

Phase 1			Phase 2		Between Runs stratum	Within Runs stratum			Treatment
v	r_b	DF of animals	n_R	n_γ		$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	
2	2	8	2	4	0	No(1 DF)	1	Yes	1
	3	4	3		1	No(1 DF)	2	No(1/9)	8/9
	4	6	4		1	No(1 DF)	4	Yes	1
	5	8	5		2	No(1 DF)	5	No(1/25)	24/25
	6	10	6		2	No(1 DF)	7	Yes	1
	7	12	7		3	No(1 DF)	8	No(1/49)	48/49
	8	14	8		3	No(1 DF)	10	Yes	1
	9	16	9		4	No(1 DF)	11	No(1/81)	80/81
	10	18	10		4	No(1 DF)	13	Yes	1

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v	r_b	DF of animals	n_R	n_γ		$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	
2	4	6	2	8	0	No(3 DF)	3	Yes	1
	6	10	3		1	No(3 DF)	6	No(1/9)	8/9
	8	14	4		1	No(3 DF)	10	Yes	1
	10	18	5		2	No(3 DF)	13	No(1/25)	24/25

Phase 1			Phase 2		Between Runs stratum	Within Runs stratum			Treatment	
v	r_b	DF of animals	n_R	n_γ		$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
3	2	3	3	4	1 (1)	No (1 DF)	1	Yes	1, 3/4	6/7
	4	9	6		2 (2)	No (1 DF)	6	Yes	15/16(2)	15/16
	6	15	9		4 (2)	No (1 DF)	10	Yes	23/24, 7/8	0.9148
	8	21	12		5 (2)	No (1 DF)	15	Yes	15/16 (2)	15/16
	10	27	15		7 (2)	No (1 DF)	19	Yes	19/20, 9/10	0.9243

Phase 1			n		Phase 2		Between Runs stratum		Within Runs stratum				Treatment	
v	r _b	DF of animals			n _R	n _γ	DF of animals		γ ⊥ A	Residual DF		γ ⊥ τ	e _{(τ)i}	E _τ
3	4	9	24		3	8	1 (1)		No (3 DF)	5	5	Yes	1, 15/16	30/31
	8	21	48		6		2 (2)		No (3 DF)	16	16	Yes	63/64 (2)	63/64

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v	r _b	DF of animals			n _R	n _γ	DF of animals		γ ⊥ A	Residual DF		γ ⊥ τ	e _{(τ)i}	E _τ
4	2	4	16		4	4	1		No (1DF)	2	3	Yes	1	1
	3	8	24		6		2		No (1DF)	5	7	No (1/9)	1(2), 8/9	24/25
	4	12	32		8		3		No (1DF)	8	11	Yes	1	1
	5	16	40		10		4		No (1DF)	11	15	No (1/25)	1(2), 24/25	72/73
	6	20	48		12		5		No (1DF)	14	19	Yes	1	1
	7	24	56		14		6		No (1DF)	17	23	No (1/49)	1(2), 48/49	0.993
	8	28	64		16		7		No (1DF)	20	27	Yes	1	1
	9	32	72		18		8		No (1DF)	23	31	No (1/81)	1(2), 80/81	0.995
	10	36	80		20		9		No (1DF)	26	35	Yes	1	1

Phase 1			Phase 2		Between Runs stratum	Within Runs stratum				Treatment	
v	r_b	DF of animals	n	n_R	n_γ	DF of animals	$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
4	2	4	16	2	8	0	No (3DF)	2	No (1/2)	1,1/2(2)	3/5
	3	8	24	3		1	No (3DF)	4	No (1/9)	8/9 (3)	8/9
	4	12	32	4		1	No (3DF)	8	Yes	1	1
	5	16	40	5		2	No (3DF)	11	No (1/25)	24/25(3)	24/25
	6	20	48	6		2	No (3DF)	15	No (1/18)	1, 17/18(2)	51/53
	7	24	56	7		3	No (3DF)	18	No (1/49)	48/49(3)	48/49
	8	28	64	8		3	No (3DF)	22	Yes	1	1
	9	32	72	9		4	No (3DF)	25	No (1/81)	80/81(3)	80/81
	10	36	80	10		4	No (3DF)	29	No (1/50)	1, 49/50(2)	0.9866

