Kevin Chan MA415 Assignment 2: Basic R Exercise 2

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Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$
(b) Replace the third column of A by the sum of the second and third columns

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First, produce A

```
A <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A%%A%%A
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Then, add the columns 2 and 3 and assign the sum to the third column

```
A[,3] <- A[,2] + A[,3]

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

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2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \cdots & \cdots & \cdots \\ 10 & -10 & 10 \end{bmatrix}$$

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```r
btb <- matrix(c(10, -10, 10), b=T, nc=3, nr=15)
crossprod(btb)
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

Now, create the 6 x 6 matix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is matE, a 6x6 matrix of 0's followed by row(matE) and col(matE)

```r
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)

# Note what the functions row() and col() do
row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    2    2    2    2    2    2
## [3,]    3    3    3    3    3    3
## [4,]    4    4    4    4    4    4
## [5,]    5    5    5    5    5    5
## [6,]    6    6    6    6    6    6
```

```r
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2    3    4    5    6
## [2,]    1    2    3    4    5    6
## [3,]    1    2    3    4    5    6
## [4,]    1    2    3    4    5    6
## [5,]    1    2    3    4    5    6
## [6,]    1    2    3    4    5    6
```

```r
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
## [6,]    5    4    3    2    1    0
```

```r
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

---

4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
a <- 0:4
A <- outer(a,a,"+")
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```r
FiveA <- outer(0:4,0:4,"+")%%5
FiveA
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```r
FiveB <- outer(0:9,0:9,"+")%%10
FiveB
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```r
FiveC <- outer(0:8,0:8,"-")%%9
FiveC
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation $\mathbf{Ax} = \mathbf{y}$.

$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 7$
$2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 = -1$
$3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 = -3$

$4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 = 5$
$5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 = 17$

```
yVec <- c(7,-1,-3,5,17)
AMat <- matrix(0,nr=5, nc=5)
AMat <- abs(col(AMat)-row(AMat))+1
xVec <- solve(AMat,yVec)
xVec
```

```
## [1] -2  3  5  2 -4
```

```
# To verify that xVec is indeed the solution to the system of equations.
AMat%*%xVec
```

```
##      [,1]
## [1,]    7
## [2,]   -1
## [3,]   -3
## [4,]    5
## [5,]   17
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)

```
set.seed(75)
aMat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```
apply(aMat, 1, function(x){sum(x>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
which(apply(aMat, 1, function(x){sum(x==7)==2}))
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater tnan 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for exalmple, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```
aMatColSums <- colSums(aMat)
which(outer(aMatColSums, aMatColSums, "+") > 75, arr.ind=T)
```

```
##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
## [5,]   8   6
## [6,]   2   8
## [7,]   6   8
```

```
## [8,]   8   8
```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```r
logicMat <- outer(aMatColSums, aMatColSums, "+") > 75
logicMat[lower.tri(logicMat,diag=T)] <- F
which(logicMat, arr.ind=T)
```

```
##      row col
## [1,]   2   6
## [2,]   2   8
## [3,]   6   8
```

8. Calculate

(a) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```r
sum((1:20)^4) * sum(1/(3+(1:5)))
```

```
## [1] 639215.3
```

```r
# or

sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```r
sum(((1:20)^4)/(3+outer(1:20,1:5,"*")))
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10}\sum_{j=1}^{i}\frac{i^4}{(3+ij)}$

```r
sum(outer(1:10,1:10,function(i,j){(i>=j)*i^4/(3+i*j)}))
```

```
## [1] 6944.743
```