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Calculus 2 Assignment

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Contents

I Problem 1.

Let $z = f(x) = x^4 - 2x^2 - y^3 + 3y$.

(a) Draw the graph of the function.

(b) Draw the contour plot of the function. Point out the local extreme and the saddle point on that figure.

(c) Find the exact local extreme and saddle point (using calculus technique).

Theory:

We all know that the main uses of ordinary derivatives is to find maximum and minimum values (extreme values). Similar with multivariable function, we can you partial derivatives to do the same thing. Suppose we have a two-variable function f is continuous on interval (a, b) and it is said that:

A function f of two variables has a local maximum at (a, b) if $f(x, y) \le f(a, b)$ when (x, y) is near (a, b). This means that $f(x, y) \le f(a, b)$ for all points (x, y) in some disk with center (a, b). The number $f(a, b)$ is called a local maximum value. If $f(x, y) \ge f(a, b)$ when (x, y) is near (a, b), then f has a local minimum at (a, b) and $f(a, b)$ is a local minimum value.

If the inequalities in the above definition hold for all points (x, y) in the domain of f, then f has an absolute maximum or local minimum at (a, b).

Therefore, If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

But in some problem, There's a point called saddle point which located at the origin. Thus, we have another tool called **Second Derivatives Test** to determine which points are minimum, maximum and saddle.

Suppose the second partial derivatives are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let:

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.
If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local minimum.
If $D < 0$, then $f(a, b)$ is not a local extrema, and the point (a, b) is called a saddle point of f.
Note: If $D = 0$, the test give no information: f could have a local maximum or local minimum at (a, b), or (a, b) could be a saddle point of f.

Finding the local extreme values of the function $f(x, y)$:

- Identifying critical points for the given function $f(x, y)$.

  - Find the partial derivatives with respect to x and to y.
  - Set each partial derivative equal to zero.

– Solve the system of equations to get critical points (x_0, y_0).

- Consider $D(x_0, y_0)$.

  – Find all second derivatives of $f(x_0, y_0)$.
  – Identify by the Second Derivative Test $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

