Basel III / FRTB-Compliant VaR & ES for Electricity Trading

1. Introduction

Electricity markets exhibit extreme volatility, fat-tailed return distributions, and strong dependence on weather, demand and renewable output. Under Basel III's Fundamental Review of the Trading Book (FRTB), these characteristics materially affect capital requirements and model approval.

A disciplined, regulatory-aligned risk framework is required to ensure compliance, support limit setting, and guide decisions in this structurally uncertain market.

1.2 Objective:

Develop a Basel III / FRTB-aligned risk measurement framework that:

- Quantifies downside risk using Value at Risk (VaR) and Expected Shortfall (ES).
- Meets FRTB standards.
- Validates model performance via backtesting and P&L attribution.
- Assesses resilience under market-specific stress scenarios.

1.3 Portfolio and Data Description

- Instruments: Hourly day-ahead prices.
- Market: Electricity (power) trading.
- Data frequency: hourly aggregation.
- Risk Horizon: 1-day (internal VaR, P&L attribution, operational risk control and daily risk limit) and 10-day (regulatory VaR / ES).

1.4 Portfolio Return and Loss Construction

Let

- n be the number of traded power instruments (Baseload, Peakload, Quarterly, Monthly, and Hourly forwards)
- $P_{i,t}$ be the settlement price of instrument i at time t
- $r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$ be the log-return (due to additive property)
- w_i be the exposure-weighted position of instrument i
  (MW * delivery hours * price sensitivity)

The portfolio return is:

$$R_{P,t} = \sum_{i=1}^{N} w_i r_{i,t} = w^{\mathrm{T}} r_t$$

The portfolio loss is defined as:

$$L_t = -R_{P,t}$$

Negative returns because returns and losses move in opposite directions.

2. Risk Measures

2.1. Value at Risk (VaR)

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2.1.1 Historic VaR

Let $\alpha \in (0, 1)$ denote the confidence level.

VaR at confidence level α denotes the threshold that will not be exceeded with probability α over a chosen horizon. This method takes all historical portfolio losses, sorts them from best to worst, finds the α-quantile, and returns that value as historical VaR

$$VaR_{\alpha}^{Hist} = inf\{l \in \mathbb{R}: \mathbb{P}(L \ \leq l) \geq \alpha\}$$

Characteristics:

- Uses the empirical distribution of historical losses
- Directly captures price spikes, skewness, and fat tails
- Well-suited for electricity markets where returns are highly non-normal

2.1.2 Parametric (Variance-Covariance) VaR

The parametric VaR method assumes that portfolio returns follow a normal distribution.

$$r_t \backsim \mathcal{N}(\mu, \Sigma): \quad r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ \vdots \\ r_{n,t} \end{bmatrix}, \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}, \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots \\ \sigma_{21} & \sigma_2^2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

where:

- r_t = vector of asset returns (column vector containing returns of all instruments in the portfolio at time t)
- μ = vector of expected returns (average return of each instrument)
- Σ = covariance matrix of returns (captures individual volatilities and volatilities between instruments)
- Returns are jointly normally distributed

Portfolio Mean (expected portfolio return), $\boldsymbol{\mu_P}$

$$\mu_P = w^{\mathrm{T}}\mu$$

- w = vector of portfolio weights or exposures

Portfolio variance, $\boldsymbol{\sigma_P^2}$

$$\sigma_P^2 = w^{\mathrm{T}}\Sigma w$$

- Captures how volatilities and correlations aggregate into portfolio risk

Portfolio standard deviation

$$\sigma_P = \sqrt{w^T \Sigma w}$$

- Easy to interpret.

Parametric VaR Formula

$$VaR_\alpha^{Para} = -\left(\mu_P + \sigma_P \Phi^{-1}(\alpha)\right)$$

where:

- $\Phi^{-1}(\alpha)$ = z-score (critical value from the standard normal distribution that corresponds to the chosen confidence level α)
- Negative sign expresses VaR as a positive loss

Characteristics:

- Relies on strong normality assumptions making it unreliable in markets with fat tails and extreme volatility such as electricity markets
- Poor tail-risk representation
- Sensitive to covariance estimation

2.1.3 Monte Carlo VaR

This method estimates portfolio risk by simulating many possible future return scenarios rather than relying only on historical data or assuming a closed-form distribution.

$$r^{(m)} \backsim \mathcal{N}(\mu, \Sigma)$$

Where:

$r^{(m)}$: simulated return vector for scenario m

The simulation preserves correlations between assets, and each simulation produces a hypothetical loss or gain for the portfolio.

$$VaR_\alpha^{MC} = \text{empirical } \alpha\text{-quantile of } \left\{L^{(m)}\right\}$$

$$L^{(m)} = -w^{\mathrm{T}}\mathrm{r}^{(m)}$$

- Sort the simulated losses
- Take the α-quantile
- This gives the Monte Carlo VaR estimate

Characteristics:

- More robust for long or complex portfolios
- Computationally intensive
- Well-suited for electricity markets with extreme volatility

2.2 Expected Shortfall (ES)

GRAPH

Expected Shortfall measures the average loss in the worst $\alpha\%$ of cases. That is, the average loss beyond VaR:

$$\mathrm{ES}_\alpha = \mathbb{E}[L \mid L \geq \mathrm{VaR}_\alpha]$$

Parametric ES (Normal Distributions)

If returns are assumed normal:

$$\mathrm{ES}_\alpha^{Para} = -\left(\mu_P + \sigma_P \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}\right)$$

Where:

$\phi(\cdot)$ standard normal density (electricity prices have extreme spikes. The normal density $\phi(\cdot)$ captures tail thickness, making ES more informative than VaR)

Characteristics:

- Required under Basel III / FRTB
- Better suited for fat-tailed markets like electricity
- Measures average tail loss, not just a threshold

Backtesting and Model Validation

Backtesting checks whether the VaR model performs as expected.

