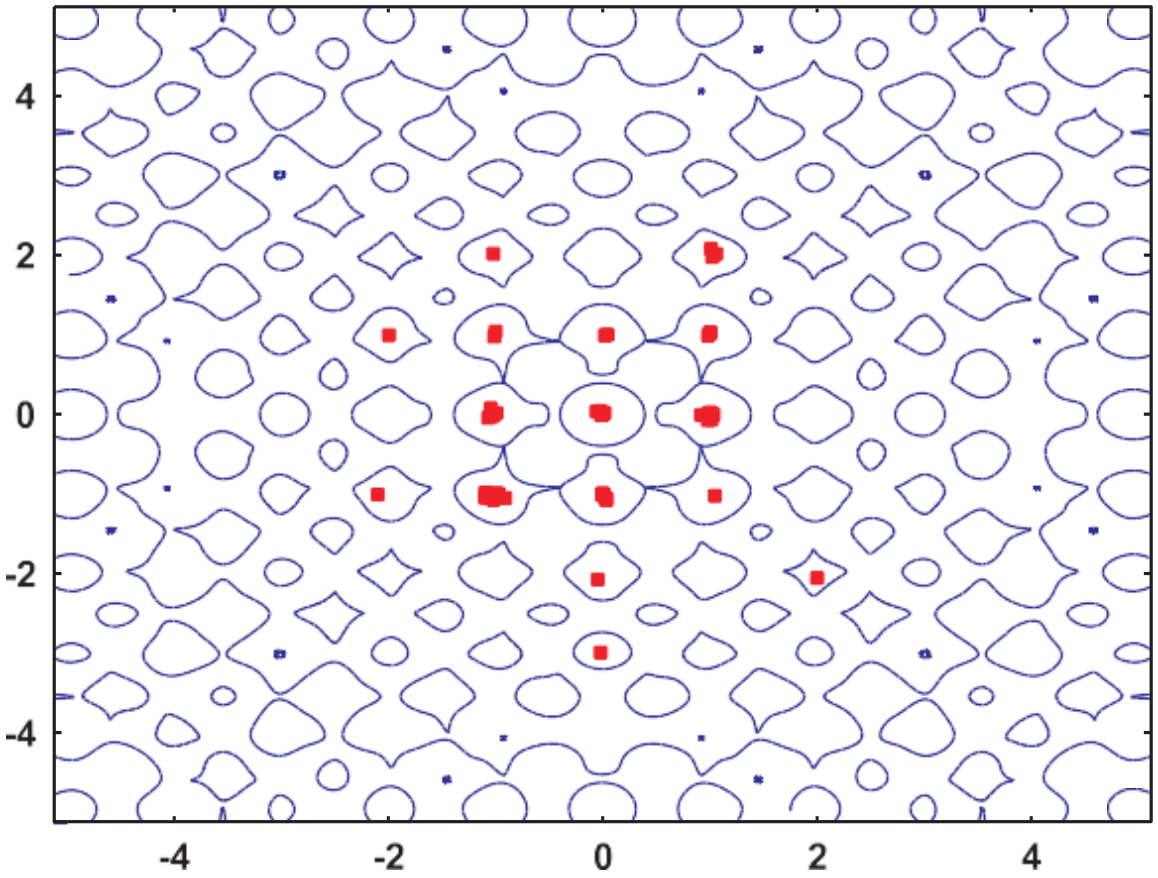
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 125: Similar individuals in local optima**



