Logistic regression, curvilinear regression, and the interpretation of results

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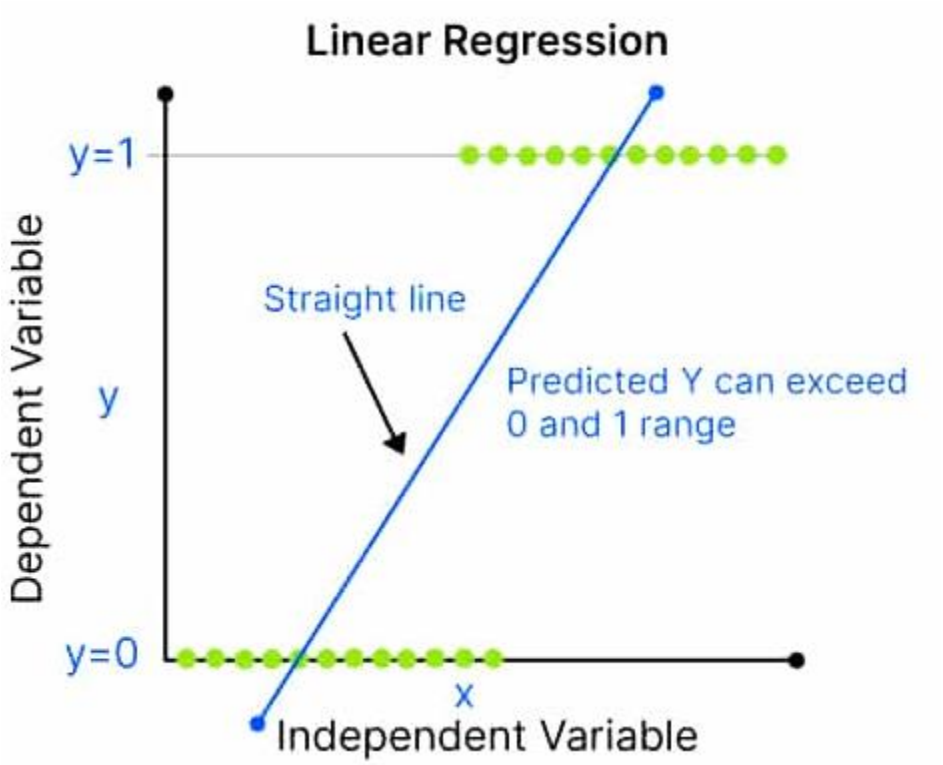
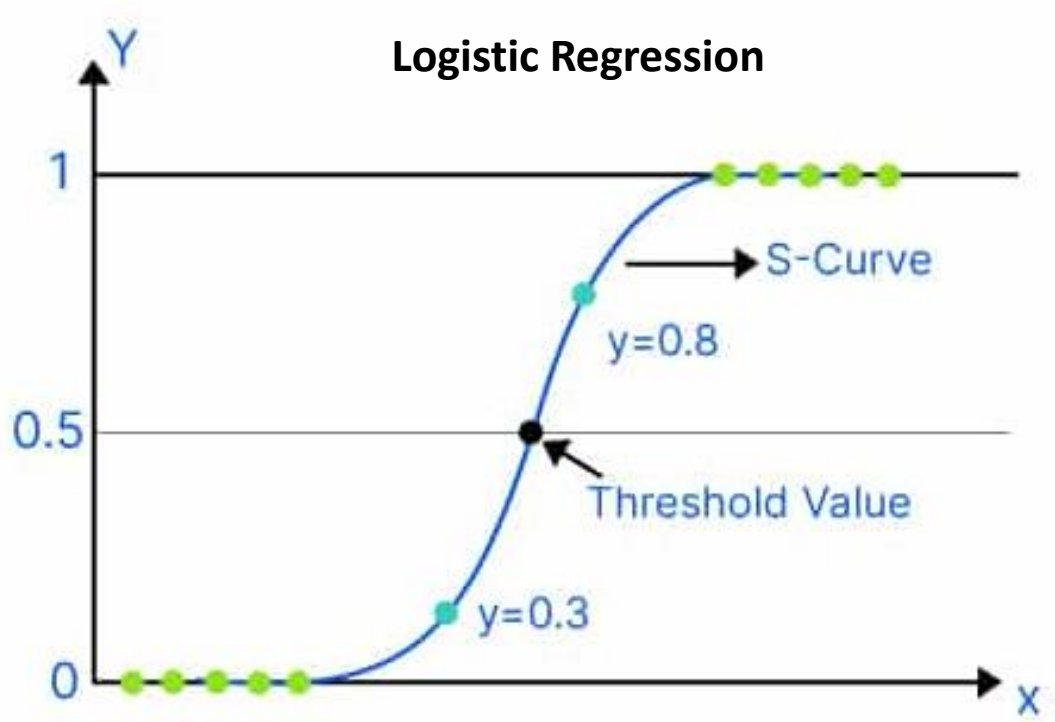
I. Introduction