MATLAB code:

```
%calculation
syms x y z
z = x.^2 + 2*y.^2 + 3*x.*y.^3 - y.^3;

%differentiate z
fx = diff(z,x,1);
fy = diff(z,y,1);
fxx = diff(z,x,2);
fyy = diff(z,y,2);
fxy = diff(fx,y,1);

%create a system equation
eqns = [ fx == 0, fy == 0];

%define the variables for system equation
vars = [x y];

%solve the system equation
[solx, soly] = vpasolve(eqns, vars);

%only choose the real value
solx_real = solx(imag(solx)==0);
soly_real = soly(imag(solx)==0);


%display section
disp(['z = f(x,y) = ', char(z)])
disp(['fx = ', char(fx)])
disp(['fy = ', char(fy)])
disp(['fxx = ', char(fxx)])
disp(['fyy = ', char(fyy)])
disp(['fxy = ', char(fxy)])

%c
```

```matlab
for n=1:length(solx_real)
    fxx_val = subs(fxx,[x,y],[solx_real(n),soly_real(n)]);

%calculate fxx, fyy, fxy values
fyy_val = subs(fyy,[x,y],[solx_real(n),soly_real(n)]);
fxy_val = subs(fxy,[x,y],[solx_real(n),soly_real(n)]);
D = fxx_val*fyy_val - fxy_val*fxy_val;

fprintf('\nFor point(%s,%s)\n',solx_real(n),soly_real(n))
    if D > 0
        fprintf('D = fxx*fyy-fxy^2 = %s\n',D)
        fprintf('So D > 0\n')
        fprintf('Therefore point (%s,%s) is local extreme\n',...
            solx_real(n),soly_real(n))
        if fxx_val > 0
            fprintf('And fxx = %s\n',fxx_val)
            fprintf('So fxx > 0\n')
            fprintf('Therefore this is a local minimum point\n')
        else
            fprintf('And fxx = %s\n',fxx_val)
            fprintf('So fxx < 0\n')
            fprintf('Therefore this is a local maximum point\n')
        end
    elseif D < 0
        fprintf('D = fxx*fyy-fxy^2 = %s\n',D)
        fprintf('So D < 0\n')
        fprintf('Therefore point (%s,%s) is a saddle point\n',...
            solx_real(n),soly_real(n))
    else
        fprintf('D = fxx*fyy-fxy^2 = %s\n',D)
        fprintf('So conclusion for point (%s,%s) \n',solx_real(n),...
            soly_real(n))
    end
end
```
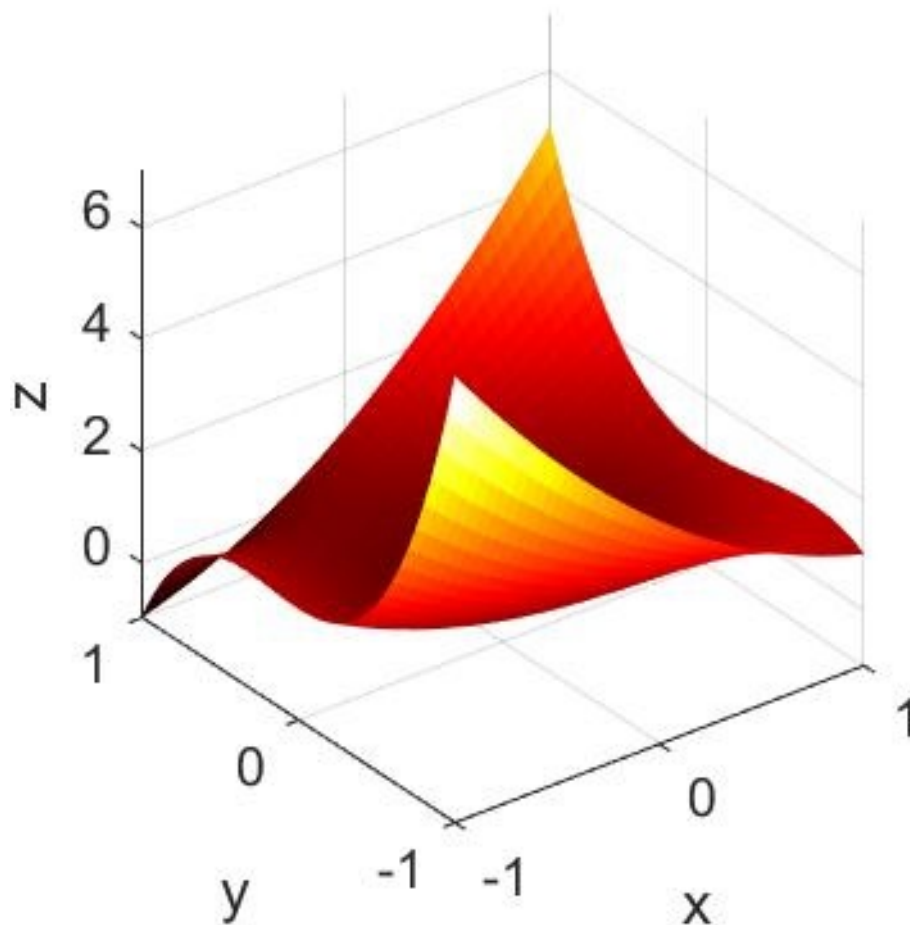
Figure 1: The graph of the function $z = f(x) = x^4 - 2x^2 - y^3 + 3y$.

```
syms x y z
[x,y] = meshgrid(-1:0.05:1,-1:0.05:1);
z = x.^2 + 2*y.^2 + 3*x.*y.^3 - y.^3;

%plot 2 figure in 1 window
tiledlayout(2,1);

%a
nexttile
surf(x,y,z,'edgecolor', 'none');
colormap hot;    %the higher value, the hotter color
xlabel('x');
ylabel('y');
zlabel('z');
title('GRAPH OF THE FUNCTION')
pbaspect([1,1,1])
```

```
%b
nexttile
contour(x,y,z,200)
hold on
for n =1:length(solx_real)
    plot(solx_real(n),soly_real(n),'*')
end
xlabel('x');
ylabel('y');
title({'CONTOUR PLOT THE FUNCTION,','LOCAL EXTREME AND SADDLE POINT'
    })
pbaspect([1,1,1])
```
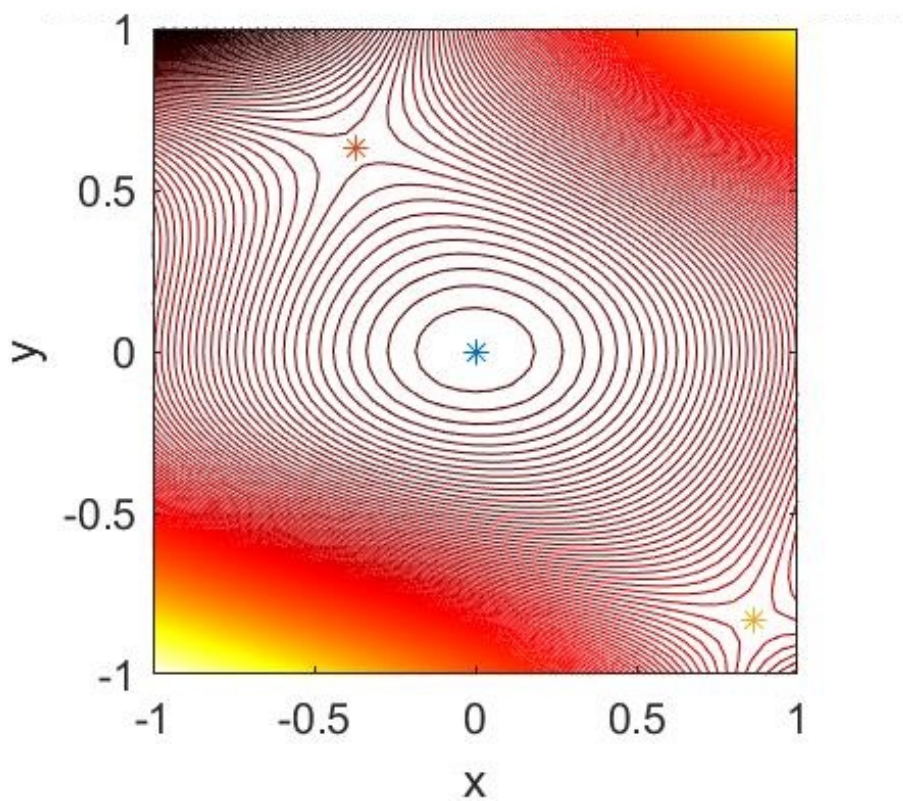
Figure 2: The contour plot the function $z = f(x) = x^4 - 2x^2 - y^3 + 3y$.

II Problem 2.

Find the maximum and minimum values of $z = 2x^2 - 2xy + y^3$ subject to the single constraint $x^2 + y^2 = 4$.

