Treatment sum of squares in the Within Blocks stratum is $\boldsymbol{q}'\Omega\boldsymbol{q}$, where Ω is the inverse of the treatment information matrix, i.e. $X'(I-P_B)X$ which can be expand as

$$\begin{aligned}
X'(I-P_B)X &= X'X - X'Z(Z'Z)^{-1}Z'X && (1)\\
&= rI - N\left(\frac{1}{k}I\right)N' && (2)\\
&= rI - \frac{1}{k}[(r-\lambda)I + \lambda J] && (3)\\
&= rI - \frac{r-\lambda}{k}I - \frac{\lambda}{k} && (4)\\
&= \frac{rv(k-1)}{k(v-1)}(I-K) && (5)\\
&= rE(I-K) && (6)
\end{aligned}$$

where

$$\lambda = \frac{r(k-1)}{v-1}$$

and

$$E = \frac{v(k-1)}{k(v-1)}.$$

$$\begin{aligned}
\boldsymbol{q}'\Omega\boldsymbol{q} &= y'(I-P_B)X[X'(I-P_B)X]^{-}X'(I-P_B)y && (7)\\
&= y'(I-P_B)X\frac{1}{rE}(I-K)X'(I-P_B)y && (8)\\
&= \frac{1}{rE}[y'(I-P_B)XX'(I-P_B)y] - [y'(I-P_B)XKX'(I-P_B)y] && (9)\\
&= \frac{1}{E}[y'(I-P_B)X\frac{1}{r}X'(I-P_B)y] && (10)\\
&= \frac{1}{E}[y'(I-P_B)X(X'X)^{-1}X'(I-P_B)y] && (11)\\
&= \frac{1}{E}[y'(I-P_B)P_\tau(I-P_B)y] && (12)\\
& && (13)
\end{aligned}$$

For the Between Blocks stratum,

$$\begin{aligned}
X'(P_B - K)X &= X'Z(Z'Z)^{-1}Z'X - X'KX && (14)\\
&= N\left(\frac{1}{k}I\right)N' - rK && (15)\\
&= \frac{1}{k}[(r-\lambda)I + \lambda J] - rK && (16)\\
&= \frac{r-\lambda}{k}I - \frac{\lambda}{k} - rK && (17)\\
&= r\left(\frac{v-k}{k(v-1)}I - \frac{v(k-1)}{k(v-1)}K\right) - rK && (18)\\
&= r[(1-E)I - EK] - rK && (19)\\
&= r(1-E)I - r(1-E)K && (20)\\
&= r(1-E)(I-K) && (21)\\
& && (22)
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{q}'\Omega\boldsymbol{q} &= y'(P_B - K)X[X'(P_B - K)X]^{-}X'(P_B - K)y && (23)\\
&= y'(P_B - K)X\frac{1}{r(1-E)}(I-K)X'(P_B - K)y && (24)\\
&= \frac{1}{1-E}[y'(P_B - K)X\frac{1}{r}X'(P_B - K)y] && (25)\\
&= \frac{1}{1-E}[y'(P_B - K)X(X'X)^{-1}X'(I - P_B)y] && (26)\\
&= \frac{1}{1-E}[y'(P_B - K)P_\tau(P_B - K)y] && (27)\\
& && (28)
\end{aligned}$$