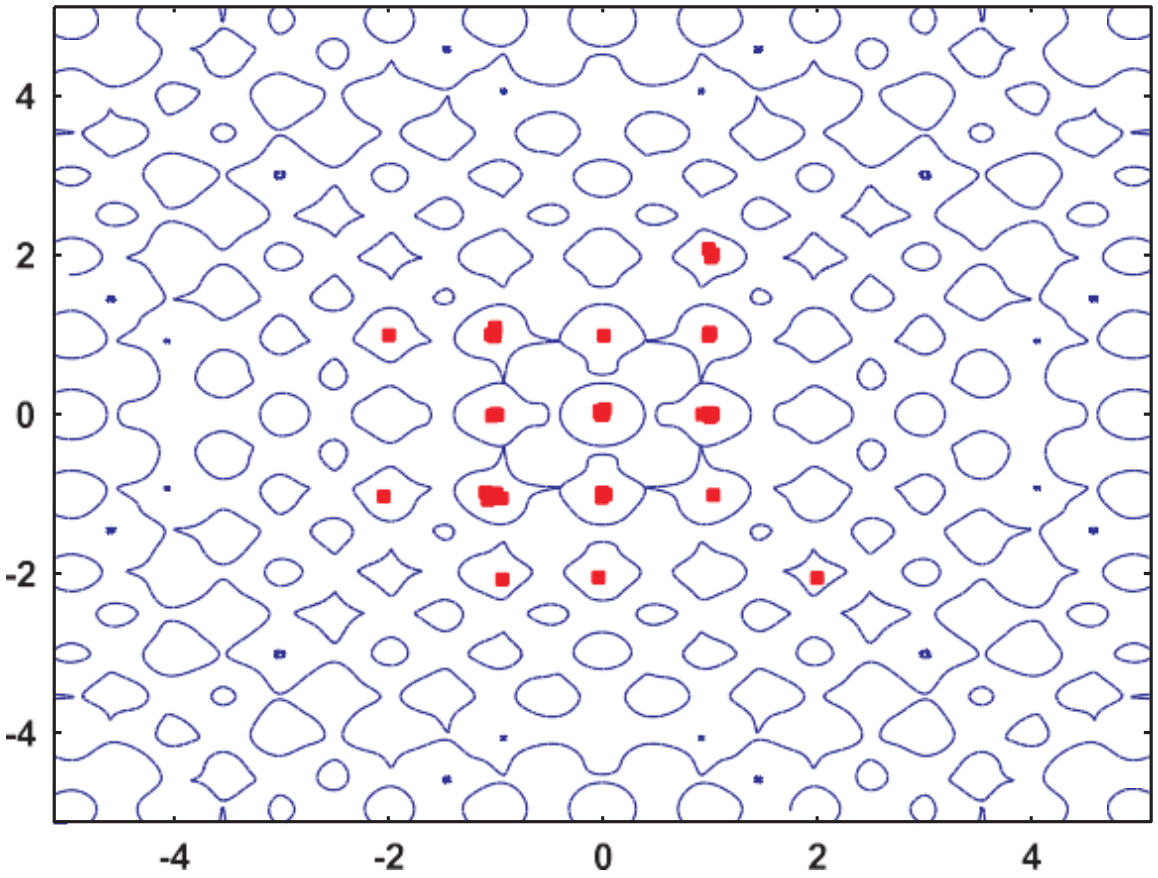
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 200 ..**