Phase 1			Phase 2		Between Runs stratum	Within Runs stratum				Treatment	
v	r_b	DF of animals	n	n_R	n_γ	DF of animals	$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
5	2	5	20	5	4	2 (2)	No (1 DF)	2	Yes	1(2), 7/8, 5/8	0.8434
	4	15	40	10		4 (4)	No (1 DF)	10	Yes	15/16(4)	15/16
	6	25	60	15		7 (4)	No (1 DF)	17	Yes	23/24(2), 11/12 5/6	0.9137
	8	35	80	20		9 (4)	No (1 DF)	25	Yes	15/16(4)	15/16
	10	45	100	25		12 (4)	No (1 DF)	32	Yes	19/20(2), 37/40, 7/8	0.9240

Phase 1			n	Phase 2		Between Runs stratum	Within Runs stratum			Treatment	
v	r_b	DF of animals		n_R	n_γ		$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
5	4	15	40	5	8	2 (2)	No (3 DF)	10	Yes	1(2), 15/16(2)	30/31
	8	35	80	10		4 (4)	No (3 DF)	28	Yes	0.994 (2), 0.959(2)	0.9763

Phase 1			n	Phase 2		Between Runs stratum	Within Runs stratum				Treatment	
v	r _b	DF of animals		n _R	n _γ		γ ⊥ A	Residual DF	γ ⊥ τ	e _{(τ)i}	E _τ	
6	2	6	24	6	4	2 (2)	No (1 DF)	3	3	Yes	1(3), 3/4(2)	0.8824
	3	12	36	9		4 (4)	No (1 DF)	7	7	No (1/9)	11/12(2), 8/9, 3/4(2)	0.8370
	4	18	48	12		5 (4)	No (1 DF)	12	13	Yes	1, 15/16(2), 13/16(2)	0.8937
	5	24	60	15		7 (5)	No (1 DF)	16	18	No (1/25)	0.953, 9/10, 0.8836, 0.8235, 4/5	0.8686
	6	30	72	18		8 (4)	No (1 DF)	21	25	Yes	1, 7/8 (4)	0.8974
	7	36	84	21		10 (5)	No (1 DF)	25	30	No (1/49)	13/14, 0.9164, 6/7(2), 0.8489	0.8804
	8	42	96	24		11 (5)	No (1 DF)	30	36	Yes	15/16 (2), 7/8 (3)	0.8990
	9	48	108	27		13 (5)	No (1 DF)	34	42	No (1/81)	0.9272, 11/12, 0.8872, 31/36, 0.8399	0.8852
	10	54	120	30		14 (5)	No (1 DF)	39	48	Yes	0.9 (5)	0.9

Phase 1			n	Phase 2		Between Runs stratum	Within Runs stratum				Treatment	
v	r _b	DF of animals		n _R	n _γ		γ ⊥ A	Residual DF	γ ⊥ τ	e _{(τ)i}	E _τ	
6	2	6	24	3	8	1 (1)	No (3 DF)	2	2	No (1/3)	1, 3/4, 2/3(3)	0.7317
	4	18	48	6		2 (2)	No (3 DF)	13	13	Yes	1(3), 15/16(2)	0.9740
	6	30	72	9		4 (4)	No (3 DF)	23	23	No (4/81)	0.9792, 0.9601, 0.9421 0.9375 0.9033	0.9438
	8	42	96	12		5 (4)	No (3 DF)	34	35	Yes	1, 63/64(2), 61/64(2)	0.9746
	10	54	120	15		7 (5)	No (3 DF)	44	46	No (4/225)	39/40, 0.974, 0.962 19/20, 0.949	0.9619

Phase 1			n	Phase 2		Between Runs stratum	Within Runs stratum				Treatment	
v	r _b	DF of animals		n _R	n _γ		γ ⊥ A	Residual DF	γ ⊥ τ	e _{(τ)i}	E _τ	
7	2	7	28	7	4	3 (3)	No (1 DF)	3	3	Yes	1(3), 7/8, 5/8, 1/2	0.7749
	4	21	56	14		6 (6)	No (1 DF)	14	14	Yes	7/8 (6)	7/8
	6	35	84	21		10 (6)	No (1 DF)	24	28	Yes	7/8(5), 19/24	0.8599
	8	49	112	28		13 (6)	No (1 DF)	35	42	Yes	7/8 (6)	7/8
	10	63	140	35		17 (6)	No (1 DF)	45	56	Yes	7/8(5), 33/40	0.8663

Phase 1			n	Phase 2		Between Runs stratum	Within Runs stratum			Treatment	
v	r_b	DF of animals		n_R	n_γ		$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
7	4	21	56	7	8	3 (3)	No (3 DF)	15	Yes	1(3), 31/32(2), 7/8	0.9666
	8	49	112	14		6 (6)	No (3 DF)	40	Yes	63/64 (6)	0.9844