- While linear regression is good for prediction, we cannot use it to solve a problem of classification in the outcome (DV).





II. The problem of classification

- Binary classification (0 or 1): When the outcome variable has only two categories, such as success or failure, high or low, etc.
  - Use simple or binomial logistic or logit or probit regression.
- Multinomial classification: When the outcome has multiple categories but no rank order, such as you can go to college (academic), trade school (vocational), or into the workforce (general) based on your SES.
  - Must use multinomial logistic regression.
- Ordinal classification: The DV has multiple categories with a rank order, such as on a 0-5 scale; low, medium, high; unemployed, part-time employed, full-time employed, etc.
  - Use ordinal or ordered logistic regression.

III. When to use which type of logistic regression?

- Depends on:

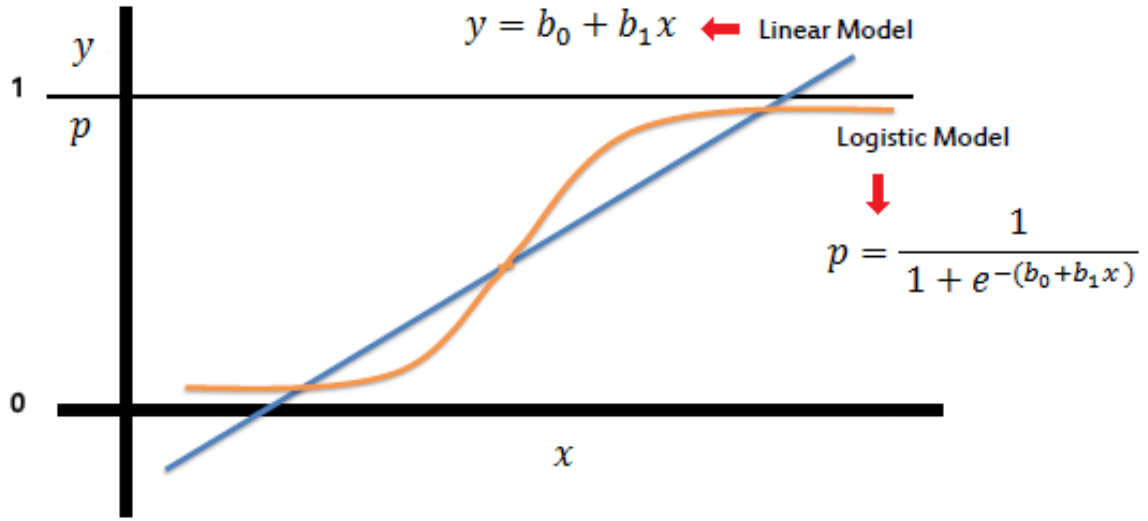
Types of Logistic Regression Models

	Binomial Logistic Regression	Multinomial Logistic Regression	Ordinal Logistic Regression
Number of Categories for Response Variable	2	3 or more	3 or more
Does Order of Categories Matter?	No	No	Yes

- Sometimes, logistic regression is problematic when the IVs are categorical based on some arbitrary categories (e.g., age from continuous to uneven categories like young, adult, old).
- In this case, curvilinear regression can be a better fit to the data [example at the end].

IV. Simple or binomial logistic regression

- Simple logistic regression is like simple linear regression.
- But the curve is constructed using the natural logarithm of the "odds" of the DV, rather than the probability.
- By this transformation, the logistic regression equation can be written in terms of an odds ratio.
- Taking the natural log of both sides, we can write the equation in terms of log-odds (logit).
- The coefficient (b_1) is the amount the logit (log-odds) changes with a one unit change in *x*.



$$\frac{p}{1-p} = \exp(b_0 + b_1 x) \qquad logit(p) = \ln\left(\frac{p}{1-p}\right)$$

$$ln\left(\frac{p}{1-p}\right) = b_0 + b_1 x \qquad p = \frac{1}{1 + e^{-logit(p)}}$$