 (a) *Using Lagrange multiplier method.*

  (b) *Using contour plot (Draw the contour plot of the function and the constraint curve in the same figure).*

Theory:

To find the maximum and minimum value of $f(x,y) = 2x^2 - 2xy + y^3$ subject to the single constraint $g(x,y) = x^2 + y^2 = 4$. We need to use the Lagrange multiplier method.

Lagrange multiplier method is a technique for finding a maximum or minimum values of a multivariable function $f(x, y, z, ...)$ subject to a constraint of the form $g(x, y, z, ...)$.

For example, a function of 2 variables can be used to explain the method. We get started by finding the extreme values of $f(x, y)$ subject to a constraint $g(x, y)$. In other words, we find the points satisfying the condition that points (x, y) lie on the level curve $g(x, y) = k$. The figure below shows this curve together with several level curves of $f(x, y)$. These have the equations $f(x, y) = c$ where $c = 7, 8, 9, 10, 11$.
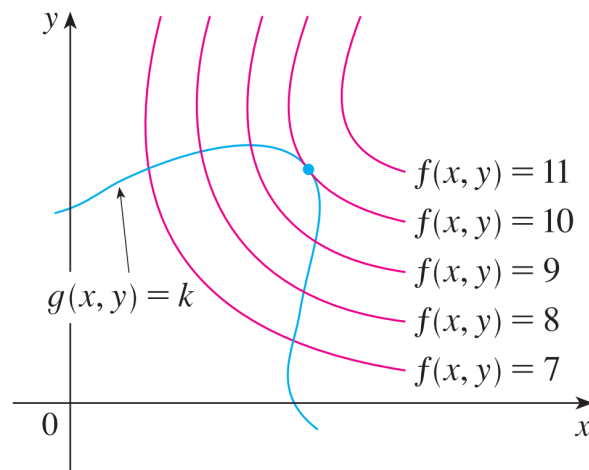
The graph show that the maximum value occupied at the point where $f(x, y) = 10$ intersecting $g(x, y) = k$. Analyzing the graph, we can easily see the fact that the maximum value appears when those 2 functions touch each other and they have the same tangent line. Thus the normal lines at the point (x_0, y_0) where they touch are identical. So the gradient vectors are parallel; Therefore, $\nabla f(x_0, y_0) = \lambda g(x_0, y_0)$ where ∇ is a scalar.

For some scalar λ. The scalar parameter is called a Lagrange multiplier. The procedure based on the above equation is as follows. We have from the chain rule,

$$\frac{df(x,y)}{dxdy} = \frac{\partial f(x,y)}{\partial x}dx + \frac{\partial f(x,y)}{\partial y} = 0, \quad \frac{dg(x,y)}{dxdy} = \frac{\partial g(x,y)}{\partial x}dx + \frac{\partial g(x,y)}{\partial y}$$

Multiplying the second equation by and add to first equation yield:

$$\left(\frac{\partial f(x,y)}{\partial x} + \frac{\partial g(x,y)}{\partial x}\right) dx + \left(\frac{\partial f(x,y)}{\partial y} + \frac{\partial g(x,y)}{\partial y}\right) dy = 0$$

Then we can rewrite the expression as the vector equation:

$$\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$$

Thus, the maximum and minimum values of $f(x,y)$ subject to the constraint $g(x_0, y_0) = 0$ can be found by solving the following set of equations.

$$\frac{\partial f(x,y)}{\partial x} + \frac{\partial g(x,y)}{\partial x} = 0$$

$$\frac{\partial f(x,y)}{\partial y} + \frac{\partial g(x,y)}{\partial y} = 0$$

$$g(x,y) = 0$$

Then what is the purpose of the Lagrange multiplier method? The answer is that the purpose of the Lagrange multiplier method is to optimize a function subject to certain constraints. It is commonly used in fields such as economics, physics, and engineering to solve optimization problems with constraints.

Solving Problem using Lagrange multiplier method:
We all know that the formula $\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$ will be used to solve this problem. But first, we need to define which $f(x,y)$ and $g(x,y)$ are.

$$Let \quad f(x,y) = z = 2x^2 - 2xy + y^3 \quad and \quad g(x,y) = x^2 + y^2 - 4$$

Therefore, we obtain a system of equations:

$$\begin{cases} x - 2y + 2\lambda x & = 0 \quad (1) \\ 2x + 3y^2 + 2\lambda x & = 0 \quad (2) \\ x^2 + y^2 - 4 & = 0 \quad (3) \end{cases}$$

Solve the equation (1) with respect to x, we get:

$$x = \frac{y}{2 + \lambda}$$

Take that value of x, we put it into the equation (2):

$$3y^2 + 2\lambda y - \frac{2y}{2+\lambda} = 0 \quad (4)$$

Then we substitute x into the equation (3), right here we solve for y:

$$\frac{y^2}{(2+\lambda)^2} + y^2 = 4$$

$$y^2 = \frac{4}{1 + \frac{1}{(2+\lambda)^2}}$$

$$y = \pm \frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}}$$

Plug $y = -\dfrac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}}$ to equation (4) then we obtain:

$$3\frac{4}{1 + \frac{1}{(2+x)^2}} + 2\lambda \frac{-2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} - \frac{-4}{(2+\lambda)\sqrt{1 + \left(\frac{1}{2+\lambda}\right)^2}} = 0$$

Then we obtain 2 values of λ:

$$\lambda = 3.13935 \quad and \quad \lambda = -2.31197$$

With $y = \dfrac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}}$ we have another equation:

$$3\frac{4}{1 + \frac{1}{(2+x)^2}} + 2\lambda \frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} - \frac{4}{(2+\lambda)\sqrt{1 + \left(\frac{1}{2+\lambda}\right)^2}} = 0$$