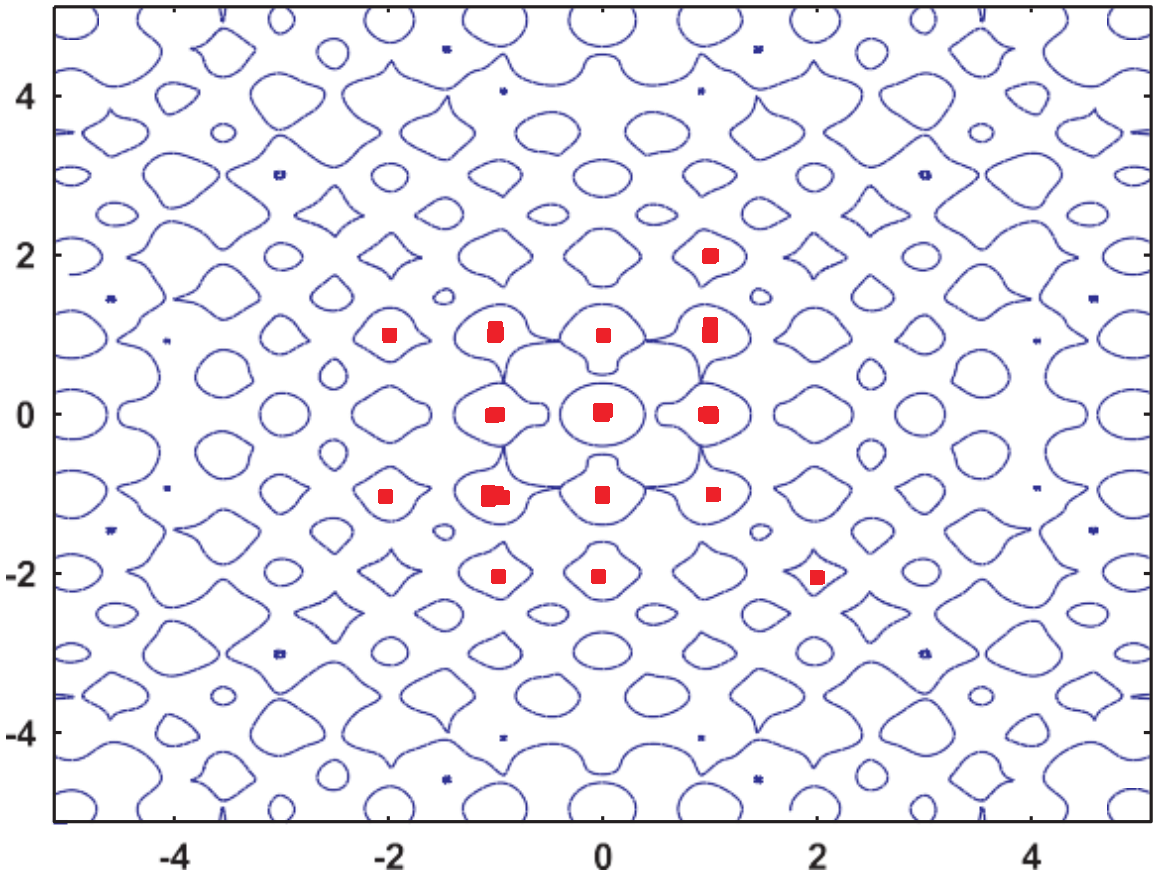
A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 250 ..**



A Numerical Optimization Example:
Minimizing a Function ..

