University of Technology - HCMC
Faculty of Applied Mathematics

Linear Algebra Project
Semester 222 (2022 - 2023)

Instruction

The students work in a group and write a report for the given project. (See the team information).

Using Matlab or Python to solve the following problems and write a report.

The report must have 3 parts:

i) The theory and algorithm (as your understanding);
ii) The Matlab or Python commands (it isn't allowed any direct command to solve the problem, explain important steps);
iii) The results and conclusion.

Project 1

Problem 1. A code breaker intercepted the encoded message below.

45 -35 38 -30 18 -18 35 -30 81 -60 42 -28 75 -55 2 -2 22 -21 15 -10

Let the inverse of the encoding matrix be $A^{-1} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

(a) You know that $[45 \quad -35]A^{-1} = [10 \quad 15]$ and $[38 \quad -30]A^{-1} = [8 \quad 14]$. Write and solve two systems of equations to find w, x, y, and z.

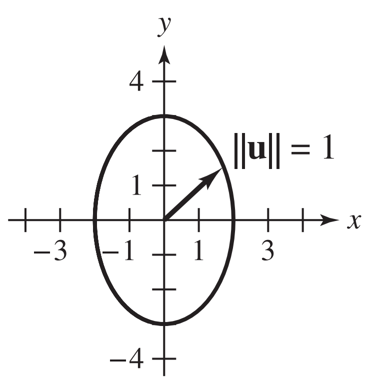
(b) Decode the message.

Problem 2. Construct an inner product in $\mathbb{R}^n$. In that inner product, write a program to input any number of vectors in $\mathbb{R}^n$ and return the orthogonal basis and orthonormal basis of the subspace spanned by these vectors. (Use Gram - Schmidt process). From that, given any vector in $\mathbb{R}^n$, find the coordinates in that basis and find the length of the vector.

Problem 3. In $\mathbb{R}^2$, the weighted inner product is given by

$$\langle x, y \rangle = ax_1y_1 + bx_2y_2$$

where a and b are positive. Find a weighted inner product such that the graph represents a unit circle as



In that inner product space, reflect that unit circle about an input plane.

Project 2

Problem 1. The cryptogram below was encoded with a 2×2 matrix. The last word of the message is __SUE. What is the message?

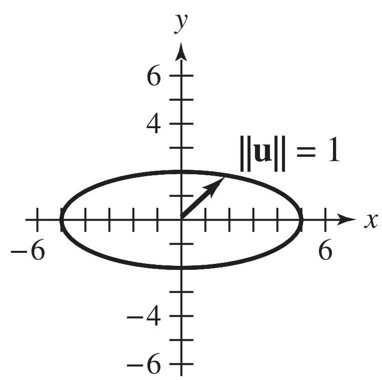
5 2 25 11 -2 -7 -15 -15 32 14 -8 -13 38 19 -19 -19 37 16

Problem 2. Construct an inner product in $\mathbb{R}^n$. In that inner product, write a program to input any number of vectors in $\mathbb{R}^n$ and return the orthogonal basis and orthonormal basis of the subspace spanned by these vectors. (Use Gram - Schmidt process). From that, given any vector in $\mathbb{R}^n$, find the coordinates in that basis and find the length of the vector.

Problem 3. In $\mathbb{R}^2$, the weighted inner product is given by

$$\langle x, y \rangle = a x_1 y_1 + b x_2 y_2$$

where a and b are positive. Find a weighted inner product such that the graph represents a unit circle as



In that inner product space, reflect that unit circle about an input plane.