Also, two other values of λ:

$$\lambda = -1,03873 \quad and \quad \lambda = -3.1317$$

Last step is to compute value of x and y according to each value of λ:
With $\lambda = 3.13935$:

$$y = -\frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} = -\frac{2}{\sqrt{1 + \frac{1}{(2+3.13935)^2}}} = -1.96318$$

$$x = \frac{y}{2+\lambda} = \frac{-1.96318}{2+3.13935} = -0.38199$$

With $\lambda = -2.31197$:

$$y = -\frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} = -\frac{2}{\sqrt{1 + \frac{1}{(2-2.31197)^2}}} = -0.595631$$

$$x = \frac{y}{2+\lambda} = \frac{-0.595631}{2 - 2.31197} = 1.90925$$

With $\lambda = -1,03873$:

$$y = -\frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} = -\frac{2}{\sqrt{1 + \frac{1}{(2-1,03873)^2}}} = 1.38602$$

$$x = \frac{y}{2+\lambda} = \frac{1.38602}{2 - 1,03873} = 1.44186$$

With $\lambda = -3.13172$:

$$y = -\frac{2}{\sqrt{1 + \frac{1}{(2+\lambda)^2}}} = -\frac{2}{\sqrt{1 + \frac{1}{(2-3.13172)^2}}} = 1.49874$$

$$x = \frac{y}{2+\lambda} = \frac{1.49874}{2 - 3.13172} = -1.3243$$

After obtaining all value of x, y in each case of, we evaluate f at all the point (x, y). The largest of those values is the maximum value of f; the smallest is the minimum value of f.

$$f(-0.38199, -1.96318) = 2(-0.38199)^2 - 2(-0.38199)(-1.96318) + (-1.96318)^3$$
$$= -8.77424$$

$$f(1.90925, -0.595631) = 2(1.90925)^2 - 2(1.90925)(-0.595631) + (-0.595631)^3$$
$$= 9.35357$$

$$f(1.44186, 1.38602) = 2(1.44186)^2 - 2(1.44186)(1.38602) + (1.38602)^3$$
$$= 2.82364$$

$$f(-1.3243, 1.49874) = 2(-1.3243)^2 - 2(-1.3243)(1.49874) + (1.49874)^3$$
$$= 10.8436$$

Therefore:

- Maximum value of $z = 2x^2 - 2xy + y^3$ subject to the single constraint $x^2 + y^2 = 4$ is $f(-1.3243, 1.49874) = 10.8436$

- Minimum value of $z = 2x^2 - 2xy + y^3$ subject to the single constraint $x^2 + y^2 = 4$ is

$f - 0.38199, -1.96318 = -8.77424$

MATLAB code: This MATLAB code demonstrates how to plot the contour of a function and a constraint curve in the same figure. Specifically, the function is $z = 2x^2 - 2xy + y^3$ and the constraint curve $x^2 + y^2 = 4$ in the same figure.

```matlab
% Define the range of values for x and y
x = linspace(-5, 5, 100);
y = linspace(-5, 5, 100);

% Create a grid of x and y values
[X, Y] = meshgrid(x, y);

% Evaluate the function z at each point on the grid
Z = 2*X.^2 - 2*X.*Y + Y.^3;

% Plot the contour of the function
contour(X, Y, Z, 100);
hold on

% Plot the constraint x^2 + y^2 = 4
theta = linspace(0, 2*pi, 100);
r = 2; % set the radius of the circle
x_circle = r*cos(theta); % compute x coordinates
y_circle = r*sin(theta); % compute y coordinates
plot(x_circle, y_circle, 'k','Color','red', 'LineWidth', 2)

% Set labels
xlabel('x');
ylabel('y');
title('Contour plot of z = 2x^2 - 2xy + y^3 and constraint x^2 + y^2 =
    4');

% Adjust the aspect ratio of the plot
pbaspect([1 1 1])
```
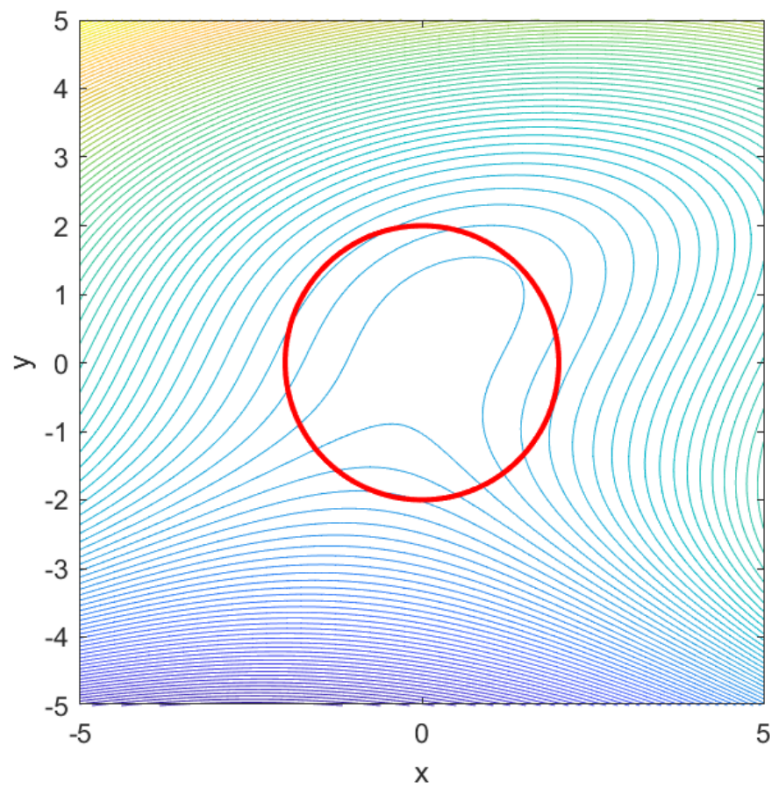
Figure 3: The contour plot of the function $z = 2x^2 - 2xy + y^3$ and the constraint $x^2 + y^2 = 4$.