V. Multinomial logistic regression

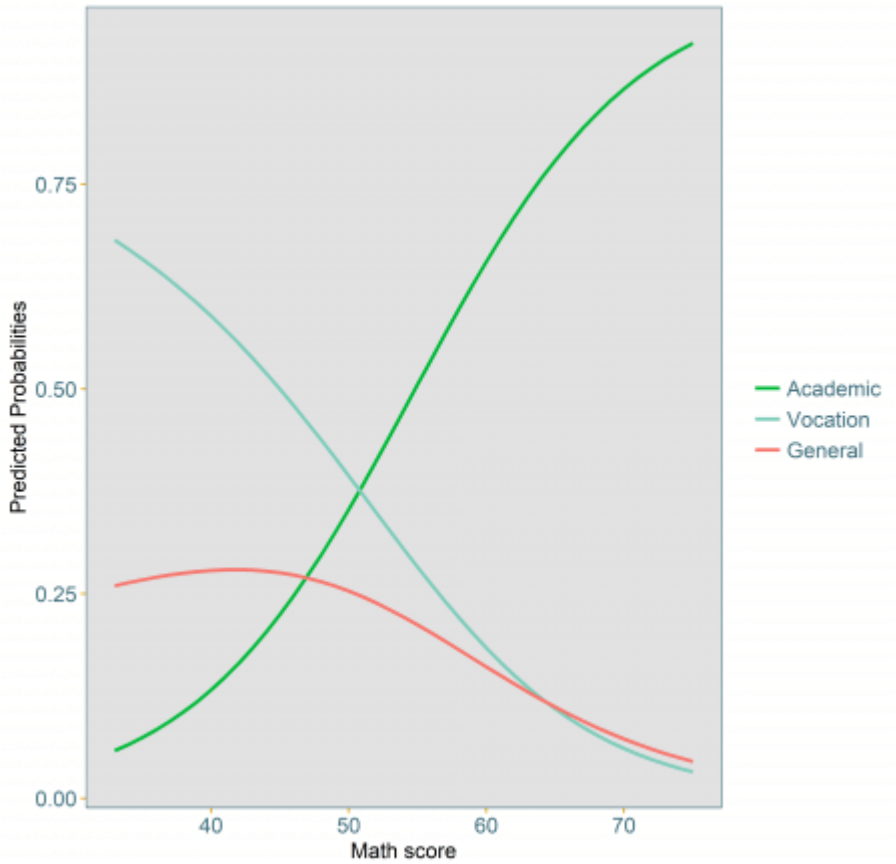
- Like multiple linear regression, logistic regression can handle any number of numerical and/or categorical variables.
- If the DV is a categorical variable with more than two categories where order does not matter, we can write:

$$p = \frac{1}{1 + e^{-(b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_p x_p)}}$$

$$\hat{P}(Y_i = \text{academic}) = \frac{\exp[-5.0391 + 0.1099x_i]}{1 + \exp[-5.0391 + 0.1099x_i] + \exp[2.8996 - 0.0599x_i]}$$

$$\hat{P}(Y_i = \text{vocational}) = \frac{\exp[2.8996 - 0.0599x_i]}{1 + \exp[-5.0391 + 0.1099x_i] + \exp[2.8996 - 0.0599x_i]}$$

$$\hat{P}(Y_i = \text{general}) = \frac{1}{1 + \exp[-5.0391 + 0.1099x_i] + \exp[2.8996 - 0.0599x_i]}$$



