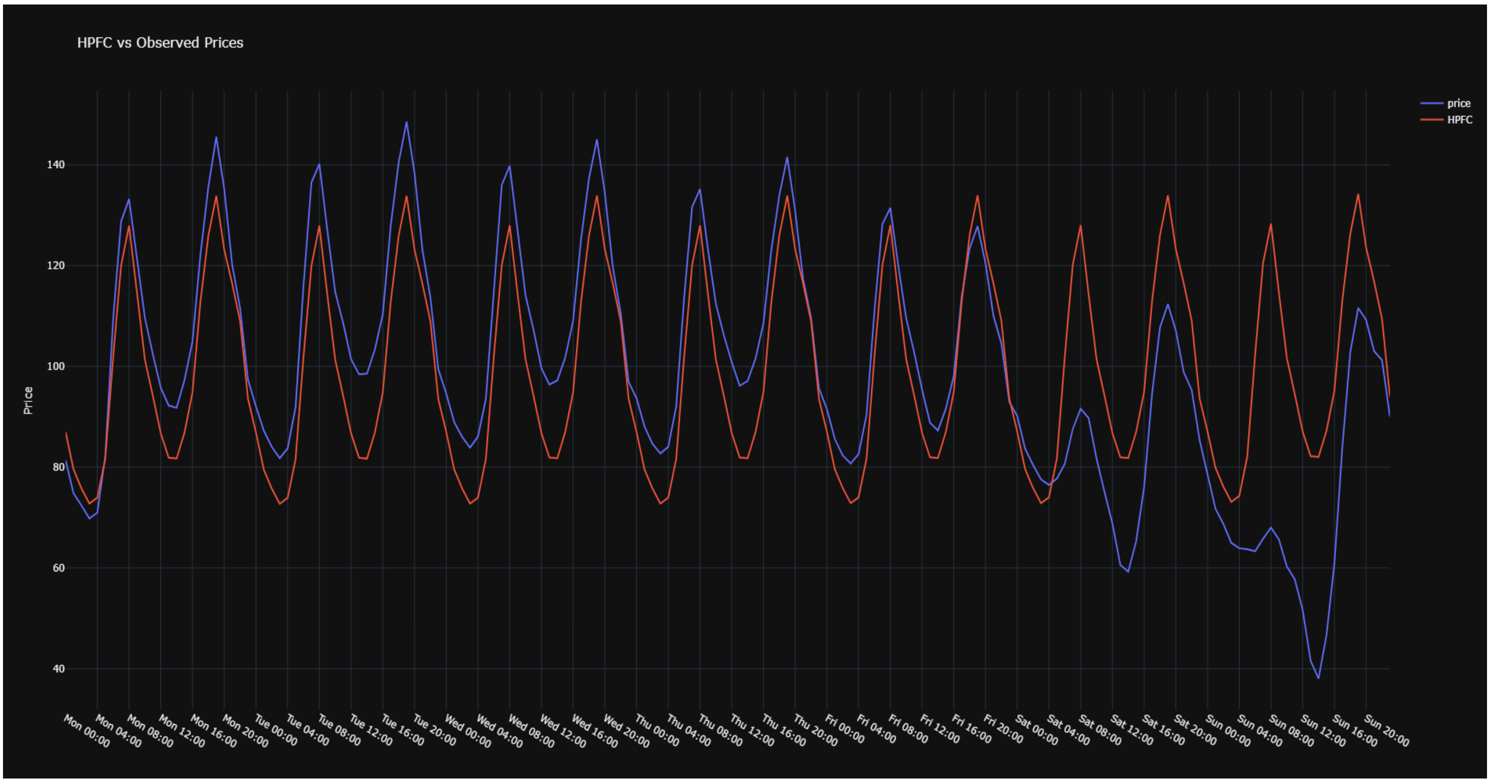
HOURLY PRICE FORWARD CURVE

A market-consistent, two-stage modelling framework



Chamberlin Keuleck

keuleckchamberlin@gmail.com

1. Executive Summary

Objective:

Design a robust and market-consistent Hourly Price Forward Curve (HPFC) for electricity markets that:

- Preserves traded weekly prices
- Captures realistic intraday patterns
- Integrates renewable(solar) effects
- Is stable, interpretable, and production-ready

Solution:

A two-stage linearized modelling framework:

- Stage 1: Shape predictions (Peak and Off-peak) on normalized prices
- Stage 2: Level (Price) reintroduction via weekly scaling

Outcome:

- Arbitrage-free hourly curves
- Transparent peak/off-peak structure
- Direct applicability to trading, risk, and valuation

2. Problem Statement & Hypothesis

2.1 Problem

Electricity forward prices are traded at aggregate levels (weekly base, peak), while risk, valuation, and operations require hourly granularity.

Naively fitting hourly prices leads to:

- Instability
- Overfitting
- Loss of market consistency

2.2 Working Hypothesis

Hourly electricity prices can be decomposed into a stable intraday shape and a weekly price level.

If this is true, then:

- Shapes can be predicted robustly
- Levels can be imposed exogenously
- The resulting HPFC is both realistic and arbitrage-free

3. Methodology

Stage	Purpose	Output
1	Learn intraday Structure	Normalized shapes
2	Introduce market prices	Final HPFC

3.1. Stage 1 - Learn Intraday Structure

Variable	**Definition**	**Purpose**
t	Absolute hour index (1-hour granularity)	Base time resolution
$k(t)$	Hour-of-day (1-24)	Captures daily cycles
$\mathcal{W}$	Calendar week (168 hours)	Forecast horizon
S_t	Observed spot price	Empirical input
$R_t \in [0, 1]$	Normalized solar production	Renewable impact
$b(t)$	Seasonal bucket (summer, winter, transitions)	Seasonal differentiation
x_k, y_k	Peak/off-peak factors	Intraday structure
β_b	Solar sensitivity per bucket	Adjusts for renewables

3.1.1. Shape Predictions (Linearized)

- $\mu_{\mathcal{W}}$: Weekly mean

$$\mu_{\mathcal{W}} = \frac{1}{168} \sum_{t \in \mathcal{W}} S_t$$

- $\bar{S}_{\mathcal{W}}^{\mathcal{P}}$: Weekly peak-hour mean:

$$\bar{S}_{\mathcal{W}}^{\mathcal{P}} = \frac{1}{|\mathcal{P}(\mathcal{W})|} \sum_{t \in \mathcal{P}(\mathcal{W})} S_t$$

- $\bar{S}_{\mathcal{W}}^{\mathcal{O}}$: Weekly off-peak-hour mean:

$$\bar{S}_{\mathcal{W}}^{o} = \frac{1}{|\mathcal{O}(\mathcal{W})|} \sum_{t \in \mathcal{O}(\mathcal{W})} S_t$$

Where:

- $\mathcal{P}(\mathcal{W})$ = set of peak hours (Mon-Fri, 08-20)
- $\mathcal{O}(\mathcal{W})$ = set of off-peak hours (Mon-Fri, 01-07 & 21-24 and weekends)

- Normalize spot prices within each week:

$$s_t = \frac{S_t}{\mu_{\mathcal{W}}}, \quad t \in \mathcal{W}$$

By definition, s_t is dimensionless and satisfies:

$$\frac{1}{168}\sum_{t\in\mathcal{W}} s_t = 1$$

3.1.2 Intraday Shape Model Normalization:

For hour t in week $\mathcal{W}$ with hour-of-day $k(t)$:

$$f_t^{shape} = \alpha_{\mathcal{W}}^{\mathcal{P}} x_k + \alpha_{\mathcal{W}}^{o} y_k$$

With

$$\alpha_{\mathcal{W}}^{\mathcal{P}} = \frac{\bar{S}_{\mathcal{W}}^{\mathcal{P}}}{\mu\_\mathcal{W}}, \quad \alpha_{\mathcal{W}}^{0} = \frac{\bar{S}_{\mathcal{W}}^{o}}{\mu\_\mathcal{W}}$$