Here is a line-by-line explanation of the code:

- `this = to_do();` - TO DO

III   Problem 3.

Let C be the intersection of the surface $x^2 + y^2 + z^2 = 9$ and the cylinder $x^2 + 3y^2 = 4$, $z > 0$.

(a) Draw the surfaces and the curve C.

(b) Find the length of the curve.

(c) At any given point (x_0, y_0, z_0) belongs to the curve, draw the unit tangent vector.

Theory: Finding the length of a curve using line integral.
Suppose that a curve C is defined by equation $y = f(x)$, where f is continuous and $a \leq x \leq b$. We obtain a polygonal approximation to C by dividing the interval $[a, b]$ into n subintervals with endpoints $x_0, x_1, ..., x_n$ and equal with Δx. If $y_i = f(x_i)$, and then the point $P_i(x, y)$ lies on C and the polygon with vertices $P_0, P_1, ..., P_n$, Illustrated in the below figure, is an approximation to C.
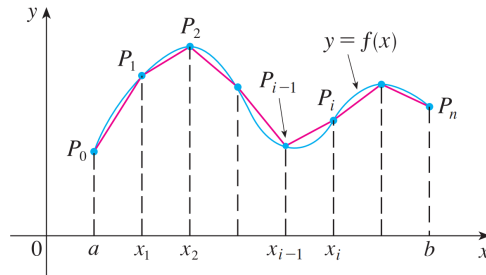
The length L of is approximately the length of this polygon and the approximation gets better as we let n increase. Therefore we define the length L of the curve C with equation $y = f(x)$, $a \leq x \leq b$, as the limit of the lengths of these inscribed polygons (if the limit exists):

$$L = \lim_{n \to \infty} \sum_{i=1}^{n} |P_{i-1} P_i|$$

If we let $\Delta x_i = x_i - x_{i-1}$ and $\Delta y_i = y_i - y_{i-1}$, then

$$|P_{i-1} P_i| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

By applying **the Mean Value Theorem** to f on the interval $[x_{i-1}, x_i]$, we find that there is a number x_i* between x_{i-1} and x_i such that

$$f(x_i) - f(x_{i-1}) = f(x)(x_i - x_{i-1})$$

$$\to \Delta y_i = f'(x_i) \Delta x$$

Thus we have:

$$|P_{i-1} P_i| = \sqrt{1^2 + [f'(x)]^2} dx$$

By the definition of a definite integral. This integral exists because the function $g(x) = \sqrt{1^2 + [f'(x)]^2}$ is continuous. Thus we have proved the following theorem: If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{1^2 + [f'(x)]^2} \, dx$$

Then we have another assumption that C can be described by the parametric equations $x = f(t)$ and $y = g(t)$, $\alpha \leq x \leq \beta$, where $\dfrac{dx}{dt} = f'(t) > 0$. This means that C is traversed once, from left to right, as t increased from α to β and $f(\alpha) = a$, $f(\beta) = b$. We obtain:

$$L = \int_a^b \sqrt{1^2 + [f'(x)]^2} \, dx = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dx$$

The length of space curve is defined in exactly the same way. Suppose that the curve has the vector equation $r(t) = \langle f(t), g(t), h(t) \rangle$, $a \leq x \leq b$, or, equvalently, the parametric equations $x = f(t), y = g(t), z = h(t)$, where f', g' and h' are continuous. If the curve is traversed exactly once as t increases from a to b, then it can be shown that its length is

$$L = \int_a^b \sqrt{[f'(x)]^2 + [g'(x)]^2 + [h'(x)]^2} \, dx = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dx$$

Tangent vectors:

In mathematics, a tangent vector is a vector that is tangent to a curve or surface at a given point. Tangent vectors are described in the differential geometry of curves in the context of curves in $\mathbb{R}^n$. More generally, tangent vectors are elements of a tangent space of a differentiable manifold. Tangent vectors can also be described in terms of germs. Formally, a tangent vector at the point x is a linear derivation of the algebra defined by the set of germs at x.

Let $r(t)$ be a parametric smooth curve. The tangent vector is given by $r'(t)$ where we have used a prime instead of the usual dot to indicate differentiation with respect to parameter t. The unit tangent vector is given by

$$T(x) = \frac{r'(x)}{|r(x)|}$$

MATLAB code:

a) Draw the surfaces and the curve C.

```
% Plot the the surface x^2 + y^2 + z^2 = 9
f = @(x, y, z) x.^2 + y.^2 + z.^2 -9;
fimplicit3( f , [-3 3 -3 3 0 5 ] , 'c' );  % plot the surface in cyan
    color
daspect([1 1 1]) % set the aspect ratio of axes to 1 1 1
xlabel('x');
ylabel('y');
zlabel('z');
```

```
hold on;  % retain the plot for the next plot
```
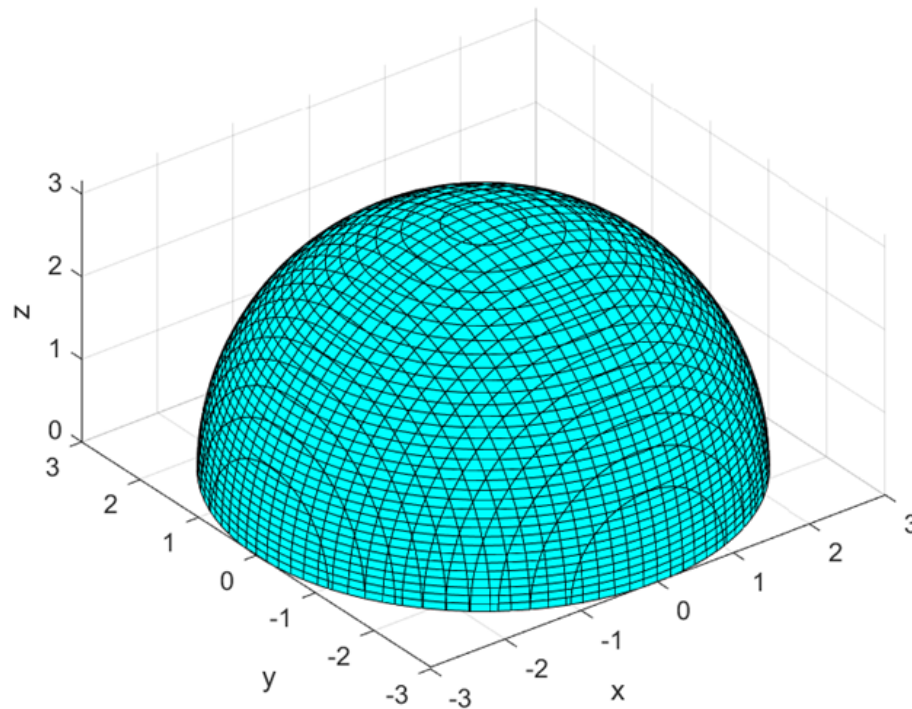

Figure 4: The surface $x^2 + y^2 + z^2 = 9$

```
% Plot the surface x^2 +3y^2 = 4
g = @(x, y) x.^2 + 3*(y.^2) -4;
fimplicit3 (g, [-3 3 -3 3 0 5 ], 'g');  % plot the surface in green
    color
daspect([1 1 1]) % set the aspect ratio of axes to 1 1 1
xlabel('x');
ylabel('y');
zlabel('z');
hold on;  % retain the plot for the next plot
```
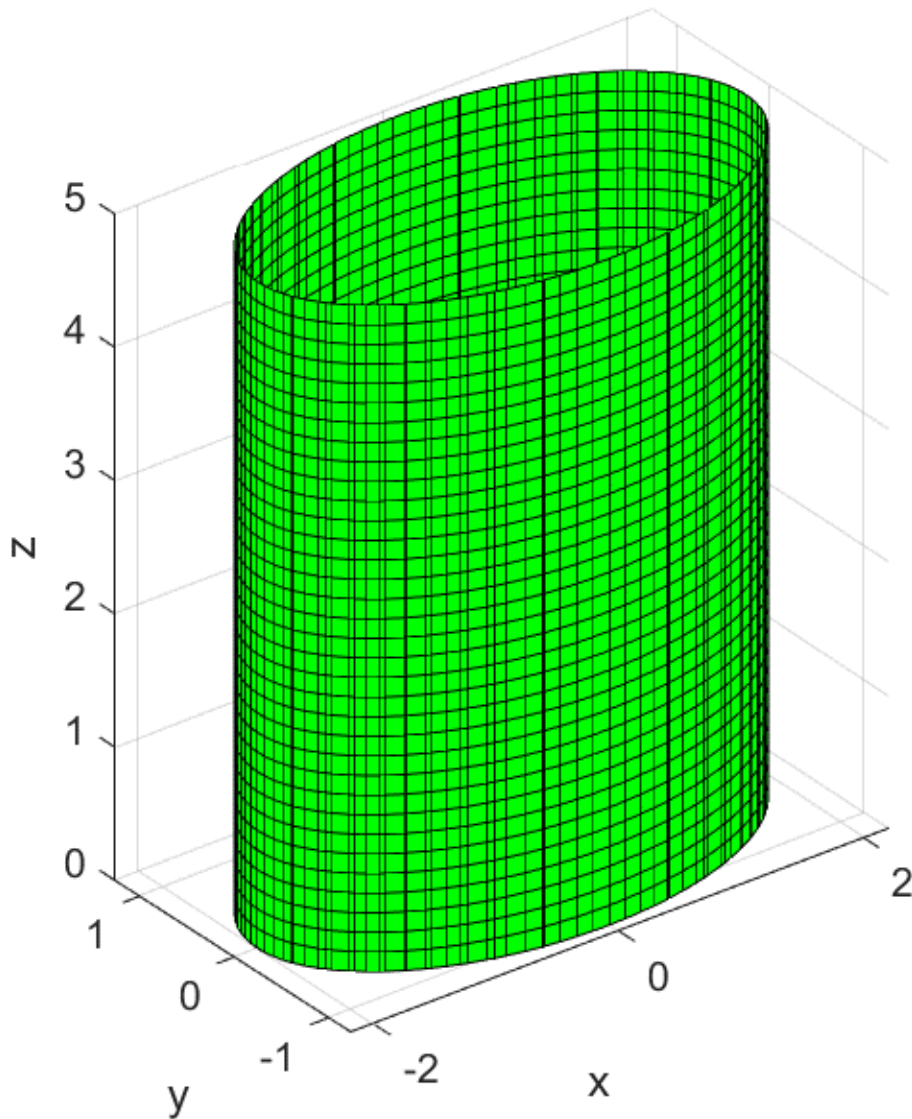
Figure 5: The cylinder $x^2 + 3y^2 = 4$

```matlab
% Draw the curve C as the interection of 2 surfaces:
syms t;
X = cos(t).*2;
Y = sin(t).*(2/sqrt(3));
syms t;
X = 2.*cos(t) ;
Y = sin(t).*(2/sqrt(3) );
Z = sqrt(9 - 4.*cos(t).^2 - 4/3.*sin(t)^2);   % calculate the z-
    coordinate of the curve
curveC = fplot3 (X, Y, Z, [ 0 2*pi ], 'linewidth', 5);
curveC.Color = 'r';   % plot the curve with red color
daspect([1 1 1]) % set the aspect ratio of axes to 1 1 1
```

```
xlabel('x');
ylabel('y');
zlabel('z');
title("Intersection curve of the surfaces x^2 + y^2 + z^2 = 9 and the
    cylinder x^2 + 3y^2 = 4, z > 0")
```
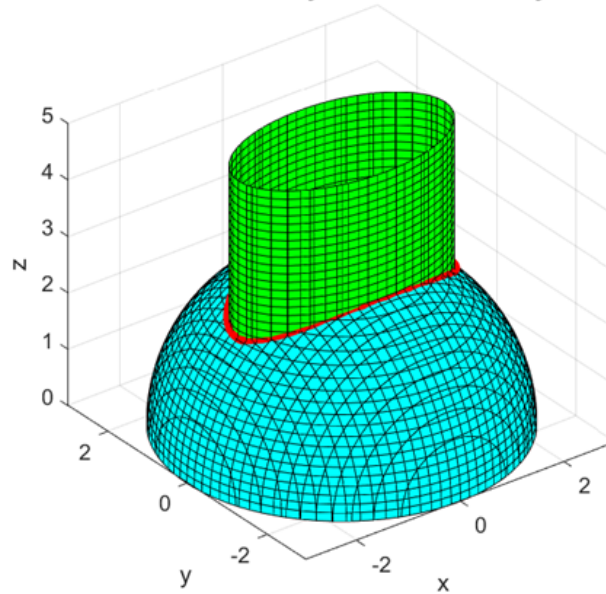