- **Population after generation 300: Concentration at optima**



DE (*Differential Evolution*): *the* Algorithm

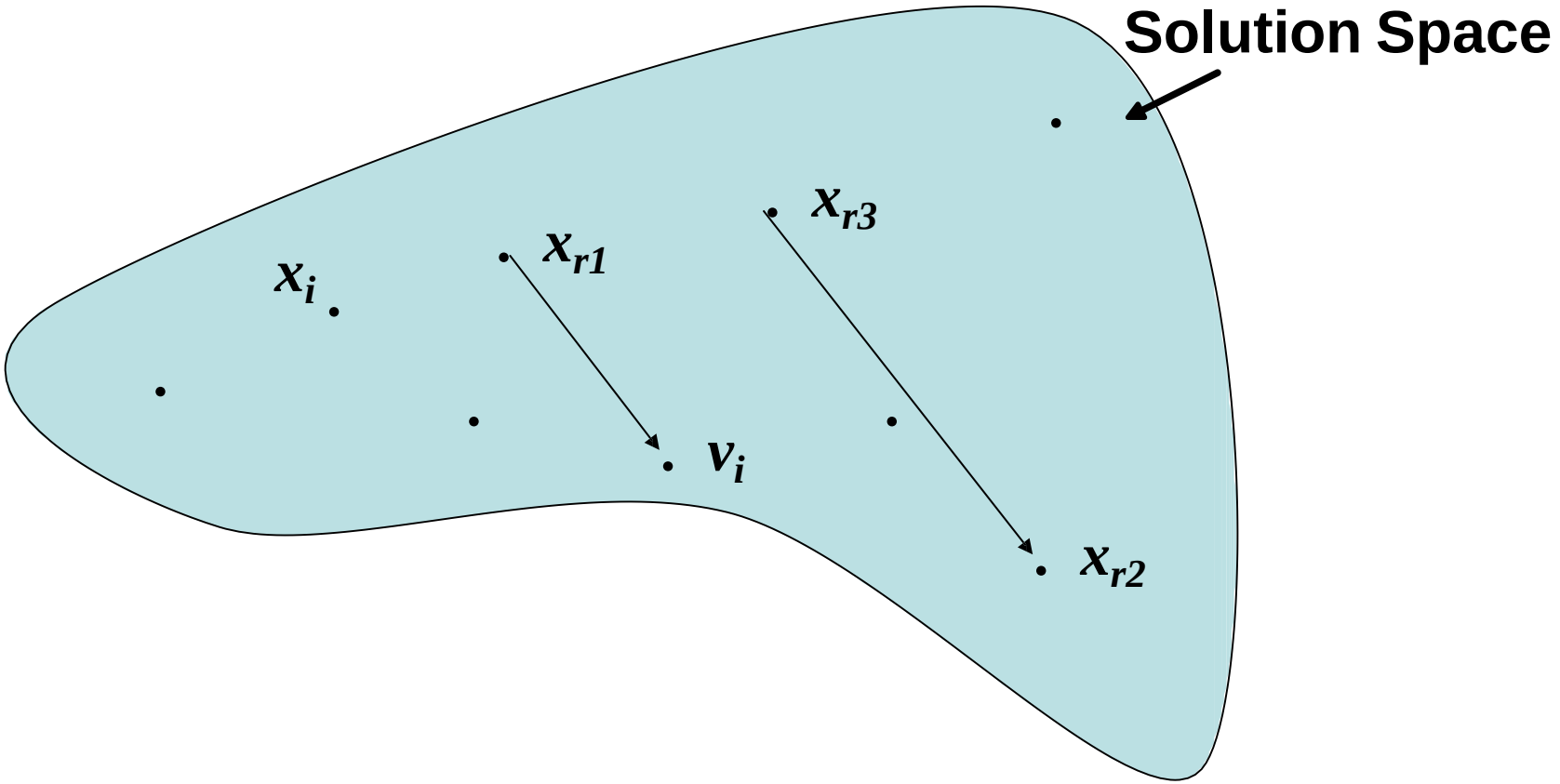
- Select four parents (p_1, p_2, p_3, p_4)
- Calculate the difference vector between two parents: $p_1 - p_2$
- Add the difference to the third parent vector with weight F:
  - $p_3 + F(p_1 - p_2)$
- Crossover between the new vector and fourth parent with gene selection probability "crossover rate" CR
  - **If (Math.random() > CR)** $x_i = x_i (p_4)$
  - **else** $x_i = x_i (p_3 + F (p_1 - p_2))$
- Compare the new child with fourth parent (p_4), the more fit will reach the next generation.

DE: *the* Algorithm ..