VaR Exceedance Indicator:

$$I_t = \begin{cases} 1, & L_t > \mathrm{Var}_{\alpha,\mathrm{t}} \\ 0, & \text{otherwise} \end{cases}$$

$I_t = 1$ means VaR was breached

$N = \sum_{t=1}^{T} I_t$ be the number of exceedances

T be the number of observations

Kupiec Proportion of Failures (POF) Test

This test checks whether the number of VaR breaches matches what the confidence level predicts.

$$LR_{\mathrm{POF}} = -2\ln\left(\left[\frac{(1-\alpha)^N \alpha^{T-N}}{(1-\hat{p})^N \hat{p}^{T-N}}\right]\right) \backsim \chi^2(1)$$

Where:

$\hat{p} = N/T$= observed breach frequency

Interpretation

- If breaches occur too often, then VaR underestimates risk
- If breaches occur too rarely, then VaR is overly conservative

Characteristics of the Kupiec Test:

- Only checks the number of exceedances
- Does not check whether exceedances cluster
- Tests unconditional coverage

Christoffersen Independence Test

Kupiec checks how many breaches occur.

Christoffersen checks when they occur.

Purpose:

- Tests whether VaR breaches are independent over time
- Detects clustering, which often happens in stressed markets, volatility regimes and electricity price spikes

Interpretation:

If breaches cluster, it could mean:

- volatility increased suddenly
- correlations changed
- the model didn't update quickly
- VaR is too slow or too smooth
- The model underestimates risk during stressed periods

Characteristics of the Christoffersen Test:

- Tests independence of exceedances

- Detects volatility clustering
- Complements the Kupiec test
- Essential for regime markets with regime shifts

Stress Testing and Scenario Analysis

Stress testing evaluates portfolio risk under extreme (what-if scenario) but plausible market conditions.

Volatility Stress (Scarcity Pricing)

$$\Sigma^{stress} = k^2\Sigma$$

Where:

Σ^{stress} = stressed covariance matrix

k= volatility-scaling factor

Why k^2?

Covariance is defined as:

$$Cov(X,Y) = \sigma_X\sigma_Y\rho_{XY}$$

If each volatility increases by k, then:

- $\sigma_X \to k\sigma_X$
- $\sigma_Y \to k\sigma_Y$

So, covariance becomes:

$$(k\sigma_X)(k\sigma_Y)\rho_{XY} = k^2\sigma_X\sigma_X\rho_{XY}$$

That's why the entire covariance matrix must be multiplied by k^2.

Stressed Portfolio Volatility

$$\sigma_P^{stress} = k\sigma_P$$

σ_P=original portfolio volatility

σ_P^{stress}=stressed portfolio volatility

Why only k?

Portfolio volatility is a single standard deviation, not a covariance. So, it scales linearly, not quadratically.

Characteristics:

- Simple and transparent
- If VaR explodes under stress, the portfolio is highly sensitive to volatility shocks.

Correlation Stress (Market Coupling Risk)

Correlation stress models a situation where markets become more tightly correlated.

$$\rho_{i,j}^{stress} = \min(\rho_{ij} + \Delta_\rho \, , 1)$$

Where:

ρ_{ij}= original correlation between the instrument i and j

Δ_ρ= correlation stress add-on (e.g, +0.1, +0.2, +0.3)

$\rho_{i,j}^{stress}$= stressed correlation

$\min(\cdot \, , 1)$= ensures correlations never exceed the maximum possible value of 1

Constructing the Covariance Matrix

$$\Sigma^{stress} = DR^{stress}D$$

Where:

R^{stress}= stressed correlation matrix

$D = \text{diag}(\sigma_1, \ldots, \sigma_N)$, a diagonal matrix containing the volatilities of each instrument

Σ^{stress}= stressed covariance matrix

Characteristics of Correlation Stress

- When correlations rise, diversification benefits shrink or vanish
- Produces realistic worst-case scenarios
- Higher correlation → higher covariance → higher portfolio risk

Models joint price spikes across hours, maturities, and regions.

Key Risk Management Insights

- VaR alone is not enough, ES is essential for electricity markets
- Monte Carlo VaR is the only method flexible enough for non-linear, spiky markets
- Covariance and correlation structures drive portfolio risk more than individual volatilities
- Stress testing reveals vulnerabilities that VaR and ES cannot.
- Backtesting is non-negotiable, VaR must prove itself in real data
- Correlation stress is the most realistic stress for power markets
- Exposure vector clarity is crucial for accurate risk attribution

- Model transparency and simplicity matter as much as mathematical sophistication

Business Relevance

- Protects P&L from extreme electricity price movements
- Enables informed trading and hedging decisions
- Strengthens portfolio resilience

Regulatory Relevance

- Aligns with Basel III / FRTB expectations
- Demonstrates model governance and validation discipline
- Supports ICAAP / ILAAP and internal capital models
- Meets expectations for stress testing and scenario analysis
- Ensures transparency and defensibility of risk numbers