Figure 6: The surfaces and the curve C

```
% Draw the curve C as the interection of 2 surfaces without line:
syms t;
X = cos(t).*2;
Y = sin(t).*(2/sqrt(3));
Z = sqrt(9 - 4.*cos(t).^2 - 4/3.*sin(t)^2);   % calculate the z-
    coordinate of the curve
curveC = fplot3 (X, Y, Z, [ 0 2*pi ], 'linewidth', 3);
curveC.Color = 'b';   % plot the curve with blue color
daspect([1 1 1]) % set the aspect ratio of axes to 1 1 1
xlabel('x');
ylabel('y');
zlabel('z');
title("The intersection curve C of two surfaces");
```

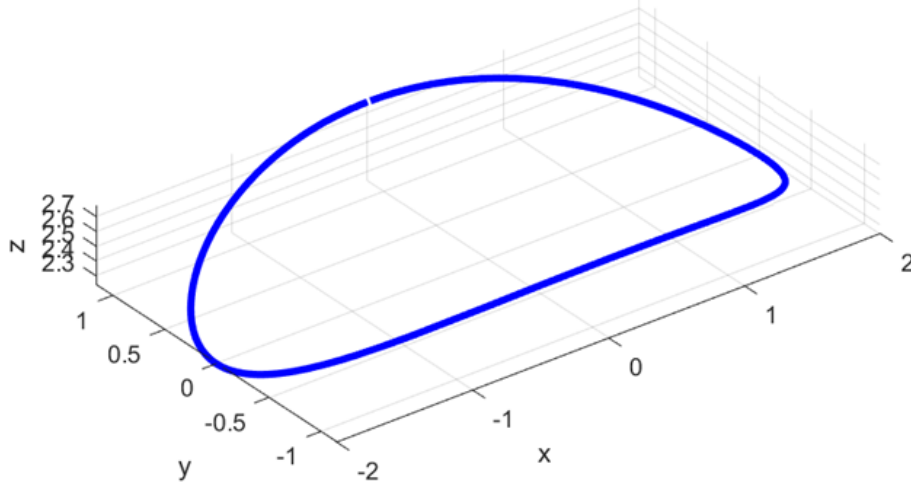**The intersection curve C of two surfaces**

Figure 7: The intersection curve C

b) Find the length of the curve C.

Solving by hand:

First we represent x and y through parameter t:

$$\begin{cases} x = 2\cos(t) \\ y = \dfrac{2}{\sqrt{3}}\sin(t) \\ 0 \le t \le 2\pi \end{cases} \tag{1}$$

Substituing x and y into the equation of the cylinder $x^2 + 3y^2 = 4$, we have:

$$z = \sqrt{9 - 4\cos(x)^2 - \frac{4}{3}\sin(t)^2}$$

Therefore the curve C is defined as:

$$\begin{cases} x = 2\cos(t) \\ y = \dfrac{2}{\sqrt{3}}\sin(t) \\ z = \sqrt{9 - 4\cos(x)^2 - \dfrac{4}{3}\sin(t)^2} \end{cases} \tag{2}$$