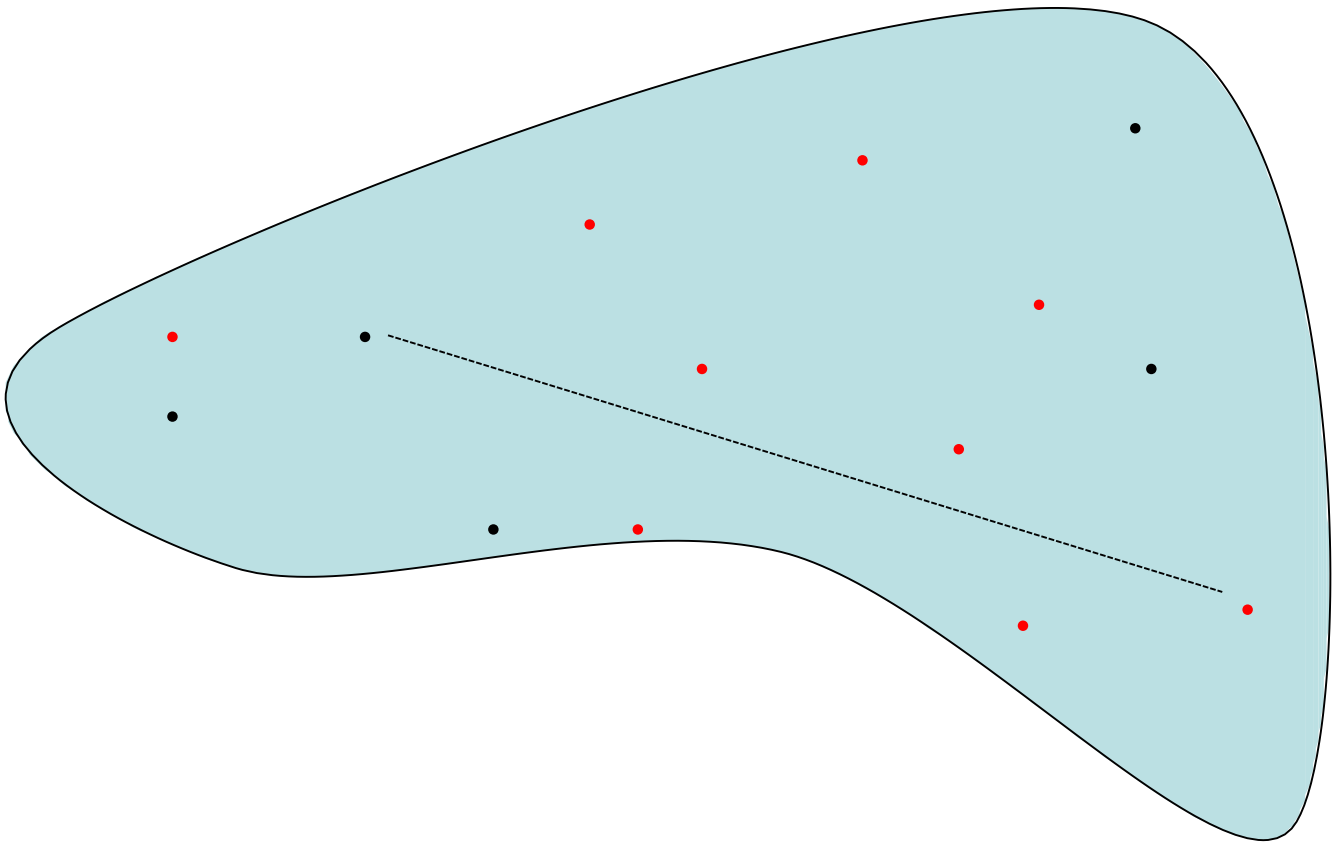
- Start with ***NP*** randomly chosen solution vectors.
- For each ***i*** in ***(1, .., NP)***, form a ‘mutant vector’
- $\boldsymbol{v_i = x_{r1} + F \cdot (x_{r2} - x_{r3})}$
- Where ***r1***, ***r2***, and ***r3*** are three mutually distinct randomly drawn indices from ***(1, .., NP)***, and also distinct from ***i***, and $\mathbf{0 < F \leq 2}$ *(F is a real and constant value).*

DE: *forming* a Mutant Vector ..