VI. Ordinal or ordered logistic regression

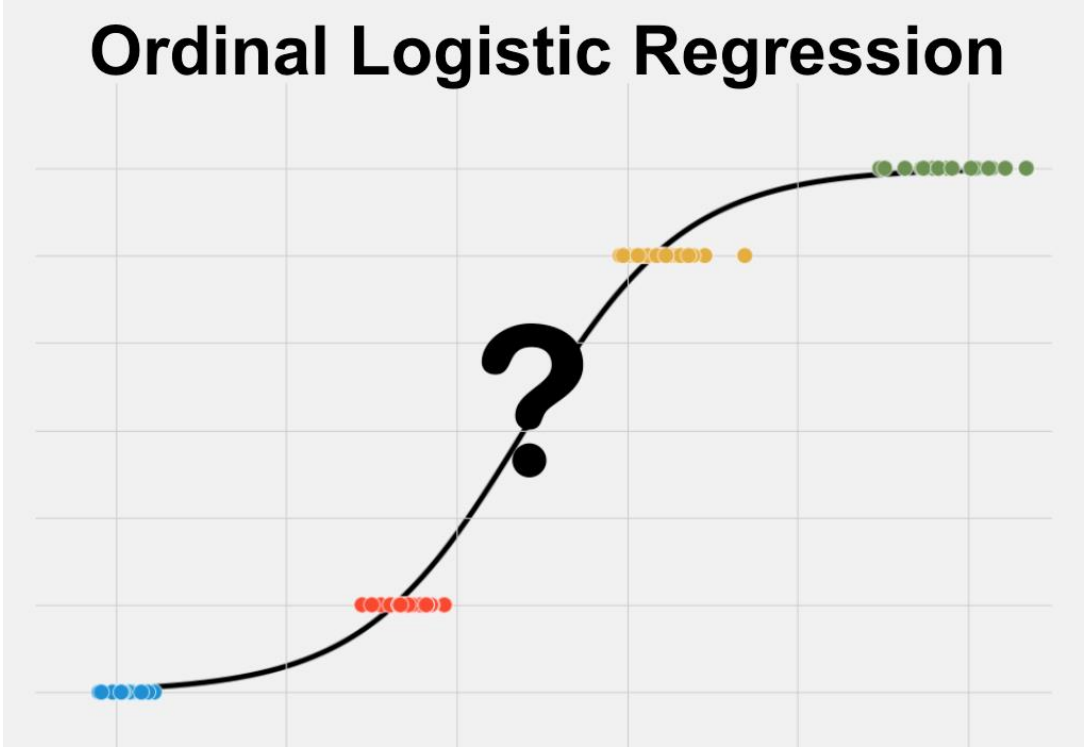
- Suppose, in a survey the proportions of respondents who would answer "poor", "fair", "good", "very good", and "excellent" health are respectively p1, p2, p3, p4, p5.
- The logarithms of the odds of answering in certain ways are:

$$\text{poor}, \quad \log \frac{p_1}{p_2+p_3+p_4+p_5}, \quad 0$$

$$\text{poor or fair}, \quad \log \frac{p_1+p_2}{p_3+p_4+p_5}, \quad 1$$

$$\text{poor, fair, or good}, \quad \log \frac{p_1+p_2+p_3}{p_4+p_5}, \quad 2$$

$$\text{poor, fair, good, or very good}, \quad \log \frac{p_1+p_2+p_3+p_4}{p_5}, \quad 3$$



VII. Interpretation of results

```
use https://stats.idre.ucla.edu/stat/data/hsb2, clear

generate honcomp = (write >=60)
logit honcomp female read science

Iteration 0:   log likelihood = -115.64441
Iteration 1:   log likelihood = -84.558481
Iteration 2:   log likelihood = -80.491449
Iteration 3:   log likelihood = -80.123052
Iteration 4:   log likelihood = -80.118181
Iteration 5:   log likelihood =  -80.11818

Logit estimates                                   Number of obs   =        200
                                                  LR chi2(3)      =      71.05
                                                  Prob > chi2     =     0.0000
Log likelihood =  -80.11818                       Pseudo R2       =     0.3072

------------------------------------------------------------------------------
     honcomp |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   1.482498   .4473993     3.31   0.001     .6056111    2.359384
        read |   .1035361   .0257662     4.02   0.000     .0530354    .1540369
     science |   .0947902   .0304537     3.11   0.002      .035102    .1544784
       _cons |   -12.7772    1.97586    -6.47   0.000    -16.64982   -8.904589
------------------------------------------------------------------------------
```