The length of the curve C is:

$$\begin{aligned} L &= \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt \\ &= \int_0^{2\pi} \sqrt{[-2\sin(t)]^2 + \left[\frac{2}{\sqrt{3}}\cos(t)\right]^2 + \left[\frac{8\sin(t)\cos(t)}{3(15 + 8\sin t)^2}\right]^2}\, dt \\ &= 10.3677 \end{aligned}$$

Solving by MATLAB:

```
% Calculate the length of the intersection curve C
func = @(x) sqrt((-2.*sin(x)).^2 + (2./sqrt(3).*cos(x)).^2 + (((8/3).*
    cos(x).*sin(x))./sqrt((8/3).*sin(x).^2 + 5)).^2);

% Use the integral function to evaluate the integral of the integrand
    over the interval [0, 2pi]
Length = integral(func, 0, 2*pi);

% Display the length of the curve C
fprintf('Length of the curve C is: ');
disp(Length);
```

Result:

```
>> % Calculate the length of the intersection curve C
func = @(x) sqrt((-2.*sin(x)).^2 + (2./sqrt(3).*cos(x)).

% Use the integral function to evaluate the integral of
Length = integral(func, 0, 2*pi);

% Display the length of the curve C
fprintf('Length of the curve C is: ');
disp(Length);
Length of the curve C is:    10.3677
```

Figure 8: The length of the curve C by MATLAB

c) At any given point (x_0, y_0, z_0) belongs to the curve, draw the unit tangent vector

We have (C):

$$
\begin{cases}
x = 2\cos(t) \\
y = \dfrac{2}{\sqrt{3}}\sin(t) \\
z = \sqrt{9 - 4\cos(x)^2 - \dfrac{4}{3}\sin(t)^2}
\end{cases}
\tag{3}
$$

Let:

$$r(t) = 2\cos(t)\hat{i} + \frac{2}{\sqrt{3}}\sin(t)\hat{j} + \sqrt{9 - 4\cos(x)^2 - \frac{4}{3}\sin(t)^2}$$

$$r'(t) = -2\sin(t)\hat{i} + \frac{2}{\sqrt{3}}\cos(t)\hat{j} + \frac{8\sin(t)\cos(t)}{3(15 + 8\sin t)^2}$$

Therefore, we can implement this MATLAB code:

21

```matlab
% Plot the unit tangent vector at anypoint on the curve C
t = linspace(0, 2*pi, 100);

% Vector r(t)s components : x_t , y_t , z_t
x_t = 2.*cos(t);
y_t = 2./sqrt(3).*sin(t);
z_t = sqrt(9 - 4.*cos(t).^2 + 4./3.*sin(t).^2);

% Vector r'(t) components : m, n, p
x = -2.*sin(t);
y = 2./sqrt(3).*cos(t);
z = (8.*cos(t).*sin(t))./sqrt(3.*(8.*sin(t).^2 + 15))

l = sqrt(x.^2 + y.^2 + z.^2);  % length of the tangent vector

m = -2*sin(t)./l;
n = (2/sqrt(3).*cos(t))./l;
p = (8.*cos(t).*sin(t))./sqrt(3.*(8.*sin(t).^2 + 15))./l;

figure
quiver3(x_t, y_t, z_t, m, n, p);  % plot the unit tangent vector at
    each point on the curve
daspect([1 1 1]) % set the aspect ratio of axes to 1 1 1
xlabel('x');
ylabel('y');
zlabel('z');
title('Unit tangent vectors to C');
```
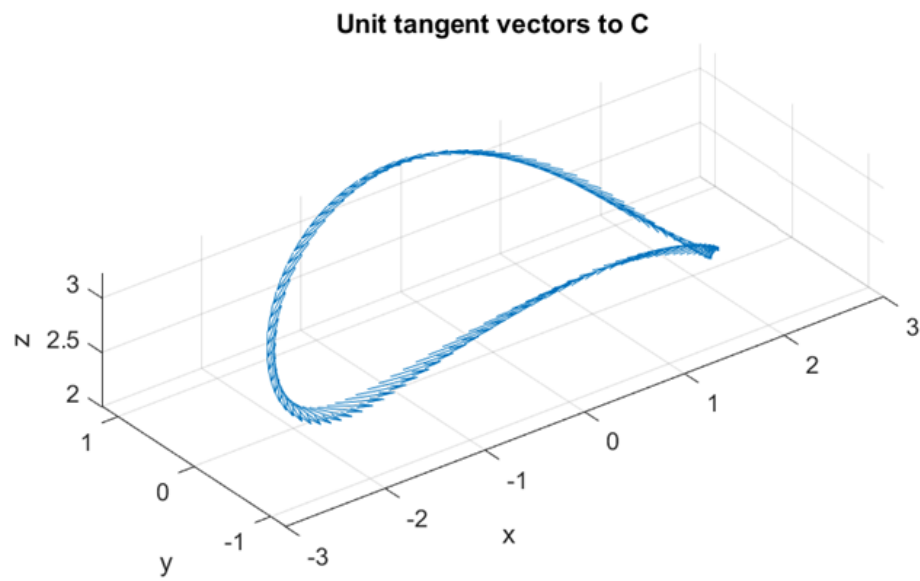
Figure 9: Unit tangent vectors to C

Code Explanation:

- `this = to_do();` - TO DO

References

[1] Nguyen Dinh Huy, Nguyen Ba Thi, Le Xuan Dai, Dau The Phiet, Ngo Thu Luong, and Tran Ngoc Diem. *Giao Trinh GIAI TICH 2*. NXB DHQG, 2009.

[2] James Stewart. *Calculus*. Cengage Learning, 8th edition, 2015.