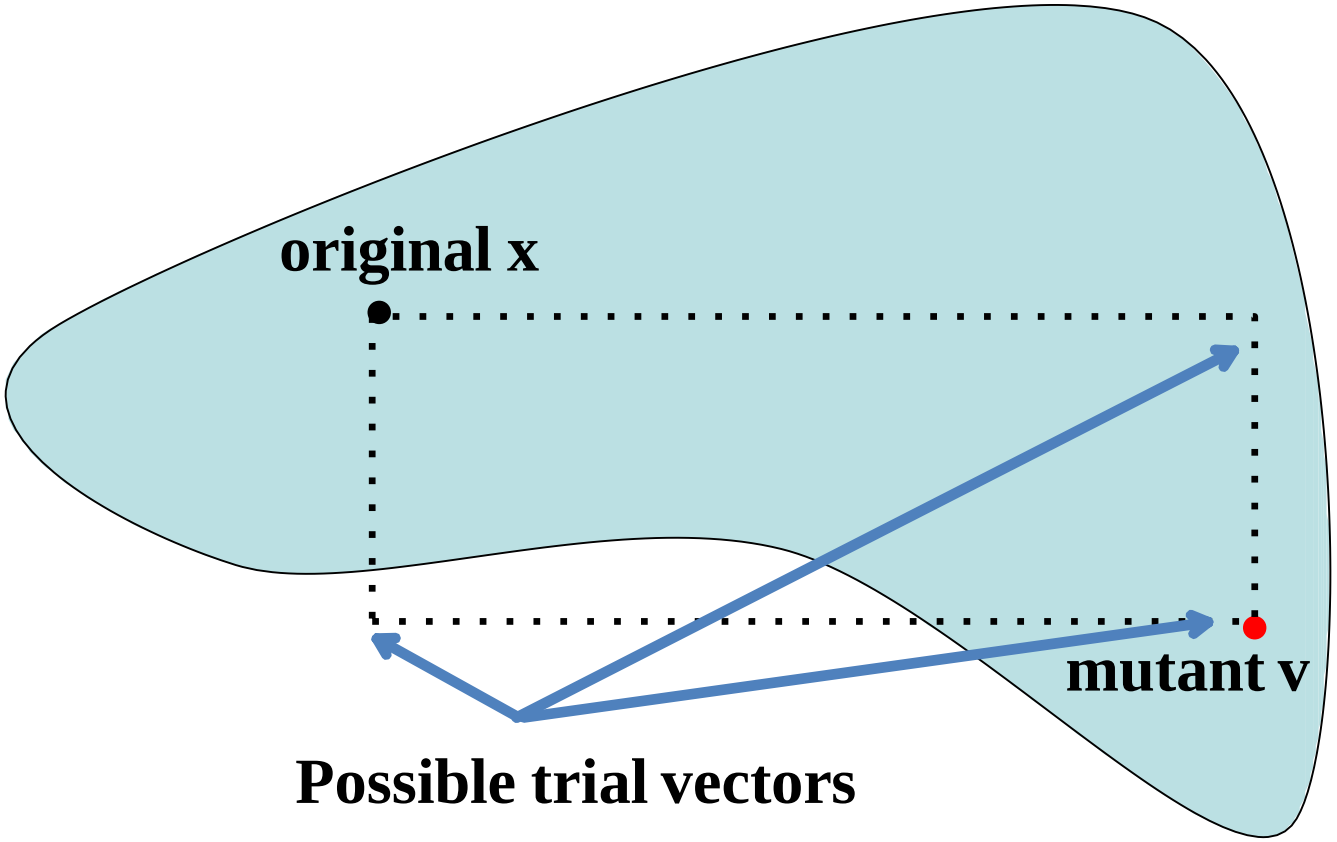
$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$



DE: *from* Old Points to Mutants ..



DE: Crossover x_i & v_i to form a Trial Vector ..