Phase 1			n		Phase 2		Between Runs stratum		Within Runs stratum			Treatment	
v	r_b	DF of animals			n_R	n_γ	DF of animals	$\gamma \perp A$	Residual DF		$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
8	2	8	32		8	4	3 (3)	No (1 DF)	4	4	Yes	1(4), 3/4(2), 1/2	0.8077
	3	16	48		12		5 (5)	No (1 DF)	10	10	No (1/9)	1, 11/12(2), 8/9, 3/4(2), 2/3	0.8261
	4	24	64		16		7 (7)	No (1 DF)	16	16	Yes	0.963 (2), 0.875 (2), 0.7866 (2), 0.75	0.8498
	5	32	80		20		9 (7)	No (1 DF)	22	24	No (1/25)	9/10(3), 43/50, 4/5(3)	0.8489
	6	40	96		24		11 (6)	No (1 DF)	28	33	Yes	1, 5/6(6)	0.8537
	7	48	112		28		13 (7)	No (1 DF)	34	40	No (1/49)	6/7(6), 41/49	0.8542
	8	56	128		32		15 (7)	No (1 DF)	40	48	Yes	0.9192(2), 0.875, 0.8308(2), 0.8125(2)	0.8550
	9	64	144		36		17 (7)	No (1 DF)	46	56	No (1/81)	8/9(2), 71/81 , 5/6 (3)	0.8546
	10	72	160		40		19 (7)	No (1 DF)	52	64	Yes	0.9, 0.8854(2), 17/20 (2), 0.8146(2)	0.8559

Phase 1			n	Phase 2		Between Runs stratum		Within Runs stratum			Treatment	
v	r_b	DF of animals		n_R	n_γ	DF of animals		$\gamma \perp A$	Residual DF		$e_{(\tau)i}$	E_τ
8	2	8	32	4	8	1		No (3 DF)	4	5	1(4), 3/4(2), 1/2	0.8077
	3	16	48	6		2		No (3 DF)	11	13	1(4), 8/9(3)	0.9492
	4	24	64	8		3		No (3 DF)	18	21	1(7)	1
	5	32	80	10		4		No (3 DF)	25	29	1(4), 24/25(3)	0.9825
	6	40	96	12		5		No (3 DF)	32	37	1(4), 35/36(2), 17/18	0.9837
	7	48	112	14		6		No (3 DF)	39	45	1(4), 48/49(3)	0.9912
	8	56	128	16		7		No (3 DF)	46	53	1(7)	1
	9	64	144	18		8		No (3 DF)	53	61	1(4), 80/81(3)	0.9947
	10	72	160	20		9		No (3 DF)	60	69	1(4), 99/100(2), 49/50	0.9942

Table 1: Summary table of optimal designs for the Phase 1 experiments arranged with BIBD

Phase 1					n		Phase 2		Between Runs stratum		Within Runs stratum			Treatment	
v	n_C	r_b	DF of animals	E_τ			n_R	n_γ	DF of cages	DF of animals	$\gamma \perp A$	Residual DF	$\gamma \perp \tau$	$e_{(\tau)i}$	E_τ
4	4	3	5	8/9	24	24	6	4	0	2	Yes	3	5	8/9(3)	8/9
5	5	4	11	15/16	40	40	10	4	4(4)	0	No(1 DF)	10	10	15/16(4)	15/16
6	6	5	19	24/25	60	60	15	4	3(3)	4(3)	Yes	13	14	0.938, 9/10, 0.874, 0.822, 0.786	0.8606
7	7	6	29	35/36	84	84	21	4	6(6)	4(4)	No(1 DF)	21	21	0.921, 0.874, 0.863, 0.840, 0.814, 0.780	0.8462
7	7	4	15	7/8	56	56	14	4	6(6)	0	No(1 DF)	14	14	7/8(6)	7/8
8	8	7	41	48/49	112	112	28	4	5(5)	8(5)	Yes	29	32	13/14(2), 6/7, 41/49(2), 11/14(2)	0.8478
4	4	3	5	8/9	24	24	3	8	0	1	Yes	4	5	8/9(3)	8/9
5	5	4	11	15/16	40	40	5	8	2(2)	0	No(3 DF)	8	8	15/16(4)	15/16
7	7	4	15	7/8	56	56	7	8	3(3)	0	No(3 DF)	12	12	7/8(6)	7/8
8	8	7	41	48/49	112	112	14	8	0	6	Yes	35	41	48/49(7)	48/49