This classification is arbitrary

Iteration log or LL

VII. Interpretation of results…

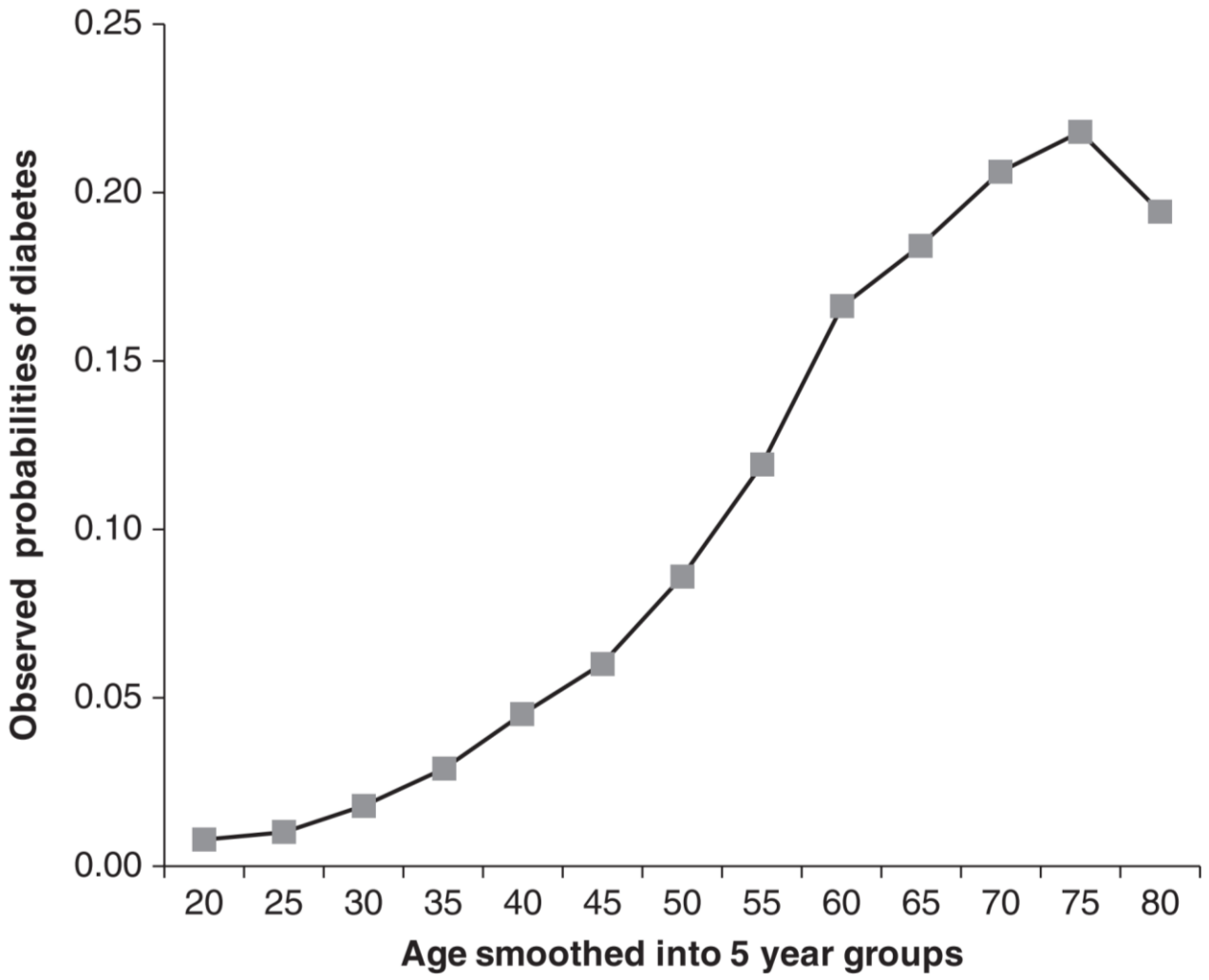
- The coefficients are given in log-odds units.
- They are often difficult to interpret, so they are often converted into odds ratios, by using “logistic” command or writing “or” option if you use “logit” command.
- Alternatively, use “probit” that gives you coefficients like OLS.

```
. logistic honcomp female

Logistic regression                               Number of obs   =        200
                                                  LR chi2(1)      =       3.94
                                                  Prob > chi2     =     0.0473
Log likelihood =  -113.6769                       Pseudo R2       =     0.0170

------------------------------------------------------------------------------
     honcomp | Odds Ratio   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   1.918168   .6400451     1.95   0.051     .9973827    3.689024
------------------------------------------------------------------------------
```

Odds ratio <1 means less likely, whereas >1 means more likely. OR cannot be negative.

VIII. Curvilinear or logistic?

- If existing theory suggests a curvilinear pattern, do not use logistic function; use a curvilinear model (usually, by adding a squared or cubed term to the linear regression equation).
- Theory: The probability of being diagnosed with diabetes is low in early life, then it accelerates at later ages, finally slowing down.



VIII. Curvilinear or logistic?

- Curvilinear regression fits the observed probabilities better than logit (log-odds of linear coefficients).