DE: Crossover x_i & v_i to form a Trial Vector ..

$$x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5})$$
$$v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5})$$
$$u_i = (\_, \_, \_, \_, \_) \ ?$$

- For each component of the vectors, draw a random number in $U[0,1]$, call this $rand_j$.
- CR *(Crossover Rate)* is the crossover-constant / cutoff which has to be determined by the user. Let $0 < CR < 1$.
- If $rand_j \leq CR$, then $u_{ij} = v_{ij}$, else $u_{ij} = x_{ij}$.
- To ensure at least some crossover, one (or more) component(s) of u_i is/are selected at random to be from v_i .
  - So, $rnbr_j$ is a randomly chosen index $\in$ 1, 2, .., D.
  - If $rnbr_j = j$, then $u_{ij} = v_{ij}$

DE: Crossover x_i & v_i to form a Trial Vector ..

$$x_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5})$$

$$v_i = (v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5})$$

E.g., Index 1 is randomly selected as a definite crossover.

So, for example, we may have:

$rand_5$ <= CR, so it crossed over too.

$$u_i = (v_{i1}, x_{i2}, x_{i3}, x_{i4}, v_{i5})$$

DE: *the* Selection ..

If the objective-value *(fitness-function)* COST(u_i) is better than COST(x_i)

then u_i replaces x_i in the next generation

otherwise, we keep x_i

DE: *an* Example ..

Parent 1:	0.12	0.65	0.45	0.32	0.77
Parent 2:	0.16	0.02	0.77	0.34	0.31
Difference *(Parent 1 – Parent 2)*:	-0.04	0.63	-0.32	-0.02	0.46
Difference * F (= 0.5 *for example*)	-0.02	0.32	-0.16	-0.01	0.23
Parent 3:	0.45	0.33	0.23	0.77	0.81
Donor Vector (Mutant Vector):	0.43	0.65	0.07	0.76	1.00* (*overflow)
Parent 4 (Target Vector):	0.23	0.77	0.43	0.88	0.96
New child (Trial Vector): *Crossover Mutant & Target Vectors*	0.43	0.77	0.07	0.76	0.96

The *basic* DE algorithm ..

Initialize all agents *X* with random positions in search space

repeat

for each agent *X* in the population **do**

Pick three agents r_1, r_2, r_3

Pick a random index between 1 and *NP*

Compute the agent's potentially new position

for each component of the agent **do**

Copy or not copy based on the cases discussed earlier

end for .. end for

until termination criterion is met *(e.g. number of iterations performed, or adequate fitness reached).*

Pick the agent from the population that has the lowest cost and return it as the best found candidate solution.

The *highlights* of DE ..

- In the beginning, all candidate solutions are spread uniformly in search space X.
- The differential information (x_{r2}-x_{r3}) is large ⇒ large search steps.
- As the population converges, the candidate solutions are located closer to each other.
- The distances (x_{r2}-x_{r3}) decrease ⇒ the search steps get smaller too.
- Very simple way to perform self-adaptation without needing any additional parameter or operation.
- The step-width is self-organizing from the structure of the population and self-adapting along the optimization process.

The *highlights* of DE ..

- A population of NP solution vectors are successively updated by addition, subtraction, and component swapping, until the population converges, hopefully to the optimum.
- No derivatives are used.
- Very few parameters to set.
- A simple and apparently very reliable method.

Hybridization of EAs ..

- The common trend of EAs is hybrids between different evolutionary algorithms.
- These hybrids are relatively easy to do, since most EAs can use the same population table and the same gene presentation.
- Hybrids are mainly done in order to avoid the shortcomings of one algorithm:
  - – E.g. In the GA/DE hybrid, GA acts like a global searcher, and DE acts as a local searcher. Thus, GA finds peaks, and DE finds the highest point of that peak.

More information:

- DE homepage *(including practical advice, source codes, etc.)*:

  http://www.icsi.berkeley.edu/~storn/code.html

- Practical Advice:

  *(e.g., start with NP = 10 * D, and CR = 0.9, F = 0.8)*

Thanks! … *Questions?*