- **Solar Adjustment**

Including additive solar effects:

$$\tilde{f} = f_t^{shape} - \beta_{b(t)} R_t \ , \qquad \beta_b \geq 0$$

- **Weekly Shape Normalization**

To maintain weekly mean = 1 after solar adjustment:

$$\bar{f}_{\mathcal{W}} = \frac{1}{168}\sum_{t\in\mathcal{W}} \tilde{f}_t$$

$$\hat{s}_t = \frac{\tilde{f}_{\mathcal{W}}}{\bar{f}_{\mathcal{W}}}$$

By construction:

$$\frac{1}{168}\sum_{t\in\mathcal{W}} \hat{S}_t = 1$$

3.1.3 Intraday Structure Optimization

$$\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{\beta}} \sum_{\mathcal{W}\in\mathcal{T}} \sum_{t\in\mathcal{T}} (\hat{\boldsymbol{s}}_t - \boldsymbol{s}_t)^2 + \boldsymbol{\lambda}_x \sum_{k=1}^{23} (\boldsymbol{x}_{k+1} - \boldsymbol{x}_k)^2 + \boldsymbol{\lambda}_y \sum_{k=1}^{23} (\boldsymbol{y}_{k+1} - \boldsymbol{y}_k)^2 + \boldsymbol{\lambda}_{\beta} \sum_{b} \boldsymbol{\beta}_b^2$$

Interpretation:

- First term: Fits the modelled price $\hat{s}_t$ to the observed spot price s_t. This ensures realism.
- Second and third terms: Smoothness penalties on $x_k$and y_k. They discourage sharp jumps between consecutive hours, producing a realistic intraday curve.
- Fourth term: Regularization on seasonal factors β_b, preventing extreme values.

This is a convex quadratic program, meaning it has a unique global minimum and can be solved efficiently with standard optimization tools.

Constraints:

1- Peak/off-peak support:

$$x_k = 0 \quad for\ k \notin \{8, \dots, 20\}, \quad y_k = 0 \quad for\ k \in \{8, \dots, 20\}$$

2- Normalization:

$$\sum_{k=1}^{24} x_k = 1, \qquad \sum_{k=1}^{24} y_k = 1,$$

3- Non-negativity:

$$x_k \geq 0, \quad y_k \geq 0, \quad \beta \geq 0$$

This normalization separates price shape from price level, making the model identifiable, stable and market consistent.

3.2 Stage 2 - Introduce market prices

- Compute raw hourly forecast (price units):

$$F_t^{raw} = \bar{S}_{\mathcal{W}}^{\mathcal{P}} x_k + \bar{S}_{\mathcal{W}}^{o} y_k - \mu_{\mathcal{W}} \beta_{b(t)} R_t$$

- Compute weekly mean:

$$\bar{F}_{\mathcal{W}} = \frac{1}{168} \sum_{t \in \mathcal{W}} F_t^{raw}$$

- Apply weekly scaling:

$$\hat{S}_t = \mu_{\mathcal{W}} \frac{F_t^{raw}}{\bar{F}_{\mathcal{W}}}$$

This ensures:

$$\frac{1}{168} \sum_{t \in \mathcal{W}} \hat{S}_t = \mu_{\mathcal{W}}$$

- $\hat{S}_t$ is the final HPFC

4. Implementation & Scalability

4.1. Extensions

- Include weekend and holiday effects.
- Additional regressors: Energy Demand $\mathcal{D}_t$, wind energy W_t, fuel spreads, large energy storage facilities.
- Seasonal variations: Peak and Off-peak factors per seasonal bucket.

5. Business Impact

5.1. Who uses this?

- Trading desks
- Risk management
- Valuation & PnL attribution

5.2. Why it matters

- Better hedging accuracy
- Transparent assumptions
- Reduced model risk

6. Key Takeaway

By separating shape from level, this model achieves both statistical robustness and market consistency aligning quantitative modelling with how electricity markets are treated.